Satellite Attitude Control

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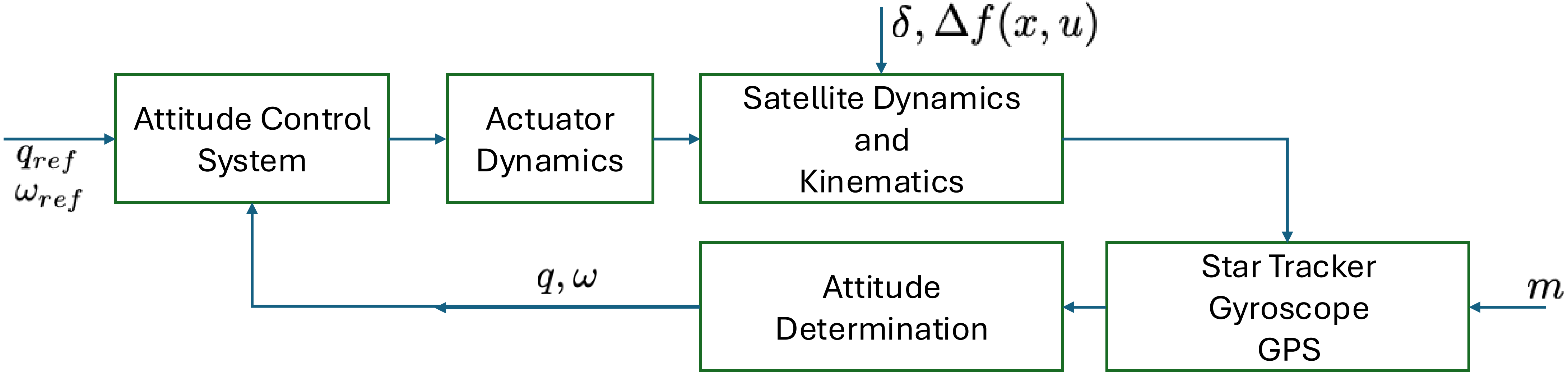
Why Control?

- To date – we've studied
  - Kinematics, dynamics, passive stabilization and attitude determination

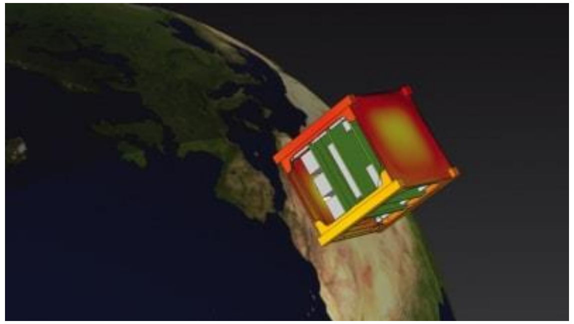


In real systems, there are disturbances, measurement/sensor noise system unknowns

Without feedback control, distribuances eventually will destabilize any satellite

Reasons to Change Attitude

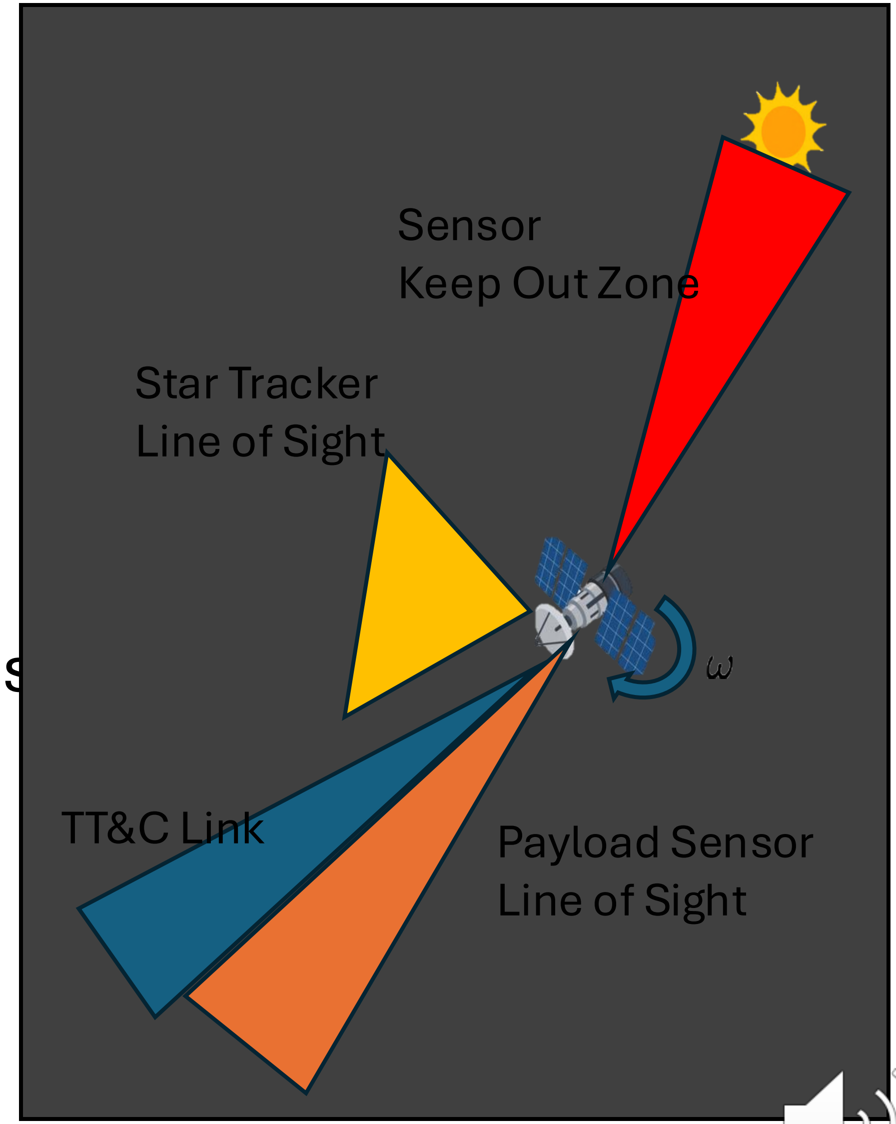
- Mission/Science objectives require it
- Maintain satellites orientation
- Regulate power charging
- Manage communication with ground stations
- Cross-link communications
- Thermal Management



Pointing Requirements

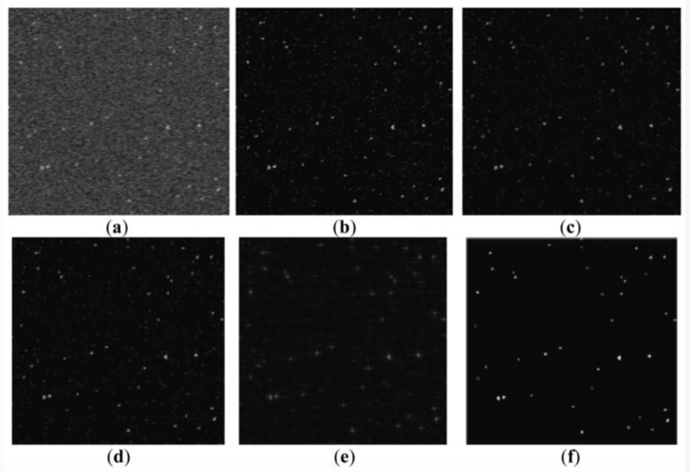
Requirements – goals for system design

- Line of sight requirements
  - For payloads, pointing requirements to gather information
  - For Star Trackers, angular diversity is needed for accuracy
  - Communication systems
- Spacecraft attitude exclusion zone geometries
  - Some optical sensors cannot point at sun
  - Star Trackers cannot point at sun
- Angular velocity constraints
  - Avoiding damage of solar arrays there is usually a limit on angular velocity, or change in vel



Pointing Requirements (cont.)

