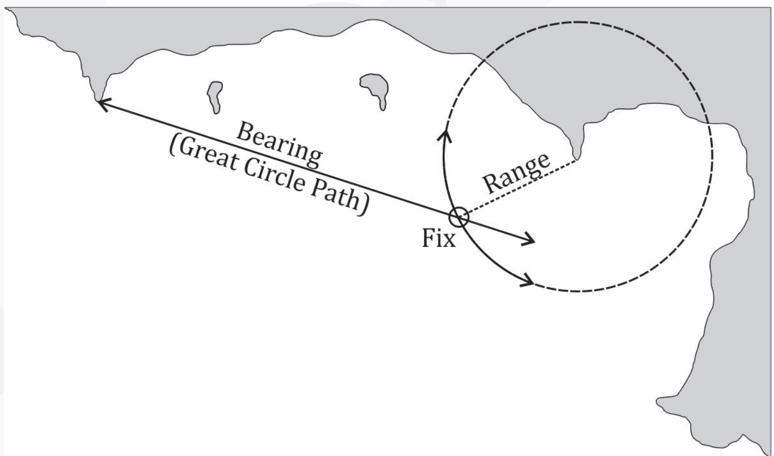
Position Lines

A position line generally is a line that contains the ship's position somewhere on it. When taking the bearing of a charted object to obtain its position, a straight line is drawn from the object with obtained degrees; the ship's position is somewhere on that line, which is called the position line. This line of bearing is basically a part of a great circle that passes through the charted object and the ship's position. When taking the range from a charted object to obtain the ship's position, a small circle can be drawn with the object at its the centre; the range is the radius, and the circle is called the **position circle**. Instead of drawing the whole circle, a part of it in the vicinity of the ship's position is drawn; this small arc of the part of the position circle is called the **position line**. The position line is therefore a part of a great or a small circle.



Position Circle by Horizontal Angle

Horizontal angle $\alpha < 90°$

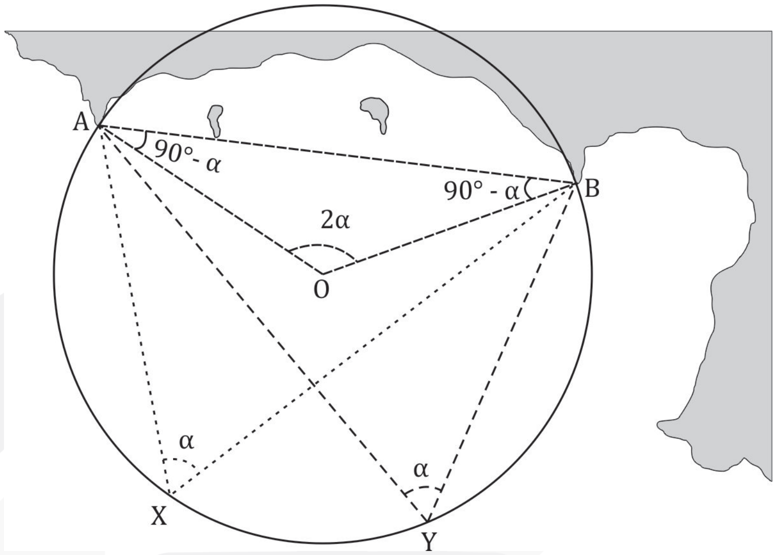
Let the horizontal angle between two shore-charted objects, A and B, be α, where α is less than 90°. Draw the line to join A and B. As shown in the figure next page, the angle on a chord (AB) at the centre of the circle is twice any angle at the circumference of the circle that is subtended by the same chord.

$$\angle AOB = 2\angle AXB = 2\angle AYB = 2\alpha$$

Hence, the triangle AOB having OA equals OB, because they are the radius of the circle, then:

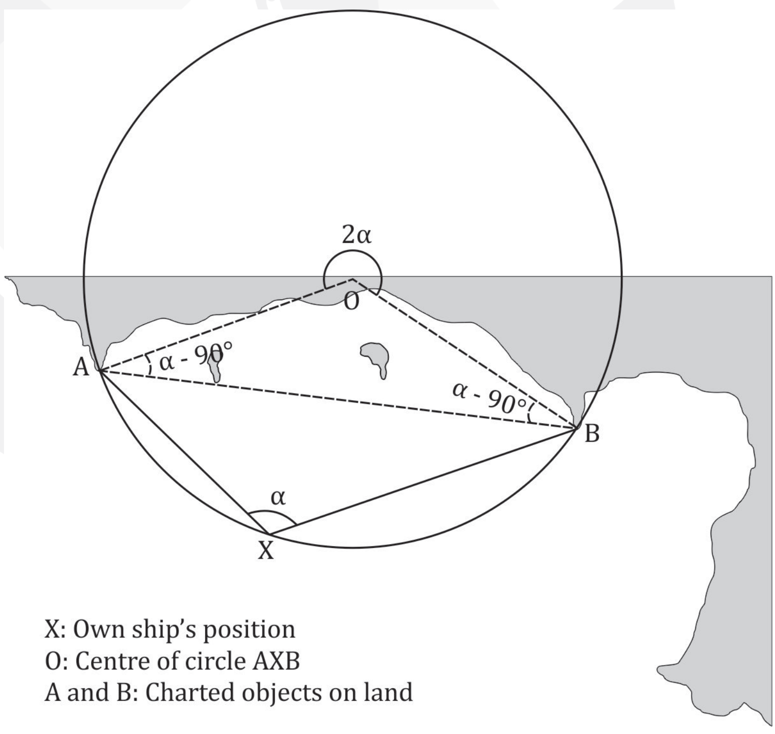
$$\angle OAB = \angle OBA = \frac{180° - 2\alpha}{2} = 90° - \alpha$$

So, by knowing the horizontal angle between two charted objects, we can draw the arc of the circle which contains the position of the ship.



X and Y: Own ship's position
A and B: Charted objects on land
O: Centre of the circle ABYX

Horizontal angle **α > 90°** In this case, the ship is on the opposite side of the chord from the centre of the circle. Similarly, as shown in the figure, when the horizontal angle is greater than 90°.



X: Own ship's position
O: Centre of circle AXB
A and B: Charted objects on land

$$\angle AOB_{(\text{Outside}\,\Delta\,AOB)} = 2\angle AXB \;\Rightarrow\; \angle AOB_{(\text{In}\,\Delta\,AOB)} = 360° - 2\angle AXB$$

Hence, the triangle AOB having OA equals OB because they are the radius of the circle, then:

$$\angle OAB = \angle OBA = \frac{180° - \left(360° - 2\angle AXB\right)}{2} = \frac{2\angle AXB - 180°}{2} = \alpha - 90°$$

Procedure to obtain position circle

1. Use the sextant or compass to obtain the angle between two charted objects. The advantage of using the compass to take the bearing of an object is that any compass error will have no effect on the position line, and the bearing of the objects can be obtained.

2. Draw the line to join the positions of the objects (AB).

3. From each object's position, if the horizontal angle between two objects is less than 90°, then construct the line on the same side of the ship's position that makes an angle of (90°− α) with the line joining two objects. If the horizontal angle between two objects is more than 90°, then construct the line on the opposite side of the ship's position that makes an angle of (α − 90°) with the line joining the two objects. The point of intersection of two lines will form the centre of the circle.

4. Draw the circle; it should pass through the positions of two objects. This is the position circle. The ship's position is somewhere in the position circle.

5. The ship's position can be obtained by drawing the bearing of one of the objects obtained earlier by compass; the intersection of the position circle and the bearing line is the fix of the ship's position. For sextant use, and to eliminate any unknown error of the compass, repeat for another horizontal angle to obtain another position circle. The intersection of the two position circles is the ship's position.

