Proceedings of the Institution of Mechanical Engineers, Part P: Journal of Sports Engineering and Technology
http://pip.sagepub.com/

An analytical model for track cycling

Richard Lukes, John Hart and Steve Haake

Proceedings of the Institution of Mechanical Engineers, Part P: Journal of Sports Engineering and Technology 2012 226: 143 originally published online 9 March 2012
DOI: 10.1177/1754337111433242

Special Issue Article

Institution of MECHANICAL ENGINEERS

Journal of SPORTS ENGINEERING AND TECHNOLOGY

An analytical model for track cycling

Proc IMechE Part P:
J Sports Engineering and Technology
226(2) 143–151

DOI: 10.1177/1754337111433242
pip.sagepub.com
SAGE

Richard Lukes, John Hart and Steve Haake

Abstract

This paper presents the full derivation of an analytical model for track cycling. The model takes into account the unique aspects of track cycling associated with riding around a velodrome. These include, riding upon a banked track and the resulting tyre scrubbing effects, and the tipping motion of a cyclist passing through a corner with the resulting centripetal forces. Validation was provided using SRM^{TM} power crank data and split times obtained for an elite national cyclist in a 4 km pursuit competition. Results have shown the model to over-predict cyclist performance with a discrepancy of 0.7 s in a finals event and 4.3 s, less than 2% error, in a qualifying race. It is believed this may be attributable to discrepancies in atmospheric variables. However the model has proved capable of predicting the velocity increase, specifically associated with track cycling, as a cyclist passes through a bend. The model is useful for analysis of the physics of track cycling, and can be used to quantitatively predict performance dependent upon bicycle efficiencies, tyre type and venue conditions, in a racing scenario.

Keywords

Track cycling, velodrome, analytical model, lap sim

Date received: 2 September 2011; accepted: 23 November 2011

Introduction

The use of analytical models in the prediction of performance is commonplace within sports, and the sport of cycling is no different. Competitive cyclists and cycling teams spend extensive amounts of time, effort and money developing the ‘best’ equipment, determining the ‘optimal’ aerodynamic position, and optimising the overall efficiency of the cyclist. Without the use of analytical models, however, it is not possible to ascertain fully the overall affect of each of these on a possible race outcome. Analytical models have found widespread use in research and competitive cycling, in assessment of both the mechanical and physiological aspects of the sport, for a number of years. In 1988, van Ingen Schenau[1] presented a well laid-out derivation of an equation of motion for cycling, used to examine the power of a cyclist. When van Ingen Schenau presented the derivation the only other method of assessing a cyclist’s power output was through the use of a friction-loaded ergonometer.[2] This had the distinct disadvantage of being a laboratory-based piece of equipment, unusable in real-world applications. The necessity for analytical models capable of assessing a cyclist’s power outside of the laboratory was clear. In the early 1990s the power-measuring crank was developed which could be used in a real-world environment to assess cyclist power. The high accuracy and functionality of the power measurement crank afforded the opportunity for researchers stringently to validate analytical models, as seen in Keen[3], Jones and Passfield[4] and Broker et al.[5] The development of this device did not, however, make the use of analytical models completely redundant, as they can be used to understand more than just a cyclist’s power output. De Groot et al.[6] and Pons and Vaughan[7] used analytical models to relate the mechanical and physiological aspects of cycling. For example, De Groot et al. demonstrated through analytical modelling that for a given oxygen consumption, a cyclist produces more power than a swimmer or ice-skater. Models have also been developed to assess the performance of equipment and position, which is of great interest in competitive cycling. Olds et al.[8] developed a comprehensive equation that included coefficients to account for wind, altitude, humidity, tyre pressure, wheel radius, rotational kinetic energy, and the tactical positioning of

Centre for Sports Engineering Research, Sheffield Hallam University, UK

Corresponding author:
John Hart, Centre for Sports Engineering Research, Sheffield Hallam University, A129 Collegiate Hall, Collegiate Crescent, Sheffield S10 2BP, UK.
Email: john.hart@shu.ac.uk

drafting. Olds et al. validated this model using 41 riders in a 26 km time trial, to within a 95% accuracy of actual time, and went on to use the model for the assessment of various parameters. The cycling World Hour Record has also been the subject of assessment using analytical models. Basset et al.[9] derived a model that showed that the then-holder of the record, Chris Boardman, had used less power to claim the record than the second placed man, Tony Rominger, due to Rominger's inferior aerodynamics. None of these analytical models, however, were concerned with the specific physics associated with track cycling. These include riding upon a banked track and the resulting tyre scrubbing effects and the tipping motion of a cyclist passing through a corner and the resulting centripetal forces. Martin et al.[10] and Lukes et al.[11] both presented track cycling analytical models in 2006. However, to account for the effects of cornering, Martin et al.[10] approximated the track as being circular, rather than the traditional straights and bends formation. Lukes et al.[11] presented a model in its early stages, with albeit a generic but geometrically correct track, accounted for tipping in the bends, but assumed a generic power profile. Underwood and Jermy[12] also presented a track cycling model that used an SRM™ power profile for a female elite pursuit cyclist. The model accounted for rider power losses associated with acceleration of the legs, and thus required cyclist cadence. This paper presents a full derivation of a track cycling model, detailing assumptions and providing explanation of the associated physics of cycling around a banked track and bends. The model accounts for individual efficiencies associated with different parts of the bicycle, and associated losses, including increased rolling resistance due to tyre scrubbing. The model is validated using data obtained for an elite national cyclist, competing in a 4 km pursuit.

