Conic Sections

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Further Mathematics Support Programme - WJEC A-Level Further Mathematics
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Conic Sections

- Parabola

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- Parabola
- Hyperbola

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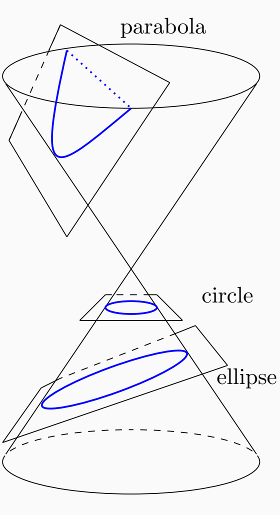
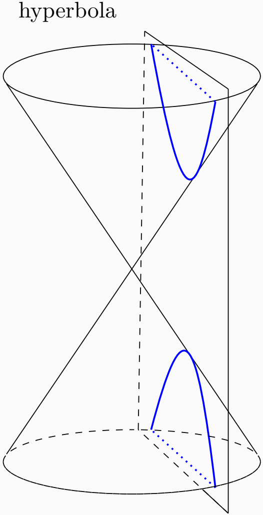
- Hyperbola
- Ellipse (*special case: circle*)

Conic Sections





The reason they're called conic sections...

Key Words

- Eccentricity (usually denoted by e)

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- Focus/Foci

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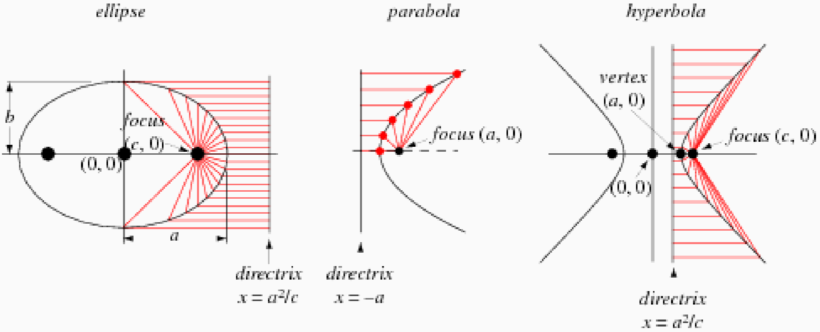
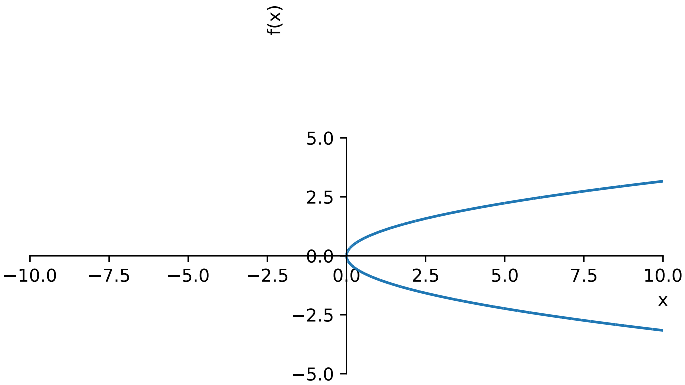
- Focus/Foci
- Directrix/Directrices
- Asymptotes

Directrix



ellipse
parabola
hyperbola
b
focus
(c, 0)
(0, 0)
a
directrix
x = a²/c
focus (a, 0)
directrix
x = −a
vertex
(a, 0)
focus (c, 0)
(0, 0)
directrix
x = a²/c

Properties of Conic Sections - Parabola



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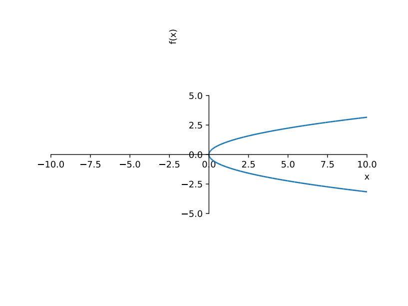
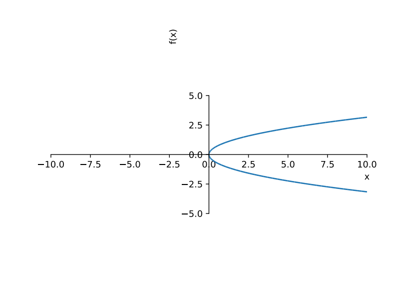


