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An improved simulated annealing for solving the linear constrained optimization problems

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Abstract

In this paper, we propose an improved simulated annealing (ISA), a global optimization algorithm for solving the linear constrained optimization problems, of which the main characteristics are that only one component of current solution is changed based on the Gaussian distribution in each iteration and ISA can directly solve the linear constrained optimization problems. By solving 6 benchmark functions with the lower and upper bounds constraints and 6 functions with linear constraints, ISA is superior to the classical techniques for solving the problems with lower and upper bounds and reduces greatly the number of function evolutions compared with GENOCOP with the same precision condition.

Keywords: Simulated annealing; Linear constrained optimization problem; Tabu search; GENOCOP

1. Introduction

Many problems in virtually all fields of physical science as well as in chemistry, technology, economics, logistics, travel scheduling and in the design of microprocessor circuitry involve global optimization. In most cases they consist not only of a nonlinear objective function that has to be optimized, but of a number of linear or nonlinear constraints as well that must be satisfied by the solution. Several gradient algorithms were developed for handling the constrained optimization problems [1–3]. This is usually done by converting the original problem into unconstrained one for which gradient methods are applied with some modifications. Despite the research made progress in global optimization in recent years, gradient optimization algorithms have only been able to tackle special formulations because they depend on the existence of derivatives, and they are insufficiently robust in discontinuous, vast multimodal [4]. Because the nature of many of these problems is complex and most gradient methods are local in scope, it is of great importance to investigate other methods for many real world problems.

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The heuristic methods are developed for solving the global constrained optimizations problems [5,6]. For example, an excellent study on comparing evolutionary algorithms for constrained optimization problems has been published by Michalewicz [4]. Among these methods, the methods based on penalty functions have proven to be the most popular. However, penalty function methods are also characterized by serious drawbacks, since small values of the penalty coefficients drive the search outside the feasible region and often produce infeasible solution [4].

Simulated annealing (SA), a very excellent heuristic global optimization algorithm, was firstly proposed by Kirkpatrick et al. [7]. The name of the algorithm derived from an analogy between the simulation of the annealing of solid first proposed by Metropolis et al. [8] and the strategy of solving combinatorial optimization problems. Simulated annealing and its variations are becoming more and more popular in solving the optimization problems. A pseudo-code of the simulated annealing procedure is given in Refs. [9–12]. But it is very less study of simulated annealing for solving the constrained optimization problems. This increases the interesting of some researchers to further study for solving the constrained optimization problems.

In this paper, we put forward a new algorithm and named it as improved simulated annealing (ISA), which is superior to the classical global methods for solving the constrained optimization problems as demonstrated. The main characteristics of which are that only one component of the current solution is changed in each iteration and ISA can directly solve the linear constrained optimization problem without penalty function methods. The rest of this paper is organized as follows. In Section 2, we describe the new algorithm. By solving the six benchmark functions with the lower and upper constraints and six functions with the linear constraints, the results of ISA comparing with the classical global methods are presented in Sections 3 and 4, respectively. Finally, Section 5 concludes with some remarks and future research directions.

2. An improved simulated annealing for the constrained optimization problems

Consider the following global optimization problems

$$(\mathrm{P}) \quad \begin{cases} \min & f(x), \\ \text{s.t.} & x \in \Omega, \end{cases} \tag{2.1}$$

where $\Omega \subset R^n$ and f is a real-valued function.

When $\Omega := \{x \in R^n | x(i) \in [a_i, b_i],\ i = 1, 2, \ldots, n\}$, (P) is an optimization problem with lower and upper bounds. When $\Omega := \{x \in R^n | Cx \leqslant d\}$, where C is a $m \times n$ matrix and d is a vector, (P) is a linear constrained optimization problem.

In this section, we put forward an improved simulated annealing (ISA) for solving the optimization problems with lower and upper bounds and the linear constraints.

