ME 596 Spacecraft Attitude Dynamics and Control

Mission Analysis for Attitude Dynamics and Control

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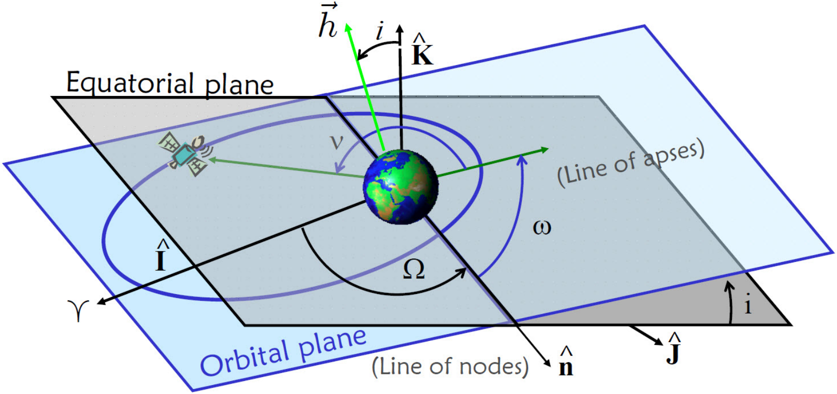
One-Minute Course On Orbital Mechanics



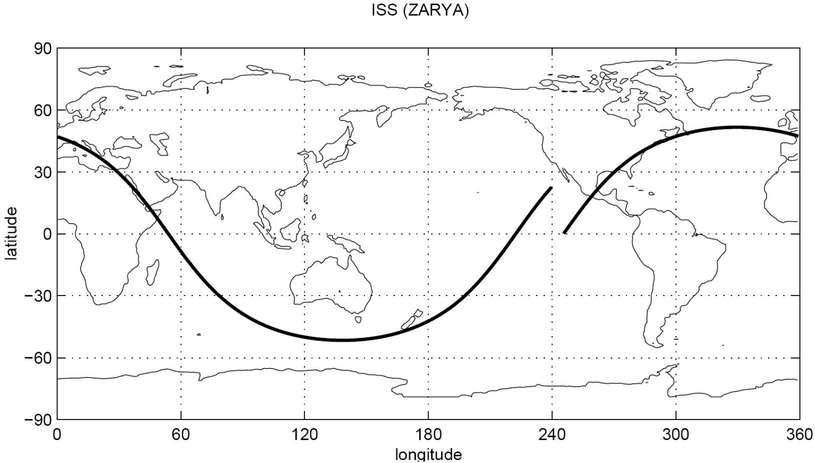
- semimajor axis (size of orbit), a;
- eccentricity (shape of orbit), e;
- inclination, i;
- Longitude of ascending node, Ω ;
- argument of periapsis, ω ;
- true anomaly, ν ;

Orbital Mechanics (continued)

- Semimajor axis a determines the size of the ellipse
- Eccentricity e determines the shape of the ellipse
- Two-body problem
  - a, e, i, Ω, and ω are constant
  - 6th orbital element is the angular measure of satellite motion in the orbit; 2 angles are commonly used:
    – True anomaly, ν
    – Mean anomaly, M
- In reality, these elements are subject to various perturbations
  - Earth oblateness (J_2)
  - atmospheric drag
  - solar radiation pressure
  - gravitational attraction of other bodies

Sub-Satellite Point, Ground Track

As satellite orbits the Earth, the sub-satellite point (SSP) traces a ground track



Algorithm for SSP, Ground Track

- Compute position vector in Earth-Centered Inertial reference frame (ECI)
- Determine Greenwich Sidereal Time (GST) θ_g at epoch, θ_{g0}
- Latitude is $\delta_s - \sin^{-1}(r_3/r)$
- Longitude is $L_s = \tan^{-1}(r_2/r_1) - \theta_{g0}$
- Propagate position vector in “the usual way”
- Propagate GST using $\theta_g = \theta_{g0} + \omega_\oplus(t - t_0)$, where $\omega_\oplus$ is the angular velocity of the Earth

Reference: James R. Wertz and Wiley J. Larson (editors), *Space Mission Analysis and Design*, 3rd Edition, Microcosm Press, El Segundo, California, 1999

Algorithm 2.1

Initialize

orbital elements: a, e, i, ω, Ω, ν_0
Greenwich sidereal time at epoch: θ_{g0}
period: $P = 2\pi\sqrt{a^3/\mu}$
number of steps: N
time step: $\Delta t = P/(N-1)$

for $j = 0$ to $N - 1$

Compute

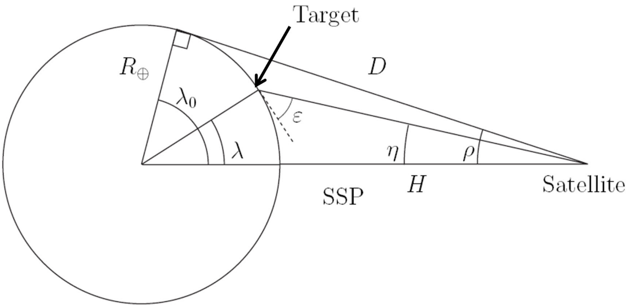
Greenwich sidereal time: $\theta_g = \theta_{g0} + j\omega_{\oplus}\Delta t$
position vector: $\mathbf{r}$ (details on next slide)
latitude: $\delta_s = \sin^{-1}(r_3/r)$
longitude: $L_s = \tan^{-1}(r_2/r_1) - \theta_g$

