$$\mu_{sun} = 132{,}712{,}440{,}018$$
$$\mu_{Merc} = 22{,}032$$
$$\mu_{Ven} = 324{,}859$$
$$\mu_{Ear} = 398{,}600$$
$$\mu_{Moon} = 4{,}902.8$$
$$\mu_{Mars} = 42{,}828$$
$$\mu_{Jup} = 126{,}686{,}534$$
$$\mu_{Sat} = 37{,}931{,}187$$
$$\mu_{Ura} = 5{,}793{,}939$$
$$\mu_{Nep} = 6{,}836{,}529$$
$$\mu_{Plu} = 871$$
$$\ddot{\vec{r}} = -\frac{\mu}{r^3}\vec{r} + \frac{T}{m}\frac{\vec{v}}{v}$$

$$\vec{h} = \vec{r} \times \vec{v}$$
$$h = r v_{\perp}$$
$$r + re\cos\theta = \frac{h^2}{\mu}$$
$$r = \frac{h^2}{\mu}\frac{1}{1 + e\cos\theta}$$
$$v_{\perp} = \frac{\mu}{h}(1 + e\cos\theta)$$
$$v_r = \frac{\mu}{h} e \sin\theta$$
$$r_p = \frac{h^2}{\mu}\frac{1}{1+e} = a(1-e)$$

$$r_a = \frac{h^2}{\mu}\frac{1}{1-e} = a(1+e)$$
$$\tan\gamma = \frac{v_r}{v_{\perp}} = \frac{e\sin\theta}{1 + e\cos\theta}$$
$$P = \frac{h^2}{\mu}\ (at\ \theta = 90^{\circ})$$
$$\varepsilon = \frac{v^2}{2} - \frac{\mu}{r} = -\frac{1}{2}\frac{\mu^2}{h^2}(1 - e^2)$$
$$v_{esc} = \sqrt{\frac{2\mu}{r}}$$

Circular: $v = \sqrt{\frac{\mu}{r}}$

$$\varepsilon = -\frac{\mu}{2r}$$

Elliptical: $\frac{h^2}{\mu} = a(1-e^2)$

$e = -\cos\beta$ where $\beta = \theta_{sem-min\,ax}$

$$\varepsilon = -\frac{\mu}{2a}$$
$$e = \frac{r_a - r_p}{r_a + r_p}$$
$$b = a\sqrt{1-e^2}$$

$$T = \frac{2\pi}{n}, n = \sqrt{\frac{\mu}{a^3}}$$
$$\bar{r_\theta} = \sqrt{r_a r_p}$$

Parabolic: $v = \sqrt{\frac{2\mu}{r}}$

$\varepsilon = 0 \quad \gamma = \frac{\theta}{2}$

Hyperbolic:

$$\theta_{inf} = \cos^{-1}\left(\frac{-1}{e}\right) for\ \frac{\pi}{2} < \theta < \pi$$
$$\sin\theta_{inf} = \frac{\sqrt{e^2-1}}{e}$$

$$a = \frac{h^2}{\mu}\frac{1}{e^2-1}$$
$$r = \frac{a(e^2-1)}{1 + e\cos\theta}$$
$$\varepsilon = \frac{\mu}{2a}$$
$$v_{inf} = \sqrt{\frac{\mu}{a}}$$
$$C3 = {v_{inf}}^2$$
$$h = \frac{\mu}{v_{inf}}\sqrt{e^2-1}$$
$$b = \Delta = a\sqrt{e^2-1}$$

Canonical Units:

$1DU = R_E = 6378.14\ km$

$1\ TU = 806.8\ s$

$\mu = 1$

Perifocal Frame:

$$\vec{r} = r\cos\theta\,\hat{p} + r\sin\theta\,\hat{q}$$
$$\vec{v} = \frac{\mu}{h}[-\sin\theta\,\hat{p} + (e + \cos\theta)\hat{q}]$$

Orbit Position as f(t):

Circular: $\theta = \frac{\mu^2}{h^3}t = \frac{2\pi}{T}t$

Elliptical: $M_e = \frac{2\pi}{T}t = \frac{\mu^2}{h^3}(1-e^2)^{3/2}t$

$$\tan(E/2) = \sqrt{\frac{1-e}{1+e}}\tan(\theta/2)$$
$$M_e = E - e\sin E$$
$$r = a(1 - e\cos E)$$

Parabolic: $M_p = \frac{1}{2}\tan\frac{\theta}{2} + \frac{1}{6}\tan^3\frac{\theta}{2}$

$$\tan\frac{\theta}{2} = \left[3M_p + \sqrt{\left(3M_p\right)^2 + 1}\right]^{1/3} - \left[3M_p + \sqrt{\left(3M_p\right)^2 + 1}\right]^{-1/3}$$

