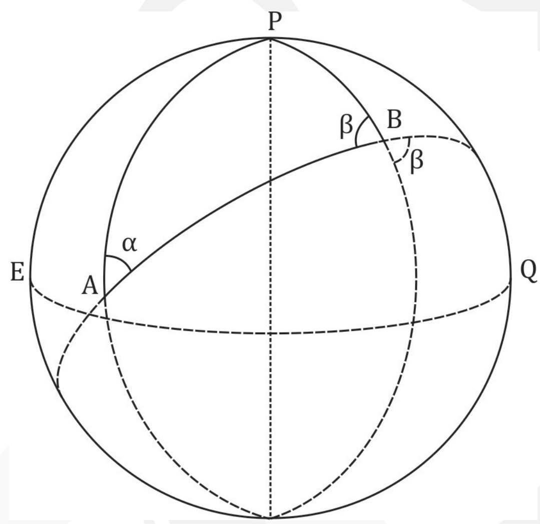
Great Circle Sailing

Great Circle Sailing is used for long ocean passages. For this purpose, the earth is considered a perfect spherical shape; therefore, the shortest distance between two points on its surface is the arc of the great circle containing two points. As the track is the circle, so the course is constantly changing, and the track must be broken down into a series of short rhumb lines at frequent intervals that can be used to sail on the Mercator chart. Doing this, the navigator would use the Gnomonic charts combined with the Mercator charts to draw the sailing track.



EQ	Equator	AB	Great circle track
P	Pole	α	Great circle initial course
PA	Polar distance of A	β	Great circle final course
PB	Polar distance of B		

Procedure to use Gnomonic and Mercator Charts for Great Circle Sailing

1. Plot departure and destination positions on the gnomonic chart; join two positions, since the great circle appears as a straight line on the gnomonic chart.

2. Choose the specific interval meridian along the track where the course will be changed. Then plot the positions of intersection of the track and the meridian chosen on the Mercator chart.

3. Join all the plotted positions on the Mercator chart by a series of rhumb lines; the course and distance between each position can be solved by the plane sailing method.

As the great circle track line is plotted on the gnomonic chart, the vertex and the chosen intermediate positions can be read off directly from the chart. However, this is not as accurate as the calculation which will be shown later in this section.

Great circle distance

$$\cos AB = \cos PA \cos PB + \sin PA \sin PB \cos P$$
$$= \sin Lat._A \sin Lat._B + \cos Lat._A \cos Lat._B \cos D.long._{AB}$$

$$\mathbf{D_{AB} = \cos^{-1}\left(\sin Lat._A \sin Lat._B + \cos Lat._A \cos Lat._B \cos D.Long._{AB}\right)}$$

Initial course

$$\cos\alpha = \frac{\cos PB - \cos PA \cos AB}{\sin PA \sin AB} = \frac{\sin Lat_B - \sin Lat_A \cos D_{AB}}{\cos Lat_A \sin D_{AB}}$$

$$\boldsymbol{\alpha = \cos^{-1}\left(\frac{\sin Lat._B - \sin Lat._A \cos D_{AB}}{\cos Lat._A \sin D_{AB}}\right)}$$

The initial course also can be calculated by following formulas:

$$\boldsymbol{\alpha = \sin^{-1}\left(\frac{\cos Lat._B \times \sin D.Long._{AB}}{\sin D_{AB}}\right)}$$

or

$$\boldsymbol{\alpha = \tan^{-1}\left(\frac{\sin D.\ Long._{AB}}{\cos Lat._A \tan Lat._B - \sin Lat._A \cos D.\ Long._{AB}}\right)}$$

Final course

$$\cos\beta = \frac{\cos PA - \cos PB \cos AB}{\sin PB \sin AB} = \frac{\sin Lat_A - \sin Lat_B \cos D_{AB}}{\cos Lat_B \sin D_{AB}}$$

$$\mathbf{B = arccos\left(\frac{\sin Lat_A - \sin Lat_B \cos D_{AB}}{\cos Lat_B \sin D_{AB}}\right)}$$

The final course also can be calculated by following formulas:

$$\boldsymbol{\beta = \sin^{-1}\left(\frac{\cos Lat._A \times \sin D.Long._{AB}}{\sin D_{AB}}\right)}$$

or

$$\boldsymbol{\beta = \tan^{-1}\left(\frac{\sin D.\ Long._{AB}}{\cos Lat._B \tan Lat._A - \sin Lat._B \cos D.\ Long._{AB}}\right)}$$

The great circle calculations of initial and final courses result in quadrantal notation as cardinal compass. Corrected quadrant must be named in order to avoid mistakes when converting into three-figure notation (0°-360°).

Rules to name Initial and Final Courses of a Great Circle

The initial course always has same name as the initial latitude and east or west direction of the course. The final course always has the opposite name from final latitude unless initial position and final position are in different hemispheres, when the final will have same name as final latitude and east or west direction of the course.

Summary

Direction of course: **Easterly**

Final Latitude \ Initial Latitude	**North**	**South**
North	Initial course: **NE** Final course: **SE**	Initial course: **SE** Final course: **NE**
South	Initial course: **NE** Final course: **SE**	Initial course: **SE** Final course: **NE**

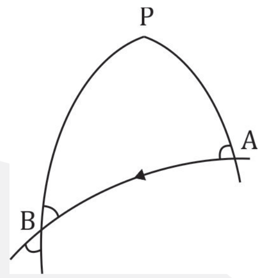
Direction of course: **Westerly**

Final Latitude \ Initial Latitude	**North**	**South**
North	Initial course: **NW** Final course: **SW**	Initial course: **SW** Final course: **NW**
South	Initial course: **NW** Final course: **SW**	Initial course: **SW** Final course: **NW**

Initial and final courses can also be found by using ABC tables or ABC computations, just like solving the azimuth of a celestial body by considering one position as the observer's position and another as position of the celestial body. The azimuth would be the initial or final course, depending which is designated. For example, in order to find initial course, the initial position is considered as the observer's position, and the final position as the celestial position. Conversely, for finding the final course; the final position is considered as the observer's position, and initial position as the celestial position. The course would be named as "C", and direction is the hour angle which is D. Long. between two positions.

Example 1 Find the distance, initial course and final course:

From A: 56°20′N 008°12′W
To B: 52°12′N 057°10′W



$$\text{Lat.}_A = 56°20'\text{N} \qquad \text{Lat.}_B = 52°12'\text{N}$$

$$\text{D.Long.}_{AB} = 57°10' - 8°12' = 48°58'(\text{W})$$

Distance

$$D_{AB} = \cos^{-1}\left(\sin \text{Lat.}_A \sin \text{Lat.}_B + \cos \text{Lat.}_A \cos \text{Lat.}_B \cos \text{D. Long.}_{AB}\right)$$
$$= \cos^{-1}\left(\sin 56°20' \sin 52°12' + \cos 56°20' \cos 52°12' \cos 48°58'\right)$$
$$= 28°16.5'$$

Distance = 1696.5 miles

Initial course

$$\alpha = \cos^{-1}\left(\frac{\sin \text{Lat.}_B - \sin \text{Lat.}_A \cos D_{AB}}{\cos \text{Lat.}_A \sin D_{AB}}\right)$$
$$= \cos^{-1}\left(\frac{\sin 52°12' - \sin 56°20' \cos 28°16.5'}{\cos 56°20' \sin 28°16.5'}\right) = 77°25.4' = 77.4°$$

Initial course = N77.4°W = 282.6°T

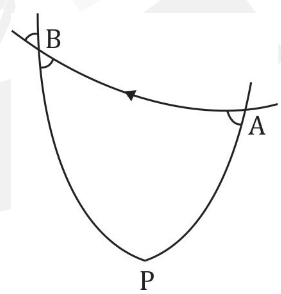
Final course

$$\beta = \cos^{-1}\left(\frac{\sin \text{Lat}_A - \sin \text{Lat}_B \cos D_{AB}}{\cos \text{Lat}_B \sin D_{AB}}\right)$$
$$= \cos^{-1}\left(\frac{\sin 56°20' - \sin 52°12' \cos 28°16.5'}{\cos 52°12' \sin 28°16.5'}\right) = 61°58.7' = 62.0°$$

Final course = S62.0°W = 242°T

Example 2 Find the distance, initial course and final course of great circle sailing:

From A: 33°22′S 113°08′E
To B: 10°51′S 049°16′E



$$\text{Lat.}_A = 33°22'\text{S} \qquad \text{Lat.}_B = 10°51'\text{S}$$

$$\text{D. Long.}_{AB} = 113°08' - 49°16' = 63°52'(\text{W})$$

Distance

$$D_{AB} = \cos^{-1}\left(\sin Lat._A \sin Lat._B + \cos Lat._A \cos Lat._B \cos D.\ Long._{AB}\right)$$
$$= \cos^{-1}\left(\sin 33°22' \sin 10°51' + \cos 33°22' \cos 10°51' \cos 63°52'\right)$$
$$= 62°18.1'$$

Distance = 3738.1 miles

Initial course

$$\alpha = \cos^{-1}\left(\frac{\sin Lat._B - \sin Lat._A \cos D_{AB}}{\cos Lat._A \sin D_{AB}}\right)$$
$$= \cos^{-1}\left(\frac{\sin 10°51' - \sin 33°22' \cos 62°18.1'}{\cos 33°22' \sin 62°18.1'}\right) = 95°13.8' = 95.2°$$

