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Stability of Periodically Modulated Rotating Boundary Layers

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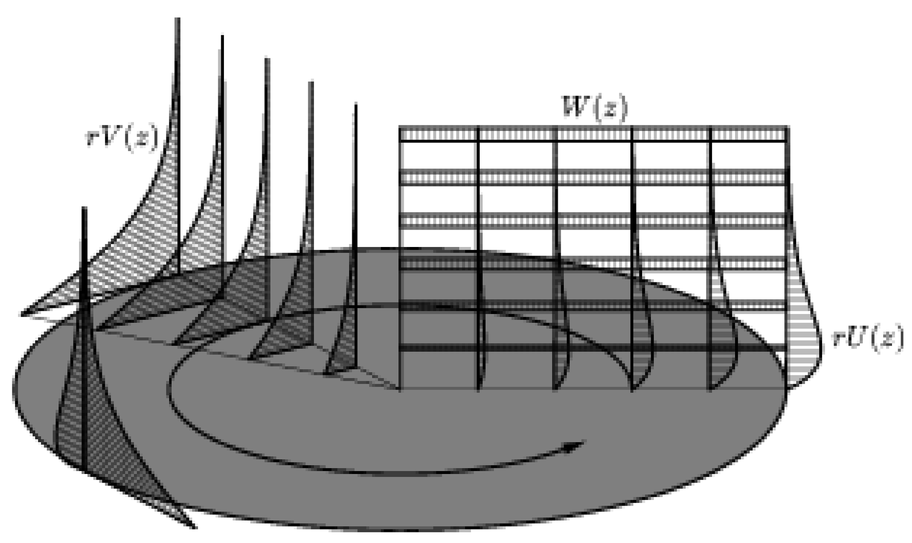
Rotating Disks ≈ Swept Wings

The **steady** rotating disk contains an archetypal model for a **three-dimensional boundary layer**.

Some stability properties are **similar** to those found in the flow over a **swept wing**.

Therefore, any **stabilising techniques** could prove effective in delaying **turbulent onset** on swept wings.

The rotating disk is much **easier** to work with **experimentally** than swept wings.



Steady flow over a disk rotating in a stationary fluid

Other Points of Interest

- **Reynolds number** ≡ **local radius** — the further away from the centre, the more unstable the flow.
- The base flow admits an **exact similarity solution** to the Navier-Stokes equations:

$$\mathbf{U}_B = \left(\frac{r}{R}F, \frac{r}{R}G, \frac{1}{R}H\right)$$

where (F, G, H) are the solutions to a system of ODEs.

Stabilising Technique - Modulation

We adapt the motion of the disk by way of **periodic modulation** of the surface in the azimuthal plane.

Motivated by **channel flow**, where adding oscillations can be **stabilising** [1].

We alter the **azimuthal boundary condition** of base flow so that:

$$G(z=0,\tau) = 1 + U_w \cos\left(\frac{\varphi}{R}\tau\right) \qquad (1)$$

where U_w gives a measure of the wall velocity and φ the frequency.

Stability Analysis

- Analyse behaviour of **infinitesimal disturbances** to the flow of the form:

$$\mathbf{U} = \mathbf{U}_B + \epsilon\hat{\mathbf{u}}$$

- Solve Navier-Stokes equations via a velocity-vorticity formulation [2].

Floquet theory:

- Take **Floquet mode approximation**

$$p(r,\theta,z,\tau) = \hat{p}(z,\tau)e^{\mu\tau}e^{i(\alpha r + n\theta)} \qquad (2)$$

- Decompose $\hat{p}$ into **harmonics** such that

$$\hat{p} = \sum_{k=-\infty}^{\infty} \hat{p}_n(z)e^{ik\tau}$$

- Gives system of **eigenvalue problems**:

$$\sum_{k=-\infty}^{\infty} \mathcal{L}_k\{\mu, \alpha; n, R, U_w, \varphi\}e^{ik\tau} = 0 \qquad (3)$$

Direct numerical simulations:

- Retain **full** temporal and radial structure

$$p(r,\theta,z,\tau) = \hat{p}(r,z,\tau)e^{in\theta}$$

- Evolve some pre-determined disturbance via a time-marching procedure.