Position circle by rising and dipping distance

When the ship is making landfall, select the highest structure charted on land, e.g., a lighthouse, especially at night time when the light from the lighthouse can be observed. The distance from the lighthouse to the ship's position can be calculated by formula:

$$\text{Distance} = 2.08\sqrt{h} + 2.08\sqrt{H}$$

h Height of eyes in metres

H Height of structure above sea level in metres

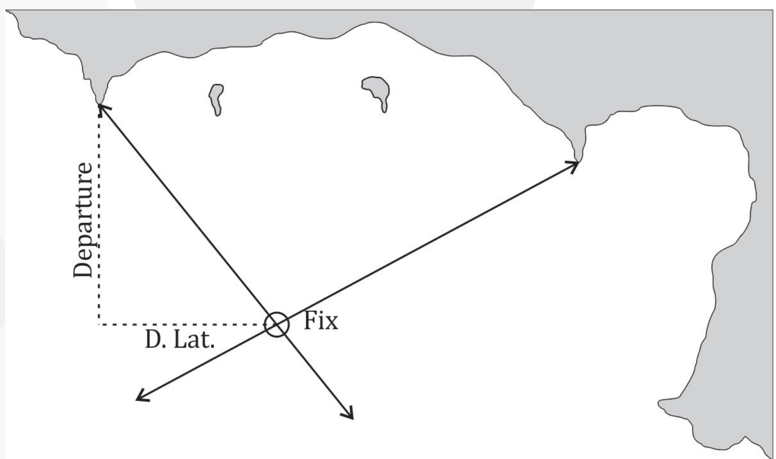
Example 1 A ship is making landfall. The navigator observes the light of a lighthouse at the horizon. Find the distance to land, if the height of the lighthouse is 45 m above sea level and the eye height is 20 m:

$$\text{Distance} = 2.08\sqrt{h} + 2.08\sqrt{H} = 2.08\sqrt{20} + 2.08\sqrt{45} = 23.3 \text{ miles}$$

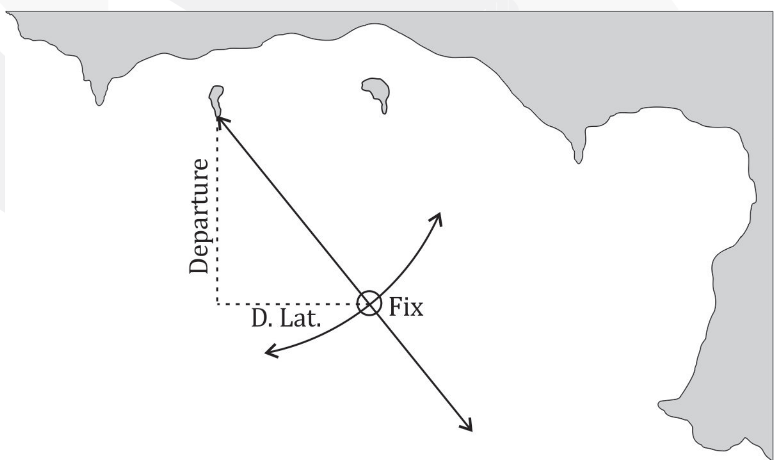
Fixing the Ship's Position

The position line tells us only that the ship's position is somewhere on that line, but not exactly where she is. To obtain the ship's position, another position line is necessary in order to form a fix. The fix can be formed by two bearings, and is the intersection of two great circles or of two ranges, which is the intersection of two small circles or the combination of a bearing and a range. When the fix is obtained and drawn on the chart, the coordinates of the ship's position can be read from the chart. Otherwise, D. Lat. and Dep. can be calculated in relation to a known object to figure out the ship's position, as shown in the figure below.

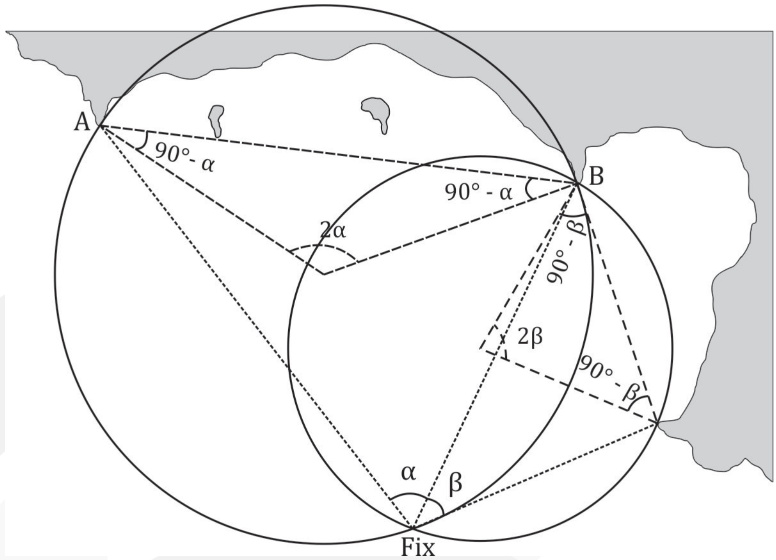
Fix by two bearings



Fix by a bearing and a range of single object



Fix by two Horizontal Angle Circles



Running Fix

This method obtains a fix from the bearings of one or two charted objects and the course and speed.

