Lifetime of Cosmic-Ray Muons

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Abstract

Using a scintillation method with coincidence counting, the lifetime of the muon went under investigation. Muons travelling to the Earth's surface are stopped in the scintillator and decay. The times at which they decay are measured and sorted into channels of time width (2.1222 ± 0.0005) x 10^{-8} s. Due to the chemical composition of the scintillator, the lifetime of the muon lied between the accepted value for a free and positive lifetime of $\tau^{+¿¿}$= 2.197120 ± 0.000077 μs, and the value for a negative muon captured in carbon τ_c= τ^-= 2.045 ± 0.002 μs. [3] Using a non-linear fitting process in Mathematica 8, the observed lifetime was τ_{obs} = 2.13 ± 0.01 μs**.** With these values, a muon charge ratio of positive to negative muons was calculated and found to be 1.36 ± 0.36.

Introduction and Theory

The term cosmic ray was given its name because they were initially thought to be electromagnetic radiation. Cosmic rays are actually energetic particles with outer space origins. As these cosmic rays travel through space they can penetrate and interact with the Earth's atmosphere. This interaction causes a production of numerous secondary particles as well as photons. In this study, the elementary positive and negative muon particles were under study. As muons travel through the Earth's atmosphere, they reach the surface with a lower energy, but are still observable. The muon, a fundamental constituent of matter, is unstable and undergoes relatively fast radioactive decay. As the muon decays it breaks up into either an electron or a positron alongside two neutrinos. The plastic scintillator used in this experiment detects this decay as muons travel through the Earth's atmosphere and happen to pass through and rest in the scintillator. Only if a muon has lost the majority of its energy in its travels through the atmosphere will be detected by the scintillator.

The decay of an elementary particle, like the muon, exhibits the same behaviour as the decay of any radioactive material. So, the same law governs both. If the rate of decay is assumed to be constant, then the process of a single particle decaying is governed by:

$$\frac{dN}{dt} = -\propto N \quad (1)$$

By integrating (1) with respect to time, the relation can be shown to be (2) below where $\propto = 1/\tau$ and τ is the lifetime of the particle.

$$N(t) = N_o \exp(-t/\tau) + N_b \quad (2)$$

Here, N(t) is the number of particles at a given time, t, and N_0 is the initial amount of particles at t = 0, and N_b is just a constant from the integration, which is equal to zero in an ideal decay.

As a free particle, both negative and positive muons have a known lifetime of 2.197120 ± 0.000077 μs. However, this changes when they decay in condensed matter, like the scintillator in the experiment. The positive muons still decay as a free particle, but the negative muons have a slightly different lifetime. Because negative muons can form mesonic atoms reactions in the nucleus of the particles in the condensed matter may occur. In this case, it would be carbon. The Bohr model of the atom is given by (2) below.

$$r = \frac{4\pi\varepsilon n^2 h^2}{mZe} \quad (3)$$

In this model, r is the radius of the orbit, m is the mass of the electron, Z is the atomic number, and everything else are known constants. The radius of an orbiting negative muon would only differ due to its larger mass compared to an electron. This causes the radius of the orbit of the negative muon to be significantly smaller. The probability of the muon causing a nuclear reaction, which is related to the probability of that muon being inside of the volume of the atom, increases as Z also increases. Because the atoms involved in the scintillator are carbon, it is expected to find a lifetime for the muon to be somewhere between the value of a positive muon and the value when a negative muon interacts with carbon, which would be smaller than the positive muon.

Apparatus

As cosmic-ray muons travel down toward the Earth's surface, in this experiment, a plastic scintillator is used to detect the decaying of the muons. The schematic for the experimental set up can be seen in the Appendix as **Figure 1**. As the muons slow down upon reaching the scintillator, a flash of light is produced inside. If and when the muons come to a stop inside the scintillator, a second flash of light is produced when they decay. These two events producing two light pulses are separated by a certain time interval. The anode pulses of the photomultiplier are separated by the same time interval due to the transit time for all pulses being equal. The amplitudes of these pulses vary depending on a number of reasons. The energy that is lost in the scintillator during the decay event which causes the pulse, as well as the varying efficiency of the collection of the light pulses due to the large volume of the scintillator.

