

# Neural Networks

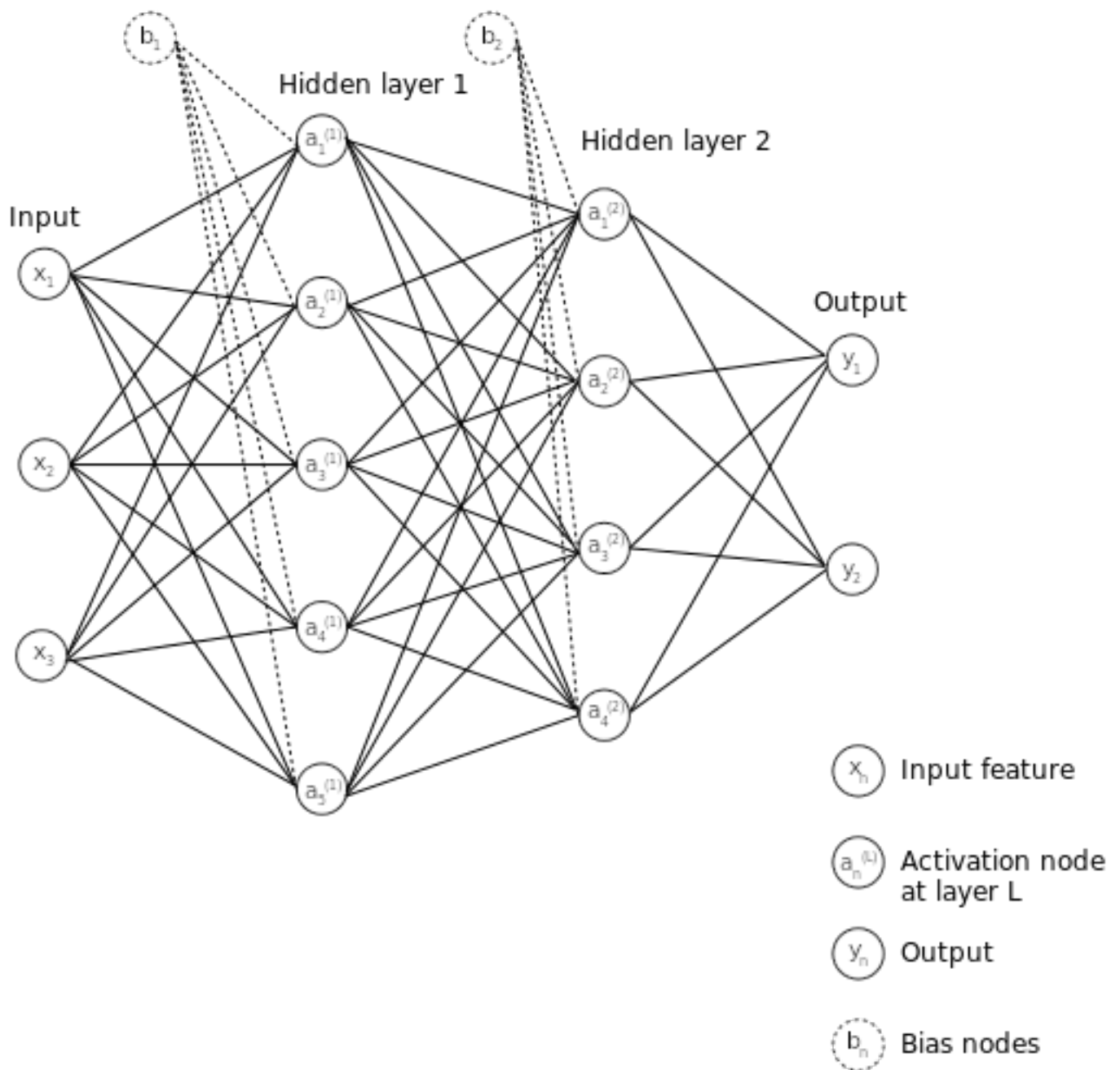
## Table of Contents

What are neural networks?	1
The components and notation of a neural network	1
Training a neural network using forward propagation	3
Linear transformation	3
Non-linear transformations using an activation function	5
A full round of forward propagation	6
Evaluating the model fit using the cost function	8
Cross-Entropy cost function	8
Optimizing neural network parameters using backward propagation	8
Optimizing parameters $W$ and $b$ using gradient descent	8
Finding the gradients $dW$ and $db$ using back propagation	9
A full round of back propagation	9
Running the neural network for the ovarian cancer data set	10
Training and optimizing the shallow neural network	10
Evaluating the costs	11
Assessing model accuracy	11
Hold out accuracy	11
Cross validation accuracy	11
Issues in training neural networks	12
Initializing values for $w$ and $b$	12
Setting $w$ and $b$ to be all zeros	12
Setting $w$ and $b$ to be all ones	12
Setting $w$ and $b$ to be large random numbers	12
Symmetry breaking	13
Overfitting	13
Data scaling	13
Multiple cost minima	13
Benchmarking against a logistic regression classifier	13
Logistic regression	13
Shallow neural network	14
Results	14
Tuning neural network model components	15
Alternative Activation functions	15
Rectified Linear Units (ReLU) Activation Function	15
Leaky ReLU Activation Function	15
Benchmarking different activation functions	16
Summary	16
Useful reading and references	17

## What are neural networks?

**Neural networks** are a set of algorithms that were inspired by neural circuits in the brain, and have exploded in popularity due to advances in computing technology. The central idea of neural networks is to take linear combinations of inputs as new features, then model the output as a nonlinear function of those features. We'll work on building our own neural network and perform classification on a breast cancer dataset.

