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Variance Reduction at Scale

**Improving IMC variance reduction methods
for thermal transport problems**



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Introduction & Theory

Thermal Radiative Transport (TRT)

- $\frac{1}{c} \frac{\partial I}{\partial t}(\vec{r}, \vec{\Omega}, \nu, t) + \vec{\Omega} \frac{\partial I}{\partial \vec{r}}(\vec{r}, \vec{\Omega}, \nu, t) + \sigma_a(\vec{r}, \nu, T)I(\vec{r}, \vec{\Omega}, \nu, t)$
 $= 2\pi\sigma_a(\vec{r}, \nu, T)B(\nu, T) + \frac{Q}{2}(\vec{r}, \nu, t)$
- $c_\nu(\vec{r}, T) \frac{\partial T}{\partial t}(\vec{r}, t) = \int_0^\infty \int_{-1}^1 \sigma_a(\vec{r}, \nu', T) [I(\vec{r}, \vec{\Omega}', \nu', t) - 2\pi B(\nu', T)] d\vec{\Omega}' d\nu'$

Implicit Monte Carlo (IMC)

- Developed by Fleck and Cummings in 1971 [1] to solve the TRT equations
- Uses ‘effective scattering’ events to model absorption/re-emission
- Two major approximations:
 1. Semi-implicit discretization of time
 2. Linearizes the TRT equations – not fully physical

Introduction & Theory Cont.

Standard Variance Reduction Methods

- IMC is stochastic – there is inherent uncertainty in the solution
- Variance reduction methods are implemented to improve simulation efficiency while producing equivalent unbiased results (e.g. implicit capture, splitting, Russian roulette, weight windows, etc.) [2]

Next Event Estimators (NXTEVT)

- Effective for limited particle histories, and large mean free paths [3] – i.e. few scattering events and few transported particles
- Method: ‘points’ particles toward region of interest and scores the tally accd. to

$$\phi(\mu) = w_0 \frac{e^{-\int_{\vec{r}}^{\vec{r}'} \Sigma_t(s) ds}}{S \cdot \mu} \quad \text{where } \mu \text{ is angle between } \vec{\Omega} \text{ & } \vec{r}', \text{ the surface normal,}$$

Σ_t is the total cross-section in the material, and S is the tally surface area

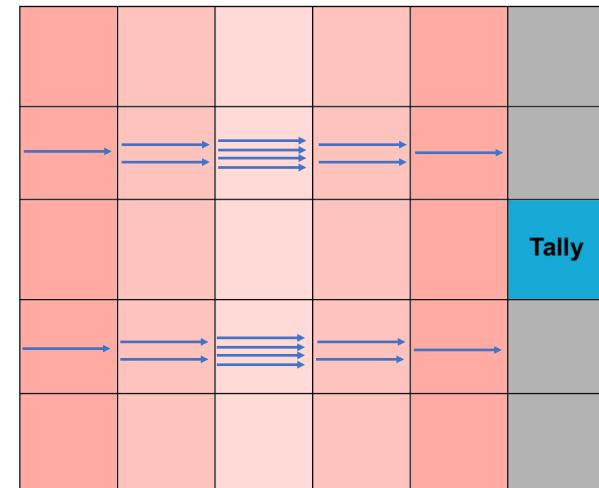


Figure 1:

A visualization as to why splitting and Russian roulette are ineffective in cases of interest to NXTEVT. Particles not initially directed towards the tally won't likely reach it.

Research Objective & Motivation

Motivation

- We attempt to produce a light curve with Monte Carlo from a simplified model of a supernova (*SN2006gy*) interacting with its circumstellar medium [4]
- The opacities of the supernova ejecta and ejecta-CSM Shock do not lend itself well to standard VRMs or the NXTEVT estimator
- The multi-physics involved require a large number of particles for answer to converge – makes the simulation computationally expensive

Objective

- Investigate the advantages of a response function based VRM for problems modeling supernova-CSM interactions
- Provide high-quality simulation data to compare with physical observation

Method and Technical Approach

Response Function Method Overview

- Run two problems – an inverse and a forward problem:
- The inverse transport problem generates the response function value (σ_r) for each cell
- The forward transport problem is run, using the σ_r values to tally at a particle's birth and every subseq. scatter event

Inverse Transport

- Initialize the particle uniformly on the tally surface, and direct it towards the source via cosine-distribution
- Trace particle through the mesh, calculating σ_r based on:
 - Weighted σ_a value the particle encounters based on the distance it travels through each cell
 - Weighted σ_a value for each cell based on every particle that passes through $\rightarrow \sigma_r$