Procedure to obtain a running fix

1. Calculate time and distance made good;

2. Plot the position line of the first bearing;

3. Plot the position line of the second bearing;

4. From any point on the first position line, lay off the course and distance made good toward the second position line;

5. From another end of the course and distance made good line, plot the transferred first position line. The intersection of the transferred position line and the second position line is the fix when the second bearing taken;

6. From the fix, transfer the course and distance made good line. The intersection between the course and distance made good line and the first position line is the fix when first bearing was taken.

Example 2

Running Fix with single charted object

At 1400, the bearing of the island on the port bow is 040°. At 1430, the same island is now on the port quarter, bearing 310°. Plot the fix at 1430 and at 1400 if the course is 080°T and the speed is 10 knots:

1. Calculate time and distance made good;

$$\text{Time of steaming} = 1430 - 1400 = 30'$$
$$\text{Distance} = \text{Speed} \times \text{Time} = 10\,\text{knots} \times 30' = 5\,\text{miles}$$

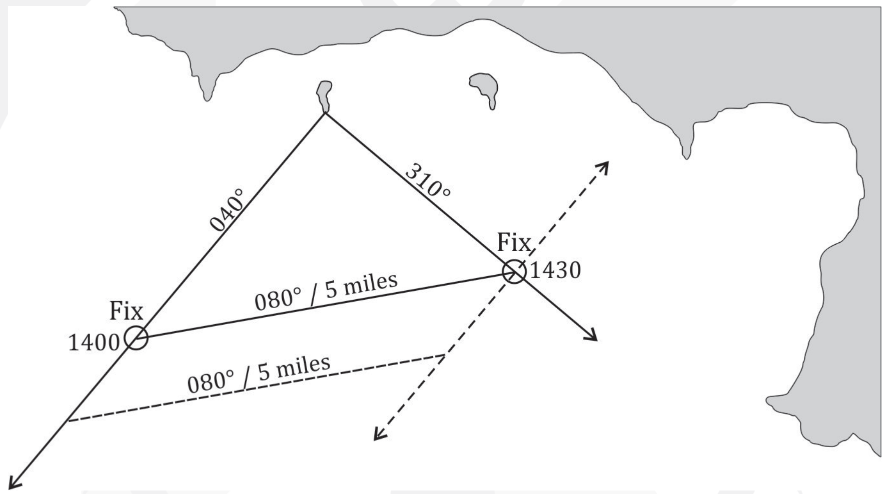
2. Plot the position line of the first bearing (040°) at 1400;

3. Plot the position line of the second bearing (310°) at 1430;

4. From first position line, lay off the course 080° and distance 5 miles;

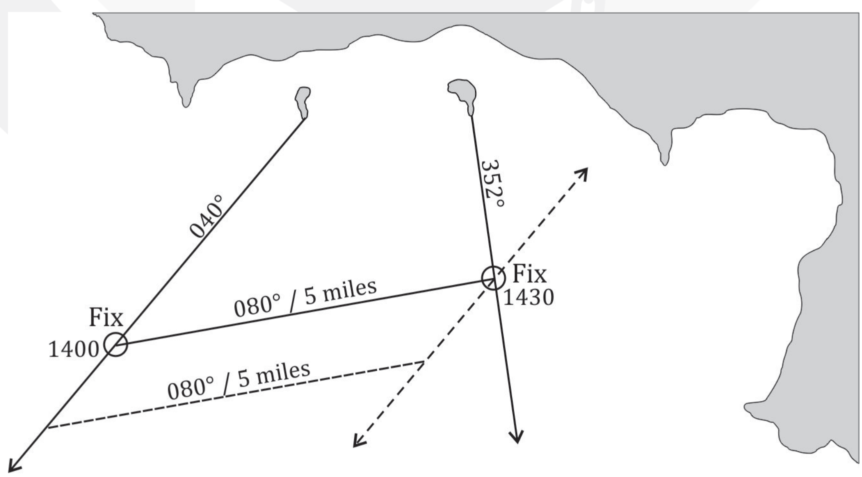
5. From the end of the course and distance made good line, plot the transferred first position line (dotted line). The intersection of the transferred position line and the second position line is the fix at 1430;

