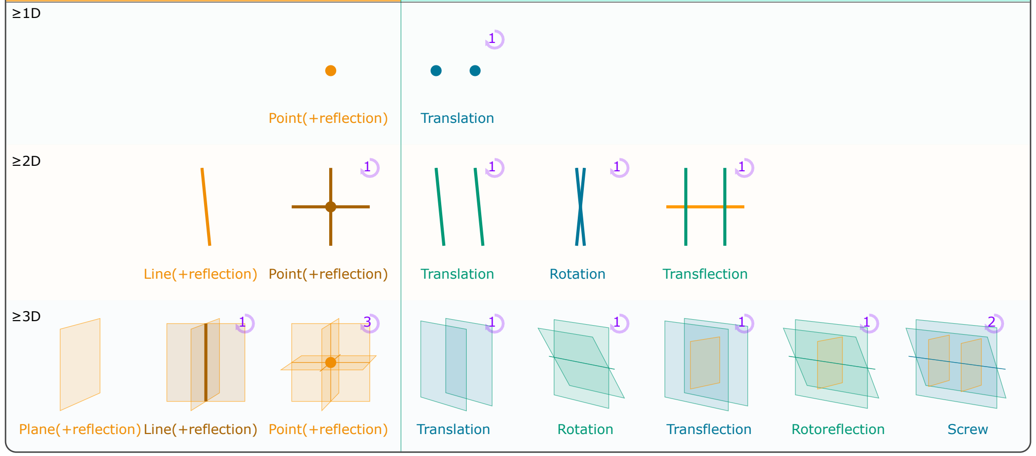
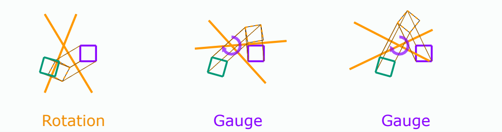
k-blades = Elements & Reflections = A ∧ B ∧ … ∧ Z	k-reflections = Transformations = AB…Z

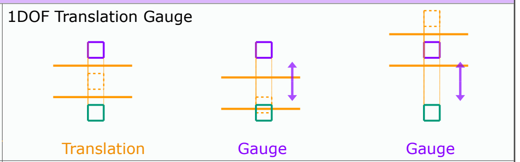


Geometric Gauges.

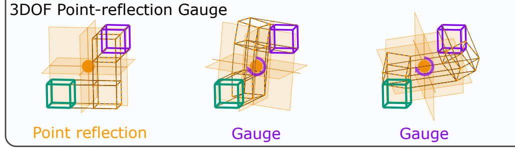
1DOF Rotation Gauge



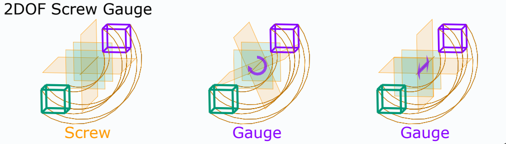
1DOF Translation Gauge



3DOF Point-reflection Gauge

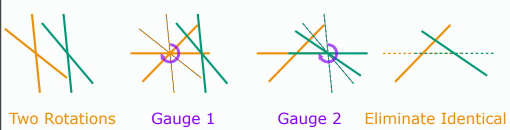


2DOF Screw Gauge

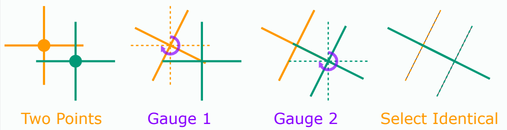


Geometric Products - How they work.

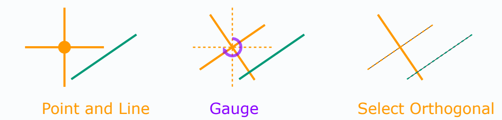
Geometric Product ab → Eliminate Identical



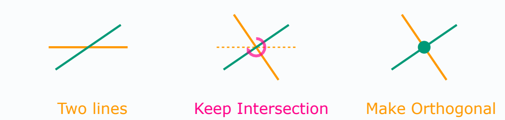
Regressive Product a∨b → Select Identical



Inner Product a•b → Select Orthogonal

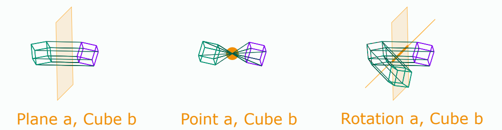


Outer Product a∧b → Make Orthogonal

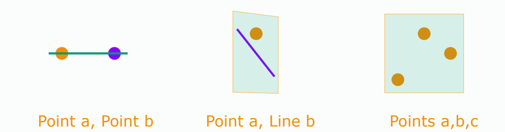


Geometric Products - what they do.