During the experiment, sometimes chance coincidences may be recorded and caused by background gamma radiation being emitted from surrounding sources. The time intervals in which these coincidences happen are of a wide range, which include the range of interest of involving the pulses from the muon decay. One goal is to suppress this background gamma radiation as much as possible below a given threshold amplitude. A constant fraction discriminator module is used to do this, allowing only pulses which correspond to events in the scintillator depositing certain amounts energy.

The pulses which correspond to energy deposits above the discriminator level are brought to a time-to-amplitude converter's (TAC) start and stop inputs. The pulse generated from the TAC has an amplitude which is proportional to the time different of the start and stop signals corresponding to the events in the scintillator. A muon entering the scintillator corresponds to the starts pulse, and the decaying of that muon corresponds to the stop pulse, which stops the TAC. Any stop pulse out of range according to the TAC is discarded. When the stop signal is discarded, the TAC resets and waits for another start signal to be fed into it. A multi-channel analyzer (MCA) is used to digitize the TAC output which then is displayed as a pulse-height spectrum.

Experiment

To start off the experiment, the TAC was set to a 40 μs range and the MCA was set with 2048 channels. In order to confirm the timing of the calibration, the output signal was displayed on an oscilloscope and then hooked back up to the TAC. Using a range of about 1 to 20 μs, ten different periods were used to determine their associated channel numbers. These measurements can be seen in **Table 1** in the Appendix where the associated uncertainties were determined by the fluctuation in period as well as the width of the associated channels.

In order to minimize the effect of the background radiation, the discriminator level had to be set accordingly. By placing a ^{60}Co source on the scintillator, the background radiation was artificially increased and the discriminator level was set accordingly to 2.5 V in order to reject most of the background signal. Before fully running the experiment, both the singles and the doubles rate were measured. The singles rate is simple the total number of start pulses, whereas the doubles rate is the rate of both a start and stop pulse together, signifying a decaying particle. The singles rate was measured and found to be 31.75 ± 0.01 counts/sec. By taking a hot ^{137}Cs source, which has a lower decay energy than^{60}Co, the singles rate did not appear to change significantly, confirming that the discriminator level was set sufficiently. As well, the doubles rate was measured to be 0.13 ± 0.04 counts/sec .The accumulation of data lasted for the span of a week.

Results and Analysis

The data accumulated over the span of slightly more than a week, totalling at 612825 ± 1 s. The collected data was converted from N vs. channel number to an N vs. t set of data using the slope of the fitted line of the data shown in **Table 1** in the Appendix. The slope of this fitted line is effectively the time width of each bin. With the high number of counts acquired over the time duration, the amount of data provided an attempt for a nonlinear fit. This nonlinear fit can be made in the form presented in (2).

