June 2006

3. The function f is defined on the domain $(-\infty, 0) \cup (0, \infty)$ by

$$f(x) = \frac{1}{x(x^2+1)}.$$

(a) Show that f is strictly decreasing over the interval $(0, \infty)$.   [3]

(b) State, giving a reason, whether f is even or odd or neither even nor odd.   [2]

(c) State the equation of each of the asymptotes on the graph of f.   [2]

(d) Sketch the graph of f.   [2]

June 2007

6. The function f is defined by

$$f(x) = \frac{x^2+4}{x}.$$

(a) Find the coordinates of the stationary points on the graph of f.   [4]

(b) Find the equation of each of the two asymptotes.   [2]

(c) Sketch the graph of f.   [2]

(d) Find $f(A)$ where A is the interval $[1, 5]$.   [4]

June 2008

7. The function f is defined by

$$f(x) = \frac{5 - 3x}{(x-1)(x-3)}.$$

(a) Express $f(x)$ in partial fractions. [3]

(b) Obtain an expression for $f'(x)$ and hence show that there are no stationary points on the graph of f. [3]

(c) Sketch the graph of f. State

   (i) the coordinates of all the points of intersection of the graph and the coordinate axes,

   (ii) the equations of all the asymptotes. [7]

(d) Find $f^{-1}(A)$ where A is the interval $(0, 1)$. [5]

June 2009

8. The function f is defined by

$$f(x) = \frac{x(x+3)}{x-1}.$$

(a) Show that $f(x)$ can be written in the form

$$ax + b + \frac{c}{x-1}$$

   where a, b, c are constants to be found. [3]

(b) Find the coordinates of the stationary points on the graph of f. [4]

(c) State the equation of each of the asymptotes on the graph of f and sketch the graph of f. [4]

(d) Find $f^{-1}(A)$, where A is the interval $[0, 10]$. [5]

June 2010

6. The function f is defined by

$$f(x) = \frac{x}{(x-1)^2}.$$

(a) Find the coordinates of the stationary point on the graph of f. [4]

(b) State the equation of each of the asymptotes of the graph of f. [2]

(c) Sketch the graph of f. [2]

(d) Find $f^{-1}(A)$, where A is the interval $[0, 2]$. [5]

June 2011

8. The function f is defined by

$$f(x) = \frac{(x+1)^2}{(x-1)(x-2)} .$$

(*a*) Prove that $f(x)$ can be written in the form

$$1 - \frac{4}{x-1} + \frac{9}{x-2} .$$

Hence find expressions for $f'(x)$ and $f''(x)$. [7]

(*b*) Find the coordinates of the stationary points on the graph of f and classify each point as a maximum or minimum. [6]

(*c*) State the equation of each of the asymptotes on the graph of f. [2]

(*d*) Sketch the graph of f. [3]

June 2012

6. The function f is defined by
$$f(x) = \frac{2}{x-3} + x - 6.$$

(*a*) Determine the coordinates of the points where the graph of f intersects the coordinate axes. [5]

(*b*) Find the coordinates of the stationary points on the graph of f. [5]

(*c*) State the equation of each of the asymptotes on the graph of f. [2]

(*d*) Sketch the graph of f. [2]