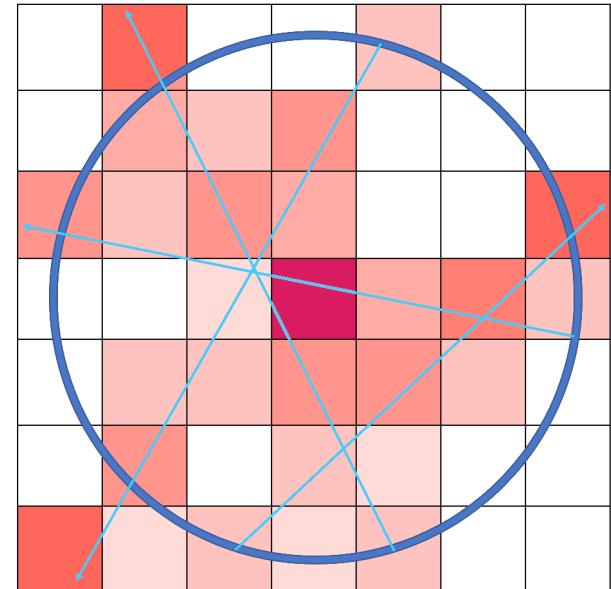


Figure 2:

A visualization of the inverse transport method. The source is the purple cell, and a darker shade of red corresponds to a higher σ_r value.

Method Cont. & Testing

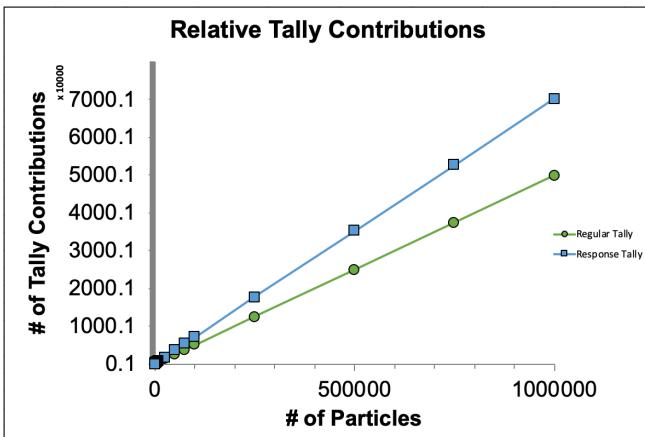
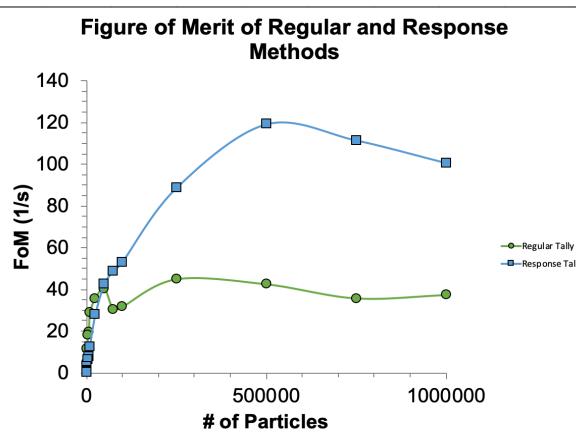
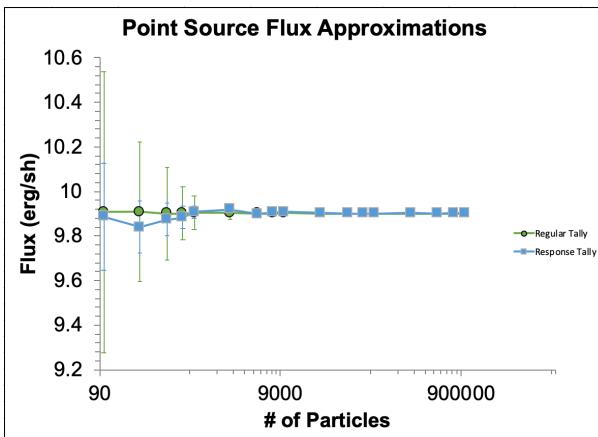
Forward Transport

- Upon the particles creation, a contribution is added to the tally accd. to:

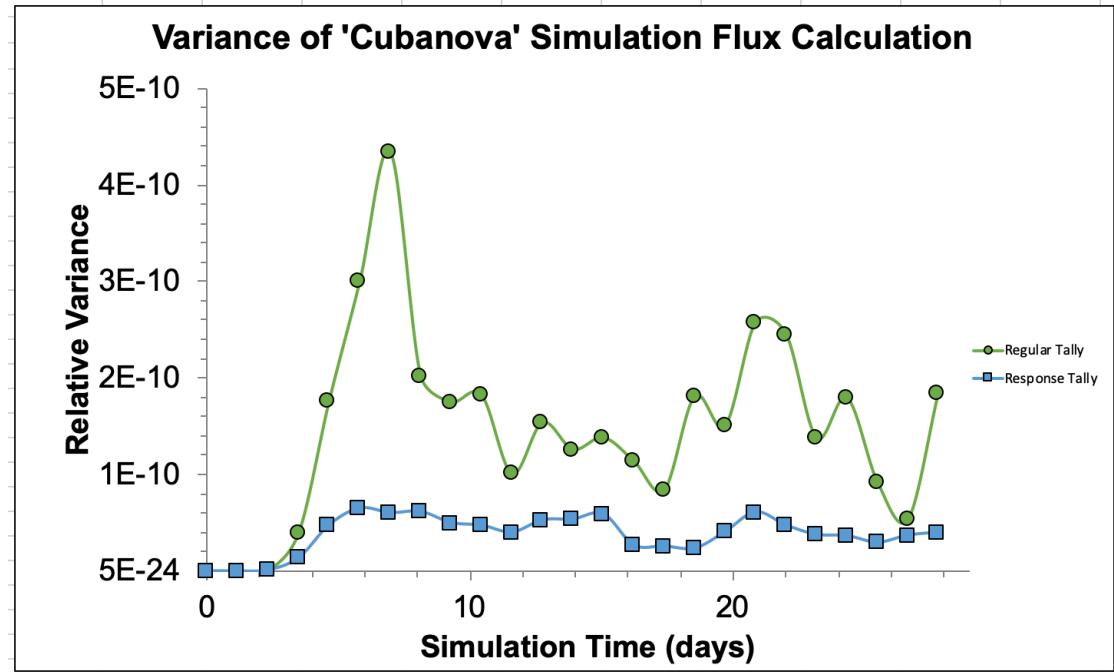
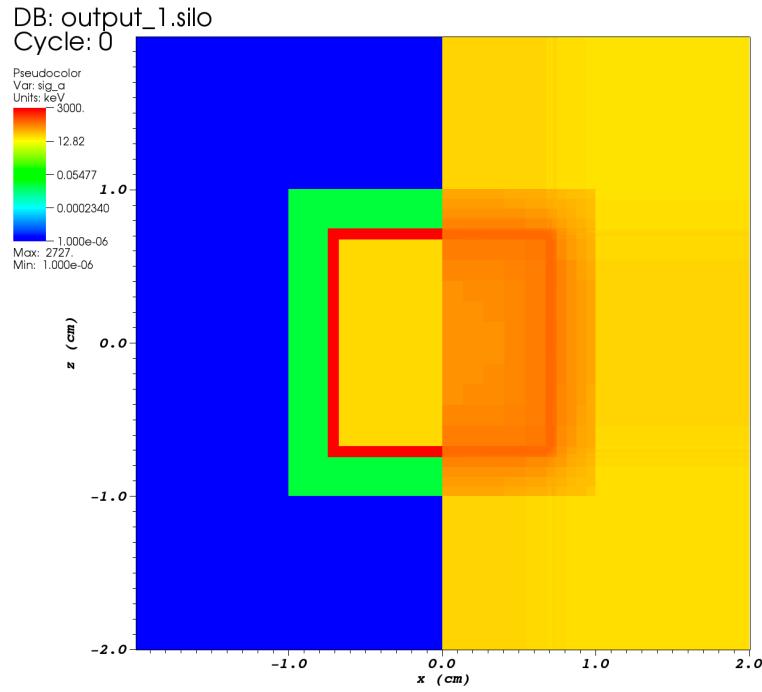
$$\text{Contribution} = E_{\text{particle}} e^{-(\sigma_r + \frac{1}{c\Delta t})d_{\text{tally}}}$$

- Transport the particle through the mesh. If the particle is scattered, add a contribution to the tally as above
- Repeat 1 & 2 for the duration of the timestep or until absorbed. Repeat for all particles