Mathematical model

Mechanically the fundamental equation to describe cycling is relatively straightforward, with only a few well-defined losses. The equation developed for track cycling will draw on different aspects of previous analytical models, which will be stated as the formulation progresses.

Fundamental principles

This model assumes two different states of riding; in the straights, and in the bends. In the straights it is assumed the cyclist travels upright and in a straight line, from the exit of one corner to the entry of the next. The cyclist will be subject to the forces as shown in Figure 1 and resolved in equation (1), an expansion of Newton's second law of motion. the force subscripts are as follows: A is acceleration, T is transmitted, D is aerodynamic, R is rolling, N is normal contact and W is weight. This is the traditional starting point of the majority of models previously cited.

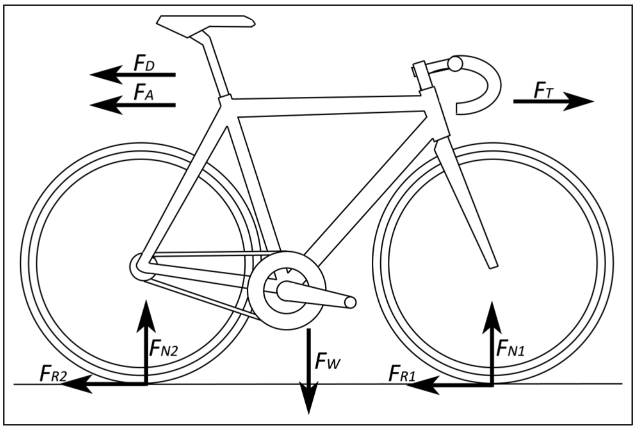

Figure 1. Forces on bicycle.

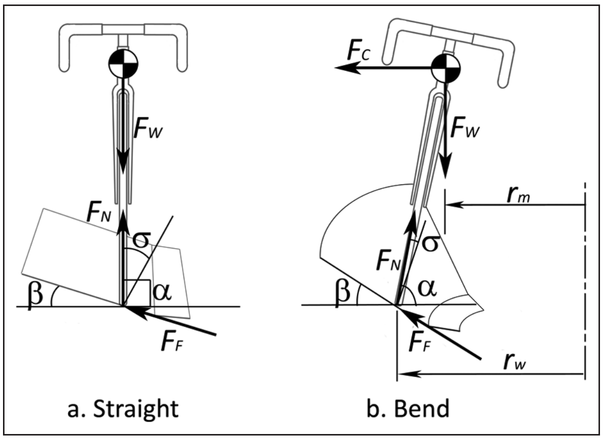

Figure 2. Forces on bicycle in straights and bends.

$$F_A = ma = F_T - F_D - F_R \tag{1}$$

The cyclist experiences a centripetal force in the bends explained by the free body diagrams, as shown head on to the cyclist, in Figure 2. It can be seen that throughout both the straights and the bends, a velodrome track has a banking angle, β. As a result there will be a lateral friction force, F_F, whenever the angle of the cyclist relative to the track, camber angle, $\sigma \neq 0°$.The cyclist's tipping angle is represented relative to the horizon as α.

In the straights it is assumed the cyclist travels in an upright position ($\alpha = 90°$) with no tipping, hence

$$F_N = F_W \tag{2}$$

Track banking has implications for the rolling resistance of the cyclist in the straights which will be discussed below. Through the bends, the constant change in velocity results in an angular acceleration and an additional centripetal force, F_C. To maintain balance the cyclist tips, moving the centre of mass (CoM) towards the centre of the track. This movement is important as the CoM will travel less distance than the base of the wheels, and the cyclist works to propel the CoM around the track. As the wheels travel the exact distance of a respective event, the result will be that the cyclist's speed as measured at the wheels increases in the bends. This principle has been confirmed by professional cyclists during interviews. When questioned about the general sensation of riding around a velodrome, a former World champion explained, 'the straights are like going up-hill, and the bends are like going down-hill'. As the cyclist tips in the bends, the radius of curvature at the wheel, r_{wh}, will never be less than the radius of curvature for the centre of mass, r_m. Hence, in the bends

$$F_N = F_C \cos\alpha + F_W \sin\alpha = m\frac{v^2}{r_m}\cos\alpha + mg\sin\alpha \quad (3)$$

