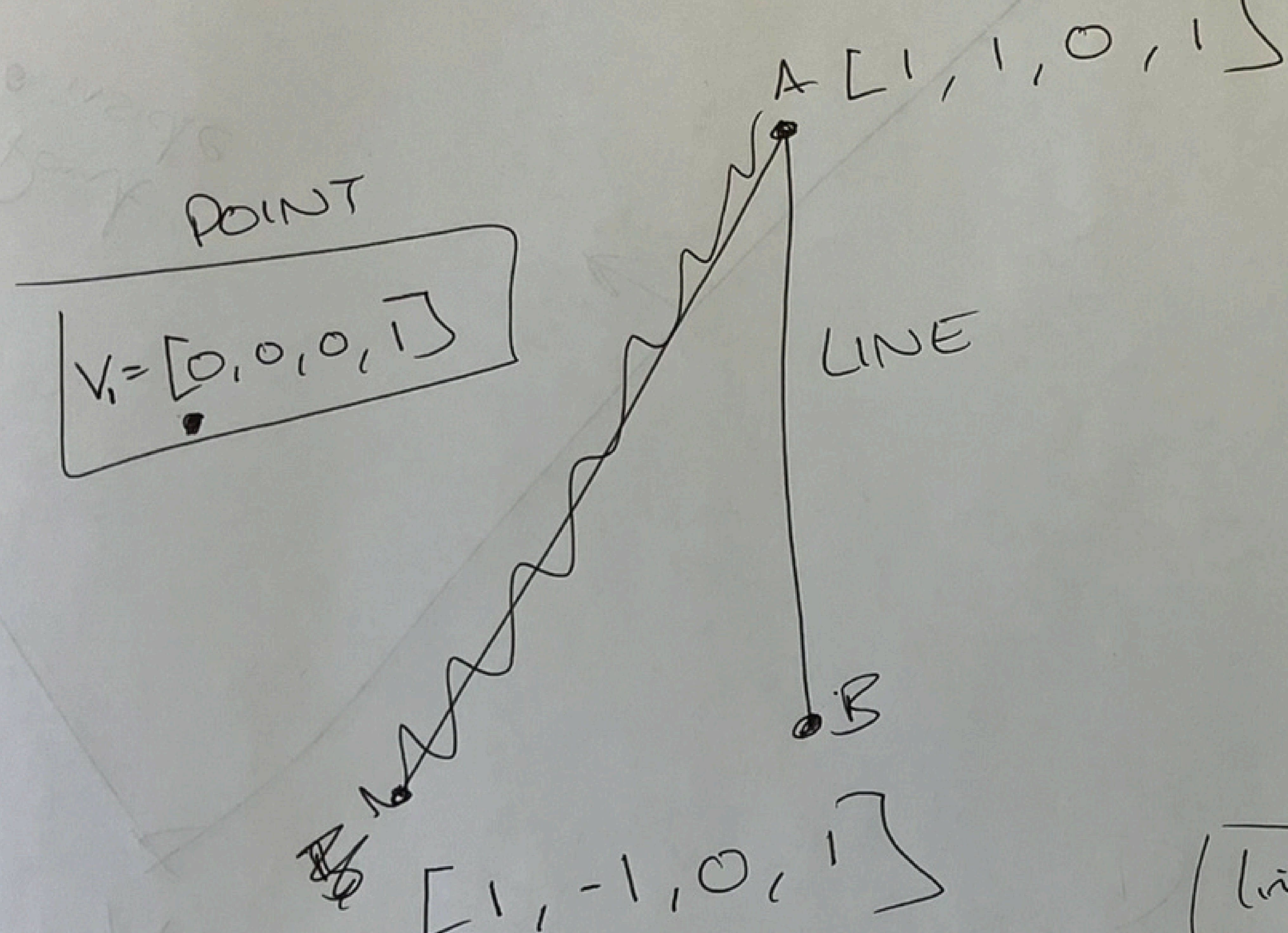
2024-04-30 UN & SF

How does the oriented distance between a point and a line work? Let's try this example:



Line $(\vec{AB}, \vec{A}\times\vec{B})$ ← in Plücker coordinates

$= A \vee B$

$= \star(\star A \wedge \star B)$

$= \star(\star(e_{032} + e_{013} + e_{123}) \wedge \star(e_{032} - e_{013} + e_{123}))$

$= \star((e_1 + e_2 + e_0) \wedge (e_1 - e_2 + e_0))$

$= \star(0 + -e_{12} + e_{10} + e_{21} + 0 + e_{20} + e_{01} - e_{02})$

$= \star(-e_{12} - \cancel{e_{01}} - e_{12} - e_{02} + \cancel{e_{01}} - e_{02})$

