Graphing Functions

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Further Mathematics Support Programme - WJEC A-Level Further Mathematics
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Graph Sketching - Asymptotes

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Step 5. Complete the sketch.

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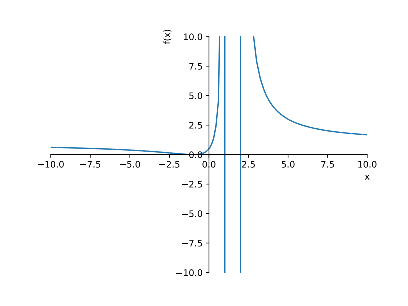
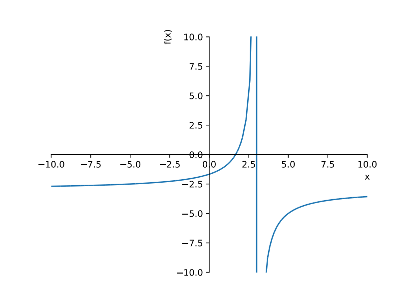


$$y = 1 - \frac{4}{x-1} + \frac{9}{x-2}$$



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    $$\implies \text{Horizontal asymptote is } y = x - 4$$

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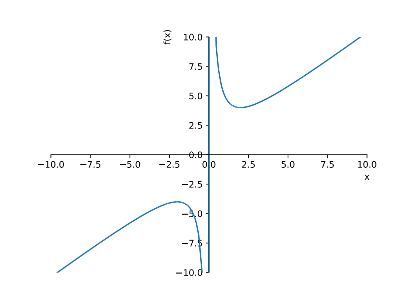

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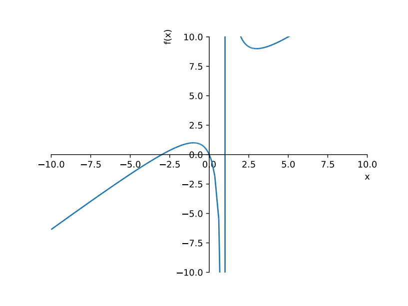

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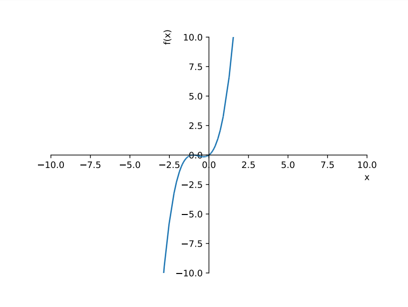

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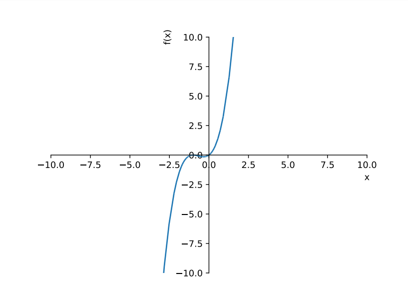

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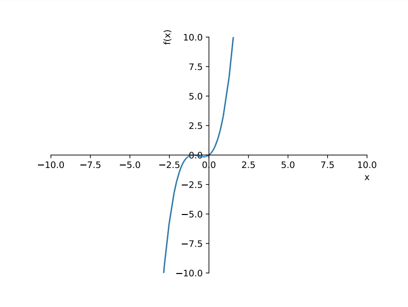

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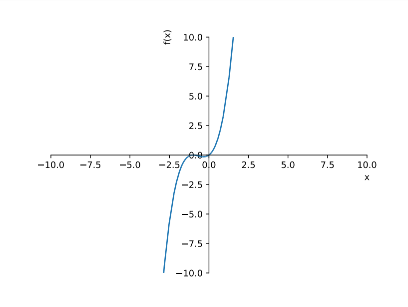

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Second derivative test:

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- Can you think of any examples of odd and even functions?

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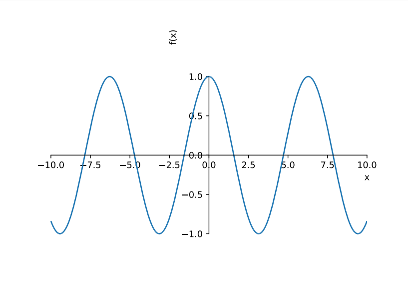

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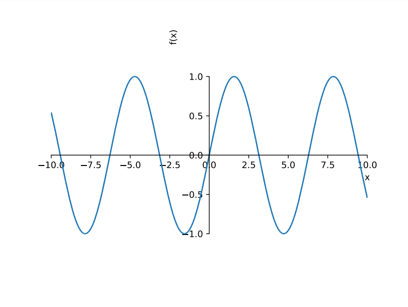

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Other Properties of Graphs - Image and Inverse Image

- The **image** of a set under a function is the set of values which that function maps the element of the set to. Consider

$$f(x) = x^2 + x + 1 \quad \text{for} \quad x \in [-2, 1] \tag{8}$$

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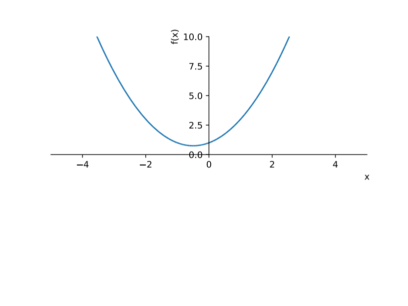

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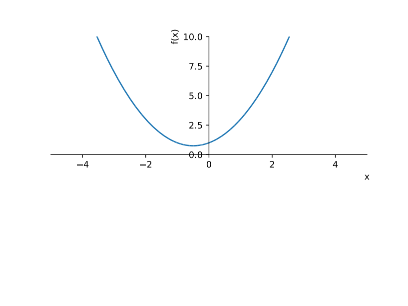

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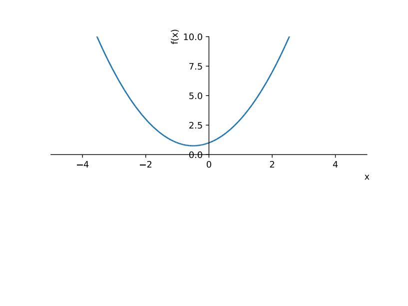

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- The **image** of the set $[-2, 1]$ is the part of the y-axis enclosed by the function in this x-range.
- In this case, $f([-2, 1]) = \left[\frac{3}{4}, 3\right]$