2.1. *ISA for solving the lower and upper bounds problems (ISA-I)*

ISA-I is as follows:

Step 1: Randomly generate an initial feasible solution x_0. Initialize the highest temperature $T_{\max}$, the lowest temperature $T_{\min}$ and the inner iteration times $L_{\max}$. Set $T = T_{\max}$ and $k = 0$. Set the best solution $x^* = x_0$ and the best value $f^* = f(x^*)$.

Step 2: While $T > T_{\min}$

(a) While $k \leqslant L_{\max}$

(1) Generate the new solution z by the formula

$$z(l_k) = x_k(l_k) + \alpha \times (b_{l_k} - a_{l_k}) \times N_k(0, 1)$$

l_k is randomly chosen from the set $\{1, 2, \ldots, n\}$. $N_k(0, 1)$ denotes a normally distributed one-dimensional random number with mean zero and standard deviation one. Other components of z are same as those of x_k. α is a variable which decreases by the formula $\alpha = \alpha \times e^{-\beta}$. In this paper $\beta = 1.01$, the initial value of α is equal to 1. If $\alpha < 10^{-4}$, then $\alpha = 1$.

(2) z can be a feasible solution by the formula:

$$z(l_k) = \begin{cases} a_{l_k} + (z(l_k) - b_{l_k}) & \text{if } z(l_k) > b_{l_k}, \\ b_{l_k} - (a_{l_k} - z(l_k)), & \text{if } z(l_k) < a_{l_k}, \\ z(l_k), & \text{if } a_{l_k} \leqslant z(l_k) \leqslant b_{l_k}. \end{cases} \tag{2.1.1}$$

(3) Evaluate the change in energy level $\Delta E^* = f(z) - f^*$ and $\Delta E = f(z) - f(x_k)$.
(4) If $\Delta E^* \leqslant 0$ update the best solution $x^* = z$ and the best value $f^* = f(x^*)$.
(5) If $\Delta E \leqslant 0$ update current state with new state, $x_{k+1} = z$.
(6) If $\Delta E > 0$ update current state with new state with probability $e^{\left(\frac{-\Delta E}{T}\right)}$.
(7) $k = k + 1$.

(b) $L_{\max} = L_{\max} + L$, $k = 0$.
(c) Decrease temperature T according to annealing schedule by formula $T := \delta \times T$.

Step 3: Output the best solution x^* and the best value f^*.

Remark. The distinction between ISA-I and SA is that only one component of x_k is changed in each iteration.

2.2. ISA for solving the linear constrained problems (ISA-II)

Based on the characteristic of ISA-I, we propose the ISA for solving the linear constrained problems (ISA-II). Firstly, we deal with the linear constraints by the following method.

$C\,x \leqslant d$ can be wrote as follow:

$$\begin{cases} c_{11}x(1) + c_{12}x(2) + \cdots + c_{1n}x(n) \leqslant d_1, \\ c_{21}x(1) + c_{22}x(2) + \cdots + c_{2n}x(n) \leqslant d_2, \\ \cdots \\ c_{m1}x(1) + c_{m2}x(2) + \cdots + c_{mn}x(n) \leqslant d_m. \end{cases} \tag{2.2.1}$$

For certain l_k, the inequalities (2.2.1) is equivalent to

$$\begin{cases} c_{1l_k}x(l_k) \leqslant d_1 - c_{11}x(1) \cdots - c_{1l_k-1}x(l_k - 1) - c_{1l_k+1}x(l_k + 1) \cdots - c_{1n}x(n), \\ c_{2l_k}x(l_k) \leqslant d_2 - c_{21}x(1) \cdots - c_{2l_k-1}x(l_k - 1) - c_{2l_k+1}x(l_k + 1) \cdots - c_{2n}x(n), \\ \cdots \\ c_{ml_k}x(l_k) \leqslant d_m - c_{m1}x(1) \cdots - c_{ml_k-1}x(l_k - 1) - c_{ml_k+1}x(l_k + 1) \cdots - c_{mn}x(n). \end{cases} \tag{2.2.2}$$

We can solve the lower bound a_{l_k} and the upper bound b_{l_k} of $x(l_k)$ by the inequalities (2.2.2).