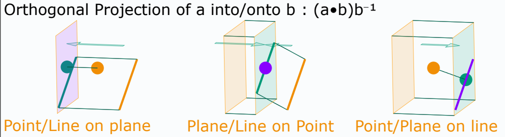
Sandwich Product aba⁻¹ → conjugate/action



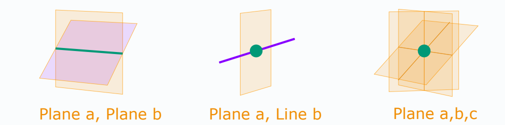
Regressive Product a∨b → join



Orthogonal Projection of a into/onto b : (a•b)b⁻¹

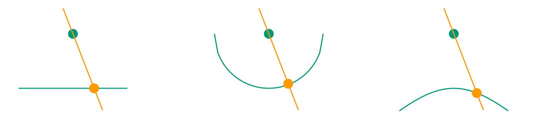


Outer Product a∧b → meet



Projective Geometric Algebra

Cheat Sheet by Steven De Keninck



$\mathbb{R}_{d,0,1}$ Euclidean $\mathbf{e}_0^{\,2} = 0$ | $\mathbb{R}_{d+1}$ Elliptic $\mathbf{e}_0^{\,2} = 1$ | $\mathbb{R}_{d,1}$ Hyperbolic $\mathbf{e}_0^{\,2} = -1$

GA Basics

Orthogonal Basis Vectors	$\mathbf{e}_i^{\,2} \in \{+1,-1,0\} \quad \mathbf{e}_i\mathbf{e}_j = -\mathbf{e}_j\mathbf{e}_i$
Basis Blade	Outer product of basis vectors $\mathbf{e}_i \wedge \mathbf{e}_j = \mathbf{e}_{ij}$
Pseudoscalar I	$I = \mathbf{e}_0\mathbf{e}_1 ... \mathbf{e}_d = \mathbf{e}_{01..d}$
Dual basis blade x^*	$xx^* = I$
k**-blade**	Outer product of k vectors
k**-reflection**	Geometric product of k normalized vectors
k**-vector**	Linear combination of k blades
Multivector	Sum of k-vectors

$$\mathbf{x} = \langle\mathbf{x}\rangle + \langle\mathbf{x}\rangle_1 + ... + \langle\mathbf{x}\rangle_{d+1}$$

where $\langle\mathbf{x}\rangle_k$ selects the k-vector part of $\mathbf{x}$ (with grade k)

Involute $\hat{x}$	$\mathbf{e}_i \leftrightarrow -\mathbf{e}_i$
Reverse $\tilde{x}$	$\mathbf{e}_{i..k} \leftrightarrow \mathbf{e}_{k..i}$
Dual x^*	$\mathbf{e}_i \leftrightarrow \mathbf{e}_i^*$
Norm $\lVert x\rVert$ **and Ideal norm** $\lVert x\rVert_\infty$	$\lVert x\rVert = \sqrt{x\tilde{x}}, \quad \lVert x\rVert_\infty = \lVert x^*\rVert$

Geometric Products

Geometric Product between k-vector x and l-vector y

$$xy = \langle xy\rangle_{|k-l|} + \langle xy\rangle_{|k-l|+2} + ... + \langle xy\rangle_{k+l}$$

Inner Product	$x \cdot y = \langle xy\rangle_{\lvert k-l\rvert}$
Outer Product	$x \wedge y = \langle xy\rangle_{k+l}$
Commutator Product	$x \times y = \frac{1}{2}(xy - yx)$
Regressive Product	$(x \vee y)^* = x^* \wedge y^*$
Sandwich Product	$-1^{kl}xyx^{-1}$
Orthogonal projection of x **onto/into** y	$(x\cdot y)y^{-1}$

$\mathbb{R}_{3,0,1}$ - 3D Euclidean PGA

Plane	$ax+by+cz+d=0$	$a\mathbf{e}_1 + b\mathbf{e}_2 + c\mathbf{e}_3 + d\mathbf{e}_0$
Point	(x,y,z)	$(x\mathbf{e}_1 + y\mathbf{e}_2 + z\mathbf{e}_3 + \mathbf{e}_0)^*$
Line	Plücker coordinates $a..f$	$a\mathbf{e}_{01} + b\mathbf{e}_{02} + c\mathbf{e}_{03} + d\mathbf{e}_{23} + e\mathbf{e}_{31} + f\mathbf{e}_{12}$