6. From the fix at 1430, transfer the course and distance made good line. The intersection between the course and distance made good line and the first position line is the fix at 1400.



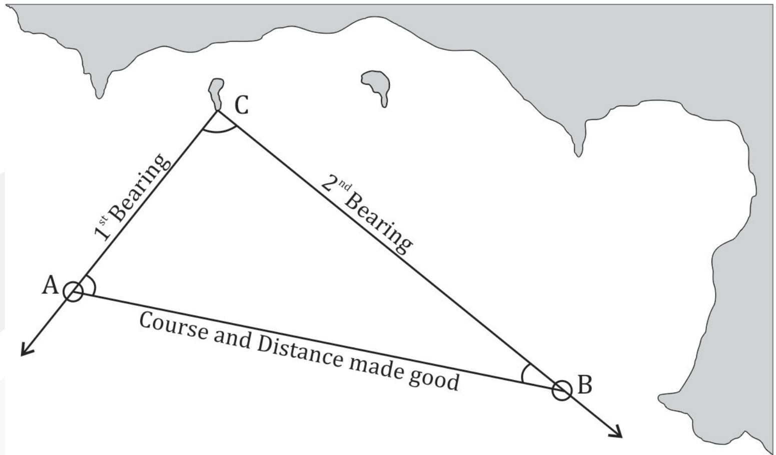
Running Fix with two charted objects

The running fix with two charted objects is similar to forming a running fix for the single charted object, except that the second position line 347° is plotted from the second charted object.



Running Fix Triangle

Using a single charted object, the fix can also be found by calculating the range of the bearing.



Procedure to obtain a fix by triangle

1. Calculate the distance made good;
2. Plot the position line of the first and second bearings;
3. Calculate the range of the first bearing to obtain the first fix;
4. From the first fix, lay off the course and distance made good; this line should contact the second position line, and the point of intersection is the second fix. The second fix can also be obtained by calculating the range similarly to the first range, then joining two fixes; it should be the line that represents the course and distance made good.

Range Calculation

From Δ ABC

$$\frac{\sin C}{AB} = \frac{\sin A}{BC} = \frac{\sin B}{AC}$$

C Angle between two bearings
A Angle between the course made good and 1st bearing
B Angle between the course made good and 2nd bearing
AB Distance made good
AC 1st range
BC 2nd range

$$AC = \frac{AB \sin B}{\sin C} \qquad BC = \frac{AB \sin A}{\sin C}$$

$$1^{st}\ \text{Range} = \frac{\text{Distance} \times \sin\left(\text{Angle between course and } 2^{nd}\ \text{bearing}\right)}{\sin\left(\text{Angle between two bearings}\right)}$$

$$2^{nd}\ \text{Range} = \frac{\text{Distance} \times \sin\left(\text{Angle between course and } 1^{st}\ \text{bearing}\right)}{\sin\left(\text{Angle between two bearings}\right)}$$

If the angle between two bearings is 90°, then sinC equals 1, therefore:

$AC = AB\sin B$ $\qquad$ $BC = AB\sin A$

$$1^{st}\ \text{Range} = \text{Distance} \times \sin\left(\text{Angle between course and } 2^{nd}\ \text{bearing}\right)$$

$$2^{nd}\ \text{Range} = \text{Distance} \times \sin\left(\text{Angle between course and } 1^{st}\ \text{bearing}\right)$$

Example 3

At 0900, a charted object was observed bearing 030° T; at 1000, the same object bears 290° T. Course made good is 080° T, speed 6 knots. Find the position of the fix at 0900 and 1000:

1. Calculate distance made good;

$$\text{Distance} = 6 \times 1 = 6\,\text{miles}$$

2. Plot the position line of the first bearing; (030°); and second bearing (290°);

3. Calculate the range of the first bearing to obtain the first fix;

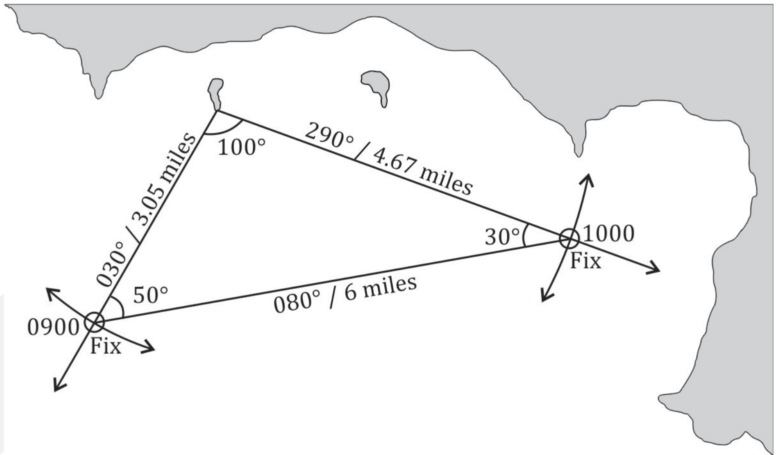
$$1^{st}\ \text{Range} = \frac{\text{Distance} \times \sin\left(\text{Angle between course and } 2^{nd}\ \text{bearing}\right)}{\sin\left(\text{Angle between two bearings}\right)}$$

$$= \frac{6 \times \sin 30°}{\sin 100°} = 3.05\,\text{miles}$$

4. From the first fix, lay off the course and distance made good to obtain the second fix. The second fix can also be obtained by calculating as follows:

$$2^{nd}\ \text{Range} = \frac{\text{Distance} \times \sin\left(\text{Angle between course and } 1^{st}\ \text{bearing}\right)}{\sin\left(\text{Angle between two bearings}\right)}$$

$$= \frac{6 \times \sin 50°}{\sin 100°} = 4.67\,\text{miles}$$

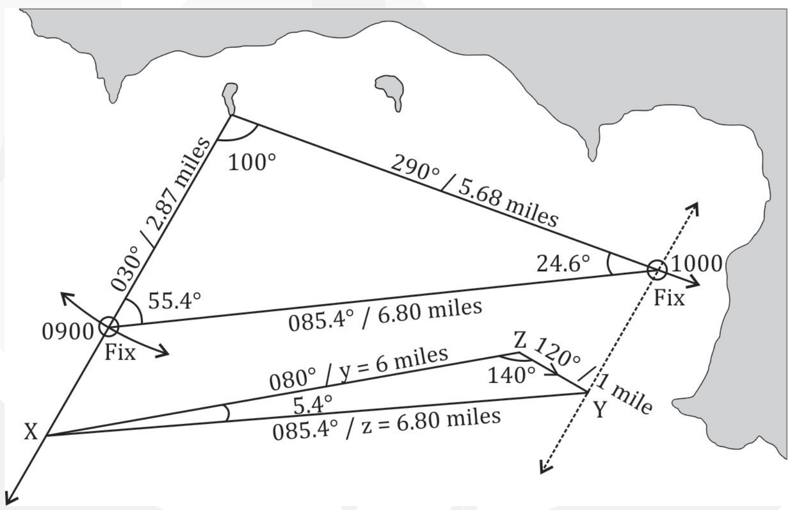


Running Fix with Current

If the current has an effect, then the set and drift is laid off at the end of the course and distance to find the course and distance made good, as shown in the figure, before plotting the transferred position line.

Example 4

Same as example 3; with current setting 120° with the drift of 1 mile per hour.