## The components and notation of a neural network



There are several flavors of neural network architectures, each one having their own advantages and disadvantages for solving different problems. The diagram above illustrates a **feed-forward neural network**, where information flows through from the input to the output, and no information feeds back into itself. This is a rather simple neural network that works for several real-world applications.

**Each component of the feed-forward neural network is described below:**

Input layer: the data ( $X_{\text{train}}$ ) we are training the neural network on, where each  $n^{\text{th}}$  column or feature vector represents a node in the network diagram. The diagram above shows 3 features in the input layer.

Hidden layer(s): where the features are transformed into new data representations via linear and non-linear transformations

- Each **activation node** ( $a_n$ ) represents our newly determined complex feature vector. A single hidden layer can consist of several activation nodes. The number of nodes and hidden layers affects the model output.
- The **bias nodes** ( $b_n$ ) can be a scalar or vector of values that further influences the model fit.

Output layer: the final computed values after processing the input data through the hidden layers.

The diagram above shows a neural network containing two hidden layers. These are considered to be **deep neural networks**. To illustrate the main concepts underpinning neural networks, we'll be dealing with a **shallow neural network** that contains 1 hidden layer.

## Training a neural network using forward propagation

The training phase of a neural network is called **forward propagation**. It consists of iteratively performing a linear transformation, followed by a non-linear transformation for each hidden layer, until we reach the output layer.

### Linear transformation

Let's first use an artificial dataset consisting of 50 observations and 6 features to demonstrate what forward propagation does to our data.

**The first step in training a neural network is to perform a linear transformation of the data** to compute a new "feature". The formula for the transformation is

$$Z = X^T w + b$$

which is equivalent to the following equations:

$$\begin{aligned} z_1 &= x_1 w_1 + b_1 \\ z_2 &= x_2 w_2 + b_2 \\ &\vdots \\ z_6 &= x_6 w_6 + b_6 \end{aligned}$$

$X$  has the predictors spanning the columns, and the observations span the rows. This is the same form as the linear regression problem, and just like our regression problem, we're trying to estimate two parameters: the **weights** ( $w$ ) and the **bias** ( $b$ ) values.

For now, let's see how the two parameters  $w$  and  $b$  impact the solution  $Z$ , and then plot the relationship between  $X$  and  $Z$ . We'll run linear transformations with 1, 2, and 3 activation nodes of different values, as well as biases equal to all zeros, ones, and random values. The plot is generated by an accessory function `plotLSLines` located in the lesson directory.

```
% !!! IMPORTANT: ADD ALL FUNCTIONS TO MATLAB PATH !!!  
addpath(genpath('.'))
```

```

% Initialize inputs using random numbers from default seed
rng('default');
X = normrnd(0, 1, [6, 50]);

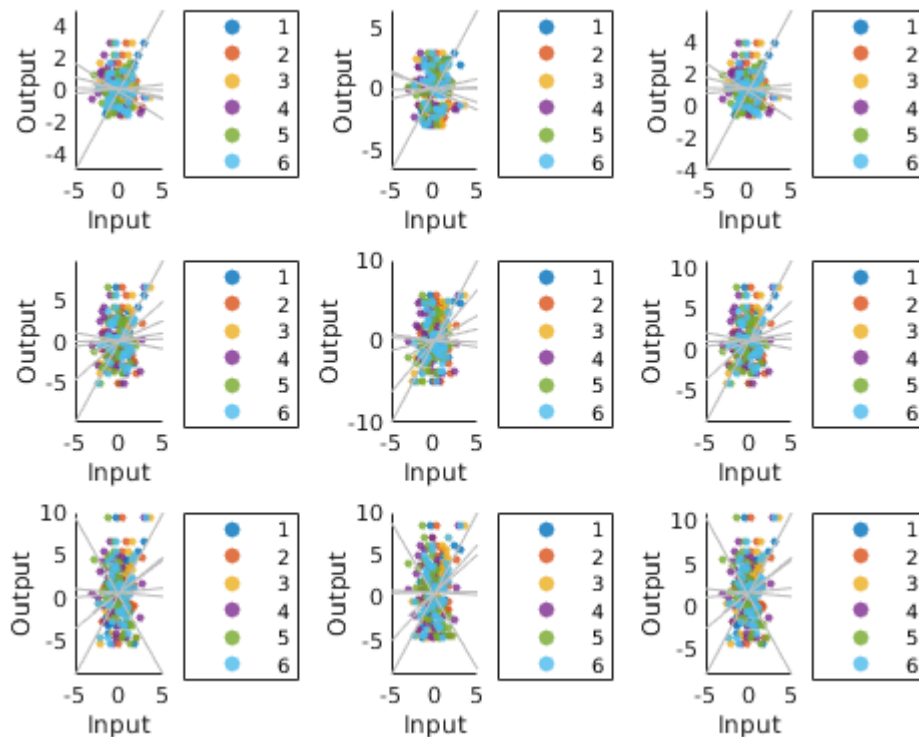
% Example with three different w and b vectors
wArray = [[1; 0; 0; 0; 0; 0], ... % One activation node
          [1; 0; 2; 0; 0; 0], ... % Two activation nodes
          [1; 0; 2; 0; -2; 0]];    % Three activation nodes

bArray = [zeros([1, 50]); ...      % All zeros
          randn([1, 50]); ...      % Random values
          ones([1, 50])];          % All ones

% Construct several values for Z by changing the weights and bias
% parameters
for i = 1:size(wArray, 2)
    for j = 1:size(bArray, 1)
        b = bArray(j, :);
        w = wArray(:, i);
        Z{i, j} = X' * w + b';
    end
end

% Plot least-squared lines for each feature in each plot
plotLSLines(X', Z);