ISA-II is as follows:

The main difference lies in step (1) between ISA-I and ISA-II. Step (1) of ISA-II is:

(1) Generate the new solution z by: randomly choose a $l_k \in \{1, 2, \ldots, n\}$, then for $j = 1, \ldots, n$

$$z(j) = \begin{cases} x_k(j) + \eta \times (b_j - a_j) \times \text{rand}_k, & j = l_k,\ b_{l_k} \neq \infty \text{ and } a_{l_k} \neq \infty, \\ x_k(j) + \eta \times \text{rand}_k, & j = l_k,\ b_{l_k} = \infty \text{ or } a_{l_k} = \infty, \\ x_k(j), & j \neq l_k, \end{cases} \tag{2.2.3}$$

$$\eta = \beta \times \eta, \tag{2.2.4}$$

where the lower bound a_{l_k} and the upper bound b_{l_k} of $z(l_k)$ is determined by the inequalities (2.2.2). rand_k is a uniformly distributed random number of the interval $[-1, 1]$. $\beta = 0.90$. The initial value of η is equal to 1. If $\eta < 10^{-4}$, then $\eta = 1$.

Other steps of ISA-II are same as those of ISA-I.

We know that ISA-II can solve the lower and upper bounds of the changed component of x in current iteration by the inequalities (2.2.2) because the other component of x are not changed. So ISA-II can directly solve the linear constrained optimization problems without penalty methods.

3. Numerical results for the lower and upper bounds constrained problems

In order to examine the performance of the ISA-I, we apply it to function optimization problem. The six benchmark functions in our numeric experiments are listed in Appendix A. The values of all algorithm parameters for solving the six functions are listed in Table 1.

Table 1
Parameters of numeric experiments

Function	T_{max}	T_{min}	δ	L_{max}	L	α	β
GP	10	0.01	0.94	2	1	1	1.01
BR	10	0.01	0.80	2	1	1	1.01
HN3	10	0.01	0.88	2	1	1	1.01
HN6	10	0.01	0.92	2	1	1	1.01
RA	10	0.01	0.84	2	1	1	1.01
SH	10	0.01	0.98	2	1	1	1.01

Table 2
Average number of objective function evaluations used by six methods to optimize six functions

Method	GP	BR	Hn3	Hn6	RA	SH
PRS	5125	4850	5280	18,090	5964	6700
MS	4400	1600	2500	6000	N/A	N/A
SA1	5439	2700	3416	3975	N/A	241215
SA2	563	505	1459	4648	N/A	780
TS	486	492	508	2845	540	727
ISA-I	311	329	355	1534	466	286

Table 3
Parameters of numeric experiments

Function	T_{max}	T_{min}	δ	L_{max}	L
f_1	10	0.001	0.97	10	1
f_2	10	0.001	0.97	10	1
f_3	10	0.001	0.97	10	1
f_4	10	0.001	0.93	10	1
f_5	10	0.001	0.97	10	1
f_6	10	0.001	0.90	10	1

Table 4
All results of function f_1–f_6 have been averaged over 10 runs, where 'N', 'mean best', 'best' and 'worst' stand for the numbers of function evaluations, mean best objective values, the best objective values and the worst objective values, respectively

f	$f(x^*)$	ISA-II				GENOCOP [4]	
		N	Best	Worst	Mean best	N	Best
f_1	−213	48,783	−212.9999992	−212.9996850	−212.9999182	70,000	−213
f_2	−47.760765	48,783	−47.7337246	−47.6640605	−47.710603	70,000	−47.760765
f_3	−15	48,783	−14.9996449	−14.9987972	−14.9992149	70,000	−14.999965
f_4	−4.5142	9271	−4.5141991	−4.4483659	−4.5027098	35,000	−4.5142
f_5	−11	48,783	−10.7648797	−10.4339709	−10.5707308	70,000	−11
f_6	−1	4708	−0.9999936	−0.9911025	−0.9981324	35,000	−1

$f(x^*)$ is the global optimal value of function f.

