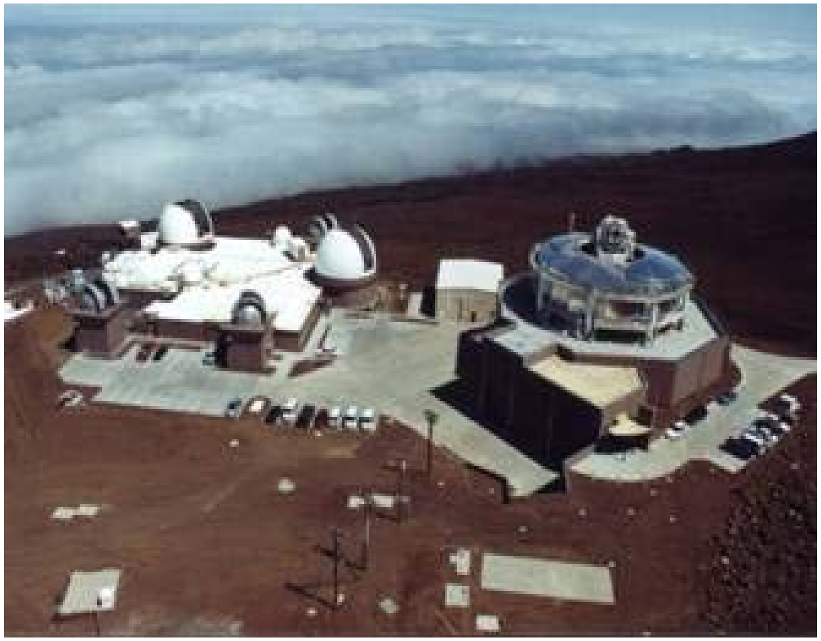
Sensing

Passive Tracking

- Week 1 slide lecture discussed active and passive tracking (we'll focus on passive tracking in this course)
- Primary sensing modes are RF (e.g. radars) & optical (e.g. telescopes)
- Most passive sensors are terrestrial, although some are in space
- We also discussed sensor networks (e.g. SSN, ISON) & the challenges they face (e.g. sensor tasking, data association)
- This week we will go into these issues in further detail

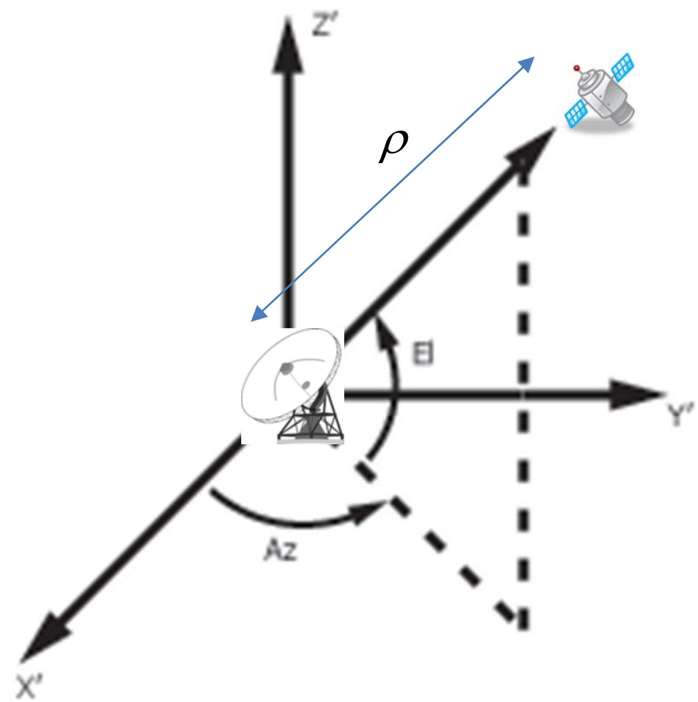
Topocentric-Horizon Frame (Az & El)