basis	1	$\mathbf{e}_1$	$\mathbf{e}_2$	$\mathbf{e}_3$	$\mathbf{e}_0$	$\mathbf{e}_{01}$	$\mathbf{e}_{02}$	$\mathbf{e}_{03}$	$\mathbf{e}_{12}$	$\mathbf{e}_{31}$	$\mathbf{e}_{23}$	$\mathbf{e}_{032}$	$\mathbf{e}_{013}$	$\mathbf{e}_{021}$	$\mathbf{e}_{123}$	I
metric	+1	+1	+1	+1	0	0	0	0	-1	-1	-1	0	0	0	-1	0
dual	I	$\mathbf{e}_{032}$	$\mathbf{e}_{013}$	$\mathbf{e}_{021}$	$\mathbf{e}_{123}$	$\mathbf{e}_{23}$	$\mathbf{e}_{31}$	$\mathbf{e}_{12}$	$\mathbf{e}_{03}$	$\mathbf{e}_{02}$	$\mathbf{e}_{01}$	$-\mathbf{e}_1$	$-\mathbf{e}_2$	$-\mathbf{e}_3$	$-\mathbf{e}_0$	1
reverse	1	$\mathbf{e}_1$	$\mathbf{e}_2$	$\mathbf{e}_3$	$\mathbf{e}_0$	$-\mathbf{e}_{01}$	$-\mathbf{e}_{02}$	$-\mathbf{e}_{03}$	$-\mathbf{e}_{12}$	$-\mathbf{e}_{31}$	$-\mathbf{e}_{23}$	$-\mathbf{e}_{032}$	$-\mathbf{e}_{013}$	$-\mathbf{e}_{021}$	$-\mathbf{e}_{123}$	I
involute	1	$-\mathbf{e}_1$	$-\mathbf{e}_2$	$-\mathbf{e}_3$	$-\mathbf{e}_0$	$\mathbf{e}_{01}$	$\mathbf{e}_{02}$	$\mathbf{e}_{03}$	$\mathbf{e}_{12}$	$\mathbf{e}_{31}$	$\mathbf{e}_{23}$	$-\mathbf{e}_{032}$	$-\mathbf{e}_{013}$	$-\mathbf{e}_{021}$	$-\mathbf{e}_{123}$	I

Join points v, w **and line** ℓ	$\ell = v \vee w, \quad p = v \vee \ell$
Meet planes p, q **and line** ℓ	$\ell = p \wedge q, \quad v = p \wedge \ell$
Distance point P **to any** x	$\lVert\overline{P} \vee \overline{x}\rVert$
Angle between same grade x, y	$\cos^{-1}(\overline{x}\cdot\overline{y})$
Distance between parallel x, y	$\lVert\overline{x}\wedge\overline{y}\rVert_\infty$
Common Normal Line ℓ_1, ℓ_2	$\overline{\ell_1 \times \ell_2}$

Motors - General

Motor/Versor Squared Norm	$M\tilde{M} = s + tI$
Study inverse square root	$\frac{1}{\sqrt{s+tI}} = \frac{1}{\sqrt{s}} + \frac{t}{2\sqrt{s}^3}I$
Normalize Motor/Versor	$\overline{M} = \frac{M}{\sqrt{M\tilde{M}}}$
Square Root of Motor	$\sqrt{M} = \overline{1+M}$

Rotors - Group & Algebra

Logarithm of Motor $\quad \log M = b\langle M\rangle_2 + c\langle M\rangle_2 I$

$$a = \frac{1}{1-\langle M\rangle^2},\; b = \sqrt{a}\,\cos^{-1}\langle M\rangle,\; c = a\langle M^*\rangle(1 - b\langle M\rangle)$$

Exponential of Bivector $\quad e^B = c + sB + tBI + msI$

$$l = \langle B^2\rangle,\; m = \langle BB^*\rangle,\; a = \sqrt{l},\; c = \cos a,\; s = \frac{\sin a}{a},\; t = m\frac{c-s}{l}$$

Cayley Transform $\quad B = \frac{1-M}{1+M}, \quad M = \frac{1-B}{1+B}$

Rotors - Constructing/Finding

Rotor from Yaw, Pitch, Roll	$R = e^{\frac{-r\mathbf{e}_{12}}{2}} e^{\frac{-p\mathbf{e}_{23}}{2}} e^{\frac{-y\mathbf{e}_{31}}{2}}$
Rotate α **around line** $\bar{\ell}$	$e^{\frac{-\alpha}{2}\ell}$
Translate δ **along line** $\bar{\ell}$	$e^{\frac{-\delta}{2}\mathbf{e}_0\ell\mathbf{e}_0^*}$

Invariant Factorisation $M = TR = RT$ $\quad T = 1 + \frac{\langle M\rangle_4}{\langle M\rangle_2}, \quad R = M\tilde{T}$