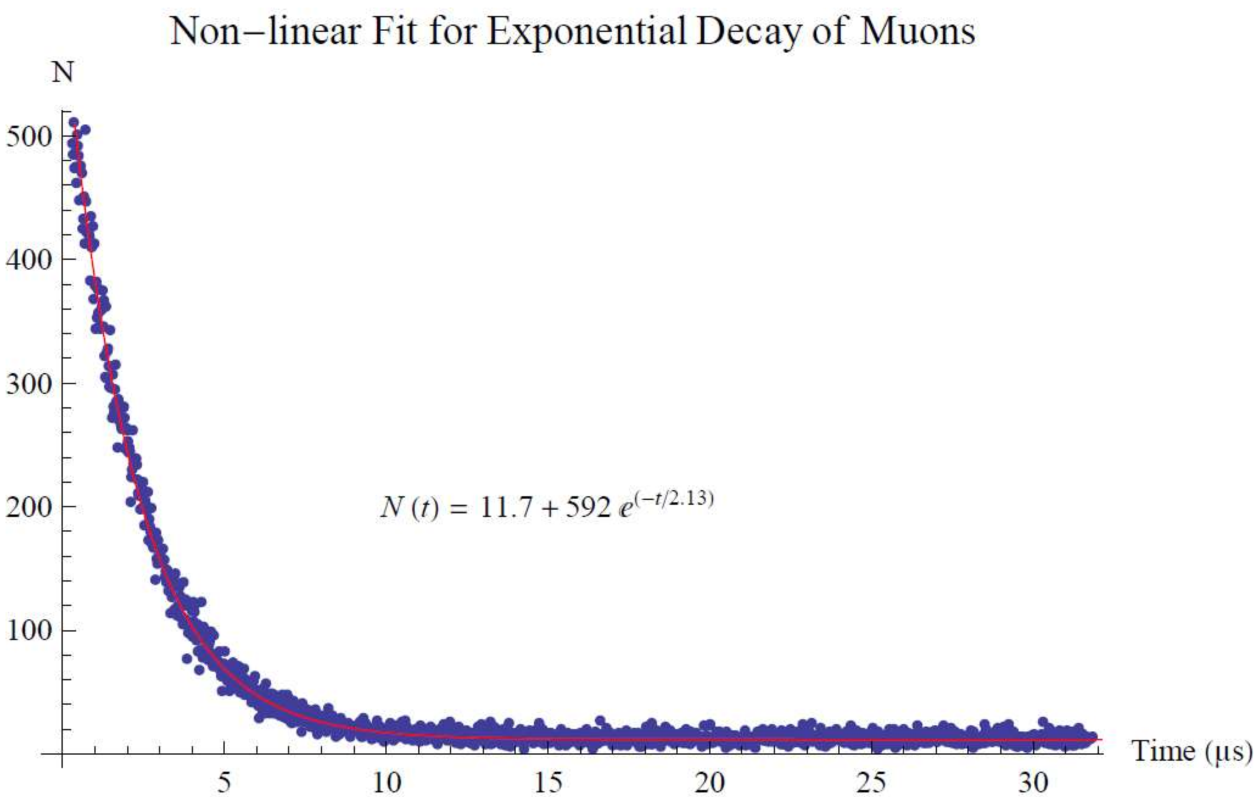
For nuclear counting experiments such as this, each measurement follows the Poisson distribution. In this experiment, with a given rate at which random pulses occur μ in a given time interval x, this distribution calculates the probability P of finding an integer number of pulses using an integer number of time. The representation of this probability is shown in (4).

$$P=\frac{\mu^x}{x!}e^{-\mu} \quad (4)$$

Because of the large number of counts acquired in each time width channel, this Poisson distribution can be approximated as a Gaussian. In order to fit the data, a fitting function F(x) (ie. Equation (2)) must be chosen in order for the χ^2 to be minimized. In (5), y_i is the measured value at time interval (channel) i with standard deviation σ_i, and $F(x_i)$ is the value of the point at i using the fitting function (2). For this muon counting experiment y_i and σ_i^2 both become n_i because the standard deviation for a measurement made by counting is $\sqrt{n_i}$.

$$\chi^2 = \sum_{i=1}^{N} \frac{\left(y_i - F\left(x_i\right)\right)^2}{\sigma_i^2} \quad (5)$$

In order to minimize the χ^2 , partial derivatives are made with respect to every parameter in (2), however, because of the nonlinearity of F(x), the fit to the data was determined numerically by implementing least squares fitting in Mathematica 8 (see Appendix). By doing so, **Plot 1** below was produced.



Plot 1: Exponential decay of positive and negative muons over the course of a week.

In order to make this fit, the first few data points which exhibited an increase with channel number were discarded as unreliable for fitting the exponential curve. Using Mathematica 8, the three parameters in (2) were found to be with associated standard deviations determined from the numerical method in the fitting process:

$$N_b = 11.7 \pm 0.1$$
$$N_0 = 592 \pm 4$$
$$\tau = 2.13 \pm 0.01\ \mu s$$

In order to determine if these parameters fit the data well, a chi-square test was performed. (exemplified in the Appendix). This test is done by calculating [1]:

$$\beta = \sum_{i=1}^{N} \frac{\left(n_i - F\left(x_i\right)\right)^2}{F\left(x_i\right)} \cong 1735.68$$

With this value, a reduced chi-square value was calculated. For this, the number of degrees of freedom is necessary. For the fit, 1486 channels were used, and 3 parameters were used for the fitting. This gave 1483 degrees of freedom. By dividing β by this value the reduced chi-square value is calculated to be 1.17.

$$Prob\left(\chi^2 \geq 1.17\right) \cong 0.0005\%$$

With a probability of having χ^2greater or equal to 1.17 being so small, there is no doubt in the fitted curve obtained through Mathematica 8.