```



From left to right, the biases are changing from all zeros, to random, to all ones. From top to bottom, the number of weights in the model vary from 1 to 3.

We created a new feature  $Z$ , which is linear combination of 6 features, all rolled into 1 single feature. As you can see, each  $w$  value highly impacts the distribution of the data in the new feature  $Z$ , and thus our final decision when performing classification for a given observation. More on that later.

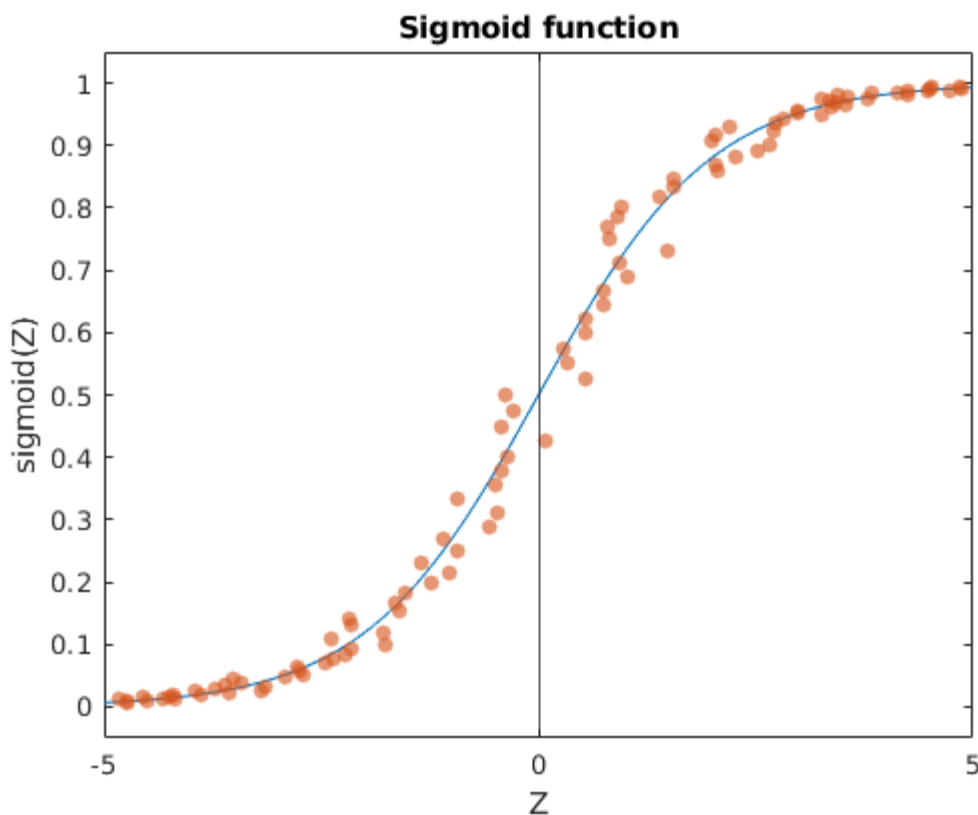
## Non-linear transformations using an activation function

**The second step involves transforming the data again using a non-linear function.** In this particular example, let's use the sigmoid function.

To review, data can be transformed to the characteristic 'S' curve using the following equation:

$$\text{sigmoid}(Z) = A = \frac{1}{1 + e^{-Z}}$$

```
plotSigmoidFunction()
```



This function is useful for binary classification, as the logit transformation scales these values in a range of [0, 1].