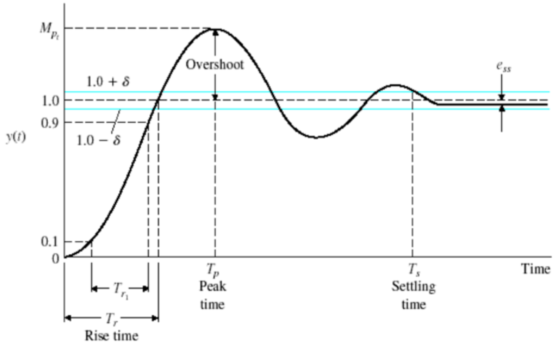
- Jitter – specified bound on high frequency angular motion
  - Used to prevent blurring of sensor data
- Drift – limit on slow, low frequency angular motion
  - May move off target with infrequent command inputs
- Range – area of angular motion over which attitude determination and control performance must be met
  - Example: Attitude within 30^{o} of nadir, when rotational rates are less than 2 ^{o}/sec



Zhang W, Quan W, Guo L. Blurred Star Image Processing for Star Sensors under Dynamic Conditions. *Sensors*. 2012; 12(5):6712-6726. https://doi.org/10.3390/s120506712

Goal of Attitude Control

- Stability
  - Predictable behavior – start close stay close
  - Convergence - tracker reference points
- Time-based Metrics
  - Settling-time – within 10% of reference
  - Rise-time – transitory response
  - Peak-time - maximum overshoot time
- Robustness
  - the ability of a system to maintain its performance under uncertainties, disturbances, and variations in the environment or system parameters



Will discuss this more later

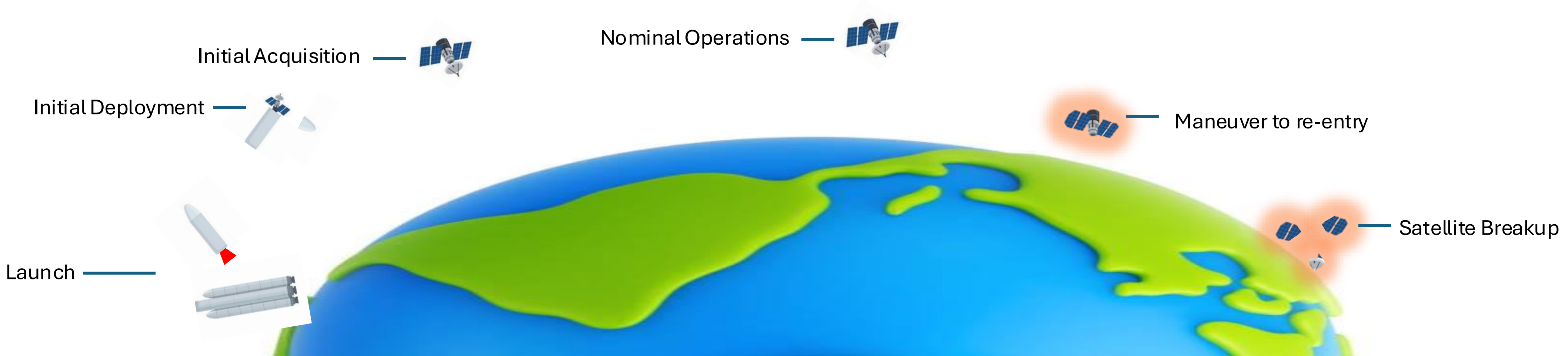
Types of Attitude Control Systems

Active Control Systems

Passive control – as previously discussed in the course allows for some stability and disturbance rejection

However, depending on the mission, active control may need to be deployed

- Launch Phase – Initial deployment and attitude control to align with launch trajectory
- Early Orbit Phase – Detumble (if necessary) initial acquisition/communication, actuator and sensor check out, solar panel orientation
- Operational Phase – Focus on mission centric attitude maneuvering
- End of Life Phase – Disposal/re-entry maneuvers



Mechanisms for Attitude Control

The different types of actuators are

- Thrusters – based on expelling mass, typically impulsive
- Momentum Wheels (Reaction Wheels) – Exchanging angular momentum
- Control Moment Gyros – inertia wheels that actuate their axis of rotation through motorized gimbals to apply change in momentum
- Magnetic Torquers – uses magnetic fields to produce torque

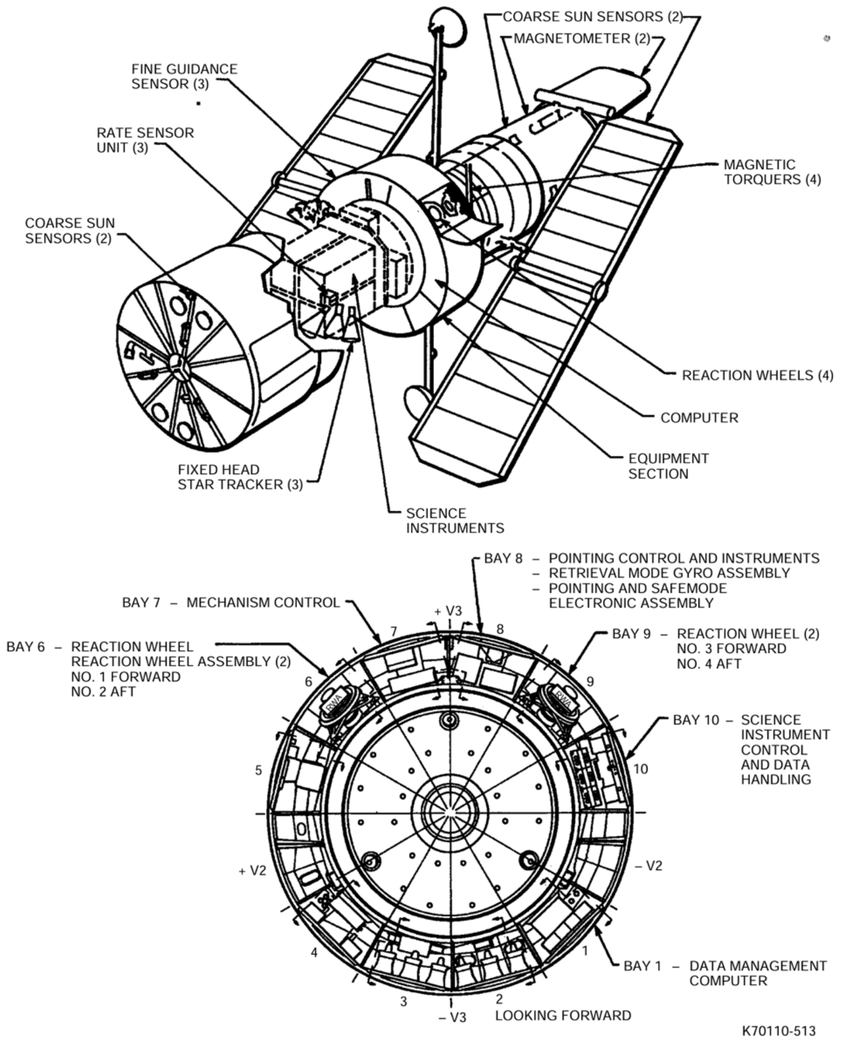
Not unusual to have several kinds of actuators

- Increase in complexity
- Increase in resiliency through redundancy

Redundancy Example

Hubble Space Telescope

- On launch
  - 4 Reaction Wheels
  - 4 Magnetorquers
- Malfunctions - only two operational reaction wheels
  - 1 axis is not controllable
  - Still possible for full attitude control, which will be discussed later



NASA, HUBBLE SPACE TELESCOPE SYSTEMS

Mechanisms for Attitude Control

	Typical Accuracy
• Spin Stabilization	(depends)
• Gravity Gradient Stabilization	(~5°)
• Magnetic Torquers	(~5°)
• 3-axis stabilization	
• Trusters	(0.1° – 0.5°)
• Momentum Wheels or Reaction Wheels	(0.001° - 1°)
• Control Moment Gyros	(0.001° - 1°)

