**The University of New Mexico**

**School of Engineering**

**Electrical and Computer Engineering Department**

**ECE 535 Satellite Communications**

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Module # 3: Problems 2.19, 2.23, 2.24, 2.25, 2.27

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2.19: Determine the Julian days for the following dates and times: midnight March 10, 1999; noon, February 23, 2000; 16:30 h, March 1, 2003; 3 P.M., July 4, 2010.

* March 10, 1999 midnight
  + Y = 1999, M = 3, D = 10, UT = 0
  + JD = 3671999 - int(7(1999 + int((3 + 9)/12))/4) + int(2753/9) + 10 + 1721013.5 + 0
  + JD = 734633 - int(72000/4) + 91 + 10 + 1721013.5
  + JD = 734633 - 3500 + 101 + 1721013.5
  + JD = 2451231.5
* February 23, 2000 noon
  + M = 14, Y = 1999, D = 23, UT = 12
  + UT/24 = 0.5
  + JD = 3671999 - int(7(1999 + int((14 + 9)/12))/4) + int(27514/9) + 23 + 1721013.5 + 0.5
  + JD = 734633 - int(72000/4) + 427 + 23 + 1721013.5 + 0.5
  + JD = 734633 - 3500 + 450 + 1721013.5 + 0.5
  + JD = 2451603.5
* March 1, 2003 16:30
  + Y = 2003, M = 3, D = 1, UT = 16.5
  + UT/24 = 0.6875
  + JD = 3672003 - int(7(2003 + int((3 + 9)/12))/4) + int(2753/9) + 1 + 1721013.5 + 0.6875
  + JD = 734101 - int(72004/4) + 91 + 1 + 1721013.5 + 0.6875
  + JD = 734101 - 3507 + 92 + 1721013.5 + 0.6875
  + JD = 2452694.1875
* July 4, 2010 15:00
  + Y = 2010, M = 7, D = 4, UT = 15
  + UT/24 = 0.625
  + JD = 3672010 - int(7(2010 + int((7 + 9)/12))/4) + int(2757/9) + 4 + 1721013.5 + 0.625
  + JD = 737670 - int(72011/4) + 213 + 4 + 1721013.5 + 0.625
  + JD = 737670 - 3517 + 217 + 1721013.5 + 0.625
  + JD = 2455387.625

2.23: The Molnya 3-(25) satellite has the following parameters specified: perigee height 462 km; apogee height 40,850 km; period 736 min; inclination 62.8°. Using an average value of 6371 km for the earth’s radius, calculate (a) the semimajor axis and (b) the eccentricity. (c) Calculate the nominal mean motion n0 . (d) Calculate the mean motion. (e) Using the calculated value for a, calculate the anomalistic period and compare with the specified value. Calculate (f ) the rate of regression of the nodes, and (g) the rate of rotation of the line of apsides.

* semimajor axis
  + a = (rp + ra) / 2
  + a = (6833 + 47221) / 2
  + a = 27027 km
* eccentricity
  + e = (ra - rp) / (ra + rp)
  + e = (47221 - 6833) / (47221 + 6833)
  + e = 40388 / 54054
  + e ~= 0.747
* Calculate the nominal mean motion n0
  + n0 = sqrt(mu / a^3)
  + n0 = sqrt(398600 / 27027^3)
  + n0 = sqrt(398600 / 1.976e13)
  + n0 ~= 4.48e-5 rad/s
* Calculate the mean motion
  + n = 2pi/ T
  + T = 44160 s
  + n = 2pi / 44160
  + n ~= 1.423e-4 rad/s
* calculate the anomalistic period
  + T\_anom = 2pi / n0 = 2pi/ 4.48e-5
  + T\_anom = 140180 s = 2336 min
  + T = 2 \* pi / n
  + T = 2 \* 3.1416 / 1.423e-4
  + T ~= 44160 sec = 736 min
* the rate of regression of the nodes
  + W\_dot = -1.5 \* J2 \* (Re^2 / a^2) \* n0 \* cos(i) / (1 - e^2)^2
  + W\_dot = -1.5 \* 1.08263e-3 \* (6371^2 / 27027^2) \* 4.48e-5 \* cos(62.8) / (1 - 0.747^2)^2
  + W\_dot = -1.5 \* 1.08263e-3 \* 0.0555 \* 4.48e-5 \* 0.460 / 0.195
  + W\_dot = ~-1.1e-7 rad/s
* the rate of rotation of the line of apsides.
  + w\_dot = 0.75 \* J2 \* (Re^2 / a^2) \* n0 \* (5 \* cos(i)^2 - 1) / (1 - e^2)^2
  + w\_dot = 0.75 \* 1.08263e-3 \* 0.0555 \* 4.48e-5 \* (5 \* 0.460^2 - 1) / 0.195
  + w\_dot = 0.75 \* 1.08263e-3 \* 0.0555 \* 4.48e-5 \* 0.06 / 0.195
  + w\_dot ~= 6.9e-9 rad/s

2.24: Repeat the calculations in Prob. 2.23 for an inclination of 63.435°.

