advanced

1. **Describe how one can calculate the advanced heuristic value for any state of the puzzle**.

Summary: In addition to the Manhattan distance measure for the 2×2 block, we augment the heuristic function with the lower bound of the number of steps it takes to move a block below the 2×2 block away.

Details: We can simplify our problem by removing all blocks other than the 2×2 block and the blocks beneath it (including those that are partially beneath). Our cost function is the number of steps took, so let's consider different cases:

- If a 1×1 block lies beneath the 2×2 block, then we need 1 step to move it to one side of the board, so the 1×1 will no longer prevent the 2×2 block from moving down.
- For a 1×2 (vertical) block, similarly, we can move it to the side of the board. So this also takes 1 step.
- For a 2×1 (horizontal) block, we have two case: either the block is fully beneath (completely under the 2×2 block), or the block is partially underneath. For both cases, we need *at least* 2 steps to move it away. Note that this is a lower bound.
- For an empty spot, we don't need to take any step.

For the case that the 2×2 block is already on the bottom-level, but it is not in the goal position, we calculate the number of steps based on blocks (pieces) in the goal region.

The number of steps described above serve as the additional heuristic values. Let h_1 be the Manhattan distance function and h_2 be this additional function, then $h(n) = h_1(n) + h_2(n)$, and h is my advanced heuristic function.

To calculate $h_2(n)$, we can iterate through every piece to find the 2×2 block. Here, we automatically have h_1. Then, we check every piece under the 2×2 block and add up different heuristic values. Since our board is 4×5, we can calculate the heuristic value for any state n in constant time.

2. **Why is your advanced heuristic admissible**?

As I described above, h_2 is a *lower bound* to remove all the pieces beneath the 2×2 block, and h_1 the *least* number of steps to move the 2×2 block to the goal state (Manhattan distance). In any game state, we need to firstly remove all obstacles and then move the 2×2 block to the goal, so this process is strictly bounded below by h. Therefore, h never overestimates the cost, and h is admissible.

3. **Why does your advanced heuristic dominate the Manhattan distance heuristic**?

In any game state, we either have some obstacles underneath the 2×2 block, have some obstacles in the goal region, or it is the goal state, so h_2 will always be positive, except in the goal state.

To be rigorous, in the case that the game starts with the goal region only consists of empty spots and the 2×2 block (only 1 move is required to win), my heuristic does not dominate the Manhattan distance heuristic. This should be fine because I don't think there's any other heuristic that's both dominating and admissible in this case.