Euclidean Factorisation $M = TR_{origin}$ $\quad T = 1 + \langle MM^*\rangle_2^*, \quad R_{origin} = \tilde{T}M$

$$R_{origin} = \langle M\rangle + \langle M\rangle_{\mathbf{e}_{12}} + \langle M\rangle_{\mathbf{e}_{31}} + \langle M\rangle_{\mathbf{e}_{23}}, \quad T = M\tilde{R}_{origin}$$

Motor k**-blade** y **to** k**-blade** x	$\sqrt{\frac{x}{y}} = \overline{1+\frac{x}{y}} = \frac{\overline{x \pm y}}{y}$
Direct motor points p_i **to** q_i	$M_{i+1} = \sqrt{\frac{q_1 \vee .. \vee q_i}{q_1 \vee .. \vee q_{i-1} \vee M_i p_i \tilde{M}_i}}\, M_i$
Interpolating motor R → S	$e^{t\log S\tilde{R}}R$
Average of $M_{1...n}$ **from guess** G_0	$G_{g+1} = \exp\left(\frac{1}{n}\sum_{j=1}^{n}\log(M_j\tilde{G}_g)\right)G_g$

Kinematics & Dynamics

Forque = Force + Torque = line	$f = d$-2 vector ℓ
Velocity = lin + ang = hyperline	$v =$ 2-vector ℓ
Inertial duality $x^\star$	$x^\star = x^*$ with coefficient scale
Euler's equations	$\dot{M} = \frac{1}{2}Mv, \quad \dot{v} = (f + v \times v^\star)^{-\star}$
Time derivative of point, line, plane x	$\dot{x} = x \times v$
Kinetic Energy	$e = \frac{1}{2}v^\star \vee v$

Polygons and Meshes

Vertices v	$v = (x\mathbf{e}_1 + y\mathbf{e}_2 + z\mathbf{e}_3 + \mathbf{e}_0)^*$
Edge Lines ℓ	$\ell = v_i \vee v_j$
Face Planes f	$f = v_i \vee v_j \vee v_k$
Mesh Surface Area	$\mathbf{a} = \frac{1}{2}\sum\lVert f_i\rVert$
Mesh Volume	$\mathbf{v} = \frac{1}{6}\lVert\sum f_i\rVert_\infty$

More info at **https://bivector.net**

$\mathbb{R}_{3,0,1}$ - Cayley Table

1	e_0	e_1	e_2	e_3	e_{01}	e_{02}	e_{03}	e_{12}	e_{31}	e_{23}	e_{021}	e_{013}	e_{032}	e_{123}	I
e_0	0	e_{01}	e_{02}	e_{03}	0	0	0	$-e_{021}$	$-e_{013}$	$-e_{032}$	0	0	0	I	0
e_1	$-e_{01}$	1	e_{12}	$-e_{31}$	$-e_0$	e_{021}	$-e_{013}$	e_2	$-e_3$	e_{123}	e_{02}	$-e_{03}$	I	e_{23}	e_{032}
e_2	$-e_{02}$	$-e_{12}$	1	e_{23}	$-e_{021}$	$-e_0$	e_{032}	$-e_1$	e_{123}	e_3	$-e_{01}$	I	e_{03}	e_{31}	e_{013}
e_3	$-e_{03}$	e_{31}	$-e_{23}$	1	e_{013}	$-e_{032}$	$-e_0$	e_{123}	e_1	$-e_2$	I	e_{01}	$-e_{02}$	e_{12}	e_{021}
e_{01}	0	e_0	$-e_{021}$	e_{013}	0	0	0	e_{02}	$-e_{03}$	I	0	0	0	$-e_{032}$	0
e_{02}	0	e_{021}	e_0	$-e_{032}$	0	0	0	$-e_{01}$	I	e_{03}	0	0	0	$-e_{013}$	0
e_{03}	0	$-e_{013}$	e_{032}	e_0	0	0	0	I	e_{01}	$-e_{02}$	0	0	0	$-e_{021}$	0
e_{12}	$-e_{021}$	$-e_2$	e_1	e_{123}	$-e_{02}$	e_{01}	I	-1	e_{23}	$-e_{31}$	e_0	e_{032}	$-e_{013}$	$-e_3$	$-e_{03}$
e_{31}	$-e_{013}$	e_3	e_{123}	$-e_1$	e_{03}	I	$-e_{01}$	$-e_{23}$	-1	e_{12}	$-e_{032}$	e_0	e_{021}	$-e_2$	$-e_{02}$
e_{23}	$-e_{032}$	e_{123}	$-e_3$	e_2	I	$-e_{03}$	e_{02}	e_{31}	$-e_{12}$	-1	e_{013}	$-e_{021}$	e_0	$-e_1$	$-e_{01}$
e_{021}	0	e_{02}	$-e_{01}$	$-I$	0	0	0	e_0	e_{032}	$-e_{013}$	0	0	0	e_{03}	0
e_{013}	0	$-e_{03}$	$-I$	e_{01}	0	0	0	$-e_{032}$	e_0	e_{021}	0	0	0	e_{02}	0
e_{032}	0	$-I$	e_{03}	$-e_{02}$	0	0	0	e_{013}	$-e_{021}$	e_0	0	0	0	e_{01}	0
e_{123}	$-I$	e_{23}	e_{31}	e_{12}	e_{032}	e_{013}	e_{021}	$-e_3$	$-e_2$	$-e_1$	$-e_{03}$	$-e_{02}$	$-e_{01}$	-1	e_0
I	0	$-e_{032}$	$-e_{013}$	$-e_{021}$	0	0	0	$-e_{03}$	$-e_{02}$	$-e_{01}$	0	0	0	$-e_0$	0

