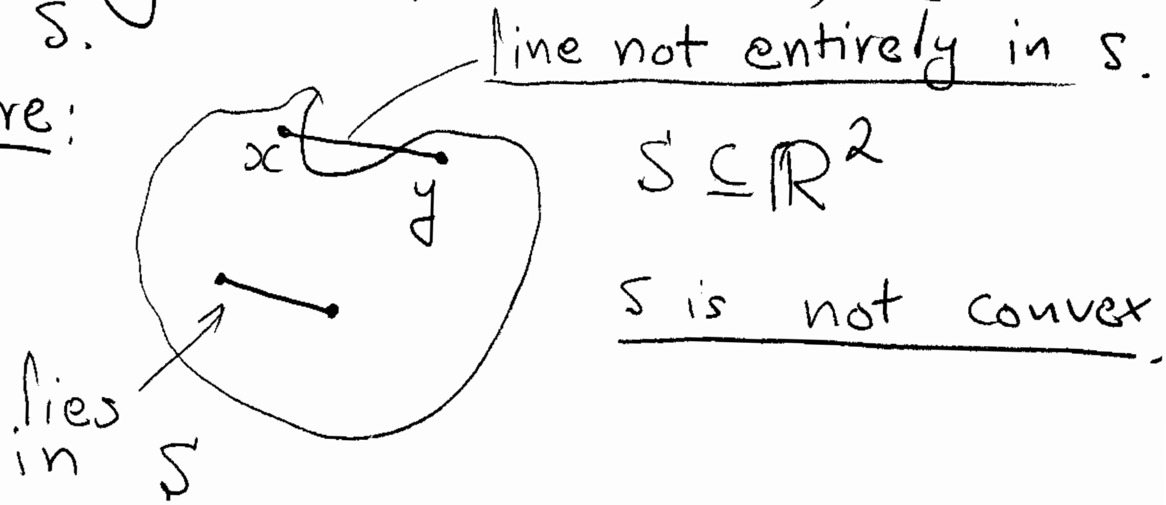
Convexity of Sets and functions

Sets: $S \subseteq \mathbb{R}^n$ is a convex set if a straight line connecting two points in S, lies entirely in S.

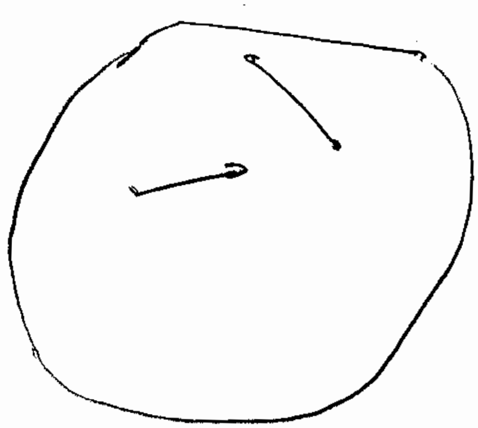
Picture:



$S \subseteq \mathbb{R}^2$

S is not convex.

You can always find a convex extension to S (keep all lines in the new set)



This is convex.

We require: $x, y \in S' \Longrightarrow \alpha x + (1-\alpha) y \in S'$, $\alpha \in [0,1]$.

Convex functions

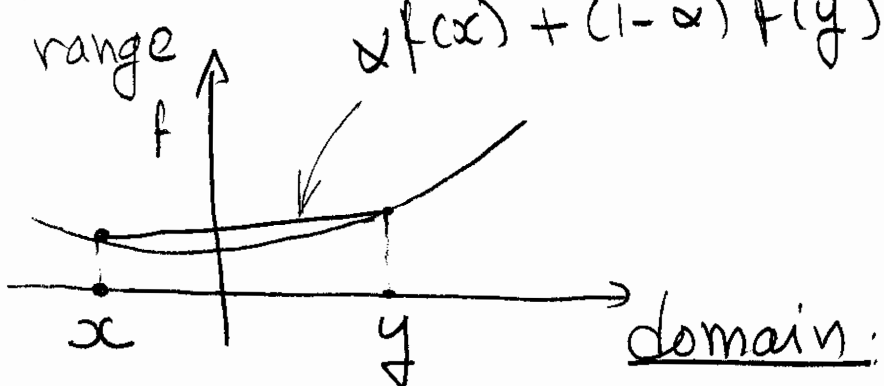
Suppose that the domain of f is convex: $f: D \to \mathbb{R}$, and D is convex. Let x, y be any two points in the domain. If f also satisfies:

$$f(\alpha x + (1-\alpha) y) \leq \alpha f(x) + (1-\alpha) f(y)$$

for all $\alpha \in [0,1]$,

then f is convex.

A picture:



"function lies below the lines"

Convex-programming

Assume that:

- objective function is **convex**,
- equality constraints $c_i(\cdot)$, $i \in \mathcal{E}$ are **linear**,
- inequality constraints $c_i(\cdot)$, $i \in \mathcal{I}$ are **concave**

Then this special case is covered by convex programming.

$\Rightarrow$ Allows strong claims regarding convergence.