Targeted Campaigns with Ambiguity-Averse Voters*

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Abstract

Two candidates compete for elective office by choosing groups of voters to target for informative advertising. Voters are ambiguity-averse, and might abstain in the face of low information. The candidates' targeting decisions can thus affect a voter in several ways: a favorable signal can lead a voter to switch his vote from one candidate to the other, can mobilize a voter who previously planned to abstain, or can lead a supporter of the opposing candidate to abstain. I characterize equilibrium targeting strategies in this setting, and derive several comparative statics. The targeted voters will be more partisan the lower is the initial level of information, and campaigns will focus more on mobilization, rather than attack, when prior information is limited, when the voters are more polarized, and when undecided voters can be more precisely categorized by their partisan leanings.

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Candidates for elected office work hard to reach specific subgroups of the electorate. According to a popular textbook, "Campaigns are not designed to reach everyone. Targeting involves categorizing different groups of voters, identifying their political preferences, and designing appeals to which they are likely to respond. It is the foundation of virtually every aspect of campaign strategy" (Herrnson 2000, p. 189). Despite the importance of targeted campaigns, political scientists have largely ignored the strategic issues involved.[1] The main exception has been the special case of presidential elections with the Electoral College. But targeting is important in all elections, even those (the overwhelming majority) with a single district. And traditional accounts of politicians' motives emphasize that incumbents set policy with an eye toward the groups expected to support them in the election (Fenno 1978). Thus the incentives to target some groups can ultimately lead to those groups getting better representation than others do.

Most work on the Electoral College studies candidates who focus on close states, because these states are most likely to be pivotal in the election. A similar intuition suggests that candidates should focus on "swing voters", voters who do not have strong prior attachments and who could vote either way. But politicians often talk about "mobilizing the base", suggesting that going after swing voters is not always a good idea. What determines the optimal tradeoff between these strategies?

To gain some insight into this problem, I construct an integrated theoretical account of targeting, participation, and vote choice. Candidates can provide informative signals to subsets of the voters. This information can affect voters in several ways. A favorable signal can lead a voter to switch his vote from one candidate to the other, or it can lead a voter to change his participation decision. This diversity of voter responses creates a rich menu of strategies for a candidate: she can try to convert supporters of the other candidate to her side, or to shift turnout in a favorable direction, either by mobilizing voters who lean her way but plan to not vote or by demobilizing supporters of the other candidate.

The model allows this diversity of voter responses because a voter might abstain in the face of low quality information. The idea is captured well by Richard Posner (2003), discussing his own

[1] The related question of incentives for targeted redistribution has received more attention (Cox and McCubbins 1986; Dixit and Londregan 1996). The conclusion discusses how the results of this paper differ from those contributions, and how those results can contribute to debates about redistribution.

decision to not vote in the 2000 presidential election:

> People have a pretty good idea of their own interests, or at least a better idea than officials do. But often they have a poor idea of how those interests will be affected by the forthcoming election. That was my own situation in regard to the 2000 election, and I am better informed about political matters than the average American. I did not have a clear sense of which candidate was on balance likely to deliver more of the things that I seek from the federal government, and so I didn't bother to vote. [pp. 168–9]

Several important studies of voter behavior suggest that Posner's abstention in the face of ambiguous information is part of a widespread phenomenon. A classic illustration is Bartels's (1988) study of presidential primaries. He finds that survey respondents are less likely to place candidates on issue scales the less informed they are, and that respondents who refuse to place candidates on issue scales are less likely to participate in politics. He finds, furthermore, that one of the main things that happens over the course of the campaign is that citizens learn more about the candidates, and so become more likely to turnout. He writes "Voters do not cast their ballots for candidates they do not feel that they know, at least superficially" (p. 57). These findings have been reinforced by the observational study of Lassen (2005) and the experimental study of Horiuchi, Imai and Taniguchi (2005), both of whom find that citizens who are randomly selected to receive information about the candidates in an election are more likely to vote than are citizens who do not get the information.

Here, I show how to capture this behavior in a simple model of ambiguity-averse voters, and then embed these voters into a model of targeted informative campaigning by two candidates. (The next section provides background on ambiguity aversion and discusses some relevant empirical evidence.) In the model, a candidate faces a tradeoff between mobilizing voters who might lean her way but lack a "clear sense" of whom to support, and targeting voters who favor the other candidate. How the candidates resolve this tradeoff depends, in part, on how responsive to new information are supporters of the opposing candidate. If these voters are relatively unresponsive, the most the candidate can hope for is that they switch from supporting the opponent to abstaining. In this case, its better to stick with the undecided voters who lean the candidate's way, because they are more likely to move in the appropriate direction. If the opponent's voters are more responsive, on the other hand, then they might actually change their vote intention towards the targeting candidate. If this happens, the candidate gets a double benefit—not only does the opponent lose a voter, as in the previous case, but she gains one as well. This double benefit must be weighed against the higher probability that the undecided voter will move in the correct direction.

Comparative statics follow from this tradeoff because the voters' responsiveness emerges endogenously from the informational environment. The comparative statics make several contributions to studies of campaigns. For example, the model predicts that, the lower is the initial level of voter information, the more partisan will be the targeted voters. In other words, candidates "play to their base" in low information environments, while they "play to the center" in high information environments. This suggests that equilibrium targeting strategies will vary across elections for different offices—consider, for example, the different levels of information voters have about presidential candidates and candidates for state legislatures. As such, the model implies that empirical work focused on targeting in national campaigns might have limited relevance for most campaigns.

The model makes other novel predictions, including that campaigns will be more likely to focus on mobilization (rather than conversion or demobilization of opponent's supporters) when prior information is limited and when voters are more polarized, and that enhanced ability for candidates to identify the partisan leanings of undecided voters will lead to more mobilization, while enhanced ability to identify which partisans are undecided will not affect the equilibrium strategies. The model also clarifies how campaign strategy and voters' aversion to casting uninformed ballots interact to determine turnout.

1 Information and Abstention

As emphasized above, this paper's results depend on voters abstaining in the face of low quality information. As Feddersen and Pesendorfer (1996) and Ghirardato and Katz (2002) argue, this mechanism can explain the well-documented phenomenon of roll off—turnout is higher for offices that are more prominent even when the races are on the same ballot. From a cost-of-voting perspective, roll off is hard to explain, because the cost must be paid to vote in any one of the simultaneous elections, and completing the rest of the ballot has negligible additional cost. It is easy, on the other hand, to motivate significant differences in how much information a voter has about the candidates in different races. Thus we are lead to consider models in which voting is costless, but information matters for turnout.[2]

But there is a problem. An outcome-oriented Bayesian voter who faces no cost of voting and does not learn from his vote being pivotal will always have a payoff from abstaining between the payoffs from voting for the candidates. This makes optimal abstention nongeneric in this setting. (See Proposition 1 of Ghirardato and Katz (2002) for a formal statement and proof.) The next subsection discusses existing formal models in which this reasoning breaks down and places this paper's approach in the context of those papers. The following subsection reviews some of the empirical evidence bearing on the foundational assumptions of my approach. Readers eager to see the model can skip to the next section with no loss in continuity.

1.1 Existing Theoretical Approaches

The formal literature offers two approaches to make turnout sensitive to the voters' information.

The first approach, pioneered by Feddersen and Pesendorfer (1996), maintains standard assumptions about voter preferences and information processing, and derives abstention from strategic considerations. Feddersen and Pesendorfer (1996) assume that voters are both differentially informed and strategically sophisticated. In that case, a poorly informed voter who conditions on his vote being pivotal believes that his vote was for the wrong candidate, whichever candidate he voted for. This inference leads him to abstain with positive probability.

The second approach considers voters who deviate from one of the standard assumptions about decision-making. Several such deviations have been considered. Merlo (2006) and Achen (2005) assume that voters worry that they will feel regret about *ex-post* incorrect votes, and so abstain if their fear of making a choice that turns out to be wrong is great enough. Ghirardato and Katz (2002) study a voter who is averse to *ambiguity*—"uncertainty about probability, created by missing information that is relevant and could be known" (Frisch and Baron 1988). They show that a voter who is ambiguity averse has a strict preference to abstain for an open set of parameter values.

[2]This is not to say that informational models are the only viable alternative to the classical pivotal voter model—the ethical voter models of Edlin, Gelman and Kaplan (2007) and Feddersen and Sandroni (2006) are another interesting alternative. I focus on informational explanations not because the ethical voter approach is unattractive but because the informational approach is naturally linked to informative advertising.

Importantly, risk aversion is not on the list of reasons that voters might abstain due to bad information. Ghirardato and Katz's (2002) demonstration that a Bayesian voter with no cost of voting almost surely votes makes no assumptions about the voter's risk tolerance. To see the intuition for this result, notice that abstaining is also a risky act for an outcome-motivated voter. Of course, with risk aversion, a voter's evaluation of one or both candidates will be worse when information is limited, and this very well might affect a voter's preference between the two candidates, as in Alvarez (1997) and Bartels (1986). Still, a Bayesian voter can always decide which of the two candidates is better, and once she has done so, she will perceive abstaining as bearing the risk of causing the less favored candidate to win. Modeling voters who abstain in response to their uncertainty about candidates requires moving beyond the standard Bayesian model.

Like Ghirardato and Katz (2002), I assume directly that voters are ambiguity averse. But I use a different specification—Bewley's (2002) model of "Knightian uncertainty"—rather than the maximin expected utility model that Ghirardato and Katz (2002) adopt from Gilboa and Schmeidler (1989). This difference reflects the different questions addressed. Ghirardato and Katz focus on showing that abstention can be optimal with a minimal deviation from the standard Anscombe and Aumann (1963) framework, and they do not go on to discuss the dynamics of a campaign or the problem of integrating their voters with strategic candidates. My setup, by contrast, makes the possibility of abstention trivial, and allows me to focus on questions of elite strategy.

Bewley's (2002) model captures ambiguity aversion by allowing preferences to be incomplete. This means that a voter cannot always decide which of two candidates is better, in sharp contrast to a Bayesian voter. This modification of the of the standard model is a natural one for applications to voter behavior, because it allows a voter to answer "I don't know" to questions about how she plans to vote. Furthermore, incompleteness arises naturally from ambiguity. If a voter has a *set* of beliefs and only chooses undominated alternatives, then his preferences are incomplete. For example, a voter might entertain one belief that implies the Democratic candidate is better than the Republican candidate with probability 2/3 *and* a second belief that implies the Republican candidate is better than the Democratic candidate with probability 2/3. In this case, the voter cannot rank the two candidates.

When a voter cannot rank the two candidates, his preferences alone do not determine what he does. In such cases, the model is closed with an "inertia" assumption—there is a status quo action and the decision maker sticks with the status quo unless some other action is better according to all of his beliefs. I assume that the status quo is abstention, so a voter who cannot rank the two candidates does not vote at all. This is a natural assumption given the evidence that poorly informed voters rarely vote.[3]

1.2 Evidence on Ambiguity and Incompleteness

The model will be more valuable the greater is the evidence for the psychological traits it embodies. What is the evidence for ambiguity aversion and incomplete preferences in political behavior?