Types of Disturbance - Stationary & Impulsive

Stationary:

- **Stationary** with respect to disk motion.
- In Floquet mode expansion (2), set $\mu = 0$.

- This is the instability mechanism that is relevant to **swept-wing** flow.

Impulsive:

- Prescribe wall motion:

$$\zeta(r,\tau) = e^{\lambda r^2}e^{-\sigma\tau^2} \qquad (4)$$

and track development of impulse both temporally and radially.

- This is of interest to more general instability structures, including **transition to turbulence**.

Assumptions:

- In order to demonstrate the **stabilising effects** of the modulation on the steady case, we want to keep **close** to the steady case.
- Therefore, we constrain the wall velocity, U_w in (1) to be:

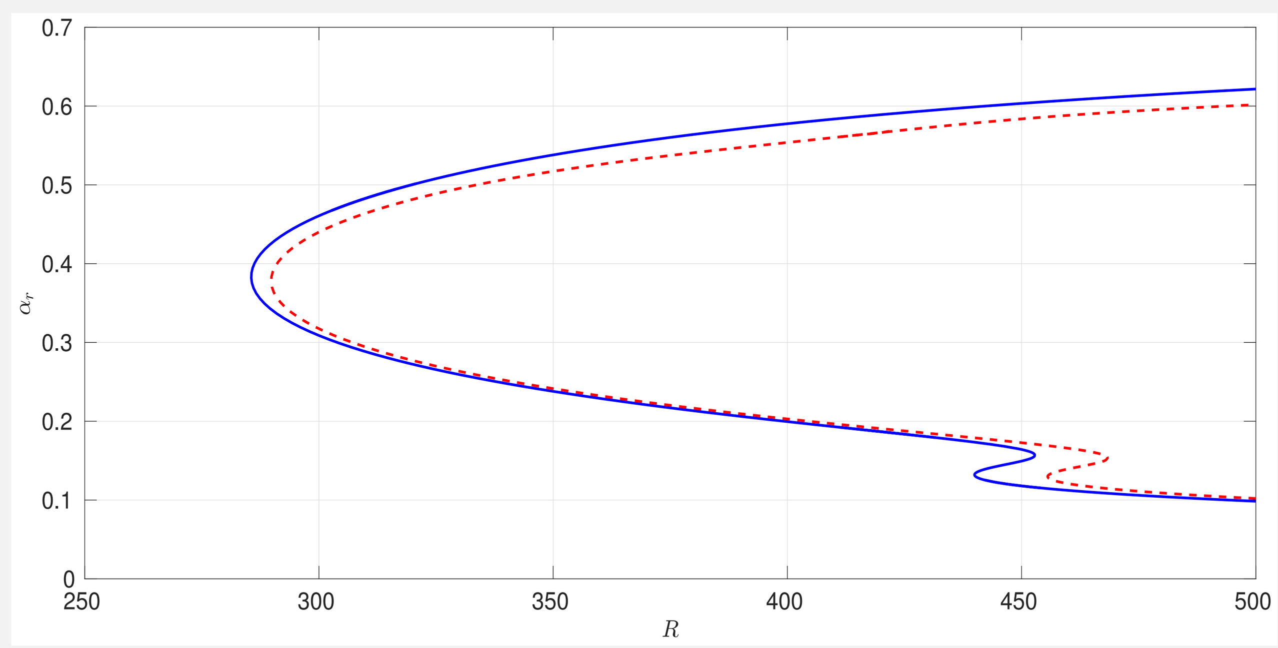
$$U_w \leq 0.2$$

so that the deviation from the steady case is not more than 20%.