1. Calculate distance made good;

$$\text{From } \Delta XYZ: \quad z^2 = x^2 + y^2 - 2(x)(y)\cos Z$$

$$\text{Distance made good} = \sqrt{1^2 + 6^2 - 2(1)(6)\cos 140^\circ}$$
$$= 6.80 \text{ miles}$$

2. Calculate course made good;

$$\frac{\sin X}{x} = \frac{\sin Z}{z} \qquad \therefore \quad X = \sin^{-1}\left(\frac{x \times \sin Z}{z}\right) = \sin^{-1}\left(\frac{1 \times \sin 140^\circ}{6.80}\right) = 5.4^\circ$$

$$\text{Course made good} = 080^\circ + 5.4^\circ = 85.4^\circ$$

3. Plot the position line of the first bearing; (030°); and second bearing (290°);

4. Calculate the range of the first bearing to obtain the first fix;

$$1^{st}\ \text{Range} = \frac{\text{Distance} \times \sin(\text{Angle between course and } 2^{nd}\ \text{bearing})}{\sin(\text{Angle between two bearings})}$$

$$= \frac{6.80 \times \sin 24.6^\circ}{\sin 100^\circ} = 2.87\,\text{miles}$$

5. From the first fix, lay off the course and distance made good to obtain the second fix. The second fix can also be obtained by calculating as follows:

$$2^{nd}\ \text{Range} = \frac{\text{Distance} \times \sin(\text{Angle between course and } 1^{st}\ \text{bearing})}{\sin(\text{Angle between two bearings})}$$

$$= \frac{6.80 \times \sin 55.4^\circ}{\sin 100^\circ} = 5.68\,\text{miles}$$

Running Fix with Position Circles

Procedure to obtain a fix by two position circles

1. Calculate distance made good;

2. From the observed object, lay off the course and distance made good line;

3. Using the end point of the course and distance made good line as the centre of the circle; plot the first range;

4. From the observed object as the centre of the circle, plot the second range. The intersection of two position circles is the second fix;

5. From the second fix, transfer the course and distance made good line; the point at the end of this line is the first fix.

Example 5 At 1300 a tip of an island was observed on the port bow, range 3.8 miles. At 1330, the same island has a range of 3.2 miles. The ship is making course 080° T, speed 10 knots. Plot the fix at 1330 and 1300:

1. Calculate distance made good;

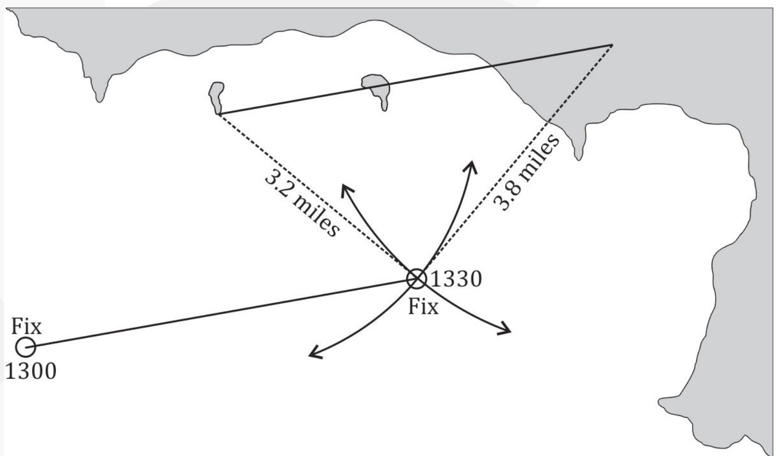
$$\text{Distance} = 10 \times (1330 - 1300) = 5\,\text{miles}$$

2. From the tip of island, lay off the course and distance made good line (080° T, 5 miles);

3. Using the end point of the course and distance made good line as the centre of the circle; plot the first range (3.8 miles);

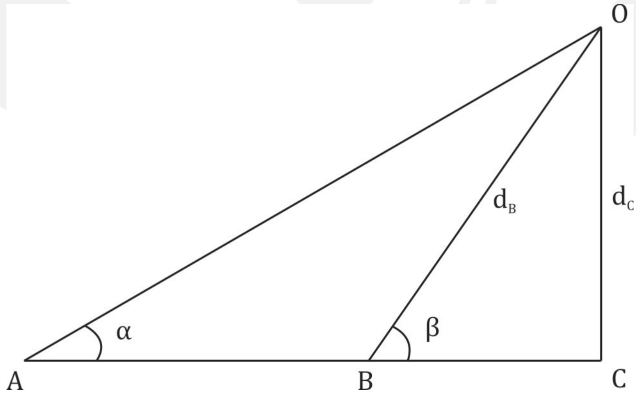
4. From the observed object as the centre of the circle, plot the second range (3.2 miles). The intersection of two position circles is the fix at 1330;

5. From the fix at 1330, transfer the course and distance made good line; the point at the end of this line is the fix at 1300.