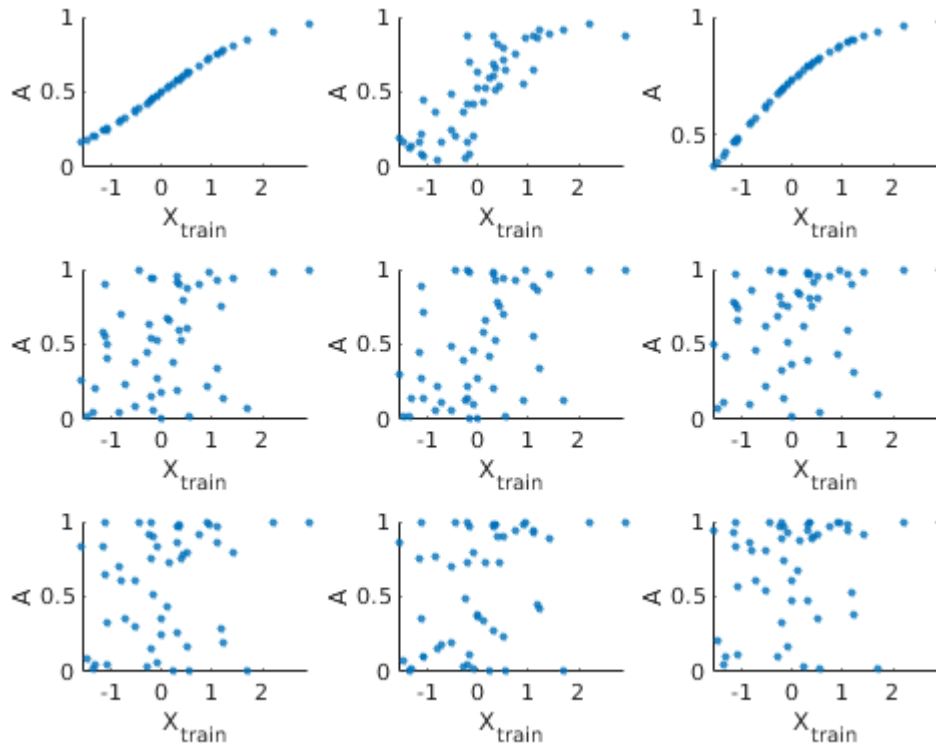
To implement the sigmoid function in MATLAB, we can use the `activationFnc` function provided in the project folder. How the function works is shown below:

```
function A = activationFnc(Z, fnc)
...
switch fnc
...
case 'Sigmoid'
A = 1.0 ./ (1.0 + exp(-Z));
```

```
end
end
```

Let's now transform our Z array using the sigmoid function and see the results of the transformation.

```
% Compute y using the Z cell array
for i = 1:size(Z, 1)
    for j = 1:size(Z, 2)
        y{i, j} = activationFnc(Z{i, j}', 'Sigmoid');
    end
end
plotSigmoidFunction(X', y);
```



From left to right, the biases are changing from all zeros, to random, to all ones. From top to bottom, the number of weights in the model vary from 1 to 3.

As we add more weights, the features tend to become more complex. However, a clearer decision boundary starts to form as values start to converge to either 0 or 1. This non-linear mapping from our combined feature  $Z$  to our output  $y$  shows how neural networks work at a fundamental level. Next, we will apply forward propagation to a real dataset.

## A full round of forward propagation

Let's now put all of these concepts together to run a full round of forward propagation using a breast cancer datasets built into MATLAB.

The dataset can be called using the `load_cancer_dataset` function, and a comprehensive description of the dataset can be found by typing in `help_cancer_dataset`. A brief description of the dataset:

- The input is a matrix of 9 features from 699 biopsies. The features include attributes such as adhesion, cell size, and cell shape. These are stored in the `cancerInputs` variable.
- The `cancerTargets` variable is a 2 x 699 matrix, where the first column corresponds to benign, and the second column corresponds to malignant. Thus, we will perform binary classification.

To save space in the code blocks, an accessory function `loadCancerData` has been made to perform the following steps:

1. Loads the dataset and target variables
2. Split the dataset and target variables into training and test datasets (Default split is 80% training / 20% testing)
3. Standardize the input datasets to have a mean = 0 and standard deviation of 1

```
clear all;
[Xtrain, Ytrain, Xtest, Ytest] = loadCancerData();
```

To keep the example relatively simple, we'll use only 2 activation nodes, as this may be enough to capture enough differences in a binary classification problem. Additionally, we'll randomly choose values for  $w$  and  $b$ . Later we'll discuss why this random initialization is important for both the weights and the biases.

```
clear all;
[Xtrain, Ytrain, Xtest, Ytest] = loadCancerData();

rng('default');
% Weights and biases for input -> hidden
w1 = randn([size(Xtrain, 2), 2]) * 0.01;
b1 = randn([size(Xtrain, 1), 1]) * 0.01;
```

Let's now run forward propagation, first from the input layer to the hidden layer:

```
% Run forward propagation for input -> hidden
actFnc = 'Sigmoid';
Z1      = Xtrain * w1 + b1;
A1      = activationFnc(Z1, actFnc);
```

Now we need to run forward propagation one more time, from the hidden layer to the output layer. We will use the values `A1` as the input for this round. Additionally, we will not bias our answers in this layer.

```
% Weights for hidden -> output
w2 = rand(2, 1) * 0.01;

% Run forward propagation for hidden -> output
Z2 = A1 * w2;
A2 = activationFnc(Z2, actFnc);

% Since we've made our way to the output layer, A2 is our final predicted y
```

```
% value  
ypred = A2;
```

That's a full round of forward propagation! Now we need to now see how well our model performed using a cost function.

