8 April 2024 U.N.

Scott is worried abt my DQMath function that converts the dual quaternion into a matrix. The function extracts the translation vector & orientation quaternion and returns the matrix $T \times R$, where T is the translation matrix & R is the rotation matrix.

This is odd as the view matrix is built in the order $R \times T$, so why does it work when I return $T \times R$?

Let's try an example to see whats going on.

Say I have a translation of $[0, 0, 2, 1]$ and a rotn of 180° abt z

In matrix form $T = \begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&2 \\ 0&0&0&1 \end{pmatrix}$ & $R = \begin{pmatrix} -1&0&0&0 \\ 0&-1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix}$

$$M_1 = TR = \begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&2 \\ 0&0&0&1 \end{pmatrix}\begin{pmatrix} -1&0&0&0 \\ 0&-1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix} = \begin{pmatrix} -1&0&0&0 \\ 0&-1&0&0 \\ 0&0&1&2 \\ 0&0&0&1 \end{pmatrix}$$

$$M_2 = RT = \begin{pmatrix} -1&0&0&0 \\ 0&-1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix}\begin{pmatrix} 1&0&0&0 \\ 0&1&0&0 \\ 0&0&1&2 \\ 0&0&0&1 \end{pmatrix} = \begin{pmatrix} -1&0&0&0 \\ 0&-1&0&0 \\ 0&0&1&2 \\ 0&0&0&1 \end{pmatrix}$$

Both matrices are the same in this case! The origin ends up at

$$O' = \begin{pmatrix} -1&0&0&0 \\ 0&-1&0&0 \\ 0&0&1&2 \\ 0&0&0&1 \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \\ 1 \end{pmatrix}$$

in both TR & RT cases as the translation & rotation are along / abt the same axis.

But what if the translation & rot^n are along / abt different axes? Say translation is [2, 0, 0, 1] instead and rot^n is still 180° abt z?

$$M_1 = TR = \begin{pmatrix} 1&0&0&2 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix} \begin{pmatrix} -1&0&0&0 \\ 0&-1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix} = \begin{pmatrix} -1&0&0&2 \\ 0&-1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix}$$

$$M_2 = RT \quad \begin{pmatrix} -1&0&0&0 \\ 0&-1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix} \begin{pmatrix} 1&0&0&2 \\ 0&1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix} = \begin{pmatrix} -1&0&0&-2 \\ 0&-1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix}$$

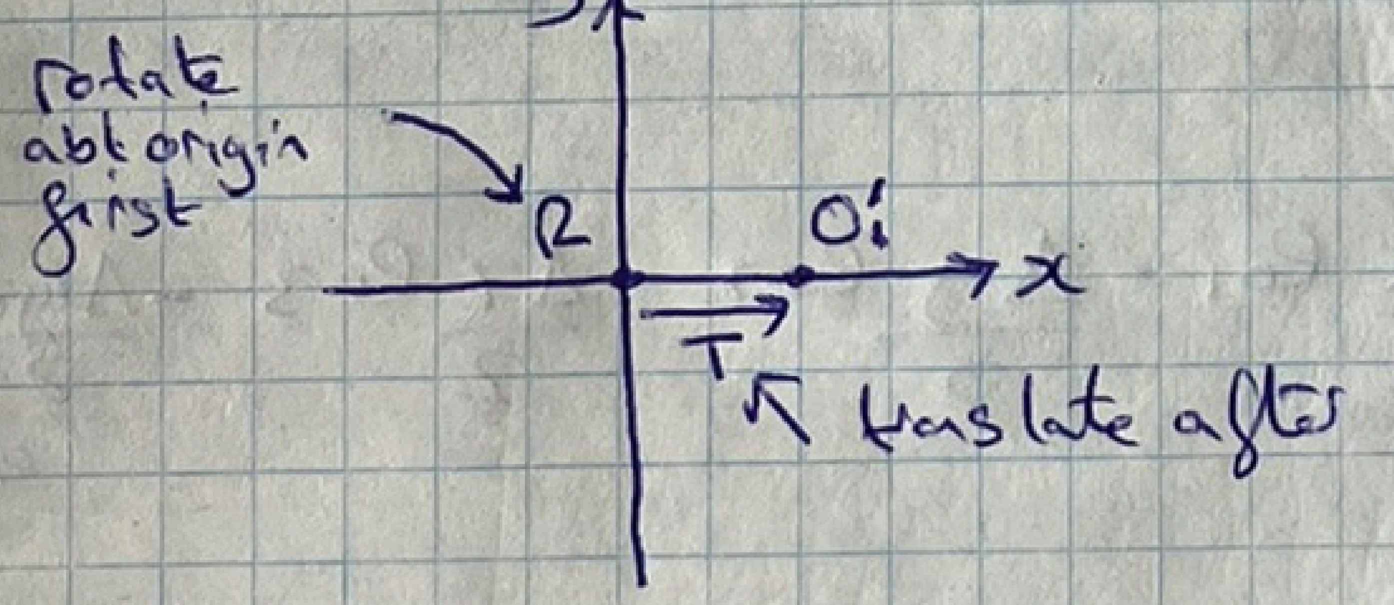
In this case the matrices are different! For the first case, the matrix $M_1 = TR$ moves the origin by:

$$O_1' = \begin{pmatrix} -1&0&0&2 \\ 0&-1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$ which is 2 units right