Stationary Disturbances

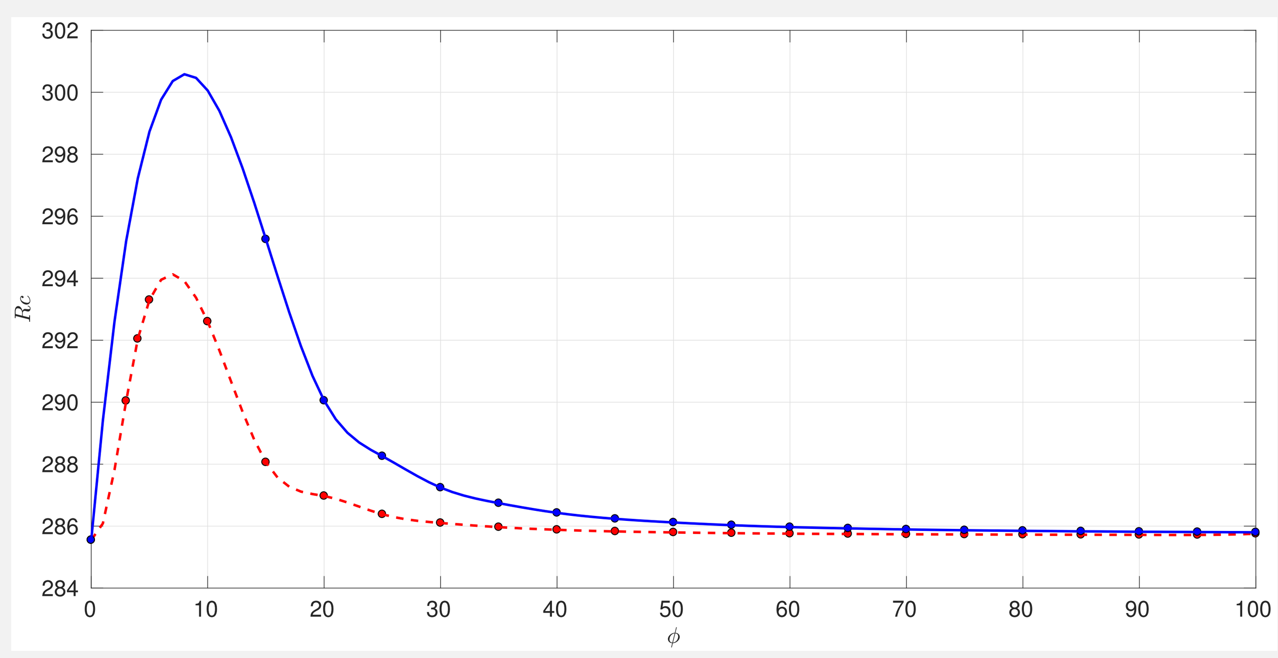
Floquet theory:

- Specify $\mu = 0$ and solve (3) for α.
- The neutral curve defines the critical value of α such that $\mathfrak{Im}(\alpha) = 0$.
- Any parameter values **inside** the curve are **unstable**.
- The **critical Reynolds number**, R_c, is defined as the smallest Reynolds number for which the flow is unstable. In the steady case, $R_c \approx 286$.
- The following figure shows a comparison between the steady neutral curve and that for parameter values $U_w = 0.2$, $\varphi = 16$ for stationary disturbances.



Comparison between steady (—) and $U_w = 0.2$, $\varphi = 16$ (- - -) neutral curves for stationary disturbances.

- The following figure shows a comparison between the critical Reynolds number R_c for $U_w \in \{0.1, 0.2\}$ and $\varphi \in [0, 100]$.

