Retirement Income Analysis
with scenario matrices

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16. Lockbox Annuities

Variable Annuities

Here are some excerpts from the U.S. Securities and Exchange Commission publication *Variable Annuities: What You Should Know*, available online.

> *A variable annuity is a contract between you and an insurance company, under which the insurer agrees to make periodic payments to you, beginning either immediately or at some future date. You purchase a variable annuity contract by making either a single purchase payment or a series of purchase payments.*

> *A variable annuity offers a range of investment options. The value of your investment as a variable annuity owner will vary depending on the performance of the investment options you choose. The investment options for a variable annuity are typically mutual funds that invest in stocks, bonds, money market instruments, or some combination of the three.*

> *... variable annuities let you receive periodic payments for the rest of your life (or the life of your spouse or any other person you designate).*

> *... variable annuities have a death benefit if you die before the insurer has started making payments to you, your beneficiary is guaranteed to receive a specified amount – typically at least the amount of your purchase payments.*

> *...variable annuities are tax-deferred*

As these descriptions suggest, variable annuities can be used for either the accumulation or the decumulation of retirement savings. Our focus is on the latter and we will concentrate on annuities purchased at the present time or accumulated in prior years, with an initial value and payments beginning either immediately (our year 1) or at some time after a deferral period during which no further funds are invested. And, since we choose to leave income tax issues to others, we will consider only the cost of such annuities and the possible payments that may be received, without regard to the tax status of any cash flows.

To return to the SEC publication:

> *At the beginning of the payout phase you may choose to receive … a stream of payments at regular intervals (generally monthly)... Under most annuity contracts, you can choose to have your annuity payments last for a period that you set (such as 20 years) or for an indefinite period (such as your lifetime or the lifetime of you and your spouse or other beneficiary). During the payout phase, your annuity contract may permit you to choose between receiving payments that are fixed in amount or payments that vary based on the performance of mutual fund investment options.*
>
> *The amount of each periodic payment will depend, in part, on the time period that you select for receiving payments. Be aware that some annuities do not allow you to withdraw money from your account once you have started receiving regular annuity payments.*

Here we will focus on contracts in which payments last for one or more lifetimes with the amount received varying based on the performance of some sort of investments, generally packaged in the form of one or more mutual funds. Moreover, we will generally assume that withdrawals are limited to those determined by the terms of the contract.

Not surprisingly, there are costs for such services. Quoting from the SEC publication, these can include:

__Mortality and expense risk charge__ – This charge is equal to a certain percentage of your account value, typically in the range of 1.25% per year. This charge compensates the insurance company for insurance risks it assumes under the annuity contract. Profit from the mortality and expense risk charge is sometimes used to pay the insurer's costs of selling the variable annuity, such as a commission paid to your financial professional for selling the variable annuity to you.

__Administrative fees__– The insurer may deduct charges to cover record-keeping and other administrative expenses. This may be charged as a flat account maintenance fee (perhaps $25 or $30 per year) or as a percentage of your account value (typically in the range of 0.15% per year).

__Fees and Charges for Other Features__– Special features offered by some variable annuities, such as … a guaranteed minimum income benefit, …often carry additional fees and charges.

Variable annuities typically combine aspects of investment and insurance. Again, from the SEC publication:

*A common feature of variable annuities is the **death benefit**. If you die, a person you select as a beneficiary (such as your spouse or child) will receive the greater of: (i) all the money in your account, or (ii) some guaranteed minimum (such as all purchase payments minus prior withdrawals)....*

Some variable annuities allow you to choose a "stepped-up" death benefit. Under this feature, your guaranteed minimum death benefit may be based on a greater amount than purchase payments minus withdrawals. For example, the guaranteed minimum might be your account value as of a specified date, which may be greater than purchase payments minus withdrawals if the underlying investment options have performed well. The purpose of a stepped-up death benefit is to "lock in" your investment performance and prevent a later decline in the value of your account from eroding the amount that you expect to leave to your heirs. This feature carries a charge, however, which will reduce your account value.

Variable annuities sometimes offer other optional features, which also have extra charges. One common feature, the guaranteed minimum income benefit, guarantees a particular minimum level of annuity payments, even if you do not have enough money in your account (perhaps because of investment losses) to support that level of payments. Other features may include long-term care insurance, which pays for home health care or nursing home care if you become seriously ill.

You may want to consider the financial strength of the insurance company that sponsors any variable annuity you are considering buying. This can affect the company's ability to pay any benefits that are greater than the value of your account in mutual fund investment options, such as a death benefit, guaranteed minimum income benefit, long-term care benefit, or amounts you have allocated to a fixed account investment option.

This chapter will describe and analyze aspects of one possible type of variable annuity, combining lockbox investment with mortality (longevity) insurance. Later chapters will discuss alternative strategies for investing and spending accumulated retirement savings, both with and without the benefit of mortality and/or investment insurance. Warning: there is no single approach that is obviously superior for everyone. That is why we have developed analytic methods that can help inform retirees' choices.

Lockbox Annuities

Here we focus on a product not available when this was written. That said, it would seem that there should be no major impediment to its introduction, although nothing is simple in the world of investment and insurance legislation and regulation. Our idea is to combine a mutual fund built using lockboxes with an insurance policy that guarantees the receipt of the values in such lockboxes if and only if one or more insured individuals is/are alive. The insurance company thus bears mortality (longevity) risk but no investment risk. Although we could include the possibility of creating lockboxes that can provide payments after the insured persons' death, our example will not include them.

While such a product could be based on a mutual fund with lockboxes containing m-shares, we will deal simpler cases in which each lockbox contains only TIPS and/or shares in the market portfolio of tradable world bonds and stocks. It should be a relatively simple matter for an insurance company, possibly in concert with a mutual fund company, to construct and offer such a product.

In effect, such an approach would have multiple lockboxes for each year. To take a simple example, assume that a lockbox variable annuity is to cover one person's life and that 1,000 such policies are issued to people of the same age and sex. There would be 1,000 lockboxes for the first year, when all are alive. Then, if mortality tables indicate that 1% of the insured are likely to die in the first year, there would be 990 lockboxes for year 2. Similarly, there might be 970 lockboxes for year 3, and so on.

The author's graphic abilities do not extend to the creation of a diagram with thousands of lockboxes. Here, instead is a crude illustration for a case in which four people are insured, with one expected to expire in each year.

Year 1	Year 2	Year 3	Year 4
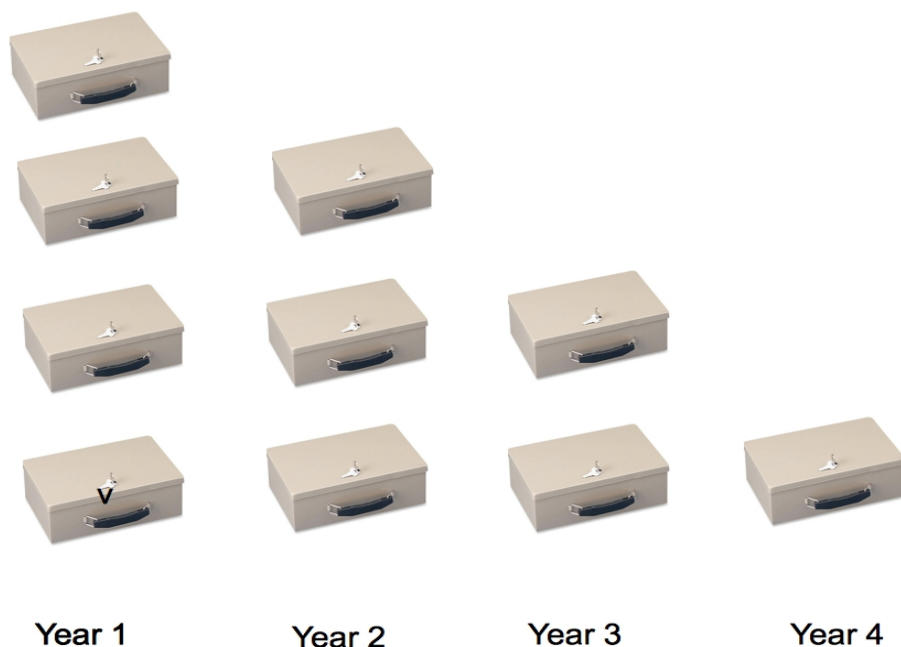
The key point is that each of the lockboxes with a given maturity date contains the same mix of TIPS and the market portfolio, although the contents of boxes for different maturity years will typically differ. At the beginning of each year, all of the boxes with the current maturity date are opened. If the number of recipients alive equals the expected value, each will receive the contents of a box and all is well. But if there are more insured people than expected, the insurance company will have to pay some of them from other funds. And, if there are fewer than expected, the insurer will have some money left over.