The only things that depend on the inclination for this problem are W\_dot and w\_dot. For brevity here are the answers, the same steps as the previous problem were used.

* a = 27027 km
* e ~= 0.747
* n0 ~= 4.48e-5 rad/s
* n ~= 1.423e-4 rad/s
* T\_anom = 2pi / n0 = 2pi/ 4.48e-5
* T\_anom = 140180 s = 2336 min
* T = 2 \* pi / n
* T = 2 \* 3.1416 / 1.423e-4
* T ~= 44160 sec = 736 min
* W\_dot ~= -1.07e-7 rad/s
* w\_dot ~= 0 rad/s

2.25: Determine the orbital condition necessary for the argument of perigee to remain stationary in the orbital plane. The orbit for a satellite under this condition has an eccentricity of 0.001 and a semimajor axis of 27,000 km. At a given epoch the perigee is exactly on the line of Aries. Determine the satellite position relative to this line after a period of 30 days from epoch.

* n0 = sqrt(398600 / 27000^3)
* n0 = sqrt(398600 / 1.9683e13)
* n0 = sqrt(2.025e-8)
* n0 = 4.5e-4 rad/s
* T = 2 \* 3.1416 / 4.5e-4
* T = 13962 s = 233 min = 3.88 h
* cos(i)^2 = 1 / 5
* cos(i)^2 = 0.2
* cos(i) = 0.447
* i = acos(0.447) = 63.435 deg
* angle traveled = 4.5e-4 \* (180 / 3.1416) \* (30 \* 24 \* 3600)
* angle traveled = 0.0258 \* 2,592,000 = 66834 deg
* angle traveled mod 360 = 66834 - 360 \* 185 = 234 deg

2.27: A satellite has an inclination of 90° and an eccentricity of 0.1. At epoch, which corresponds to time of perigee passage, the perigee height is 2643.24 km directly over the north pole. Determine (a) the satellite mean motion. For 1 day after epoch determine (b) the true anomaly, (c) the magnitude of the radius vector to the satellite, and (d) the latitude of the subsatellite point.

import math

mu = 398600          # Earth's gravitational parameter, km^3/s^2

Re = 6371            # Earth radius, km

hp = 2643.24         # Perigee height, km

e = 0.1              # Eccentricity

i\_deg = 90           # Inclination in degrees

t\_day = 1            # Time after epoch in days

rp = Re + hp

a = rp / (1 - e)

n0 = math.sqrt(mu / a\*\*3)

t = t\_day \* 24 \* 3600

M = (n0 \* t) % (2 \* math.pi)

E = M

for \_ in range(100):

    E\_new = E + (M - (E - e \* math.sin(E))) / (1 - e \* math.cos(E))

    if abs(E\_new - E) < 1e-8:

        break

    E = E\_new

tan\_nu\_2 = math.sqrt((1 + e) / (1 - e)) \* math.tan(E / 2)

nu = 2 \* math.atan(tan\_nu\_2)

if nu < 0:

    nu += 2 \* math.pi

r = a \* (1 - e \* math.cos(E))

i = math.radians(i\_deg)

lat = math.asin(math.sin(i) \* math.sin(nu))

lat\_deg = math.degrees(lat)

print(f"Semimajor axis a = {a:.2f} km")

print(f"Mean motion n0 = {n0:.6e} rad/s")

print(f"Mean anomaly M = {M:.6f} rad")

print(f"Eccentric anomaly E = {E:.6f} rad")

print(f"True anomaly nu = {nu:.6f} rad = {math.degrees(nu):.2f} deg")

print(f"Radius vector r = {r:.2f} km")

print(f"Subsatellite latitude = {lat\_deg:.2f} deg")

Semimajor axis a = 10015.82 km

Mean motion n0 = 6.298523e-04 rad/s

Mean anomaly M = 4.153758 rad

Eccentric anomaly E = 4.073483 rad

True anomaly nu = 3.995382 rad = 228.92 deg

Radius vector r = 10613.08 km

Subsatellite latitude = -48.92 deg