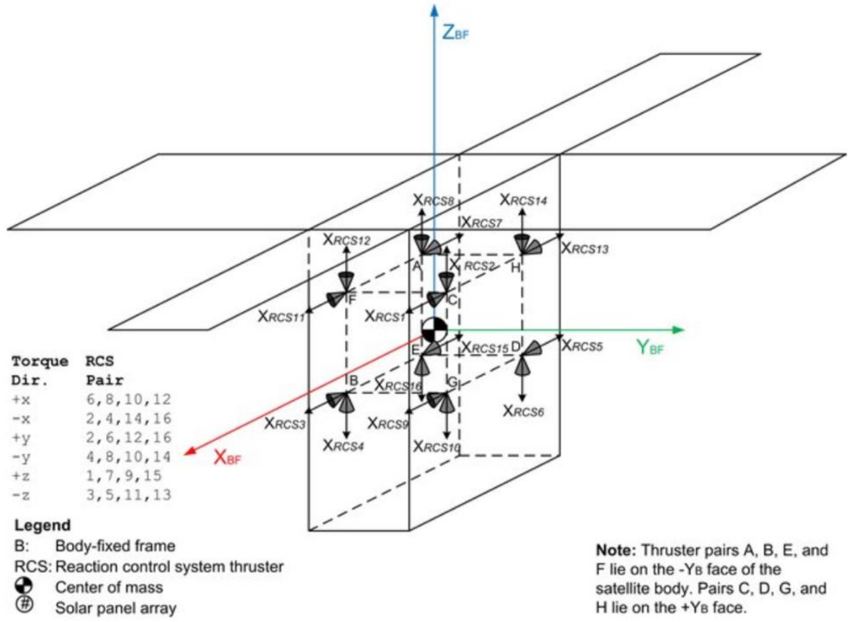
Fine attitude is also specified using arcseconds, arcminute notation where

Arcminute is $\frac{1}{60}$ of a degree   Arcsecond is $\frac{1}{60}$ of an arcminute

Example: 1 degree, is 60 arcmin (60’), 3600 arcsec (3600’)

Thrusters

- Typically, thrusters are grouped in pairs
  - Induces pure moment (ideal)
  - No orbit change
- Thrusters are typically impulsive
  - All on or all off
- Thrusters can easily overheat
- Provides discrete units of angular momentum (spin up)



$$\Delta h = F \Delta x \Delta t$$

Thrusters (cont)

- Thrusters may alter orientation and angular velocity

Only two sets of thrusters are needed to achieve any orientation

For example (Euler angles):

- Rotate about $\hat{b}_3$ until $\hat{b}_1$ lays in $\hat{a}_2 - \hat{a}_1$ plane
- Rotate about $\hat{b}_1$ until $\hat{b}_2$ lays in $\hat{a}_2 - \hat{a}_1$ plane
- Rotate about $\hat{b}_3$ until $\hat{b}_1 = \hat{a}_1$ and $\hat{b}_2 = \hat{a}_2$

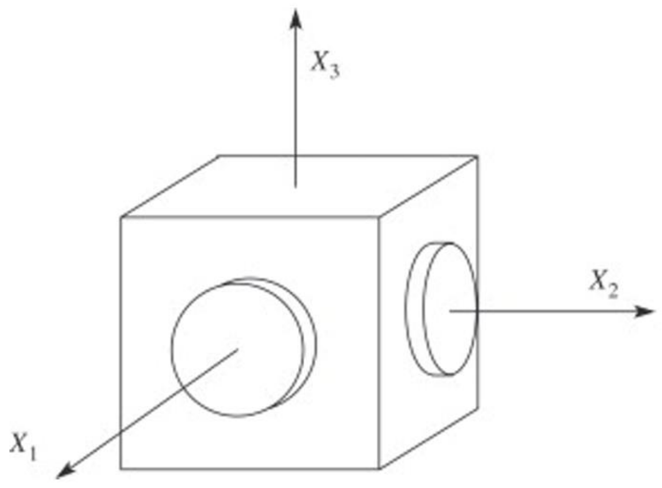
However, complex – better to have 3 sets of thrusters to minimize fuel

Momentum Wheels

As seen in the previous module, a momentum wheel leverages the **conservation of momentum**

$$I_x(\omega_f + \omega_s) + J_x \omega_s = 0$$

where I_x is the moment of inertia of the flywheel and J_x moment of inertia of the spacecraft about the x-axis

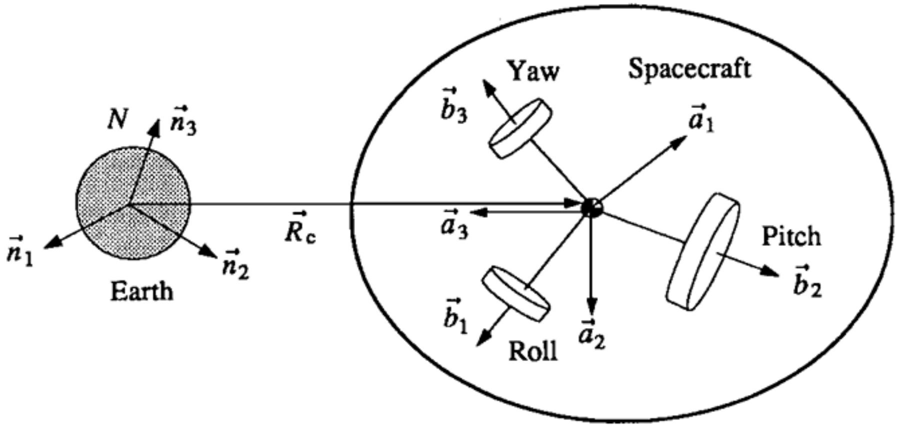


The conservation of momentum assumes what?

Example

Assume the example in the figure, with reaction wheels about each principal axis

$$
\begin{aligned}
I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 + \dot{h}_1 + \omega_2 h_3 - \omega_3 h_2 &= M_1 \\
I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 + \dot{h}_2 + \omega_3 h_1 - \omega_1 h_3 &= M_2 \\
I_3\dot{\omega}_3 + (I_2 - I_1)\omega_2\omega_1 + \dot{h}_3 - \omega_2 h_1 + \omega_1 h_2 &= M_3
\end{aligned}
$$



The angular momentum of the wheels is denoted as $h_i = \omega_{R_i} J_{R_i}$

Changing ω_{R_i} induces a moment on the spacecraft, implying that these are the **control inputs**

Example (cont.)

Typically, in control system design we use u to designate the control input in the dynamics, following this, we have the following dynamics

$$
\begin{aligned}
I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 &= u_1 + M_1 \\
I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 &= u_2 + M_2 \\
I_3\dot{\omega}_3 + (I_2 - I_1)\omega_2\omega_1 &= u_3 + M_3
\end{aligned}
$$

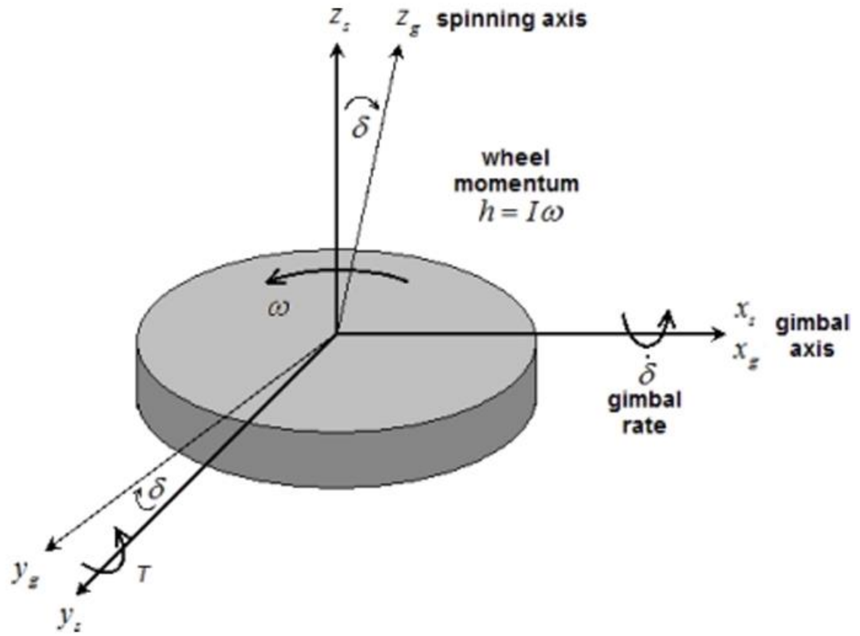
where

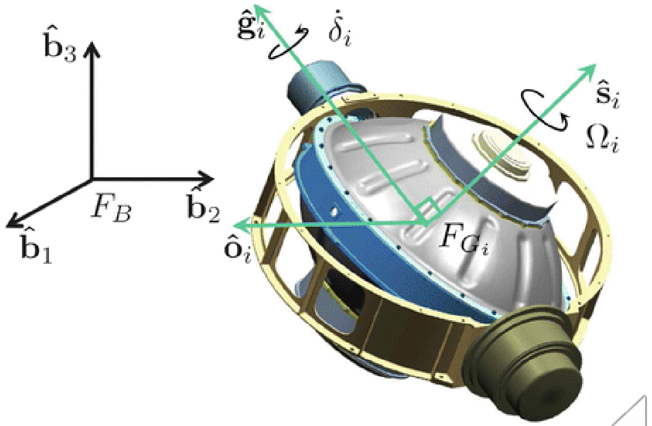
$$
\begin{aligned}
u_1 &= -\dot{h}_1 - \omega_2 h_3 + \omega_3 h_2 \\
u_2 &= -\dot{h}_2 - \omega_3 h_1 + \omega_1 h_3 \\
u_3 &= -\dot{h}_3 - \omega_1 h_2 + \omega_2 h_1
\end{aligned}
$$

Goal is to design a control law for h_i .