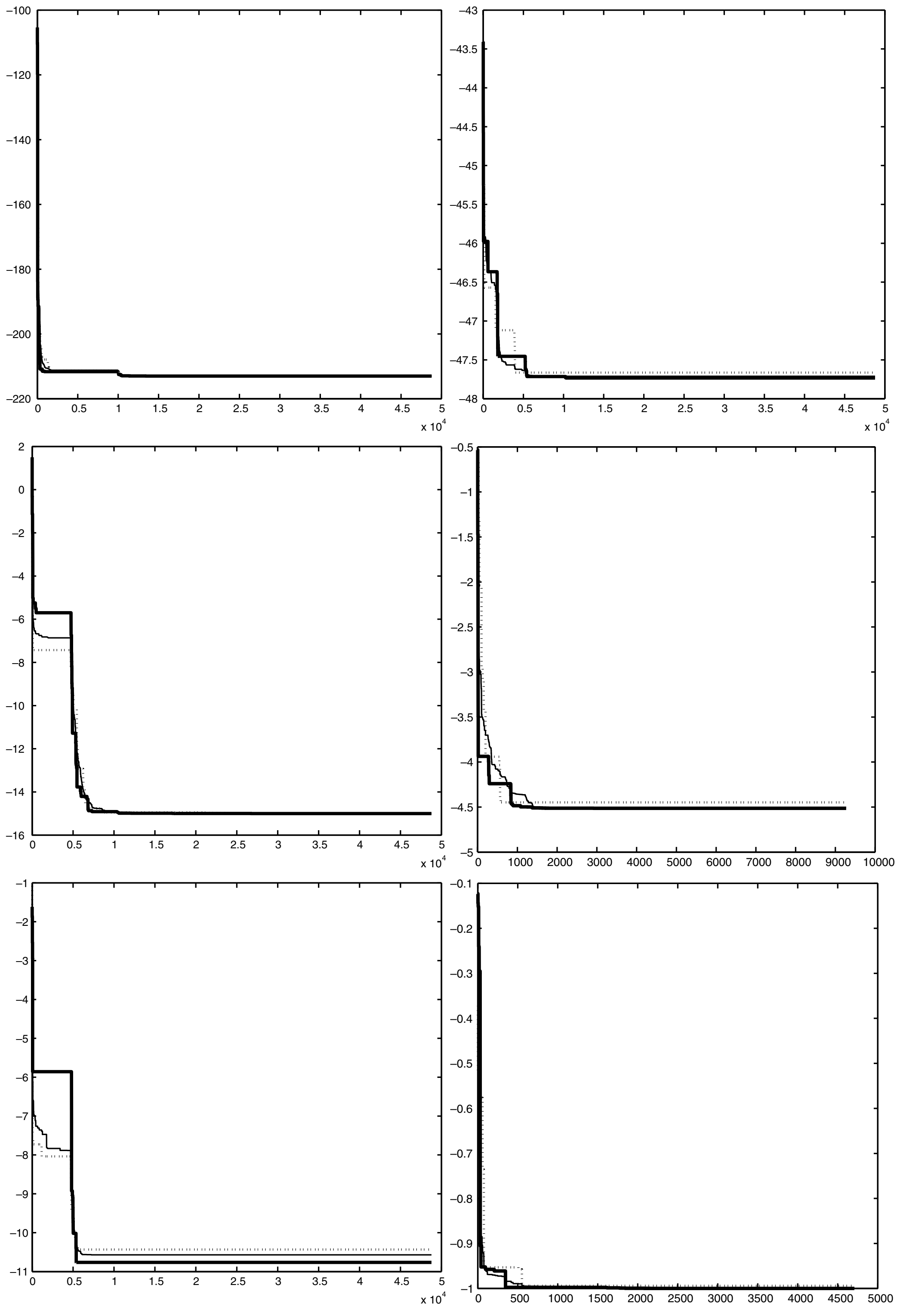

Fig. 1. Performance of ISA with test functions f_1–f_6. The thick solid line, the dotted line and the thin solid line indicate the runs with best objective value, with the worst value and with the mean value, respectively.