Here, as with other types of annuities, the insurance company is subject to what we have called *actuarial table risk.* This may be borne by equity owners in a public company, by others holding annuity or insurance policies in a mutual company, or by the people holding the policies in a tontine. But if a policy is to be free of any default risk, there will be a cost.

These preliminaries concluded, we turn to the construction of a program to construct such lockbox annuities.

Lockbox Annuity Functions

As is our custom, we will develop two functions: in this case, an *iLBAnnuity_create* function to create an iLBAnnuity data structure, and *iLBAnnuity_process* function to process the data structure then add the resulting incomes to the client incomes matrix.

To keep matters simple, we require that a matrix with the contents of the required lockboxes be provided as an input for the *iLBAnnuity_process* function. In our example we will use the matrix in a *combinedLockboxes.proportions* data element, produced by combining equal parts of lockboxes designed to provide approximately similar income distributions from year 2 onward (AMD2) with lockboxes designed to be optimal for the same marginal utility (CMU). But one could construct a matrix in some other manner, as long as it (1) contains relative proportions for TIPS in the first row, and for the market portfolio in the second row and (2) has as many columns as there are years in the *client.incomesM* matrix. There are many possibilities.

For our example, we add the following statements from the previous chapter to the *SmithCase* program:

```
% create and process AMDnLockboxes
  AMDnLockboxes = AMDnLockboxes_create( );
  AMDnLockboxes.showProportions = 'n';
  AMDnLockboxes = AMDnLockboxes_process(AMDnLockboxes, market, client );

% create and process CMU Lockboxes
  CMULockboxes = CMULockboxes_create;
  CMULockboxes.showProportions = 'n';
  CMULockboxes.initialMarketProportion = 0.5;
  CMULockboxes = CMULockboxes_process(CMULockboxes, market, client);

% combine lockboxes with equal weights
  combinedLockboxes = combinedLockboxes_create( );
  combinedLockboxes.componentLockboxes = {AMDnLockboxes CMULockboxes};
  combinedLockboxes.componentWeights = [0.5 0.5 ];
  combinedLockboxes.title = 'Lockbox Proportions for 0.5*AMD2 + 0.5*CMU ';
  combinedLockboxes.showCombinedProportions = 'y';
  combinedLockboxes =combinedLockboxes_process( combinedLockboxes, client );
```

We now have a set of lockbox proportions (*combinedLockboxes.proportions*) for our annuity.

Creating a Lockbox Annuity Data Structure

Now to the details. First, the function to create a lockbox annuity data structure:

```
function iLBAnnuity = iLBAnnuity_create( );
    % create a Lockbox Annuity data structure
    % uses only TIPS and market holdings

    % relative payments from lockboxes (2* client number of years)
        %   row 1: tips
        %   row 2: market portfolio
        iLBAnnuity.proportions = [ ];
    % first income year
        iLBAnnuity.firstIncomeYear = 1;
    % relative incomes in first post-guarantee year for personal states 1, 2, 3 and 4
        iLBAnnuity.pStateRelativeIncomes = [ 0.5 0.5 1.0  0 ];
    % graduation ratio of each real income distribution relative to the prior
    %   distribution
        iLBAnnuity.graduationRatio = 1.00;
    % retention ratio for investment returns for tips and market portfolio
    %   = 1 - expense ratio
    %   e.g. expense ratio = 0.10% per year,retentionRatio = 0.999
        iLBAnnuity.retentionRatios = [ 0.999  0.999 ];
    % ratio of value invested in lockboxes to initial cost
        iLBAnnuity.valueOverCost = 0.90;
    % cost
        iLBAnnuity.cost = 100000;

    end
```

The first data element, *iLBAnnuity.proportions*, is to be filled with the proportions in TIPS and the market portfolio for the chosen lockboxes. The second element, *ILBAnnuity.firstIncomeYear*, allows for deferring the income payments. If this element is set to the default value of 1, payments begin immediately (at the beginning of year 1). If a later year is specified, there will be no income payments until the indicated year and thereafter.

The next four data elements are similar (but not identical) to those used for the *iFixedAnnuity* data structures. The vector *ILBAnnuity.pStateIncomes*, indicates the relative magnitudes of desired incomes for each personal state from 1 to 4. The last value in the vector should be zero unless an inheritance payment is desired. The next data element, *iLBAnnuity.graduationRatio*, provides for the graduation of real incomes from year to year. For example, a value 0.99 indicates a desire to have the distributions of income decrease by 1% each year.

Since this type of annuity includes aspects of mortality insurance and investment, two types of expenses need to be considered. The first concerns the fees charged each year based on the values of the investments. For most mutual funds and ETFs these are deducted monthly, with the amount determined by taking a specified percentage of the month-end assets. Typically, the *expense ratio* for such a fund is specified as a percentage of average net assets. Thus an expense ratio of 1.00% per year indicates that each year roughly 1% of the assets will be removed and transferred to the fund manager. (In the finance industry, this is sometimes termed *100 basis point (bips),* with one basis point equal to 1/100'th of one percent). Since we use annual values, the expense ratio is applied to the year-end value of an asset holding, then deducted from it.

We choose to express the impact of expenses by indicating the proportion of year-end fund value that will be retained by the owner. Thus if the expense ratio is 1%, the *retention ratio* is 0.99, indicating that each year the owner retains 99% of the value of the fund. Some think that a 1% expense ratio is relatively harmless, but this is not so. I analyzed the effects of expenses in detail in an article in the March/April 2013 issue of the *Financial Analysts Journal,* titled "The Arithmetic of Investment Expenses" (available online at *www.cfapubs.org*). Here is a simple example that makes the point.

Assume an investment produces a total return (ending value / beginning value) of R_t in year t. Let the retention ratio be r. Then at the end of *n* years, the ending value net of expenses will be:

$$r R_1 r R_2 ... r R_n$$

or

$$r^n (R_1 R_2 ... R_n)$$

The parenthesized expression is the cumulative (gross) return for *n* years. Thus the ratio of the ending value with expenses to the amount that would have been obtained without expenses is r^n . An investment with an expense ratio of 1.0% held for 20 years will provide an ending value of 0.99^{20} or 0.8179 times the amount that would have been obtained without expenses. But an investment with an expense ratio of 0.10% would provide an ending value of 0.999^{20} or 0.9802 times the cumulative before-expense return. Assuming similar gross returns, the canny index investor could have almost 20% more money to spend after 20 years of net returns.

Our variable, *iLBAnnuity.retentionRatios*, should have a vector of two retention ratios, the first for investments in TIPS, and the second for investments in the market portfolio.

The SEC publication quoted earlier did not discuss expenses that might be charged by an investment manager, whether an independent mutual fund or some division of the insurance company providing the annuity. But it is important to include such expenses when evaluating any variable annuity, hence our retention ratios.

The next two elements, *iLBAnnuity.valueOverCost,* and *iLBAnnuity.cost,* are similar to the corresponding elements for a fixed annuity. For example, if the *valueOverCost* is 0.90 and the *Cost* is $100,000, $90,000 will be invested in the lockboxes, with the relative lockbox amounts scaled so that the present value of the incomes received and fees paid equals the product of the two elements.

Processing a Lockbox Annuity Data Structure

It is not a simple matter to do the calculations required to cover all possible combinations of the elements in our lockbox annuity data structure. We will take, in turn, each of several sections of the function that does so. Those reluctant to follow the details of programs may still wish to skim some of the descriptions to get a sense of the economic assumptions employed.