Transmitted force, drag and rolling resistance

The acceleration of the cyclist is the sum of the transmitted force and losses attributable to aerodynamic drag and rolling resistance. Transmitted force, F_T, can be represented by equation (4), where F is the force applied by the cyclist and η is the efficiency of the bicycle

$$F_T = \eta F = \eta_{dt}\eta_b\eta_{st}\frac{P_{cyc}}{v} \quad (4)$$

where, η is a function of bicycle stiffness, η_{st}, chain efficiency, η_{dt}, and bearings efficiency, η_b. It is assumed that these values remain constant, irrespective of speed. The force applied by the cyclist is equivalent to the cyclist's power, P_{Cyc}, divided by the velocity of the bicycle, v.

Aerodynamic drag is described by equation (5), where C_D is drag coefficient, A is frontal area and ρ is air density

$$F_D = C_D A\frac{1}{2}\rho v^2 \quad (5)$$

Rolling resistance, F_R, will occur due to the inelastic deformation of the pneumatic tyres. This will vary according to the normal contact force and the rolling resistance coefficient, μ_R. Here it is assumed that μ_R is constant. This assumption is based on Kyle,[13] who found rolling resistance to be near-constant in cycling. Rolling resistance of a track cyclist is described as

$$F_R = F_N\mu_R C_S \quad (6)$$

where C_s is a tyre 'scrubbing' correction coefficient. As the track surface is banked, the cyclist has to point the front wheel slightly up the slope to maintain a straight line of motion (Figure 3). Since the front wheel no longer travels precisely in the direction of travel of the bicycle, the front wheel not only rolls but additionally slips. This slipping phenomenon is known as 'scrubbing' in track cycling, and causes an increase in the overall rolling resistance of the wheel. Substituting equation (3) into equation (6) to account for tipping

$$F_R = \left(m\frac{v^2}{r_m}\cos\alpha + mg\sin\alpha\right)\mu_R C_S \quad (7)$$

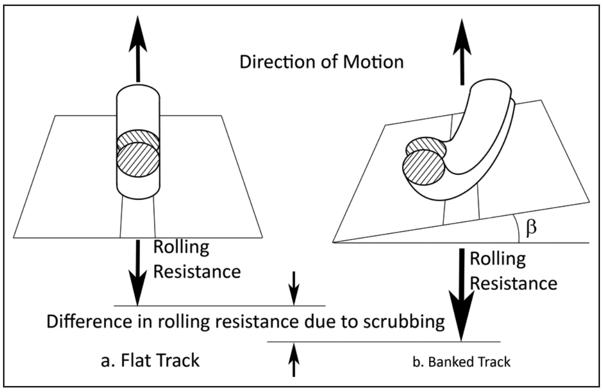

Figure 3. Cutaway through wheel showing effect of cycling on banked track – 'scrubbing'.

Equation for track cycling

Placing the terms derived in equations (4), (5) and (7) into equation (1) the governing equation for the track cycling model is

$$ma = \left(\eta_{st}\eta_b\eta_{dt}\frac{P_{cyc}}{v}\right) - \left(C_D A\frac{1}{2}\rho v^2\right) - \left(\left(m\frac{v^2}{r_m}\cos\alpha + mg\sin\alpha\right)\mu_R C_S\right) \quad (8)$$

Motion through bends

Cyclist motion through the bends and the tipping that results is an important part of the track model development. Relocation of the CoM, through tipping, maintains a balance of forces at the pivot, between tyre–track contact. Velodrome tracks are purposely banked to keep a cyclist at speed as perpendicular to the track surface as possible ($\alpha = \beta$, $\sigma = 0°$) and prevent the bicycle slipping. This is only achievable at a specific velocity, v_{OPT}, dependant upon the designed track camber. When $v > v_{OPT}$, the tendency will be for the bicycle to want to slip up the slope, so the cyclist tips further ($\alpha < \beta$) to counteract increasing F_C (Figure 4). At $v < v_{OPT}$ the tendency will be to slip down the slope as the cyclist tips less ($\alpha > \beta$). The position of the CoM, can be found through calculation of α in the bends. Assuming the cyclist is in equilibrium whilst travelling around a bend, the moment at the pivot will be zero, and the components of F_C and F_W perpendicular to the bicycle will be equal, giving the equation for α as

$$\alpha = \tan^{-1}\left(\frac{F_W}{F_C}\right) = \tan^{-1}\left(\frac{gr_m}{v^2}\right) \quad (9)$$

