

Figure 1: The layout of nuSTORM facility.

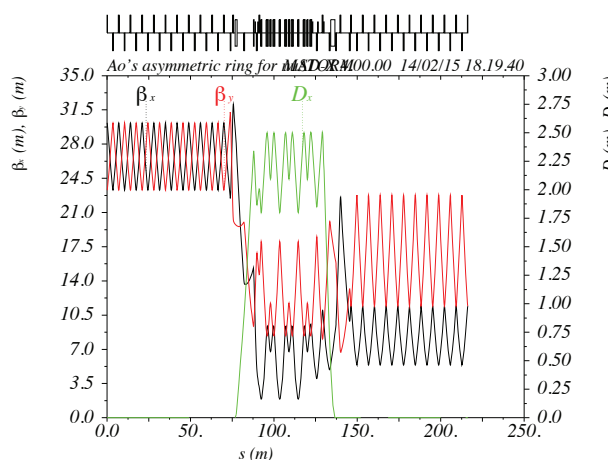


Figure 2: The linear optics of the proposed ring design with combined function magnets in the arcs.

The natural higher order nonlinearities combine with those induced by the sextupoles and play significant roles in the beam stability. Therefore, choosing and correcting the most significant terms as done conventionally in other designs does not work the most efficiently. In order to find the balance among corrections of all the nonlinearities that limit the acceptance of the beam, it is straightforward to apply a numerical algorithm to find the best configuration. This is a single objective optimization, which is to maximize the acceptance of a muon beam with a 2 mm Gaussian transverse admittance, and a uniform momentum spread within  $3.8 \pm 10\%$  GeV/c. The variables are the strengths, lengths, and locations of the sextupolar fields. Since the concept of “sextupole families” does not apply in this design, namely all the variables are independent of each other, a meta-heuristic algorithm that thoroughly searches the parameter space for the global optimum can be implemented.

The Simulated Annealing (SA) algorithm is known for its efficiency in performing a satisfactory global search, especially when limited computing resources are available [4]. In the SA algorithm, each set of variables determines a “state”

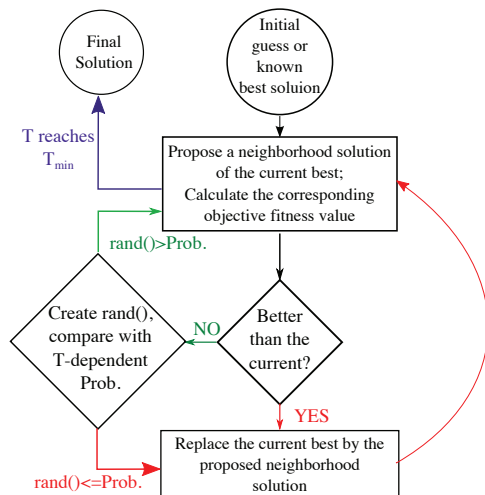


Figure 3: The flowchart of the basic SA algorithm.

of the problem, characterized by the fitness function. For each iteration of one search, the algorithm probabilistically decides whether or not to move to a new state, even if the new state is worse than the current state. This probability function is called the "acceptance probability function". However, this brings one commonly recognized drawback to the algorithm, that this process may result in a temporary deterioration, and possible loss of an optimum that has already been found. The original algorithm is purely serial, and it is difficult to implement any parallelism in a single search. The flowchart of the basic SA algorithm is shown in Fig. 3.

In order to expedite the optimization, a Message Passing Interface (MPI) process was added in the algorithm, to enable a group of simultaneous independent search. The MPI enhanced SA algorithm is more likely to find the global optimum. In this paper, we describe the optimization procedure in details, including the setup and the results, and present the recent progress on applying the technique to the optimization of the stage 3 of the 6-dimensional muon cooling channel [5].

## OPTIMIZATION PROCEDURE

### The MPI Enhanced SA

In the MPI enhanced SA algorithm, each MPI rank (processor) performs its individual search for the global optimum, **starting from independent random initial guess**. In each iteration, the results from all the ranks are compared, and the best one is gathered to the "mom rank". The "all-time" best solution kept by the mom rank can only be replaced by the best solution in this iteration if the new one is even better. The mom rank then focuses on searching the neighborhood area of the best solution in the parameter space. There are two advantages of using the enhanced algorithm in our study. On the one hand, since the acceptance of the ring is calculated by multiparticle tracking, running multiple tracking simulations accelerates the whole optimization by the same

number of times with the total number of MPI ranks. On the other hand, the mom rank keeps track of the all-time best solution from all the independent search, without being affected by the probabilistic deterioration. The success of this technique has been confirmed by the widely used test functions.

### Optimization Setup

We use a population of 120 individuals in each iteration, each of which calls MAD-X to run tracking with its own sextupole parameters. The acceptance probability function depends on the current "temperature" parameter, of which the initial value is set to  $T_0 = 2$ , and gradually "cools" to  $T_f = 0.5$  with a cooling factor of 0.95, or  $T_{i+1} = 0.95T_i$ . For each individual search, the probability of moving from a solution (a set of sextupole parameters)  $a$  to a solution  $b$  is  $e^{(\Omega_b - \Omega_a)/T_i}$ , provided that  $\Omega_b < \Omega_a$ , where  $\Omega$  is the acceptance corresponding to each solution and  $T_i$  is the temperature at the  $i$ th iteration. If the new acceptance given by  $b$  is better than that of  $a$ ,  $a$  will be replaced by  $b$  as the current solution. The mom rank controls the all-time best in all the individual search. The computation was done on NERSC [6], with 120 cores requested in each iteration. 5000 particles are tracked for 100 turns in the PTC tracking module in MAD-X [7] in determining the acceptance. Since muons in the nuSTORM storage ring decay to  $\sim 15\%$  after 100 turns, the acceptance of each run is calculated by the number of survived particles after 100 turns without decay.