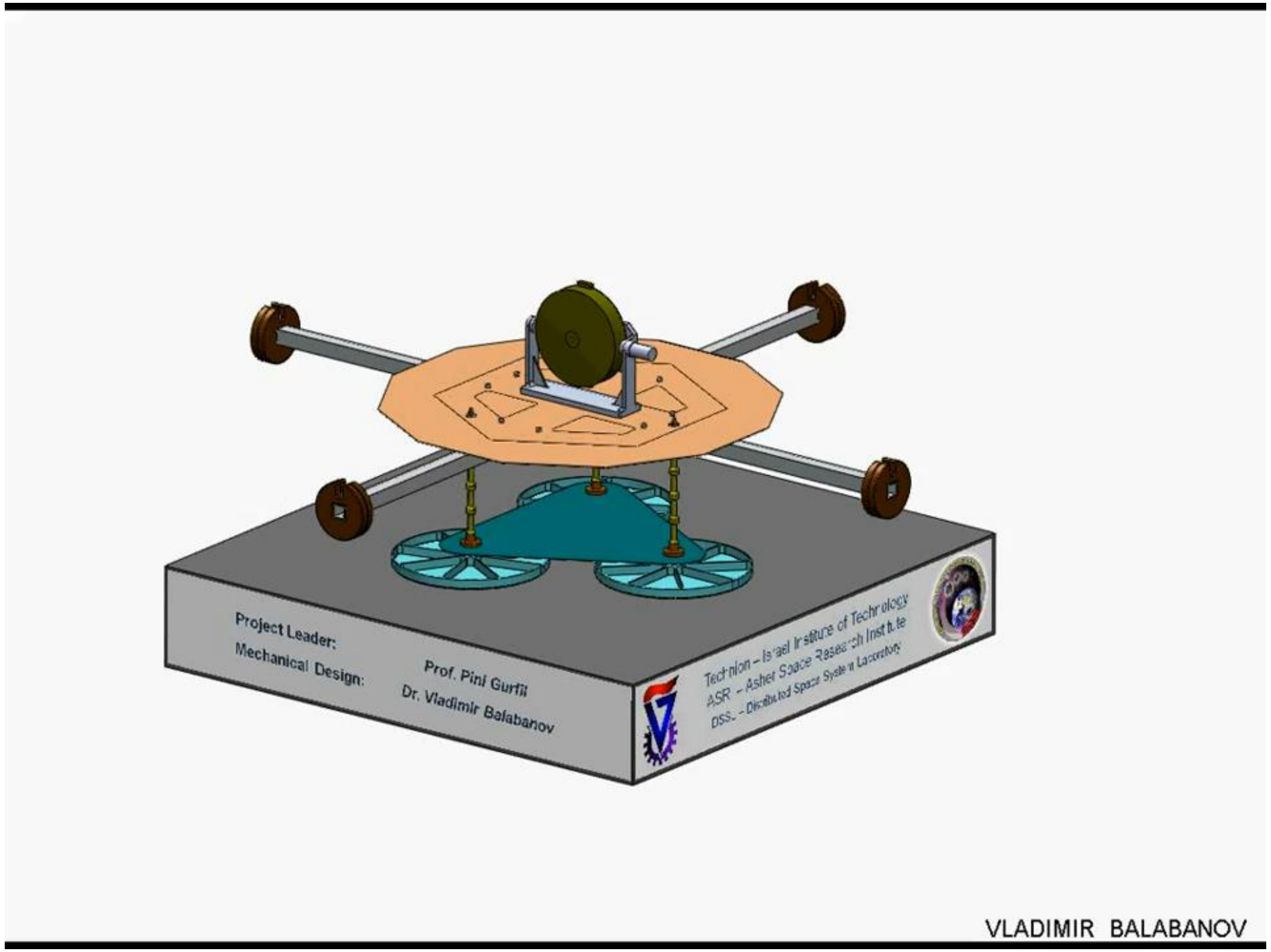
Control Moment Gyros (CMG)

- CMGs are different than reaction wheels
  - Fixed rate of rotation (ω_{CMG}) – takes some time to spin up
  - Direction of the angular momentum vector will vary
- Control is achieved by rotation of the gyroscope through the angle δ
- Single Gyroscope
  - can be used for two axis stabilization





Control Moment Gyro Simulation



Control Moment Gyro Saturation

- Saturation occurs when the torque output of a SMG reaches its maximum limit
- Causes:
  - High Attitude Rates – Rapid changes in the spacecraft's orientation
  - Control Algorithms – Control strategies may require more torque than available
- Effects
  - Loss of Control – Spacecraft may be unable to achieve the desired attitude or may respond unpredictably
- Mitigation Strategies
  - Saturation Management – Developing algorithms that predict and prevent saturation
  - Redundant/heterogeneous systems – Thrusters and Magnetorquers can be used to dampen spacecraft and desaturate CMGs

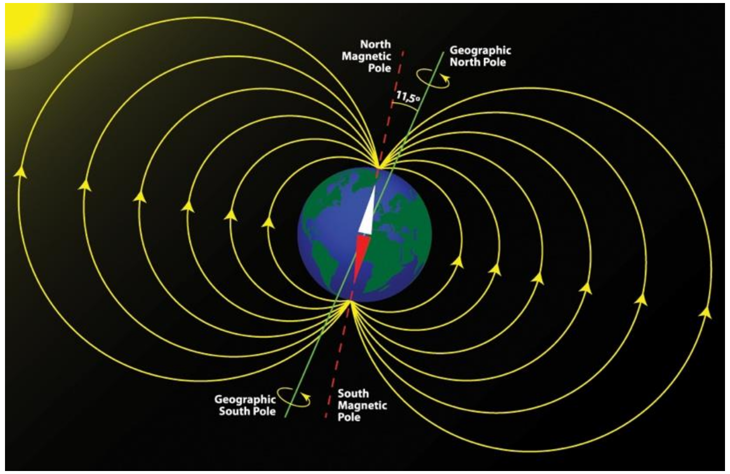
Issue is not unique to CMGs, this also occurs with reaction wheels.

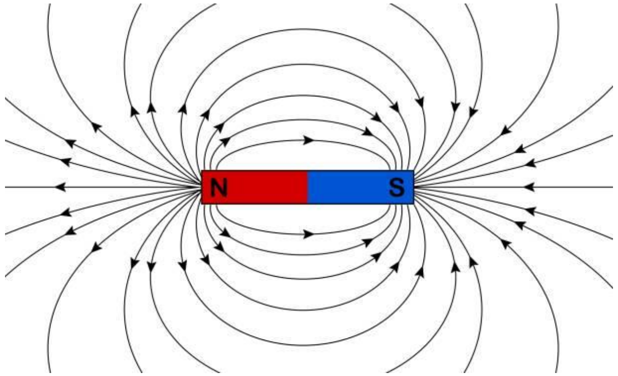
Magnetic Torquers (Magnetorquers)

- Magnetic Field of earth can be used to provide attitude control.
- Let the magnetic field

$$\vec{\mathbf{B}}_e(x, y, z)$$

From fundamental first principles, if you have two magnets moving around each other, a force is produced.





