

Conic Sections

Scott Morgan

Further Mathematics Support Programme - WJEC A-Level Further Mathematics
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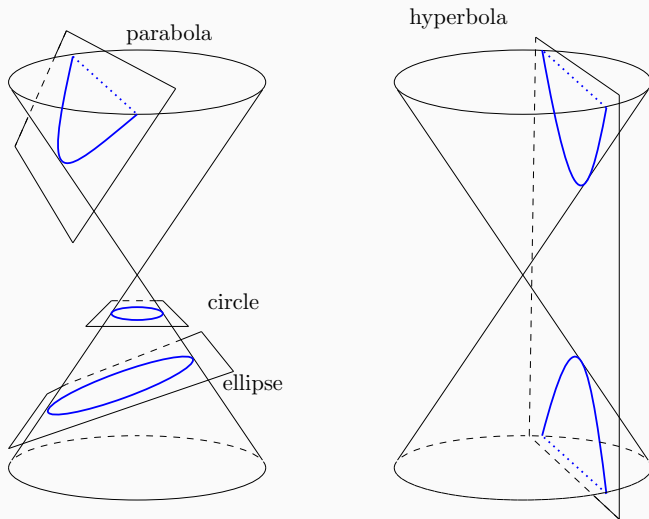
- Parabola

- Parabola
- Hyperbola

Conic Sections

- Parabola
- Hyperbola
- Ellipse (*special case: circle*)

Conic Sections



The reason they're called conic sections...

Key Words

- Eccentricity (usually denoted by e)

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- Focus/Foci

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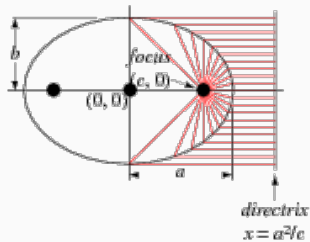
- Eccentricity (usually denoted by e)
- Focus/Foci
- Directrix/Directrices

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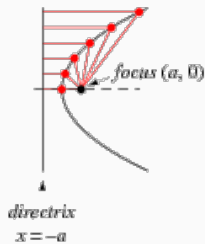
- Eccentricity (usually denoted by e)
- Focus/Foci
- Directrix/Directrices
- Asymptotes

Directrix

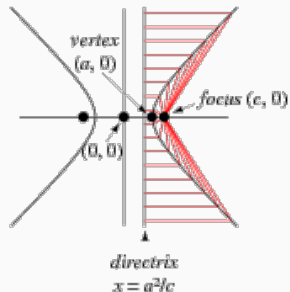
ellipse



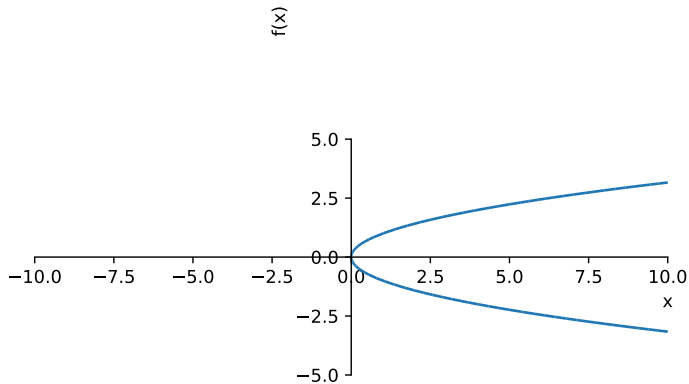
parabola



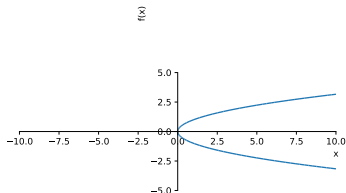
hyperbola



Properties of Conic Sections - Parabola

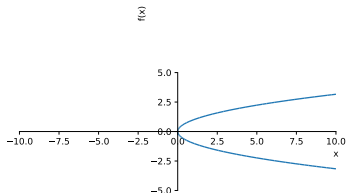


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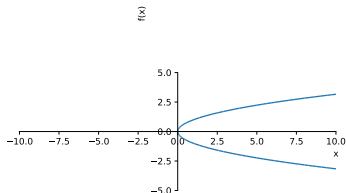
- Equation (centre (Q, R)):
$$(y - R)^2 = 4a(x - Q)$$
- Parametric form:
$$(Q + at^2, R + 2at)$$

Properties of Conic Sections - Parabola



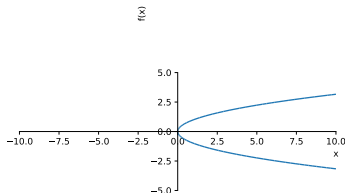
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Properties of Conic Sections - Parabola



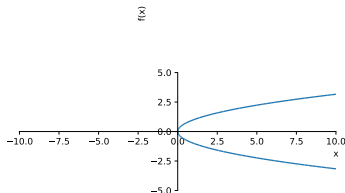
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Properties of Conic Sections - Parabola