### Optimization Result

The acceptance of a full-size beam described in the Introduction section was increased from 58% to 67% by the optimization. The optimum sextupole configuration found by the algorithm does not suppress any one of the nonlinearity terms to 0, but finds a balance instead. The result is shown in Fig. 4, where the average number of turns particles survive in each momentum bin are plotted for the lattices before and after the correction.

It can be noticed that there are still stopbands on the momentum spectrum, at  $|\delta| = 0.1$  and  $\delta \approx 0.03$ , which were not compensated by the correction like at  $\delta \approx -0.05$ . To prove that the algorithm did not stop at a local optimum, another optimization was done. In this case, a new beam with the same transverse admittance, but flat momentum distribution with  $\delta \in [0.02, 0.04]$ , was generated and used as the initial distribution. From results shown in Fig. 4, the stopband can be compensated by the sextupoles with the same optimization constraints in the case with a full-size beam. The acceptance in the new case was increased from as low as 16% to 88%. However, with this sextupole configuration, the acceptance of the full-size beam drops from 58% to 37%. The correction effect for the special beam is shown in Fig. 5. It shows that the correction is capable of removing the stopband, which however was not favored by the algorithm since it destroys the full acceptance.

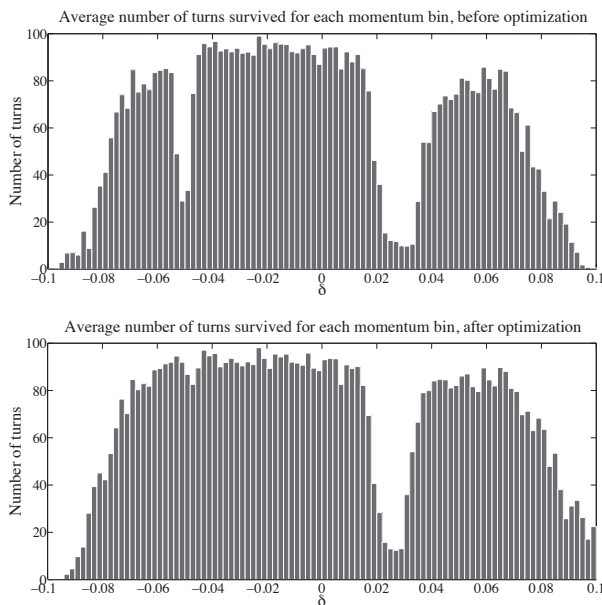


Figure 4: The average number of turns survived for particles in each momentum bin for the full-size beam. Upper: non-optimized lattice without correction; Lower: lattice with optimized sextupole correction.

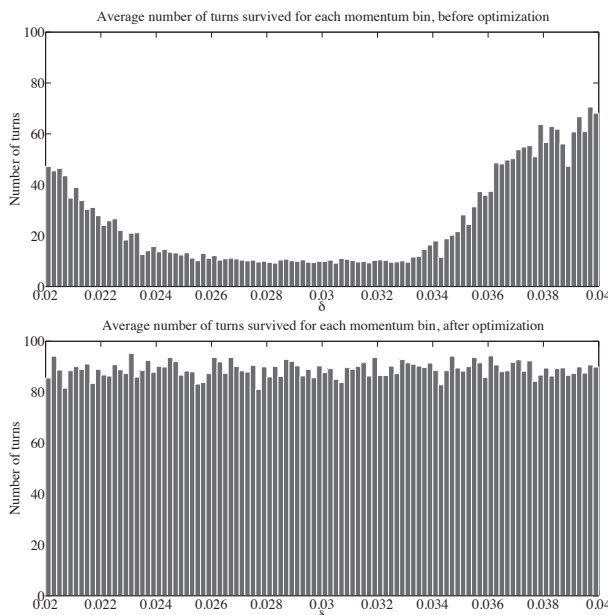


Figure 5: The average number of turns survived for particles in each momentum bin for the special beam. Upper: non-optimized lattice without correction; Lower: lattice with optimized sextupole correction for this beam.

### Application to the 6D Muon Cooling Channel

The rectilinear cooling channel proposed to serve as the only scheme that can reduce the emittance of the muon beam in a short time for a muon collider [5]. Specifically, the stage 3 of the channel could be used to provide 6D cooling demonstration for the extracted muon beam at nuSTORM [1].

In order to so, not only the cell parameters need to be tuned for the nuSTORM muon beam, but also appropriate matching needs to be done to get the beam cooled. We plan to apply the aforementioned algorithm to the design, while as the first step of the study, the functionality of the algorithm was tested by optimizing the current stage 3 of the channel.

We chose two fitness functions as two different optimization goals in two separate cases in this study. The variables in the optimization are parameters such as the tilt angles for the coils, coil strengths, wedge absorber dimensions, and so forth. One of the fitness function characterizes the effective 6D cooling speed, while the other characterizes the number of cells required to achieve the benchmark cooling. In both of the cases, the fitness functions were increased by a factor of 4, which had included the scaling factor based on the particle loss.

## CONCLUSIONS

It is shown in this paper that the MPI enhanced Simulated Annealing algorithm can be applied to the optimization of sextupole correction for the nuSTORM muon decay ring, of which the design aims at achieving both large phase space and large momentum acceptance. The algorithm can find a balanced compromise among corrections of all the significant nonlinearities, and converge to a configuration that yields the largest acceptance. The algorithm was also tested by the optimization of the cell parameters for the 6D muon cooling channel. We found significant improvement on the cooling efficiency, either characterized by the effective cooling speed, or the number of cells required to achieve the cooling.

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