Here is the function heading, some comments and the first set of statements:

```
function client = iLBAnnuity_process( iLBAnnuity, client, market );
  % creates LB annuity income matrix and fees matrix
  % then adds values to client incomes matrix and fees matrices

  % the lockbox proportions matrix can be computed by AMDnLockboxes_process
  %   or in some other manner. The first row is TIPS proportions, the second is Market
  %   proportions, and there is a column for each year in the client matrix

  % get number of scenarios and years
    [nscen nyrs] = size( client.pStatesM );
  % set initial lockbox proportions
    proportions = iLBAnnuity.proportions;
  % reset proportions to adjust for graduation and retention ratios
    gr =  iLBAnnuity.graduationRatio;
    rrs = iLBAnnuity.retentionRatios;
    for row = 1:2
      factors = ( gr/rrs(row) ) .^ ( 0: nyrs-1 );
      proportions( row, : ) = factors .* proportions( row, : );
  end;
```

We begin by setting the number of rows (scenarios) and columns (years) in the matrices with which we will be working. Next we set a local variable to the matrix of proportions provided in the lockbox annuity data structure. We need to adjust these to take into account both the desired graduation ratio and the retention ratios. The next set of statements do this separately for the TIPS relative amounts (in the first row of the proportions matrix) and the market portfolio relative amounts (in the second row). Each of the initial proportions is multiplied by a factor that accounts for the cumulative effect of the graduation ratio and the expense ratio. If, for example, if the graduation ratio were 1.0 and the retention ratio 0.99, we would increase each proportion by an amount equal to 1/.99 each year in order to obtain approximately the desired distribution of incomes each year. If instead the graduation ratio were 1.02, we would increase each proportion by an amount equal to 1.02/0.99 to the obtain the desired growth in returns net of expenses.

The next task is to accommodate deferred annuities by making the lockboxes empty for the years (if any) before payments are to begin:

```
% set lockbox proportions to zero for any excluded years
  firstyear = iLBAnnuity.firstIncomeYear;
  if  firstyear > 1
    proportions( :, 1:firstyear-1 ) = zeros( 2, firstyear-1 );
  end;
```

Since at this stage the proportions are all relative, there is no need to change any of the magnitudes of the entries for a post-deferral period.

The next section creates matrices of cumulative returns net of expenses – one for TIPS, the other for the market portfolio. First we compute returns net of expenses, with each entry equal to the net return for a year. Then we compute matrices of cumulative net returns as of the beginning of each year:

```
% create matrices of returns net of expenses
  NrfsM =  iLBAnnuity.retentionRatios(1)*market.rfsM;
  NrmsM = iLBAnnuity.retentionRatios(2)*market.rmsM;
% create matrices of cumulative returns net of expenses
  m = cumprod( NrfsM, 2 );
  cumNrfsM = [ ones( nscen, 1 )  m( :, 1:nyrs-1 ) ];
  m = cumprod( NrmsM, 2 );
  cumNrmsM = [ ones( nscen, 1 )  m( :, 1:nyrs-1 ) ];
```

The stage is now set for creating a matrix of relative fees associated with investment expenses:

```
% create matrices with proportions in market and rf in each row
   xfm =   ones( nscen, 1 ) * proportions( 1, : );
   xmm = ones( nscen, 1 ) * proportions( 2, : );
 % compute net incomes for lockbox relative proportions
   boxIncsM  =  xfm.*cumNrfsM   + xmm.*cumNrmsM;
 % compute  incomes if there were no expenses
   gboxIncsM = xfm.*market.cumRfsM  + xmm.*market.cumRmsM;
 % set fees to differences
   feesM = gboxIncsM - boxIncsM;
```

The first two statements compute matrices with the relative proportions invested in each of the two assets each year in every row, reflecting the fact that the proportions are the same for every scenario. It is then straightforward to create one matrix of the same size with the net incomes after expenses and another matrix with the gross incomes if there were no expenses. The desired matrix of fees is then derived by simply taking the differences in the entries in these two matrices.

It is important to understand the economics behind this computation. In a sense, we compute the fee for a given year in a specific scenario as the difference between the income that would have been obtained if there had been no expenses (with a retention ratio of 1.0) to that obtained after expenses (using the actual retention ratio). This is equivalent to assuming that for each asset, the fee charged in a given year is reinvested by the insurance company in the asset in the lockbox, with the total return for each subsequent year retained by the company. In the year that the lockbox is opened, the cumulative values of all the fees so obtained are then taken by the insurance company; it is this amount that we compute and add to the fees matrix.

The next task is to create the relative incomes and fees for each personal state. This is fairly straightforward, although a bit tedious. We begin by making certain that the maximum value in the vector of relative incomes for personal states 1 through 4 is 1.0 (although probably not really needed, it feels like a good thing to do). We then set up two matrices with zero values – one for relative incomes, the other for relative fees. The remaining statements do the calculations for each of the personal states. For each state, we find the relative income *relInc*, and compute matrix *psmat* with 1.0 in each scenario/year cell in which the personal state is germane and a zero in every other cell. By multiplying every cell in this matrix by the corresponding cell in the *boxIncsM,* we obtain a matrix with incomes for only the scenario/year combinations for the personal state in question. This matrix, multiplied by the relative income for the personal state provides matrix *psIncsM* with relative incomes for that personal state. We then add the entries in this matrix to those in the *relIncsM* which will include incomes for all personal states. The next two statements repeat the procedure for the fees matrices.

```
% set up relative incomes matrix and relative fees matrix
    psRelIncs = iLBAnnuity.pStateRelativeIncomes;
    psRelIncs = psRelIncs / max( psRelIncs );
    relIncsM = zeros( nscen, nyrs );
    relFeesM = zeros( nscen, nyrs );
    for ps = 1:4
      relInc =  psRelIncs( ps );
      psmat =   ( client.pStatesM == ps );
      psIncsM = relInc * ( psmat .* boxIncsM );
      relIncsM = relIncsM + psIncsM;
      psFeesM = relInc * ( psmat .* feesM );
      relFeesM = relFeesM + psFeesM;
    end; % for ps = 1:4
```

Thus far all the computations have been based on the relative amounts of TIPS and the market portfolio invested in each of the lockboxes. It is now time to convert them to dollar amounts. The computations are relatively simple:

```
% convert relative incomes to dollar incomes
    pvbase = sum( sum(  ( relIncsM + relFeesM ) .* market.pvsM ) );
    totval = iLBAnnuity.cost * iLBAnnuity.valueOverCost;
    incsM =  relIncsM * ( totval / pvbase );
    feesM =  relFeesM * ( totval / pvbase );
```

First we compute the present value of all the cash flows, for both incomes and fees. Then we compute the total value that can be funded by multiplying the ratio of value over cost by the cost of the annuity (both set when the ILBAnnuity data structure was created). We then multiply each entry in the incomes matrix and each entry in the fees matrix by this ratio. Voila! The present value will equal the desired amount.

The remaining statements update the client incomes and fees matrices. The computed incomes are added to the former, and the computed investment expenses fees and the insurance fee to the latter:

```
% add incomes and fees to client incomes and fees matrices
    client.incomesM = client.incomesM + incsM;
    client.feesM     = client.feesM     + feesM;

  % add insurance fee to fee matrix
    insFee = iLBAnnuity.cost * ( 1 – iLBAnnuity.valueOverCost );
    client.feesM( :, 1 ) = client.feesM( :, 1 ) + insFee;

end
```

All the work completed, we end the *iLBAnnuity_process* function.

Lockboxes and Fixed Annuities

While lockbox annuities using the *AMDnLockboxes.proportions* did not exist in the real world when this was written, there are annuities with some lockbox characteristics. Recall that *iLBAnnuity.proportions* is a matrix with two rows and as many columns as there are years covered by an annuity. The top row indicates the relative amounts invested in TIPS, the bottom row the relative amounts invested in the market portfolio. Now, assume that the proportions matrix has zero values in every entry in the bottom row, so that only TIPS are to be in the lockboxes. When *iLBAnnuity.process* is executed, the result will be a fixed real annuity. And, as discussed in Chapter 10, such annuities do exist.

To take one example, imagine that the goal is to produce a fixed annuity with the same real income each year for each relevant personal state. If the projected real value-relative for TIPS (*market.rf)* is, say, 1.01, then the entries in the first row of the proportions matrix could be set to:

$$1 \quad \frac{1}{1.01} \quad \frac{1}{1.01^2} \quad \frac{1}{1.01^3}, \dots$$

with all the entries in the second row (market proportions) equal to 0. And, of course, we could include a graduation ratio, different relative incomes for different personal states, and a cost to compensate the insurance company for administrative costs and for bearing actuarial table risk.