1.2.1 Experimental Background for Ambiguity

The seminal demonstration that some people dislike acting on ambiguous information was Daniell Ellsberg's (1961) urn experiments. Here is the simplest version of Ellsberg's thought experiment. The subject chooses to draw a ball from one of two urns. She is told that the first urn contains exactly 50 red and 50 black balls. The second urn also contains 100 balls, but the subject is not told what fraction are red and what fraction are black. (She does know that every ball is either red or black.) In one stage of the experiment, the subject wins $10 if she draws a red ball. In the second stage, she wins $10 if she draws a black ball. Most subjects strictly prefer to draw from the first urn in both treatments. This is inconsistent with probabilistic beliefs (assuming more money is preferred to less). A strict preference for urn 1 in the first treatment would imply that the subject's subjective expectation of the number of red balls in the second urn is less than 50, while a strict preference for urn 1 in the second treatment would imply the subjective expectation is greater than 50. But both of these cannot be true if the subject's beliefs about the composition of the second urn are probabilistic, since the two expectations would have to sum to 100. (Risk aversion cannot account for these results since there are only two prizes.)

Ellsberg's result has been replicated many times, often with real payoffs. (Camerer and Weber (1992) review this literature.) Slovic and Tversky (1974) show that explaining the normative arguments against ambiguity aversion does not reduce Ellsberg outcomes, and Curley, Yates and

[3] Another natural status quo would have a voter who cannot rank the two candidates vote for the candidate of a specific party, as in the "standing decision" model of party identification. In section 2.4, I show that this status quo leads to a voter who is observationally equivalent to a Bayesian with "extra" leanings toward the favored party.

Abrams (1986) show that people are strictly ambiguity averse—the results cannot be explained by assuming that subjects are indifferent.

Most of the early studies on ambiguity were based on experiments with physical randomization devices, like drawing balls from an urn. But ambiguity aversion has also been found in studies based on real-world uncertainty. Heath and Tversky (1991) find that subjects are averse to ambiguous bets about the outcomes of political and sporting events. They also find that the degree of ambiguity aversion varies with the level of the subject's background knowledge—the aversion to ambiguous bets is greater the less informed the subject is about the issue. They call this the "competence hypothesis". Fox and Tversky (1995) find that subjects are more averse to ambiguous bets when they know that more informed people are making similar choices. They call this the "comparative ignorance hypothesis". Applied to elections, both the competence hypothesis and the comparative ignorance hypothesis suggest that voters will be more ambiguity averse in elections for less visible offices, since, in these elections, there is less background information.

1.2.2 Voters and Incomplete Preferences

Ambiguity aversion seems to be a robust feature of human decision making. But the model goes beyond just ambiguity aversion—as in Posner's description, the specific expression of ambiguity aversion is incomplete preferences over the candidates. I next review the extant empirical evidence linking these two features of decision-making.

A few papers in political science have tried to directly test the hypothesis that voters have complete, transitive preferences. The seminal article is Brady and Ansolabehere (1989) (also see the follow-up work by Hansen (1998) and Alvarez and Kiewiet (2005)). They use data from a poll of 1034 California adults in March 1976 and a self-administered questionnaire completed in March 1983 by 170 University of California at Berkeley students, faculty, and staff. In each case, the respondents were asked about their preferences over each pair of candidates in the then current Democratic presidential primary contest. They find that between 60% and 70% of the respondents have fully rational preferences and fewer than 10% have intransitive strict preferences.[4] So just

[4] More specifically, they show that 60% to 70% have negatively transitive strict preferences. Since the strict preferences are asymmetric by the construction of the surveys, this means that the weak preferences are complete and transitive. See Proposition 2.2 of Kreps (1990).

	LOW	Actual	HIGH
Prefer Hart	40.1%	21.6	38.3
Prefer Mondale	24.1	62.3	58.6
No Preference	35.8	16.1	3.1

Table 1: Entries in the column labeled "Actual" are the percentage of respondents in Brady and Ansolabehere (1989) who expressed the preference in the row label. The other two columns are simulated results from a statistical model estimated on the responses, where each voter is assumed to have the same level of knowledge about each candidate—"LOW" knowledge about Mondale is reduced to that about Hart and "HIGH" when knowledge about Hart is raised to that about Mondale.

under 1/3 of the respondents have strict preferences that are transitive, but weak preferences that are not both transitive and complete. They write:

> We do not think that this means that people are irrational about their interests; rather it means that people sometimes do not know enough about their interests to make a commitment one way or another. [p. 158]

To test this conjecture about voter information, they constructed measures of uncertainty based on self-reported knowledge and on willingness to place the candidates on issue scales. They find that more uncertainty leads to a lower likelihood of negative transitivity. To give a sense of the substantive magnitude of this information effect, they simulate the preferences (in 1983) over former Vice-President Walter Mondale and second-term Senator Gary Hart under two counterfactual scenarios: each voter's knowledge of Hart raised to that of Mondale (HIGH), and each voter's knowledge of Mondale reduced to that of Hart (LOW). Table 1 shows the results.

The effect of knowledge on full rationality is substantively large, with a swing from 35.8% to 3.1% of voters with complete preferences. Also, this simulation shows the different effects of ambiguity aversion and risk aversion. Moving from LOW to actual to HIGH monotonically increases overall information, and the probability of no preference decreases monotonically, as implied by ambiguity aversion. On the other hand, the same change has a nonmonotonic effect on the *relative* information advantage of Mondale. And just as risk aversion predicts, Mondale's relative popularity is nonmonotonic.

Brady and Ansolabehere force the subjects' preferences to be complete by declaring them indifferent when they say "I'm really not sure" (these are the exact words from their 1983 survey).[5] However, they recognize that what they call indifference may be a form of incompleteness:

> When indicating indifference a person may be making an equal commitment to each alternative or indicating insufficient knowledge to make a commitment. Indifference may indicate equal preference or it may indicate no preference. Intransitive preferences, then, suggest a deep-seated confusion about one's interests whereas intransitive indifference may only indicate temporary ignorance about one's interest. (p. 147)

I adopt the incompleteness interpretation because it leads to a tractable model that explicitly links the degree of incompleteness to the informational level.

2 The Model

2.1 Candidates and Voters

Two candidates, R and L, compete for an elected office. Each candidate maximizes her expected margin of victory.

There is a continuum of voters, with measure 1. These voters are divided into K types, where all voters of a given type have the same prior beliefs and utilities. With only two candidates, it suffices to keep track of the difference in utility that a voter has for candidates R and L. So, without loss of generality, I normalize payoffs so that a type k voter gets utility θ_k from R and utility 0 from L. If θ_k is positive, then a type k voter prefers R. These payoffs are independent across types.

A voter does not know θ, but only that $\theta \in \{-1, 1\}$.[6] Since the treatment of the uncertainty is novel, I defer its discussion to the next subsection.

A voter who abstains gets a payoff between the two payoffs of voting for one or the other candidate; for concreteness, I take it to be $\delta\theta$, where $0 < \delta < 1$.[7] In most models of turnout, the

[5] See Mandler (2005) on the relationship between incompleteness and intransitivity with this convention.

[6] It might appear that restricting θ to have only two possible values is restrictive. But in the context of the rest of the model, it is not. Since voting is costless, the scale of the payoff for each action has no decision-theoretic meaning—changing the numbers will not affect the model's ability to capture any behavior.

[7] Notice that these voters are risk neutral—as emphasized in the introduction, risk aversion on its own could not produce my results.

payoff for abstaining lies between the payoff from voting for the better candidate and the worse candidate. For example, in the classical model of outcome-oriented voters, a voter ranks actions by their probability of producing the favored candidate, conditional on the voter being pivotal. This probability is greatest when voting for the favored candidate, and least when voting for the worse candidate.

Voting is costless, so a voter who knows θ has a dominant strategy to vote for R if $\theta = 1$ and to vote for L if $\theta = -1$. The assumption of costless voting, which is standard in models that focus on informational issues (e.g. Feddersen and Pesendorfer (1996) and Ghirardato and Katz (2002)), isolates the effects of ambiguity and ambiguity aversion on voting behavior and candidate strategy. Adding a cost of voting, constant across elections, would not affect the results.

The candidates compete by sending informative messages to subsets of voters. Specifically, the electorate is partitioned into 4 targetable groups, and each candidate can send a message targeted to at most one group.[8] Groups and types are not coextensive—a group can contain more than one type of voter, and voters of one type can be spread across several groups. Let β_{kg} be the measure of voters of type k in group g, and write $\beta_g = \sum_k \beta_{kg}$.

If a group is targeted, each member gets a signal $s \in \{\underline{s}, \overline{s}\}$. The signal likelihoods are

$$\Pr(\overline{s} \mid \theta = 1) = \Pr(\underline{s} \mid \theta = -1) = q > \frac{1}{2}.$$

These signals are conditionally independent across the voters within a group, and if a voter gets signals from both candidates then these signals are conditionally independent of each other.[9] Without a signal, a voter learns nothing during the campaign.

Notice that candidates to not get to condition their targeting decisions on the realization of the signals; instead, they must choose which group to target knowing that there is some risk of the signal being a negative one. This assumption allows me to keep the voters' updating simple because it means that voters who do not get signals will not try to infer anything about what a candidate knew about the signals she did not send. As a substantive matter, the assumption does not seem troubling, as it just reflects the risk that any strategy might backfire.

[8]Many of the results hold for any number of groups. I restrict attention to 4 groups from the outset because that is the number in all of the examples below.

[9]I follow the usual practice of extending the idea of independence to a continuum of random variables by making the law of large numbers into a definition.

2.2 Voter Beliefs

A voter's problem is complicated by the fact that he does not know θ, his true payoff to voting for the R candidate. The standard approach to uncertainty about θ would assume that a voter believes $\theta = 1$ with probability p and $\theta = -1$ with probability $1 - p$, and that he votes for R if $p > 1/2$ and for L if $p < 1/2$. This representation implies that a voter generically has a strict preference over candidates (Ghirardato and Katz 2002).

In contrast, I assume that a voter might have incomplete preferences. Following Bewley (2002), I model this by representing his beliefs with a convex set of probabilities, $\mathcal{P}$. A voter prefers R to L if $p > 1/2$ for all $p \in \mathcal{P}$, he prefers L to R if $p < 1/2$ for all $p \in \mathcal{P}$, and he is unable to rank the candidates if neither condition holds. If the set of priors is a singleton, then this collapses to the standard model with complete preferences.[10]

It is convenient to parameterize the set of beliefs as

$$\mathcal{P} = [m - r, m + r],$$

where m is the midpoint of the set of priors and r is the radius of the set of priors. Thus m measures the partisan leanings of the voters, while r measures ambiguity aversion.

In the standard Bayesian model, new information is modeled as the realization of a signal whose likelihood is known up to the unknown parameter, and beliefs are updated with Bayes's rule. Following Bewley's (2002) and Halpern's (2003, §3.3) treatment of information, the voters here get a signal with known likelihood, and beliefs are updated prior-by-prior, using Bayes's rule.[11]

Let

$$\mathcal{S} = \{\emptyset, \underline{s}, \overline{s}, \underline{s}\underline{s}, \underline{s}\overline{s}, \overline{s}\overline{s}\}$$

be the set of possible signal realizations a voter might receive, and let $\tilde{s}$ denote a generic element of this set. I do not distinguish signals based on which candidate sent them because candidates have no private information for voters to extract from targeting decisions.

[10] The appendix outlines the decision-theoretic background of this type of multi-prior model. Readers who are not interested in axiomatic decision theory can skip that discussion without any loss of continuity.

[11] As Ghirardato and Katz (2002) point out, Bayesian updating prior-by-prior is inappropriate with the maximin EU form of multiprior utility—that updating rule can lead to time-inconsistency in that framework. Since Bewley's model keeps the independence axiom, this time consistency problem does not arise in Bewley's model, and prior-by-prior updating is appropriate.