- Week 2 slide lecture discussed how Topocentric-Horizon (TH) Frame is useful for ground sensors
- Direction from a sensor to a space object is often characterized by azimuth (Az) & elevation (El) angles
- In TH coordinates x_{TH}, y_{TH}, z_{TH}:

$$Az = \tan^{-1}\left(\frac{y_{TH}}{x_{TH}}\right), El = \sin^{-1}\left(\frac{z_{TH}}{\rho}\right)$$

where range ρ to the object is given by

$$\rho = \sqrt{x_{TH}{}^2 + y_{TH}{}^2 + z_{TH}{}^2}$$



Azimuth & elevation depicted in the TH Frame

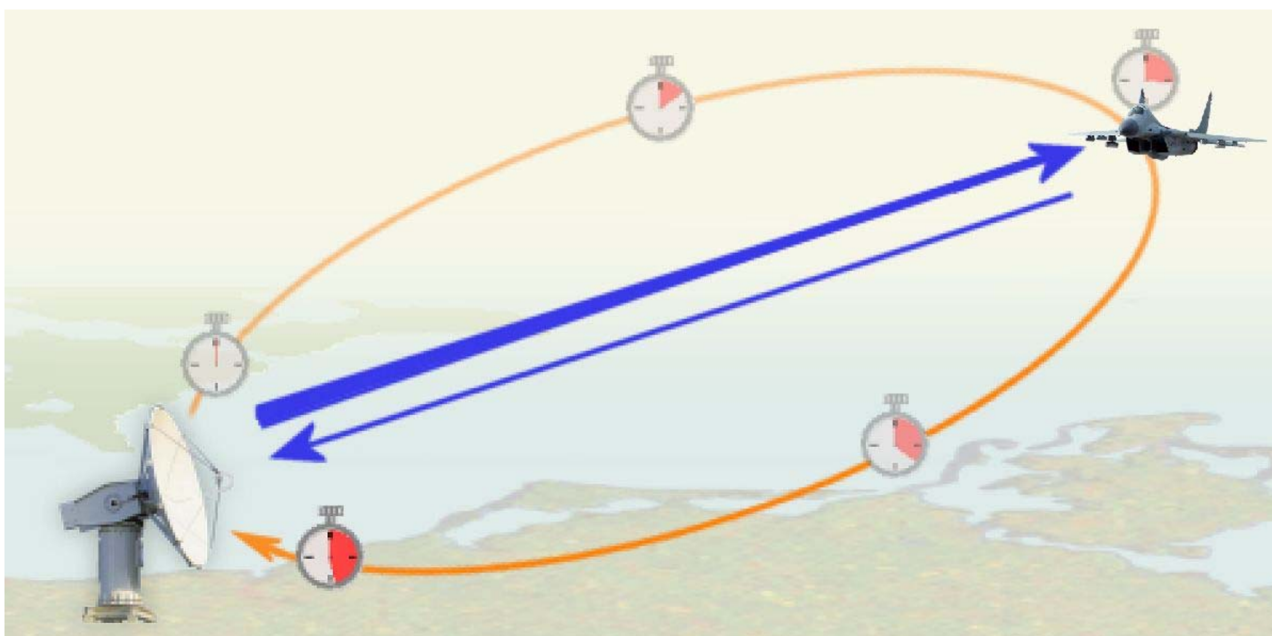
Topocentric-Horizon Frame (Az & El)

- Note that R, Az, & El comprise a spherical coordinatization of the TH Frame

- Az & El are analogous to latitude & longitude in the ECEF Frame & RA & Dec in the Celestial Frame
  - Az, longitude, & RA all indicate the azimuth or "compass" aspect of direction
  - El, latitude, & Dec all indicate the elevation above or below a reference plane