Initial course = S95.2°W = 275.2°T

Final course

$$\beta = \cos^{-1}\left(\frac{\sin Lat_A - \sin Lat_B \cos D_{AB}}{\cos Lat_B \sin D_{AB}}\right)$$
$$= \cos^{-1}\left(\frac{\sin 33°22' - \sin 10°51' \cos 62°18.1'}{\cos 10°51' \sin 62°18.1'}\right) = 57°52.1' = 57.9°$$

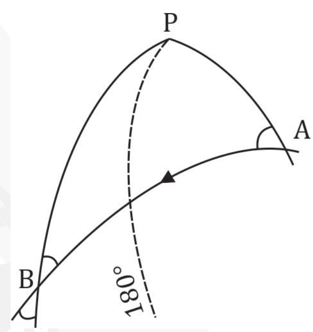
Final course = N57.9°W = 302.1°T

Example 3

Find the distance and initial course of great circle sailing from Vancouver to Guam:

Crossing 180° meridian

Vancouver (A): 49°12′N 122°50′W
Guam (B): 13°30′N 145°15′E



$Lat._A = 49°12'N$ $Lat._B = 13°30'N$

$$D.Long._{AB} = 360° - (122°50' + 145°15')$$
$$= 91°55'(W)$$

Distance

$$D_{AB} = \cos^{-1}\left(\sin Lat._A \sin Lat._B + \cos Lat._A \cos Lat._B \cos D.\ Long._{AB}\right)$$
$$= \cos^{-1}\left(\sin 49°12' \sin 13°30' + \cos 49°12' \cos 13°30' \cos 91°55'\right)$$
$$= 81°03.4'$$

Distance = 4863.4 miles

Initial course

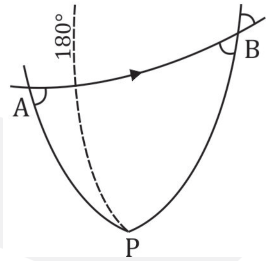
$$\alpha = \cos^{-1}\left(\frac{\sin Lat._B - \sin Lat._A \cos D_{AB}}{\cos Lat._A \sin D_{AB}}\right)$$
$$= \cos^{-1}\left(\frac{\sin 13°30' - \sin 49°12' \cos 81°03.4'}{\cos 49°12' \sin 81°03.4'}\right) = 79°40.1' = 79.7°$$

Initial course = N79.7°W = 280.3°T

Example 4 Find the distance and initial course of great circle sailing from Bluff Harbour to Easter Island:

Crossing 180° meridian

Bluff Harbour (A): 46°20′S 169°10′E
Easter Island (B): 26°25′S 105°15′W



$$\text{Lat.}_A = 46°20'\text{S} \qquad \text{Lat.}_B = 26°25'\text{S}$$

$$\begin{aligned}\text{D.Long.}_{AB} &= 360° - (169°10' + 105°15') \\ &= 85°35'(\text{E})\end{aligned}$$

Distance

$$\begin{aligned}D_{AB} &= \cos^{-1}\left(\sin\text{Lat.}_A \sin\text{Lat.}_B + \cos\text{Lat.}_A \cos\text{Lat.}_B \cos\text{D. Long.}_{AB}\right) \\ &= \cos^{-1}\left(\sin 46°20' \sin 26°25' + \cos 46°20' \cos 26°25' \cos 85°35'\right) \\ &= 68°19.1'\end{aligned}$$

$$\text{Distance} = 4099.1\,\text{miles}$$

Initial course

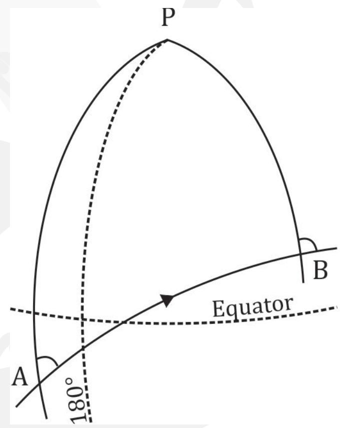
$$\begin{aligned}\alpha &= \cos^{-1}\left(\frac{\sin\text{Lat.}_B - \sin\text{Lat.}_A \cos D_{AB}}{\cos\text{Lat.}_A \sin D_{AB}}\right) \\ &= \cos^{-1}\left(\frac{\sin 26°25' - \sin 46°20' \cos 68°19.1'}{\cos 46°20' \sin 68°19.1'}\right) = 73°55.5' = 73.9°\end{aligned}$$

$$\text{Initial course} = \text{S}73.9°\text{E} = 106.1°\text{T}$$

Example 5 Find the distance, initial course and final course of great circle sailing:

Crossing the Equator and 180° meridian

From A: 17°S 170°E
To B: 22°N 110°W



$$\text{Lat.}_A = 17°\text{S} \qquad \text{Lat.}_B = 22°\text{N}$$

$$\begin{aligned}\text{D.Long.}_{AB} &= 360° - (170° + 110°) \\ &= 80°(\text{E})\end{aligned}$$

Since the name of the latitude of the destination is contrary to the latitude of departure, then the latitude of destination is treated as a negative quantity.

Distance

$$\begin{aligned}D_{AB} &= \cos^{-1}\left(\sin\text{Lat.}_A \sin\text{Lat.}_B + \cos\text{Lat.}_A \cos\text{Lat.}_B \cos\text{D. Long.}_{AB}\right) \\ &= \cos^{-1}\left(\sin 17° \sin(-22°) + \cos 17° \cos(-22°) \cos 80°\right) \\ &= 87°27.2'\end{aligned}$$

$$\text{Distance} = 5247.2\,\text{miles}$$

Initial course

$$\alpha = \cos^{-1}\left(\frac{\sin \text{Lat.}_B - \sin \text{Lat.}_A \cos D_{AB}}{\cos \text{Lat.}_A \sin D_{AB}}\right)$$

$$= \cos^{-1}\left[\frac{\sin(-22^\circ) - \sin 17^\circ \cos 87^\circ 27.2'}{\cos 17^\circ \sin 87^\circ 27.2'}\right]$$

$$= 113^\circ 56.1' = 113.9^\circ$$

Initial course $= \text{S}113.9^\circ\text{E} = 066.1^\circ\text{T}$

Final course

$$\beta = \cos^{-1}\left(\frac{\sin \text{Lat}_A - \sin \text{Lat}_B \cos D_{AB}}{\cos \text{Lat}_B \sin D_{AB}}\right)$$

$$= \cos^{-1}\left[\frac{\sin 17^\circ - \sin(-22^\circ)\cos 87^\circ 27.2'}{\cos(-22^\circ)\sin 87^\circ 27.2'}\right]$$

$$= 70^\circ 30.7' = 70.5^\circ$$

Final course $= \text{N}70.5^\circ\text{E} = 070.5^\circ\text{T}$

Example 6 Compare rhumb line distance with great circle distance:

Dunedin (A): 45°44′S 171°15′E
Panama (B): 7°30′N 79°21′W

Rhumb line sailing

Lat._A	45°44′S	M.P._A	3075.80	Long._A	171°15′E
Lat._B	7°30′N	M.P._B	448.24	Long._B	79°21′W
D. Lat.	3194′(N)	D.M.P.	3524.04	D. Long.	6564′(E)

$$C = \tan^{-1}\left(\frac{\text{D.Long.}}{\text{D.M.P.}}\right) = \tan^{-1}\left(\frac{6564}{3524.04}\right) = 61.8^\circ$$

$$\text{Distance} = \frac{\text{D. Lat.}}{\cos C} = \frac{3194'}{\cos 61.8^\circ} = 6159.1\,\text{miles}$$

Great circle sailing

$\text{Lat.}_A = 45^\circ 44'\text{S}$ $\text{Lat.}_B = 07^\circ 30'\text{N}$

$$\text{D.Long.}_{AB} = 360^\circ - (171^\circ 15' + 79^\circ 21') = 109^\circ 24'$$

$$D_{AB} = \cos^{-1}(\sin \text{Lat.}_A \sin \text{Lat.}_B + \cos \text{Lat.}_A \cos \text{Lat.}_B \cos \text{D. Long.}_{AB})$$

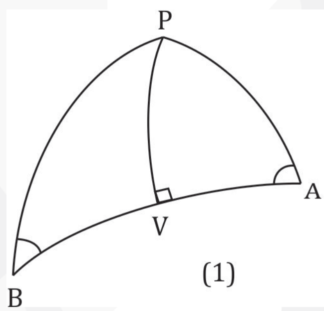
$$= \cos^{-1}(\sin 45^\circ 44' \sin 7^\circ 30' + \cos 45^\circ 44' \cos 7^\circ 30' \cos 109^\circ 24')$$

$$= 108^\circ 51.9'$$

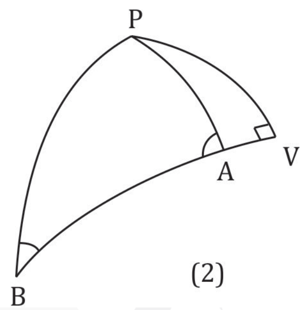
Distance $= 6531.9\,\text{miles}$

Difference $= 6531.9 - 6159.1 = 372.8\,\text{miles}$

Vertex The arc of a great circle will always curve towards the nearest pole and away from the equator. The vertex is the point on a great circle that is closest to the pole; by knowing the latitude of the vertex, if it is too high (which is usually associates with ice, fog, cold and severe weather), the navigator might have to modify the passage plan for a safer voyage. There are two vertices on a great circle, 180° apart; the nearer vertex is usually the chosen one for navigational calculation. The vertex's latitude is always numerically equal to or greater than the latitude of any other point on the great circle, including the latitude of departure and destination. At the vertex, the great circle is running in a direction of 090°/270°. Knowing the position of the vertex also helps in calculating the position of any intermediate position on the track of a great circle. In the spherical triangle APB, if angles A and B are less than 90°; the vertex will lie inside the triangle between A and B, as shown in the figure (1) below, and the ship's track passes through the vertex. If either A or B is greater than 90°, the vertex will lie outside the spherical triangle and on the side of the angle which is greater than 90°, as shown in the figure (2) below, and the ship's track does not pass through the vertex.