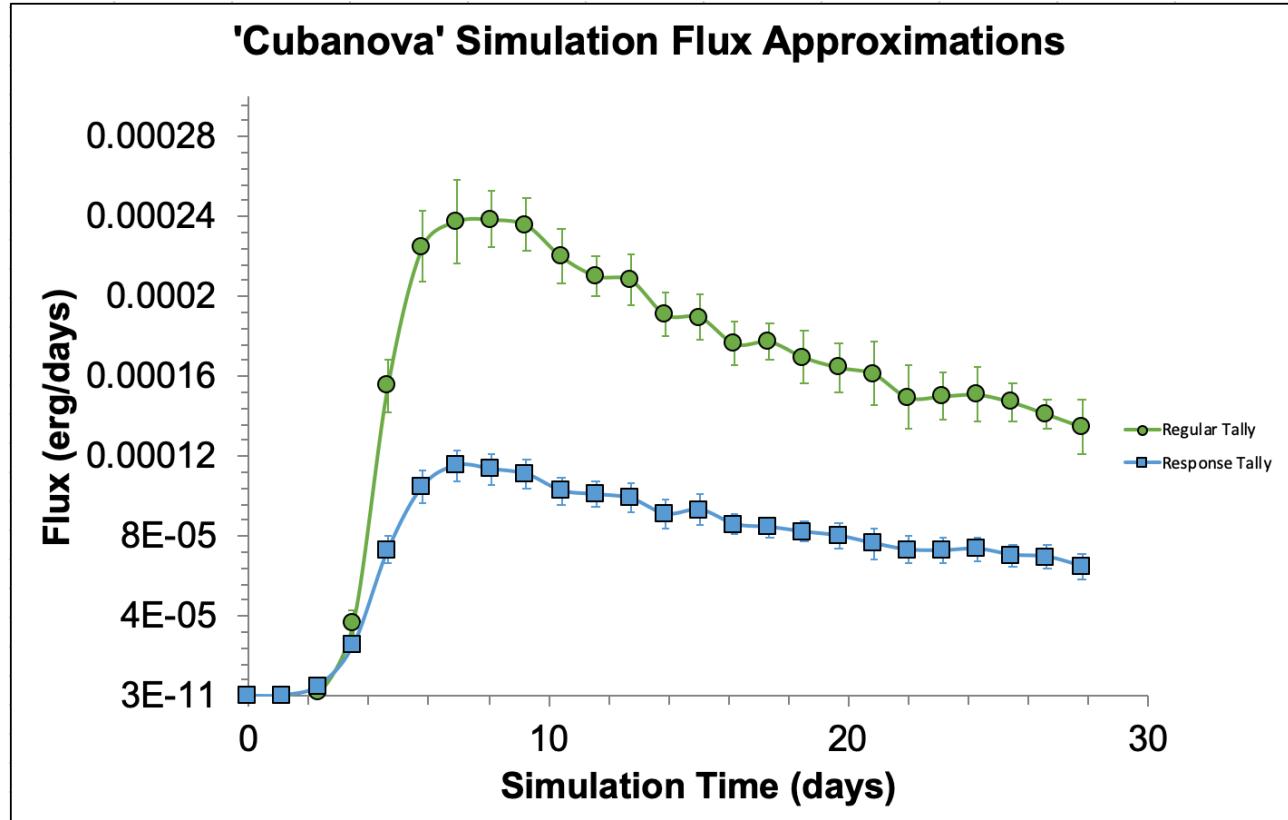
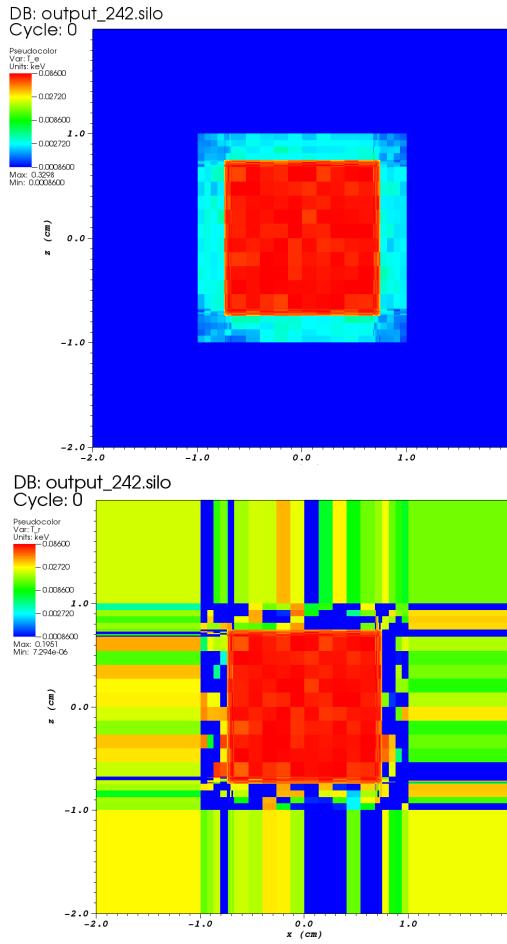
Test Problem