A voter who gets n signals with realization $\tilde{s}$ updates each of his priors $p \in \mathcal{P}$ according to Bayes's rule:

$$\mu(\tilde{s}; p) = \frac{p \Pr(\tilde{s} \mid n, \theta = 1)}{p \Pr(\tilde{s} \mid n, \theta = 1) + (1 - p) \Pr(\tilde{s} \mid n, \theta = -1)}.$$

Because this updating rule is continuous and increasing in p, the voter's updated beliefs are an interval $[\underline{\mu}(\tilde{s}; m, r), \overline{\mu}(\tilde{s}; m, r)]$ with endpoints

$$\overline{\mu}(\tilde{s}; m, r) = \frac{(m + r) \Pr(\tilde{s} \mid n, \theta = 1)}{(m + r) \Pr(\tilde{s} \mid n, \theta = 1) + (1 - m - r) \Pr(\tilde{s} \mid n, \theta = -1)}$$

and

$$\underline{\mu}(\tilde{s}; m, r) = \frac{(m - r) \Pr(\tilde{s} \mid n, \theta = 1)}{(m - r) \Pr(\tilde{s} \mid n, \theta = 1) + (1 - m + r) \Pr(\tilde{s} \mid n, \theta = -1)}.$$

Closing the model requires pinning down the candidates' beliefs about the likelihoods of the various signals. Assume that each candidate believes that a signal sent to group k has probability $\sigma_k = m_k q + (1 - m_k)(1 - q)$ of being $\overline{s}$. That is, the candidates (point) beliefs about a type are the midpoint of the type's beliefs. Substantively, this means that the probability a message is successful for a candidate is increasing in the prior likelihood that the voter really favors the candidate.[12]

2.3 A Voter's Decision Rule

As is typical in models with a continuum of voters, equilibrium does not pin down voter behavior, since no voter is ever pivotal. I follow standard practice and simply assume that voters use a strategy that would be undominated in a large, finite electorate. I elaborate on this a bit more than is usual, since the decision theory is nonstandard.

If the beliefs in $\mathcal{P} \mid \tilde{s}$ are not unanimous and the voter cannot rank the two candidates, then the preferences alone do not determine his actions. In this case, a Bewley-voter takes a *status-quo* action. I take the status-quo to be abstention, so a voter whose beliefs are not unanimous abstains.

[12] A seemingly richer model would give a candidate several varieties of message to send to each group. But this extra generality would not add anything. Sending any message that is not the most likely to work on the targeted group would be a dominated strategy. Thus the results would be the same as long as the success probability of the best message to send to a group is increasing in the likelihood that members of the group favor the candidate.

Thus a voter's decision rule is:

$$\begin{array}{rl} \text{vote } R \text{ if} & \underline{\mu}(\tilde{s}; m, r) > \frac{1}{2} \\ \text{vote } L \text{ if} & \overline{\mu}(\tilde{s}; m, r) < \frac{1}{2} \\ \text{abstain} & \text{otherwise.} \end{array}$$

Note that I am not assuming that the payoff to abstaining is zero—it is still $\delta\theta$. What's going on is that, if the posteriors are not unanimous, then the voter falls back on the status quo, abstention.

2.4 The "Standing Decision" Model

As mentioned in the introduction, another plausible specification of a voter's status-quo action is to vote for a particular party. This reflects the classical idea that party identification represents a standing decision to vote for the party unless race-specific factors intervene (Key 1966). It turns out that such a voter is observationally equivalent to a voter who is not ambiguity averse at all, but leans strongly toward his favored party.

To see this consider a voter with upper expectation $\overline{\mu}$ and lower expectation $\underline{\mu}$, and who votes for R when he cannot rank the two candidates. His decision rule is to vote

$$\begin{array}{ll} L & \text{if } \overline{\mu}(\tilde{s}; m, r) < 1/2 \\ R & \text{if } \overline{\mu}(\tilde{s}; m, r) > 1/2. \end{array}$$

But this is identical to the decision rule of a Bayesian with prior $m + r$. Thus allowing for status-quo actions consistent with the standing decision model adds nothing, since voters can already be ambiguity neutral.

3 Equilibrium

3.1 Definition of Equilibrium

Recall that a candidate can target at most one group of voters. Let t_L denote the group that candidate L targets and let t_R denote the group that R targets. If candidate c targets no group, set $t_c = 0$. Then feasibility requires that $t_c \in \{0, 1, 2, 3, 4\}$ for candidate $c \in \{R, L\}$.

A voter's decision rule is a map

$$\nu : \mathcal{S} \times [0, 1] \times [0, 1/2] \to \{-1, 0, 1\},$$

where $\nu(\tilde{s};m,r)=1$ (respectively 0, -1) means that a voter with priors (m,r) and signal realization $\tilde{s}$ votes for R (respectively abstains, L). The discussion above implies that the voting rule is:

$$\nu(\tilde{s},m,r)=\begin{cases} 1 & \text{if } \underline{\mu}(\tilde{s};m,r)>\frac{1}{2} \\ -1 & \text{if } \overline{\mu}(\tilde{s};m,r)<\frac{1}{2} \\ 0 & \text{otherwise.} \end{cases}$$

Let $n_g(t_R,t_L)$ be the number of signals group g gets when the candidate strategies are (t_R,t_L):

$$n_g(t_R,t_L)=|\{c\in\{R,L\}\text{ st } t_c=g\}|.$$

(For a finite set X, I denote the cardinality of X by $|X|$.)

If group g gets n signals, then its net contribution to R's margin of victory is

$$\pi_g(n)=\sum_k \beta_{kg}\sum_{\tilde{s}}\Pr(\tilde{s}\mid m_k,r_k,n)\nu(\tilde{s};m_k,r_k),$$

and R's overall margin of victory is

$$\pi(t_R,t_L)=\sum_g \pi_g(n_g(t_R,t_L)).$$

(I suppress the dependence of π on the various belief parameters when no confusion can result.)

An *equilibrium* is a pair (t_L^*,t_R^*) such that:

1. $\pi(t_R^*,t_L^*)\geq\pi(t_R,t_L^*)$ for all feasible t_R and

2. $\pi(t_R^*,t_L^*)\leq\pi(t_R^*,t_L)$ for all feasible t_L.

3.2 Characterization of Pure Equilibria

As is common in models of informative campaigns (see, eg, Schultz (2007)), equilibria typically involve each group being targeted by at most one candidate.

Definition 1 *A pure-strategy equilibrium is* specialized *if* $t_L=g\neq 0$ *implies* $t_R\neq g$.

So, in a specialized equilibrium, at most one candidate targets each group. While it is possible for there to be a pure equilibrium that is not specialized, this requires a very special configuration of parameters. In particular, the following condition must fail.

Definition 2 *The voters are* strictly responsive to signals *if, for all groups g, $\pi_g(2) \neq \pi_g(1)$.*

One implication of strict responsiveness to signals is that, within each group, some type changes their behavior given some configuration of signals—if all types abstain, say, no matter what, then the group obviously fails the condition. In addition to this non-triviality implication, strict responsiveness simply rules out certain knife-edged parameter configurations.

Lemma 1

1. *If there is a pure-strategy equilibrium, then there is a specialized pure strategy equilibrium.*
2. *If the voters are strictly responsive to signals, then any pure strategy equilibrium must be specialized.*

(Proofs of all lemmata are in Appendix B.)

The idea behind the proof is simple. Except for knife-edged cases, one candidate does better when group g gets only one signal instead of two. Thus a profile in which both candidates target g is unstable—the candidate who does better with only one signal will deviate.

The specialization result yields a useful partial characterization of equilibrium targeting. When a candidate targets a group in a specialized equilibrium, the candidate gets the contribution to margin of victory that the group gives her with one signal rather than the contribution that the group gives her with no information at all. Define the difference in these pluralities as

$$\delta_g = \pi_g(1) - \pi_g(0),$$

and set $\delta_0 = 0$. I sometimes abuse notation and write, eg, δ_{t_R} to mean δ_g if $t_R = \{g\}$ and δ_0 if $t_R = \emptyset$.

Proposition 1 *If (t_R, t_L) is a specialized equilibrium, then δ_{t_R} (δ_{t_L}) is maximal (minimal) in $\{\delta_g\}_{g=0}^{4}$.*

Proof At the specialized profile (t_R, t_L), candidate R's margin of victory is $\sum_{k=1}^{4} \pi_k(0) + \delta_R + \delta_L$. Candidate R wants to maximize this expression while candidate L wants to minimize it. It's obvious from this expression that, in a specialized equilibrium, $\delta_R \geq 0$ and $\delta_L \leq 0$—if one of these were not true, then the candidate in question could deviate to targeting the empty group, which has $\delta_0 = 0$.

Now assume, seeking a contradiction, that (t_R, t_L) is a specialized equilibrium in which there is a group g such that $\delta_g > \delta_R \geq 0$. Candidate L is not targeting group g, because $0 \geq \delta_L$. Thus if candidate R deviates to target group g, her payoff increases by $\delta_g - \delta_R > 0$. This is a profitable deviation, so the original profile was not an equilibrium after all. Thus δ_R must be maximal in a specialized equilibrium. A similar argument shows that δ_L must be minimal. □

The maximality condition may look like a simple restatement of equilibrium, but it is not. Equilibrium does not require that $\delta_{t_R} \geq \delta_{t_L}$ because moving the group targeted by L from 0 to 1 signals in not a feasible choice for R. On the other hand, equilibrium does require the additional condition that R not want to deviate to t_L. The condition in the Proposition is nonetheless important because it identifies an easily found candidate equilibrium. Furthermore, the candidate is typically unique:

Corollary 1 *If $g \neq g'$ implies $\delta_g \neq \delta_{g'}$ and $\delta_g \neq 0$ for all g, then there is at most one specialized equilibrium.*

Because the empty group is feasible and has $\delta_0 = 0$, the maximality result means that R will only consider targeting groups with $\delta_g \geq 0$. Which groups are these? To build some intuition, consider this question in the special case of internally homogenous groups. Formally:

Definition 3 *A group is* homogenous *if all members of the group are of the same type.*

In such a case, the incentives are particularly clear. In the general case, the incremental benefit from targeting a group will be a mixture of the benefits discussed below.

Consider first types initially planning to vote R. Any change induced by a new signal is detrimental to the R candidate's margin of victory, so such types will never be targeted by R. Conversely, any change in the plan of a type initially planning to vote L helps R, so these types are possible targets for R. Intuitively, campaign activity can only push away voters who are initially supporters of the candidate, so targeting such voters has a negative return. Similarly, voters initially supporting the other candidate can only be pushed in the right direction, so targeting them has a positive return.

Finally, consider a homogenous group g that initially plans to abstain. Recall that $\sigma(m) = mq + (1-m)(1-q)$ is the probability that a signal for a group with prior midpoint m is $\overline{s}$, and write $\mathbb{I}[p]$ for the indicator function that takes the value 1 if p is true and 0 if p is false. Since $\pi_g(0) = 0$ for a group of initial abstainers,

$$\delta_g = \pi_g(1) = \beta_g \left(\sigma(m)\mathbb{I}[\underline{\mu}(\overline{s}; m, r) > 1/2] - (1 - \sigma(m))\mathbb{I}[\overline{\mu}(\underline{s}; m, r) < 1/2]\right).$$

Lemma 2 *For types who abstain absent a signal, the increment to R's margin of victory with one signal is nondecreasing in the prior midpoint, and is strictly increasing if at least one signal leads the type to turnout.*

Because a voter with prior midpoint 1/2 treats the two candidates symmetrically, δ_g is zero for any group with midpoint 1/2. Thus Lemma 2 implies that a group with initial intention not to vote is targetable by R only if $m > 1/2$. Intuitively, if someone plans to abstain, information can help by mobilizing him for you, but can hurt if it mobilizes him for your opponent. The location of his prior midpoint determines which response is more likely.