## Evaluating the model fit using the cost function

Next, we need to evaluate how well our model has performed in it's initial pass through using a cost function. Because we are performing a binary classification problem, we can use the Cross-Entropy cost function.

### Cross-Entropy cost function

Recall the formulation for the Cross-Entropy cost function:

$$J = -\frac{1}{m} \sum_{i=1}^m [-y^{(i)} \log(y_{\text{pred}}^{(i)}) - (1 - y^{(i)}) \log(1 - y_{\text{pred}}^{(i)})]$$

where  $m$  = number of observations,  $y_{\text{train}}$  = the response variable, and  $y_{\text{pred}}$  = the predicted value for  $y_{\text{train}}$  from forward propagation. The **cost** value can be computed using the code from the `costFunction`.

```
function J = costFunction(Ytrain, ypred, L)  
m = size(Ytrain, 1);  
switch L  
...  
case 'Log-Loss'  
J = (1/m) .* ((-Ytrain' * log(ypred) - (1 - Ytrain)' * log(1 - ypred)));  
end  
end
```

```
J = costFunction(Ytrain, ypred, 'Log-Loss');
```

The cost value you should have outputted should be 0.6891. Now that we have a cost value, we can start to optimize our neural network algorithm using backward propagation.

## Optimizing neural network parameters using backward propagation

After computing the cost, we need to optimize the model parameters ( $w$  and  $b$ ) until the error is either below a given threshold we assign, or after a certain number of iterations. This can be done using an optimization algorithm we covered earlier in the course called **gradient descent**, where we're adjusting the model parameters by following the gradient that minimizes the model error.

Specifically for neural networks, we need to **back propagate** and compute the gradients **dW** and **db** using the variables we computed in forward propagation.

### Optimizing parameters $w$ and $b$ using gradient descent

Recall that the objective of gradient descent is to minimize the cost by updating the parameters in the model. Because we have two parameters **W** and **b**, we have two equations to update:



$$W = W - (\alpha \cdot dW)$$

$$b = b - (\alpha \cdot db)$$

where  $\alpha$  is the learning rate hyperparameter, a constant value that controls how fast we move along the gradient.

## Finding the gradients $dW$ and $db$ using back propagation

To propagate the loss  $J$  with respect to the parameters  $W$  and  $b$ , we need to compute the gradients  $dW$  and  $db$ . Two differential equations are used:

$$\frac{\partial J}{\partial W} = \frac{\partial J}{\partial y} \cdot \frac{\partial y}{\partial Z} \cdot \frac{\partial Z}{\partial W}$$

$$\frac{\partial J}{\partial b} = \frac{\partial J}{\partial y} \cdot \frac{\partial y}{\partial Z} \cdot \frac{\partial Z}{\partial b}$$

We won't go over the derivation for each term, but in practice, we would work these differential equations individually, multiply them together as shown above, and carry out gradient descent for each layer and each parameter in the neural network.

After working out the differential equations, we get the following set of equations that are needed to compute  $dw2$ ,  $dw1$ , and  $db1$ .

### Back propagating from the output layer to the hidden layer:

$$dZ2 = A2 - Y_{\text{train}}$$

$$dW2 = dZ2 \cdot A1^T \cdot \frac{1}{m}$$

### Backpropagating from the hidden layer to the input layer:

$$dZ1 = W2^T \cdot Z2 \cdot (1 - A2^2)$$

$$dW1 = dZ1 \cdot X_{\text{train}}^T \cdot \frac{1}{m}$$

$$db1 = \frac{1}{m} \sum_{i=1}^n (dZ1)$$

Note that the derivative of the sigmoid function is:

$$g'(Z1) = \text{sigmoid}(Z1) \cdot (1 - \text{sigmoid}(Z1)).$$

This is equivalent to

$$g'(Z1) = (1 - A2^2)$$

## A full round of back propagation

Now let's codify the expressions above to run a full round of back propagation. First we need to initialize constants that will be used in back propagation. A good starting point for the learning rate is 0.05.

```
% Initialize constants that will be used in the following expressions
```

```
alpha = 0.05;
m = size(Xtrain, 1);
```

First let's back propagate from the output layer to the hidden layer:

```
% Back propagation from output layer back to hidden layer
dZ2 = A2 - Ytrain;
dW2 = (1/m) .* (A1' * dZ2);
```

Next let's back propagate from the hidden layer to the input layer:

```
% Back propagation from hidden layer to the input layer
dZ1 = (w2 * dZ2') .* (1 - A1.^2)';
dW1 = (1/m) .* Xtrain' * dZ1';
db1 = (1/m) .* sum(dZ1', 2);
```

And finally, let's update the values using gradient descent:

```
% Update w2, w1, and b1 using the gradients we just computed
w2 = w2 - (alpha .* dW2);
w1 = w1 - (alpha .* dW1);
b1 = b1 - (alpha .* db1);
```

We have successfully back propagated to get better values for  $w$  and  $b$ . To train the neural network even more, we would iterate the entire process of forward propagation, computing the cost, and backward propagation several times until we reach the minimum cost value.