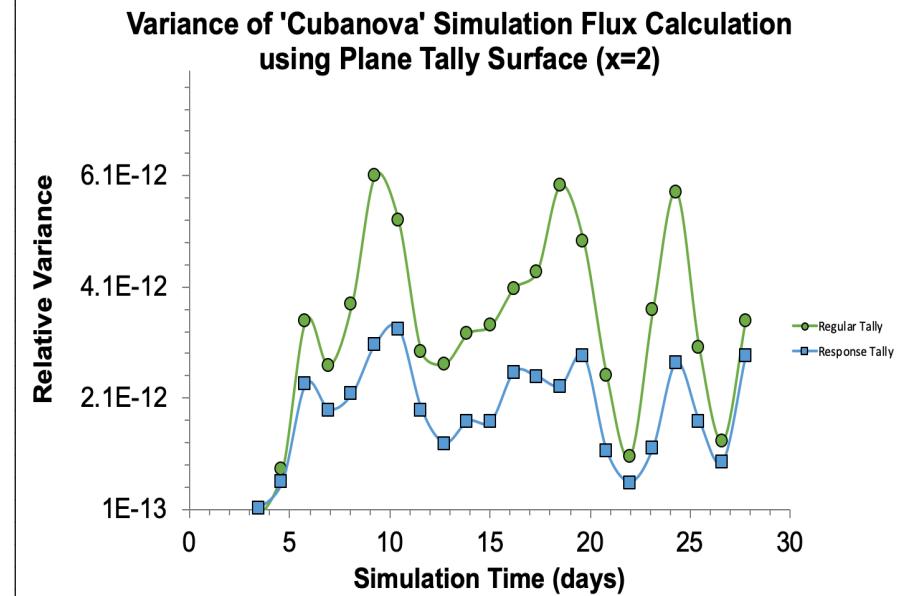
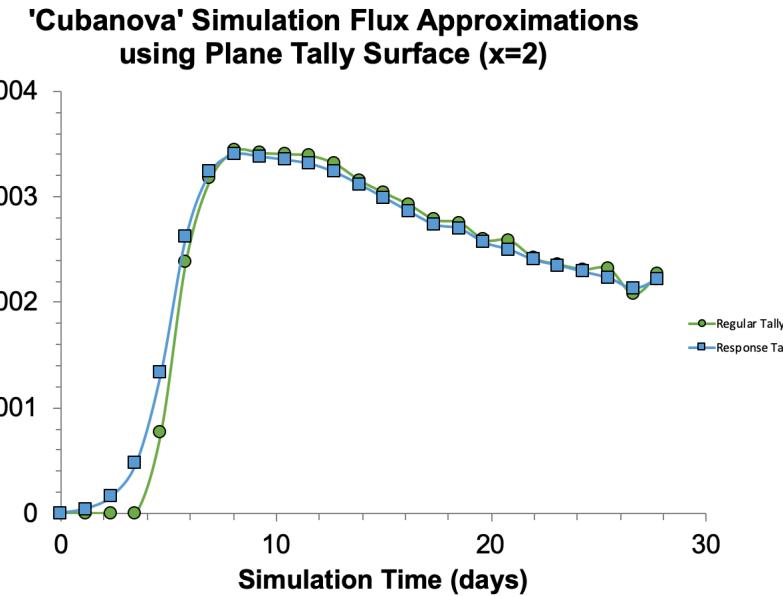
Supernova Applications