Hyperbolic: $M_h = e\sinh F - F = \frac{\mu^2}{h^3}(e^2-1)^{3/2}t$

$$\tan\frac{\theta}{2} = \sqrt{\frac{e+1}{e-1}}\tanh\frac{F}{2}$$
$$r = a(e\cosh F - 1)$$

Orbital Elements:

$$i = \cos^{-1}\frac{h_z}{h} \qquad 0 \le i \le 180deg$$
$$\vec{N} = \hat{k} \times \vec{h}$$
$$N_y \ge 0: \Omega = \cos^{-1}\frac{N_x}{N}$$
$$N_y < 0: \Omega = 360 - \cos^{-1}\frac{N_x}{N}$$

$$\vec{e} = \frac{1}{\mu}\left[\vec{v} \times \vec{h} - \mu\frac{\vec{r}}{r}\right]$$
$$e_z \ge 0: \omega = \cos^{-1}\left(\frac{\vec{N}\cdot\vec{e}}{Ne}\right)$$
$$e_z < 0: \omega = 360 - \cos^{-1}\left(\frac{\vec{N}\cdot\vec{e}}{Ne}\right)$$

$v_r \geq 0: \theta = \cos^{-1}\left(\frac{\vec{e}\cdot\vec{r}}{er}\right)$

$v_r < 0: \theta = 360 - \cos^{-1}\left(\frac{\vec{e}\cdot\vec{r}}{er}\right)$

$R_1(\emptyset) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\emptyset & \sin\emptyset \\ 0 & -\sin\emptyset & \cos\emptyset \end{bmatrix}$

$R_2(\emptyset) = \begin{bmatrix} \cos\emptyset & 0 & -\sin\emptyset \\ 0 & 1 & 0 \\ \sin\emptyset & 0 & \cos\emptyset \end{bmatrix}$

$R_3(\emptyset) = \begin{bmatrix} \cos\emptyset & \sin\emptyset & 0 \\ -\sin\emptyset & \cos\emptyset & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\vec{r} = r\left[\cos\delta\cos\alpha\,\hat{I} + \cos\delta\sin\alpha\,\hat{J} + \sin\delta\,\hat{K}\right]$

$I_{sp} = \frac{T}{\dot{m}_e g_0}$

$\Delta v = I_{sp} g_0 \ln\frac{m_0}{m_f}$

Plane Change:

Circular: $\Delta v = 2v_i \sin\frac{\alpha}{2}$

Ellip: $(\Delta v)^2 = (v_{r2} - v_{r1})^2 + {v_{\perp 2}}^2 + {v_{\perp 1}}^2 - 2v_{\perp 1}v_{\perp 2}\cos\delta$, $\cos\delta = \hat{u}_{\perp 1}\cdot\hat{u}_{\perp 2}$

$(\Delta v)^2 = {v_1}^2 + {v_2}^2 - 2v_1 v_2[\cos\Delta\gamma - \cos\gamma_2\cos\gamma_1\,(1-\cos\delta)]$

Changing $\boldsymbol{\Omega}$: $\cos\alpha = \cos i_i \cos i_f + \sin i_i \sin i_f \cos\Delta\Omega$

$\sin A_{LA} = \sin(\omega + \theta) = \frac{\sin i_f \sin\Delta\Omega}{\sin\alpha}$

Inclination change Δv split:

$\Delta v = \Delta v_a + \Delta v_b = \sqrt{{v_{tr,a}}^2 + {v_i}^2 - 2v_i v_{tr,a}\cos(S\Delta i)} + \sqrt{{v_{tr,b}}^2 + {v_f}^2 - 2v_f v_{tr,b}\cos\big((1-S)\Delta i\big)}$

$S = \frac{1}{\Delta i}\tan^{-1}\left[\frac{\sin\Delta i}{\frac{v_i v_{tr,a}}{v_f v_{tr,b}} + \cos\Delta i}\right]$, circular: $\frac{v_i v_{tr,a}}{v_f v_{tr,b}} = \sqrt{R^3}, R = \frac{r_f}{r_i}$

Relative Motion:

$\left(\ddot{\vec{r}}_{rel}\right)_R = \ddot{\vec{r}}_{rel} - \dot{\vec{\omega}}_R \times (\vec{r}_{rel})_R - 2\vec{\omega}_R \times \left(\dot{\vec{r}}_{rel}\right)_R - \vec{\omega}_R \times (\vec{\omega}_R \times (\vec{r}_{rel})_R)$

Circ: $\vec{\omega}_R = \omega\widehat{W} = \sqrt{\frac{\mu}{{r_{tgt}}^3}}\widehat{W}$

$$\left(\ddot{\vec{r}}_{rel}\right)_R = -\omega^2\left\{x\hat{R} + y\hat{S} + z\widehat{W} - 3x\hat{R}\right\} + \vec{F} + 2\omega\dot{y}\hat{R} - 2\omega\dot{x}\hat{S} + \omega^2 x\hat{R} + \omega^2 y\hat{S}$$