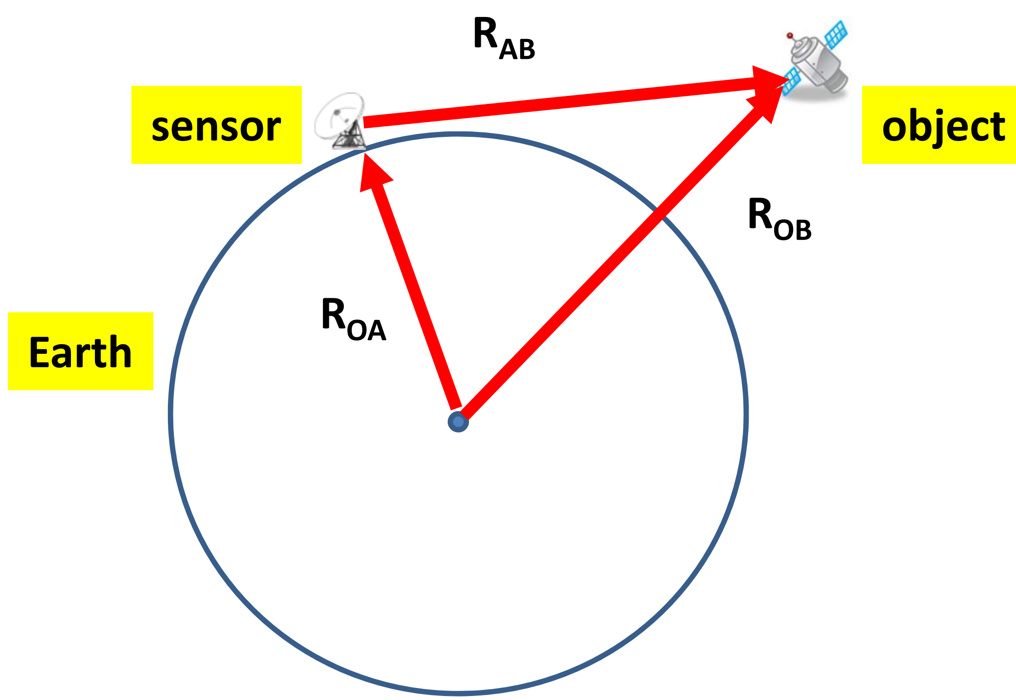
RF (Radar) Tracking Basics

- In the most basic sense, radars sense range to an object by transmitting an electromagnetic signal (i.e. pulse) outward & measuring the time of the signal's reflection back to the sensor → multiplying half the travel time by speed of light yields range to the object
- The stronger a radar's signal power, the more accurately it can sense range (& over longer distances)



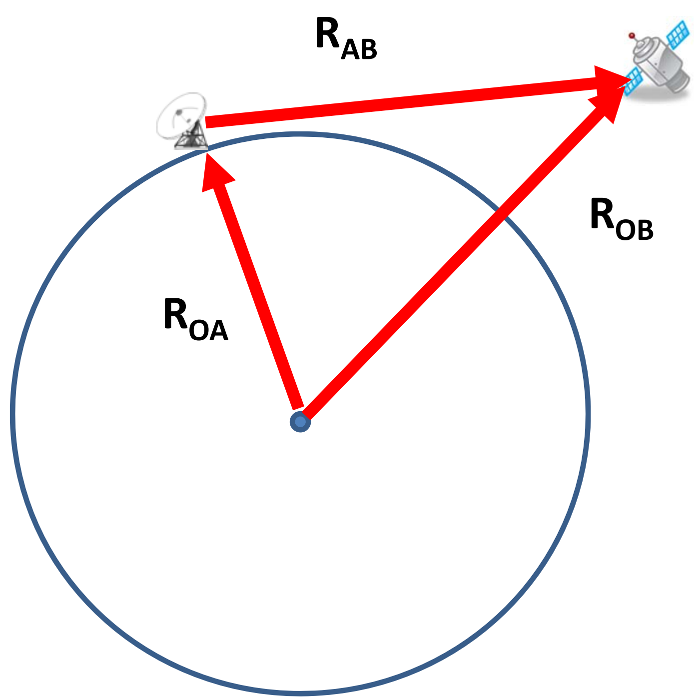
Range Measurement

- How does a range measurement to an object relate to its orbit?
- The sketch below shows Earth's center at O, a radar at A, & an object at B
- Clearly $\bar{R}_{OA} + \bar{R}_{AB} = \bar{R}_{OB}$, where $\bar{R}_{OA}$ is the sensor's position vector from Earth's center, $\bar{R}_{OB}$ is the object's position vector from Earth's center, & $\bar{R}_{AB}$ is the position vector from sensor to object



Range Measurement

- The object's position vector $\bar{R}_{OB}$, along with its inertial velocity vector, comprise its Cartesian state vector, which is one way to express the object's orbit
- The range measurement to the object from the sensor is R_{AB} (the magnitude of $\bar{R}_{AB}$), therefore range is intrinsically related to (& driven by) the object's orbital motion



Range Measurement

- Some radars can also measure the Doppler return of the signal reflected back from an object
- Comparing the received frequency to the transmitted frequency yields the Doppler shift, a direct function of range rate, which is intrinsically related to the object's position & velocity vectors
- Thus, range rate, like range itself, serves as a useful measurement in determining the object's orbit
- One qty that is not easily discernible from radar is the direction (i.e. line-of-sight) to the object
  - Due to the wide beam pattern of most radars, one can only determine the direction from which a signal return came in a very broad sense
  - Some radars are even omni-directional, in which case determining direction is virtually impossible