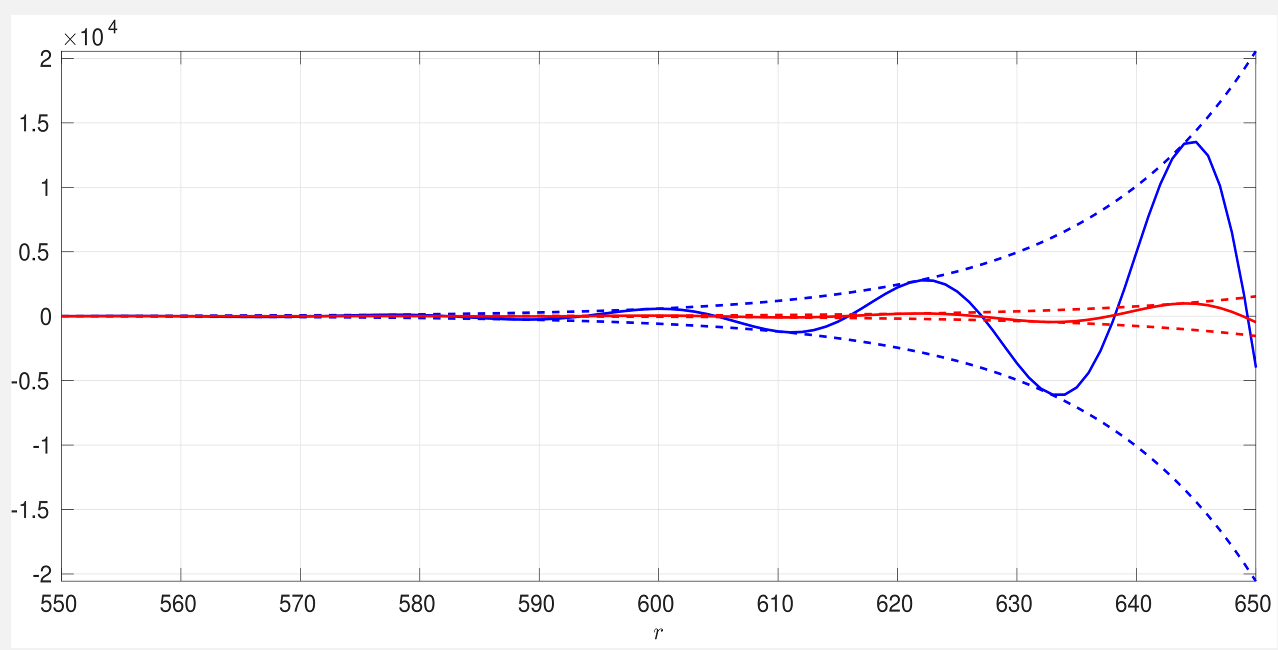


Comparison between critical values of R for $U_w = 0.1$ (—) and $U_w = 0.2$ (- - -) for stationary disturbances across a range of φ.

- There is a somewhat **optimum** range of φ for which stabilisation is maximised.
- This phenomenon is as yet **unexplained**.

Direct numerical simulations:

- Track **radial evolution** of stationary disturbance via time-marching procedure.



Comparison between steady (—) and $U_w = 0.2$, $\varphi = 16$ (—) radial wavetrains.

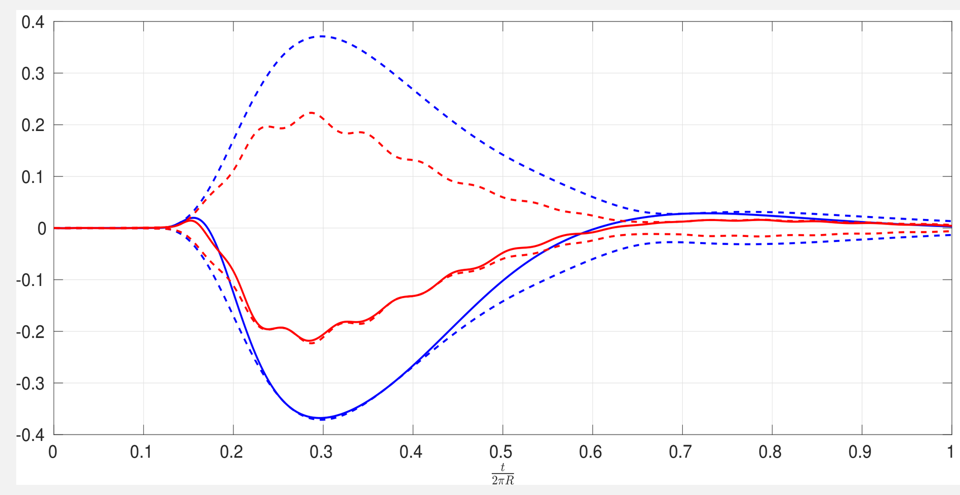
- We see a clear reduction in radial growth rate as expected.

