Sec. 2.5 Globally Convergent Methods for Solving One Equation in One Unknown

(from section 2.5 of Dennis & Schnabel)

Suppose that we are looking for the root x_* in $f(x_*) = 0$.

Assume:

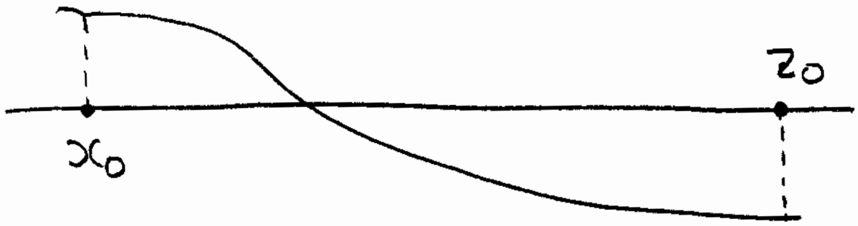
* x_0 and z_0 are given to us, and $\text{sign}(f(x_0)) \neq \text{sign}(f(z_0))$,
* There is only one root in $[x_0, z_0]$.

(First condition is easy to check)

The bisection method may then be used to provide an interval around the root.

1. Setup

$f(x_0) > 0$



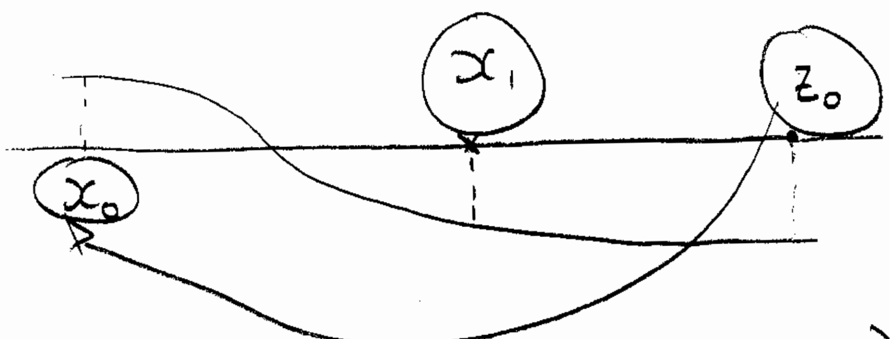
$f(z_0) < 0$.

2. Pick the middle point:

$$x_1 = \frac{(x_0 + z_0)}{2}$$

and "move z_1" so that:

$$\text{sign}(f(x_1)) \neq \text{sign}(f(z_1)).$$



z_0 can move to $z_1 = x_0$, or

stay: $z_1 = z_0$.

3. Repeat 2 until the interval is small "enough".

Pseudocode:

Given x_0, z_0 with $f(x_0) \cdot f(z_0) < 0$.

for $k = 0, 1, 2, \dots,$ do

$$x_{k+1} = \frac{x_k + z_k}{2}$$

$$z_{k+1} = \begin{cases} x_k, & \text{if } f(x_k) \cdot f(x_{k+1}) < 0 \\ z_k, & \text{otherwise.} \end{cases}$$