ME 596 Spacecraft Dynamics: Homework Assignment 4

Your submission must be submitted as a report, by which I mean your work must be written out as it would be in a report, showing and explaining all your work, and clearly stating your answers using complete sentences and references as appropriate, and the entire assignment must be neatly written or typeset.

You may use `Matlab` or other programming environment to perform the calculations in all of the problems in this assignment, but you must show your work and provide results of intermediate calculations.

Note: Including a `Matlab` program does not qualify as "showing your work."

You must include your name, course number, and assignment number at the top of the first page.

You must upload your work as a *single* pdf file by 11:59 PM on the due date.

1. The orientation of a spacecraft B relative to an inertial frame is given through the 3-2-1 Euler angle sequence, with initial conditions:

$$\boldsymbol{\theta}_B = [10 \quad -15 \quad 30]^\mathsf{T} \text{ degrees}$$

   The attitude control system makes the spacecraft rotate with angular velocity

$$\boldsymbol{\omega}^{Bi} = [1 \;\; 1 \;\; 1]^\mathsf{T} \text{ rad/s}$$

   expressed in $\mathcal{F}_B$.

   Integrate the quaternion equations of motion to obtain $\bar{\mathbf{q}}(t)$, for $t = 0$ to 20 seconds.

   Make a presentation-quality plot of the elements of $\bar{\mathbf{q}}(t)$ over the given time range.

2. Two pairs of photocells are arranged in a perpendicular configuration, and measure sun direction angles of $\alpha_1 = 45°$ and $\alpha_2 = 60°$. Calculate the components of the unit sun vector in sun sensor frame, $\mathcal{F}_s = \left\{\hat{\mathbf{n}}_1, \hat{\mathbf{n}}_2, \hat{\mathbf{t}}\right\}$.

   The sensor frame $\mathcal{F}_s$ is oriented in the spacecraft body frame, $\mathcal{F}_b$, by $\mathbf{R}^{sb}$, which is a 3-2-1 Euler angle rotation sequence through the angles $\theta_1 = 45°$, $\theta_2 = 180°$, and $\theta_3 = 90°$ from $\mathcal{F}_b$ to $\mathcal{F}_s$. Calculate the components of the unit sun vector in $\mathcal{F}_b$.

   The spacecraft generates power using a single solar panel. The components of the unit normal vector of the panel expressed in $\mathcal{F}_b$ are $\mathbf{p}_b = (0 \quad -1 \;\; 0)^\mathsf{T}$. The panel generates the maximum amount of power when it is perpendicular to the incoming sunlight, so we want to know the angle between the sun vector and the unit normal vector of the panel. Calculate the angle between $\hat{\mathbf{s}}$ and $\hat{\mathbf{p}}$.

3. Consider two unit vectors $\hat{\mathbf{v}}_1$ and $\hat{\mathbf{v}}_2$. (These vectors could be sun vector, magnetic field vector, Earth horizon vector, or any other vector for which a model and a sensor are available.) The components of $\hat{\mathbf{v}}_1$ and $\hat{\mathbf{v}}_2$ expressed relative to the inertial frame are known to be

$$\begin{aligned} \mathbf{v}_{1i} &= \begin{pmatrix} 0.6651 & 0.7395 & 0.1037 \end{pmatrix}^\mathsf{T} \\ \mathbf{v}_{2i} &= \begin{pmatrix} 0.8190 & 0.5670 & 0.0875 \end{pmatrix}^\mathsf{T} \end{aligned}$$

   Sensors on a spacecraft measure the components of $\hat{\mathbf{v}}_1$ and $\hat{\mathbf{v}}_2$ relative to the body frame to be

$$\begin{aligned} \mathbf{v}_{1b} &= \begin{pmatrix} 0.5629 & -0.3243 & -0.7592 \end{pmatrix}^\mathsf{T} \\ \mathbf{v}_{2b} &= \begin{pmatrix} 0.5770 & -0.5186 & -0.6299 \end{pmatrix}^\mathsf{T} \end{aligned}$$

   Use the TRIAD algorithm with $\hat{\mathbf{v}}_1$ as the "exact" measurement and determine the rotation matrix $\mathbf{R}^{bi}$. Repeat using $\hat{\mathbf{v}}_2$ as the "exact" measurement.

   If the "true" rotation matrix $\mathbf{R}^{bi}$ is given by a 3-2-1 Euler angle rotation sequence through the angles $\theta_1 = 30°$, $\theta_2 = -60°$, and $\theta_3 = 45°$, calculate the attitude error for each case of applying TRIAD. Express your answer as a principal Euler angle; i.e., calculate the Euler angle of the rotation matrix relating the estimated $\mathcal{F}_b$ and the true $\mathcal{F}_b$.

   Using your calculated errors, which sensor do you think is more accurate (the one that provides $\mathbf{v}_{1b}$ or the one that provides $\mathbf{v}_{2b}$)? Justify your reasoning.

4. Repeat Problem 3 using the $\mathbf{q}$-Method to determine the rotation matrix $\mathbf{R}^{bi}$. Assume that the two measurements are equally weighted, $w_1 = w_2 = 1$.

   Calculate the attitude error as a principal Euler angle, and compare the error determined in this problem to the errors calculated in Problem 3.

   Calculate the value of Wahba's loss function $J(\mathbf{R}^{bi})$ and compare the value determined in this problem to the values for the rotation matrices calculated in Problem 3.

5. Repeat Problem 3 using the QUEST algorithm to determine the rotation matrix $\mathbf{R}^{bi}$. Assume that the two measurements are equally weighted, $w_1 = w_2 = 1$.

6. Write a paragraph describing your observations and conclusions about using TRIAD, $\mathbf{q}$-Method, and QUEST.