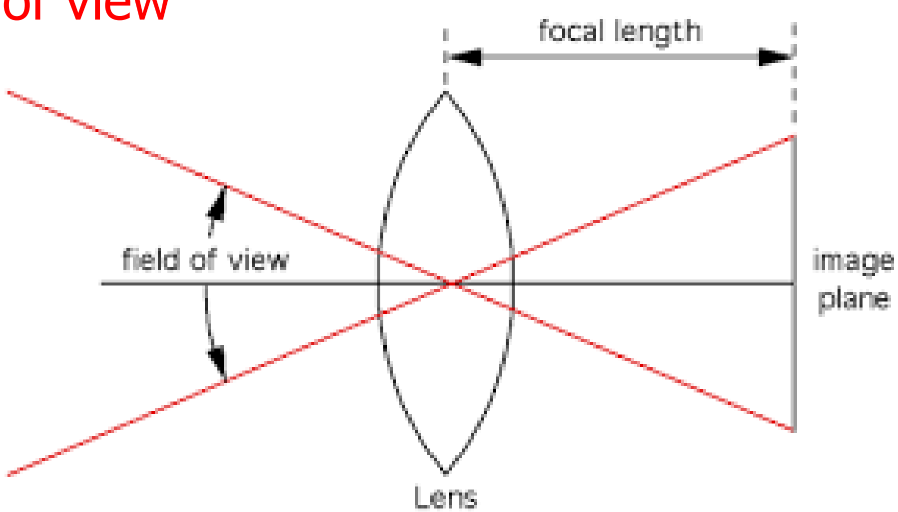
Optical (Telescope/Camera) Basics

- In this course, we will focus primarily on optical (line-of-sight) sensing rather than RF (range) sensing
- A telescope equipped with a camera can capture an image of space
- Generally, this image will contain multiple celestial objects (e.g. stars)
- The goal of optical SSA is of course to capture Earth-orbiting objects in the images so the line-of-sight (or alternately Az & El) of the vector from the sensor to the object can be measured
- Distinguishing Earth-orbiting objects from celestial objects in an image can be challenging (& is the focus of Week #7 of the course); for now, we will assume that we know which objects in an image are satellites

LOS Measurement

(NOTE: the following explanation pertains to grayscale images; for color images the explanation is more detailed)

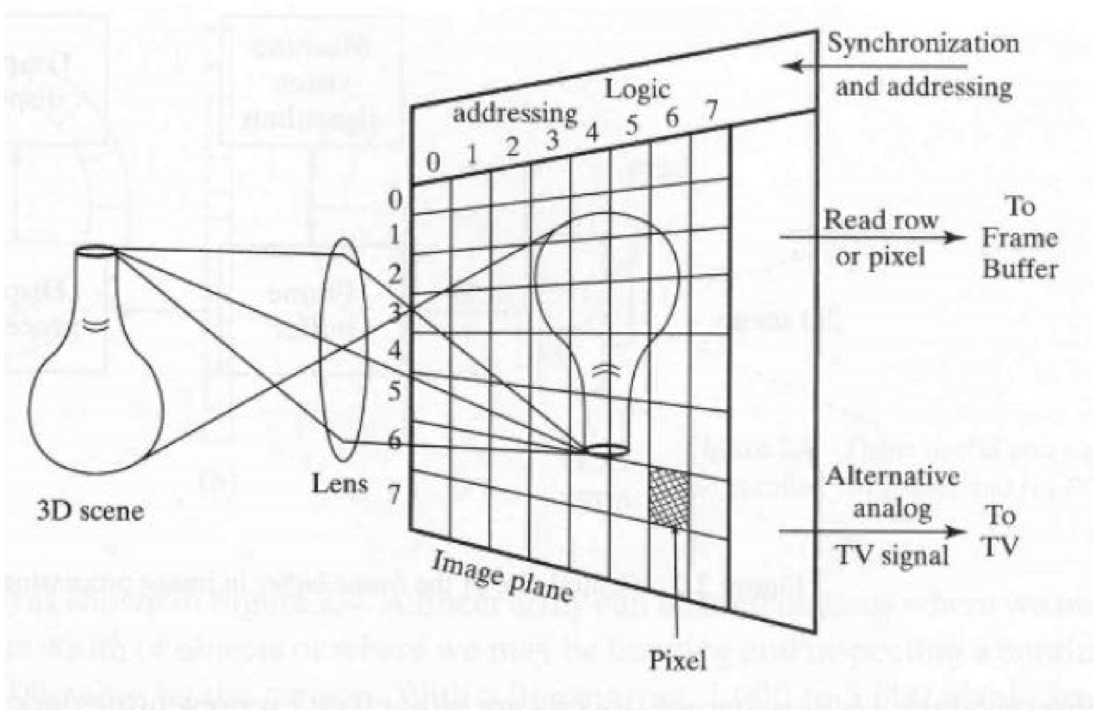
- How does one determine line-of-sight (LOS) to an object from a space image?
- If the object were to appear exactly in the center (i.e. boresight) of an image, then the LOS would simply be the pointing direction of the telescope at the time it took the image
- Practically, objects will not appear exactly on the boresight, so one must consider the relationship between the image plane, focal length, & field of view