If the length between the pivot and the CoM, L_m, is known, the radius of curvature of the CoM, r_m, through the bends is

$$r_m = r_{wh} - (L_m\cos\alpha) \quad (10)$$

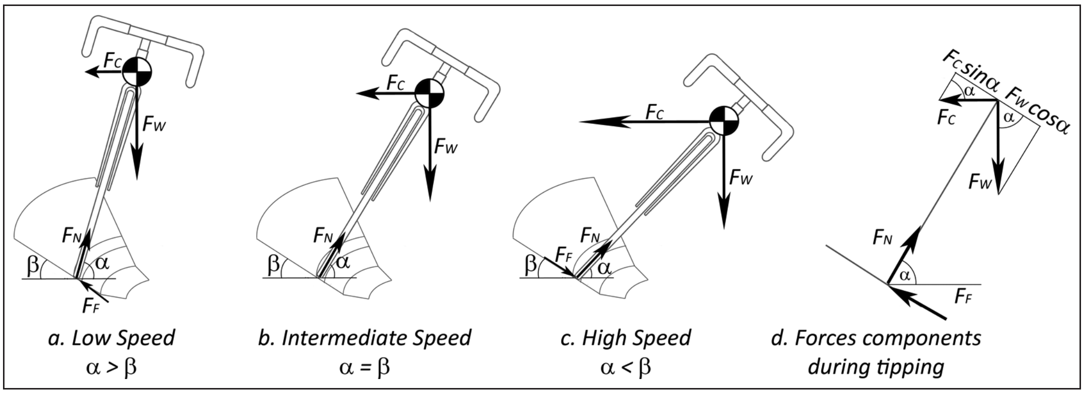

Figure 4. Tipping of bicycle and associated forces.

Distance travelled by the wheels, x_{wh}, in relation to the distance travelled by CoM, x_m, is

$$x_{wh} = x_m \frac{r_{wh}}{r_m} \tag{11}$$

This relationship is the final fundamental principle for the track cycling analytical model.

Model input parameters

Having defined the fundamental relationships governing the model, input parameters are defined as follows.

Environmental conditions

Air density is calculated based on velodrome pressure, P, temperature, T, and relative humidity, h

$$\rho = \frac{P}{T}\left(\frac{1-hs}{R_{air}} + \frac{hs}{R_{water}}\right) \tag{12}$$

where, R_{air} and R_{water} are specific gas constants and s is the saturation factor of water in air, specified as

$$s = 0.0069e^{0.0476T} \tag{13}$$

If the local pressure is unknown, altitude (h_{alt}) can be used to approximate pressure, thus

$$P = 10^{\left(5-\left(h_{alt}/15500\right)\right)} \tag{14}$$

Cyclist mass, drag and area

Total mass is divided into the mass of the bicycle (m_b), and of the cyclist (m_c). To calculate aerodynamic drag force, F_D, drag coefficient, C_D, and frontal area, A, are required. C_D for the cyclist can be specified using wind tunnel data, or calculated using computational fluid dynamic data if a geometric model of the cyclist exists. The frontal area can either be specified or approximated using the relationship presented by Basset et al.[9]

$$A = 0.0293H^{0.725}m_c^{0.425} + 0.0604 \tag{15}$$

where H is cyclist height. Equation (15) does not take into account riding position and should, therefore, be used only when no other data is available. In this study the geometry of an elite national cyclist and bicycle had been previously acquired using non-contact laser scanning. The height of the CoM above the pivot point, L_m, could, therefore, be determined using CAD and an approximation for density (density of the cyclist and bicycle was taken as constant). The position of CoM will be approximate only.

Rolling resistance

Rolling resistance coefficient, μ_R, and scrubbing constant, C_S, are needed to determine rolling resistance. Values for μ_R have been taken from Kyle,[13] which provided μ_R for four different racing tyres (μ_R = 0.0016 $\sim$ 0.0026). Kyle also investigated the influence of steering angle and found in the straights a track cyclist on a 250 m track compensated steering angle by approximately 1°, and that the front wheel oscillated by 2.5 per pedal stroke. Using data taken from Kyle[13] it was possible to determine that the increase in rolling resistance for a track cyclist due to scrubbing in the straights is 9.7%, giving the scrubbing constant $C_S = 1.097$. In the straights the average camber angle is $\sigma = 13.5°$. Assuming a linear relationship between camber angle and increase in rolling resistance due to scrubbing, μ_S, gives the relationship

$$C_S = 1 + \sigma\mu_S \tag{16}$$

μ_S is then determined as 0.0072. The effect of speed on rolling resistance of a bicycle has been assumed to be negligible and this is in agreement with the work of Whitt and Wilson[14] and Hennekam.[15]

Efficiency

Losses due to flexibility in the frame and components are accounted for by the factor η_{st}. The modelled bicycle is predominantly constructed using carbon fibre, known for its high stiffness and strength characteristics. It is assumed the frame is, therefore, around 99%