## Running the neural network for the ovarian cancer data set

Now that we have all the steps required to train, evaluate, and optimize a shallow neural network, we would iterate this until the cost value reaches a minimum. Each iteration where we run a single round of forward and backward propagation is called an **epoch**. This is another hyperparameter that we can tune to assess model performance.

The SNN function for the shallow neural network is modularized to the three steps we performed: forward propagation, evaluating the cost, and back propagation. The values from forward propagation are stored in a structure data type called `cache`. Similarly, the values from back prop are stored in the `backwardCache` variable. Further documentation is included in the SNN function.

## Training and optimizing the shallow neural network

Let's now use this function to train a neural network using the ovarian cancer dataset:

```
clear all;
[Xtrain, Ytrain, Xtest, Ytest] = loadCancerData();

% Initialize other important model parameters
numOfActiveNodes = 2;
alpha = 0.2;
epoch = 1000;
actFnc = 'Sigmoid';
```

```
randomstate = true;

% Run the neural network
model = SNN(Xtrain, Ytrain, numOfActiveNodes, alpha, epoch, actFnc, randomstate);
```

## Evaluating the costs

Now that we have a trained model, let's evaluate the results. First, let's plot the cost with respect to the number of iterations we made in the model:

```
% Plot of the cost:
plot(1:length(model.J), model.J);
title(strcat('Cost with \alpha = ', string(alpha)));
xlabel('Number of iterations');
ylabel('Cost (J)');
```

As you can see, while it appears that the model is approaching a minimum cost value, it looks like there is still room for improvement as we have not reached the global minimum. Later on, we'll optimize the model hyperparameters as well to get the best shallow neural network model.

## Assessing model accuracy

### Hold out accuracy

Next, let's use the test set to come up with a prediction. To do that using our neural network, we can use the `predict` function, which takes the test data and outputs predictions for classification.

```
ypred = predict(model, Xtest, Ytest, 'SNN');
```

Now that we have a prediction from our neural network, we can evaluate the model accuracy. To do that, we need to transform the `ypred` field in the model to be either 0 or 1. For the decision boundary, let's use mean value of  $y_{pred}$ .

- If the value is greater than the `mean(ypred)`, we'll assign that value to be 1 (malignant)
- Otherwise, we'll assign that value to be 0 (benign).

We'll also plot the confusion matrix and report the hold out accuracy using the `plotConfusionMatrix` code in the directory.

```
summary = plotConfusionMatrix(Ytest, ypred);
```

The average accuracy across all classes is 96.43%, which is a decent score for a single hold out. Additionally, the accuracy for predicting cancer (96.36%) and healthy (96.47%) is pretty good as well.

### Cross validation accuracy

Of course, the hold out accuracy itself is not necessarily the best measure to evaluate the neural network. Let's also run 10-fold cross validation and see how our model does on average. The `crossValidate` function is used to compute k-fold cross validation with two classes

```
kfold = 10;
type = 'SNN';
```

```
Summary = crossValidate(Xtrain, Ytrain, model, kfold, type);
```

The average accuracy across all classes is 96.77%, which is pretty good considering we did not even pursue further optimization of the hyperparameters yet. Additionally, the accuracy for predicting cancer (92.46%) and healthy (98.83%) is pretty high.

## Issues in training neural networks

While neural networks are really popular for artificial intelligence research and applications, there are issues with training them. We'll discuss some of these issues in this section.

### Initializing values for w and b

Earlier, we initialized our values for w and b to be small random numbers. Is this the best choice for w and b? We discuss 3 alternative cases: if w and b are all zeros, all ones, and large random numbers.

#### Setting w and b to be all zeros

We'll go back to our synthetic dataset with 50 observations and 6 features and plot the results when w and b are all zero.

```
% Initialize inputs using random numbers from default seed
rng('default');
Xtrain = normrnd(0, 1, [50, 6]);

% Example with three different w and b vectors
w = [0, 0, 0, 0, 0, 0];
b = zeros([50, 1]);
Z = Xtrain * w' + b;
plot(Z); hold on;
plot(activationFnc(Z, 'Sigmoid'));
```

If the values are all zeros, the neural network stops learning the features - they will be stuck at  $Z = 0$  and  $A = 0.5$  during every iteration.