These functions have the characteristics as unimodal/mulitimodal and low-dimensional/high-dimensional. Because of these characteristics, it is difficult to seek for the global minima. The results of ISA-I are listed in Table 2 compared with the results of the classical global optimization methods, which included pure random search (PRS) [13], multistart (MS) [14], simulated annealing based on stochastic differential equations (SA1) [14], simulated annealing (SA2) [14] and taboo search (TS) [15]. The results of ISA-I optimization of test functions are the average outcome of 100 independent runs. The reliability is excellent: in each case at least 90% of runs have been successful(with the average result less than 3% of the global minimum) except for function GP with 35% of runs.

Results in Table 2 indicate that ISA-I is reliable and efficient: more so than SA1, SA2 and TS methods. ISA-I significantly reduces the amount of blind search characteristic of earlier techniques such as PRS and MS. So our algorithm is efficient.

4. Numerical results for the linear constrained optimization problems

Six optimization problems with linear constraints are listed in the Appendix B in this paper. These problems are tested by other methods. The values of the algorithm parameters for solving the six functions are listed in Table 3. Table 4 shows the results of ISA-II compared with GENOCOP, a very excellent method for solving the constrained problems [4]. Fig. 1 shows the performance of ISA-II solving the six problems. From Table 4, we know that the ISA-II reduces numbers of function evaluations under the same precision condition. For function f_1, f_2 and f_3, the best values of ISA are the same precision as those of GENOCOP with about 2/3 of number of function evaluations of GENOCOP, but not better precision for function f_5. For function f_4 and f_6, we obtain the best values by using ISA-II with about 1/5 of number of GENOCOP. From Fig. 1, we know that ISA-II is very robust for solving the six problem because all ten runs converge to the global optimal value. So ISA-II is an effective method for handling the linear constrained optimization problems.

5. Conclusion

By applying it to the benchmark function optimization problems, our algorithm is examined and good results are obtained. Compared with the classical techniques, our algorithm is the best one. Since ISA and SA differ mainly in generating new solution, it is quite easy to apply ISA to real-world problems as SA. In addition, only one component of current solutions is changed in ISA compared with SA, so ISA can save cpu time. Numerical results illustrate that ISA is efficient, robust and easy to implement for handling linear constrained problems. ISA can directly solve linear constrained problems, and avoid converting the constrained optimization problem into the unconstrained optimization problem. One can extend the work by introducing this idea to the other heuristic algorithms.

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Appendix A

Example 1 (*GP* (*Goldstein–Price function*: $n = 2$)).

$$f(x) = [1 + (x_1 + x_2 + 1)^2(19 - 14x_1 + 3x_1^2 + 6x_1x_2 + 3x_2^2)] \\ \times [30 + (2x_1 - 3x_2)^2 \times (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1x_2 + 27x_2^2)], \quad -2 < x_i < 2, \ \ i = 1, 2.$$

The global minimum is equal to 3 and the minimum solution is $(0, -1)$. There are four local minima in the region.

Example 2 (*BR* (*Branin*: $n = 2$)).

$$f(x) = a(x_2 - bx_1^2 + cx_1 - d)^2 + e(1 - f)\cos(x_1) + e,$$

where $a = 1$, $b = 5.1/(4\pi^2)$, $c = 5/\pi$, $d = 6$, $e = 10$, $f = 1/(8\pi)$. $-5 \leqslant x_1 \leqslant 10$, $0 \leqslant x_2 \leqslant 15$. $x^* = (-\pi, 12.275)$; $(\pi, 2.275)$; $(3\pi, 2.475)$. $f(x^*) = 5/(4\pi)$. There are no more minima.

