## Loop ordering

One of the optimizations we attempted was to change the ordering of the loops. With a blocked strategy, there are two different places where the loop ordering can be changed: Changing the order in which the blocks are processed, and changing the loop ordering inside of each block multiplication.

**Inside loop ordering.** When multiplying the blocks, we see that there are three index variables: i, j, and k. Changing the ordering of these loops affects the stride and regularity at which we access memory, which can have a significant effect upon performance.

We note that there are 3! = 6 possible orderings of these loops. In order to determine which was fastest, we tested and compared all 6 on the totient node. Note that we kept the outside loop ordering constant for all of these, as we assumed that the inside and outside loop orderings were orthogonal, or at least close enough that the difference was not significant. The results can be found in the following figure:

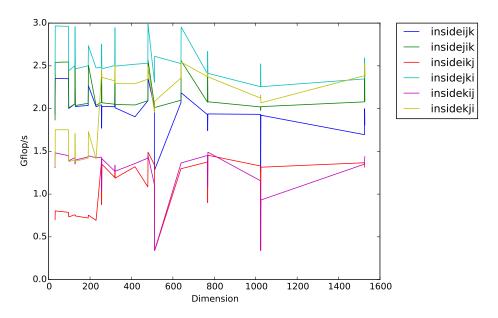


FIGURE 1. Timing results for the different inside loop orderings

Here we see that the loop ordering of j, k, i was clearly fastest, and significantly faster than the slowest loop orderings.

Outside loop ordering. Similarly, the order in which we choose which blocks to multiply can also be changed around. Again, we compared the six different possible orderings, while keeping the inside loop ordering fixed as j, k, i, which was found to be the fastest in the previous section. The timing results are found in the following figure:

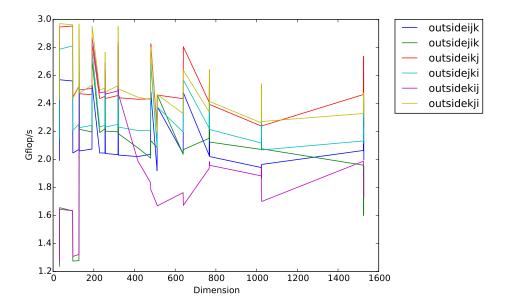


FIGURE 2. Timing results for the different outside loop orderings

Here we see that some loop orderings are definitely faster than others, but this time there is no clear best ordering as both k, j, i and i, k, j are fastest on different size matrices. We chose to go with the outside loop ordering i, k, j

Unfortunately, our results for loop ordering are mostly incompatible with the work done on copy optimization, as certain inside loop orderings are faster because of the stride of the data access, but copy optimization changes this so that data is generally accessed at unit stride.

## FUTURE WORK

There are some further possible attempts at optimization that we would like to try, but have not yet managed to implement in this first stage of the assignment.

One thing we could possibly do is to more rigorously experiment with the block size to determine which is fastest.

Another possible optimization would be to change how we handle the parts of matrices that do not fit cleanly into blocks. Currently those are handled in a naive way, but one alternative would be to pad the matrix with zeros so that all (not almost all) multiplication occurs in consistently sized blocks. If the multiplication of blocks is sufficiently well optimized, then this could be noticeably faster.