

$$23) \quad (p.e_0 + p.e_1 + p.e_2 + p.e_3) \times (q.w + q.e_{12} + q.e_{31} + q.e_{23} + q.e_{01} + q.e_{02} + q.e_{03} + q.e_{012})$$

$$= (p.e_0)(q.w)e_0 + (p.e_0)(q.e_{12})(-e_{021}) + (p.e_0)(q.e_{31})(-e_{013}) + (p.e_0)(q.e_{23})(-e_{032}) + (p.e_1)(q.w)e_1 + (p.e_1)(q.e_{12})e_2 + (p.e_1)(q.e_{31})(-e_3) + (p.e_1)(q.e_{23})e_{123} + (p.e_1)(q.e_{01})(-e_0) + (p.e_1)(q.e_{02})e_{021} + (p.e_1)(q.e_{03})(-e_{013}) + (p.e_1)(q.e_{0123})e_{032} + (p.e_2)(q.w)e_2 + (p.e_2)(q.e_{12})(-e_1) + (p.e_2)(q.e_{31})e_{123} + (p.e_2)(q.e_{23})e_3 + (p.e_2)(q.e_{01})(-e_{011}) + (p.e_2)(q.e_{02})(-e_0) + (p.e_2)(q.e_{03})e_{032} + (p.e_2)(q.e_{0123})e_{013} + (p.e_3)(q.w)e_3 + (p.e_3)(q.e_{12})e_{123} + (p.e_3)(q.e_{31})e_1 + (p.e_3)(q.e_{23})(-e_2) + (p.e_3)(q.e_{01})e_{013} + (p.e_3)(q.e_{02})(-e_{032}) + (p.e_3)(q.e_{03})(-e_0) + (p.e_3)(q.e_{0123})e_{021}$$

Grouping up we get

$$e_0 \quad (p.e_0)(q.w) - (p.e_2)(q.e_{02}) - (p.e_1)(q.e_{01}) - (p.e_3)(q.e_{03})$$

$$e_1 \quad (p.e_1)(q.w) - (p.e_2)(q.e_{12}) + (p.e_3)(q.e_{31})$$

$$e_2 \quad (p.e_1)(q.e_{12}) + (p.e_2)(q.w) - (p.e_3)(q.e_{23})$$

$$e_3 \quad (p.e_2)(q.e_{23}) + (p.e_3)(q.w) - (p.e_1)(q.e_{31})$$

$$e_{032} \quad -(p.e_0)(q.e_{23}) + (p.e_1)(q.e_{0123}) + (p.e_2)(q.e_{03}) - (p.e_3)(q.e_{02})$$

$$e_{013} \quad -(p.e_0)(q.e_{31}) - (p.e_1)(q.e_{03}) + (p.e_2)(q.e_{0123}) + (p.e_3)(q.e_{01})$$

$$e_{021} \quad -(p.e_0)(q.e_{12}) + (p.e_1)(q.e_{02}) - (p.e_2)(q.e_{01}) + (p.e_3)(q.e_{0123})$$

$$e_{123} \quad (p.e_1)(q.e_{23}) + (p.e_2)(q.e_{31}) + (p.e_3)(q.e_{12})$$