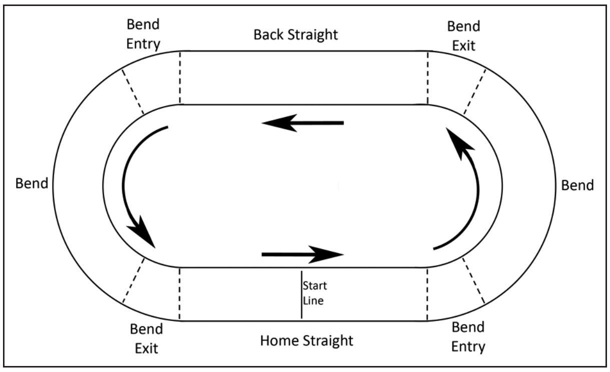

Figure 5. Track diagram.

efficient. The efficiency of the chain drive train, η_{dt}, has been the subject of study by Whitt and Wilson,[14] Burgess[16,17] and Kyle,[13] who state efficiency to be between 95% and 98.5%. As track bicycles do not have derailleur systems, with lateral chain displacement, it is believed a well maintained track chain will be at the upper end of this range. According to Whit and Wilson[14] bearings account for less than 1% of total resistance, and Kyle[13] stated that they account for between 1/20th and 1/50th of the resistance of high pressure racing tyres. According to Kyle's approximations, this would place bearing efficiency around 99.64% to 99.84%.

Velodrome geometry

Velodromes are traditionally 250 m, 333 m or 500 m in length, the majority of modern velodromes being 250 m long. In this analytical model the velodrome is 250 m in length. The specific geometry for the modelled velodrome, including track bank angle and bend curvature, was obtained in confidence from a national governing body, therefore no specific measurements will be presented. A sinusoidal function is used to describe the relationship between track position and track bank angle. The track is divided into eight regions for the calculation of radius of curvature, as shown in Figure 5. If the cyclist is in either of the straights, curvature is infinite. In the bend entry/exit the radius changes linearly from maximum to minimum, whilst in the bends it is the constant minimum. It is assumed the wheels will always follow the shortest distance around the track, and remain at constant height.

Cyclist power profile

The power profile for the cyclist can be either generic or obtained from a power measurement crank. A power profile for the elite cyclist used in this study was obtained using a SRM power measurement crank (Figure 6). A SRM crank basically comprises a series of strain gauges positioned between the crank arm and chain rings that measure torque and cadence from which power can be derived. The measured profile had a generic power profile applied to it, as the data was acquired for a specific event and ends abruptly as the cyclist crosses the finish line. This makes the use of a generic profile preferable for use in analysis.

Solution procedure

The analytical model has been compiled as a Microsoft Excel spreadsheet, using the presented equations and input parameters. Solution is obtained through performing calculations in a specific order, using a time-stepping technique.

Calculation 1: velocity of centre of mass (CoM)

The position of the cyclist on the track is first determined. This is achieved by tracking the distance travelled at the start of the current time step. Velocity at the start of each time step is based on velocity at the end of the previous time step. Transmitted force, F_T, from the cyclist is determined by dividing power by velocity. The solution, therefore, needs to be primed with an initial velocity for the first time step, at time = 0. Sensitivity of solution to this initial velocity must be determined. Having determined F_T, accounted for the relevant efficiencies and solved for F_A and F_R, the acceleration force of the

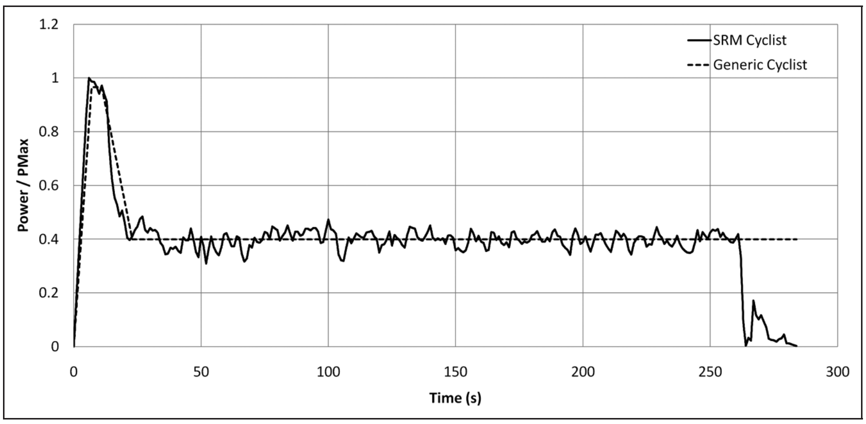

Figure 6. Cyclist power profile.

cyclist can be determined. Any change in velocity across the time step, and the final velocity at the end of the time step is thus obtained. The model assumes work done is to propel the CoM, therefore calculated velocity is for the CoM. To obtain the difference in distance travelled between the CoM and the wheels, α is calculated.