$\mathbb{R}_4$ - Cayley Table

1	e_0	e_1	e_2	e_3	e_{01}	e_{02}	e_{03}	e_{12}	e_{31}	e_{23}	e_{021}	e_{013}	e_{032}	e_{123}	I
e_0	1	e_{01}	e_{02}	e_{03}	e_1	e_2	e_3	$-e_{021}$	$-e_{013}$	$-e_{032}$	$-e_{12}$	$-e_{31}$	$-e_{23}$	I	e_{123}
e_1	$-e_{01}$	1	e_{12}	$-e_{31}$	$-e_0$	e_{021}	$-e_{013}$	e_2	$-e_3$	e_{123}	e_{02}	$-e_{03}$	I	e_{23}	e_{032}
e_2	$-e_{02}$	$-e_{12}$	1	e_{23}	$-e_{021}$	$-e_0$	e_{032}	$-e_1$	e_{123}	e_3	$-e_{01}$	I	e_{03}	e_{31}	e_{013}
e_3	$-e_{03}$	e_{31}	$-e_{23}$	1	e_{013}	$-e_{032}$	$-e_0$	e_{123}	e_1	$-e_2$	I	e_{01}	$-e_{02}$	e_{12}	e_{021}
e_{01}	$-e_1$	e_0	$-e_{021}$	e_{013}	-1	$-e_{12}$	e_{31}	e_{02}	$-e_{03}$	I	e_2	$-e_3$	e_{123}	$-e_{032}$	$-e_{23}$
e_{02}	$-e_2$	e_{021}	e_0	$-e_{032}$	e_{12}	-1	$-e_{23}$	$-e_{01}$	I	e_{03}	$-e_1$	e_{123}	e_3	$-e_{013}$	$-e_{31}$
e_{03}	$-e_3$	$-e_{013}$	e_{032}	e_0	$-e_{31}$	e_{23}	-1	I	e_{01}	$-e_{02}$	e_{123}	e_1	$-e_2$	$-e_{021}$	$-e_{12}$
e_{12}	$-e_{021}$	$-e_2$	e_1	e_{123}	$-e_{02}$	e_{01}	I	-1	e_{23}	$-e_{31}$	e_0	e_{032}	$-e_{013}$	$-e_3$	$-e_{03}$
e_{31}	$-e_{013}$	e_3	e_{123}	$-e_1$	e_{03}	I	$-e_{01}$	$-e_{23}$	-1	e_{12}	$-e_{032}$	e_0	e_{021}	$-e_2$	$-e_{02}$
e_{23}	$-e_{032}$	e_{123}	$-e_3$	e_2	I	$-e_{03}$	e_{02}	e_{31}	$-e_{12}$	-1	e_{013}	$-e_{021}$	e_0	$-e_1$	$-e_{01}$
e_{021}	$-e_{12}$	e_{02}	$-e_{01}$	$-I$	e_2	$-e_1$	$-e_{123}$	e_0	e_{032}	$-e_{013}$	-1	e_{23}	$-e_{31}$	e_{03}	e_3
e_{013}	$-e_{31}$	$-e_{03}$	$-I$	e_{01}	$-e_3$	$-e_{123}$	e_1	$-e_{032}$	e_0	e_{021}	$-e_{23}$	-1	e_{12}	e_{02}	e_2
e_{032}	$-e_{23}$	$-I$	e_{03}	$-e_{02}$	$-e_{123}$	e_3	$-e_2$	e_{013}	$-e_{021}$	e_0	e_{31}	$-e_{12}$	-1	e_{01}	e_1
e_{123}	$-I$	e_{23}	e_{31}	e_{12}	e_{032}	e_{013}	e_{021}	$-e_3$	$-e_2$	$-e_1$	$-e_{03}$	$-e_{02}$	$-e_{01}$	-1	e_0
I	$-e_{123}$	$-e_{032}$	$-e_{013}$	$-e_{021}$	$-e_{23}$	$-e_{31}$	$-e_{12}$	$-e_{03}$	$-e_{02}$	$-e_{01}$	$-e_3$	$-e_2$	$-e_1$	$-e_0$	1