$= \star(-2e_{12} - 2e_{02})$

$\boxed{\text{line} = -2e_{03} - 2e_{31}}$

Oriented distance $= \| \hat{\text{Point}} \vee \hat{\text{Line}} \|$

$= \| \hat{e}_{123} \vee (-2e_{03} - 2e_{31})^{\wedge} \|$

$= \| e_{123} \vee (-e_{03} - e_{31}) \|$

$= \| \star(\star e_{123} \wedge \star(-e_{03} - e_{31})) \|$

$= \| \star(e_0 \wedge (-e_{12} - e_{02})) \|$

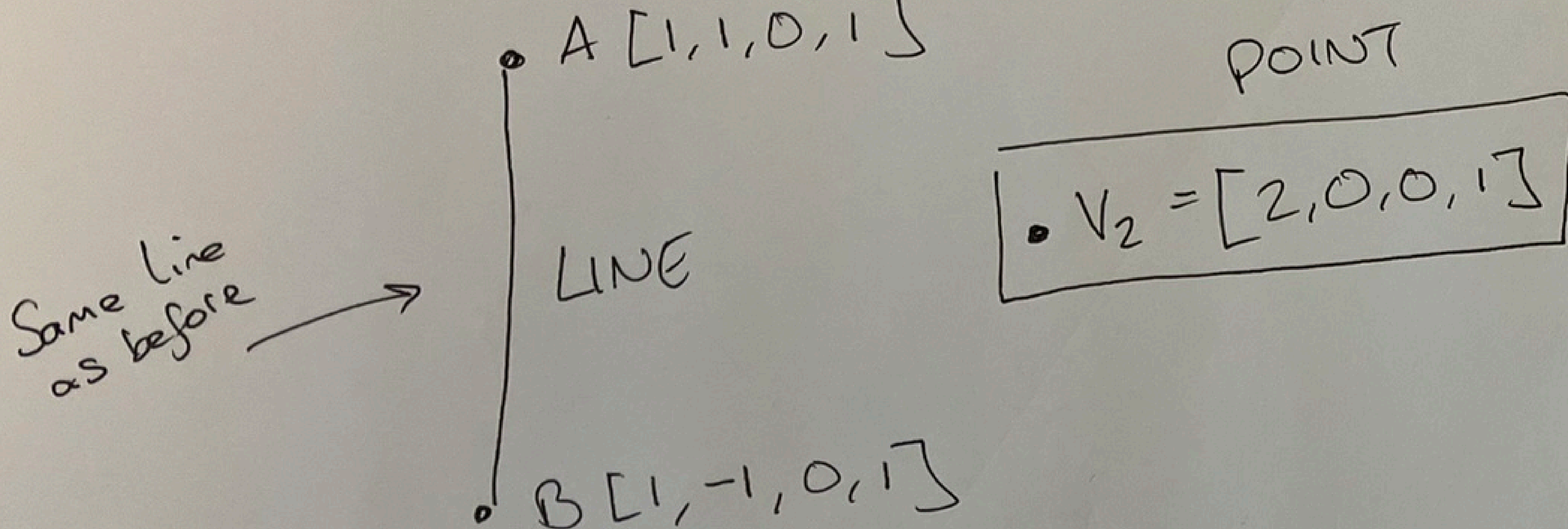
$= \| \star(-e_{012}) \|$

$= \| \star(e_{021}) \|$

$= \| e_3 \|$

For oriented dist, just take the coeff. of the e_1, e_2 or e_3. In our case $= +1$

Ok, but what if the point was on the otherside?



Then oriented distance is

$\|\widehat{\text{Point}} \vee \widehat{\text{Line}}\|$ as before

$= \|\widehat{(2e_{032} + e_{123})} \vee \widehat{(-2e_{03} - 2e_{31})}\|$

$= \|(2e_{032} + e_{123}) \vee (-e_{03} - e_{31})\|$

$= \|\star(\star(2e_{032} + e_{123}) \wedge \star(-e_{03} - e_{31}))\|$

$= \|\star((2e_1 + e_0) \wedge (-e_{12} - e_{02}))\|$

$= \|\star(0 - 2e_{102} - e_{012} + 0)\|$

$= \|\star(-2e_{021} + e_{021})\|$

$= \|\star(-e_{021})\|$

$= \|-e_3\|$

Oriented distance $= -1$

Other side of the line!