To summarize this discussion:

Proposition 2 *Assume groups are internally homogenous, and let (t_R, t_L) be a specialized equilibrium.*

- *If t_R is not the empty group, then it is either a group that initially intended to vote L or a group that initially intended to abstain and has midpoint greater than* 1/2.
- *If t_L is not the empty group, then it is either a group that initially intends to vote R or a group that initially intends to abstain and has midpoint less than* 1/2.

The different conditions making groups targetable correspond to substantively different kinds of strategies. When a candidate targets an initial abstainer, she is aiming at *mobilization.* When targeting a supporter of the other candidate, the substantive interpretation depends on how the voter responds to the good signal. If the voter will switch to abstention given the good signal, then the candidate is aiming at *demobilization.* If the voter will continue to vote but switch candidates given the good signal, then the candidate is aiming at *conversion.* To refer to targeting of the other candidate's supporters without reference to the exact aim, call it *attack.*

Although this section has provided some insights into (pure-strategy) equilibrium targeting, it is incomplete in two ways. First, I have not given conditions under which pure-strategy equilibria exist. The specialized profile identified in Proposition 1 will only be an actual equilibrium if it satisfies a *no incentive to match* condition—neither candidate should do better by deviating to the profile where one group gets two signals. This condition is not trivial because there can be a kind of nonmonotonicity in candidate preferences over the number of signals some group receives. In particular, a candidate might be best off if a certain group gets no signals, and worst off if that group gets one signal, with *two* signals giving an intermediate payoff. This is best illustrated by example.

Consider a voter who initially plans to vote for candidate R. Then candidate R can only lose if this voter gets any information, making no signals best. But if this voter is already getting one signal, sending him another can be beneficial. Assume that, with one bad signal, his posteriors fall to the point that he abstains. Conditional on this event, a second signal has a positive chance of moving him back to his prior. And the extra signal has no cost in the event that the first signal was positive, since his beliefs are then at least as favorable to R as his prior. So long as two bad signals do not tip him all the way to supporting candidate L, candidate R strictly prefers that he receive two signals rather than one.

Second, I have shown that several strategies are potentially optimal but have not given any insight into the choice between those strategies. The next section gives a complete analysis of equilibrium (including pure-strategy existence) for the case of symmetric electorates with internally homogenous groups. The following section looks at several examples to show how heterogeneous groups affect equilibrium behavior, while Appendix D explores how heterogeneity and asymmetry affect equilibrium existence.

4 Symmetric and Homogenous Electorates

Now I turn to a case for which I can give a complete analysis. I use the following notion:

Definition 4 *The electorate is* symmetric *if:*

- $m_1 = b < m_2 = s < 1/2$,
- $m_1 = 1 - m_4$ *and* $m_2 = 1 - m_3$,
- $r_g = r$ *for all* g, *and*

- $\beta_g = 1/4$ *for all* g.

(The letters are mnemonics for “base” and “swing”, respectively.)

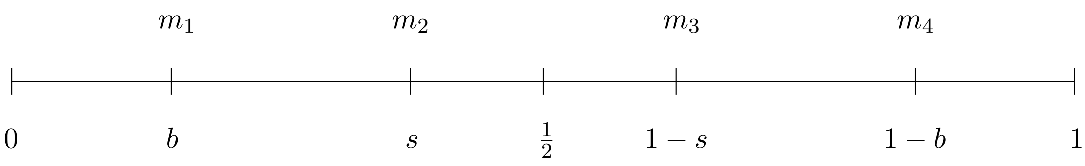

Figure 1: Prior midpoints with a symmetric electorate.

The combination of a symmetric electorate and homogenous groups gives the two candidates the same opportunities.

Lemma 3 *With a symmetric electorate and homogenous groups,* $\delta_1 = -\delta_4$ *and* $\delta_2 = -\delta_3$.

This fact is a key step in proving the main existence result:

Proposition 3 *In a symmetric environment with internally homogenous groups, there is a specialized pure-strategy equilibrium.*

Proof See Appendix C. □

This Proposition says that a pure-strategy equilibrium exists. The only possibility consistent with Lemma 1 is the profile in which R targets the group with greatest δ_g and L targets the group with least δ_g. The main step in the proof is showing that this profile does not create incentives for “matching”, that is, for one candidate to switch to targeting the same group that the opponent targets. Doing so has a potential benefit because giving an extra signal to the group targeted by the opponent might undo the effect of a positive signal for the other candidate, or an additional negative signal might enhance the beneficial effects of a negative signal from the other candidate. It turns out that these potential benefits are never worth the cost, namely foregoing the original plan. Symmetry ensures that, with the original plan, the other candidate's gain from her targeting is exactly offset by the candidate's own gain from targeting. When matching, however, the offset is only partial. The reason differs between two cases. When the other candidate is mobilizing, matching is move from a group that has a high probability of responding "correctly" to a group with a lower probability. When the other candidate is attacking, on the other hand, only one configuration of signals changes the group's behavior, and that change is in the wrong direction.

The proof makes clear that all of the incentive constraints hold as strict inequalities, so small deviations from symmetry or homogeneity will not upset the existence of a pure-strategy equilibrium. Nonetheless, the statement of the Proposition cannot be significantly improved. Appendix D presents two examples, one with asymmetry and one with heterogeneous groups, of environments with no pure strategy equilibria, even though the underlying type structure is identical to that of the current section.

Figure 2 provides an example of equilibrium targeting—it shows the strategies as a function of ambiguity aversion, r, and the signal precision, q, for locations $b = 0.2$ and $s = 0.4$.[13] This example illustrates several general features of equilibrium strategies, developed in the next few paragraphs.

The first step in finding the equilibrium targeting strategies is to find out who will turnout with no signal—by Proposition 2, this determines who the possible targets are.

Lemma 4 *1. If $r < 1/2 - s$, then all voters turnout absent a signal.*

2. If $1/2 - s < r < 1/2 - b$, then only groups 1 and 4 turnout absent a signal.

3. If $1/2 - b < r$, then no one turns out absent a signal.

This Lemma divides the parameter space into three regions, delimited in Figure 2 by the vertical lines. To the left of $q = 1/2 - s$, all voters turnout with no signal. Proposition 2 then implies that, a candidate will only consider targeting groups that currently plan to vote against her, hoping to move the group's beliefs enough to change their vote choice. Such a move is more likely the closer to 1/2 is the group's prior beliefs. So the best group to target is the moderate supporters of the other candidate, and R targets 2 and L targets 3.

Similarly, to the right of $q = 1/2 - b$, no one turns out absent a signal. This time, Proposition 2 implies that a candidate will consider only groups that lean in her direction. Again, the candidate

[13]In the examples and propositions to follow, I add the assumption that the signal must be sufficiently informative, $q > 1/2 + (s - b)$. This is because, for low informativeness, at least one of attack or mobilization will not work at all. Thus the equilibrium is determined by the combination of the strategic considerations and "technological" considerations about which voters are movable.

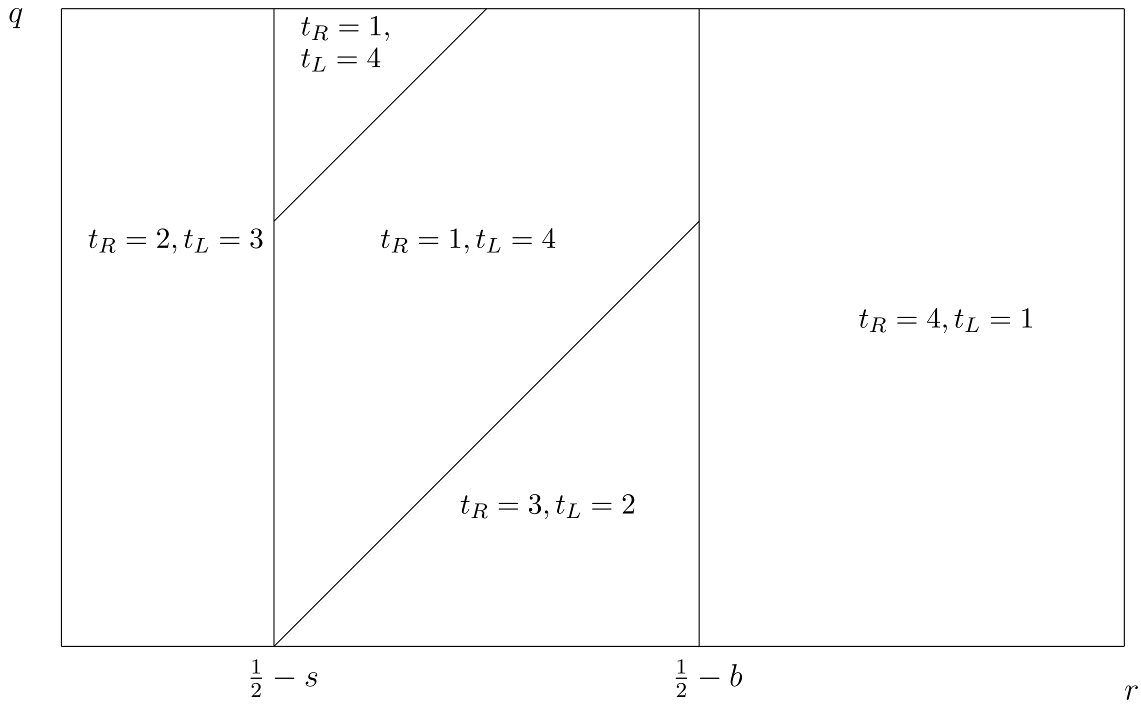

Figure 2: Equilibrium targeting profiles as a function of r and q with the locations fixed at $s = 0.4$ and $b = 0.2$. On the horizontal axis, r ranges from 0 to 1/2, while, on the vertical axis, q ranges from $1/2+(s-b)$ to 1.

goes for the group more likely to move in the favorable direction, this time the more extreme group.

This leaves the intermediate case, $1/2 - s < r < 1/2 - b$. Lemma 2 implies that the equilibrium calls for a candidate to either attack the other candidate's base or to mobilize her own moderates. The diagonal lines in the middle region divide that region into three subregions, corresponding to different degrees of voter responsiveness to signals, as outlined in the following:

Lemma 5

1. *If $r > s + q - 1$, then group 1 will never choose R and group 3 will never choose L.*
2. *If $s + q - 1 > r > b + q - 1$, then group 1 will never choose R, but group 3 will choose L if it sees signal $\underline{s}$.*
3. *If $b + q - 1 > r$, then group 1 votes R with the high signal and group 3 votes L with the low signal.*

Below the bottom diagonal, the only benefit to targeting a group is the prospect of increasing the margin of victory in the case that the group switches from L to A or from A to R. Thus successful targeting of moderates and base voters increase the margin by the same amount, but success is more likely with the moderates, and the familiar likelihood argument implies that the candidates target the moderates for mobilization.

