

Graphing Functions

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Further Mathematics Support Programme - WJEC A-Level Further Mathematics
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Graph Sketching - Asymptotes

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- Step 5.** Complete the sketch.

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$$f(x) = \frac{x}{(x - 1)^2} \quad (5)$$

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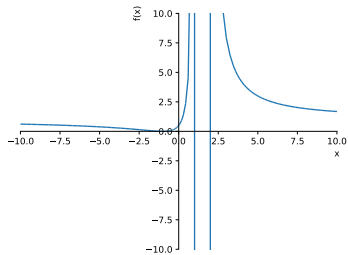
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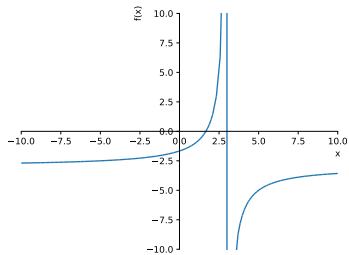
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\Rightarrow Horizontal asymptote at $y = -3$

Graph Sketching - Asymptotes



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Step 4. Consider the behaviour of the graph for numerically large x , giving the equation of any horizontal or oblique asymptotes.

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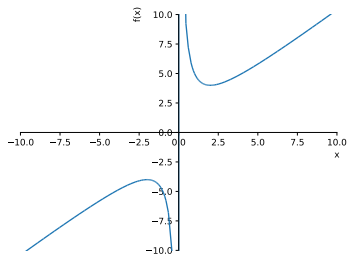
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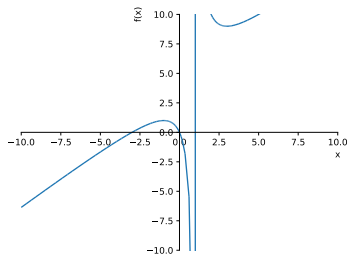
$$f(x) = \frac{x(x+3)}{x-1} = x + 4 + \frac{4}{x-1}$$

\Rightarrow Oblique asymptote is $y = x + 4$

Graph Sketching - Asymptotes



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Things to Remember

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- Horizontal and oblique asymptotes.

Stationary Points

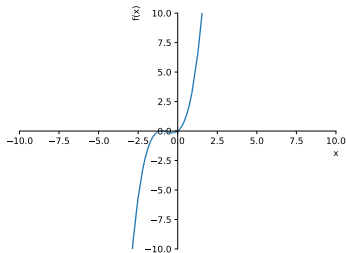
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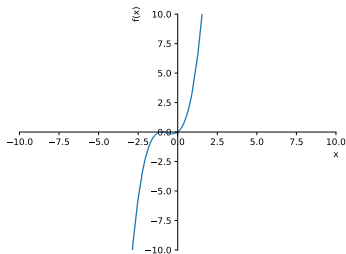
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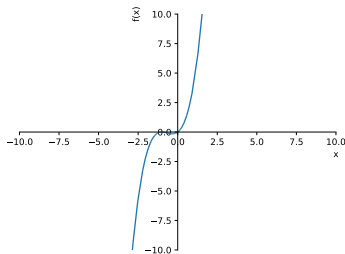


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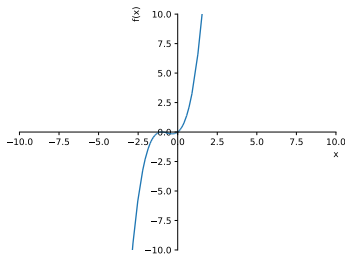
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- If $f''(x) < 0$, the stationary point at x is concave down; a **maximum**.

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$$f(x) = x^3 + 2x^2 + x$$

Second derivative test:

- If $f''(x) < 0$, the stationary point at x is concave down; **a maximum.**
- If $f''(x) > 0$, the stationary point at x is concave up; **a minimum.**

Other Properties of Graphs - Odd & Even

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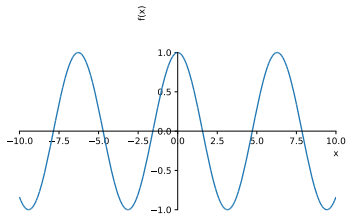
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- An **odd** function is one such that

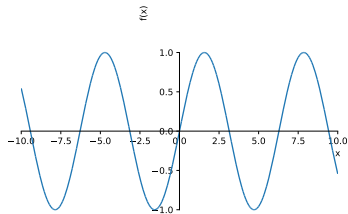
$$f(x) = -f(-x) \quad (7)$$

- Can you think of any examples of odd and even functions?

Other Properties of Graphs - Odd & Even



$$f(x) = \cos(x)$$



$$f(x) = \sin(x)$$

Other Properties of Graphs - Image and Inverse Image

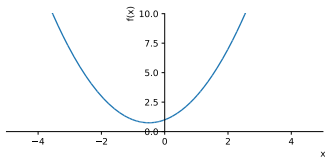
- The **image** of a set under a function is the set of values which that function maps the element of the set to. Consider

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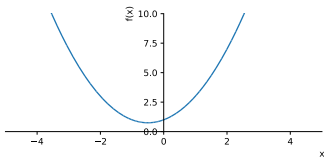


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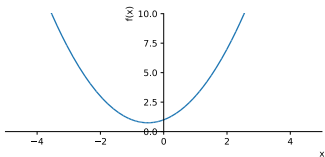
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- The **image** of the set $[-2, 1]$ is the part of the y-axis enclosed by the function in this x-range.
- In this case, $f([-2, 1]) = [\frac{3}{4}, 3]$

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