DRP Proposal

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1 Topic Outline

A study of statistical optimal transport with a view to characterising the space of probability measures, and optimisation on the space of probability measures for ML purposes. Potentially a practical implementation of the theory for example demonstration purposes.

2 Project Goals

1. Motivate Optimal Transport for Probability Theory
2. Introduce the Monge and Kantorovich problems
3. Understand the Wasserstein space of probability measures
4. Characterise the geometry on the Wasserstein space
5. Applications of Wasserstein gradient flows
6. (Stretch) Practical applications such as sgd, neural networks and transformers

3 Resources

- Books
  - Statistical Optimal Transport (Chewi, Niles-Weed, Rigollet) `https://arxiv.org/abs/2407.18163`

    Primary reference for the project, foundational optimal transport and applications. Chapters 1,7,8 + 5,6
  - An Invitation to Statistics in Wasserstein Space (Panaretos, Zemel) `https://link.springer.com/book/10.1007/978-3-030-38438-8`

    Secondary reference, alternative descriptions of Wasserstein space with a more pure emphasis. Chapters 1,2
  - Optimal Transport Old and New (Villani) `https://link.springer.com/book/10.1007/978-3-540-71050-9`

    Tertiary reference, contains more soft descriptions and historical journey of optimal transport, also contains plenty of detail when need. Chapters 1-7, 23-25

- Papers
  - Collection of papers (curated by kilianFatras) https://github.com/kilianFatras/awesome-optimal-transport
  - Recent advances in Optimal Transport for Machine Learning (Montesuma, Mboula, Souloumiac) https://arxiv.org/pdf/2306.16156
  - Flowing Datasets with Wasserstein over Wasserstein Gradient Flows (Bonet, Vauthier, Korba) https://arxiv.org/pdf/2506.07534
- Videos
  - Rigollet Seminar https://youtu.be/EBA0NyY4Myc?si=OTthkc1Arh5xc0Cw
    Introduction to optimal transport and motivation with reference to theory. First half more important
  - Gabe Khan information geometry https://youtu.be/EauDdCzxphE?si=mX4ier4BWl-ed8Zv
    Historical overview and alternate motivation
- GitHub
  - Python Optimal Transport (POT) [MIT] https://github.com/PythonOT/POT
    Open source Python library providing several solvers for optimization problems related to Optimal Transport for signal, image processing and machine learning. Provides great number of examples.
- Lecture notes
  - [Prerequisites] Measure Y2 (Rodriguez Notes), Probability for Statistics Y2 (Hallsworth notes), Markov Processes chapters 7-8 (Hairer-Li notes)
  - A user's guide to optimal transport (Ambrosio, Gigli) https://cvgmt.sns.it/media/doc/paper/195/users_guide-final.pdf
    Alternate reference for foundational optimal transport
- Seminars
  - Imperial Stochastic Analysis Seminars (Crisan contact) https://www.imperial.ac.uk/stochastic-analysis-group/activities/stochastic-analysis-seminar/
  - Imperial Statistics Seminars (Passeggeri Contact) https://www.imperial.ac.uk/statistics/seminars/statistics-seminar/

4 Indicative Prerequisites

- Foundational Probability Theory (Y1 + Y2 courses)
- Foundational Measure Theory (Y2 course)
- Couplings and transition maps

5 Possible checkpoints

1. Sufficient prerequisite checks
2. Understanding the Monge and Kantorovich problems
3. Introducing the Wasserstein Space
4. Studying applications of OT
5. Practical Implementations
6. Colloquium end of term

6 Possible concerns for the project

- Helping students with prerequisite knowledge if they are not fluent with introductory measure theory
- Managing the right speed for studying the material

7 Further comments

Hopefully this example is useful! It's not necessary to include every single part as given, but try and stick to the section structure. The level of detail is up to you, as are the number and types of resources - this will vary a lot between different types of projects.