Our *iFixedAnnuity* function did not provide for a retention ratio for the returns from TIPS since typical fixed annuities do not account separately for investment expenses (and outside investment firms are rarely utilized). To replicate such an approach, the retention ratios for the corresponding lockbox annuity would be set to 1.0.

So at least a subset of possible Lockbox Annuities does exist.

Lockbox Annuity Expenses

We have chosen to break expenses for lockbox annuities into two types: (1) those proportional to amounts invested each year and (2) all other costs, expressed as a proportion of the amount charged when the insurance is first sold. In cases in which an outside firm provides investment funds, the first type of expenses are likely to be explicit. When a variable annuity provider manages funds internally, there may be a separate charge or such expenses may be covered by providing payments with a present value sufficiently below that of the likely payments to beneficiaries. In some cases it is difficult to parse the descriptions in a contract to determine exactly what overall costs may be.

Importantly, our parameter for *valueOverCost* (sometimes termed "money's worth") may not be stated in any explicit manner. Expenses associated with the sale of an annuity, such as commissions to salespeople, may be disclosed but the present value of the amounts expected to cover ongoing operating costs, expected profits (where applicable) and a reserve for actuarial table risk will almost certainly not be stated in any available document.

What can be said is that the component of the reserve for actuarial table risk should be greater, the smaller is the estimated possible shortfall of actual mortality from that estimated in the tables used to price the annuity. Thus for the early years of annuity payments, there is relatively little risk that significantly more policy holders than predicted will live. If the policy is priced on the assumption that 99% will be alive in year 1, the worst that can happen for the insurance provider is that 100% survive. And if fewer than 99% are alive, the insurance company or mutual company policy holders will actually be better off. The implication is that, other things equal, the ratio of value over cost should be higher for an immediate annuity than for a deferred annuity for the same client or clients. But the flexibility provided by retaining assets that can be spent in the event of unpredictable needs provides an argument for deferral of annuity payments. We will expand on this idea in subsequent chapters. Suffice it to say here that an annuity's value over cost is both extremely relevant and very difficult to estimate. But estimate it we must.

Lockbox Annuities plus Social Security

As shown in Chapter 10, in both the United States and many other countries, retirees have more than one source of income. In the U.S., employees of most state and local governments and agencies have defined benefit and/or defined contribution pension plans, and most of those employed in the private sector can receive benefits from the Social Security system. Moreover, most of these sources of income are intended to provide benefits similar to those of a fixed real annuity. It is thus important to evaluate any other source of retirement income in context. We illustrate with our favorite retirees, Bob and Sue Smith.

Recall that Bob and Sue's Social Security benefits had a present value slightly greater than $955,000. For the majority of those reaching retirement age, Social Security is the most valuable asset, followed by home equity, then investable retirement savings. Bob and Sue have equity in their home but plan to remain in it for as long as possible, using some or all of the equity, if needed, to pay for long-term care or other medical costs not covered by insurance. But they have accumulated a million dollars that can be used to purchase annuities and/or invested in some other manner. The rest of the chapters in this book will consider different ways to use this money to create retirement income in addition to that provided by Social Security. Here we will examine the possibility of using it all to purchase an immediate lockbox annuity with a strategy designed to obtain income distributions in each year after year 2 similar to that obtained by holding the market portfolio in the first year.

As usual, we create a program to do the job. Statements in the first part should be familiar:

```matlab
% Smith Case_Chapter 16.m

% clear all previous variables and close any figures
  clear all;
  close all;

% create a new client data structure
  client = client_create(  );
% process the client data structure
  client = client_process( client );

% create a new market data structure
  market = market_create( );
% process the client data structure
  market = market_process( market, client );


% create social security accounts
  iSocialSecurity = iSocialSecurity_create(  );
  iSocialSecurity.state1Incomes = [ Inf    30000 ];
  iSocialSecurity.state2Incomes = [ Inf    30000 ];
  iSocialSecurity.state3Incomes = [ 44000 ];
% process social security accounts
  client = iSocialSecurity_process( iSocialSecurity, client, market );

% create AMDn lockboxes
  AMDnLockboxes = AMDnLockboxes_create(  );
  AMDnLockboxes.cumRmDistributionYear = 2;
  AMDnLockboxes.showProportions = 'y';
  AMDnLockboxes = AMDnLockboxes_process( AMDnLockboxes,  market,  client );

% create iLBAnnuity account
  iLBAnnuity = iLBAnnuity_create( );
% set iLBAnnuity cost
  iLBAnnuity.cost = 1000000;
% set annuity lockbox proportions to AMDn proportions
  iLBAnnuity.proportions = AMDnLockboxes.proportions;
% process LBAnnuity
  client = iLBAnnuity_process( iLBAnnuity, client, market );
```

The remainder of the program creates an *analysis* data structure, sets its elements as needed, and then processes it to produce the desired graphs. Here we choose to create the majority of the possible outputs, but omit the *efficient incomes* graphs, since we know what they would show and they can take a substantial amount of time to create and display. The statements follow:

```
% create analysis
  analysis = analysis_create( );

% reset analysis parameters
  analysis.animationDelays = [ 0.5 .5 ];
  analysis.animationShadowShade = .2;

  analysis.figuresCloseWhenDone = 'n';
  analysis.stackFigures = 'n';
  analysis.figureDelay = 0;

  analysis.plotIncomeDistributions = 'y';
  analysis.plotIncomeDistributionsTypes = { 'rc' };
  analysis.plotIncomeDistributionsStates = { [3]  [1 2] };
  analysis.plotIncomeDistributionsProportionShown = 0.999;


  analysis.plotYOYIncomes = 'y';
  analysis.plotYOYIncomesTypes = { 'r' };
  analysis.plotYOYIncomesStates = { [3] [1 2] };

  analysis.plotScenarios = 'y';
  analysis.plotScenariosTypes = { 'ri' };
  analysis.plotScenariosNumber = 20;

  analysis.plotRecipientPVs = 'y';

  analysis.plotIncomeMaps = 'y';
  analysis.plotIncomeMapsTypes = { 'r' };
  analysis.plotIncomeMapsStates = { [3] [1 2] };

  analysis.plotPPCSandIncomes = 'y';
  analysis.plotPPCSandIncomesSemilog = 'n';
  analysis.plotPPCSandIncomesStates = { [3] [1 2] };

  analysis.plotYearlyPVs = 'y';
  analysis.plotYearlyPVsStates = { [3] [1 2] };

% produce analysis
  analysis_process(analysis, client, market);
```

Note that we choose to show only the lowest 99.9% of the incomes when plotting distributions. This provides a maximum value for the x-axis that will show the vast majority of points while avoiding compressing the plots in order to show a few extremely large incomes.

As a practical matter, it might be desirable to have an alternative version of the *analysis_create* function that contains these settings plus others from the original version that need not be changed. If, for example, this were called *analysisVersion1_create( ),* the SmithCase program could simply include the statements:

```
% create analysis
   analysis = analysisVersion1_create( );
% produce analysis
   analysis_process(analysis, client, market);
```

One could envision having a set of such versions (of which this is number 1) of *analysis_create,* making it possible to simply choose the desired one for a particular application.