Example 3, 4 (*HN* (*Hnrtman functions*: $n = 3, 6$)).

$$f(x) = \sum_{i=1}^{4} c_i \exp\left[-\sum_{j=1}^{n} \alpha_{ij}(x_j - p_{ij})^2\right], \quad 0 \leqslant x_i \leqslant, \ i = 1, 2, \ldots, n.$$

For $n = 3$, the global minimum is equal to -3.86 and it is reached at the point (0.114, 0.556, 0.882). For $n = 6$ the minimum is -3.32 at the point (0.201, 0.150, 0.477, 0.275, 0.311, 0.657).

i	α_{i1}	α_{i2}	α_{i3}	c_i	p_{i1}	p_{i2}	p_{i3}
$n = 3$							
1	3	10	30	1	0.3689	0.1170	0.2673
2	0.1	10	35	1.2	0.4699	0.4387	0.7470
3	3	10	30	3	0.1091	0.8742	0.5547
4	0.1	10	35	3.2	0.03815	0.5743	0.8828

i	α_{i1}	α_{i2}	α_{i3}	α_{i4}	α_{i5}	α_{i6}	c_i
$n = 6$							
1	10	3	17	3.5	1.7	8	1
2	0.05	10	17	0.1	8	14	1.2
3	3	3.5	1.7	10	17	8	3
4	17	8	0.05	10	0.1	14	3.2

i	p_{i1}	p_{i2}	p_{i3}	p_{i4}	p_{i5}	p_{i6}
1	0.1312	0.1696	0.5569	0.0124	0.8283	0.5886
2	0.2329	0.4135	0.8307	0.3736	0.1004	0.9991
3	0.2348	0.1451	0.3522	0.2883	0.3047	0.6650
4	0.4047	0.8828	0.8732	0.5743	0.1091	0.0381

Example 5 (*RA* (*Rastrigin function*: $n = 2$)).

$$f(x) = x_1^2 + x_2^2 + \cos(18x_1) - \cos(18x_2), \quad -1 \leqslant x_1, \ x_2 \leqslant 1,$$

which has 50 minima in the region. The global minimum is at $x^* = (0, 0)$ where $f(x^*) = -2$.

Example 6 (*SH* (*Shubert function*)).

$$f(x) = \left\{\sum_{i=1}^{5} i\cos((i+1)x_1 + i)\right\}\left\{\sum_{i=1}^{5} i\cos((i+1)x_2 + i)\right\}, \quad -10 \leqslant x_1, \ x_2 \leqslant 10.$$

In the region the function has 760 local minima, 18 of which are global with $f = -186.7309$.

Appendix B. Ref. [4]

Test function f_1

$$\min \quad f_1(x) = -10.5x_1 - 7.5x_2 - 3.5x_3 - 2.5x_4 - 1.5x_5 - 10x_6 - 0.5\sum_{i=1}^{5} x_i^2$$

subject to

$$6x_1 + 3x_2 + 3x_3 + 2x_4 + x_5 \leqslant 6.5, \quad 10x_1 + 10x_3 + x_6 \leqslant 20, \quad 0 \leqslant x_i \leqslant 1, \quad i = 1, \ldots, 5, \ 0 \leqslant x_6.$$

The global solution is $x^* = (0, 1, 0, 1, 1, 20)$, and $f_1(x^*) = -213$.

Test function f_2

$$\min \quad f_2(x) = \sum_{j=1}^{10} x_j \left(c_j + \ln \frac{x_j}{\sum_{i=1}^{10} x_i} \right)$$

subject to

$$x_1 + 2x_2 + 2x_3 + x_6 + x_{10} = 2, \quad x_4 + 2x_5 + x_6 + x_7 = 1,$$
$$x_3 + x_7 + x_8 + 2x_9 + x_{10} = 1, \quad x_i \geqslant 0.000001, \quad i = 1, \ldots, 10,$$

where $c = (-6.089, -17.164, -34.054, -5.914, -24.721, -14.986, -24.100, -10.708, -26.663, -22.179)$. The global solution is $x^* = (0.04034785, 0.15386976, 0.77497089, 0.00167479, 0.48468539, 0.00068965, 0.02826479, 0.01849179, 0.03849563, 0.10128126)$ in Ref. [4], $f_2(x^*) = -47.760765$.