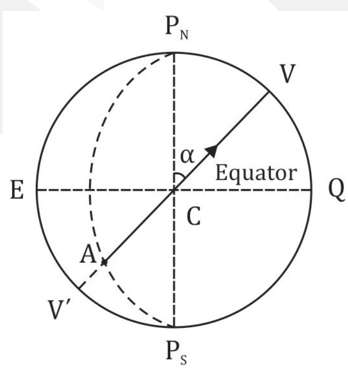


(1)



(2)

- The vertex is 90° from the point where the track of the great circle cuts the equator.
- The course where the great circle crosses the equator is equal to the co. latitude of the vertex.



A	Initial position
AV	Great circle track
V	Vertex (upper branch)
V′	Vertex (lower branch)
QV	Latitude of vertex
EQ	Equator
C	Intersection of GC track and equator
α	GC course at equator
P_NV	Co. Latitude of vertex V

$P_NV = \alpha \qquad CV = 90°$

The position of the vertex, and the distance from departure point to vertex, can be calculated by using Napier's Rules in the right angle triangle PVA:

For all formulas used for great circle vertex calculations, if the name of the latitude of any position, including the departure and destination, is contrary to the latitude of the vertex, then the latitude of those having a contrary name to the latitude of vertex is treated as a negative quantity.

Latitude of the vertex (Lat.V)

$$\sin PV = \cos(\text{co}\,A)\cos(\text{co}\,PA) = \sin A \sin PA$$

$$\Rightarrow \cos \text{Lat.}_V = \sin A \cos \text{Lat.}_A$$

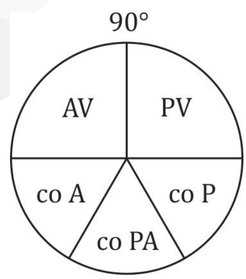
$$\mathbf{Lat._V = \cos^{-1}\left(\sin\alpha \cos Lat._A\right)} \quad \text{or} \quad \mathbf{Lat._V = \cos^{-1}\left(\sin\beta \cos Lat._B\right)}$$

Difference of longitude between departure and vertex (D. Long.$_{AV}$)

$$\sin(\text{co}\,P) = \tan PV \tan(\text{co}\,PA)$$

$$\cos P = \frac{\cot PA}{\cot PV} \quad \therefore \quad \cos \text{D.Long.}_{AV} = \frac{\tan L_A}{\tan L_V}$$

$$\mathbf{D.Long._{AV} = \cos^{-1}\left(\frac{\tan Lat._A}{\tan Lat._V}\right)}$$



D. Long. can also be found by the formula:

$$\sin(\text{co}\,A) = \cos PV \cos(\text{co}\,P)$$

$$\cos A = \cos PV \sin P$$

$$\therefore \quad \sin P = \frac{\cos A}{\cos PV} \quad \Rightarrow \quad \sin \text{D.Long.}_{AV} = \frac{\cos A}{\sin \text{Lat.}_V}$$

$$\mathbf{D.Long._{AV} = \sin^{-1}\left(\frac{\cos\alpha}{\sin Lat._V}\right)}$$

When using above formula, if the latitude of departure is contrary to the vertex, then the result has to be subtracted by 180° to get corrected D. Long.

Difference of longitude between destination and vertex (D. Long.$_{BV}$)

$$\mathbf{D.Long._{BV} = \cos^{-1}\left(\frac{\tan Lat._B}{\tan Lat._V}\right)} \text{ or } \mathbf{D.Long._{BV} = \sin^{-1}\left(\frac{\cos\beta}{\sin Lat._V}\right)}$$

Similarly, when using the above formula, if the latitude of destination is contrary to vertex, then the result has to be subtracted by 180° to get corrected D. Long.

Distance from departure position to the vertex (D$_{AV}$)

$$\sin AV = \cos(\text{co}\,PA)\cos(\text{co}\,P) = \sin PA \sin P$$

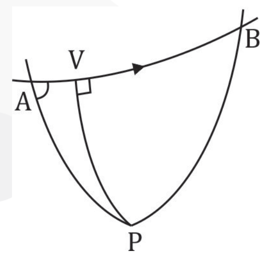
$$\therefore \quad \sin D_{AV} = \cos \text{Lat.}_A \sin \text{D.Long.}_{AV}$$

$$\mathbf{D_{AV} = \sin^{-1}\left(\cos Lat._A \sin D.\,Long._{AV}\right)} \qquad \text{or}$$

$$\sin(coP) = \cos AV \cos(co\,A)$$
$$\cos P = \cos AV \sin A$$
$$\therefore \quad \cos AV = \frac{\cos P}{\sin A} \qquad\qquad \mathbf{D_{AV} = \cos^{-1}\left(\frac{\cos D.\,Long._{AV}}{\sin\alpha}\right)}$$

Example 7 Find the great circle distance, the initial course and the position of the vertex, and also the distance from departure position to the vertex:

$$\text{From } A:\ 34°55'S \quad 56°10'W$$
$$\text{To } B:\ 33°55'S \quad 18°25'E$$



$$Lat._A = 34°55'S \qquad Lat._B = 33°55'S$$

$$D.\,Long._{AB} = 056°10' + 18°25' = 74°35'(E)$$

Distance

$$D_{AB} = \cos^{-1}\left(\sin Lat._A \sin Lat._B + \cos Lat._A \cos Lat._B \cos D.\,Long._{AB}\right)$$
$$= \cos^{-1}\left(\sin 34°55' \sin 33°55' + \cos 34°55' \cos 33°55' \cos 74°35'\right)$$
$$= 59°58.9'$$

Distance = 3598.9 miles

Initial course

$$\alpha = \cos^{-1}\left(\frac{\sin Lat._B - \sin Lat._A \cos D_{AB}}{\cos Lat._A \sin D_{AB}}\right)$$
$$= \cos^{-1}\left(\frac{\sin 33°55' - \sin 34°55' \cos 59°58.9'}{\cos 34°55' \sin 59°58.9'}\right) = 67°30.4' = 67.5°$$

Initial course = S67.5°E = 112.5°T

Vertex's position

$$Lat._V = \cos^{-1}\left(\sin\alpha \cos lat._A\right) = \cos^{-1}\left(\sin 67°30.4' \cos 34°55'\right)$$
$$Lat._V = 40°44.8'S$$
$$D.\,Long._V = \cos^{-1}\left(\frac{\tan Lat_A}{\tan Lat._V}\right) = \cos^{-1}\left(\frac{\tan 34°55'}{\tan 40°44.8'}\right) = 35°53.0'(E)$$

$$Long._V = 56°10'W - 35°53.0'(E) = 20°17'W$$

$$\text{Vertex's Position:} \quad Lat._V = 44°44.8'S \quad Long._V = 20°17.0'W$$

Distance from departure position to the vertex

$$D_{AV} = \sin^{-1}\left(\cos \text{Lat.}_A \sin \text{D.Long.}_{AV}\right) = \sin^{-1}\left(\cos 34°55' \sin 35°53.0'\right)$$
$$= 28°43.6'$$

Distance = 1723.6 miles

Practical Method for Great Circle Sailing

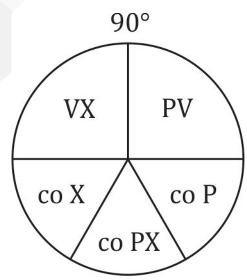
It is not practical for a ship to sail along a great circle track, because she has to change course constantly in order to follow it. Therefore, the great circle is divided into equal segments by longitudes, and is then made up of a series of rhumb lines. The rhumb lines can be plotted on the Mercator chart and followed by the ship. The rule of thumb for selecting the equal interval D. Long. from the vertex is:

"Short legs in lower latitudes, long legs in higher latitudes"

By using Napier's rules for the spherical right-angle triangle PVX:

Latitude at the meridian cuts the great circle track

$$\sin(\text{co}\,P) = \tan PV \tan(\text{co}\,PX)$$
$$\cos P = \tan PV \cot PX$$
$$\cot PX = \frac{\cos P}{\tan PV} \quad \Rightarrow \quad \tan PX = \cos P \cot PV$$
$$\therefore \tan \text{Lat.}_X = \cos \text{D.Long.}_{VX} \tan \text{Lat.}_V$$



$$\mathbf{Lat._X = \tan^{-1}\left(\cos D.Long._{VX} \tan Lat._V\right)}$$

Course at the meridian cuts the great circle track

$$\sin(\text{co}\,X) = \cos PV \cos(\text{co}\,P)$$
$$\cos X = \cos PV \sin P = \sin \text{Lat.}_V \sin \text{D.Long.}_{VX}$$
$$\mathbf{X = \cos^{-1}\left(\sin Lat._V \sin D.Long._{VX}\right)}$$