Here is a video with the results obtained by executing the above SmithCase prog
ram:

www.stanford.edu/~wfsharpe/RISMAT/SmithCase_Chapter16.mp4

There is no substitute for watching the video, but we will provide excerpts with some comments. For animated figures in which each year's results are plotted, we arbitrarily freeze the animations after 30 years have been shown.
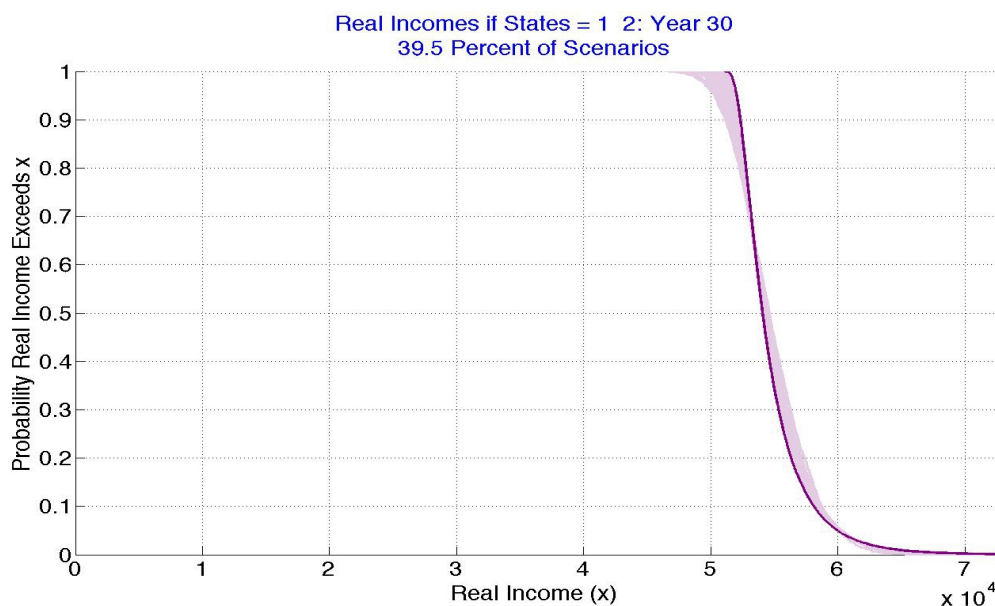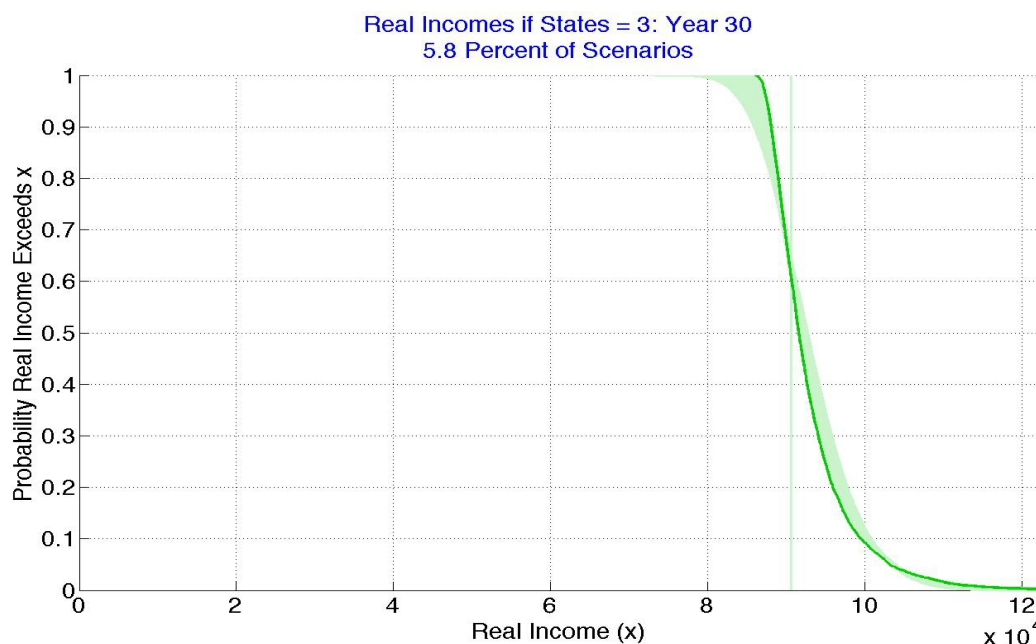
First, a few scenarios.
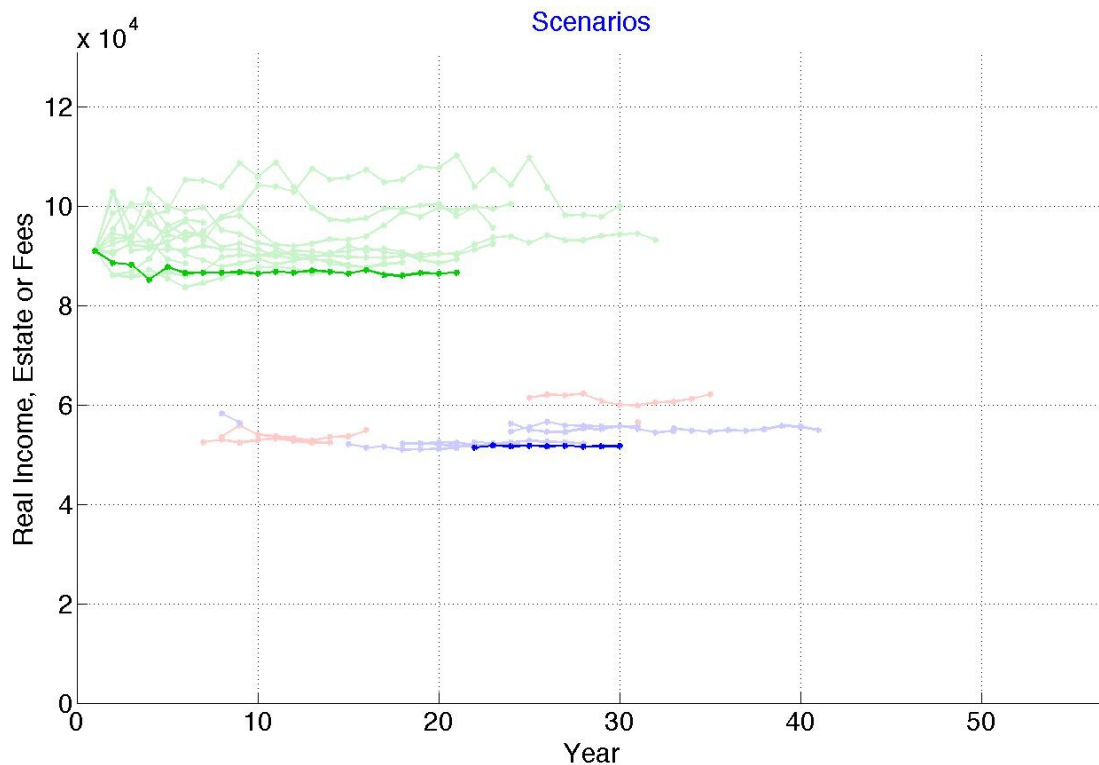
As usual, green indicates years in which both Bob and Sue are alive (personal state 3), red those in which only Bob is alive (state 1) and blue those in which only Sue is alive (state 2). In the last case shown (in the dark shade), they both live for 21 years, then Sue enjoys another 9 years. Incomes are greater when both are alive for two reasons. First, the annual real incomes from Social Security are $44,000 in state 3 and $30,000 in states 1 and 2. Second, we have constructed the lockbox annuity so that on average, income in states 1 and 2 will be half those in state 3. In the scenario shown in bold, real income is between roughly $80,000 and slightly over $90,000 when both are alive and close to $55,000 when Sue is alone. As the previously-plotted scenarios show, there can be considerable variation in incomes for a given year across scenarios. However, the anchor of Social Security leads to smaller proportional variations in total income than had the lockbox annuity been the only source of income.
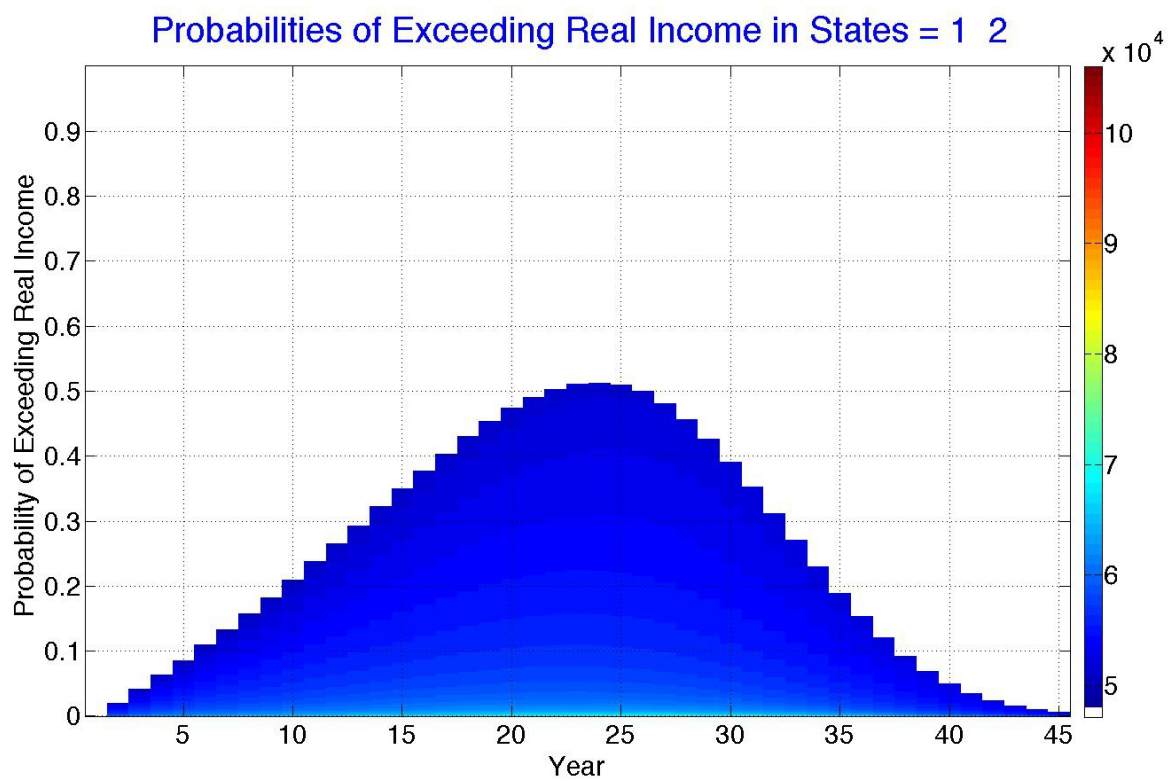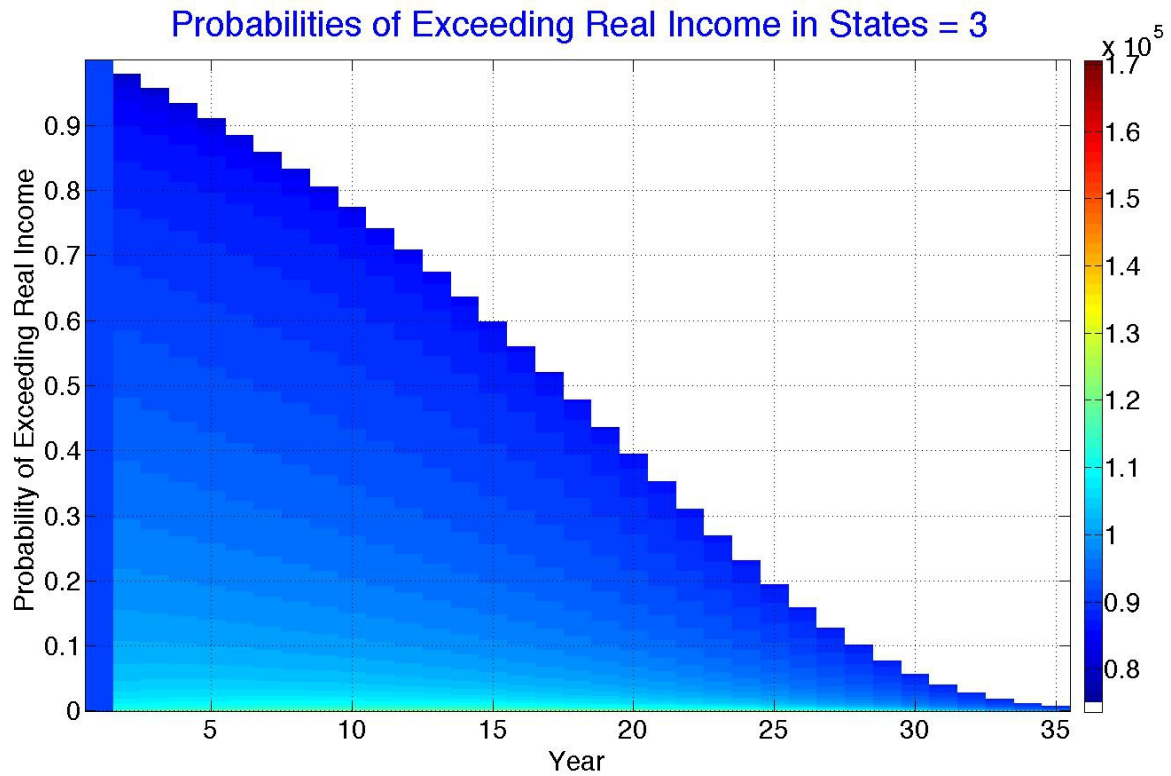
Finally, note that for these scenarios chosen randomly, income may be generated (and needed) for as long as 41 years. And as we will see, some other scenarios produce income for even more years.

