20 MAY 2023

To combine rotation & translation I need to be able to combine dual quats by multiplying

rotation then translation = Dual quat tr * Dual quat rot

How do you multiply these beasts?

$$q_1^d \times q_2^d = \left(w_1 + yz_1 e_{23} + zx_1 e_{31} + xy_1 e_{23} + dx_1 e_{01} + dy_1 e_{02} + dz_1 e_{03} + dxyz_1 e_{0123}\right) \times \left(w_2 + yz_2 e_{23} + zx_2 e_{31} + xy_2 e_{23} + dx_2 e_{01} + dy_2 e_{02} + dz_2 e_{03} + dxyz_2 e_{0123}\right)$$

$$w_1 w_2 + w_1 yz_2 e_{23} + w_1 zx_2 e_{31} + w_1 xy_2 e_{23} + w_1 dx_2 e_{01} + w_1 dy_1 e_{02} + w_1 dz_2 e_{03} + w_1 dxyz_1 e_{0123}$$

you know what, looking at Marish Todd's GDC talk it may be easier to write out this way

$$q_a^{dual} \times q_b^{dual} = \left(a.w + a.e_{23} + a.e_{31} + a.e_{23} + a.e_{01} + a.e_{02} + a.e_{03} + a.e_{0123}\right) * \left(b.w + b.e_{23} + b.e_{31} + b.e_{23} + b.e_{01} + b.e_{02} + b.e_{03} + b.e_{0123}\right)$$

$$= a.w * b.w + a.w * b.e_{23} + a.w * b.e_{31} + a.w * b.e_{23} + a.w * b.e_{01} + a.w * b.e_{02} + a.w * b.e_{03} + a.w * b.e_{0123}$$

Nah looks even more confusing. lets go with what I had before but for a & b

$$q_a^{dual} \times q_b^{dual} = \left(w_a + yz_a e_{23} + zx_a e_{31} + xy_a e_{12} + dx_a e_{01} + dy_a e_{02} + dz_a e_{03} + dxyz_a e_{0123}\right) \times \left(w_b + yz_b e_{23} + zx_b e_{31} + xy_b e_{12} + dx_b e_{01} + dy_b e_{02} + dz_b e_{03} + dxyz_b e_{0123}\right)$$

$$= w_a w_b + (w_a yz_b) e_{23} + (w_a zx_b) e_{31} + (w_a xy_b) e_{12} + (w_a dx_b) e_{01} + (w_a dy_b) e_{02} + (w_a dz_b) e_{03} + (w_a dxyz_b) e_{0123}$$
$$+ (yz_a w_b) e_{23} \mp (yz_a yz_b) e_{2323} + (yz_a zx_b) e_{2331} + (yz_a xy_b) e_{2312} + (yz_a dx_b) e_{2301}^{0123} + (yz_a dy_b) e_{2302}^{30} \mp (yz_a dz_b) e_{2303}^{20}$$
$$\mp (yz_a dxyz_b) e_{230123}^{01}$$
$$+ (zx_a w_b) e_{31} + (zx_a yz_b) e_{3123}^{12} \mp (zx_a zx_b) e_{3131} + (zx_a xy_b) e_{3112}^{32} \mp (zx_a dx_b) e_{3101}^{30} + (zx_a dy_b) e_{3102}^{0123} + (zx_a dz_b) e_{3103}^{10}$$
$$\mp (zx_a dxyz_b) e_{310123}^{02}$$
$$+ (xy_a w_b) e_{12} + (xy_a yz_b) e_{1223}^{13} + (xy_a zx_b) e_{1231}^{23} \mp (xy_a xy_b) e_{1212} + (xy_a dx_b) e_{1201}^{20} \mp (xy_a dy_b) e_{1202}^{10} + (xy_a dz_b) e_{1203}^{0123}$$
$$\mp (xy_a dxyz) e_{120123}^{03}$$
$$+ (dx_a w_b) e_{01} + (dx_a yz_b) e_{0123} \mp (dx_a zx_b) e_{0131}^{03} + (dx_a xy_b) e_{0112}^{02} + (dy_a w_b) e_{02} + (dy_a yz_b) e_{0223}^{03} + (dy_a zx_b) e_{0231}^{0123} \mp (dy_a xy_b) e_{0212}^{01}$$
$$+ (dz_a w_b) e_{03} \mp (dz_a yz_b) e_{0323}^{02} + (dz_a zx_b) e_{0331}^{01} + (dz_a xy_b) e_{0312}^{0123} + (dxyz_a w_b) e_{0123} \mp (dxyz_a yz_b) e_{012323}^{01}$$
$$\mp (dxyz_a zx_b) e_{012331}^{02} \mp (dxyz_a xy_b) e_{012312}^{03}$$