Calculation 2: α and r_m

Tipping angle, α, and radius of CoM, r_m, are determined using equations (9) and (10). As both α and r_m appear in both these equations, there is no simple solution. An iterative process is used to solve these equations, based on an initial estimation of r_m and repeated until acceptable values of α and r_m are obtained. As the cyclist only tips in the bends, α sensitive to track location is determined. The ratio of r_w to r_m is also calculated, and the camber angle of the bicycle, σ, is calculated for use in the determination of scrubbing resistance.

Calculation 3: will the cyclist slip?

The cyclist cannot remain perpendicular to the track under all conditions. A calculation is performed to determine whether the bicycle will slip at any point whilst travelling around the bends. This is determined by calculating the ratio of the side force acting on the bicycle to the normal contact force (Figure 7)

$$\frac{F_{Ni}}{F_{Nj}} = \frac{F_{Ci} - F_{Wi}}{F_{Cj} + F_{Wj}} = \frac{\left(v^2/r_w \cos\beta\right) - (g\sin\beta)}{\left(v^2/r_w \sin\beta\right) + (g\cos\beta)} \tag{17}$$

If the determined value of this force ratio does not exceed the lateral coefficient of friction, between tyre and track, the bicycle does not slip.

Calculation 4: distance

The distance travelled by the CoM is determined by multiplication of the average velocity over a time step, by the time step size. The distance travelled by the wheels is then determined. If the cyclist is in a bend the wheels travel further than the CoM. This distance is obtained using equation (11). The actual distance travelled is then determined by the addition of the wheel distance travelled over the time step to total wheel distance travelled. Wheel velocity is determined by dividing time step wheel distance travelled, by time step size.

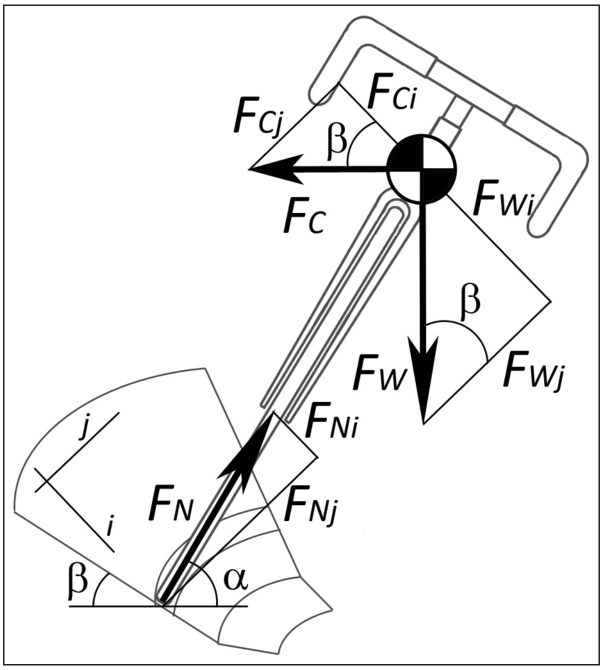

Figure 7. Forces on bicycle in bend.

Validation

The model has been validated against measured data of an elite 4 km pursuit cyclist in competition, at the velodrome modelled in this study. Data used in validation consisted of power and velocity taken from a SRM crank, and recorded split times. This validation data was supplied by the national governing body, of the cyclist in the study.

Input

In addition to the values previously detailed, model boundary conditions as listed in Table 1 were used in the validation of the model. Aerodynamic related variables, Cd and A, were obtained from wind tunnel testing, but will not be disclosed here. These values were obtained in confidence from the cyclists' national governing body, for use only on condition that they remain confidential. Examples of typical drag coefficient values for cyclists can be found in Whitt and Wilson.[14]

The solution was primed with an initial velocity of 0.05 m/s. This value was found to be optimum and to not affect the solution, either by returning a false time, or by causing divergence of the solution process. A time step of 0.05 s was specified, and this value was

Table 1. Input conditions.

Mass		Efficiencies		Venue		Tyres	
m_b	7.8 kg	η_{st}	98.5 %	Altitude, h_{ALT}	35 m	Rolling resistance, μ_r	0.0023
m_c	80 kg	η_b	99.7 %	Temperature, T	18 °C		
		η_{dt}	97 %	Humidity, h	30 %		

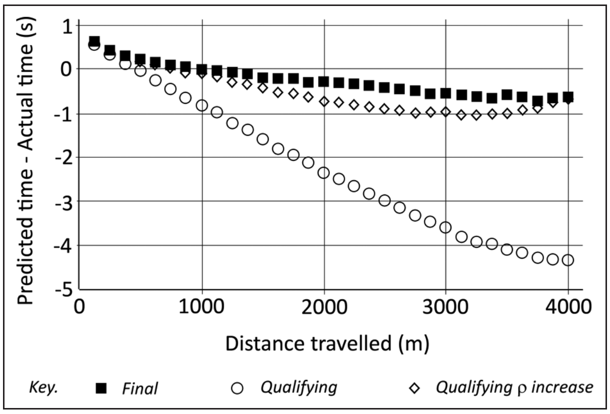

Figure 8. Difference between cyclist's split times.

found sufficiently small not to affect solution, but large enough to be efficient in solution.