- Equation (centre (Q, R)):
  $(y - R)^2 = 4a(x - Q)$
- Parametric form:
  $(Q + at^2, R + 2at)$

Properties of Conic Sections - Parabola



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- Eccentricity: $e = 1$

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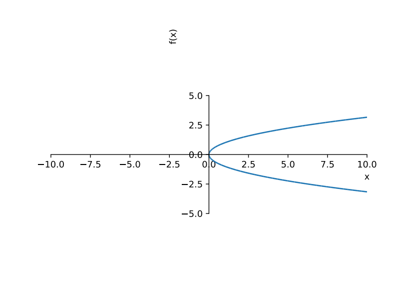

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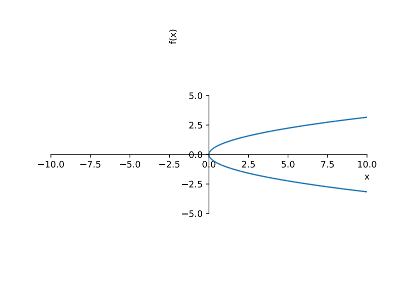

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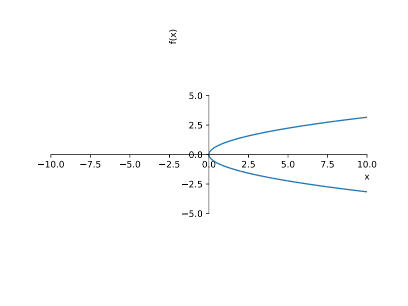

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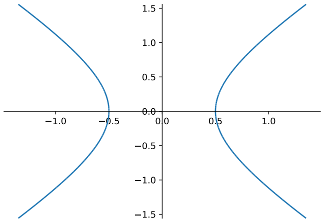
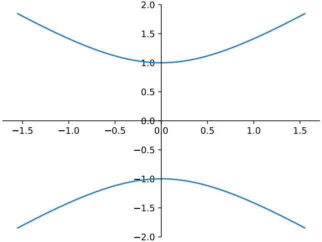
- Asymptotes: *none*

Properties of Conic Sections - Hyperbola





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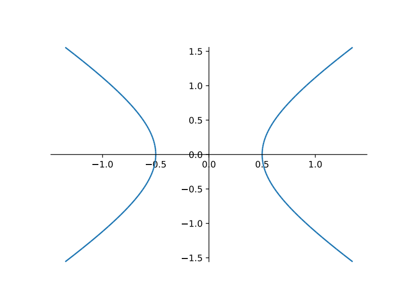

- Equation (centre (Q, R)): $\frac{(x-Q)^2}{a^2} - \frac{(y-R)^2}{b^2} = 1$
- Parametric form: $(Q + a\sec\theta, R + b\tan\theta)$, $(Q + \pm a\cosh\theta, R + b\sinh\theta)$

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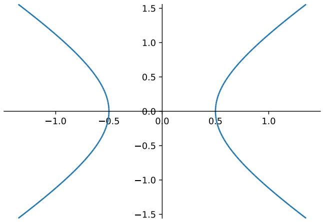

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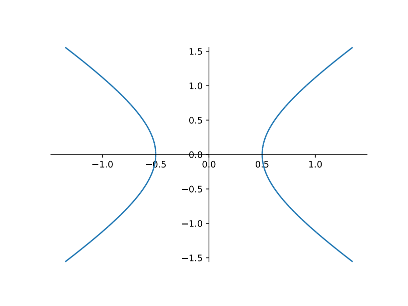

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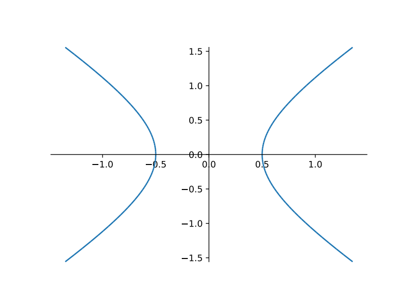