LOS Measurement

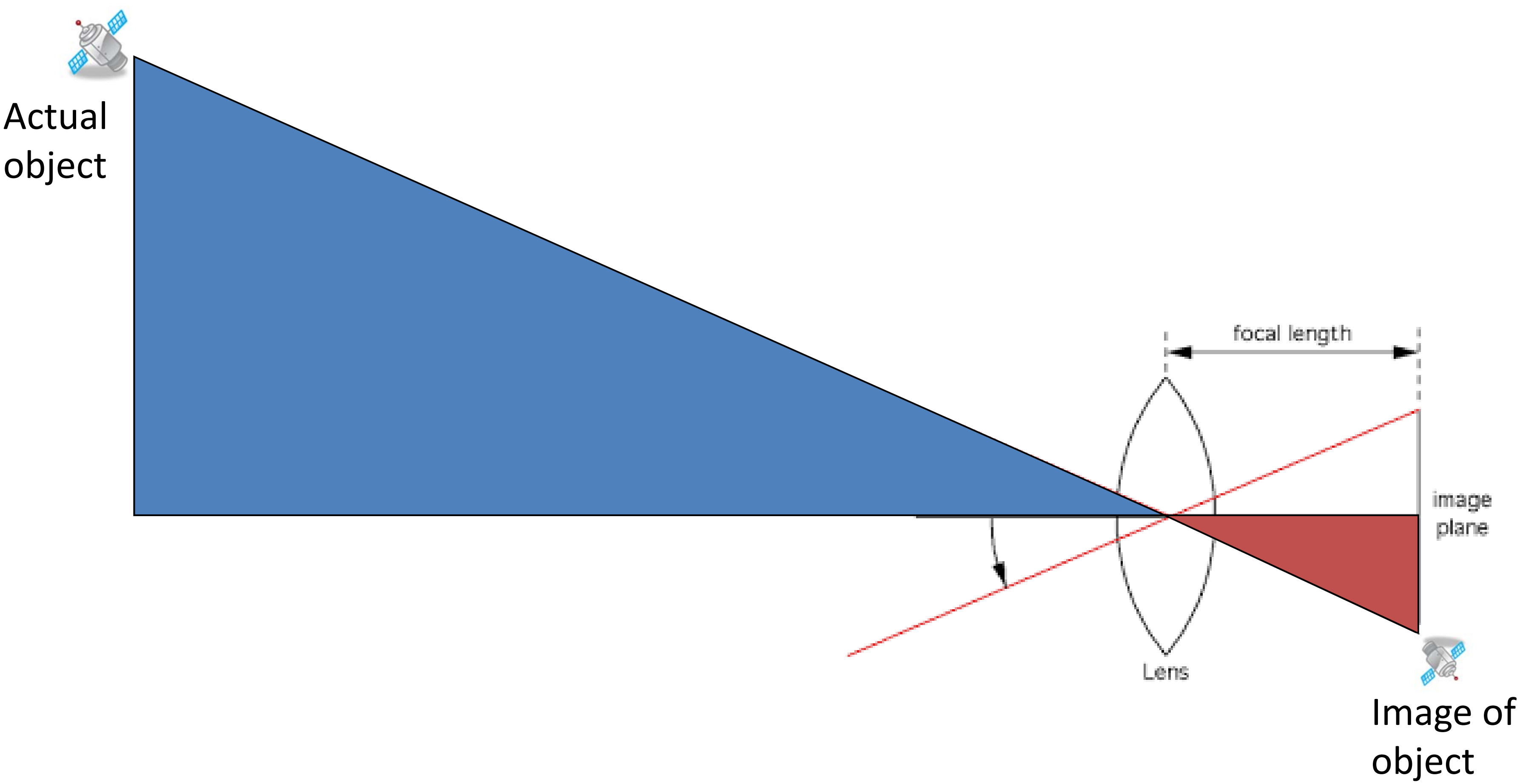
- Fundamentally, light (in the form of photons) from objects in a camera's field of view passes through the lens & is captured on the image plane, which is placed a certain distance behind the lens (the focal length)
- In a charge-coupled device (CCD) camera, the image plane consists of a matrix of pixels, each of which captures light & registers its intensity (based on # of photons)