Magnetic Torquers

- Use electromagnets to create a magnetic dipole moment.
- Maxwell's Equations led to

$$\vec{T} = \vec{M} \times \vec{\mathbf{B}}_e(x, y, z)$$

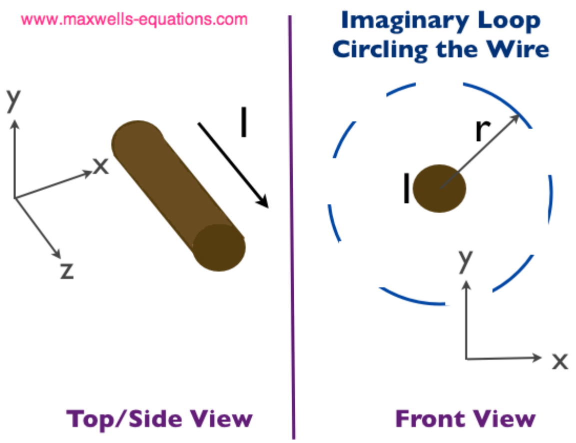
where $\vec{M}$ is the moment induced by the spacecraft

- Note that

$$\vec{\mathbf{B}}_e(x,y,z) = \begin{bmatrix} 0 & B_z & -B_y \\ -B_z & 0 & B_x \\ B_y & -B_x & 0 \end{bmatrix}$$

- is not invertible

ISIS MagneTorQuer board (iMTQ)



Magnetic Torquers

ISIS MagneTorQuer board (iMTQ)

- Magnitude of earth magnetic field is inversely proportional to the distance

$$||\vec{\mathbf{B}}_e|| \approx \frac{7.69 \cdot 10^15Wb - m}{r^3}$$

TAURUS Magnetorquer Rods

- The ISIS MagneTorQuer Board produces $0.2Am^2$ magnetic moment
- The TAURUS Magnetorquer Rods produce

  2.0 Am² up to *300 Am²*

- The angle to the field line α
- The Torque induced is given by $T = ||M||||B|| \sin\alpha$

Challenges Magnetic Torquers

- Magnetic Torquers are weak producers of torques
- Magnetic fields cannot rotate the spacecraft about a field-line
- Pitch or Yaw forces – No Roll
- Control is somewhat difficult due to field analysis

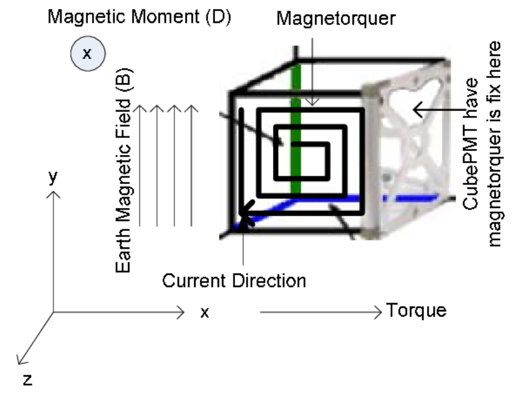


Figure 3. Earth magnetic field interaction with Magnetorquer coil magneti moment.

- Typically, not used for active attitude control
  - Mostly used to dump angular momentum over time
    - Reaction Wheels and CMGs

Quiz

What is the primary purpose of an active attitude control system in a satellite? (select all that may apply)

1. To regulate power consumption
2. To maintain or change the orientation
3. To manage communication with ground stations
4. To gather sensor measurement from different targets

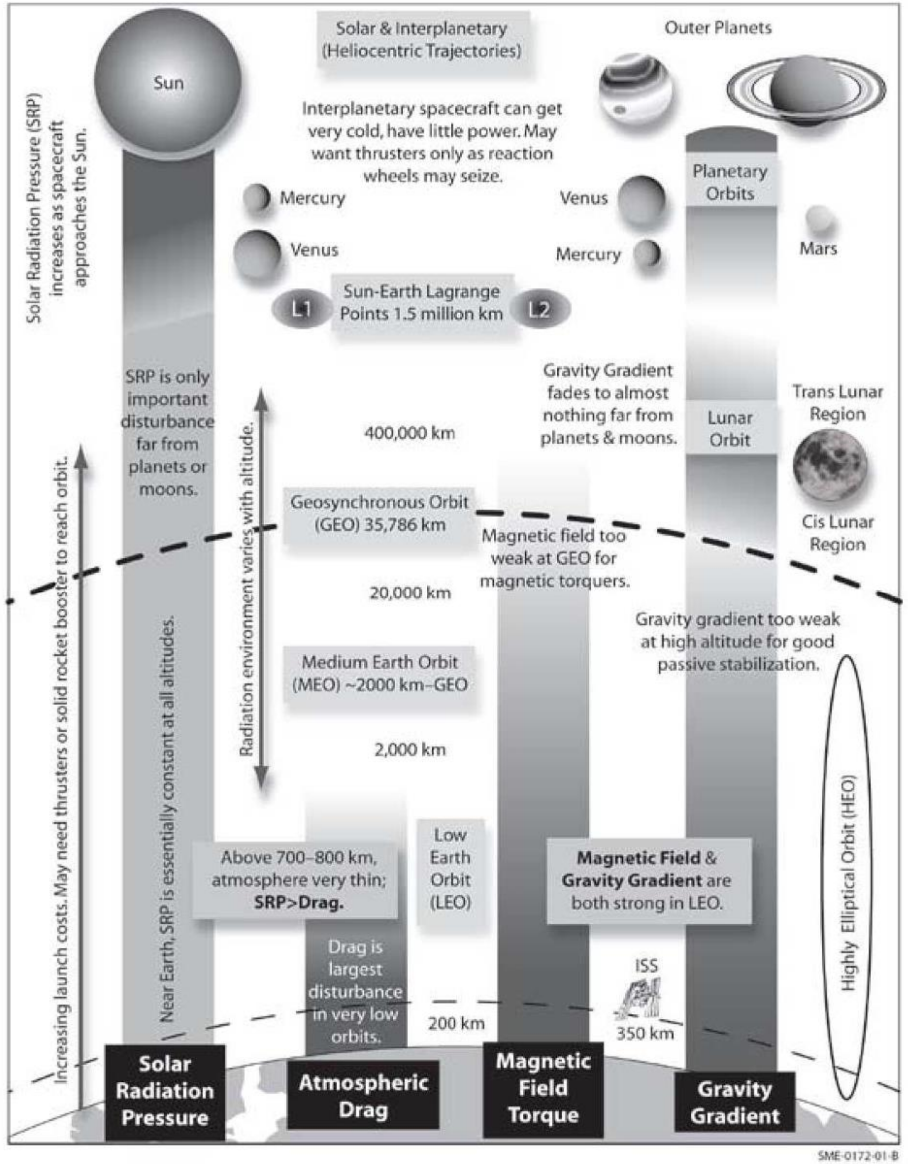
A satellite is designed with two-star trackers, and a sensor on a brittle extended boom. What types of constraints may need to be considered? (select all that may apply)

1. Solar exclusion zone
2. Line of sight constraint
3. Angular rate constraint
4. To gather sensor measurement from different targets

Environmental Distrubances

Major Disturbances

- LEO
  - Atmospheric Drag
  - Gravity Gradient
  - Strong Magnetic Field
- GEO
  - Solar Radiation Pressure



Principle Internal Disturbances

- Uncertainty of Center of Gravity
- Thruster Misalignment
- Mismatch of Thruster Outputs
- Reaction Wheel Friction
- Rotating Machinery
- Liquid Slosh
- Flexible Bodies
- Thermal Shocks

Effect on Vehicle

Unbalanced torques during firing of coupled thrusters
Unwanted torques during translation thrusting