Hill's/Cloheny-Whiltshire Equations:

$\ddot{x} - 2\omega\dot{y} - 3\omega^2 x = f_x$

$\ddot{y} + 2\omega x = f_y$

$\ddot{z} + \omega^2 z = f_z$

Soln:

$z(t) = z_0\cos\omega t + \frac{\dot{z}_0}{\omega}\sin\omega t$

$x(t) = 4x_0 + 2\frac{\dot{y}_0}{\omega} + \frac{\dot{x}_0}{\omega}\sin\omega t - \left[3x_0 + 2\frac{\dot{y}_0}{\omega}\right]\cos\omega t$

$y(t) = 2\frac{\dot{x}_0}{\omega}\cos\omega t + \left[6x_0 + 4\frac{\dot{y}_0}{\omega}\right]\sin\omega t - (6\omega x_0 + 3\dot{y}_0)t - 2\frac{\dot{x}_0}{\omega} + y_0$

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$$[\Phi_{rr}(t)] = \left[\begin{array}{cc|c} 4-3\cos nt & 0 & 0 \\ 6(\sin nt - nt) & 1 & 0 \\ \hline 0 & 0 & \cos nt \end{array}\right] \qquad [\Phi_{rv}(t)] = \left[\begin{array}{cc|c} \frac{1}{n}\sin nt & \frac{2}{n}(1-\cos nt) & 0 \\ \frac{2}{n}(\cos nt - 1) & \frac{1}{n}(4\sin nt - 3nt) & 0 \\ \hline 0 & 0 & \frac{1}{n}\sin nt \end{array}\right]$$

$$[\Phi_{vr}(t)] = \left[\begin{array}{cc|c} 3n\sin nt & 0 & 0 \\ 6n(\cos nt - 1) & 0 & 0 \\ \hline 0 & 0 & -n\sin nt \end{array}\right] \qquad [\Phi_{vv}(t)] = \left[\begin{array}{cc|c} \cos nt & 2\sin nt & 0 \\ -2\sin nt & 4\cos nt - 3 & 0 \\ \hline 0 & 0 & \cos nt \end{array}\right]$$

$$\delta\vec{r}(t) = \Phi_{rr}(t)\,\delta\vec{r}(t_0) + \Phi_{rv}(t)\,\delta\vec{v}(t_0)$$
$$\delta\vec{v}(t) = \Phi_{vr}(t)\,\delta\vec{r}(t_0) + \Phi_{vv}(t)\,\delta\vec{v}(t_0)$$

Interpl. Traj.:

$$\Delta v_{dep} = \sqrt{\frac{\mu_s}{R_1}}\left(\sqrt{\frac{2R_2}{R_1+R_2}} - 1\right)$$

$$\Delta v_{arr} = \sqrt{\frac{\mu_s}{R_2}}\left(1 - \sqrt{\frac{2R_1}{R_1+R_2}}\right)$$

$$T_{syn} = \frac{2\pi}{|n_1 - n_2|}$$

$$\emptyset = \theta_2 - \theta_1 = (\theta_{20} - \theta_{10}) + (n_2 - n_1)t$$

$$t_{trans} = \frac{\pi}{\sqrt{\mu_s}}\left(\frac{R_1+R_2}{2}\right)^{3/2}$$

$$\emptyset_0 = \pi - n_2 t_{trans}$$

$$\emptyset_f = \emptyset_0 + (n_2 - n_1)t_{trans}$$

$$t_{wait} = \frac{-2\emptyset_f}{n_2 - n_1} + NT_{syn}$$

$$\frac{r_{SOI}}{R} = \left(\frac{m_p}{m_s}\right)^{2/5}$$

$$e_{traj} = 1 + \frac{r_p {v_{inf}}^2}{\mu}$$

$$h = r_p\sqrt{{v_{inf}}^2 + 2\frac{\mu}{r_p}}$$

Rendezvous:

$$\delta = 2\sin^{-1}(1/e)$$

$$\Delta = r_p\sqrt{1 + \frac{2\mu}{r_p {v_{inf}}^2}}$$

$$v_{p-hyp} = \sqrt{{v_{inf}}^2 + 2\frac{\mu}{r_p}}$$

$$v_{p-capt} = \sqrt{\frac{\mu(1 + e_{capt})}{r_p}}$$

$$r_{p-opt} = \frac{2\mu}{{v_{inf}}^2}\frac{1-e}{1+e}$$

$$r_{a-opt} = \frac{2\mu}{{v_{inf}}^2}$$

$$\Delta v_{opt} = v_{inf}\sqrt{\frac{1 - e_{capt}}{2}}$$

$$\Delta_{opt} = r_p\sqrt{\frac{2}{1 - e_{capt}}}$$

$$\beta = \cos^{-1}(1/e)$$

Flyby:

$$\delta = 2\sin^{-1}(1/e)$$

$$\varphi_2 = \varphi_1 + \delta$$