Phew. Lets group em up according to REAL

e_{23}

e_{31}

e_{12}

e_{01}

e_{02}

e_{03}

e_{0123}

$q_a \times q_b$

REAL => $w_a w_b - yz_a yz_b - zx_a zx_b - xy_a xy_b$

PART or in code

a.w * b.w - a.e23 * b.e23 - a.e31 * b.e31 - a.e12 * b.e12

e_{23} => $w_a yz_b + yz_a w_b - zx_a xy_b + xy_a zx_b$

PART or in code

a.w * b.e23 + a.e23 * b.w - a.e31 * b.e12
+ a.e12 * b.e31

e_{31} => $w_a zx_b + yz_a xy_b + zx_a w_b - xy_a yz_b$

PART or in code

a.w * b.e31 + a.e23 * b.e12 + a.e31 * b.w
- a.e12 * b.e23

e_{12} => $w_a xy_b - yz_a zx_b + zx_a yz_b + xy_a w_b$

PART or in code

a.w * b.e12 - a.e23 * b.e31 + a.e31 * b.e23
+ a.e12 * b.w

$e_{01} \Rightarrow w_a dx_b - yz_a dxyz_b - zx_a dz_b + xy_a dy_b$
PART $+ dx_a w_b - dy_a xy_b + dz_a zx_b - dxyz_a yz_b$

or in code:

```
a.w*b.e01 - a.e23*b.e0123 - a.e31*b.e03
+a.e12*b.e02
+a.e01*b.w - a.e02*b.e12 + a.e03*b.e31
-a.e0123*b.e23
```

$e_{02} \Rightarrow w_a dy_b + yz_a dz_b - zx_a dxyz_b - xy_a dx_b$
PART $+ dx_a xy_b - dz_a yz_b - dxyz_a zx_b + dy_a w_b$

or in code:

```
a.w*b.e02 + a.e23*b.e03 - a.e31*b.e0123
-a.e12*b.e01
+a.e01*b.e12 - a.e03*b.e23 + a.e02*b.w
-a.e0123*b.e31
```

$e_{03} \Rightarrow w_a dz_b - yz_a dy_b + zx_a dx_b - xy_a dxyz$
part $- dx_a zx_b + dy_a yz_b + dz_a w_b - dxyz_a xy_b$

or in code

```
a.w*b.e03 - a.e23*b.e02 + a.e31*b.e01
-a.e12*b.e0123
-a.e01*b.e31 + a.e02*b.e23 + a.e03*b.w
-a.e0123*b.e12
```

$e_{0123} \Rightarrow w_a dxyz_b + yz_a dx_b + zx_a dy_b + xy_a dz_b$
PART $+ dx_a yz_b + dy_a zx_b + dz_a xy_b + dxyz_a w_b$

or in code:

```
a.w*b.e0123 + a.e23*b.e01 + a.e31*b.e02
+a.e12*b.e03
+a.e01*b.e23 + a.e02*b.e31 + a.e03*b.e12
+a.e0123*b.w
```