The longitude can also be selected as the equal interval distance on the great circle from the vertex, and the position can be calculated by using Napier's rules:

$$\sin(\text{co}\,PX) = \cos PV \cos VX$$
$$\cos PX = \cos PV \cos VX \qquad \therefore \sin \text{Lat.}_X = \sin \text{Lat.}_V \cos D_{VX}$$

$$\mathbf{Lat._X = \sin^{-1}\left(\sin Lat._V \cos D_{VX}\right)}$$

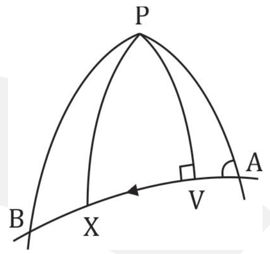
$$\sin(\text{co}\,P) = \tan(\text{co}\,PX) \tan PV$$
$$\cos P = \cot PX \tan PV \qquad \therefore \cos \text{D.Long.}_{VX} = \frac{\tan \text{Lat.}_X}{\tan \text{Lat.}_V}$$

$$\mathbf{D.\ Long._{VX} = \cos^{-1}\left(\frac{\tan Lat_X}{\tan Lat_V}\right)}$$

Example 8 Find: the great circle distance; initial course; final course; position of the vertex; and the latitudes that cut intermediate meridians at 5° intervals, starting from the departure meridian:

From A: 51°25′N 9°30′W

To B: 46°00′N 49°00′W



$$\text{Lat.}_A = 51°25'N \qquad \text{Lat.}_B = 46°00'N$$

$$\begin{aligned}\text{D.Long.}_{AB} &= 49°00'W - 9°30'W \\ &= 39°30'(W)\end{aligned}$$

Distance

$$\begin{aligned}D_{AB} &= \cos^{-1}\left(\sin \text{Lat.}_A \sin \text{Lat.}_B + \cos \text{Lat.}_A \cos \text{Lat.}_B \cos \text{D. Long.}_{AB}\right) \\ &= \cos^{-1}\left(\sin 51°25' \sin 46°00' + \cos 51°25' \cos 46°00' \cos 39°30'\right) \\ &= 26°17.1'\end{aligned}$$

$$\text{Distance} = 1577.1\,\text{miles}$$

Initial course

$$\alpha = \cos^{-1}\left(\frac{\sin \text{Lat.}_B - \sin \text{Lat.}_A \cos D_{AB}}{\cos \text{Lat.}_A \sin D_{AB}}\right)$$

$$= \cos^{-1}\left(\frac{\sin 46°00' - \sin 51°25' \cos 39°30'}{\cos 51°25' \sin 39°30'}\right) = 86°09.9' = 86.2°$$

$$\text{Initial course} = N86.2°W = 273.8°T$$

Final course

$$\beta = \cos^{-1}\left(\frac{\sin \text{Lat}_A - \sin \text{Lat}_B \cos D_{AB}}{\cos \text{Lat}_B \sin D_{AB}}\right)$$

$$= \cos^{-1}\left(\frac{\sin 51°25' - \sin 46°00' \cos 39°30'}{\cos 46°00' \sin 39°30'}\right) = 63°36.5' = 63.6°$$

$$\text{Final course} = S63.6°W = 243.6°T$$

Vertex's Position

$$\begin{aligned}\text{Lat.}_V &= \cos^{-1}\left(\sin \alpha \cos \text{lat.}_A\right) \\ &= \cos^{-1}\left(\sin 86°09.9' \cos 51°25'\right) = 51°31.1'N\end{aligned}$$

$$\text{D. Long.}_{AV} = \cos^{-1}\left(\frac{\tan \text{Lat}_A}{\tan \text{Lat}_V}\right) = \left(\frac{\tan 51°25'}{\tan 51°31.1'}\right) = 4°54.0'(W)$$

$$\text{Long.}_V = 9°30.0'W + 4°54.0'(W) = 14°24.0'W$$

$$\text{Vertex's Position:} \quad \text{Lat.}_V = 51°31.1'N \quad \text{Long.}_V = 14°24.0'W$$

Waypoints For intervals of D. Long. of 5° from departure position (A)

$$\text{Long.}_A = 9°30.0'W \qquad \text{Long.}_V = 14°24.0'W$$

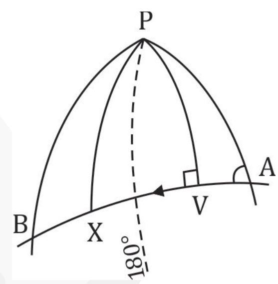
Latitude of any x position can be calculated by formula:

$$\text{Lat.}_X = \tan^{-1}\left(\cos \text{D.Long.}_{VX} \tan \text{Lat.}_V\right)$$

Position	Longitude (Long._X)	D. Long._{VX} ($\text{Long.}_X - \text{Long.}_V$)	Latitude (Lat._X)
X_1	14°30′W	0°06′	51°31.1′N
X_2	19°30′W	5°06′	51°24.5′N
X_3	24°30′W	10°06′	51°04.9′N
X_4	29°30′W	15°06′	50°32.1′N
X_5	34°30′W	20°06′	49°45.2′N
X_6	39°30′W	25°06′	48°43.4′N
X_7	44°30′W	30°06′	47°25.4′N

Example 9 Find: the great circle distance; initial course; position of the vertex; distance from departure position to the vertex; and the positions where the meridians of 140° W, 160° W, 180° and 160° E cut the track on the great circle:

From A: 48°24′N 124°44′W
To B: 34°50′N 139°50′E



$\text{Lat.}_A = 48°24'\text{N}$ $\text{Lat.}_B = 34°50'\text{N}$

$$\begin{aligned}\text{D.Long.}_{AB} &= 360° - (124°44' + 139°50') \\ &= 95°26'(\text{W})\end{aligned}$$

Distance

$$\begin{aligned}D_{AB} &= \cos^{-1}\left(\sin \text{Lat.}_A \sin \text{Lat.}_B + \cos \text{Lat.}_A \cos \text{Lat.}_B \cos \text{D. Long.}_{AB}\right) \\ &= \cos^{-1}\left(\sin 48°24' \sin 34°50' + \cos 48°24' \cos 34°50' \cos 95°26'\right) \\ &= 57°56.6'\end{aligned}$$

Distance = 4076.6 miles

Initial course

$$\begin{aligned}\alpha &= \cos^{-1}\left(\frac{\sin \text{Lat.}_B - \sin \text{Lat.}_A \cos D_{AB}}{\cos \text{Lat.}_A \sin D_{AB}}\right) \\ &= \cos^{-1}\left(\frac{\sin 34°50' - \sin 48°24' \cos 67°56.6'}{\cos 48°24' \sin 67°56.6'}\right) \\ &= 61°50.6' = 61.8°\end{aligned}$$

Initial course = N61.8° W = 298.2° T

Vertex's Position

$$\text{Lat.}_V = \cos^{-1}\left(\sin\alpha \cos \text{Lat.}_A\right) = \cos^{-1}\left(\sin 61°50.6' \cos 48°24'\right) = 54°10.3'\text{N}$$

$$\text{D. Long.}_{AV} = \cos^{-1}\left(\frac{\tan \text{Lat}_A}{\tan \text{Lat}_V}\right) = \left(\frac{\tan 48°24'}{\tan 54°10.3'}\right) = 35°35.6'(\text{W})$$

$$\text{Long.}_V = 124°44'\text{W} + 35°35.6' = 160°19.6'\text{W}$$

$$\text{Vertex's Position:}\quad \text{Lat.}_V = 54°10.3'\text{N} \quad \text{Long.}_V = 160°19.6'\text{W}$$

Distance from departure position to the vertex

$$D_{AV} = \sin^{-1}\left(\cos \text{Lat.}_A \sin \text{D.Long.}_{AV}\right) = \sin^{-1}\left(\cos 48°24' \sin 35°35.6'\right) = 22°43.9'$$

$$\text{Distance} = 1363.9 \text{ miles}$$

Position where the meridian cut the great circle track

$$\text{Lat.}_X = \tan^{-1}\left(\cos \text{D. Long.}_{VX} \tan \text{Lat.}_V\right)$$

Position	Longitude (Long._X)	D. Long._{VX} ($\text{Long.}_X - \text{Long.}_V$)	Latitude (Lat._X)
X_1	140°W	20°19.6′	52°24.4′N
X_2	160°W	0°19.6′	54°10.3′N
X_3	180°W	19°40.4′	52°31.3′N
X_4	160°E	39°40.4′	46°50.0′N

Example 10

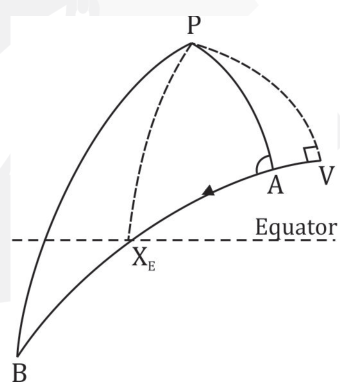
Crossing Equator

Find the great circle distance; initial course; final course; longitude where the great circle crosses the equator; nearest vertex; and the waypoints at 10° intervals from 130°W to 170°W:

$$\text{From}\quad A: \ 30°\text{N} \quad 120°\text{W}$$
$$\text{To}\quad B: \ 20°\text{S} \quad 173°\text{W}$$

$$\text{Lat.}_A = 30°\text{N} \qquad \text{Lat.}_B = 20°\text{S}$$

$$\text{D.Long.}_{AB} = 173° - 120° = 53°\ (\text{W})$$



Since the name of the latitude of the destination is contrary to the latitude of departure, then the latitude of destination is treated as a negative quantity.

