ME 596 (562) Spacecraft Attitude Dynamics and Control

Rigid Body Dynamics: Center of Mass Refresher

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Center of Mass of a System of Particles

For a system mass particles of mass $m_i, i = 1, ..., n$

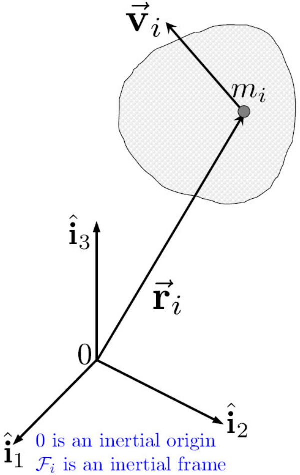
- We can easily calculate total mass

$$m = \sum_{i=1}^{n} m_i$$

- Total mass is sometimes called the "zeroth moment of inertia"
- We can easily determine the location of the center of mass

$$m\mathbf{r}_c = \sum_{i=1}^{n} m_i \mathbf{r}_i$$

where $\mathbf{r}$ is the 3×1 matrix whose components are the components of the vector $\vec{\mathbf{r}}_c$ from O to the mass center c



Center of Mass Calculations

First, let's note that there's a slight abuse of notation here, since we use i in referring to the inertial reference frame and its unit vectors, as well as in referring to the i_{th} of n particles. It should be clear which is which, but if not, ask!

Write $\vec{\mathbf{r}}_i$ as $x_i\,\hat{\mathbf{i}}_1 + y_i\,\hat{\mathbf{i}}_2 + z_i\,\hat{\mathbf{i}}_3$. Thus $\mathbf{r}_i = [x_i \;\; y_i \;\; z_i]^\mathsf{T}$

Write $\vec{\mathbf{r}}_c$ as $x_c\,\hat{\mathbf{i}}_1 + y_c\,\hat{\mathbf{i}}_2 + z_c\,\hat{\mathbf{i}}_3$. 0 Thus $\mathbf{r}_c = [x_c \;\; y_c \;\; z_c]^\mathsf{T}$

Calculate the components of $\vec{\mathbf{r}}_c$ in $\mathcal{F}_i$ as

$$\begin{aligned} x_c &= \frac{1}{m}\sum_{i=1}^{n} m_i x_i \\ y_c &= \frac{1}{m}\sum_{i=1}^{n} m_i y_i \\ z_c &= \frac{1}{m}\sum_{i=1}^{n} m_i z_i \end{aligned}$$