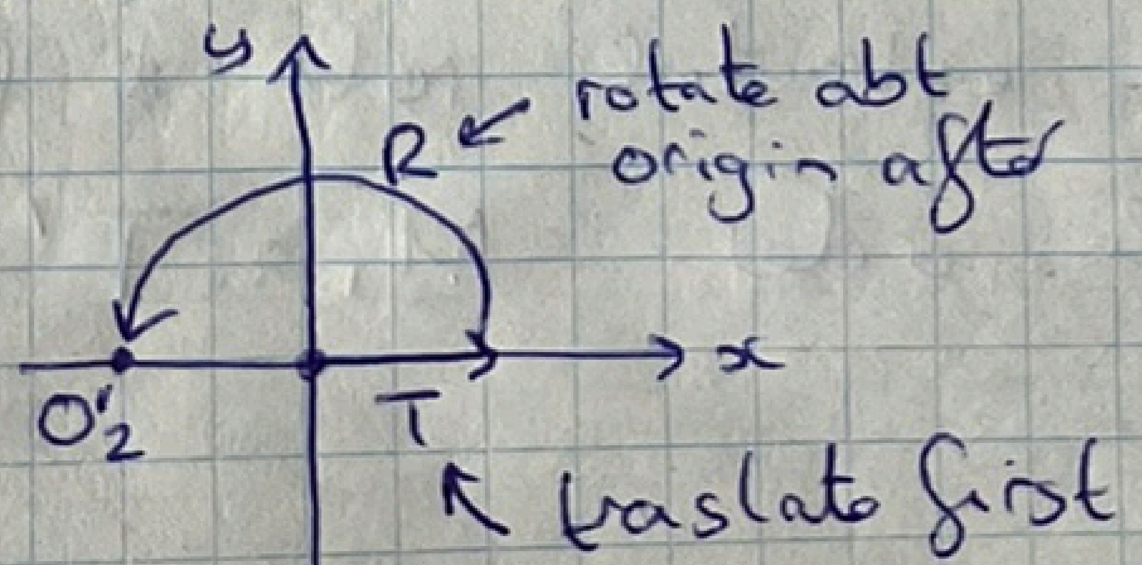
The other matrix $M_2 = RT$ moves the origin by:

$$O_2' = \begin{pmatrix} -1&0&0&-2 \\ 0&-1&0&0 \\ 0&0&1&0 \\ 0&0&0&1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -2 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$ which is 2 units left

This makes sense as in the first case we rotate first then translate right:



In the second case $M_2 = RT$, we translate to the right first then rotate



Let's do all this again using dual quaternions and see if I get the same results. ③

A traslation of 2 units along x is $1 - \frac{2}{2} e_{01}$

$T = 1 - e_{01}$

A rot^n of 180° abt z is $\cos\left(\frac{180^\circ}{2}\right) - \sin\left(\frac{180^\circ}{2}\right) e_{\cancel{23}\,12}$ oops

$R = -e_{\cancel{23}\,12}$

$M_1 = TR = (1 - e_{01})(-e_{\cancel{23}\,12}) = -e_{\cancel{23}\,12} + e_{\cancel{0123}\,02}$

$M_2 = RT = (-e_{\cancel{23}\,12})(1 - e_{01}) = -e_{\cancel{23}\,12} - e_{02}$

So the motors are different, just like the matrices were.

Now the weird thing in my code, as Scott points out, is that I convert dual quaternions as $T \times R$ in matrix form, no matter the order they originally come in.

Lets see what is going on.

$M_1 = -e_{12} + e_{02}$

Extract rot^n from the real, e_{23}, e_{31}, e_{12} part

$R' = -e_{12}$ ← angle $= a\cos(0) \times 2 = 180^\circ$

axis $= [0, 0, 1]$

Extract traslation by undoing the rot^n on the $e_{01}, e_{02}, e_{03}, e_{0123}$ part

$$\begin{aligned} T' = 1 + e_{02} \times R^{-1} &= e_{02}(-e_{21}) + 1 \\ &= e_{02}(e_{12}) + 1 \\ &= 1 - e_{01} \end{aligned}$$

A traslation is given by $1 - \frac{\Delta x}{2} e_{01} - \frac{\Delta y}{2} e_{02} - \frac{\Delta z}{2} e_{03} - \frac{\Delta xyz}{2} e_{0123}$

So $1 - e_{01}$ corresponds to a traslation of $[2, 0, 0]$

So we have the extracted rot^n & traslation that are multiplied in the order $M_1 = T'R'$.

If I convert those into matrices, I have

R' is a rotn of $180°$ abt z

T' is a traslation of $[2,0,0]$

$$M_1 = \begin{pmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & 2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \checkmark$$

exactly what I started with on page 2

Now onto the strange case:

$$M_2 = RT = -e_{12} - e_{02}$$

This matrix/dual quat comes in as RT but I'm gonna convert it to $T'R'$. How's that gonna work?

Ok, just as before on page 3, extract the rotn

$$R' = -e_{12} \leftarrow \text{angle} = \cos^{-1}(0) \times 2 = 180°$$
$$\text{axis} = [0,0,1]$$

Extract translation by undoing the rotn on the infinite part

$$T' = 1 + (-e_{02}) \times R^{-1}$$
$$= 1 - e_{02}(-e_{21})$$
$$= 1 - e_{02}(e_{12})$$
$$= 1 - -e_{01} = 1 + e_{01}$$

compare w/ $1 - \frac{\Delta x}{2} e_{01}$ gives us the translation vector $[-2,0,0]$

Notice that this traslation is different to the T' on page 3

Return this as a matrix ~~as~~ using:

$$M_2 = T'R' = \begin{pmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 & -2 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \checkmark$$

exactly what I started with on page 2

That means if you build a trasformation that traslates first & rotates after $M = RT$, I can always recreate a different order $M = T'R'$ that performs the exact same result!