June 2006

4. A hyperbola has equation

$$2x^2 - 4x - y^2 - 4y = 4.$$

(a) Find the coordinates of the centre of the hyperbola. [4]

(b) Find the coordinates of the foci and the equations of the directrices. [5]

8. The line $y = m(x - 2)$ intersects the circle $x^2 + y^2 = 1$ at the points A and B.

(a) Show that the coordinates of M, the mid-point of AB, are

$$\left(\frac{2m^2}{1+m^2},\ -\frac{2m}{1+m^2}\right).$$

[5]

(b) Find the Cartesian equation of the locus of M as m varies. [6]

June 2007

5. The ellipse E has equation

$$16x^2 + 25y^2 = 400.$$

(a) Find the coordinates of the foci of E. [4]

(b) Show that the point P with coordinates $(5\cos\theta, 4\sin\theta)$ lies on E. [1]

(c) (i) Show that the equation of the normal to E at P is

$$4y\cos\theta - 5x\sin\theta + 9\sin\theta\cos\theta = 0.$$

(ii) This normal intersects the x-axis at Q and the y-axis at R. Show that the locus of M, the mid-point of QR, is an ellipse. [10]

June 2008

5. *(a)* Show that the equation of the normal to the parabola $y^2 = 4ax$ at the point $P(ap^2, 2ap)$ is
$$y + px = ap(2 + p^2).$$ [4]

(b) This normal meets the x-axis at Q and the mid-point of PQ is R.

(i) Find the coordinates of R.

(ii) The locus of R as p varies is a parabola. Find the equation of this parabola and the coordinates of its focus. [8]

June 2009

6. The ellipse E has equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad .$$

(a) Show that the equation of the tangent to E at the point $(a\cos\theta, b\sin\theta)$ is

$$bx\cos\theta + ay\sin\theta = ab.$$ [5]

(b) This tangent meets the coordinate axes at P and Q, and the mid-point of PQ is R. Find the Cartesian equation of the locus of R as θ varies. [7]

June 2010

8. A parabola has equation

$$x^2 + 8y = 0.$$

(a) Find the coordinates of the focus and the equation of the directrix. [3]

(b) (i) Show that the point $P\ (4p, -2p^2)$ lies on the parabola for all values of p.

(ii) Find the equation of the tangent to the parabola at the point P.

(iii) Given that this tangent passes through the point $(\lambda, 2)$, show that

$$2p^2 - \lambda p - 2 = 0 \ .$$

Hence show that the two tangents to the parabola from any point on the line $y = 2$ are perpendicular. [7]

June 2011

6. The ellipse E has equation

$$2x^2 + 3y^2 - 4x + 12y + 8 = 0.$$

Find

(a) the coordinates of the centre of E, [3]

(b) the eccentricity of E, [4]

(c) the coordinates of the foci of E, [2]

(d) the equations of the directrices of E. [2]

June 2012

7. A parabola has equation

$$y^2 - 2y - 8x + 25 = 0.$$

(a) Find

(i) the coordinates of the vertex,
(ii) the coordinates of the focus,
(iii) the equation of the directrix. [6]

(b) The line $y = mx$ cuts the parabola at the points P_1 and P_2.

(i) Obtain a quadratic equation whose roots are the x-coordinates of P_1 and P_2.
(ii) Hence find the gradients of the two tangents from the origin to the parabola. [7]