- Image appears inverted on image plane
- Camera resolution is determined in part by pixel size → the smaller the pixels, the better the resolution (i.e. the more accurately one can determine the size of an object in the field of view)
- Also determined by "bit depth" (always a power of 2); for example, a bit depth of $2^8 = 256$ means intensity will register on a scale from 0 to 255



LOS Measurement

- Consider an object at a long distance from the camera
- Note that the two right triangles shown are similar (equal angles)



LOS Measurement

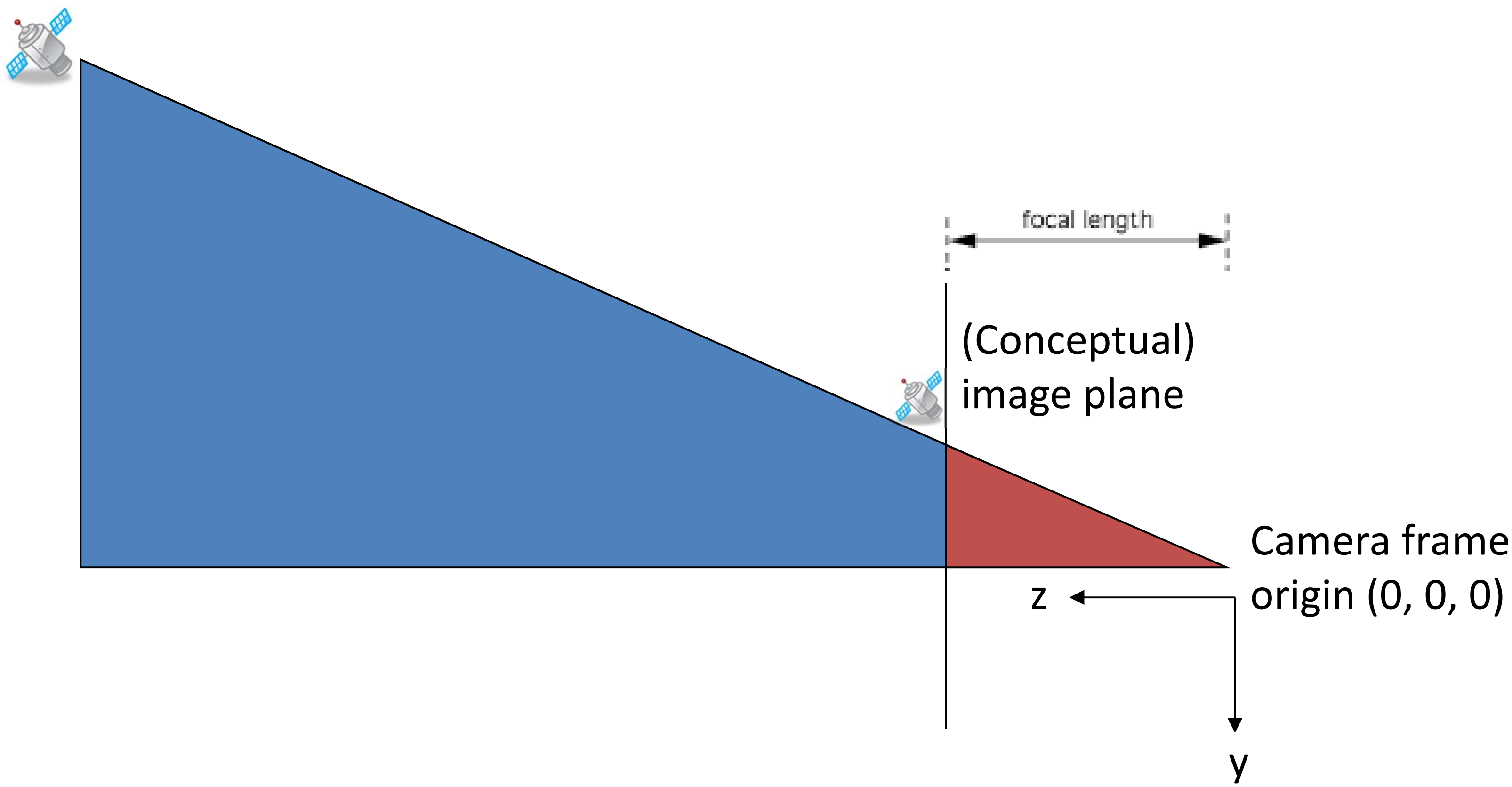
Therefore, a conceptual (but not actual) way to visualize the camera frame is as follows:

- Imagine we invert the image plane (so the image no longer appears inverted) & move it ahead of the lens by a distance equal to its original distance behind the lens (i.e. the focal length)
- Define the origin at the lens, with coordinate directions as follows:
  - x axis along the horizontal dimension of the image plane (positive to the right when looking forward)
  - y axis along the vertical dimension (positive downward)
  - z axis normal to the image plane (positive forward)

LOS Measurement

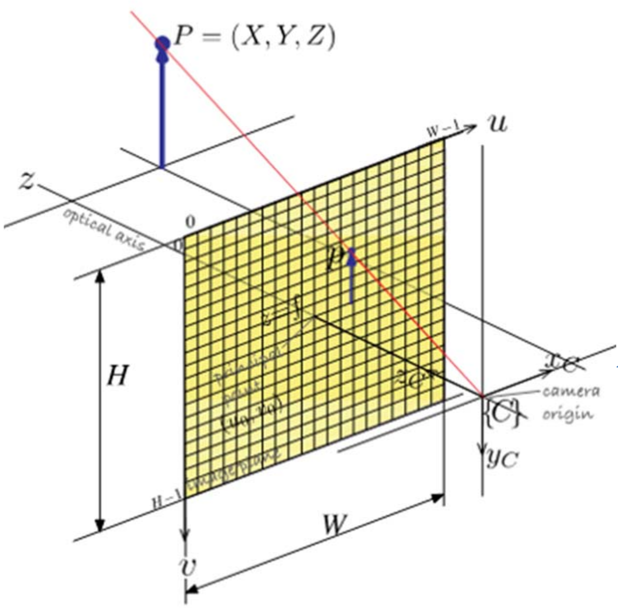
- As a result of this conceptualization, we see that the vector to the actual object & the vector to the object's image in the image plane are proportional

- Remember, we're not actually inverting or moving the image plane! (This is merely a visual device)



LOS Measurement

- Typically an object (unless it is extremely small or far away) will illuminate more than 1 pixel; in such a case, the centroid of the group of pixels can be calculated
- Because the group of pixels representing an object have varying intensities, the centroid location is usually not an integer value (i.e. sub-pixel)



This orientation of the x, y, z axes in the camera frame is often called "right-down-forward"

LOS Measurement

So once the centroid location of an object (in pixel units from the origin) is calculated, how do we turn this into LOS?

- This location, combined with the height & width of each pixel & the focal length, allows us to calculate the vector from the origin to the object's image in the image plane (which we know is proportional to the actual position vector from the sensor to the object)
- This information also allows us to calculate the camera's field of view

LOS Measurement

EXAMPLE PROBLEM:

A CCD camera has a focal length of 70mm, an image plane 1500 pixels wide by 1700 pixels high, & each pixel is 0.0075mm by 0.0075mm. The camera is installed on a telescope & takes an image of space. An object is identified in the image, & its centroid is determined to lie 328.75 pixels to the right of the boresight & 507.35 pixels below the boresight. Find the camera's total field of view & the LOS unit vector from the camera to the object expressed in the camera frame.

LOS Measurement

SOLUTION:

- First, define the camera frame in “right-down-forward” fashion, with origin (0,0) at the boresight

- Thus, the 4 corners of the image plane, in pixel units, are located at (-750, 850) → lower left, (750, 850) → lower right, (750, -850) → upper right, & (-750, -850) → upper left

- Because the width & height of each pixel is 0.0075mm, the locations of the 4 corners in physical units are (-5.625, 6.375) mm, (5.625, 6.375) mm, (5.625, -6.375) mm, & (-5.625, -6.375) mm

- Given the 70mm focal length, the half angle of the field of view in the width dimension is $\tan^{-1}(5.625/70) = 4.59^\circ$ & in the height dimension is $\tan^{-1}(6.375/70) = 5.20^\circ$ → field of view is $(2*4.59) \times (2*5.20) = 9.18^\circ \times 10.40^\circ$