$\mathbb{R}_{3,1}$ - Cayley Table

1	e_0	e_1	e_2	e_3	e_{01}	e_{02}	e_{03}	e_{12}	e_{31}	e_{23}	e_{021}	e_{013}	e_{032}	e_{123}	I
e_0	-1	e_{01}	e_{02}	e_{03}	$-e_1$	$-e_2$	$-e_3$	$-e_{021}$	$-e_{013}$	$-e_{032}$	e_{12}	e_{31}	e_{23}	I	$-e_{123}$
e_1	$-e_{01}$	1	e_{12}	$-e_{31}$	$-e_0$	e_{021}	$-e_{013}$	e_2	$-e_3$	e_{123}	e_{02}	$-e_{03}$	I	e_{23}	e_{032}
e_2	$-e_{02}$	$-e_{12}$	1	e_{23}	$-e_{021}$	$-e_0$	e_{032}	$-e_1$	e_{123}	e_3	$-e_{01}$	I	e_{03}	e_{31}	e_{013}
e_3	$-e_{03}$	e_{31}	$-e_{23}$	1	e_{013}	$-e_{032}$	$-e_0$	e_{123}	e_1	$-e_2$	I	e_{01}	$-e_{02}$	e_{12}	e_{021}
e_{01}	e_1	e_0	$-e_{021}$	e_{013}	1	e_{12}	$-e_{31}$	e_{02}	$-e_{03}$	I	$-e_2$	e_3	$-e_{123}$	$-e_{032}$	e_{23}
e_{02}	e_2	e_{021}	e_0	$-e_{032}$	$-e_{12}$	1	e_{23}	$-e_{01}$	I	e_{03}	e_1	$-e_{123}$	$-e_3$	$-e_{013}$	e_{31}
e_{03}	e_3	$-e_{013}$	e_{032}	e_0	e_{31}	$-e_{23}$	1	I	e_{01}	$-e_{02}$	$-e_{123}$	$-e_1$	e_2	$-e_{021}$	e_{12}
e_{12}	$-e_{021}$	$-e_2$	e_1	e_{123}	$-e_{02}$	e_{01}	I	-1	e_{23}	$-e_{31}$	e_0	e_{032}	$-e_{013}$	$-e_3$	$-e_{03}$
e_{31}	$-e_{013}$	e_3	e_{123}	$-e_1$	e_{03}	I	$-e_{01}$	$-e_{23}$	-1	e_{12}	$-e_{032}$	e_0	e_{021}	$-e_2$	$-e_{02}$
e_{23}	$-e_{032}$	e_{123}	$-e_3$	e_2	I	$-e_{03}$	e_{02}	e_{31}	$-e_{12}$	-1	e_{013}	$-e_{021}$	e_0	$-e_1$	$-e_{01}$
e_{021}	e_{12}	e_{02}	$-e_{01}$	$-I$	$-e_2$	e_1	e_{123}	e_0	e_{032}	$-e_{013}$	1	$-e_{23}$	e_{31}	e_{03}	$-e_3$
e_{013}	e_{31}	$-e_{03}$	$-I$	e_{01}	e_3	e_{123}	$-e_1$	$-e_{032}$	e_0	e_{021}	e_{23}	1	$-e_{12}$	e_{02}	$-e_2$
e_{032}	e_{23}	$-I$	e_{03}	$-e_{02}$	e_{123}	$-e_3$	e_2	e_{013}	$-e_{021}$	e_0	$-e_{31}$	e_{12}	1	e_{01}	$-e_1$
e_{123}	$-I$	e_{23}	e_{31}	e_{12}	e_{032}	e_{013}	e_{021}	$-e_3$	$-e_2$	$-e_1$	$-e_{03}$	$-e_{02}$	$-e_{01}$	-1	e_0
I	e_{123}	$-e_{032}$	$-e_{013}$	$-e_{021}$	e_{23}	e_{31}	e_{12}	$-e_{03}$	$-e_{02}$	$-e_{01}$	e_3	e_2	e_1	$-e_0$	-1

Matrix Forms

For a normalized rotor R, $\quad R\tilde{R} = 1$

$$R = R_0 + R_1\mathbf{e}_{01} + R_2\mathbf{e}_{02} + R_3\mathbf{e}_{03} + R_4\mathbf{e}_{12} + R_5\mathbf{e}_{31} + R_6\mathbf{e}_{23} + R_7\mathbf{e}_{0123}$$
$$R_{ij} = R_i R_j$$