Resistance that opposes control torque

Perturbs both stability and accuracy

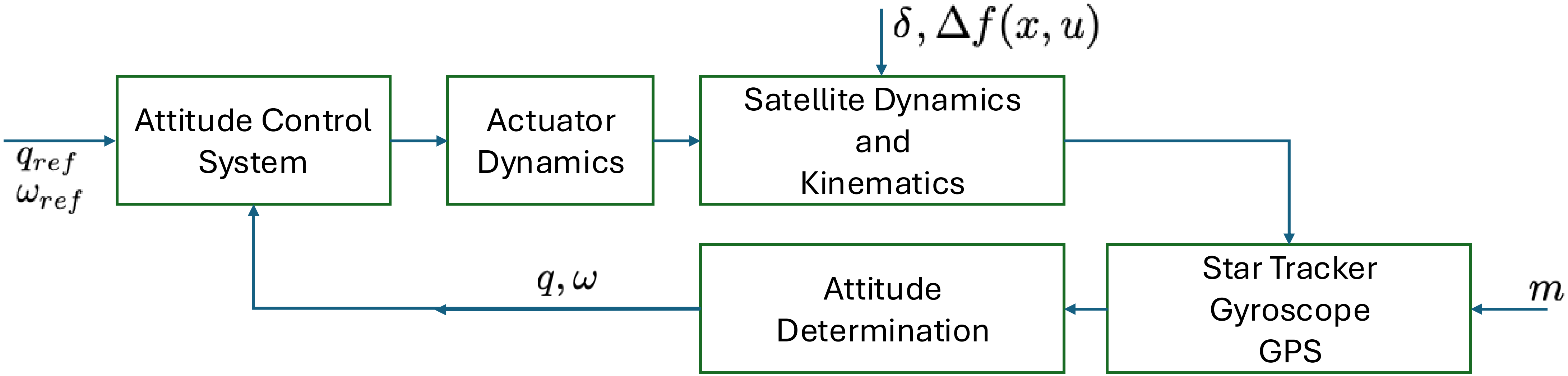
Changes in CG, torques due to liquid dynamic pressure

Oscillatory resonance at bending/twisting freq.

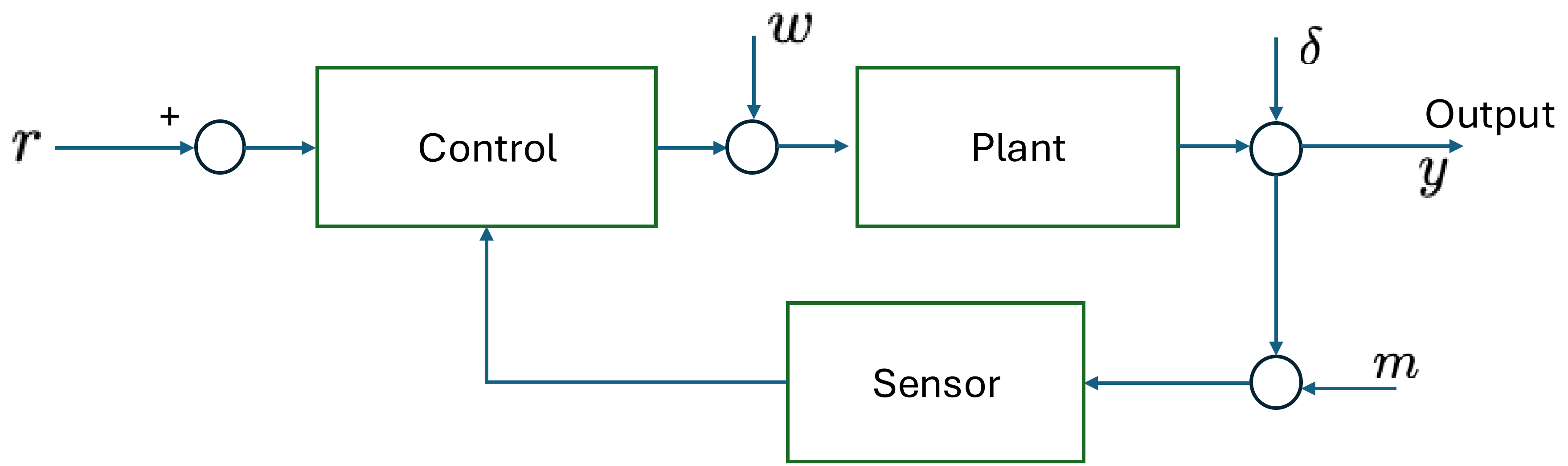
Attitude disturbance when entering/leaving umbrella

Control Systems Fundamentals

Satellite Control System Loop

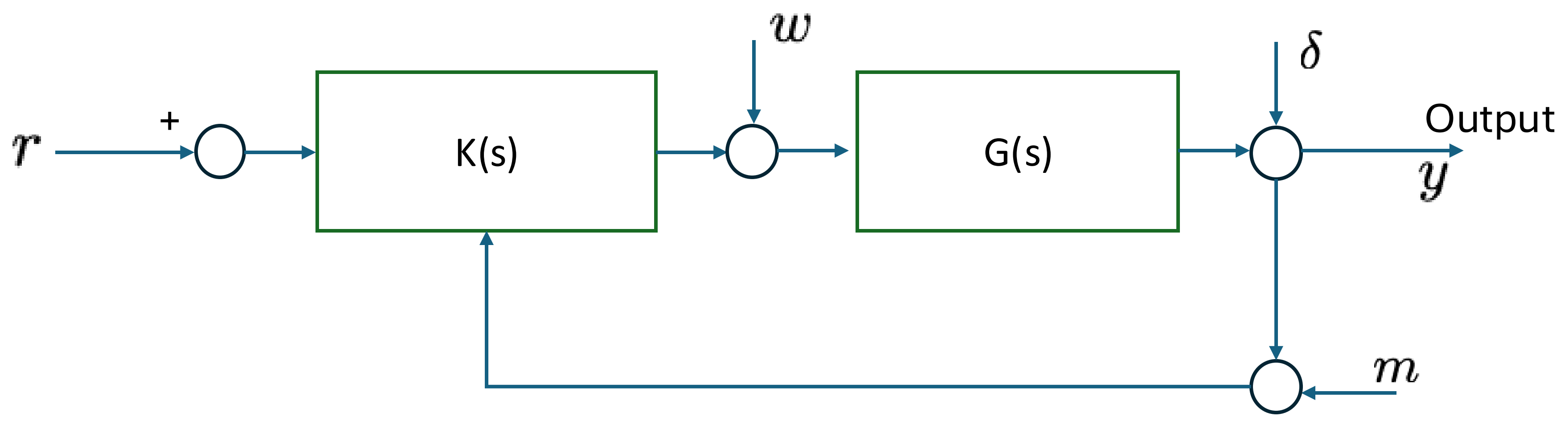


Generic Control System Feedback Loop



Plant usually contains most dynamics including, system dynamics and kinematics models, and actuator models

Generic Control System Feedback Loop



In the frequency domain, we can write the closed-loop transfer function as

$$y(s) = \frac{K(s)G(s)}{1 + K(s)G(s)} r(s) + \frac{G(s)}{1 + K(s)G(s)} w(s) + \frac{1}{1 + K(s)G(s)} d(s)$$

where $y(s) = \mathcal{L}[y(t)]$ and $\mathcal{L}[\cdot]$ is the Laplace Transform of its argument

Sensitivity Functions

From

$$y(s) = \frac{K(s)G(s)}{1+K(s)G(s)}r(s) + \frac{G(s)}{1+K(s)G(s)}w(s) + \frac{1}{1+K(s)G(s)}d(s)$$

$1 + K(s)G(s)$ is called the characteristic equation

$K(s)G(s)$ is the open loop transfer function

$S(s) = \dfrac{1}{1+K(s)G(s)}$ is the sensitivity function

$T(s) = \dfrac{K(s)(G(s)}{1+K(s)G(s)}$ is the complementary sensitivity function

Steady State Error

- Define the error as $e(t) = r(t) - y(t)$
- The steady state error can be found to be

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} se(s)$$

- The error can be written as

$$e(s) = \frac{1}{1 + K(s)G(s)} r(s)$$

- Then

$$e_{ss}(s) = \lim_{s \to 0} \frac{s}{1 + K(s)G(s)} r(s)$$

for stability it follows that $\lim_{s \to 0} K(s)G(s) = \infty$ for zero steady-state tracking error