Distance

$$D_{AB} = \cos^{-1}\left(\sin Lat._A \sin Lat._B + \cos Lat._A \cos Lat._B \cos D.\,Long._{AB}\right)$$

$$= \cos^{-1}\left[\sin 30^\circ \sin\left(-20^\circ\right) + \cos 30^\circ \cos\left(-20^\circ\right)\cos 53^\circ\right]$$

$$= 71^\circ 24.8'$$

$$\text{Distance} = 4284.8\,\text{miles}$$

Initial course

$$\alpha = \cos^{-1}\left(\frac{\sin Lat._B - \sin Lat._A \cos D_{AB}}{\cos Lat._A \sin D_{AB}}\right)$$

$$= \cos^{-1}\left[\frac{\sin\left(-20^\circ\right) - \sin 30^\circ \cos 71^\circ 24.8'}{\cos 30^\circ \sin 71^\circ 24.8'}\right]$$

$$= 127^\circ 38.9' = 127.6^\circ$$

$$\text{Initial course} = N127.6^\circ W = 232.4^\circ T$$

Final course

$$\beta = \cos^{-1}\left(\frac{\sin Lat_A - \sin Lat_B \cos D_{AB}}{\cos Lat_B \sin D_{AB}}\right)$$

$$= \cos^{-1}\left[\frac{\sin 30^\circ - \sin\left(-20^\circ\right)\cos 71^\circ 24.8'}{\cos\left(-20^\circ\right)\sin 71^\circ 24.8'}\right]$$

$$= 46^\circ 51.7' = 46.9^\circ$$

$$\text{Final course} = S46.9^\circ W = 226.9^\circ T$$

Vertex's position

In this case, the angle PAB is greater than 90°; therefore, the vertex does not lie between A and B, but outside of the A side. From the right-angled triangle PVA:

$$Lat._V = \cos^{-1}\left(\sin\alpha \cos Lat._A\right)$$

$$= \cos^{-1}\left(\sin 127^\circ 38.9' \cos 30^\circ\right) = 46^\circ 42.6' N$$

$$D.\,Long._{AV} = \cos^{-1}\left(\frac{\tan Lat_A}{\tan Lat_V}\right) = \left(\frac{\tan 30^\circ}{\tan 46^\circ 42.6'}\right) = 57^\circ 03.1'(E)$$

$$Long._V = 120^\circ\ W + 57^\circ 03.1' = 062^\circ 56.9' W$$

$$\text{Vertex's Position:}\quad Lat._V = 46^\circ 42.6'\ N \quad Long._V = 062^\circ 56.9' W$$

Longitude where great circle crosses the Equator

At the pole, the meridian of the position where the great circle crosses the equator is 90° with the meridian of the vertex. We can prove this as follows:

$$\tan Lat._X = \cos D.Long._{VX} \tan Lat._V$$

At equator, $Lat._X$ equals zero so $\tan Lat._X = 0$

$$\cos D.Long._{VX} \tan Lat._V = 0 \qquad \because Lat._V \neq 0 \quad \therefore \tan Lat._V \neq 0$$

$$\therefore \cos D.Long._{VX} = 0 \qquad \Rightarrow D.Long._{VX} = 90^\circ$$

$$\text{Longitude}_{X_E} = \text{Long.}_V + \text{D. Long.}_{VX_E}$$
$$= 62°56.9'W + 90° = 152°56.9'W$$

Meridian where great circle crosses the equator is 152°56.9′W

Positions where the meridian cut the great circle track

$$\text{Lat.}_X = \tan^{-1}\left(\cos \text{D.Long.}_{VX} \tan \text{Lat.}_V\right)$$

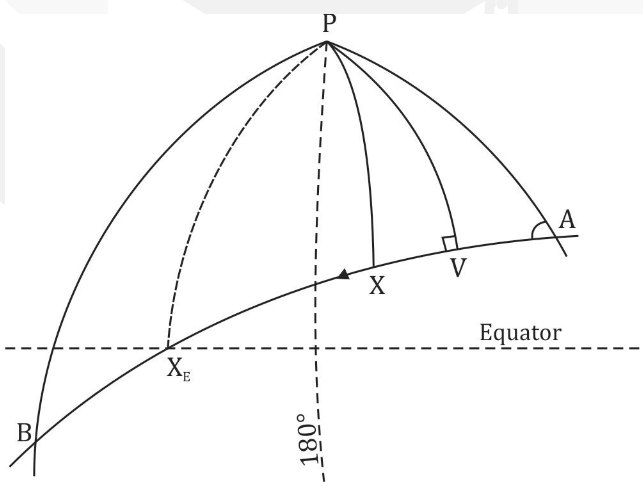
Position	Longitude (Long._X)	D. Long._{VX} ($\text{Long.}_X - \text{Long.}_V$)	Latitude (Lat._X)
X_1	130°W	67°03.0′	22°29.1′N
X_2	140°W	77°03.0′	13°22.8′N
X_3	150°W	87°03.0′	03°07.5′N
X_4	160°W	97°03.0′	07°25.5′S
X_5	170°W	107°03.0′	17°17.4′S

Example 11

Crossing Equator and 180° Meridian

From A: 45°N 100°W
To B: 30° S 130° E

Find:
1. Great circle distance
2. Initial course
3. Final course
4. Vertex's position
5. Course at equator
6. Longitude when crossing equator
7. Latitude when crossing 180° meridian
8. Course and latitude for every 10° longitude
9. Mercator sailing course and distance
10. Compare distance between sailing methods.



$$\text{Lat.}_A = 45°\text{N} \qquad \text{Lat.}_B = 30°\text{S}$$

$$\text{D.Long.}_{AB} = 360° - (100° + 130°) = 130°(\text{W})$$

Since the name of the latitude of the destination is contrary to the latitude of departure, then the latitude of destination is treated as a negative quantity.

Distance

$$\begin{aligned} D_{AB} &= \cos^{-1}\left[\sin \text{Lat.}_A \sin \text{Lat.}_B + \cos \text{Lat.}_A \cos \text{Lat.}_B \cos \text{D. Long.}_{AB}\right] \\ &= \cos^{-1}\left[\sin 45° \sin(-30°) + \cos 45° \cos(-30°) \cos 130°\right] \\ &= 138°20.8' \end{aligned}$$

$$\text{Distance} = 8300.8\,\text{miles}$$

Initial course

$$\begin{aligned} \alpha &= \cos^{-1}\left[\frac{\sin(-\text{Lat.}_B) - \sin \text{Lat.}_A \cos D_{AB}}{\cos \text{Lat.}_A \sin D_{AB}}\right] \\ &= \cos^{-1}\left[\frac{\sin(-30°) - \sin 45° \cos 138°20.8'}{\cos 45° \sin 138°20.8'}\right] \\ &= 86°32.6' = 86.5° \end{aligned}$$

$$\therefore \text{Initial course} = \text{N}86.5°\text{W} = 273.5°\text{T}$$

Final course

$$\begin{aligned} \beta &= \cos^{-1}\left[\frac{\sin \text{Lat.}_A - \sin(-\text{Lat.}_B)\cos D_{AB}}{\cos(-\text{Lat.}_B)\sin D_{AB}}\right] \\ &= \cos^{-1}\left[\frac{\sin 45° - \sin(-30°)\cos 138°20.8'}{\cos(-30°)\sin 138°20.8'}\right] \\ &= 54°35.3' = 54.6° \end{aligned}$$

$$\text{Final course} = \text{S}54.6°\text{W} = 234.6°\text{T}$$

Vertex's position

$$\begin{aligned} \text{Lat.}_V &= \cos^{-1}\left(\sin A \cos \text{Lat.}_A\right) \\ &= \cos^{-1}\left(\sin 86°32.6' \cos 45°\right) = 45°06.2'\text{N} \end{aligned}$$

$$\text{D. Long.}_{AV} = \cos^{-1}\left(\frac{\tan \text{Lat}_A}{\tan \text{Lat}_V}\right) = \left(\frac{\tan 45°}{\tan 45°06.2'}\right) = 4°52.9'(\text{W})$$

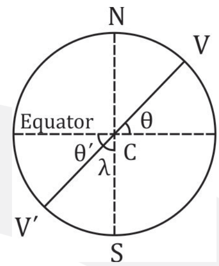
$$\text{Long.}_V = 100°00'\text{W} + 4°52.9'(\text{W}) = 104°52.9'\text{W}$$

$$\text{Vertex's Position:} \quad \text{Lat.}_V = 45°06.2'\text{N} \quad \text{Long.}_V = 104°52.9'\text{W}$$

Great circle course when crossing the Equator

At the equator, the angle θ between a great circle track and the equator equals the latitude of the vertex.

