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Apparently we can figure out the oriented distance between a point and a plane using

$$(\hat{\text{point}} \wedge \hat{\text{plane}}).e0123$$

So I need to code up the meet between a point and a plane

$$(v.e032 + v.e013 + v.e021 + v.e123) \wedge$$
$$(p.e1 + p.e2 + p.e3 + p.e0)$$

$$\begin{aligned} &= (v.e032)(p.e1)(-e0123) \\ &+ (v.e013)(p.e2)(-e0123) \\ &+ (v.e021)(p.e3)(-e0123) \\ &+ (v.e123)(p.e0)(-e0123) \end{aligned}$$

remember
$e_1 \wedge e_1 = 0$
and so on...

Wow, so there is only one term, the e0123 part of a dual quaternion.
So the result is:

$$\begin{aligned} dq.w &= 0 \\ dq.e23 &= 0 \\ dq.e31 &= 0 \\ dq.e12 &= 0 \\ dq.e01 &= 0 \\ dq.e02 &= 0 \\ dq.e03 &= 0 \\ dq.e0123 &= -(v.e032)(p.e1) \\ &\quad -(v.e013)(p.e2) \\ &\quad -(v.e021)(p.e3) \\ &\quad -(v.e123)(p.e0) \end{aligned}$$