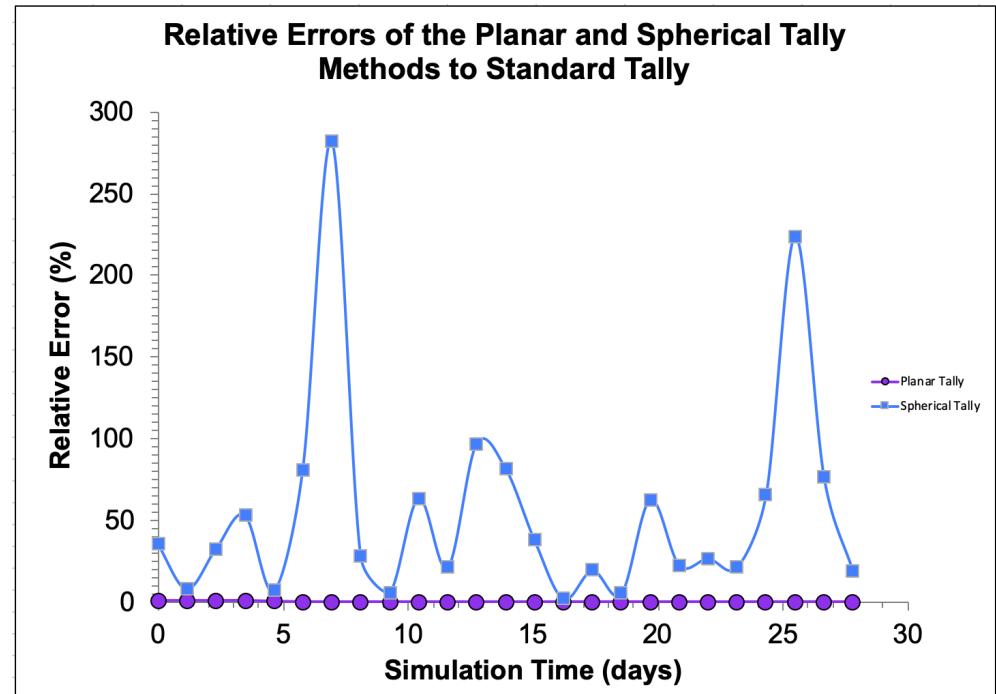
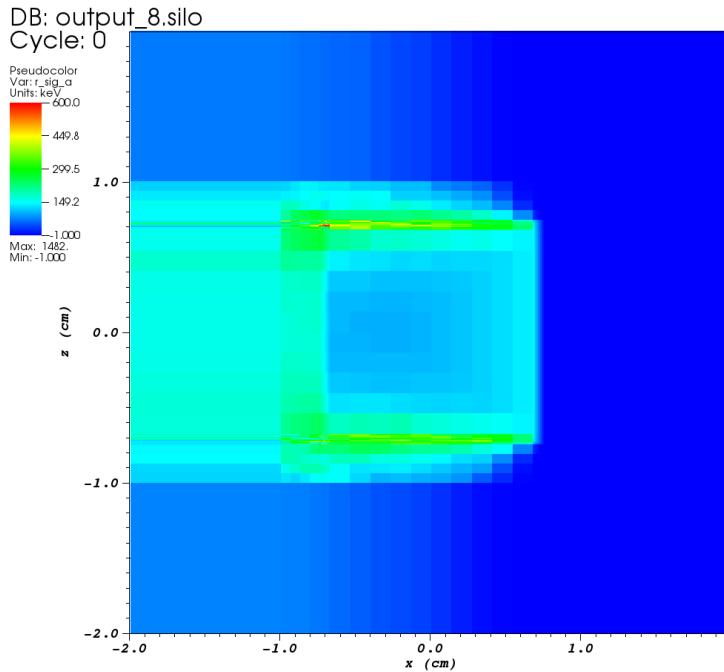
Supernova Applications Cont.



Supernova Applications Cont.



Supernova Applications Cont.



Conclusions & Future Work

- Our investigation into the use of a response function-based variance reduction method has shown a notable improvement in variance over standard methods.
- Our method can be applied to astrophysical events such as supernova interactions with its CSM
- There is a strong link between problem and tally geometry in determining how effective our method will perform
- Future work will be looking at potentially improving the directionality of the response function and its performance for other tally geometries.

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References

- [1] J.A. Fleck, Jr. and J.D. Cummings, Jr., “An implicit Monte Carlo scheme for calculating time and frequency dependent nonlinear radiation transport,” *J. Comp. Phys.* 8, pp. 313–342, (1971).
- [2] J.T. Landman, “Variance reduction strategies for implicit Monte Carlo simulations,” PhD thesis, Texas A&M University, *Texas A&M University*, (2016).
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- [4] T.J. Moriya, S.I. Blinnikov, N. Tominaga et al., “Light-curve modelling of superluminous supernova 2006gy: collision between supernova ejecta and a dense circumstellar medium,” *MNRAS* 428, pp. 1020–1035, (2013).