Test function f_3

$$\min \quad f_3(x) = 5x_1 + 5x_2 + 5x_3 + 5x_4 - 5\sum_{i=1}^{4} x_i^2 - \sum_{i=5}^{13} x_i$$

subject to

$$2x_1 + 2x_2 + x_{10} - x_{11} \leqslant 10, \quad 2x_1 + 2x_3 + x_{10} + x_{12} \leqslant 10,$$
$$2x_2 + 2x_3 + x_{11} + x_{12} \leqslant 10, \quad -8x_1 + x_{10} \leqslant 0, \quad -8x_2 + x_{11} \leqslant 0, \quad -8x_3 + x_{12} \leqslant 0,$$
$$-2x_4 - x_5 + x_{10} \leqslant 0, \quad -2x_6 - x_7 + x_{11} \leqslant 0, \quad -2x_8 - x_9 + x_{12} \leqslant 0,$$

and bounds $0 \leqslant x_i \leqslant 1$, $i = 1, \ldots, 9$, $0 \leqslant x_i \leqslant 100$, $i = 10, 11, 12$, $0 \leqslant x_{13} \leqslant 1$.

The problem have 13 variables and 9 linear constraints; the function f_3 is quadratic with its global minimum at $x^* = (1, 1, 1, 1, 1, 1, 1, 1, 1, 3, 3, 3, 1)$, where $f_3(x^*) = -15$.

Test function f_4

$$\min \quad f_4(x) = x_1^{0.6} + x_2^{0.6} - 6x_1 - 4x_3 + 3x_4,$$

subject to

$$-3x_1 + x_2 - 3x_3 = 0, \quad x_1 + 2x_3 \leqslant 4, \quad x_2 + 2x_4 \leqslant 4, \quad x_1 \leqslant 3, \quad x_4 \leqslant 1, \quad 0 \leqslant x_i, \quad i = 1, 2, 3, 4.$$

The best known global solution is $x^* = \left(\frac{4}{3}, 4, 0, 0\right)$, and $f_4(x^*) = -4.5142$.

Test function f_5

$$\min \quad f_5(x) = 6.5x_1 - 0.5x_1^2 - x_2 - 2x_3 - 3x_4 - 2x_5 - x_6$$

subject to

$$x_1 + 2x_2 + 8x_3 + x_4 + 3x_5 + 5x_6 \leqslant 16, \quad -8x_1 - 4x_2 - 2x_3 + 2x_4 + 4x_5 - x_6 \leqslant -1,$$
$$2x_1 + 0.5x_2 + 0.2x_3 - 3x_4 - x_5 - 4x_6 \leqslant 24, \quad 0.2x_1 + 2x_2 + 0.1x_3 - 4x_4 + 2x_5 + 2x_6 \leqslant 12,$$
$$-0.1x_1 - 0.5x_2 + 2x_3 + 5x_4 - 5x_5 + 3x_6 \leqslant 3, \quad x_4 \leqslant 1, \ x_5 \leqslant 1, \ \text{and} \ x_6 \leqslant 2, \ x_i \geqslant 0, \ i = 1, \ldots, 6.$$

The global solution is $x^* = (0,6,0,1,1,0)$, and $f_5(x^*) = -11$.

Test function f_6

$$\min \quad f_6(x) = \begin{cases} x_2 + 10^{-5}(x_2 - x_1)^2 - 1.0, & 0 \leqslant x_1 < 2, \\ \frac{1}{27\sqrt{3}}((x_1 - 3)^2 - 9)x_2^3, & 2 \leqslant x_1 < 4, \\ \frac{1}{3}(x_1 - 2)^3 + x_2 - \frac{11}{3}, & 4 \leqslant x_1 \leqslant 6 \end{cases}$$

subject to

$$x_1/\sqrt{3} - x_2 \geqslant 0, -x_1 - \sqrt{3}x_2 + 6 \geqslant 0, 0 \leqslant x_1 \leqslant 6, \quad \text{and} \quad x_2 \geqslant 0.$$

The function f_6 has three global solutions: $x^* = (0,0), (3, \sqrt{3})$ or $(4,0)$. In all cases $f_6(x^*) = -1$.

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