Laplace Transforms

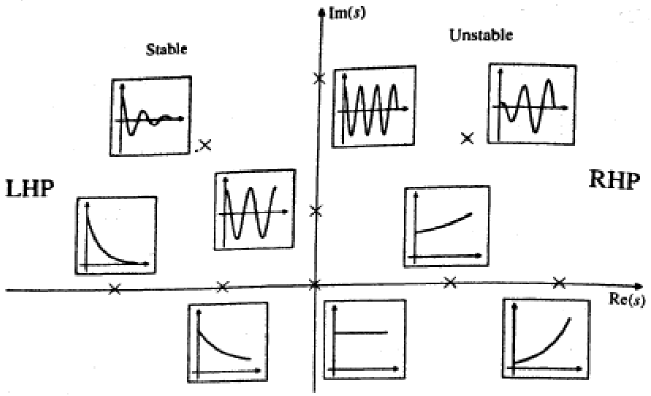
- Recall

Table of Laplace Transforms

Number	$F(s)$	$f(t), t \geq 0$
1	1	$\delta(t)$
2	$\frac{1}{s}$	$1(t)$
3	$\frac{1}{s^2}$	t
4	$\frac{2!}{s^3}$	t^2
5	$\frac{3!}{s^4}$	t^3
6	$\frac{m!}{s^{m+1}}$	t^m
7	$\frac{1}{(s+a)}$	e^{-at}
8	$\frac{1}{(s+a)^2}$	te^{-at}
9	$\frac{1}{(s+a)^3}$	$\frac{1}{2!}t^2e^{-at}$
10	$\frac{1}{(s+a)^m}$	$\frac{1}{(m-1)!}t^{m-1}e^{-at}$
11	$\frac{a}{s(s+a)}$	$1-e^{-at}$
12	$\frac{a}{s^2(s+a)}$	$\frac{1}{a}(at-1+e^{-at})$
13	$\frac{b-a}{(s+a)(s+b)}$	$e^{-at}-e^{-bt}$
14	$\frac{s}{(s+a)^2}$	$(1-at)e^{-at}$
15	$\frac{a^2}{s(s+a)^2}$	$1-e^{-at}(1+at)$
16	$\frac{(b-a)s}{(s+a)(s+b)}$	$be^{-bt}-ae^{-at}$
17	$\frac{a}{(s^2+a^2)}$	$\sin at$
18	$\frac{s}{(s^2+a^2)}$	$\cos at$
19	$\frac{s+a}{(s+a)^2+b^2}$	$e^{-at}\cos bt$
20	$\frac{b}{(s+a)^2+b^2}$	$e^{-at}\sin bt$
21	$\frac{a^2+b^2}{s\left[(s+a)^2+b^2\right]}$	$1-e^{-at}\left(\cos bt+\frac{a}{b}\sin bt\right)$

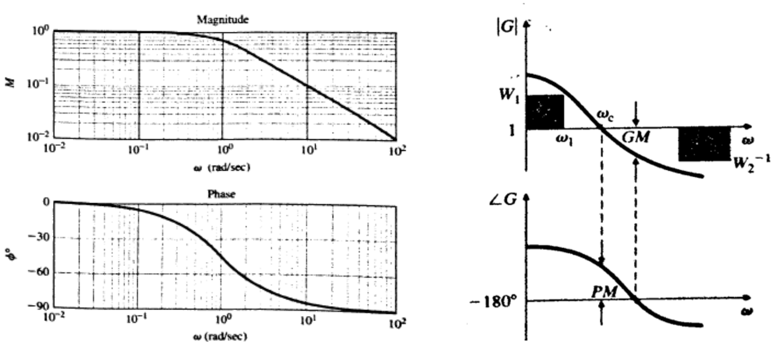
Classical Control Design

- Root Locus
  - Pole-zero locations



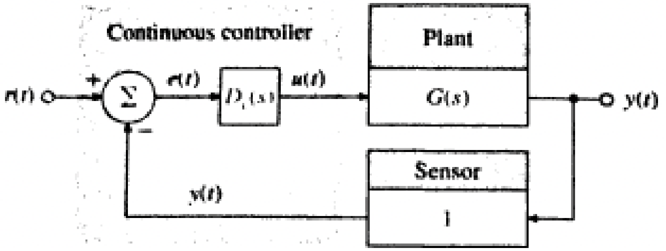
  - Design tool to place poles for the closed-loop transfer function where you need them based on time-based and frequency-based design parameters

- Bode Plots

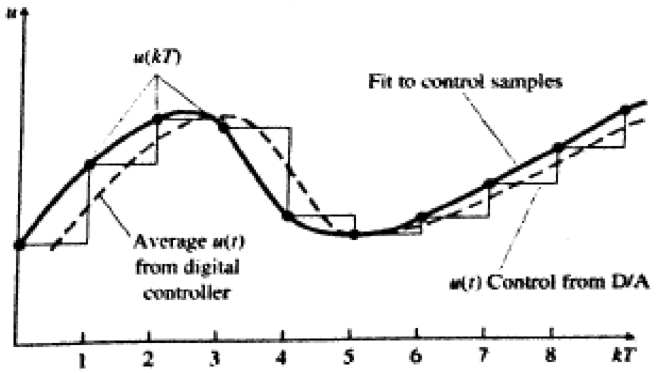


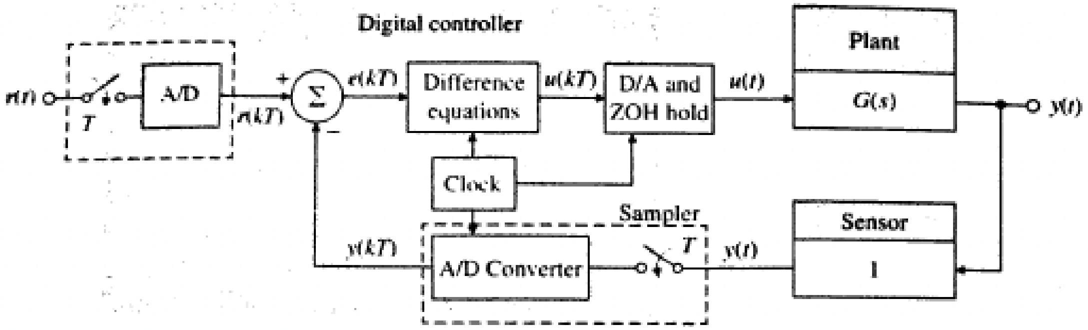
  - Analyzes frequency response of systems
  - Can be used for system identification
  - Can determine and quantify stability and robustness using phase-gain margin criterions

Discretization



The advent of computers which act at periodic “clock speeds” has led to discretized control is as another useful tool for control design





Sampling (analog to digital converter)
Rule of thumb– sampling rates should be at least 20x the bandwidth to assure good performance
Zero-order hold (digital to analog converter)

Due to the fast speed of computer systems, many people simply approximate the fast discrete time as continuous.

z-transforms

Signal $x[n]$	z-Transform $X(z)$	ROC
$\delta[n]$	1	All z
$u[n]$	$\frac{1}{1-z^{-1}}$	$\lvert z\rvert > 1$
$n\,u[n]$	$\frac{z^{-1}}{\left(1-z^{-1}\right)^2}$	$\lvert z\rvert > 1$
$a^n u[n]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert > \lvert a\rvert$
$na^n\,u[n]$	$\frac{az^{-1}}{\left(1-az^{-1}\right)^2}$	$\lvert z\rvert > \lvert a\rvert$
$-a^n u[-n-1]$	$\frac{1}{1-az^{-1}}$	$\lvert z\rvert < \lvert a\rvert$
$-na^n\,u[-n-1]$	$\frac{az^{-1}}{\left(1-az^{-1}\right)^2}$	$\lvert z\rvert < \lvert a\rvert$
$(\cos\omega_o n)u[n]$	$\frac{1-z^{-1}\cos\omega_o}{1-2z^{-1}\cos\omega_o+z^{-2}}$	$\lvert z\rvert > 1$
$(\sin\omega_o n)u[n]$	$\frac{z^{-1}\sin\omega_o}{1-2z^{-1}\cos\omega_o+z^{-2}}$	$\lvert z\rvert > 1$
$(a^n\cos\omega_o n)u[n]$	$\frac{1-az^{-1}\cos\omega_o}{1-2az^{-1}\cos\omega_o+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$
$(a^n\sin\omega_o n)u[n]$	$\frac{az^{-1}\sin\omega_o}{1-2az^{-1}\cos\omega_o+a^2z^{-2}}$	$\lvert z\rvert > \lvert a\rvert$

State Space Control Design

Consider a linear time-invariant (LTI) dynamic system

$$\dot{x} = Ax + Bu$$

consider $u = -Kx$ where *K* is a tunable controller gain.