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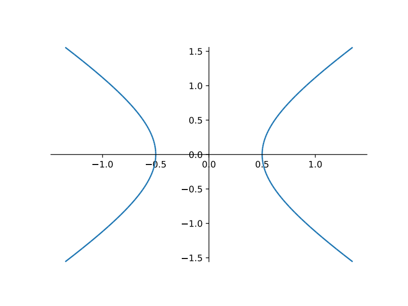

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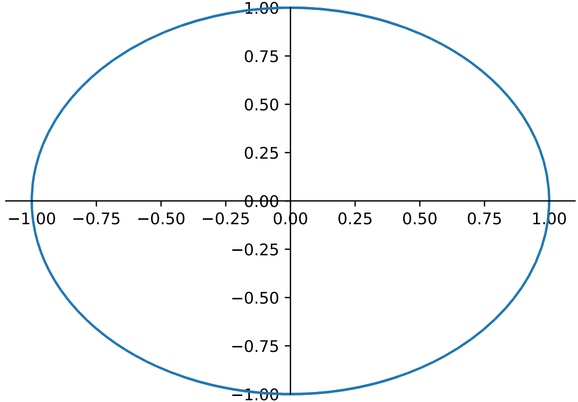
- Asymptotes: $\frac{(x-Q)}{a} = \pm\frac{(y-R)}{b}$

Properties of Conic Sections - Ellipse



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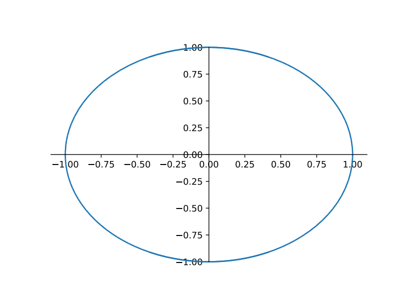

- Equation (centre (Q, R)): $\frac{(x-Q)^2}{a^2} + \frac{(y-R)^2}{b^2} = 1$
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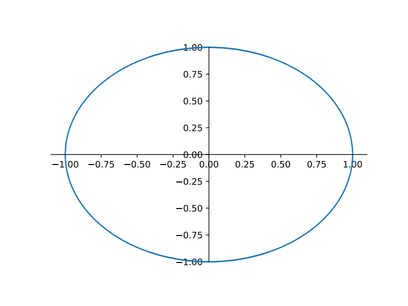

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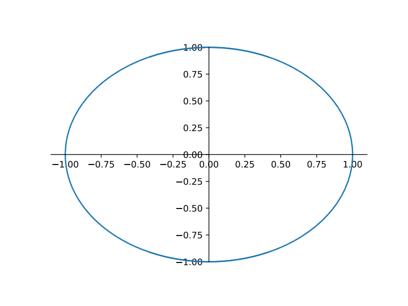

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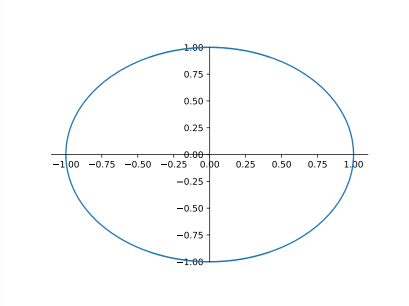

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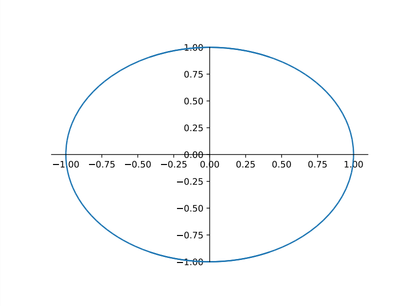

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Tangents and Normals

- The equation of the **tangent** to the curve $F(x)$ at the point (A, B) is

$$y - B = \frac{dF}{dx}(x - A) \tag{1}$$

  where the gradient $\frac{df}{dx}$ is evaluated at the point (A, B).

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- Thus, the equation of the **normal** to the curve $F(x)$ at the point (A, B) is

$$y - B = k(x - a) \tag{2}$$

  where $k = -\frac{1}{\frac{df}{dx}}$.