The matrix that transforms a point $x\mathbf{e}_{032} + y\mathbf{e}_{013} + z\mathbf{e}_{021} + w\mathbf{e}_{123}$ with coordinate vector $[x, y, z, w]$ is

$$M = \mathbf{I} + 2\begin{bmatrix} -R_{44} - R_{55} & R_{04} + R_{56} & R_{46} - R_{05} & R_{35} - R_{01} - R_{24} - R_{67} \\ R_{56} - R_{04} & -R_{44} - R_{66} & R_{06} + R_{45} & R_{14} - R_{02} - R_{36} - R_{57} \\ R_{05} + R_{46} & R_{45} - R_{06} & -R_{55} - R_{66} & R_{26} - R_{03} - R_{15} - R_{47} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

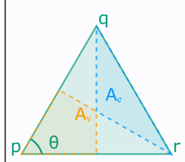
The matrix that transforms a plane $a\mathbf{e}_1 + b\mathbf{e}_2 + c\mathbf{e}_3 + d\mathbf{e}_0$ with coefficient vector $[a, b, c, d]$ is

$$M = \mathbf{I} + 2\begin{bmatrix} -R_{44} - R_{55} & R_{04} + R_{56} & R_{46} - R_{05} & 0 \\ R_{56} - R_{04} & -R_{44} - R_{66} & R_{06} + R_{45} & 0 \\ R_{05} + R_{46} & R_{45} - R_{06} & -R_{55} - R_{66} & 0 \\ R_{01} - R_{24} + R_{35} + R_{67} & R_{02} + R_{14} - R_{36} + R_{57} & R_{03} - R_{15} + R_{26} + R_{47} & 0 \end{bmatrix}$$

$\mathbb{R}_{3,0,1}$ - Overview

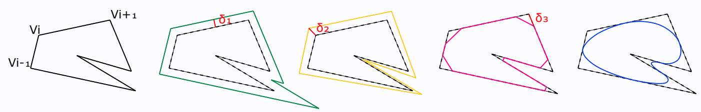
vector		bivector		trivector
e_1 e_2 e_3 e_0	1	e_{12} e_{31} e_{23} e_{01} e_{02} e_{03}	I	e_{021} e_{013} e_{032} e_{123}
Plane Reflection	Quaternion/Rotator	Translator		Point Reflection
	Dual Quaternion/Motor			
1-reflection	2-reflection			3-reflection
	4-reflection			
		Lie Algebra		
	Lie Group			
Homogeneous Plane		Line in Plücker Coordinates		Homogeneous Point

Triangle Areas



$$\cot\Theta = \frac{(p \vee q)\cdot(p \vee r)}{\|p \vee q \vee r\|}$$
$$A_{\text{tot}} = \tfrac{1}{2}\|p \vee q \vee r\|$$
$$A_v = \tfrac{1}{2}\|p \vee \overline{p+q} \vee \overline{p+q+r}\| + \tfrac{1}{2}\|p \vee \overline{p+q+r} \vee \overline{p+r}\|$$
$$A_e = \tfrac{1}{2}\|q \vee \overline{p+q+r} \vee r\|$$

Polygon Operators in n dimensions



Polygon P $\quad P = [\dots, v_{i-1}, v_i, v_{i+1}, \dots]$

Polygon Face f $\quad f = \overline{v_0 \vee v_1 \vee v_2}$

Incoming edge ℓ_{i-} $\quad \ell_{i-} = (\overline{v_{i-1} \vee v_i}) \cdot \tilde{f}$

Outgoing edge ℓ_{i+} $\quad \ell_{i+} = (\overline{v_i \vee v_{i+1}}) \cdot \tilde{f}$

Edge extrude δ_1 $\quad P = [\dots, (\ell_{i-} + \delta_1\mathbf{e}_0) \wedge (\ell_{i+} + \delta_1\mathbf{e}_0) \wedge f, \dots]$

Vertex extrude δ_2 $\quad d_i = \frac{\delta_2}{\sqrt{2}}\sqrt{1 + \ell_{i-}\cdot\ell_{i+}}\,\mathbf{e}_0$

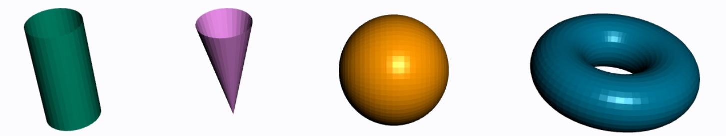
$$P = [\dots, (\ell_{i-} + d_i) \wedge (\ell_{i+} + d_i) \wedge f, \dots]$$