Next, the distributions of income.

Real Incomes if States = 3: Year 30
5.8 Percent of Scenarios

Real Incomes if States = 1 2: Year 30
39.5 Percent of Scenarios

These are similar to the ones shown in the previous chapter but plot farther to the right, since the totals include fixed real income from social security. As desired, the distributions provide a compromise between relatively similar distributions and constant implied marginal utility. As one moves to later years the worst outcomes (near the top of the graph) tend to be similar with the best (near the bottom) increasing significantly, with the intermediate outcomes (in the middle) increasing somewhat. Finally, as desired, the incomes are lower when only one of our protagonists is extant.

Here are the income maps, each of which summarizes all the distributions of income in one graph:

Probabilities of Exceeding Real Income in States = 3

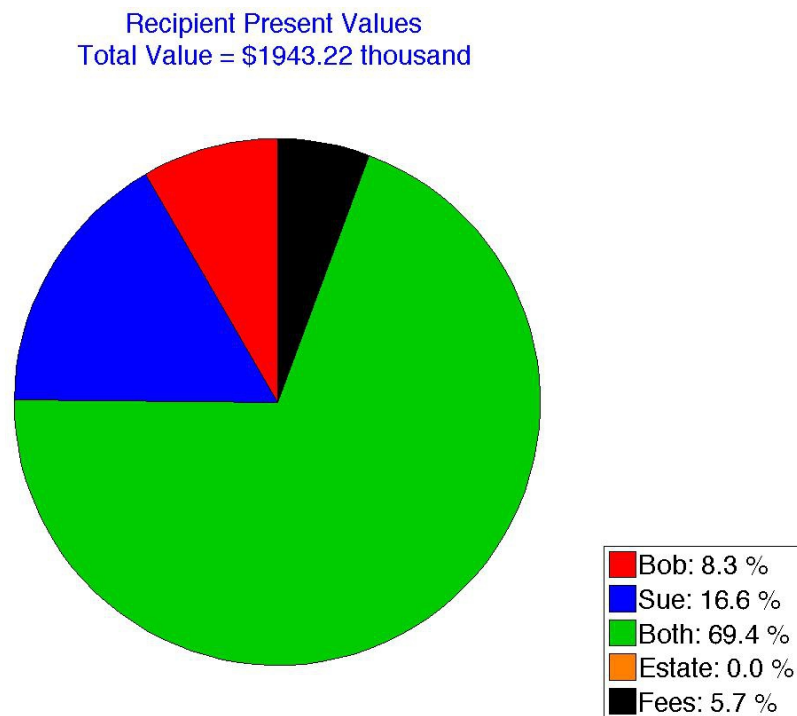Probabilities of Exceeding Real Income in States = 1 2

As in the previous graphs, the range of possible incomes (indicated by the different colors) is greater in the later years. The second graph shows that it is possible for someone (1 or 2) to be alive for as long as 45 years. There is at least a very small chance (measured by the height of the curve at the far right) that either Bob or Sue could be cashing checks from Social Security and the insurance policy in a year far in the future.