Impulsive Disturbances

- Specify wall motion of the form (4), centered around some radial location r_f.
- Track temporal development at **fixed radial location**.
- The rotating disk contains an **absolute instability** for $R > 507, n \approx 68$ [3].
- Thus, for these parameter values, any disturbance will **grow at all radial locations for all times**.

Convective instability:

- The following figure shows a comparison between the temporal development at a fixed radial location of an impulsive disturbance centered at the convectively unstable parameter values $R = r_f = 500$ and $n = 32$.

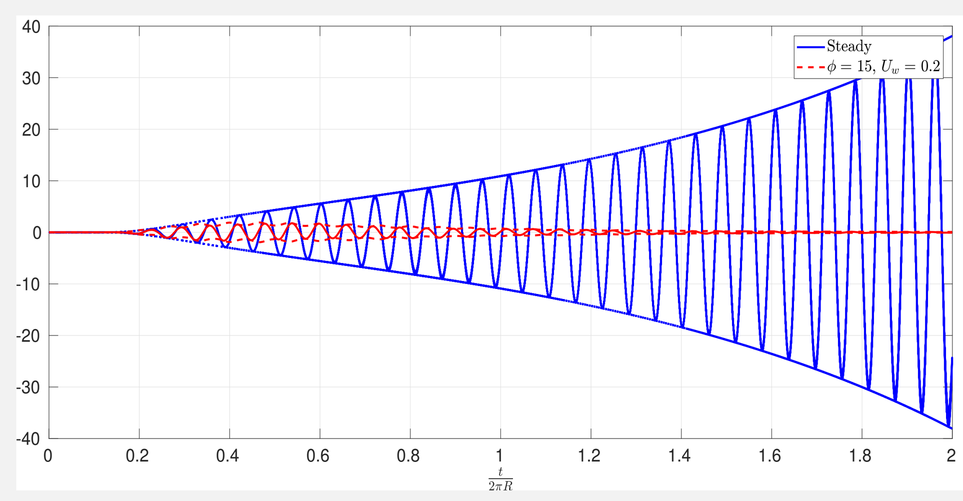


Comparison between steady (—) and $U_w = 0.2$, $\varphi = 16$ (- - -) temporal development measured at radial location $r = 550$ for an impulsive forcing centered at $R = r_f = 500$ with $n = 32$.

- A clear reduction in temporal growth rates is visible, which persists across a range of φ.
- This indicates that the stabilisation is much more **robust** than is indicated by the **single temporal mode** study conducted by the analysis of stationary disturbances.

Absolute instability:

- The following figure shows a comparison between the temporal development at a fixed radial location of an impulsive disturbance centered at the absolutely unstable parameter values $R = r_f = 525$ and $n = 68$.



Comparison between steady (—) and $U_w = 0.2$, $\varphi = 16$ (—) temporal development measured at radial location $r = 600$ for an impulsive forcing centered at $R = r_f = 550$ with $n = 68$.

- Again, a clear reduction in temporal growth rates is visible, which persists across a range of φ.
- This indicates that the absolute instability is also stabilised, and in some cases, for a certain set of parameter values, **eliminated entirely**.

Future Directions

Short-term:

- Conduct more thorough **parametric analysis** to illustrate all aspects of stabilising behaviour.
- Provide plausible **physical explanations** for modulation effects.
- Study parallels between temporal modulation and periodic surface roughness, which is of particular interest to the aerospace industry.

Long-term:

- Study **torsional oscillations** of the disk surface.
- Explore connections with **hydrodynamic voltammetry at rotating and rocking disk electrodes**.

References

[1] THOMAS, C., BASSOM, A., BLENNERHASSETT, P. & DAVIES, C. 2010 The linear stability of oscillatory Poiseuille flow in channels and pipes *Proc. R. Soc.* **467**, 2643-2662.

[2] DAVIES, C. & CARPENTER, P. W. 2001 A novel velocity-vorticity formulation of the Navier-Stokes equations with applications to boundary layer disturbance evolution. *J. Comput. Phys.* **172**, 119-165.

[3] LINGWOOD, R. J. 1995 Absolute instability of the boundary layer on a rotating disk. *J. Fluid Mech.* **299**, 17-33.