A Fix by Angles on the Bow

A vessel is steaming the course along AC track, at speed V knots. At the time t_A, from position A, a stationary object at O is observed bearing at an angle α° off the port bow; and at the time $t_{B,}$ from position B, the same object is observed bearing at an angle β° off the port bow.



With the above information, we can calculate:

- Distance from object to B position;
- Distance when the object is abeam;
- The time when the object will be abeam.

At any angles of α and β

$$d_B = \frac{AB\sin\alpha}{\sin(\beta - \alpha)} \qquad d_C = \frac{AB\sin\alpha\sin\beta}{\sin(\beta - \alpha)}$$

and $$\Delta t_{BC} = \frac{60 d_B}{V\tan\beta} \quad \text{or} \quad \Delta t_{BC} = \frac{\Delta t_{AB}\sin\alpha\cos\beta}{\sin(\beta - \alpha)}$$

At the angles where β = 2α

$$d_B = AB \qquad d_C = AB\sin\beta \qquad \Delta t_{BC} = t_A\cos\beta$$

Example 6 Vessel is steaming at 16 knots. At 1345 hrs, a lighthouse is bearing 22° off the bow. At 1405 hrs, the lighthouse is bearing 45° off the bow. Find the distance from the light to the vessel at 1405 hrs, the distance when the lighthouse is abeam, and the time when the lighthouse will be abeam:

$$\Delta t_{AB} = 1405 - 1345 = 20 \text{ minutes}$$

$$AB = V \cdot \Delta t_{AB} = \frac{18 \times 20}{60} = 6 \text{ miles}$$

Distance from the light at 1405 hrs:

$$d_B = \frac{AB\sin\alpha}{\sin(\beta - \alpha)} = \frac{6\sin 22^\circ}{\sin(45^\circ - 22^\circ)} = 5.75 \text{ miles}$$

Distance from the light when it is abeam:

$$d_C = \frac{AB\sin\alpha\sin\beta}{\sin(\beta - \alpha)} = \frac{6\sin 22^\circ\sin 45^\circ}{\sin(45^\circ - 22^\circ)} = 4.1 \text{ miles}$$

Time when the light will be abeam:

$$\Delta t_{BC} = \frac{60 d_C}{V\tan\beta} = \frac{60 \times 4.1}{18\tan 45^\circ} = 13.7 \approx 14 \text{ minutes}$$

or

$$\Delta t_{BC} = \frac{\Delta t_{AB} \sin\alpha \cos\beta}{\sin(\beta - \alpha)} = \frac{20 \sin 22° \cos 45°}{\sin(45° - 22°)} = 13.6 \approx 14 \text{ minutes}$$

$$t_C = t_B + \Delta t_{BC} = 1405 + 14 = 1419 \text{ hrs}$$

Example 7 A vessel is steaming at 16 knots. At 1130 hrs, a lighthouse is bearing 15° off the bow; at 1145 hrs, the lighthouse is bearing 30° off the bow. Find the distance from the lighthouse to the vessel at 1145 hrs, the distance when the lighthouse is abeam, and the time when the lighthouse will be abeam:

$$\Delta t_{AB} = 1145 - 1130 = 15 \text{ minutes}$$

$$AB = V \cdot \Delta t_{AB} = \frac{16 \times 15}{60} = 4 \text{ miles}$$

Distance from the lighthouse at 1145 hrs:

$$d_B = AB = 4 \text{ miles}$$

Distance from the lighthouse when it is abeam:

$$d_C = AB \sin\beta = 4 \sin 30° = 2 \text{ miles}$$

Time when the lighthouse will be abeam:

$$\Delta t_{BC} = \Delta t_{AB} \cos\beta = 15 \cos 30° = 13 \text{ minutes}$$

$$t_C = t_B + \Delta t_{BC} = 1145 + 13 = 1158 \text{ hrs}$$