Play in the other two subregions, on the other hand, typically depends on the precision of the signal. Above the lower diagonal, undecided voters can be mobilized for either side. Thus, the benefit of targeting them is diluted—there is some risk they will actually be turned out for the opponent. This makes targeting the extremists more attractive. In this case, the benefit from targeting moderates is decreasing in the probability that they get the wrong signal, that is, it is increasing in q.

Above the top diagonal, voters who plan to vote for one candidate will jump all the way to supporting the other candidate with just one signal. This increases the relative benefit targeting base voters even more because the gain in case of a good signal doubles. Again, this gain comes only when the signal goes against the prior, so it is more likely when q is small.

So there is generally a tradeoff between the higher probability of success with moderates and the greater risk inherent in targeting them. In Figure 2, the tradeoff is always resolved in the same way—attack is optimal. But this is not a general result—consider Figure 3. In that picture, there is only one diagonal line dividing the middle range of ambiguity aversion—even with perfectly precise signals, the base voters will not vote contrary to their prior. But now, the upper subregion is itself split in two by a horizontal line. Below that line, the likelihood of the good signal to the moderates is low enough that the extra risk means the candidates would rather target base voters. Above the line, on the other hand, the risk is low enough for moderates to be targeted.

Turning these considerations into an exact description of the equilibrium targeting strategies is straightforward, but not terribly enlightening. Rather than following that path, I offer comparative static results to show that the situations depicted in Figures 2 and 3 are typical.

Start with comparative statics on ambiguity aversion, r. The next Proposition formalizes two features illustrated in Figures 2 and 3. First, mobilization is more likely the greater is ambiguity aversion. Second, both mobilization and attack are more likely to be aimed at base voters the greater is ambiguity aversion.

Proposition 4 *Fix locations b and s and signal precision $q \geq 1/2 + (s - b)$. There are critical*

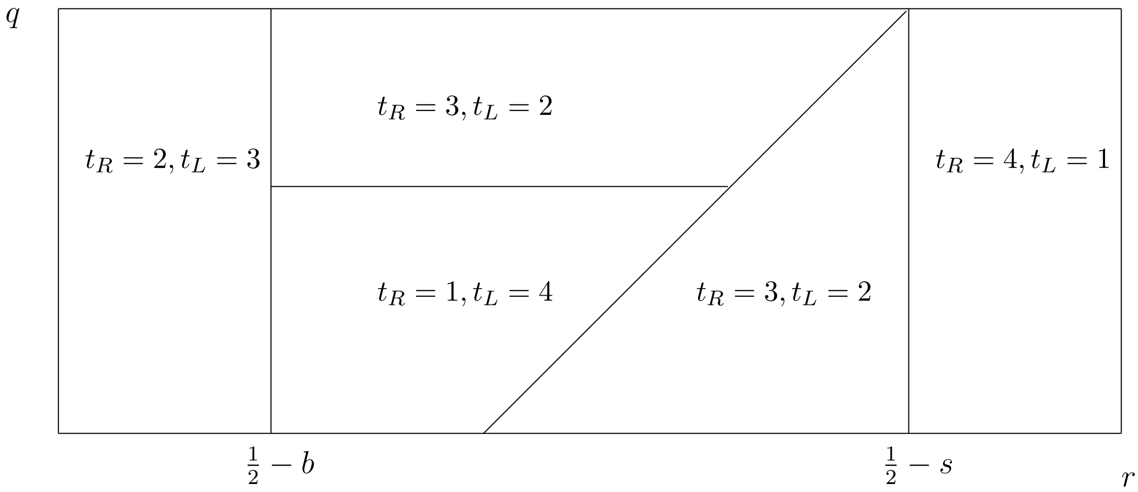

Figure 3: Equilibrium targeting profiles as a function of r and q with the locations fixed at $s = 0.4$ and $b = 0.1$. On the horizontal axis, r ranges from 0 to 1/2, while, on the vertical axis, q ranges from $1/2 + (s - b)$ to 1.

values $\overline{r} = 1/2 - b$, $\underline{r} = 1/2 - s$, and r^* such that strategies in a specialized equilibrium are

- *target own base voters if ambiguity is high* $[(t_R, t_L) = (4, 1)$ *if* $r > \overline{r}]$;
- *target own swing voters* $[(t_R, t_L) = (3, 2)]$ *if* $\overline{r} > r > r^*$;
- *target other-side base voters* $[(t_R, t_L) = (1, 4)]$ *if* $r^* > r > \underline{r}$; *and*
- *target other-side swing voters if ambiguity is low* $[(t_R, t_L) = (2, 3)$ *if* $r < \underline{r}]$.

Two notes. First, r^* need not lie between $\underline{r}$ and $\overline{r}$, as seen in the top part of Figure 2. Second, when r is equal to one of the critical values, there are ties in the δ_g, so there are multiple specialized equilibria.[14]

Proof Consider first the case of $r < \underline{r} \equiv 1/2 - s$. By Proposition 2 and Lemma 4, candidate L can restrict attention to groups 1 and 2. To see that group 2 is better to target than group 1, note that if the high signal induces group 1 to vote R, then it induces group 2 to vote R, and that if the high signal induces group 1 to abstain, then it induces group 2 to either abstain or vote R. (These follow from monotonicity of $\overline{\mu}$ and $\underline{\mu}$ in the priors.) Finally, the probability of the high signal is greater for group 2, because $d\sigma/d\alpha > 0$. Together, these imply that targeting 2 makes a bigger contribution to R's margin of victory.

[14]For example, if $r = \overline{r} \neq r^*$, then $(4, 1)$, $(3, 2)$, $(4, 2)$, and $(3, 4)$ are all specialized equilibria.

If $r > \overline{r} \equiv 1/2 - b$, then an argument just like the previous one shows that R will target group 4.

To study the incentives when $\frac{1}{2} - s < r < \frac{1}{2} - b$, use the following:

Lemma 6

$$\begin{aligned}\delta_3 - \delta_1 &= \mathbb{I}[r > s + q - 1](\sigma_3 - \sigma_1) \\ &\quad + \mathbb{I}[s + q - 1 > r > b + q - 1](2\sigma_3 - 1 - \sigma_1) \\ &\quad + \mathbb{I}[r < b + q - 1](2\sigma_3 - 1 - 2\sigma_1).\end{aligned}$$

Notice that

$$\sigma_3 - \sigma_1 > 2\sigma_3 - 1 - \sigma_1 > 2\sigma_3 - 1 - 2\sigma_1,$$

where the first inequality follows from $2\sigma_3 - 1 = \sigma_3 - (1 - \sigma_3)$.

Because the σs do not depend on r, the expression in Lemma 6 makes it easy to do the comparative statics on r for a fixed q: increasing r only affects which indicator function is equal to one, with larger values of r making the probability difference with positive weight larger. Thus the difference $\delta_3 - \delta_1$ is nondecreasing in r, and there is a cutoff r^* such that swings are targeted if and only if $r > r^*$. □

Proposition 4 can be interpreted in terms of a comparison of different types of elections. Based on Heath and Tversky's (1991) competence hypothesis, voters should be more ambiguity averse the worse is their background knowledge about the race. Thus, as the background informational condition varies, the partisanship of the marginal voters changes as well. In Presidential elections, there are high-information public signals, so more partisan voters will not be marginal. On the other hand, in races for lower offices, information will be lower and the marginal voters will be more partisan. This will produce swings in the degree that parties chase swing voters (m close to 0) versus playing to their base (m far from 0).

It's clear from Figure 3 that changes in the signal precision do not lead to such straightforward comparative statics. When signals are imprecise, the choice is between mobilizing swing voters or demobilizing the opponent's base. Swing voters are more attractive because they are more likely to change. More precise signals can make a swing voter turnout for the opponent if the signal is negative, and can make a base supporter of the opponent convertible. Thus a change in the signal precision that changes the set of possible voter responses makes attack more attractive. Within a region where the set of possible responses are fixed, on the other hand, more precise signals reduce the risk that swing voters will be pushed the wrong way, making swing voters more attractive.

Proposition 5

1. *Changes in signal precision that change the set of possible voter responses can reduce the likelihood of mobilization.*

2. *If the signal becomes more precise but possible voter responses do not change, then mobilization becomes more likely.*

Proof

1. For the first claim, it suffices to give an example. Let

$$(b, s, r) = (0, .45, .1,).$$

   For $q < .65$, then $r > s + q - 1$, and the optimal strategy is mobilization. But for

$$.65 < q < \frac{2 - 2s}{3 - 4s} \approx .85,$$

   the optimal strategy is to attack the other candidate's base.

2. There are three subcases, one for each possible value of the indicator variable in the expression in Lemma 6.

   - If $r > s + q - 1$, then group 1 will never choose R and group 3 will never choose L. Thus R will target 3 for all values of q.

   - If $s + q - > r > b + q - 1$, then R will target 3 iff

$$\sigma_3 - (1 - \sigma_3) = 2\sigma_3 - 1 > \sigma_1.$$

     Substitute the definitions of σ_1 and σ_3 and rearrange to rewrite this as

$$q > \hat{q} \equiv \frac{2 - 2s - b}{3 - 4s - 2b}.$$

- If $b + q - 1 > r$, then R will target 3 if and only if

$$2\sigma_3 - 1 > 2\sigma_1$$

or

$$q > \hat{\hat{q}} \equiv \frac{3 - 2s - 2b}{4 - 4s - 4b}.$$

□

Finally, it's also interesting to ask what happens in response to changes in what the figures hold fixed, namely the locations b and s. Say that the electorate is *more polarized* if both s and b decrease, at least one strictly.

Proposition 6 *An increase (decrease) in polarization makes campaigns more (less) likely to focus on mobilization.*

Proof It's immediate that a decrease in s leads to a decrease in $\underline{r}$ and that a decrease in b leads to a decrease in $\overline{r}$. Both of these changes sometimes change the equilibrium strategies from attack to mobilization, and never lead to the reverse change. So I'll be done as soon as I show that r^* is decreasing in both b and s.

Recall that $\sigma_3 = (1 - s)q + s(1 - q)$, and that this expression is decreasing in s. Similarly, $\sigma_1 = bq + (1 - b)(1 - q)$, and this expression is increasing in b. Thus a decrease in both s and b increases all of the differences involving the σs, tending to increase the difference $\delta_3 - \delta_1$. At the same time, these decreases move the positive indicator variable in the direction of larger values of the difference. Thus the difference increases uniformly when polarization increases, making mobilization more likely. □

The intuition is simple: polarization simultaneously increases the chance of successful mobilization and decreases the change of successful demobilization/conversion.

5 Internally Heterogenous Groups

It's natural to wonder how important the assumption of homogenous groups is to the results of the previous section. This question is particularly salient given campaign professionals' recent excitement about the prospect of "microtargeting"—using detailed statistical information and modern

	Fraction with prior midpoint			
Group	b	s	$1-s$	$1-b$
1	λ	$1/4-\lambda$	0	0
2	$1/4-\lambda$	λ	0	0
3	0	0	λ	$1/4-\lambda$
4	0	0	$1/4-\lambda$	λ

Table 2: Joint distribution of types and groups for the heterogeneous partisan example.

computing power to *refine* the profiles of groups that can be targeted (Gertner 2004; Malchow N.d.). While the practitioner literature is focused on the candidate-level improvement in the marginal benefits of targeting, it is natural for campaigns and elections scholars to wonder if microtargeting will have any systematic effects. Addressing this phenomenon requires a model in which the heterogeneity of targetable groups can vary.

