Trigonometry, Sets & Functions

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Further Mathematics Support Programme - WJEC A-Level Further Mathematics
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Properties of Real Functions

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- A function can be *neither* even *nor* odd.
- Can you think of any examples?

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- $f(-x) = \frac{-x}{(-x^2)+1} = -\left(\frac{x}{x^2+1}\right) = -f(x)$
- Hence, the function is odd.

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- $f(-x) = e^{-x} + 1$ which is neither $f(x)$ or $-f(x)$.
- Hence, the function is neither even nor odd.

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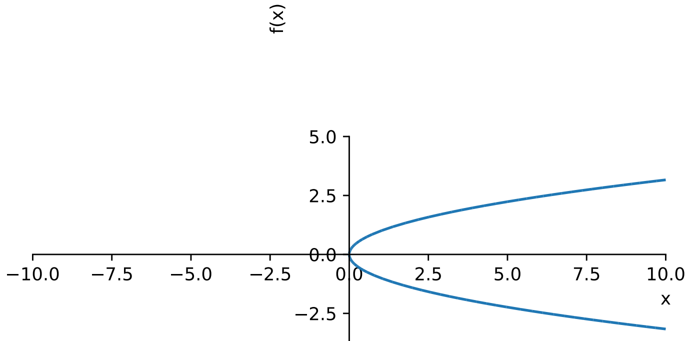
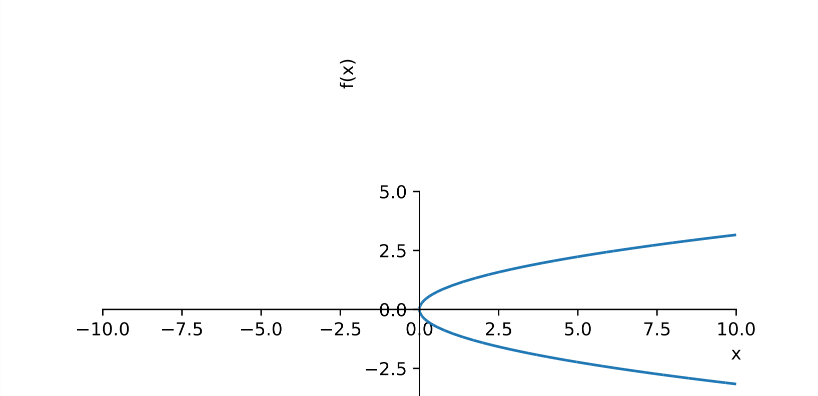
- The function $f(x) = \frac{x+1}{x-1}$ has a discontinuity at $x = 1$.



Continuous & Discontinuous

- A function $f(x)$ is called continuous if the graph of the function consists of a single unbroken line.

- Consider the function $f(x) = \begin{cases} x^2 - 2x + 4, & x > 2 \\ -x^2 + 6x - 7, & x \leq 2 \end{cases}$



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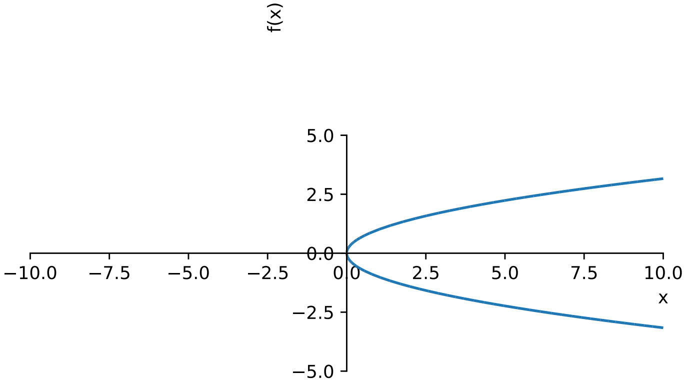

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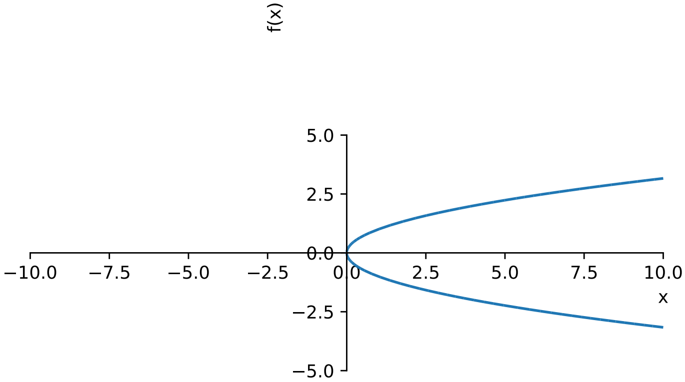

Real Functions - Increasing/Decreasing

- Differentiating a function is often a good way to test these properties.

June 2007 - Q8

- The function f is defined on the domain $(0, 2)$ by

  $$f(x) = \begin{cases} 4x^2, & 0 < x < 1 \\ (x+1)^2, & 1 \leq x < 2 \end{cases}$$

  1. Determine whether or not f is continuous when $x = 1$.
  2. Show that f is a strictly increasing function.
  3. Obtain an expression for $f^{-1}(x)$ on each part of its domain.

June 2011 - Q3

- The piecewise function f is defined by

  $$f(x) = \begin{cases} -x^2 + 6x - 7, & x \leq 2 \\ x^2 - 2x + 4, & x > 2 \end{cases}$$

  1. Determine whether or not f is continuous for all values of x.
  2. Determine whether or not f is a strictly increasing function.
  3. The interval $[1, 3]$ is denoted by A. Determine $f(A)$.

2. Trigonometry

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$$\tan(\theta) = A \implies \theta = \pm p + 2n\pi, \quad \text{for any integer } n$$

Examples

- Find the general solution, in degrees, to each of the following questions:
    1. $\sin(x) = 0.3$
    2. $\tan(x) = 1.5$
    3. $\cos(x) = -0.7$
    4. $\sin(x) = -0.6$

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- Use the *half-angle* formula when you have *only* cos or sin:

$$\sin(a\theta) + \sin(b\theta) = c$$
$$\cos(a\theta) + \cos(b\theta) = c$$

June 2006 - Q5

- By putting $t = \tan\left(\frac{\theta}{2}\right)$, find the general solution of the equation:
    1. $3\cos(\theta) + 4\sin(\theta) = 3 - \tan\left(\frac{\theta}{2}\right)$

June 2009 - Q4

- Find the general solution to the equation:
    1. $\sin(\theta) + \sin(2\theta) + \sin(3\theta) = 0$