ME 594
Midterm Exam

SHOW ALL OF YOUR WORK. If you utilize MATLAB or similar software, you may attach code and/or command line output to SUPPLEMENT YOUR WRITTEN WORK. Include the following statement at the top of your assignment:

"I ATTEST THAT I HAVE NEITHER GIVEN NOR RECEIVED HELP (other than from the instructor) ON THIS ASSIGNMENT."

For this assignment, you will apply Laplace's method of initial orbit determination by performing the following steps:

- Define an object in a particular Earth orbit, in terms of the parameters defining the object's orbit (e.g. classical orbit elements or position/velocity vector) at a chosen epoch time t_0, including units for all quantities (e.g. km for lengths/positions, km/sec for velocities, degrees for angles, etc)
- Define a location (latitude/longitude) for your optical sensor
- Choose three measurement times and propagate the object's motion (assuming 2-body gravity) from t_0 to each of these times, calculating the sensor-to-object line-of-sight vector at each measurement time (NOTE: although normally the line-of-sight vector is first measured in the sensor's TH frame, you may directly calculate it in the ECI frame and leave it expressed in the ECI frame for the duration of the assignment)
- Using this information, execute Laplace's method, choosing one of the roots of the resulting 8th-order polynomial and substituting that back into the equations to solve the object's position and velocity vector at t^* (this can be any time you choose, likely t_1, t_2, or t_3)
- Compare your solution to the object's true orbit and comment on the accuracy of this method for your scenario

Your submitted work should include a careful outlining of each step above, with the quantities asked for in each step clearly marked (e.g. your object's orbit parameters, location of your sensor, chosen measurement times, etc). You may want to include several sketches if this will help explain your process.

For 10 BONUS POINTS, perturb the three line-of-sight vectors slightly (to represent measurement error) and re-calculate the orbit solution. How does this compare with the original solution? (NOTE: since each line-of-sight vector should remain a unit vector even if it is perturbed, an easy way to ensure this is to first convert the unperturbed line-of-sight vector to RA and Dec angles, perturb those values slightly, then convert back to a unit vector.)

Following is some further guidance that may be helpful:

- A straightforward way to define a "true" orbit and generate sensor LOS measurements for the object on that orbit is as follows:
    - Choose values for each of the 6 orbit elements (a simple choice would be a circular and/or equatorial orbit, etc) as well as an epoch time (can be any time you choose)
    - Convert these orbit element values to ECI position and velocity vector components ("r-bar" and "v-bar") as discussed in Sec 2.5 of the textbook

    - Using these values as initial conditions, numerically integrate the 2-body equations of motion (Slide 7 of Module 4 lecture) to each measurement time using the ode45 function in MATLAB or a similar numerical integration routine
- In calculating the ECI coordinates of your sensor location, you may assume a spherical Earth, just as we have done in HW assignments.
- You may choose any sensor location and any measurement times you wish, but the combination of sensor location and times should be such that the object is within the sensor's field of regard for all 3 times. A good choice of times to start with, in checking for field of regard compliance, might be 0.1, 0.2, and 0.3 of the orbit period (tweaking as necessary until field of regard compliance at all 3 times is achieved).
- The epoch time of your Laplace solution orbit can be any time you choose (I mentioned in lecture that a commonly chosen time is the middle measurement time), whereas the epoch time of the true orbit will be the time you chose above (prior to any of the measurement times). However, a simple way to compare your solution orbit to the true orbit is to convert the solution orbit (which is expressed as ECI position and velocity) to orbit elements as discussed in Sec 2.5 of the textbook, and compare those values to the true orbit elements you originally chose above. The true anomaly (orbit angle) of the two orbits will not agree because of their different epoch times; but if the Laplace method yields an accurate solution orbit, the other 5 orbit element values of the two orbits should agree fairly well.
- For a unit vector defined in the ECI frame as $u_x\hat{\imath} + u_y\hat{\jmath} + u_z\hat{k}$, the formulas to convert to RA and Dec (and vice versa) are given by:
    - $RA = tan^{-1}\left(\frac{u_y}{u_x}\right), Dec = sin^{-1}(u_z)$
    - $u_x = \cos(Dec)\cos(RA), u_y = \cos(Dec)\sin(RA), u_z = \sin(Dec)$
- The rubric that accompanies this assignment clearly indicates what elements you should include in your submission: a definition of your object's orbit, a choice of sensor with LOS measurements generated, all steps of the Laplace method with values inserted correctly and math calculations performed correctly, and a comparison of the solution orbit with your true (chosen) orbit. Show the roots of the 8th-order polynomial and clearly indicate the root you choose as the correct one.