LOS Measurement

SOLUTION (cont'd):

- The centroid location in physical units is (328.75*0.0075, 507.35*0.0075) mm = (2.47,3.81) mm
- Given the 70mm focal length, the position vector from the sensor to the object (in camera frame coordinates) is proportional to

$$\bar{r} = [2.47 \quad 3.81 \quad 70]mm$$

- Thus, the LOS unit vector from the camera to the object in camera frame coordinates is

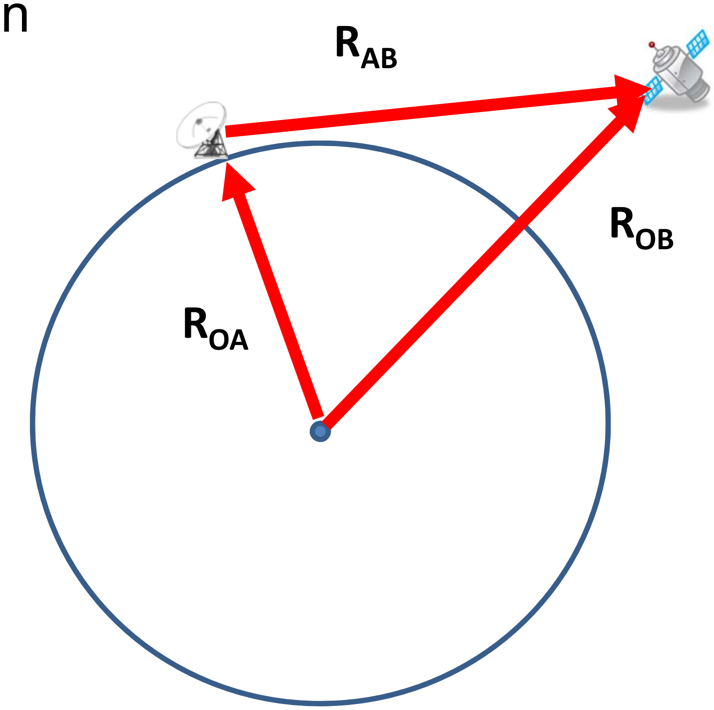
$$\hat{u} = \frac{\bar{r}}{r} = \frac{[2.47 \quad 3.81 \quad 70]}{\sqrt{2.47^2 + 3.81^2 + 70^2}} = [0.035 \quad 0.054 \quad 0.998]$$

LOS Measurement

- Now that we know how to calculate LOS from a sensor to an object in camera frame coordinates, how does this measurement relate to the object's orbit?
- First, note that if we knew the DCM (i.e. frame rotation) between the camera frame & the TH Frame of the sensor, we could then express the LOS vector in TH coordinates
  - If the sensor were pointing directly above ("up") & the camera were aligned such that the image plane had one dimension oriented North-South & the other oriented East-West → then the camera frame would be aligned with the TH Frame!
  - Call this the sensor's "home" orientation; in order to image an object, if the sensor were then shifted from this home orientation to its desired orientation through a series of known elementary rotations (i.e. rotations about x, y, or z), we could use formulas from last week to calculate the DCM between camera & TH Frames
- We also learned last week how to calculate the DCM between the TH & ECI Frames → thus, we are able to convert a sensor-to-object LOS from camera frame coordinates to ECI coordinates!

LOS Measurement

- Consider again the sketch below → recall that sensor-to-object range is the magnitude of $\bar{R}_{AB}$
- Whereas, sensor-to-object LOS is the direction of $\bar{R}_{AB}$
- Therefore LOS, like range, is intrinsically related to (& driven by) the object's orbital motion



LOS Measurement

- Just as radars cannot discern the direction to an object, optical devices cannot discern the range to the object

- Since range together with direction comprise the full sensor-to-object position vector, it would seem that SSA efforts would be greatly enhanced by co-locating a radar device with an optical device at sites all over the world!

- In reality, this is rarely the case—why?
  - If our goal were to determine only the instantaneous position of an object (& from measurements made at only one time), certainly co-located radar & optical devices would be the best way to achieve this goal
  - But, as we will see next week, the goal of orbit determination instead is to determine an object's orbital trajectory (not just its position at one instant)
  - And in fact, measurements of a single type (range or LOS), if made at multiple times, are quite effective at achieving this goal!