To explore these issues, I will consider examples in which the voters are divided into the same four types as in the symmetric homogeneous example. Now, however, I will allow the groups to be internally heterogeneous. Comparing this case with the previous results is informative about improvements in the targeting technology, since “homogenizing” the groups can be interpreted as improved targeting. This modeling approach automatically builds in the reason for the practitioner’s excitement—targeting a mixture of two types of voter will not be as effective as targeting the more attractive of those two types. But how will candidate responses to the new incentives affect the qualitative nature of campaigns?

5.1 Heterogeneous Partisan Groups

For the first example, the joint distribution of group and type is given in Table 2, where $1/4 \geq \lambda \geq 1/8$. In this example, the candidates can perfectly tell who leans right and who leans left, but they cannot perfectly distinguish between base and swing leaners. As in the previous section, the groups can be ordered in terms of how strongly they lean toward the R candidate on average. If, for some vector of parameters (b, s, q, r), the equilibrium strategies involve targeting the groups in the same location relative to this order as in the homogenous case, say that the strategy is *qualitatively the* *same* as in the homogeneous case.

Throughout this section, I abuse notation slightly and let $\hat{\delta}_g$ denote the responsiveness of group g, reserving the symbol with no hat for the type resposivenesses calculated before. From Lemma 3,

$$\hat{\delta}_1 \equiv \lambda\delta_1 + (1-\lambda)\delta_2 = -\lambda\delta_4 - (1-\lambda)\delta_3 \equiv -\hat{\delta}_4$$

and

$$\hat{\delta}_2 \equiv (1-\lambda)\delta_1 + \lambda\delta_2 = -(1-\lambda)\delta_4 - \lambda\delta_3 \equiv -\hat{\delta}_3.$$

If $r < 1/2 - s$, then

$$\delta_2 > \delta_1 > 0 > \delta_4 > \delta_3,$$

so $\hat{\delta}_2$ is (at least weakly) greater than any other net benefit. This means that when ambiguity is low, each candidate targets the other side's weakest supporters. If $r > 1/2 - b$, then

$$\delta_4 > \delta_3 > 0 > \delta_2 > \delta_1,$$

so $\hat{\delta}_4$ is (at least weakly) greater than any other net benefit. This means that when ambiguity is high, each candidate targets the group that leans most strongly towards her. In each case, the strategy is qualitatively the same as in the case with perfect targeting.

In fact, this qualitative sameness holds in this example even in the case where $1/2 - s < r < 1/2 - b$. In this case δ_3 and δ_1 are positive, while δ_2 and δ_4 are negative. Thus a more refined analysis is needed to sign the $\hat{\delta}_g$. The analysis of the perfect targeting case makes it reasonable to guess that either group 1 or group 3 will be optimal for candidate R. By the definition of $\hat{\delta}$,

$$\begin{aligned}\hat{\delta}_3 &= \lambda\delta_3 + (1-\lambda)\delta_4 \\ &= \lambda\delta_3 - (1-\lambda)\delta_1\end{aligned}$$

and

$$\begin{aligned}\hat{\delta}_1 &= \lambda\delta_1 + (1-\lambda)\delta_2 \\ &= \lambda\delta_1 - (1-\lambda)\delta_3.\end{aligned}$$

Thus $\hat{\delta}_3 > \hat{\delta}_1$ if and only if

$$\lambda\delta_3 - (1-\lambda)\delta_1 > \lambda\delta_1 - (1-\lambda)\delta_3,$$

	Fraction with prior midpoint			
Group	b	s	$1-s$	$1-b$
1	1/4	0	0	0
2	0	λ	$1/4-\lambda$	0
3	0	$1/4-\lambda$	λ	0
4	0	0	0	1/4

Table 3: Joint distribution of types and groups for the heterogeneous moderate example.

or

$$\delta_3 > \delta_1.$$

But this says that targeting the diluted version of group 3 is optimal exactly when targeting the pure version is optimal in the homogeneous case.

Better targeting of the form that better separates partisans of one candidate into those already mobilized and those still in play does not lead to differences in the patterns of targeting. This is not the only way that better targeting can manifest itself, however. Improvements in the ability of candidates to separate undecided moderate voters into those who lean left and those who lean right do change optimal strategies.

5.2 Heterogeneous Moderate Groups

For the second example, the joint distribution of group and type is given in Table 3, where, again, $1/4 \geq \lambda \geq 1/8$.

The cases of high or low ambiguity are again easy. Since the extreme groups are not diluted, a candidate still has access to her optimal strategy from the case of perfect targeting. And the alternatives have become worse from dilution. Thus, there is no change in the optimum.

There is a change in the intermediate case, however. With imperfect targeting, group 3 is targeted only if

$$\hat{\delta}_3 = (2\lambda - 1)\delta_3 > \delta_1.$$

Because $2\lambda - 1 < 1$, this means that a candidate is less likely to target swing voters when targeting is imperfect.

This means that an improvement in targeting that makes candidates better able to divide undecided voters into those who lean towards the candidate and those who lean the other way will have a systematic effect on qualitative nature of targeting strategies. In particular, this kind of targeting improvement will lead campaigns to substitute away from demobilization/persuasion in favor of more mobilization of undecided voters. This contrasts with the case of better characterization of which partisans are turned out and which are not, where there is no change in the qualitative nature of strategies.

6 Turnout

The results so far have focused on the comparative statics of candidate strategy. But, with endogenous participation we can also study the comparative statics of turnout. There are two points to address: the comparative statics of turnout as the primitives vary, and the effectiveness of campaigns at manipulating turnout.

First, a formula for expected turnout. The expected share of group g voters who turn out given targeting strategies t_R and t_L is

$$\tau_g(t_R, t_L) = \sum_k \beta_{kg} \sum_{\tilde{s}} \Pr(\tilde{s} \mid m_k, r_k, n_g(t_R, t_L)) \left|\nu(n_g(t_R, t_L), \tilde{s}; m_k, r_k)\right|,$$

and overall net turnout is

$$\tau(t_R, t_L) = \sum_g \tau_g(t_R, t_L).$$

This differs from the definition of R's margin of victory only in that it uses the absolute value of the voting rule ν.

To build intuition, consider the symmetric homogenous example. When r is close to 0, all voters turn out regardless of their signals, and turnout is at its maximum of 1. When $r > 1/2 - b$, so candidates target their own bases for mobilization, a group 4 voter votes R after $\overline{s}$ and votes A after $\underline{s}$. Given this, voters in groups 1 and 4 vote with probability σ_4 and voters in groups 2 and 3 vote with probability 0. Thus overall expected turnout is $\sigma_4/2$. So turnout must be decreasing "on average"—as r increases from 0 to 1/2, turnout falls from 1 to $\sigma_4/2 < 1/2$.

The change in turnout as r increases is composed of two parts. First there is the "natural" decline—the fall off that would happen with no campaign at all. For small r, all voters have a clear favorite candidate and will thus turnout. When r crosses $1/2 - s$, the moderates start to entertain the possibility that either candidate might give the higher expected payoff, and they start to stay home. When r goes even further and crosses $1/2 - b$, the same thing happens to the moderates. At that point, turnout is zero. So absent a campaign, turnout would be nonincreasing.

The second factor influencing turnout is the candidate targeting behavior. The impact of this factor is typically not monotonic—the direction of the effect depends on whether the candidates engage in mobilization, which increases turnout, or attack, which does not increase turnout and may decrease it. Recall that there is a critical level of ambiguity where campaigns switch from focusing on attack to focusing at mobilization. Now, just above and just below this critical level, the levels of turnout absent a campaign are typically the same, but below the critical level campaign activity works to reduce turnout (at least weakly) while above the critical level campaign activity works to increase turnout.

Note well, this does not say that attacks always reduce turnout. Recall that a candidate may have two different motivations for attack—demobilization and conversion. To see how this matters, consider small r. When r is very small ($r < 1 - q - s$) a moderate with a contrary signal moves to the other candidate, not abstention. Thus when r is small enough, the campaign has no effect on turnout, although it does affect individual-level vote choices.

These results may help shed some light on the empirical work on the demobilizing effects of negative campaigns. Although campaign messages in the model are not differentiated in any semantic way, so variation in "tone" is not captured, the model's distinction between mobilization-oriented and attack-oriented messages does capture some of the incentives associated with "going negative".[15] The comprehensive review of the empirical literature by Lau et al. (1999) shows that the evidence on the demobilizing effects of negative campaigns is ambiguous—some studies find demobilization effects but others find that elections following negative campaigns actually have greater turnout. Although the model predicts that attack-oriented campaigns (at least weakly) demobilize voters, the correlation between the attack/mobilization decision and turnout can have either sign in equilibrium. This is because candidates choose negative campaigns when they have a better shot at demobilizing supporters of the opponent than at mobilizing latent supporters. When

[15] See Polborn and Yi (2006) and Mattes (2006) for models of the choice between positive and negative campaign messages that better capture the semantic valences of messages.

background conditions lead to relatively high expected turnout, there are relatively few mobilizable voters, and candidates are more likely to choose negative campaigns. Note that this explanation for the positive correlation has additional testable implications, namely that negative campaigning should be predicted by pre-campaign variables that predict high turnout. In the model, these include voter polarization and voter information.

7 Conclusion

This paper has characterized equilibrium campaigns when candidates can target specific groups for informative advertising. The strategies have several comparative statics properties. The targeted voters will be more partisan the lower is the initial level of information. Campaigns will focus more on mobilization, rather than attack, when prior information is limited, when the voters are more polarized, and when undecided voters can be more precisely categorized by their partisan leanings. These results flow from two ideas. First, the voters' aversion to casting ballots when they perceive ambiguity means that there two "pivots"—one between voting L and abstaining and another between abstaining and voting R. Second, changes in a voter's information change the distance between these two pivots, changing the relative benefits of targeting around each pivot.

The results about the partisanship of targeted voters suggest some conjectures about a model enriched to include promises of targeted transfers, namely that low informational environments will feature transfers targeted to partisans, rather than moderates. This conjecture differs greatly from the results in the traditional models of targeted redistribution (Lindbeck and Weibull 1987; Dixit and Londregan 1996).[16] In these models, parties target their resources to "swing voters"—voters who are evenly balanced between the two parties. These voters are the most likely to change their vote decision.[17] Ansolabehere and Snyder (2003) test this prediction with data on the distribution of expenditures by state governments. Contrary to the prediction, they find that the party in power skews the resource allocation in favor of districts that traditionally support the party, rather than to districts that have supported both parties at different times. They also find that regions that receive extra funds have greater turnout in the subsequent election. This suggests that the basic

[16]Schultz (2007) shows that targeted redistribution promises and informational advertising are complements.

[17]Strömberg (2002) finds supportive evidence for U.S. Presidential elections.

logic of the swing voter model is correct—parties target the groups where the marginal increase in net votes is greatest, but it suggests that turnout is a more relevant margin in state-level elections.

Not all of the theoretical work predicts swing voter outcomes. Dixit and Londregan (1996) also consider a "core support" version of their model. It is based on a stylized depiction of big-city machine politics. In this model, one of the parties controls the bureaucracy, and consequently has an advantage in targeting pork. Furthermore, the machine has better connections to its base of support than to swing voters. They suggest this as a model of pre-civil service municipal politics, so it doesn't apply to the kind of evidence that Ansolabehere and Snyder (2003) find. Cox and McCubbins (1986) present a model in which a party's "home base" is more responsive to spending, but they do not derive this assumption from a model of voters.