$\theta = \text{Latitude of vertex}$



$$\begin{aligned}\text{Course} &= 180° + \lambda \\ &= 180° + \text{coLat.}_V \\ &= 180° + (90° - 45°06.2') \\ &= 224°53.8' \\ &= 224.9°\text{T}\end{aligned}$$

Longitude of position at Equator

The meridian of the position at the equator and the meridian of the vertex would make an angle of 90° at the pole.

$$\text{Longitude} = 360° - (104°52.9' + 90°) = 165°07.1'\text{E}$$

Latitude when crossing 180° Meridian

$$\text{Lat.}_X = \tan^{-1}(\cos \text{D.Long.}_{VX} \tan \text{Lat.}_V)$$

$$\text{D.Long.}_{VX} = 180° - 104°52.9' = 75°07.1'$$

$$\text{Lat.}_X = \tan^{-1}(\cos 75°07.1' \tan 45°06.2') = 14°27.2'\text{N}$$

Latitude at every 10° of Longitude

$$\text{Lat.}_X = \tan^{-1}(\cos \text{D.Long.}_{VX} \tan \text{Lat.}_V)$$

Position	Longitude (Long._X)	D. Long._{VX} ($\text{Long.}_X - \text{Long.}_V$)	Latitude (Lat._X)
X_1	110°W	5°07.1′	44°59.4′N
X_2	120°W	15°07.1′	44°05.7′N
X_3	130°W	25°07.1′	42°15.7′N
X_4	140°W	35°07.1′	39°23.0′N
X_5	150°W	45°07.1′	35°18.4′N
X_6	160°W	55°07.1′	29°51.3′N
X_7	170°W	65°07.1′	22°53.6′N
X_8	180°	75°07.1′	14°27.2′N
X_9	170°E	85°07.1′	4°52.9′N
X_{10}	160°E	95°07.1′	5°07.0′S
X_{11}	150°E	105°07.1′	14°40.1′S
X_{12}	140°E	115°07.1′	23°04.5′S

Rhumb line course and distance at every 10° of Longitude

	Long.	Lat.	Mean Lat.	D. Lat.	Dep.	Co.	Dist.
X_0	100°W	45°00.0′N					
			44°59.7′N	0.6	424.3	269.9°	424.3
X_1	110°W	44°59.4′N					
			44°32.6′N	53.7	427.6	262.8°	431.0
X_2	120°W	44°05.7′N					
			43°10.7′N	110.0	437.5	255.9°	451.1
X_3	130°W	42°15.7′N					
			40°49.4′N	172.7	454.0	249.2°	485.7
X_4	140°W	39°23.0′N					
			37°20.7′N	244.6	477.0	242.9°	536.1
X_5	150°W	35°18.4′N					
			32°34.9′N	327.1	505.6	237.1°	602.2
X_6	160°W	29°51.3′N					
			26°22.5′N	417.7	537.5	232.1°	680.7
X_7	170°W	22°53.6′N					
			18°40.4′N	506.4	568.4	228.3°	761.3
X_8	180°	14°27.2′N					
			9°40.1′N	574.3	591.5	225.9°	824.4
X_9	170°E	4°52.9′N					
			0°07.1′S	599.9	600.0	225.0°	848.5
X_{10}	160°E	5°07.0′S					
			9°53.6′S	573.1	591.1	225.9°	823.3
X_{11}	150°E	14°40.1′S					
			18°52.3′S	504.4	567.8	228.4°	759.5
X_{12}	140°E	23°04.5′S					
			26°32.3′S	415.5	536.8	232.3°	678.8
X_{13}	130°E	30°00.0′S					

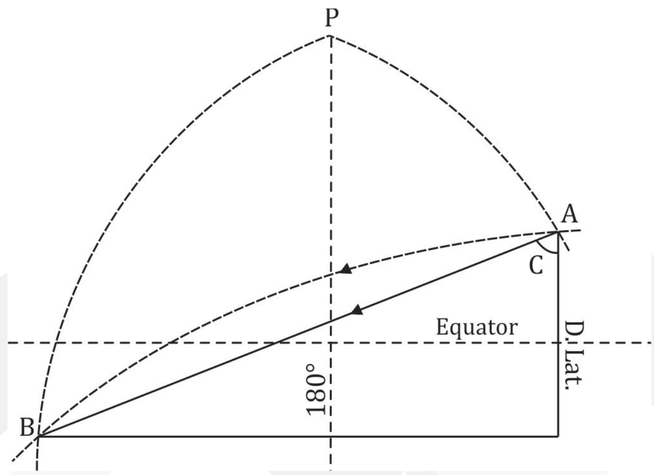
Total distance: 8306.9 miles

Mercator sailing course and distance

From A: 45°N 100°W
To B: 30° S 130° E

$Lat._A$	45°N	$M.P._A$	3013.38	$Long._A$	100°W
$Lat._B$	30°S	$M.P._B$	1876.67	$Long._B$	130°E
D. Lat.	4500′(S)	D.M.P.	4890.05	D. Long.	7800′(W)

$$C = \tan^{-1}\left(\frac{D.Long.}{D.M.P.}\right) = \tan^{-1}\left(\frac{7800}{4890.05}\right) = 57.9°$$



$$\text{Course} = S57.9°W + 180° = 237.9°T$$

$$\text{Distance} = \frac{\text{D. Lat}}{\cos C} = \frac{4500}{\cos 57.9°} = 8471.8 \text{ miles}$$

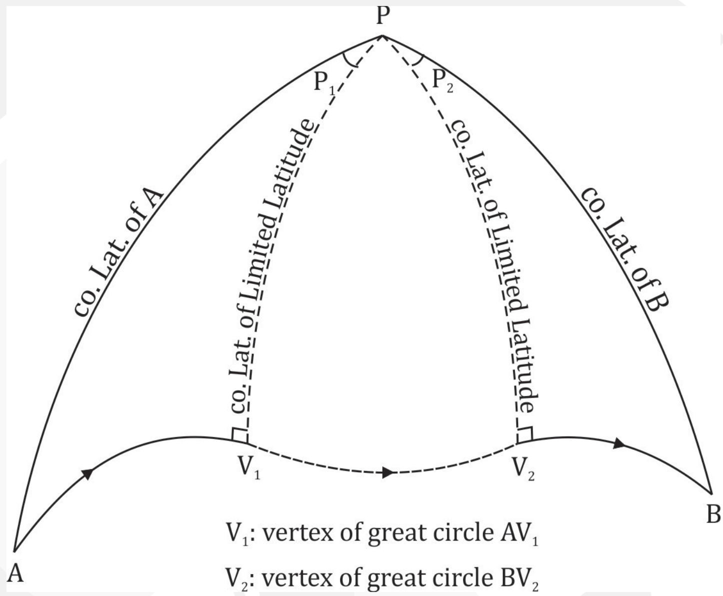
Compare distances between methods of sailings

Great circle distance: 8300.8 miles
Series rhumb line course: 8306.9 miles
One rhumb line course: 8471.8 miles

We can see there is not much difference when breaking down the great circle track by a series of rhumb lines for convenience (6.1 miles difference), but it still saves compared to Mercator sailing 164.9 miles.

Composite Great Circle Sailing

The great circle track is always curved toward the nearest pole, where its vertex is the point nearest to the pole. In very high latitudes, the track of a great circle cannot go beyond a certain latitude due to navigational restrictions, e.g., ice, fog, severe weather, etc. In such cases, the sailing track of a great circle has to be modified. The track then consists of the combined parts of great circles and the parallel of limiting latitude, which is called Composite Great Circle Sailing. So, composite great circle sailing is a combination of great circle sailing and parallel sailing.

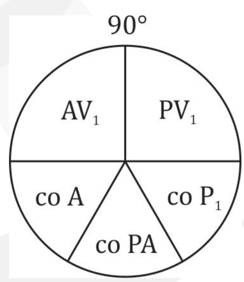


Initial course

$$\sin PV_1 = \cos(co\,A)\cos(co\,PA) = \sin A \sin PA$$

$$\sin A = \frac{\sin PV_1}{\sin PA} = \frac{\cos Lat._V}{\cos Lat._A}$$

$$\boldsymbol{\alpha = \sin^{-1}\left(\frac{\cos Lat._V}{\cos Lat._A}\right)}$$



Final course

$$\sin PV_2 = \cos(co\,PB)\cos(co\,B) = \sin PB \sin B$$

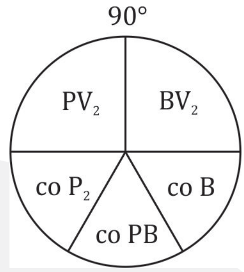
$$\sin B = \frac{\sin PV_2}{\sin PB} = \frac{\cos Lat._V}{\cos Lat._B}$$

$$\boldsymbol{\beta = \sin^{-1}\left(\frac{\cos Lat._V}{\cos Lat._B}\right)}$$

D. Long. between departure, destination positions and vertices

$$\sin\left(\text{co}\,P_1\right) = \tan PV_1 \tan\left(\text{co}\,PA\right)$$

$$\cos P_1 = \tan PV_1 \cot PA = \frac{\tan \text{Lat.}_A}{\tan \text{Lat.}_V}$$



$$\mathbf{D.Long._{AV_1} = \cos^{-1}\left(\frac{\tan Lat._A}{\tan Lat._V}\right)}$$

$$\sin\left(\text{co}\,P_2\right) = \tan PV_2 \tan\left(\text{co}\,PB\right)$$

$$\cos P_2 = \tan PV_2 \cot PB = \frac{\tan \text{Lat.}_B}{\tan \text{Lat.}_V}$$

$$\mathbf{D.Long._{BV_2} = \cos^{-1}\left(\frac{\tan Lat._B}{\tan Lat._V}\right)}$$