Results

Results can be seen in Figure 8 which show a comparison between the split times as predicted by the model, and the times taken during a qualifying race, and a finals race. The results show that the model predicts a slower first 375 m of the event, than reality. After this point accuracy varies significantly between the predictions for qualifying and finals. The model over predicts performance of the cyclist times, resulting in a 0.7 s discrepancy in the finals and a 4.3 s discrepancy in the qualifying race; however, this is still less than 2% error.

Discussion

It is believed discrepancies between actual and predicted times may be attributable to either interpolation of the power profile, atmospheric conditions or cyclist path. Interpolation of the generic power profile at the start of the race is required, as the sampling data rate of the SRM is low in the first five seconds. This interpolation may escalate uncertainties in input power. The model is highly dependent upon the input power, which has a significant effect on the predicted result. However, as the prediction of the qualifying and finals race both used this same profile, it would not alone explain the difference in the discrepancies between each of these events.

The atmospheric conditions were assumed to have remained constant at the event. However, if the air density on the day was to rise by 4.6%, (a temperature reduction of 3 °C and local pressure increase of 35 mbar, not an improbable occurrence), then the discrepancy in the qualifying time would also reduce to 0.7 s (see Figure 8). The final possibility is that the cyclist did not follow the shortest path around the velodrome, which the model assumes. If the cyclist took a wider path through the bends or drifted in the straights, this would account for time discrepancies.

It is possible to further validate the model through comparison of SRM data with predicted bicycle velocity. Figure 9 shows a comparison between the measured bicycle velocity, from the qualifying race, and the predicted model velocity. SRM data was not available for the finals race, so comparison with model predictions cannot be compared for this race.

Both profiles reveal the acceleration of the cyclist through the bends of the race. Peaks in the data indicate that the cyclist is passing through a bend, and troughs indicate a straight. The model is observed to slightly over-predict velocity in the straights (0.1 s ~ 0.2 s) compared to the measured values. This has the effect that the cyclist reaches the bends faster in the model than in reality, resulting in a phase shift between the peaks and troughs. In the final stages of the event the model is observed to under-predict velocity in both straights and bends.

To conduct a more stringent validation of the model it will be necessary to obtain detailed data of atmospheric conditions during the capture of cyclist data with the SRM crank. A more detailed, higher frequency, power profile and velocity data from the SRM crank will also be required. Additionally, to date the model has only been validated with a single rider; if suitable data were obtained for additional riders, then this could be used to further strengthen the validation.

The validation has illustrated that the model predicts race performance with reasonable accuracy. It has also proven that the increase of velocity in the bends, specifically associated with track cycling, can be modelled accurately. This makes the model suitable for use in comparative, predictive studies of racing performance.

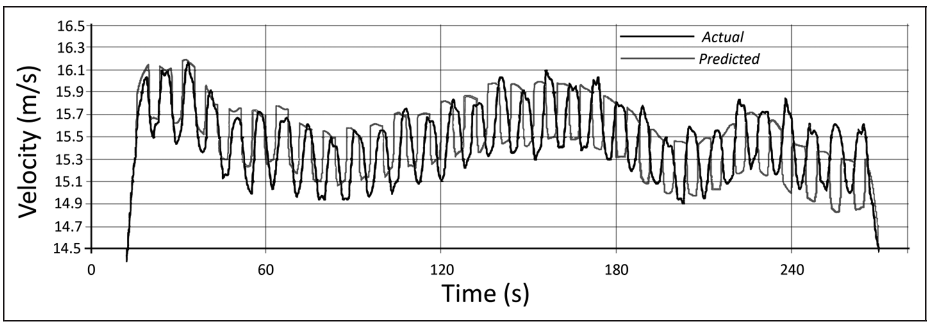

Figure 9. Velocity profile for the bicycle in the qualifying event.

For example, the influence of tyre type upon possible race outcome can be examined. However the application of the model is wider than this sole application, and it can also be used for the investigation of bicycle design, efficiencies, aerodynamics and venue variables upon the possible outcome of racing scenarios.