#### Setting w and b to be all ones

Let's see what happens when we set w and b to be all ones:

```
% Example with three different w and b vectors
w = [1, 1, 1, 1, 1, 1];
b = ones([50, 1]);
Z = Xtrain * w' + b;
plot(Z); hold on;
plot(activationFnc(Z, 'Sigmoid'));
```

As you can see, our features (blue) end up being transformed to log-odds in the range  $[0, 1]$  (red) by the activation function. Thus, the neural network algorithm becomes a logistic regression classifier when using the sigmoid activation function.

#### Setting w and b to be large random numbers

Finally, let's complete this exercise by seeing what happens when the random values are large.

```
% Example with three different w and b vectors
```

```
w = randn([1, 6])*10;  
b = randn([50, 1])*10;  
Z = Xtrain * w' + b;  
plot(activationFnc(Z, 'Sigmoid'));
```

Values start to converge to either 0 or 1 really fast, but this can simply be biased by the random number generated. Thus, large numbers are also a terrible way to initialize these values.

### Symmetry breaking

Symmetry breaking is a phenomenon where very small, random fluctuations in a system are able to influence the outcome. By randomly initializing  $w$  and  $b$  to be small random numbers, we don't break the algorithm from the beginning and we allow the neural network to approach a better solution through back propagation / gradient descent.

### Overfitting

Recall that our objective in machine learning is to build a model that balances the model's ability to minimize bias and variance. Neural networks are relatively complex models, and are susceptible to model overfitting - that is, they are susceptible to learning from noisy data. This is especially true when the number of predictors  $p$  is much greater than the number of observations  $n$ .

To address this issue, regularization is usually imposed in the loss function to select the predictors that best explain the variance of the dataset. Thus, we reduce the amount of noise in the dataset, allowing the neural network to better capture the trends in the data.

### Data scaling

It is good practice to standardize all the features to have a mean of zero and standard deviation of one. This ensures that the inputs are treated equally by the random starting weights.

### Multiple cost minima

The error function may not be convex - in other words, there may be multiple minima that gradient descent can converge to. To try to get the best solution, different learning rates should be explored.

## Benchmarking against a logistic regression classifier

A neural network is a more flexible model compared to other models, in that it performs both linear and nonlinear transformations on the data. Let's see how well our shall model performs against a logistic regression classifier, a less flexible model that only performs nonlinear transformations on the dataset.

### Logistic regression

Recall that the formulation for a logistic regression problem is 1) applying the sigmoid function to our linear equation  $Z = X^T \beta$  to transform the values to log-odds, then 2) finding the value of  $\beta$  using the maximum likelihood objective function. Thus the equation we're trying to solve for a logistic regression problem is:

$$y = \frac{1}{1 + e^{-X^T \beta}}$$

Let's load up the dataset again:

```
clear all;
[Xtrain, Ytrain, Xtest, Ytest] = loadCancerData();
```

Now let's set some of the other hyperparameters similar to the shallow neural network.

```
% Initialize other important model parameters
alpha = 0.5;
actFnc = 'Sigmoid';
randomstate = true;
epoch = 1000;
m = size(Xtrain, 2);
theta = randn([size(Xtrain, 2), 1]) * 0.01;
b = randn([size(Xtrain, 1), 1]);
```

To set the coefficients Beta and b, we'll use the same method we used for the shallow neural network:

```
logistic = logRegress(Xtrain, Ytrain, ...
    theta, b, alpha, epoch);
```

If you're interested in the implementation of this logistic regressor, you can check out the `logRegress` function.

Let's feed the model into the `predict` function to get the hold out values:

```
ypred = predict(logistic, Xtest, Ytest, 'Logistic');
logisticCM = plotConfusionMatrix(Ytest, ypred);
```

The overall hold out accuracy is 96.43%. Let's see how our neural network compares.

## Shallow neural network

Let's re-train our neural network and compare the two results

```
% Train model
clearvars -except logisticCM;
[Xtrain, Ytrain, Xtest, Ytest] = loadCancerData();

% Initialize other important model parameters
numOfActiveNodes = 2;
alpha = 0.5;
epoch = 1000;
actFnc = 'Sigmoid';
randomstate = true;

% Run the neural network
nn = SNN(Xtrain, Ytrain, numOfActiveNodes, alpha, epoch, actFnc, randomstate);

ypred = predict(nn, Xtest, Ytest, 'SNN');
snnCM = plotConfusionMatrix(Ytest, ypred);
```

The overall hold out accuracy is 96.43%.

## Results

The logistic regression classifier and the shallow neural network have the same hold out accuracy, which at least shows that neural networks can perform equal, if not worse than a logistic classifier. However, neural networks are expected to perform better with more complex data, because the model itself is more complex.

We'll see this in the next lecture, where we'll work with imaging data to classify histological slides into cancer subtypes.