Distance from departure position and destination to limiting latitude

$$\sin\left(\text{co}\,PA\right) = \cos AV_1 \cos PV_1$$

$$\cos PA = \cos AV_1 \cos PV_1$$

$$\cos AV_1 = \frac{\cos PA}{\cos PV_1} = \frac{\sin \text{Lat.}_A}{\sin \text{Lat.}_V}$$

$$\mathbf{AV_1 = \cos^{-1}\left(\frac{\sin Lat._A}{\sin Lat._V}\right)}$$

$$\sin\left(\text{co}\,PB\right) = \cos BV_2 \cos PV_2$$

$$\cos PB = \cos BV_2 \cos PV_2$$

$$\cos BV_2 = \frac{\cos PB}{\cos PV_2} = \frac{\sin \text{Lat.}_B}{\sin \text{Lat.}_V}$$

$$\mathbf{BV_2 = \cos^{-1}\left(\frac{\sin Lat._B}{\sin Lat._V}\right)}$$

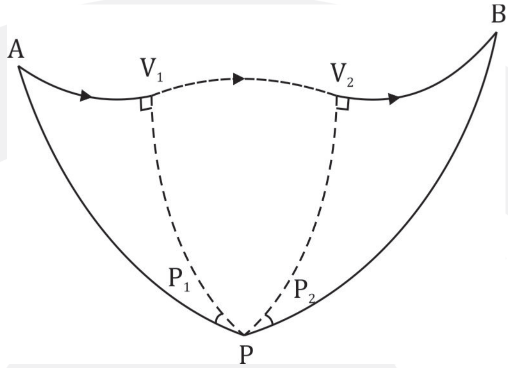
Distance along limiting latitude

$$\mathbf{V_1V_2 = D.Long._{V_1V_2} \cos Lat._V}$$

Where $\text{D.Long.}_{V_1V_2} = \text{D.Long.}_{AB} - \left(\text{D.Long.}_{AV_1} + \text{D.Long.}_{BV_2}\right)$

Example 12 Find the initial course, final course, meridians of the vertices and the total distance of the following great circle positions if the limiting latitude is 38° S:

From A: 34°55′S 056°10′W
To B: 33°55′S 018°25′E



Initial course

$$\alpha = \sin^{-1}\left(\frac{\cos \text{Lat.}_V}{\cos \text{Lat.}_A}\right) = \sin^{-1}\left(\frac{\cos 38°}{\cos 34°55'}\right) = 73°56.8'$$

Initial course = S73.9°E = 106.1°T

Final course

$$\beta = \sin^{-1}\left(\frac{\cos \text{Lat.}_V}{\cos \text{Lat.}_B}\right) = \sin^{-1}\left(\frac{\cos 38°}{\cos 33°55'}\right) = 71°43.7'$$

Final course = N71.7°E = 071.7°T

Meridians of the vertices

$$\text{D.Long.}_{AV_1} = \cos^{-1}\left(\frac{\tan \text{Lat.}_A}{\tan \text{Lat.}_V}\right) = \cos^{-1}\left(\frac{\tan 34°55'}{\tan 38°}\right) = 26°41.4'(\text{E})$$

$$\text{Long.}_{V_1} = 56°10'\text{W} - 26°41.4'(\text{E}) = 29°28.6'\text{W}$$

$$\text{D.Long.}_{BV_2} = \cos^{-1}\left(\frac{\tan \text{Lat.}_B}{\tan \text{Lat.}_V}\right) = \cos^{-1}\left(\frac{\tan 33°55'}{\tan 38°}\right) = 30°36.8'(\text{W})$$

$$\text{Long.}_{V_2} = 30°36.8'(\text{W}) - 18°25'\text{E} = 12°11.8'\text{W}$$

Distance AV_1 and BV_2

$$AV_1 = \cos^{-1}\left(\frac{\sin \text{Lat.}_A}{\sin \text{Lat.}_V}\right) = \cos^{-1}\left(\frac{\sin 34°55'}{\sin 38°}\right) = 21°31.7'$$

Distance $AV_1 = 1296.7$ miles

$$BV_2 = \cos^{-1}\left(\frac{\sin \text{Lat.}_B}{\sin \text{Lat.}_V}\right) = \cos^{-1}\left(\frac{\sin 33°55'}{\sin 38°}\right) = 24°59.9'$$

Distance $BV_2 = 1499.9$ miles

Distance V_1V_2

$$\begin{aligned} \text{D.Long.}_{V_1V_2} &= \text{D.Long.}_{AB} - \left(\text{D.Long.}_{AV_1} + \text{D.Long.}_{BV_2}\right) \\ &= \left(56°10' + 18°25'\right) - \left(26°41.4' + 30°36.8'\right) \\ &= 17°16.8' = 1036.8' \end{aligned}$$

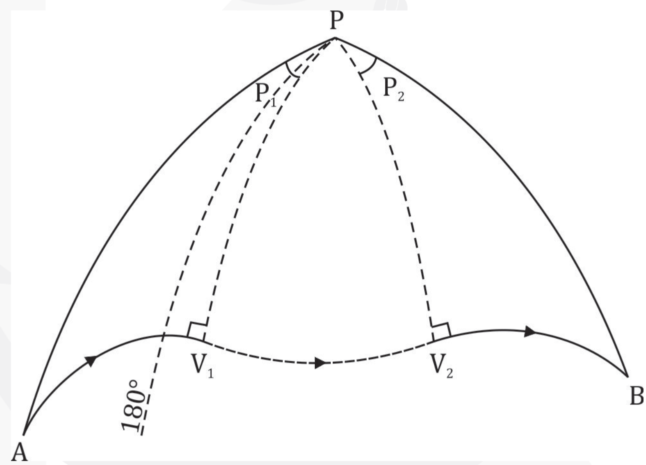
$$V_1V_2 = \text{D.Long.}_{V_1V_2} \cos \text{Lat.}_V = 1036.8' \times \cos 38° = 817'$$

$$\text{Distance } V_1V_2 = 817 \text{ miles}$$

$$\text{Total Distance} = 1296.7 + 1499.9 + 817 = 3613.6 \text{ miles}$$

Example 13 A composite great circle route from 35°40´ N 140°00´ E to 37°30´N 120°00´W. Limited latitude is 45°. Find initial and final course, longitudes of the vertices and total distance:

From	A:	35°40′N	140°E
To	B:	37°30′N	120°W



Initial course

$$\alpha = \sin^{-1}\left(\frac{\cos \text{Lat.}_V}{\cos \text{Lat.}_A}\right) = \sin^{-1}\left(\frac{\cos 45°}{\cos 35°40'}\right) = 60.5°$$

$$\text{Initial Course} = \text{N}60.5°\text{E} = 060.5°\text{T}$$

Final course

$$\beta = \sin^{-1}\left(\frac{\cos \text{Lat.}_V}{\cos \text{Lat.}_B}\right) = \sin^{-1}\left(\frac{\cos 45°}{\cos 37°30'}\right) = 63°$$

$$\text{Final Course} = \text{S}63°\text{E} = 180° - 63° = 117°\text{T}$$

Meridians (longitudes) of the vertices

$$\text{D.Long.}_{AV_1} = \cos^{-1}\left(\frac{\tan \text{Lat.}_A}{\tan \text{Lat.}_V}\right) = \cos^{-1}\left(\frac{\tan 35°40'}{\tan 45°}\right) = 44°08.2'(\text{E})$$

$$\text{Long.}_{V_1} = 360 - \left(140°\text{E} + 44°08.2'\right) = 175°51.8'\text{W}$$

$$\text{D.Long.}_{BV_2} = \cos^{-1}\left(\frac{\tan \text{Lat.}_B}{\tan \text{Lat.}_V}\right) = \cos^{-1}\left(\frac{\tan 37°30'}{\tan 45°}\right) = 39°53.1'(\text{W})$$

$$\text{Long.}_{V_2} = 120°\text{W} + 39°53.1' = 159°53.1'\text{W}$$

Distance AV_1 and BV_2

$$AV_1 = \cos^{-1}\left(\frac{\sin \text{Lat.}_A}{\sin \text{Lat.}_V}\right) = \cos^{-1}\left(\frac{\sin 35°40'}{\sin 45°}\right) = 34°27.2'$$

$$\text{Distance } AV_1 = 2067.2 \text{ miles}$$

$$BV_2 = \cos^{-1}\left(\frac{\sin \text{Lat.}_B}{\sin \text{Lat.}_V}\right) = \cos^{-1}\left(\frac{\sin 37°30'}{\sin 45°}\right) = 30°34.8'$$

$$\text{Distance } BV_2 = 1834.8 \text{ miles}$$

Distance V_1V_2

$$\text{D. Long.}_{V_1V_2} = \text{Long.}_{V_1} - \text{Long.}_{V_2}$$
$$= 175°51.8' - 159°53.1' = 15°58.7' = 958.7'$$

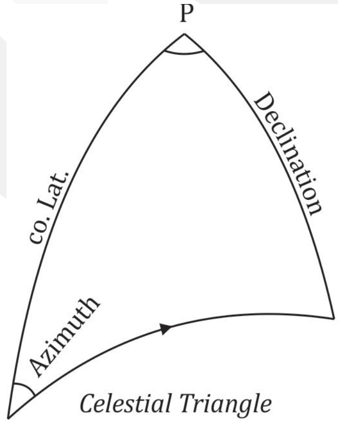
$$V_1V_2 = \text{D.Long.}_{V_1V_2} \cos \text{Lat.}_V = 958.7' \times \cos 45° = 677.9'$$

$$\text{Distance}_{V_1V_2} = 677.9 \text{ miles}$$

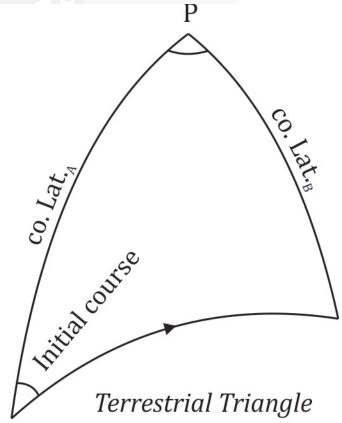
$$\text{Total Distance} = 2067.2 + 1834.8 + 677.9 = 4579.9 \text{ miles}$$

Using ABC Tables for Great Circle Sailing

The ABC tables can be used to find the initial and final courses similarly to finding the azimuth. For finding the initial course, the departure latitude is used as DR latitude; destination latitude is used as declination, and D. Long. is used as hour angle with direction east or west. Similarly for finding final course, destination latitude becomes DR latitude, departure latitude becomes destination and same hour angle except direction is opposite in initial course case. Then the course is named according to the direction (East or West), instead of the size of the hour angle.