Next, the graphs of year over year incomes after 30 years:
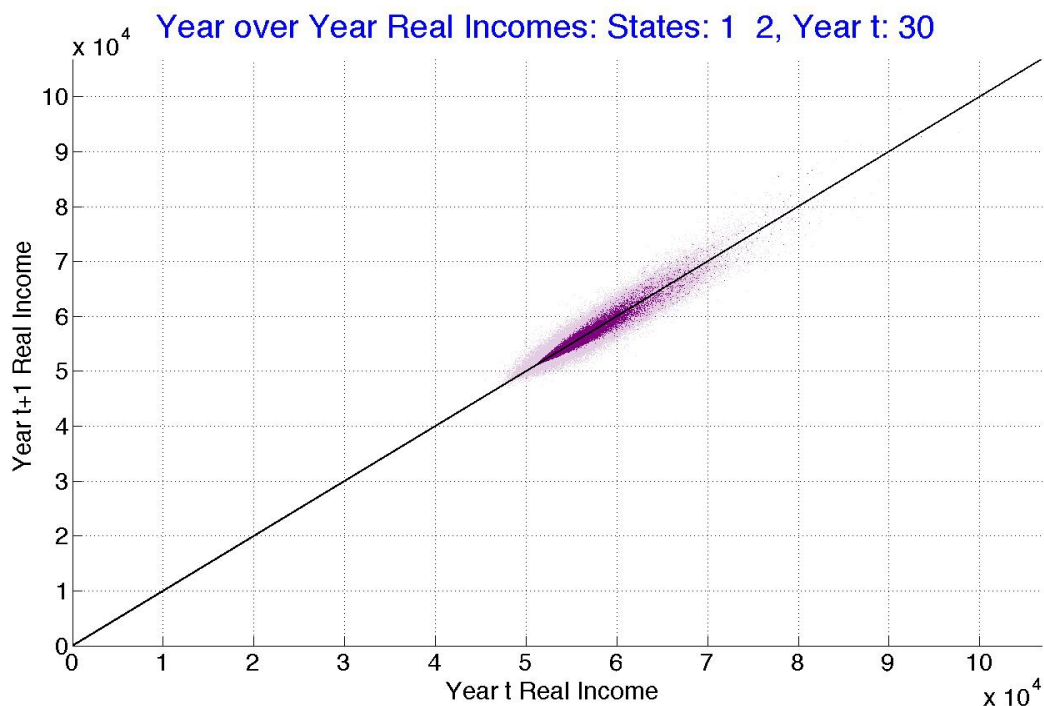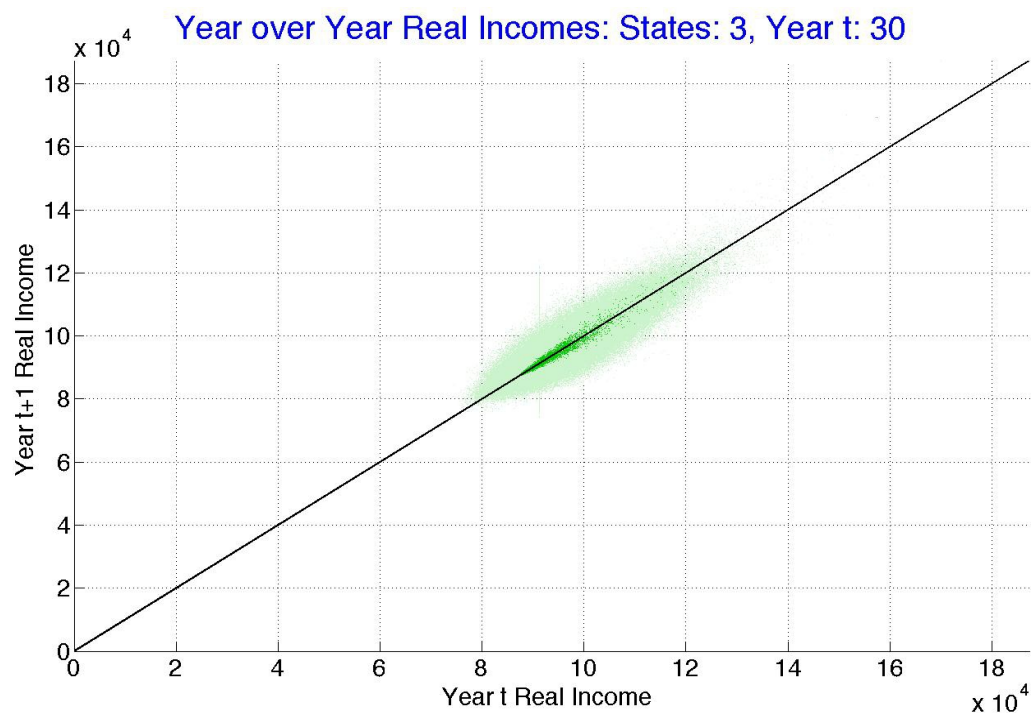
While the scatter of points around the 45 degree line shows that there is variation in income from year to year, it is relatively small (since the points fall close to the line), with little bias (since the scatter of number of points above the line seems similar to that below the line). There is year-to-year variation in income for a given personal state, but it is presumably acceptable for Bob and Sue.

A pie chart provides important information about the combination of approaches we are analyzing:

Recipient Present Values
Total Value = $1943.22 thousand

Bob: 8.3 %
Sue: 16.6 %
Both: 69.4 %
Estate: 0.0 %
Fees: 5.7 %

The combined present value of Social Security plus the lockbox annuity plus fees is close to $1.94 million. We know that the value of the annuity plus fees is $1.0 million, since that was the amount invested in it. And, as previously discussed, the present value of the possible incomes that Bob and/or Sue could receive from Social Security is roughly $0.95 million ($950,000). They are fortunate to have a similar amount available to provide additional income. Many retirees have far less.

The sizes of the wedges in the pie chart are informative. Of their total wealth, 5.7% goes to the financial industry in the form of fees. We will see far worse cases, but this is nonetheless worthy of attention. Recall also, that it is not possible to attribute fees to Social Security, but the progressive nature of its benefits could imply that the present value of its income payments is less than that of the contributions that Bob and Sue made over their working lives. Here and elsewhere, we only include fees that are separately identified by retirement income providers.

Now to the present values of the prospective income claims of the interested parties. First, the value of possible income payments to the estate is zero since both sources are annuities and we do not include any possible payments after Bob and Sue are both deceased. The present value of possible payments when both are alive is the greatest for a number of reasons. Social Security will pay less after one of the beneficiaries dies and we have designed the annuity so that it will do so as well. Moreover, in each scenario, the payments when both are alive precede any payments for years in which only one is alive, and the present value of $1 is greater, the sooner its date of receipt. Finally, the present value of possible payments when only Sue is alive is greater than that of payments when only Bob is alive because it is of course more likely that Sue will outlive Bob than vice-versa since she is both younger and female.