## Tuning neural network model components

### Alternative Activation functions

#### Rectified Linear Units (ReLU) Activation Function

While the Sigmoid activation function is easy to interpret and is simple to apply, there are issues with using it as an activation function:

1. With each iteration, the  $e^{-z}$  term in the sigmoid function becomes infinitesimally small. In back propagation, the gradient becomes really small, resulting in slower updates. This is called the **vanishing gradient problem**, and is an issue when it comes to learning model parameters.
2. Further, our values are between 0 and 1. Because the values are not zero-centered, our algorithm becomes more difficult to optimize.

Thus, the formulation of the ReLU is as follows:

$$\text{ReLU}(Z) = \begin{cases} \max(Z) & \text{if } z \text{ is positive} \\ 0 & \text{if } z \text{ is negative} \end{cases}$$

And the function has the following distribution:

```
plotReLU()
```

The ReLU has gained popularity in recent years because it takes care of the problems faced by the Sigmoid activation function, such as:

1. All of the negative values have a derivative value of 0, while for positive values, the derivative is 1. Thus, this rectifies the vanishing gradient problem.
2. It is also computationally economical, and easy to optimize.
3. Because only a few neurons would be activated (negative values are all 0), this makes the model more efficient.

To implement the ReLU using MATLAB, we can use the `activationFnc` function, using ReLU as the second argument. The ReLU is not a magic bullet, and there are limitations to it, despite it being a better activation function than other alternatives. Specifically, for values of  $Z < 0$ , the gradient will be 0 because the weights will not get adjusted during gradient descent. Thus, these neurons will not respond to back propagation. This is known as the **dying ReLU problem**.

#### Leaky ReLU Activation Function

To counteract the dying ReLU problem, the following solution has been proposed: instead of setting to value to 0 for negative numbers, we'll scale the negative values using a small weight (usually this weight, which we'll denote as epsilon ( $\epsilon$ ) is set to 0.01). This will cause a smaller gradient to flow for the negative values, and thus prevents the information loss from the ReLU.

The formulation for the leaky ReLU is

$$\text{Leaky ReLU}(Z) = \begin{cases} z & \text{if } z \text{ is positive} \\ \epsilon \cdot z & \text{if } z \text{ is negative} \end{cases}$$

The leaky ReLU has the following distribution:

```
plotLeakyReLU()
```

To implement the Leaky ReLU using MATLAB, we can use the `activationFnc` function, using Leaky ReLU as the second argument. Additionally, we can specify the scaling factor  $\epsilon$  as the third argument. The default value for  $\epsilon$  is 0.01:

### Benchmarking different activation functions

Finally, let's compare three neural networks that use all three different activation functions, and see which one performs the best:

```
% Train the 3 models
logisticAF = SNN(Xtrain, Ytrain, numOfActiveNodes, alpha, epoch, 'Sigmoid', randomstate);
ReLUAF     = SNN(Xtrain, Ytrain, numOfActiveNodes, alpha, epoch, 'ReLU', randomstate);
LeakyReLUAF = SNN(Xtrain, Ytrain, numOfActiveNodes, alpha, epoch, 'Leaky ReLU', randomstate);

% Plot the cost curves:
figure;
plot(1:length(logisticAF.J), logisticAF.J); hold on;
plot(1:length(ReLUAF.J), ReLUAF.J); hold on;
plot(1:length(LeakyReLUAF.J), LeakyReLUAF.J); hold on;
title('Cost with \alpha = 0.05');
xlabel('Number of iterations');
ylabel('Cost (J)');
legend('Sigmoid', 'ReLU', 'Leaky ReLU');
```

As you can see in the plot with the cost over the number of epochs, the ReLU and Leaky ReLU have very similar performances in this dataset. They both tend to converge to a lower minimum much faster than the Sigmoid activation function as well. However, with 1000 epochs, you can see that cost tends to oscillate after 100 epochs - this suggests that we should reduce the number of epochs to ~ 100 for optimal model performance.

## Summary

We have covered how neural networks work under the hood, and coded a shallow neural network from scratch. We also saw how modifying various parameters and hyperparameters influence the model prediction. Finally, we applied the neural network to a breast cancer dataset to classify whether a patient has a benign or malignant tumor with over 96% accuracy.



## Useful reading and references

- A [generalized tutorial for learning about neural networks](#) that is language-agnostic.
- The [various flavors of neural networks](#), and how they are currently being used in real-world settings.
- Heuristics on deciding [how many hidden layers and activation nodes](#) to initialize in a neural network.
- [Other activation functions, their usage, and generalizations about pros/cons](#).
- [Andrew Ng's CS229 deep learning notes](#) for a more in-depth understanding of the mathematics and algorithms for constructing neural networks from scratch.
- The [deep learning specialization on Coursera](#) (also taught by Andrew Ng and others at [deeplearning.ai](#)). It consists of 5 courses, all on deep learning and optimizing these models. The course is taught in Python with the specific usage of the numpy and scipy libraries.
- [Deep Learning](#) by Ian Goodfellow and Yoshua Bengio - two pioneers in modern deep learning. The book covers 1-2 semester's worth of material for not the basics of constructing and applying deep neural networks, but more cutting-edge techniques in Artificial Intelligence.