Vertex truncate δ_3 $\quad P = [\dots, v_i - \delta_3\frac{v_i - v_{i-1}}{\|v_i - v_{i-1}\|_\infty}, v_i - \delta_3\frac{v_i - v_{i+1}}{\|v_i - v_{i+1}\|_\infty}, \dots]$

Subdivide $\quad d_i = \frac{1}{2 + \sqrt{2 - \ell_{i-}\cdot\ell_{i+}}}$

$$P = [\dots, v_i - d_i(v_i - v_{i-1}), v_i - d_i(v_i - v_{i+1}), \dots]$$

3D Motor Orbits ($0 \le s, t \le 1$)



Cylinder with radius r, height h $\quad e^{-th\mathbf{e}_{02}}\, e^{s\pi\mathbf{e}_{13}}\, e^{r\mathbf{e}_{01}}$

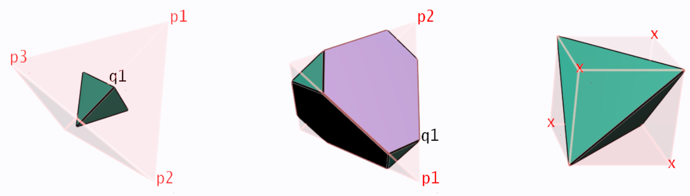
Cone with radius r, height h $\quad e^{-th\mathbf{e}_{02}}\, e^{s\pi\mathbf{e}_{13}}\, e^{sr\mathbf{e}_{01}}$

Sphere of radius r $\quad e^{t\pi\mathbf{e}_{23}}\, e^{s\frac{\pi}{2}\mathbf{e}_{13}}\, e^{r\mathbf{e}_{01}}$

Torus of inner radius r_1 and outer radius r_2 $\quad e^{s\pi\mathbf{e}_{13}}\, e^{r_2\mathbf{e}_{01}}\, e^{s\pi\mathbf{e}_{12}}\, e^{r_1\mathbf{e}_{01}}$

Polyhedron Operators

Dual : $D(x)$ | Truncate : $T_\alpha(x)$ | Alternate : $X(x)$

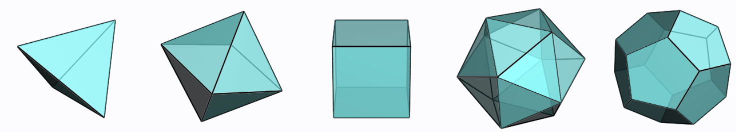


vertices $\leftrightarrow$ faces $\quad q_1 = p_1 + p_2 + p_3$

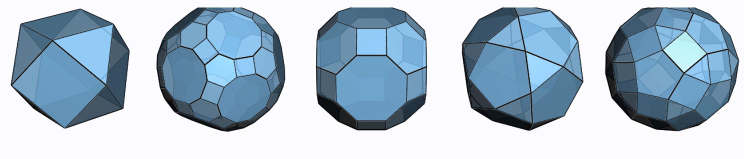
truncate vertices $\quad q_1 = (1 - \frac{\alpha}{2})p_1 + \frac{\alpha}{2}p_2$

eliminate alternating vertices

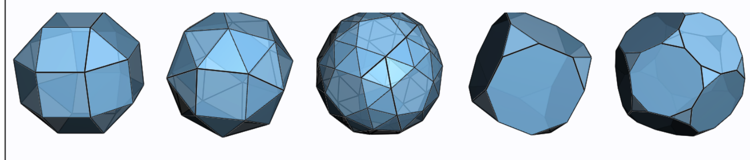
Ambo $A(x)$	$T_1(x)$	Truncate untill $q_1 = q_2$
Snub $S(x)$	$X(T_{3-\sqrt{5}}(A(x)))$	Ambo, Truncate, Alternate
Expand $E(x)$	$A(A(x))$	Ambo, Ambo



Tetrahedron x | Octahedron $A(x)$ | Cube $D(A(x))$ | Icosahedron $S(x)$ | Dodecahedron $D(S(x))$



Cuboctahedron $E(x)$ | Rhombicosi-dodecahedron $T(A(S(x)))$ | Rhombicub-octahedron $T(E(x))$ | Icosi-dodecahedron $A(S(x))$ | Small Rhombicosi-dodecahedron $E(S(x))$



Small Rhombicub-octahedron $E(A(x))$ | Snub Cube $S(D(A(x)))$ | Snub Dodecahedron $S(D(S(x)))$ | Truncated Cube $T_{3-\sqrt{5}}(D(A(x)))$ | Truncated Dodecahedron $T(D(S(x)))$