The implied marginal utility curves are next. Since we include Social Security, there are no scenarios in which Bob and/or Sue are alive without any income, so the preferred format with logarithms of values plotted on both axes can be employed.
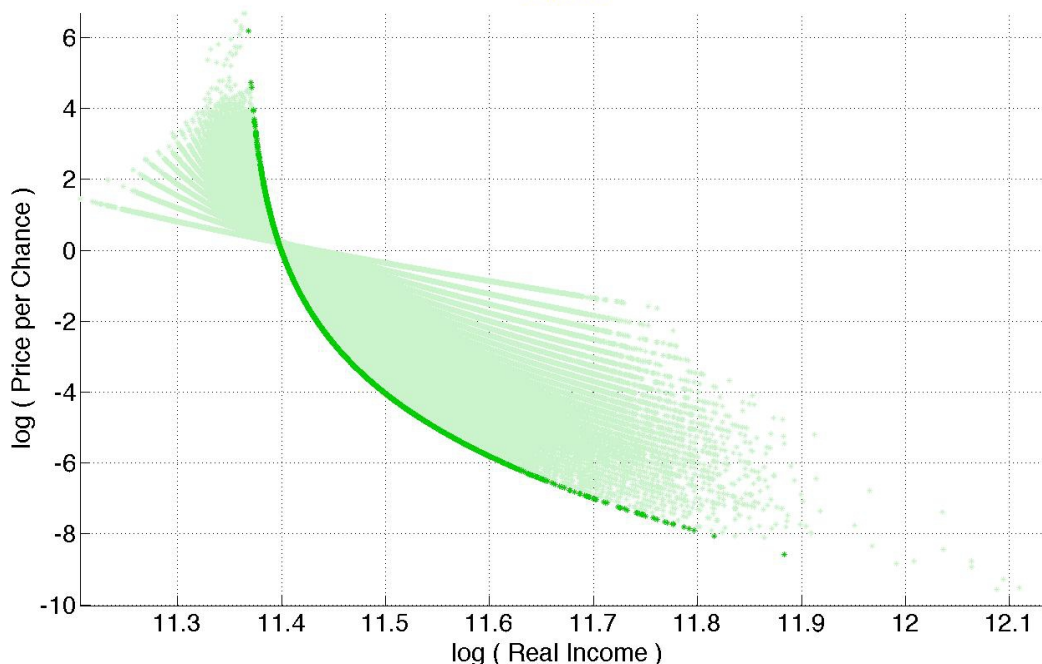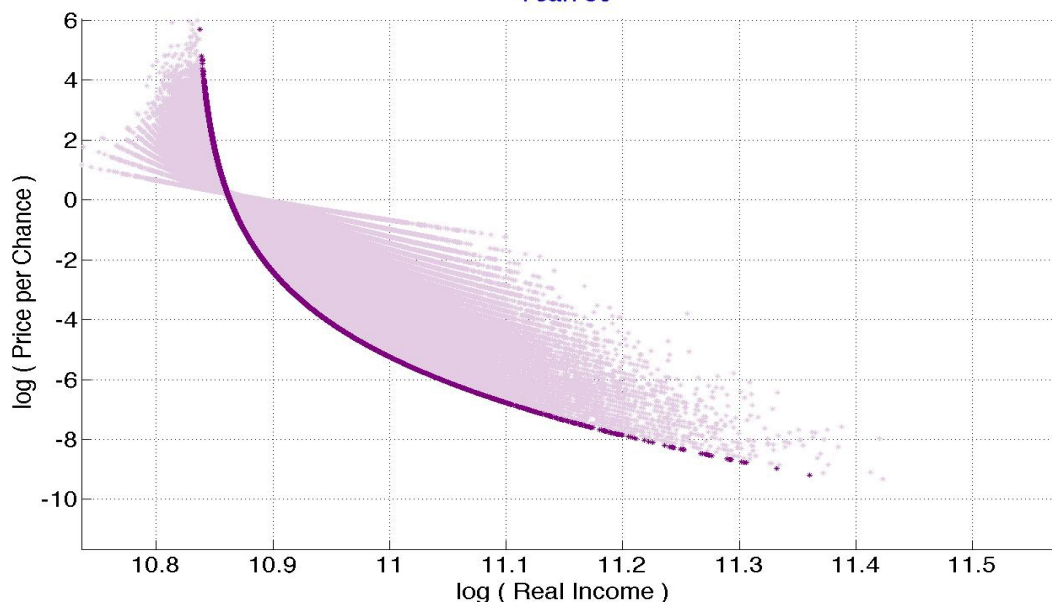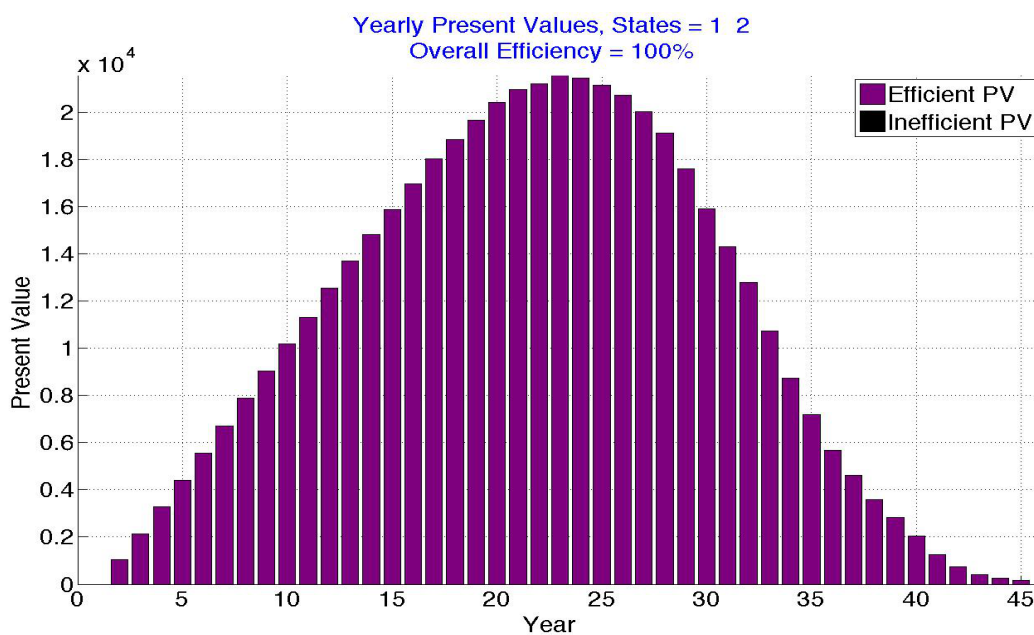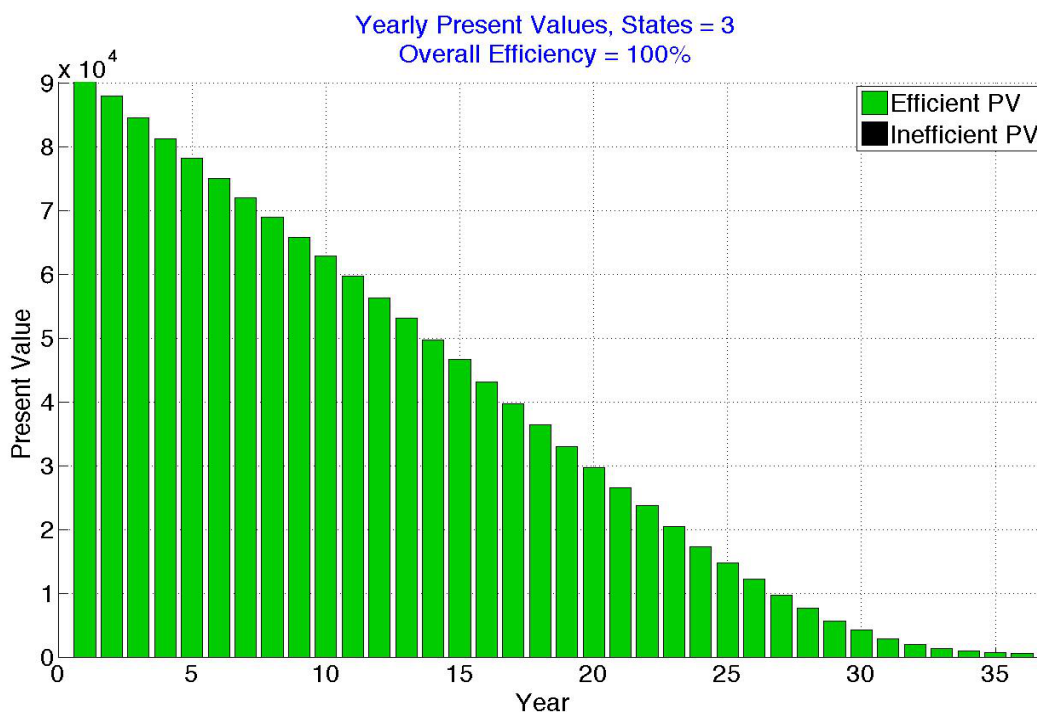

The relative risk aversion (indicated by the slope of each curve) increases as income decreases, so the curves can approach the vertical line that would indicate the income from Social Security alone. Moreover the curves differ across years, reflecting the choice an income source that is consistent with marginal utilities that differ for incomes in different years.

The next two graphs show the present values of possible incomes in each year, first for personal state 3, then for personal states 1 and 2.

Yearly Present Values, States = 3
Overall Efficiency = 100%

Yearly Present Values, States = 1 2
Overall Efficiency = 100%

No surprises here. The heights of the bars reflect both the probabilities of the personal states in different years and the diminishing present value of $1, the farther a payment is in the future.

Happily, there are no black sections in the graphs. Both Social Security and our Lockbox Annuity are completely cost-efficient – there are no cheaper ways to provide the set of possible incomes.

Other Approaches

While lockbox annuities have many attractive properties, as this is written there is one large negative – they do not exist. To be realistic, we must consider alternative approaches, of which there are many. To cover a reasonable variety of alternatives we will first focus on *spending strategies* that do not explicitly insure against longevity and/or poor financial results. As we will see, it may be desirable to adopt such a strategy as well as some type of deferred annuity. After exploring some such possibilities, we will examine variable annuities that combine longevity insurance with protection against some possible adverse investment returns. We then return to lockboxes, but as a source of spending without annuitization. A final chapter discusses retirement income advice and advisors.