The closed-loop system is then described by

$$\dot{x} = Ax - BKx = (A - BK)x$$

The characteristic equation can be defined as

$$|sI - A + BK| = 0$$

If the system is controllable, then the eigenvalues of the closed-loop system can be arbitrarily assigned.

Linear Quadratic Regulator (LQR)

LQR is an optimal control policy that finds the “best” gains for a controller

Consider an LTI system and the following cost function

$$J = \frac{1}{2}\int_0^\infty (x^\top Q x + u^\top R u)dt$$

and feedback $u = -Kx$ where $K = R^{-1}B^\top X$ by solving the Riccati equation

$$A^\top X + XA - XBR^{-1}B^\top X + Q = 0$$

Q,R must be symmetric and positive definite
(A,B) must be controllable

Challenges of Nonlinear Control Systems

Consider the following nonlinear system

$$\dot{x} = f(x, u)$$

Complex dynamics – limit cycles, bifurcations, chaotic systems

Stability Analysis – must leverage advanced tools for control design and showing stability may be nontrivial

Observability and Controllability – not as straight forward for determining systems are observable or contrallable

Robustness – may be sensitive to parameter variations and disturbances

Computational Complexity – Approaches may involve numerical methods that can be computationally intensive

Stability of Nonlinear Systems

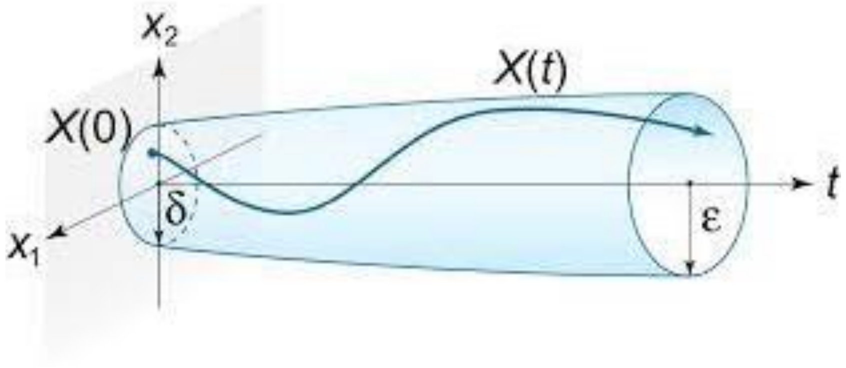
Definition 4.1 *The equilibrium point $x = 0$ of (4.1) is*

- *stable if, for each $\varepsilon > 0$, there is $\delta = \delta(\varepsilon) > 0$ such that*

$$\|x(0)\| < \delta \Rightarrow \|x(t)\| < \varepsilon, \quad \forall\, t \geq 0$$

- *unstable if it is not stable.*
- *asymptotically stable if it is stable and δ can be chosen such that*

$$\|x(0)\| < \delta \Rightarrow \lim_{t \to \infty} x(t) = 0$$



As written, stability can be quite difficult to show.

To show the origin is *stable*, then we have to show that for any value of *ε,* we much produce a value $\boldsymbol{\delta}$ such that a trajectory starting in a $\boldsymbol{\delta}$ neighborhood of the origin will never leave the *ε* neighborhood.

Lyapunov Stability

Since showing stability explicitly requires a solutions-based approach, which can be difficult for every parameter

Theorem 4.1 *Let $x = 0$ be an equilibrium point for (4.1) and $D \subset R^n$ be a domain containing $x = 0$. Let $V : D \to R$ be a continuously differentiable function such that*

$$V(0) = 0 \quad \text{and} \quad V(x) > 0 \text{ in } D - \{0\} \tag{4.2}$$

$$\dot{V}(x) \leq 0 \text{ in } D \tag{4.3}$$

Then, $x = 0$ is stable. Moreover, if

$$\dot{V}(x) < 0 \text{ in } D - \{0\} \tag{4.4}$$

then $x = 0$ is asymptotically stable. ◇

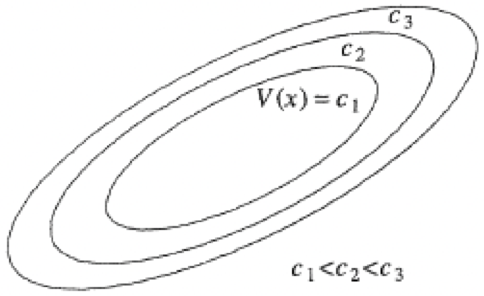


Figure 4.2: Level surfaces of a Lyapunov function.

In English:

1) Find a function *V* defined on *D*, check to make sure that *V(0) = 0* and *V(x)* is positive in *D*
2) Find the derivative of *V* , and make sure that it is negative semidefinite in *D*, then the equilibrium point is stable
3) If the derivative of *V* is negative definite outside of the equilibrium point, then the equilibrium point is asymptotically stable

Example

Consider a pendulum equation without friction

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -a \sin x_1$$

Consider the following function

$$V(x) = a(1 - \cos x_1) + \frac{1}{2} x_2^2$$

V is positive everywhere away from the origin and positive definite over the domain $-2\pi < x_1 < 2\pi$

The derivative of V is given by

$$\dot{V}(x) = a\dot{x}_1 \sin x_1 + x_2 \dot{x}_2 = a x_2 \sin x_1 - a x_2 \sin x_1 = 0$$

Which leads to the origin as being a stable point.

Simulate to verify this is the case?

Example Continued

Dynamics given by

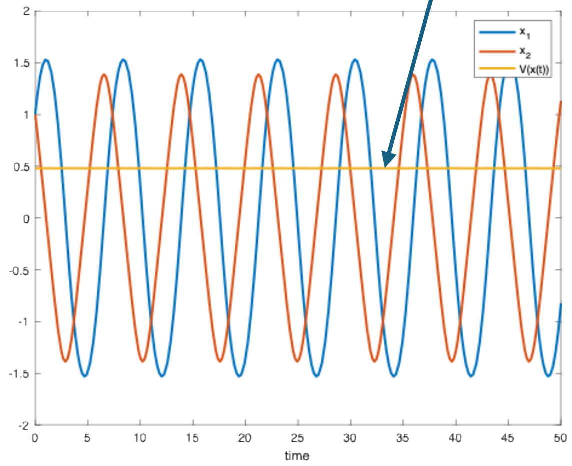
$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -a \sin x_1$$

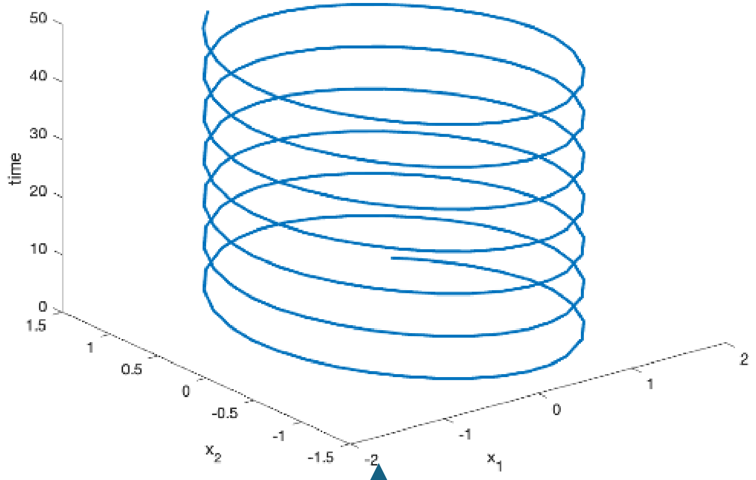
Lyapunov Function

$$V(x) = a(1 - \cos x_1) + \frac{1}{2} x_2^2$$

Numerical solution with $a = 0.5$

V is constant verifying the results





There is neither convergence nor divergence in the trajectories.

A 3D plot of the trajectory where the z-axis is time, note the trajectory is on a cycle.

Note that for nonlinear system analysis, finding V is half the battle
best place to start is $V(x) = x^\top P x$ with P being positive definite

Complexities for Control Design

- Attitude control is a complex and difficult problem
  - Nonlinear dynamics
  - Actuators have nonlinear dynamics, sometimes impulsive
  - Controllability is not always guaranteed
  - Depending on the formulations we have discontinuities or duplicative orientations
  - There exist numerous keep out zones that need to be adhered to for maneuvering

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