

## Homework 4

### Different ways of calculating the value of $\pi$

1. The value of  $\pi$  can be calculated by plotting random points on a graph in the range  $S = (-1, 1) \times (-1, 1)$  and checking whether they are within the unit circle  $C$ . Remember that the unit circle has a radius  $r = 1$  and area  $A_{circle} = \pi r^2 = \pi$ . The area of the square is  $A_{square} = 2 \times 2$ . Therefore the probability that any random point  $P_{x,y}$  in  $S$  is within the circle  $C$  is:

$$P(P_{x,y} \in C \mid P_{x,y} \in S) = \frac{A_{circle}}{A_{square}} = \frac{\pi}{4}$$

This function reads as such: *The probability that a point  $P_{x,y}$  is within the circle  $C$  given that it is within the area of the square  $S$  is equal to the ratio of the areas of  $C$  and  $S$ .* If we can calculate this probability  $P$  then we can invert this equation to find the value of  $\pi$ :

$$\pi = 4P$$

In order to estimate the value of  $P$  and consequently the value of  $\pi$ , we can start by randomly generating a list of  $N$  points in  $S$ , and checking how many are within the circle  $C$  ( $N_C$ ). The estimated probability then becomes:

$$P = \frac{N_C}{N}$$

which increases in accuracy as the number of sample points  $N$  increases. Use the package NumPy (Numerical Python) to generate points in the range  $(-1, 1) \times (-1, 1)$  and check whether they are in the circle. Remember that a point  $P_{x,y} = (x, y)$  will be in a circle of radius  $r$  when:

$$x^2 + y^2 \leq r^2$$

Note: In order to do this problem, you will need to install both `numpy` and `matplotlib`. You can install both of these packages from the command line via:

```
pip install numpy
pip install matplotlib
```