# 80 reasons you want to take the geometric product of a point/line/plane with a point/line plane in your game engine

| If you want the... | When A is a... | | When B is a... | M=AB lets you get it with... | |
|---|---|---|---|---|---|
| Intersection of a | Plane | with a | Plane | $\langle M\rangle_2$ | Remember: $\langle M\rangle_3$ means the *point* part of M; $\langle M\rangle_3$, the line part is $\langle M\rangle_2$, the plane part is $\langle M\rangle_1$ |
| | | | Line | $\langle M\rangle_3$ | |
| | Point | onto a | Plane | $\langle M\rangle_2/B$ | |
| | | | Line | $\langle M\rangle_1/B$ | |
| Projection of a | Plane | onto a | Line | $\langle M\rangle_1/B$ | |
| | | | Point | $\langle M\rangle_2/B$ | |
| | Line | onto a | Plane | $\langle M\rangle_1/B$ | |
| | | | Point | $\langle M\rangle_1/B$ | |
| | Point | to a | Plane | $\sqrt{M}$ | The principle square root of M is 1+normalize(M) |
| | | | Line | $\sqrt{\sqrt{(M^2)}}$ | $\sqrt{(M^2)}$ can be different from M if, for example, M is a planar reflection - then $\sqrt{(M^2)}$ will just be the identity |
| | | | Point | $\sqrt{M}$ | |
| | Line | to a | Plane | $\sqrt{\sqrt{(M^2)}}$ | |
| Transform that takes a | | | Line | $\sqrt{M}$ | |
| | | | Point | $\sqrt{\sqrt{(M^2)}}$ | |
| | Plane | to a | Plane | $\sqrt{M}$ | This is how you are already getting a quaternion (rotation) from a start vector to an end vector! |
| | | | Line | $\sqrt{\sqrt{(M^2)}}$ | |
| | | | Point | $\sqrt{M}$ | |
| | Rotation | to a | Point | M/A | And this is how you are applying quaternions to vertices! |
| | | | Line | M/A | |
| | | | Plane | M/A | |
| | | | Rotation | M/A | |
| | | | Translation | M/A | |
| | | | Rigid motion | M/A | |
| | Translation | to a | Point | M/A | |
| | | | Line | M/A | |
| | | | Plane | M/A | |
| | | | Rotation | M/A | |
| | | | Translation | M/A | |
| Result of applying a | | | Rigid motion | M/A | |
| | Rigid motion | to a | Point | M/A | |
| | | | Line | M/A | |
| | | | Plane | M/A | |
| | | | Rotation | M/A | |
| | | | Translation | M/A | |
| | | | Rigid motion | M/A | |
| | Planar reflection | to a | Translation | M/A | |
| | | | Rigid motion | M/A | |
| | | | Line | M/A | |
| | | | Rotation | M/A | |
| | | | Plane | -M/A | Yes, negative! It's because planar reflections and "point reflections" *both* change handedness |
| | | | Point | -M/A | |
| Angle between a | Line | and a | Line | $\mathrm{atan}(\sqrt{\lvert\langle M\rangle_2^2/\langle M\rangle_0^2\rvert}$ | $\langle M\rangle_x^2$ will be a scalar for all of these; it is the magnitude of the grade-x part of M |
| | | | Plane | $\mathrm{atan}(\sqrt{\lvert\langle M\rangle_3^2/\langle M\rangle_1^2\rvert}$ | So we are taking a ratio between the magnitudes of the lowest- and second-lowest-grade parts of a transformation from A toward B |
| Distance between a | Point | and a | Plane | $\sqrt{\lvert(\langle M\rangle_4^*)^2/\langle M\rangle_2^2\rvert}$ | |
| | | | Line | $\sqrt{\lvert(\langle M\rangle_3^*)^2/\langle M\rangle_1^2\rvert}$ | |
| | | | Point | $\sqrt{\lvert(\langle M\rangle_2^*)^2/\langle M\rangle_0^2\rvert}$ | |
| | Line | and a | Parallel plane | $\sqrt{\lvert(\langle M\rangle_3^*)^2/\langle M\rangle_1^2\rvert}$ | |
| | | | Non-parallel line | $\sqrt{\lvert(\langle M\rangle_4^*)^2/\langle M\rangle_2^2\rvert}$ | In all cases, we are taking a ratio between the magnitudes of the highest-grade parts of a transformation from A toward B |
| | | | Parallel line | $\sqrt{\lvert(\langle M\rangle_2^*)^2/\langle M\rangle_0^2\rvert}$ | |
| | | | Point | $\sqrt{\lvert(\langle M\rangle_3^*)^2/\langle M\rangle_1^2\rvert}$ | |
| | Plane | and a | Parallel plane | $\sqrt{\lvert(\langle M\rangle_2^*)^2/\langle M\rangle_0^2\rvert}$ | |
| | | | Parallel line | $\sqrt{\lvert(\langle M\rangle_3^*)^2/\langle M\rangle_1^2\rvert}$ | |
| | | | Point | $\sqrt{\lvert(\langle M\rangle_4^*)^2/\langle M\rangle_2^2\rvert}$ | |
| Derivative of a | Plane | | | $\langle M\rangle_1$ | |
| | Line | wrt a | Rotation axis line | $\langle M\rangle_2$ | If your rotation is R, your rotation axis line B is the logarithm of R, and the rotation angle is \|B\| |
| | Point | | | $\langle M\rangle_3$ | |
| | Plane | | | $\langle M\rangle_1$ | |
| | Line | wrt a | Translation axis line | $\langle M\rangle_2$ | If your translation is T, your translation axis line B is the logarithm of T, and the translation distance is \|B\| |
| | Point | | | $\langle M\rangle_3$ | |
| | Plane | | | $\langle M\rangle_1$ | |
| | Line | wrt a | Rigid motion axis | $\langle M\rangle_2$ | A rigid motion axis is a sum of a rotation axis line and a translation axis line |
| | Point | | | $\langle M\rangle_3$ | |
| Result of composing a | Rotation | with a | Planar reflection | M | |
| | | | Rotation | M | |
| | | | Translation | M | |
| | | | Rigid motion | M | |
| | Translation | with a | Planar reflection | M | |
| | | | Rotation | M | |
| | | | Translation | M | |
| | | | Rigid motion | M | Yes, this is what the geometric product fundamentally does! Note the slight difference between "compose" and "transform between" |
| | Planar reflection | with a | Planar reflection | M | |
| | | | Rotation | M | |
| | | | Translation | M | |
| | | | Screw motion | M | |
| | Rigid motion | with a | Planar reflection | M | |
| | | | Rotation | M | |
| | | | Translation | M | |
| | | | Rigid motion | M | |