Conclusions

An analytical model for track cycling has been developed. The model has drawn on the analysis of a number of previous authors. Derivation of the current model has been explained from first principles, and an equation for the motion of track cycling derived. Parameters relating to each term in the equation were deciphered and discussed. A solution procedure was presented, the analytical model having been compiled as a Microsoft Excel spreadsheet. Validation has demonstrated the accuracy of the model, and that the fundamental principles are valid. However, validation has revealed that the model has high dependency on atmospheric conditions (as does cycling in reality). It is suggested that a more stringent validation, with detailed atmospheric and cyclist data, are required. However, the model is useful for the examination of the physics of track cycling, and can be used to predict the performance of a cyclist dependent upon bicycle efficiencies, tyre type and venue conditions, in a racing scenario.

Funding

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

References

1. van Ingen Schenau GJ. Cycle power: a predictive model. *Endeavour* 1988; 12(1): 44–47.
2. Lakomy HKA. Measurements of work and power using friction-loaded cycle ergometers. *Ergonomics* 1986; 29(4): 509–517.
3. Keen P. The truth behind the race of truth. *Cycle Sport* 1994; July: 46–50
4. Jones SM and Passfield L. The dynamic calibration of bicycle power measuring cranks. In: *The engineering of sport: design and development*, Oxford: Blackwell Science, 1998, pp.265–274.
5. Broker JP. Cycling power: road and mountain. In: *high-tech cycling: the science of riding faster*, Colorado: Human kinetics, 2003, pp.147–174.
6. de Groot G, Welbergen E, Clijsen L, et al. Power, muscular work, and external forces in cycling. *Ergonomics* 1994; 37(1): 31–42.
7. Pons DJ and Vaughan CL. Mechanics of cycling. In: *Biomechanics of sport*. Boca Raton, FL: CRC Press, 1989, pp.289–315.
8. Olds TS, Norton KI, Lowe ELA, et al. Modelling road-cycling performance. *J Appl Physiol* 1995; 78(4): 1596–1611.
9. Basset DRJ, Kyle CR, Passfield L, et al. Comparing cycling world hour records, 1967–1996: modelling with empirical data. *Med Sci Sports Exer* 1999; 31(11): 1677–1685.
10. Martin JC, Gardner AS, Barras M, et al. Modelling sprint cycling using field-derived parameters and forward integration. *Med Sci Sports Exer* 2006; 38(3): 592–597.
11. Lukes R, Carre M and Haake S. Track cycling: an analytical model. In: *The engineering of sport 6, Vol 1: developments for sports*. New York: Springer, 2006, pp.115–120.
12. Underwood L and Jermy M. Mathematical model of track cycling: the individual pursuit. In: *Procedia engineering 2 – the engineering of sport 8 – engineering emotion*. London: Elsevier, 2010, vol.2(2), pp.3217–3222.
13. Kyle CR. Selecting cycling equipment. In: Burke ER (ed) *High-tech cycling, the science of riding faster*. Colorado: Human Kinetics, 2003, pp.1–48.
14. Whitt FR and Wilson DG. *Bicycling Science*. Cambridge, MA: MIT Press, 1982.
15. Hennekam W. The speed of a cyclist. *Phys Educ* 1990; 25: 141–146.
16. Burgess SC. Improving cycling performance with large sprockets. *Sports Engng* 1999; 1: 107–113
17. Burgess SC. Improving cycling performance with large sprockets. In: *The engineering of sport: design and development*. Oxford: Blackwell Science, 1998, pp.23–31.

Appendix

Notation

a	acceleration (m/s^2)
A	frontal area (m^2)
C_D	drag coefficient
CoM	centre of mass
C_s	tyre scrubbing coefficient
F_A	acceleration force (N)
F_C	centripetal force (N)
F_D	aerodynamic force (N)
F_F	lateral friction force (N)
F_N	normal contact force (N)
F_R	rolling force (N)
F_T	transmitted force (N)
F_W	weight (N)
g	gravity (m/s^2)
h	relative humidity (%)
h_{alt}	altitude (m)
H	cyclist height (m)
L_m	distance from bicycle pivot to centre of mass (m)
m	mass (kg)
m_b	bicycle mass (kg)
m_c	cyclist mass (kg)
P	pressure (Pa)
P_{cyc}	cyclist power (W)
r_m	radius of curvature of travel of centre of mass (m)
r_{wh}	radius of curvature of travel of wheels (m)
R_{air}	specific gas constant of air
R_{water}	specific gas constant of water

s	saturation factor of water
T	temperature (K)
v	velocity (m/s)
x_m	distance travelled by centre of mass (m)
x_{wh}	distance travelled by wheels (m)
α	tipping angle (°)
β	track banking angle (°)
η	bicycle efficiency
η_b	bearing efficiency
η_{dt}	chain efficiency
η_{st}	bicycle stiffness
μ_r	rolling resistance coefficient
μ_s	rolling resistance due to tyre scrubbing
ρ	density (kg/m^3)
σ	camber angle (°)