Ch 11: Nonlinear equations

11.1 Local algorithms

Thm 11.1 $r: \mathbb{R}^n \to \mathbb{R}^n$ ctsly-diff'ble in convex open set D, $x, x+p \in D$. Then:

$$r(x+p) = r(x) + \int_0^1 J(x+tp)\, p\, dt \sim \circledast$$

Define linear model based on $\circledast$:

$$M_k(p) \stackrel{def}{=} r(x_k) + J(x_k)\, p.$$

For $M_k(p) = 0$, we get:

$$0 = r(x_k) + J(x_k)\, p$$

$$\Rightarrow P_k = -J(x_k)^{-1} r(x_k)$$

Algorithm 11.1   Nonlinear Newton

Choose x_0

for $k = 0, 1, 2, \cdots$

   Solve: $J(x_k) P_k = -r(x_k)$

   $x_{k+1} \leftarrow x_k + P_k$

end

Problems:

* If $J(x_k)$ is singular, Newton step may not be defined.

* J maybe hard to get.

* $J(x^*)$ may be singular.

Concluding remarks:

* Algorithm 11.5 that uses the trust-region method with the dogleg of procedure 11-6 is what we want to consider first.