

Now for a dual quat * plane

20123)

$$(q.w + q.e23 + q.e31 + q.e12 + q.e01 + q.e02 + q.e03 + q.e0123) \\ \times (p.e0 + p.e1 + p.e2 + p.e3)$$

$$\begin{aligned} = & +(q.w)(p.e0)e0 + (q.w)(p.e1)e1 + (q.w)(p.e2)e2 \\ & + (q.w)(p.e3)e3 \\ & + (q.e23)(p.e0)(-e032) + (q.e23)(p.e1)e123 + (q.e23)(p.e2)(-e3) \\ & + (q.e23)(p.e3)e2 \\ & + (q.e31)(p.e0)(-e013) + (q.e31)(p.e1)e3 + (q.e31)(p.e2)e123 \\ & + (q.e31)(p.e3)(-e1) \\ & + (q.e12)(p.e0)(-e021) + (q.e12)(p.e1)(-e2) + (q.e12)(p.e2)e1 \\ & + (q.e12)(p.e3)e123 + (q.e1) \\ & + (q.e01)(p.e1)e0 + (q.e01)(p.e2)(-e021) + (q.e01)(p.e3)e013 \\ & + (q.e02)(p.e1)e021 + (q.e02)(p.e2)e0 + (q.e02)(p.e3)(-e032) \\ & + (q.e03)(p.e1)(-e03) + (q.e03)(p.e2)e032 + (q.e03)(p.e3)e0 \\ & + (q.e0123)(p.e1)(-e032) + (q.e0123)(p.e2)(-e013) + (q.e0123)(p.e3)(-e021) \end{aligned}$$

Group up to find

$$e0 \quad (q.w)(p.e0) \\ + (q.e01)(p.e1) + (q.e02)(p.e2) + (q.e03)(p.e3)$$

$$e1 \quad (q.w)(p.e1) - (q.e31)(p.e3) + (q.e12)(p.e2)$$

$$e2 \quad (q.w)(p.e2) + (q.e23)(p.e3) - (q.e12)(p.e1)$$

$$e3 \quad (q.w)(p.e3) - (q.e23)(p.e2) + (q.e31)(p.e1)$$

$$e032 \quad - (q.e23)(e0) - (q.e02)(p.e3) + (q.e03)(p.e2) \\ - (q.e0123)(p.e1)$$

$$e013 \quad - (q.e31)(p.e0) + (q.e01)(p.e3) - (q.e03)(p.e1) \\ - (q.e0123)(p.e2)$$

$$e021 \quad - (q.e12)(p.e0) + (q.e01)(p.e2) + (q.e02)(p.e1) \\ - (q.e0123)(p.e3)$$

$$e0123 \quad (q.e23)(p.e1) + (q.e31)(p.e2) + (q.e12)(p.e3)$$