Computing the position vector

Want position vector in Earth-Centered Earth-Fixed (ECEF) frame:

1. Compute mean anomaly (changes with time)
2. Solve Kepler's Equation for eccentric anomaly
3. Compute position vector in orbital frame
4. Rotate to inertial frame (ECI)
5. Rotate to ECEF frame

Geometry of Earth viewing



Given altitude H, we can state $\sin\rho = \cos\lambda_0 = R_\oplus/(R_\oplus + H)$, and $\rho + \lambda_0 = 90^\circ$

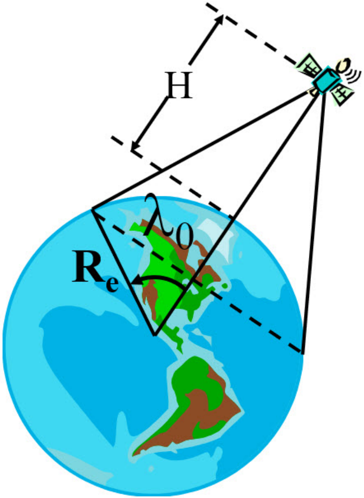
For a target with known position vector, λ is easily computed

$$\cos\lambda = \cos\delta_s \cos\delta_t \cos\Delta L + \sin\delta_s \sin\delta_5$$

Then $\tan\eta = \sin\rho \sin\lambda/(1 - \sin\rho\cos\lambda)$

And $\eta + \lambda + \varepsilon = 90^\circ$ and $D = R_\oplus \sin\lambda/\sin\eta$

Instantaneous Access Area (IAA)



Instantaneous Access Area (IAA) is the area that the spacecraft's instruments or antennas could potentially see at any instant

$$
\begin{aligned}
\text{IAA} &= K_A(1 - \cos\lambda) \\
K_A &- 2.55604187 \times 10^8 \text{ km}^2 \\
\cos\lambda &= \frac{R_\oplus}{R_\oplus + H}
\end{aligned}
$$

Example: Hubble Space Telescope

$$
\begin{aligned}
R_\oplus &= 6378 \text{ km} \\
H &= 560 \text{ km} \\
\cos\lambda &= 0.9193 \Rightarrow \lambda = 23.18^\circ \\
\text{IAA} &= 20,631,067 \text{ km}^2
\end{aligned}
$$

Error Sources

The error sources described in the table below are self-explanatory. Position and orientation errors are relevant to both *pointing* and *mapping*.

Pointing is the act of orienting the spacecraft or a sensor on the spacecraft to a specific target.

Mapping is the act of determining the geographic (or spatial) coordinates of a target in the field of view of the spacecraft or sensor.

Table 2.1: Sources of Pointing and Mapping Errors[1]

Spacecraft Position Errors		
ΔI	In- or along-track	Displacement along the spacecraft's velocity vector
ΔC	Cross-track	Displacement normal to the spacecraft's orbit plane
ΔR_S	Radial	Displacement toward the center of the Earth (nadir)
Sensing Axis Orientation Errors (in polar coordinates about nadir)		
$\Delta \eta$	Elevation	Error in angle from nadir to sensing axis
$\Delta \phi$	Azimuth	Error in rotation of the sensing axis about nadir
Other Errors		
ΔR_T	Target altitude	Uncertainty in the altitude of the observed object
ΔT	Clock error	Uncertainty in the real observation time

Error Budgets

Table 2.2: Pointing and Mapping Error Formulas[1]

Source	Magnitude	Magnitude of Mapping Error (km)	Magnitude of Pointing Error (rad)	Direction of Error
Attitude Errors:				
Azimuth	$\Delta\phi$ (rad)	$\Delta\phi D \sin\eta$	$\Delta\phi \sin\eta$	Azimuthal
Nadir Angle	$\Delta\eta$ (rad)	$\Delta\eta D / \sin\varepsilon$	$\Delta\eta$	Toward nadir
Position Errors:				
In-track	ΔI (km)	$\Delta I (R_T/R_S) \cos H$	$(\Delta I / D) \sin Y_I$	Parallel to ground track
Cross-track	ΔC (km)	$\Delta C (R_T/R_S) \cos G$	$(\Delta C / D) \sin Y_C$	Perpendicular to ground track
Radial	ΔR_S (km)	$\Delta R_S \sin\eta / \sin\varepsilon$	$(\Delta R_S / D) \sin\eta$	Toward nadir
Other Errors:				
Target altitude	ΔR_T (km)	$\Delta R_T / \tan\varepsilon$	—	Toward nadir
S/C Clock	ΔT (s)	$\Delta T V_e \cos\ \text{lat}$	$\Delta T (V_e / D) \cos\ \text{lat} \sin J$	Parallel to Earth's equator

$\sin H = \sin\lambda \sin\phi$

$\sin G = \sin\lambda \cos\phi$

$V_e = 464$ m/s (Earth rotation velocity at equator)

$\cos Y_I = \cos\phi \sin\eta$

$\cos Y_C = \sin\phi \sin\eta$

$\cos J = \cos\phi_E \cos\varepsilon$, where ϕ_E = azimuth relative to East