The model presented here provides microfoundations for the behavior similar to that assumed by Cox and McCubbins (1986) and found empirically by Ansolabehere and Snyder (2003). Explicitly modeling the relationship between voter responsiveness and information both connects the model closely to empirical studies of voter behavior and provides an explanation for differences across electorates and levels of government. An important agenda for future research is to combine these features with rich policy environments to explore the potentially interesting implications about how the decision of what level of government should provide public goods determines which groups in society get represented.

A Representing Incomplete Preferences

A ballot is what decision theorists call an *act*—a map from states of the world (θ) to consequences (attributes of the selected candidate). I follow Bewley (2002) and assume the standard Anscombe and Aumann (1963) axioms, except that completeness is weakened to hold only for constant acts.[18] Intuitively, the voter would have complete preferences were she certain of the candidates' attributes, but she may be unable to rank candidates because she is uncertain about the map between candidates and attributes.

Bewley's Theorem *If preferences satisfy all of the Anscombe and Aumann (1963) axioms except that completeness need hold only for constant acts, then the preferences are represented by a (cardinal) utility u and a* set *of probabilistic beliefs $\mathcal{P}$ in the sense that*

$$a \succ b \quad \textit{iff} \quad \mathbb{E}_p(u(a)) > \mathbb{E}_p(u(b)) \textit{ for all } p \in \mathcal{P}.$$

Why is this true? The best way to build some intuition for the result is to think first of the case with no uncertainty. Then preferences can be represented by a utility function if and only if they are complete and transitive.[19] What happens to the classical theory without completeness? Under some regularity conditions, transitive and reflexive preferences are represented by a *set* of utilities $\mathcal{U}$, where $a \succsim b$ if and only if $u(a) \geq u(b)$ for all $u \in \mathcal{U}$ (Ok 2002). This is because any reflexive and transitive order is equal to the intersection of all complete and transitive orders that contain it. (Zorn's lemma implies that the latter set is nonempty regardless of the cardinality of the set of alternatives.)

As an example, consider the alternatives $\{a, b, c\}$, and $\mathcal{U} = \{u, v\}$, where

$$u(a) = 0 \quad u(b) = 1 \quad u(c) = 2$$

and

$$v(a) = 1 \quad v(b) = 0 \quad v(c) = 2.$$

Then $c \succ a$ and $c \succ b$ since u and v agree on these comparisons, while a and b are not ranked.

[18] Ryan (2003) provides a useful survey of the Anscombe-Aumann model and various modifications, including the Bewley model that I use and the maximin expected utility model that Ghirardato and Katz (2002) use.

[19] Modulo topology.

A comparison to social choice theory may help explain this result. For any society of people with complete and transitive preferences, we can define the unanimity order, which consists of the preferences that all citizens hold in common. In general, this order will be incomplete—there will be two alternatives and two citizens such that those citizens disagree about how to rank the alternatives. The representation essentially comes from recognizing that *any* reflexive, transitive order is the unanimity order for some society, and treating that "society" as a description of an individual's preferences.

Moving from ordinal utility to expected utility, Aumann (1962) shows that the mixture space axioms, minus completeness, imply that the preference can be represented by a set of linear utility functions.

Bewley (2002) builds on Aumman's theorem to show that adding back a limited form of completeness (completeness over constant acts) is sufficient to mimic the Anscombe and Aumann (1963) proof and pin down the utility uniquely. In this case, the voter's preferences can be represented by a Bernoulli utility (unique up to an affine transformation) over outcomes and a closed, convex set of probability measures over states. An act a is better than b if and only if it leads to a greater expected utility according to all of the probability measures.

When preferences are complete, Bewley assumes that the DM chooses an alternative that is *optimal* in the sense that it is weakly preferred to all other alternatives. (If there are multiple optima, then the DM is indifferent between them, and is happy to choose any of them.) With incomplete preferences, this assumption cannot be used, since optimal choices might not exist. (This will be the case here whenever two beliefs lead to different best choices—no act dominates a *and* no act dominates b.) Instead, Bewley assumes that the DM chooses an alternative that is *maximal* in the sense that there is no other alternative that is preferred to the choice. We cannot be as flippant about multiple maximal elements as we could about multiple optima—the DM is not indifferent between the different maximal elements. Thus, to complete the model, we need to know what the voter does when not all alternatives can be ranked. Bewley assumes that there is some status quo act, and that the voter chooses this status quo unless some act dominates it. He calls this assumption "inertia". In a voting application, a natural choice for the status quo is to abstain unless some candidate dominates.

B Proofs of Lemmata

B.1 Proof of Lemma 1

The key to both claims is the following:

Lemma 7 *If (t_R, t_L) is a pure strategy equilibrium and there is a group $g \in \{1,2,3,4\}$ with $n_g(t_R, t_L) = 2$, then $\pi_g(2) = \pi_g(1)$.*

Proof If $\pi_g(2) > \pi_g(1)$, then candidate L can increase her margin of victory by deviating and not targeting g. And if the inequality is reversed, then candidate R increases her margin of victory by deviating and not targeting g. Since (t_R, t_L) is an equilibrium, neither candidate has an incentive to deviate, and $\pi_g(2) = \pi_g(1)$. □

To prove the first statement, let (t_R, t_L) be a pure-strategy equilibrium such that there is a group g with $n_g(t_R, t_L) = 2$. I will show that the specialized profile $(0, t_L)$ is also an equilibrium. Because (t_R, t_L) is an equilibrium, neither candidate benefits by switching to another group. Furthermore, because the expected R margin is the same with one and two signals, if R switches to not targeting g, its margin is unaffected. And this move does not change either candidate's incentive to target the other groups, because the gain from doing so is the same however many candidates target group g. Thus R has no strict incentive to use the freed-up funds to target some new group, and L has no strict incentive to switch to another group. Thus the new profile is also an equilibrium.

The second statement is immediate from the definitions of specialized equilibrium and strict responsiveness to signals, and the Lemma. □

B.2 Proof of Lemma 2

Fix two prior midpoints $m > m'$, both of which imply abstention absent a signal. Then the result follows from the chain of inequalities:

$$\begin{aligned}
&\beta_g \left(\sigma(m)\mathbb{I}[\underline{\mu}(\overline{s}; m, r) > 1/2] - (1-\sigma(m))\mathbb{I}[\overline{\mu}(\underline{s}; m, r) < 1/2]\right) \\
&\quad \geq \quad \beta_g \left(\sigma(m')\mathbb{I}[\underline{\mu}(\overline{s}; m, r) > 1/2] - (1-\sigma(m'))\mathbb{I}[\overline{\mu}(\underline{s}; m, r) < 1/2]\right) \\
&\quad \geq \quad \beta_g \left(\sigma(m')\mathbb{I}[\underline{\mu}(\overline{s}; m', r) > 1/2] - (1-\sigma(m'))\mathbb{I}[\overline{\mu}(\underline{s}; m, r) < 1/2]\right) \\
&\quad \geq \quad \beta_g \left(\sigma(m')\mathbb{I}[\underline{\mu}(\overline{s}; m', r) > 1/2] - (1-\sigma(m'))\mathbb{I}[\overline{\mu}(\underline{s}; m', r) < 1/2]\right).
\end{aligned}$$

To see that the first inequality holds, differentiate the expression for δ with respect to σ to get

$$\beta_g \left(\mathbb{I}[\underline{\mu}(\overline{s}; m, r) > 1/2] + \mathbb{I}[\overline{\mu}(\underline{s}; m, r) < 1/2]\right) \geq 0,$$

with strict inequality if at least one of the signals leads the voter to turnout. The second inequality follows because $\underline{\mu}$ increasing in m implies $\mathbb{I}[\underline{\mu} > 1/2]$ is nondecreasing in m; the third follows similarly from $\mathbb{I}[\overline{\mu} < 1/2]$ is nonincreasing in m. □

B.3 Proof of Lemma 3

It suffices to show that $\pi_g(n) = -\pi_{5-g}(n)$ for all n. Symmetry implies that $\sigma_1 = 1 - \sigma_4$ and $\sigma_2 = 1 - \sigma_3$. Thus, if $n \geq 1$, group 1, say, has the same likelihoods of moving in the L and R directions as does group 4 of moving in the R and L directions, respectively. Furthermore, a move is the magnitude to make 1 vote L (R) if and only if the analogous move is of the magnitude to make 4 vote for R (L).

For the $n = 0$, the claim follows from

$$m_4 - r_4 > 1/2 \text{ iff } 1 - m_1 - r_{>}1/2 \text{ iff } 1/2 > m_1 + r_1,$$

where the fist biconditional follows from symmetry. □

B.4 Proof of Lemma 4

$r < 1/2 - s$ if and only if $1/2 < 1 - s - r$ if and only if groups 2 and 3 turnout with no signal. Similarly, $r < 1/2 - b$ if and only if $1/2 < 1 - b - r$ if and only if groups 1 and 4 turnout with no signal. □

B.5 Proof of Lemma 5

A member of group 1 votes R following signal $\overline{s}$ if

$$\underline{\mu}(\overline{s}; b, r) = \frac{(b-r)q}{(b-r)q + (1-b+r)(1-q)} > \frac{1}{2},$$

which holds if and only if

$$r < b + q - 1.$$

Similarly, a member of group 3 votes L following signal $\underline{s}$ if

$$\overline{\mu}(\underline{s}; 1-s, r) = \frac{(1-s+r)(1-q)}{(1-s+r)(1-q)+(s-r)q} < \frac{1}{2},$$

which holds if and only if

$$r < s + q - 1,$$

$\square$

B.6 Proof of Lemma 6

Consider first group 1. The signal to this group is $\overline{s}$ with probability σ_1. By Lemma 5, if $r > b+q-1$, the high signal leads the group to abstain, while if $r < b+q-1$, it leads the group to vote R. With the low signal, it votes L always. Thus

$$\delta_1 = \mathbb{I}[r > b+q-1]\sigma_1 + \mathbb{I}[r < b+q-1]2\sigma_1.$$

A similar argument gives

$$\delta_3 = \mathbb{I}[r > s+q-1]\sigma_3 + \mathbb{I}[r < s+q-1](2\sigma_3 - 1).$$

Then the statement follows from a simple subtraction. $\square$

C Proof of Proposition 3

Choose $j \in \arg\max_g \{\delta_g\}$, and let $k = 5 - j \pmod 5$. There are two cases.

First consider $\delta_j = 0$. Then the profile with no targeting is a pure Nash equilibrium.

Second consider $\delta_j > 0$. Here I claim that the profile in which R targets j and L targets k is a pure Nash equilibrium.

The construction ensures that neither candidate will deviate to either the empty group or a group in $\{1,2,3,4\} \setminus \{j,k\}$. The only remaining deviation is to match the other candidate. By symmetry, it is sufficient to show that R does not do better by targeting k.

If R targets j, then R's margin of victory is the baseline (what she gets with no campaign activity at all) plus the change in the margin when each of j and k are given one signal. Because of the symmetric structure, both the baseline and the sum of the changes from j and k are 0.