From **Plot 1** it can be seen that the decay of the muons are superimposed onto a constant background rate. This background rate is given by the constant term, 11.7 ± 0.1 counts/channel. This background rate is due to chance coincident start and stop pulses, which would trigger a doubles count, giving rise to a false double. Using the singles rate 32.75 ± 0.01 counts/sec, the time width of the channel (2.1222 ± 0.0005) x 10^{-8} s, and the total analyzing time 612825 ± 1s, the background can be calculated using the equation given by Hall et al. [2] Doing this gives a constant background rate of 13.11 ± 0.03 counts/channel. It's clear that the fitted background rate and the calculated background rate do not agree within uncertainties. Because the fitted curve was seen to be good, the blame may lie in the measurement of the singles rate. Only one measurement was made when recording the singles rate, while a better to measure it would have been to do several trials and to take the average. By doing this, a more accurate singles rate would be recorded and the equation given by Hall et al would be applied again. Seeing how the two values are relatively close, it seems reasonable to assume the error lies in the measurement of the singles rate.

The accepted value for the lifetime of a decaying positive muon is $\tau^{+¿¿}$= 2.197120 ± 0.000077 μs. With the negative muons forming mesonic atoms with the carbon in the scintillator, their expected lifetime is shorter. The value for the lifetime of negative muons in carbon is given as $\tau_c = \tau^{-}$ = 2.045 ± 0.002 μs. [3] From **Plot 1** the fitted curve shows the observed lifetime of the muons was τ_{obs}= 2.13 ± 0.01 μs, which agrees with the expectation for this lifetime to be in between the values for $\tau^{+¿¿}$ and τ^{-}. Using these values, the muon charge ratio can be calculated. This ratio gives the number of positive muons to negative muons N^+/N^-. The muon charge ratio can be calculated using (6) below with its propagated uncertainty (7).

$$\rho = -\frac{\tau^{+¿}}{\tau^{-}} ¿ \qquad (6)$$

$$\delta\rho = \sqrt{\left(\frac{\partial \rho}{\partial \tau_{obs}} \delta \tau_{obs}\right)^2 + ¿¿} \qquad (7)$$

Using (6) and (7) a muon charge ratio was found to be 1.36 ± 0.36, which agrees with the charge ratio of 56% to 44% given by Galbiati and Beacom. [4]

Conclusions

The expected exponential distribution for muon decay was confirmed by the results of this experiment. The observed muon lifetime lied within the accepted values for the lifetime of a positive muon and value for the lifetime of a negative muon interacting with the carbon in the scintillator, 2.197120 ± 0.000077 μs and 2.045 ± 0.002 μs respectively. This observed lifetime was τ_{obs} = 2.13 ± 0.01 μs. The exponential decay for the muon was superimposed on constant background caused by accidental double measurements. The curve fitted background constant was 11.7 ± 0.1 counts/channel, where a calculated value using the singles rate was 13.11 ± 0.03 counts/channel. This discrepancy between these two values may be due to the unsatisfactory way in which the singles rate was measured. The goodness of the curved fit was reassured by calculating the reduced chi-square and computing the probability of a value greater and equal to this value of 1.17. Doing this showed the fit to be good.

Appendix

Figure 1: Schematic of experimental setup. [5]