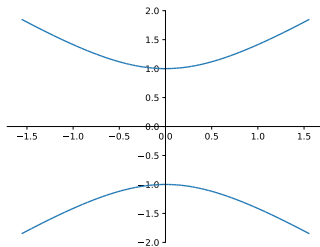
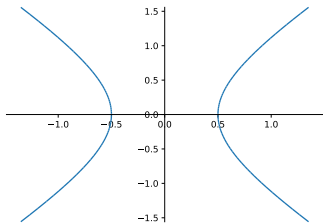
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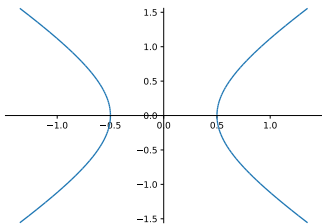


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- Asymptotes: *none*

Properties of Conic Sections - Hyperbola

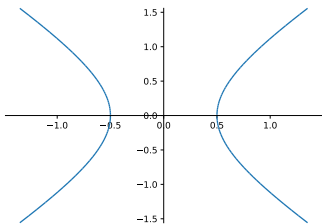


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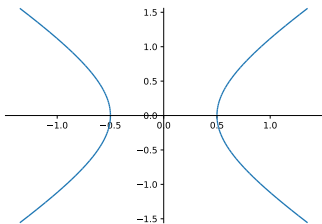
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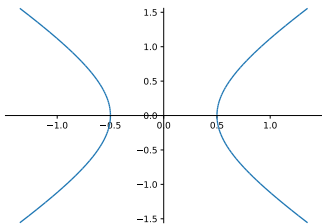
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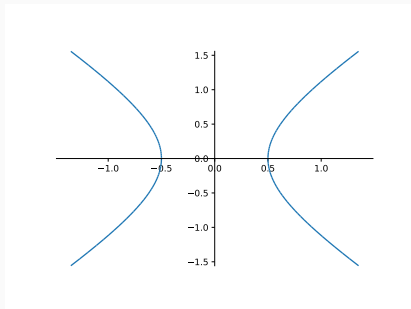
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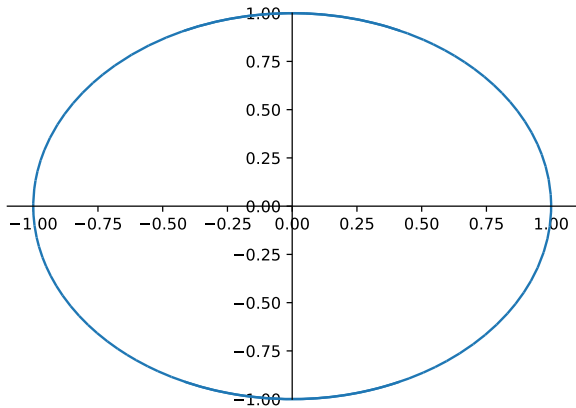
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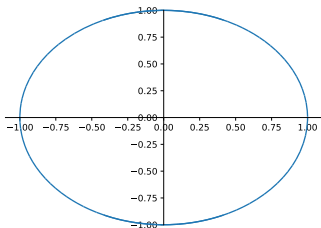


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- Asymptotes: $\frac{(x-Q)}{a} = \pm \frac{(y-R)}{b}$

Properties of Conic Sections - Ellipse

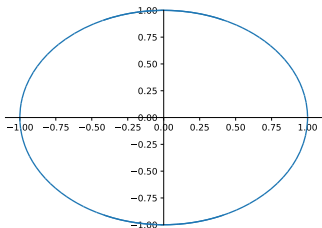


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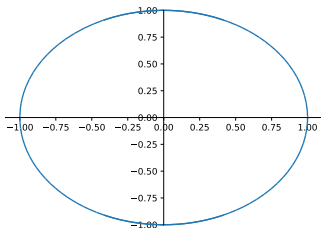
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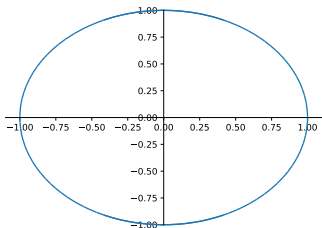
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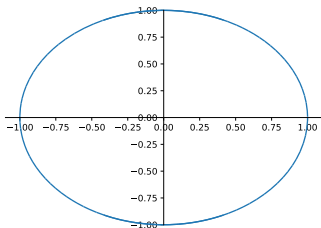
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Tangents and Normals

- The equation of the **tangent** to the curve $F(x)$ at the point (A, B) is

$$y - B = \frac{dF}{dx}(x - A) \quad (1)$$

where the gradient $\frac{df}{dx}$ is evaluated at the point (A, B) .

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- The **tangent** and the **normal** are perpendicular, so the product of their gradients is -1 .
- Thus, the equation of the **normal** to the curve $F(x)$ at the point (A, B) is

$$y - B = k(x - a) \quad (2)$$

where $k = -\frac{1}{\frac{dF}{dx}}$.