If R deviates and matches L in targeting k, then R's margin of victory is the baseline of zero plus the change in the margin when k gets two signals. Thus the deviation will be unprofitable if and only if the change in the margin given two signals is nonpositive. The change in the margin is

$$\sigma^2 \left(\nu(\overline{s}\overline{s}) - \nu(\emptyset)\right) + 2\sigma(1-\sigma)\left(\nu(\overline{s}\underline{s}) - \nu(\emptyset)\right) + (1-\sigma)^2 \left(\nu(\underline{s}\underline{s}) - \nu(\emptyset)\right).$$

Because the signals $\overline{s}$ and $\underline{s}$ cancel each other out, $\nu(\overline{s}\underline{s}) = \nu(\emptyset)$, and the change in the margin reduces to

$$\sigma^2 \left(\nu(\overline{s}\overline{s}) - \nu(\emptyset)\right) + (1-\sigma)^2 \left(\nu(\underline{s}\underline{s}) - \nu(\emptyset)\right).$$

Thus it suffices to show that

$$\sigma^2 \left(\nu(\overline{s}\overline{s}) - \nu(\emptyset)\right) \leq -(1-\sigma)^2 \left(\nu(\underline{s}\underline{s}) - \nu(\emptyset)\right).$$

By Proposition 2, there are two cases.

1. $m_k < 1/2$. In this case, L is targeting k for mobilization. This means that $\nu(\emptyset) = 0$ and $\nu(\underline{s}\underline{s}) = -1$. Also, $\nu(\overline{s}\overline{s}) \geq 0$ and $\sigma < 1/2 < 1-\sigma$. Thus

$$\sigma^2 \nu(\overline{s}\overline{s}) \leq \sigma^2 < (1-\sigma)^2,$$

   and the deviation is not profitable.

2. $m_k > 1/2$. In this case, L is targeting k for attack. This means that $\nu(\overline{s}\overline{s}) = \nu(\emptyset) = 1$ and $\nu(\underline{s}\underline{s}) \leq 0$. Thus

$$0 \leq (1-\sigma)^2 \leq (1-\sigma)^2 \left(\nu(\emptyset) - \nu(\underline{s}\underline{s})\right),$$

   and the deviation is not profitable.

□

D Examples Without Pure-Strategy Equilibria

Internally heterogeneous groups The first example is in the context of the mixed-partisan example from Section 5.1. Consider a case with parameters $(b, s, q, r, \lambda) = (.39, .49, .6, .1, 1/8 - \epsilon)$. Then there is intermediate ambiguity ($s + q - 1 > r > b + q - 1$), and mobilization is the optimal strategy. Thus the candidate for pure-strategy equilibrium is ($t_R = 3, t_L = 2$). At these parameters, a type $1-b$ voter abstains after both signals $\underline{s}$ and $\underline{ss}$ but a type $1-s$ voter abstains after $\underline{s}$ and votes L after $\underline{ss}$. Then

$$\delta_3 = -(1/4-\lambda)\sigma_4 + \lambda\sigma_3,$$

while the contribution with 2 signals is

$$-(1/4-\lambda)\sigma_4^2 + \lambda(2\sigma_3 - 1).$$

There is an incentive for L to match if

$$1 \approx \frac{\lambda}{1/4-\lambda} < \frac{\sigma_4^2}{2\sigma_3 - 1} \approx 68.$$

Thus L will deviate from the only possible equilibrium profile, and there is no pure-strategy equilibrium.

Lopsided Electorates The simplest way to introduce asymmetry is to let the measure of leftist voters be $1-\beta$ and the measure of rightist voters be $\beta \geq 1/2$, where within each group the voters are equally divided between extremists ($m = b$ and $m = 1-b$) and moderates ($m = s$ and $m = 1-s$).

I will construct an example in this framework that does not have a pure strategy equilibrium. Consider a case where no one votes absent a signal: $r > 1/2 - b$. Also assume that one favorable signal will tip group 4 to vote R, but even 2 unfavorable signals will not lead that group to vote for R. This is implied, for example, by $(b, r, q) = (0.4, 0.3, 0.6)$.

In this case, the R candidate must target group 4 in any specialized equilibrium—the text already shows this for the case of $\beta = 1/2$, and increasing β only makes group 4 more attractive. Furthermore, the L candidate will never target groups 2 or 3—for group 2 this follows from the text and the fact that changes in β to not affect the tradeoff between groups 1 and 2. And Proposition 2 implies that L cannot target 3. So the only candidate for a specialized equilibrium is for L to target 1. But she will strictly prefer to target 4 if

$$\beta\sigma_4(1-\sigma_4) > (1-\beta)(1-\sigma_1),$$

where the LHS is the probability that the signal sent by L offsets a good signal from the R candidate. Thus if

$$\beta > \frac{1-\sigma_1}{\sigma_4(1-\sigma_4) + 1 - \sigma_1},$$

there is no specialized equilibrium, and thus no pure strategy equilibrium, by Lemma 1.

References

Achen, Christopher H. 2005. "Bayesian Voters and their Turnout Decisions: Unifying the Behavioral Literature." Unpublished paper.

Alvarez, R. Michael. 1997. *Information and Elections*. Ann Arbor: University of Michigan Press.

Alvarez, R. Michael and D. Roderick Kiewiet. 2005. "Rationality and the Recall." Unpublished paper, Caltech.

Anscombe, F. J. and R. J. Aumann. 1963. "A Definition of Subjective Probability." *Annals of Mathematical Statistics* 34:199–205.

Ansolabehere, Stephen and James M. Snyder, Jr. 2003. "Party Control of State Government and the Distribution of Public Expenditures." MIT working paper.

Aumann, Robert J. 1962. "Utility Theory without the Completeness Axiom." *Econometrica* 30:445–462.

Bartels, Larry M. 1986. "Issue Voting Under Uncertainty: An Empiricial Test." *American Journal of Political Science* 30(4):709–728.

Bartels, Larry M. 1988. *Presidential Primaries and the Dynamics of Public Choice*. Princeton, NJ: Princeton University Press.

Bewley, Truman F. 2002. "Kightian Decision Theory: Part I." *Decisions in Economics and Finance* 25:79–110.

Brady, Henry E. and Stephen Ansolabehere. 1989. "The Nature of Utility Functions in Mass Publics." *American Political Science Review* 83(1):143–163.

Camerer, Colin and Martin Weber. 1992. "Recent Developments in Modeling Preferences: Uncertainty and Ambiguity." *Journal of Risk and Uncertainty* 5:325–370.

Cox, Gary W. and Mathew D. McCubbins. 1986. "Electoral Politics as a Redistributive Game." *Journal of Politics* 48(2):370–389.

Curley, Shawn P., Frank J. Yates and R. A. Abrams. 1986. "Psychological Sources of Ambiguity Avoidance." *Organizational Behavior and Human Decision Processes* 38:230–256.

Dixit, Avinash and John Londregan. 1996. "The Determinants of Sucess of Special Interests in Redistributive Politics." *Journal of Politics* 58(4):1132–1155.

Edlin, Aaron, Andrew Gelman and Noah Kaplan. 2007. "Voting as a Rational Choice: Why and How People Vote To Improve the Well-Being of Others." *Rationality and Society* 19(3):293–314.

Ellsberg, Daniel. 1961. "Risk, Ambiguity, and the Savage Axioms." *Quarterly Journal of Economics* 75:643–669.

Feddersen, Timothy J. and Alvaro Sandroni. 2006. "A Theory of Participation in Elections." *American Economic Review* 96(4):1271–1282.

Feddersen, Timothy J. and Wolfgang Pesendorfer. 1996. "The Swing Voter's Curse." *American Economic Review* 86:408–424.

Fenno, Jr., Richard F. 1978. *Home Style: House Members in Their Districts.* Boston: Little, Brown.

Fox, Craig R. and Amos Tversky. 1995. "Ambiguity Aversion and Comparative Ignorance." *Quarterly Journal of Economics* 110:585–603.

Frisch, Deborah and Jonathan Baron. 1988. "Ambiguity and Rationality." *Journal of Behavioral Decision Making* 1:149–157.

Gertner, Jon. 2004. "The Very, Very Personal is the Political." *New York Times* . Magazine, February 15.

Ghirardato, Paolo and Jonathan N. Katz. 2002. "Indecision Theory: Quality of Information and Voting Behavior." Unpublished paper.

Gilboa, Itzhak and David Schmeidler. 1989. "Maximin Expected Utility with a Non-Unique Prior." *Journal of Mathematical Economics* 18:141–153.

Halpern, Joseph Y. 2003. *Reasonong About Uncertainty.* Cambridge: MIT Press.

Hansen, John Mark. 1998. "Individuals, Institutions, and Public Preferences over Public Finance." *American Political Science Review* 92(3):513–531.

Heath, Chip and Amos Tversky. 1991. "Preference and Belief: Ambiguity and Competence in Choice under Uncertainty." *Journal of Risk and Uncertainty* 4:2–28.

Herrnson, Paul S. 2000. *Congressional Elections: Campaigning at Home and in Washington*. Washington, DC: CQ Press.

Horiuchi, Yusaku, Kosuke Imai and Naoko Taniguchi. 2005. "Designing and Analysing Randomized Experiments." Unpublished paper, Princeton University.

Key, V.O., Jr. 1966. *The Responsible Electorate: Rationality in Presidential Voting 1936–1960*. Cambridge: Harvard University Press.

Kreps, David M. 1990. *A Course in Microeconomic Theory*. Princeton: Princeton University Press.

Lassen, David Dreyer. 2005. "The Effect of Information on Voter Turnout: Evidence from a Natural Experiment." *American Journal of Political Science* 49(1):103–118.

Lau, Richard R., Lee Sigelman, Caroline Heldman and Paul Babbitt. 1999. "The Effects of Negative Political Advertisements: A Meta-Analytic Assessment." *American Political Science Review* 93(4):851–875.

Lindbeck, Assar and Jörgen W. Weibull. 1987. "Balanced-Budget Redistribution as the Outcome of Political Competition." *Public Choice* 52:273–297.

Malchow, Hal. N.d. The Targeting Revolution in Political Direct Communication: Why Old Techniques Won't Win Elections of the Future. In *The Road to Victory*, ed. Ron Faucheux. Dubuque: Kendall/Hunt pp. 148–154.

Mandler, Michael. 2005. "Incomplete Preferences and Rational Intransitivity of Choice." *Games and Economic Behavior* 50(2):255–277.

Mattes, Kyle. 2006. "Attack Politics: Who Goes Negative and Why?" Caltech working paper.

Merlo, Antonio. 2006. Whither Political Economy? Theories, Facts and Issues. In *Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress, Volume I*, ed. Richand Blundell, Whitney K. Newey and Torsten Persson. New York: Cambridge University Press pp. 381–421.

Ok, Efe A. 2002. "Utility Representation of an Incomplete Preference Relation." *Journal of Economic Theory* 104:429–449.

Polborn, Mattias K. and David T. Yi. 2006. "Informative Positive and Negative Campaigning." *Quarterly Journal of Political Science* 1(4):351–371.

Posner, Richard A. 2003. *Law, Pragmatism, and Democracy.* Cambridge: Harvard University Press.

Ryan, Matthew J. 2003. "Generalizations of SEU: A Geometric Tour of Some Non-Standard Models." Unpublished paper.

Schultz, Christian. 2007. "Strategic Campaigns and Redistributive Politics." *The Economic Journal* 117:936–963.

Slovic, Paul and Amos Tversky. 1974. "Who Accepts Savage's Axiom?" *Behavioral Science* 19:368–373.

Strömberg, David. 2002. "Optimal Campaigning in Presidential Elections: The Probability of Being Florida." IIES working paper.