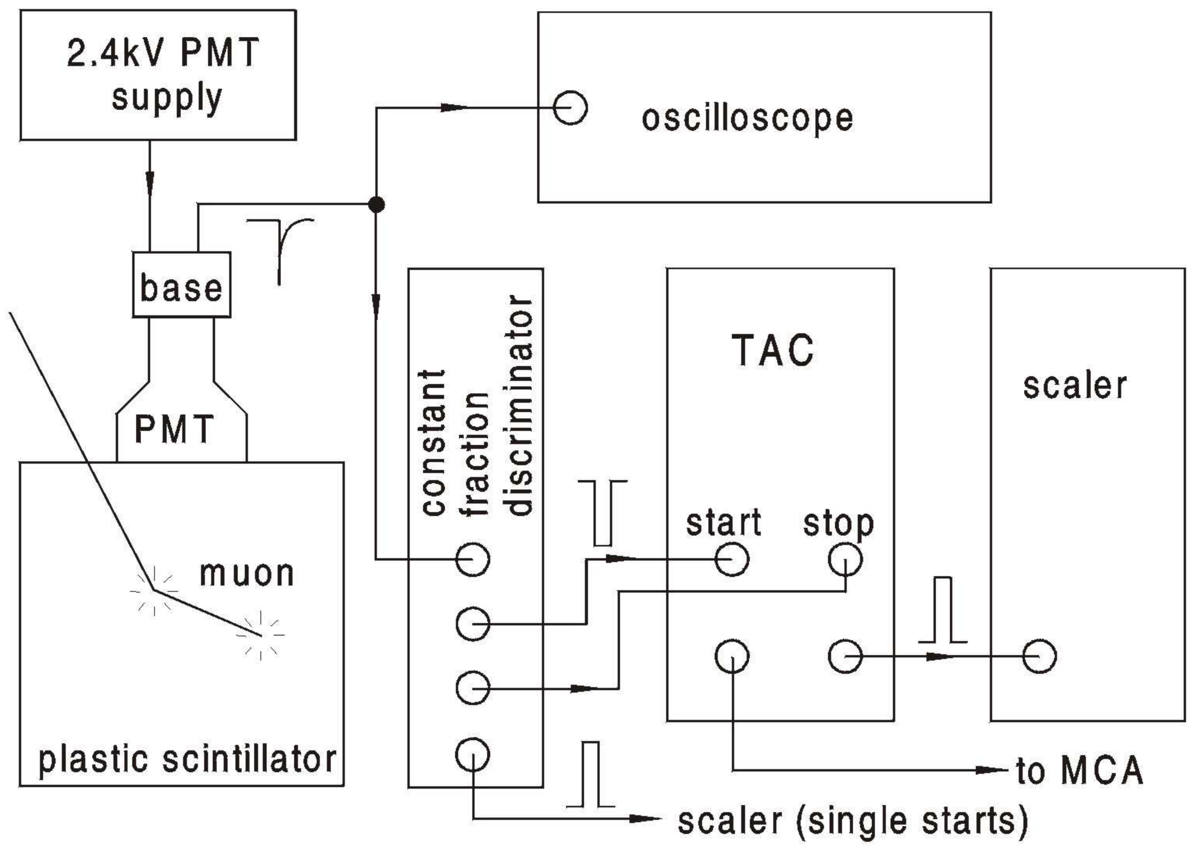


Table 1: Period with corresponding channel number.

Channel	Uncertainty	Period (μs)	Uncertainty
79	2	2.08	0.01
170	2	4.00	0.01
264	2	6.00	0.01
371	2	8.16	0.02
451	3	10.00	0.05
549	2	12.04	0.02
640	4	13.97	0.04
738	2	16.06	0.03
837	3	18.14	0.01
918	4	19.87	0.03

Fitting in Mathematica 8:

In: $Regress[ch, \{1, x\}, x]$

Out:

$\{ParameterTable \rightarrow$

	" Estimate "	" SE "	" TStat "	" PValue "
	1	0.38499364306725903	0.026306636499894558	¿

$RSquared \rightarrow 0.9999621167571056, AdjustedRSquared \rightarrow 0.9999573813517437, EstimatedVariance \rightarrow 0.001552$

Non-linear fit of exponential decay using Mathematica 8:

In: $nlm3 = NonlinearModelFit[data, a * \exp[-t/b] + c, \{a, b, c\}, t, Weights \rightarrow 1/errors^2]$

Out: FittedModel[$11.6958 + 592.388\, e^{-0.467682\, t}$]

In: $nlm3[\text{ParameterTable}]$

Out:

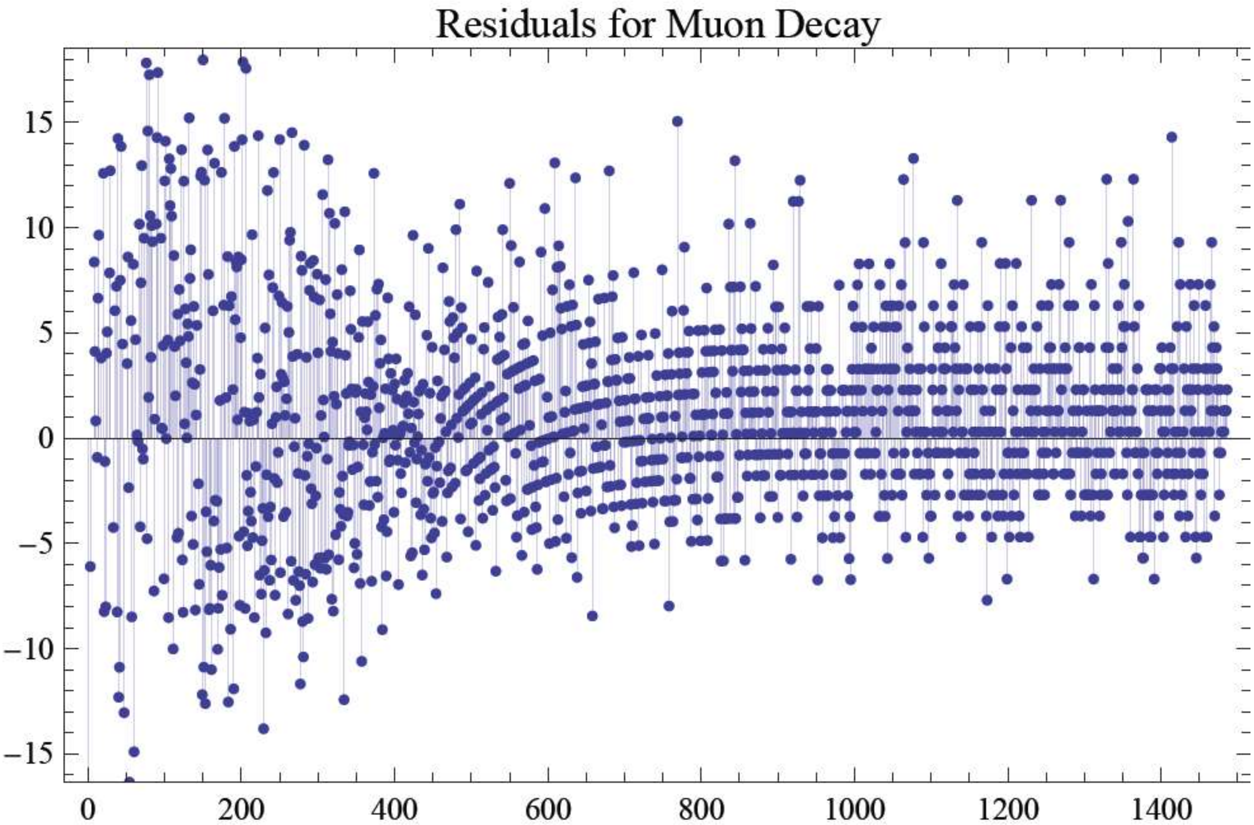
	Estimate	Standard Error	t-Statistic	P-Value
a	592.3878845889486	4.495096233897886	131.78536204001674	$3.84891546235 \times 10^{-821}$
b	2.138203143093484	0.013004566382608884	164.41941085809066	$1.87263195544 \times 10^{-954}$
c	11.695778315731761	0.11363740976815973	102.92190168354948	$1.004800908499 \times 10^{-677}$

Residuals for exponential fit for the decay of muons:

In:

$ListPlot[nlm3[\text{FitResiduals}], Frame \rightarrow True, Filling \rightarrow Axis, PlotLabel \rightarrow \text{Residuals for Muon Decay}]$

Out:



Chi-Squared test in Mathematica 8:

In: $chi = Total\left[(obs - \exp)^2 / \exp\right]$

Out: $\{1735.6809064342087\}$

In: df=1486 -3;
1-CDF[ChiSquareDistribution[df],chi]

Out: $\{0.0000051286873679057 26\}$

Bibliography

[1] Taylor, J.R., "An Introduction to Error Analysis: the Study of Uncertainties in Physical Measurements", 2nd Edition
[2]Hall et al, "A Simplified Muon Lifetime Experiment for the Instructional Laboratory", (1970)
[3] T. Suzuki, D. F. Measday and J. F. Roalsvig, "Total nuclear capture rates for negative muons" Phys. Rev. C **35**, 2212 (1987)
[4] Galbiati and Beacom, "Measuring the Cosmic Ray Muon-Induced Fast Neutron Spectrum by (n,p) Isotope Production Reactions in Underground Detectors" Physical Review C 72 (2005)
[5] Experimental schematic diagram
Queen's University PHYSICS 350 Experiment 6: Lifetime of Cosmic-Ray Muons
[6] Queens University online notes, "Physics 350: Error Analysis"