Celestial Triangle



Terrestrial Triangle

Example 14 Using the ABC table, find the initial and final courses of great circle sailing:

From A: 48°24′N 124°44′W
To B: 34°50′N 139°50′E

$$\text{D.Long.}_{AB} = 360° - (124°44' + 139°50') = 95°26'(W)$$

Initial course

$$\left.\begin{array}{rl} \text{Lat.}_B & 48°24'N \\ \text{Lat.}_A & 34°50'N \\ \text{D.Long.}_{AB} & 95°26'(W) \end{array}\right] \begin{array}{ll} A & 0.11\,N \\ B & \underline{0.70}\,N \\ C & 0.81\,N \end{array}$$

$$\text{Azimuth} = N61.7°W$$
$$\text{Initial Course} = N61.7°W = 298.3°T$$

Final course

$$\left.\begin{array}{rl} \text{Lat.}_B & 34°50'N \\ \text{Lat.}_A & 48°24'N \\ \text{D.Long.}_{AB} & 95°26'(E) \end{array}\right] \begin{array}{ll} A & 0.07\,N \\ B & \underline{1.13}\,N \\ C & 1.20\,N \end{array}$$

$$\text{Azimuth} = N45.4°E$$
$$\text{Final Course} = S45.4°W = 225.4°T$$

Example 15 Find initial course and final course of great circle sailing:

From A: 33°22′S 113°08′E
To B: 10°51′S 049°16′E

$$\text{D.Long}_{.AB} = 113°08' - 49°16' = 63°52'(W)$$

Initial course

$$\left.\begin{array}{rl} \text{Lat.}_B & 33°22'S \\ \text{Lat.}_A & 10°51'S \\ \text{D.Long.}_{AB} & 63°52'(W) \end{array}\right] \begin{array}{ll} A & 0.32\,N \\ B & \underline{0.21}\,S \\ C & 0.11\,N \end{array}$$

$$\text{Azimuth} = N84.8°W$$
$$\text{Initial Course} = N84.8°W = 275.2°T$$

Final course

$$\left.\begin{array}{rl} \text{Lat.}_B & 10°51'S \\ \text{Lat.}_A & 33°22'S \\ \text{D.Long.}_{AB} & 63°52'(E) \end{array}\right] \begin{array}{ll} A & 0.09\,N \\ B & \underline{0.73}\,S \\ C & 0.64\,S \end{array}$$

$$\text{Azimuth} = S57.9°E$$
$$\text{Final Course} = N57.9°W = 302.1°T$$

The values A, B, and azimuth can also be computed by formulas:

$$\mathbf{A = \frac{\tan Lat._A}{\tan P} \qquad B = \frac{\tan Lat._B}{\sin P} \qquad Azimuth = \tan^{-1}\left(\frac{1}{C \times \cos Lat._A}\right)}$$

Where P is difference of longitude Between A and B positions, then the above example can be solved as follows:

Initial course

$Lat._A$: latitude of observer

$Lat._B$: declination of celestial body

D. Long.: hour angle (W)

Initial course: azimuth

$$Lat._A = 33°22'S \qquad Lat._B = 10°51'S$$

$$D.Long._{AB} = P = LHA = 63°52'(W)$$

$$\left.\begin{aligned} A &= \frac{\tan Lat._A}{\tan P} = \frac{33°22'}{\tan 63°52'} = 0.323093\,N \\ B &= \frac{\tan Lat._B}{\sin P} = \frac{10°51'}{\sin 63°52'} = 0.213489\,S \end{aligned}\right] \quad \begin{array}{ll} A & 0.323093\,N \\ B & \underline{0.213489}\,S \\ C & 0.109603\,N \end{array}$$

$$Azimuth\,(\alpha) = \tan^{-1}\left(\frac{1}{C \times \cos Lat._A}\right) = \tan^{-1}\left(\frac{1}{0.109603 \times \cos 33°22'}\right)$$

$$= 84°46.2'$$

$$Azimuth = N84°46.2'W = N84.8°W$$

$$\text{Initial course} = N84.8°W = 275.2°T$$

Final course

$Lat._B$: latitude of observer

$Lat._A$: declination of celestial body

D. Long.: hour angle (opposite direction of HA in initial course)

Final course: azimuth

At final position, direction of movement of the ship is away from initial position, not toward, so the azimuth found must be added to or subtracted from 180° to find correct heading of the ship.

$$Lat._B = 10°51'N \quad Lat._A = 33°22'N$$

$$D.Long._{AB} = P = LHA = 48°58'(E)$$

$$A = \frac{\tan Lat._B}{\tan P} = \frac{10°51'}{\tan 63°52'} = 0.094034\,N \qquad B = \frac{\tan Lat._A}{\sin P} = \frac{33°22'}{\sin 63°52'} = 0.733532\,S$$

A	0.094034 N
B	0.733532 S
C	0.639498 S

$$\text{Azimuth } (\beta) = \tan^{-1}\left(\frac{1}{C \times \cos Lat._B}\right) = \tan^{-1}\left(\frac{1}{0.639498 \times \cos 10°51'}\right)$$
$$= 57°52.1'$$

$$\text{Azimuth} = S57°52.1'E = S57.9°E$$
$$\text{Final course} = N57.9°W = 302.1°T$$

Example 16 Find initial course and final course by using the ABC computation formula:

From A: 56°20′N 008°12′W
To B: 52°12′N 057°10′W

Initial course

$$Lat._A = 56°20'N \qquad Lat._B = 52°12'N$$
$$D.Long._{AB} = P = LHA = 48°58'(W)$$

$$A = \frac{\tan Lat.}{\tan P} = \frac{56°20'}{\tan 48°58'} = 1.306619\,S \qquad B = \frac{\tan Lat.}{\sin P} = \frac{52°12'}{\sin 48°58'} = 1.709061\,N$$

A	1.306619 S
B	1.709061 N
C	0.402442 N

$$\text{Azimuth } (\alpha) = \tan^{-1}\left(\frac{1}{C \times \cos Lat._A}\right) = \tan^{-1}\left(\frac{1}{0.402442 \times \cos 56°20'}\right)$$
$$= 77°25.4'$$

$$\text{Azimuth} = N77°25.4'\,W = N77.4°W$$
$$\text{Initial course} = N77.4°W = 282.6°T$$

Final course

$$Lat._B = 52°12'N \quad Lat._A = 56°20'N$$
$$D.Long._{AB} = P = LHA = 48°58'(E)$$

$$A = \frac{\tan Lat._B}{\tan P} = \frac{52°12'}{\tan 48°58'} = 1.121995\,S \qquad B = \frac{\tan Lat._A}{\sin P} = \frac{56°20'}{\sin 48°58'} = 1.990286\,N$$

A	1.121995 S
B	1.990286 N
C	0.868291 N

$$\text{Azimuth } (\beta) = \tan^{-1}\left(\frac{1}{C \times \cos \text{Lat.}_B}\right) = \tan^{-1}\left(\frac{1}{0.868291 \times \cos 52°12'}\right)$$
$$= 61°58.7'$$

$$\text{Azimuth} = N61°58.7'E = N62°E$$
$$\text{Final course} = S62°W = 242°T$$

Example 17 Find initial course and final course of great circle sailing from Suva to Honolulu:

Suva (A): 18°08′S 178°26′E

Honolulu (B): 21°19′N 157°52′W

Initial course

$$\text{Lat.}_A = 18°08'S \quad \text{Lat.}_B = 21°19'N$$
$$\text{D.Long.}_{AB} = P = LHA = 23°42'(E)$$

$$A = \frac{\tan \text{Lat.}_A}{\tan P} = \frac{18°08'}{\tan 23°42'} = 0.746053\,N$$
$$B = \frac{\tan \text{Lat.}_B}{\sin P} = \frac{21°19'}{\sin 23°42'} = 0.970820\,N$$

A	0.746053 N
B	0.970820 N
C	1.716873 N

$$\text{Azimuth } (\alpha) = \tan^{-1}\left(\frac{1}{C \times \cos \text{Lat.}_A}\right) = \tan^{-1}\left(\frac{1}{1.716873 \times \cos 18°08'}\right)$$
$$= 31°30.2'$$

$$\text{Azimuth} = N31°30.2'E = N31.5°E$$
$$\text{Initial course} = N31.5°E = 031.5°T$$

Final course

$$\text{Lat.}_B = 21°19'N \quad \text{Lat.}_A = 18°08'S$$

$$\text{D.Long.}_{AB} = P = LHA = 23°42'(W)$$

$$A = \frac{\tan \text{Lat.}}{\tan P} = \frac{21°19'}{\tan 23°42'} = 0.888943\,S$$
$$B = \frac{\tan \text{Lat.}}{\sin P} = \frac{18°08'}{\sin 23°42'} = 0.814769\,S$$

A	0.888943 S
B	0.814769 S
C	1.703712 S

$$\text{Azimuth } (\beta) = \tan^{-1}\left(\frac{1}{C \times \cos \text{Lat.}_B}\right) = \tan^{-1}\left(\frac{1}{1.703712 \times \cos 21°19'}\right)$$
$$= 32°12.8'$$

$$\text{Azimuth} = S32°12.8'W = S32.2°W$$
$$\text{Final course} = N32.2°E = 032.2°T$$