Seasonality in Cryptocurrency Volatility

May 20, 2021

1 Dataset: 1 minute OHLCV

	ETHUSDC	ETHUSDT	ETHUSD	BTCUSDT	BTCUSD	Sample Period
Kraken			O		O	Jan. 2019~now
Coinbase		O	O	O	O	Jan. 2019~now
Binance	O	O		O		Jan. 2019~now
Uniswap_v2	O					Aug, 2020~now
Sushiswap	O					Feb, 2021~now

For comparability, we choose the sample for 2019~now for centralized exchanges. We do not consider Sushiswap because the sample is too small. Pairs marked with the red circle in the above table will be dealt in this report. Volatility and volume patterns of pairs marked with the black circle show the same pattern to those with the red circle.

2 Measure of Volatility

We assume that daily return r_t follows a conditional normal distribution with conditional variance σ_t given the information set $\mathcal{F}_{t-1}$.

$$r_t|\mathcal{F}_{t-1} \sim N\left(0, \sigma_t^2\right)$$

We can estimate conditional variance by the simple quadratic variance. For example, daily conditional variance $\sigma_t^2 \approx RV_t^{day} = \sum_{h=1}^{1440}\left(\frac{p_{t,h}-p_{t,h-1}}{p_{t,h-1}}\right)^2 = \sum_{h=1}^{1440}\left(r_{t,h}\right)^2$ where $p_{t,h}$ is close price and $r_{t,h}$ is return for minute h of day t, respectively . Note that $p_{t,h-1440k} = p_{t-k,h}$

3 Seasonality

3.1 Weekly Pattern:

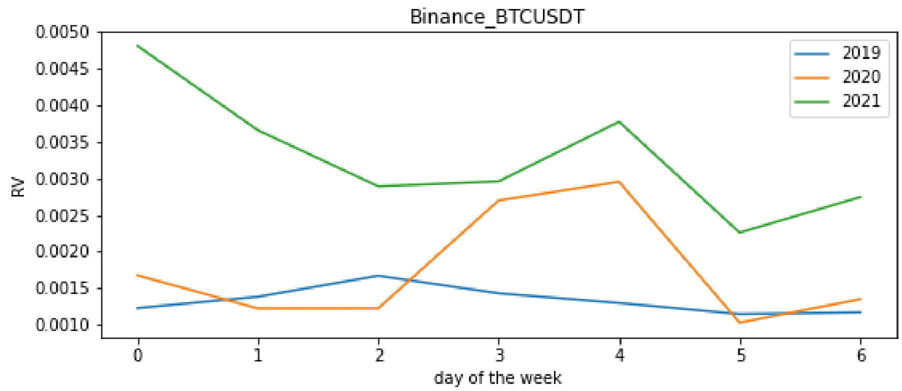
3.1.1 Volatility: 1minute RV

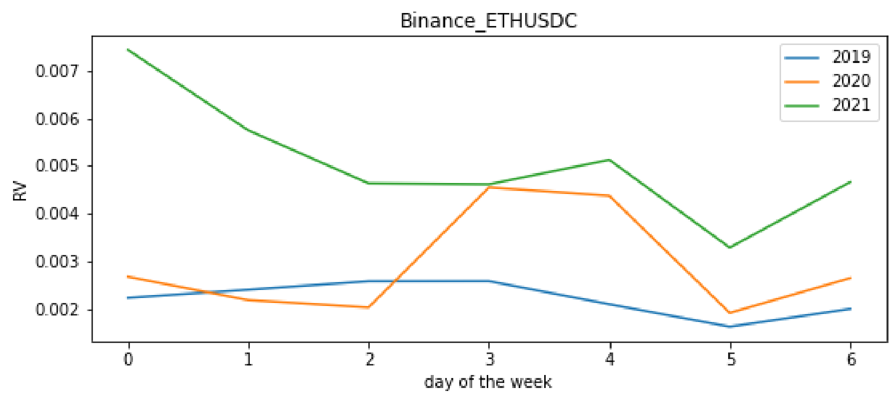
We define the mean of the realized variance for a day of the week i given a year y as

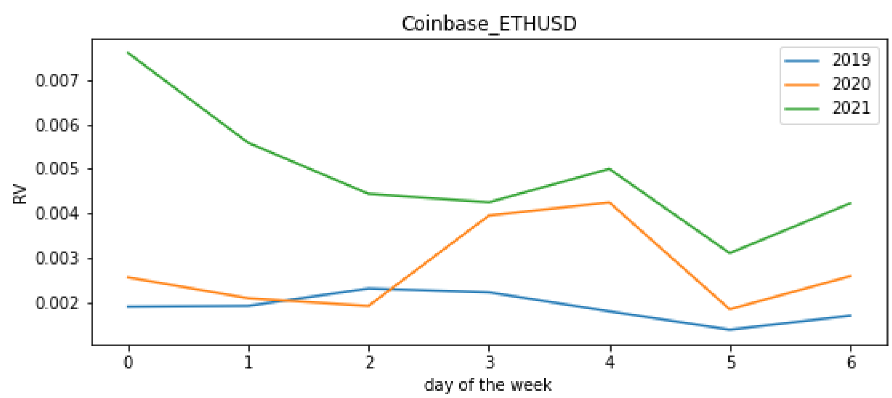
$$\overline{RV}_{y,i}^{dow} = \frac{1}{W}\sum_{w=0}^{W-1} RV_{y,7w+i}^{day}$$

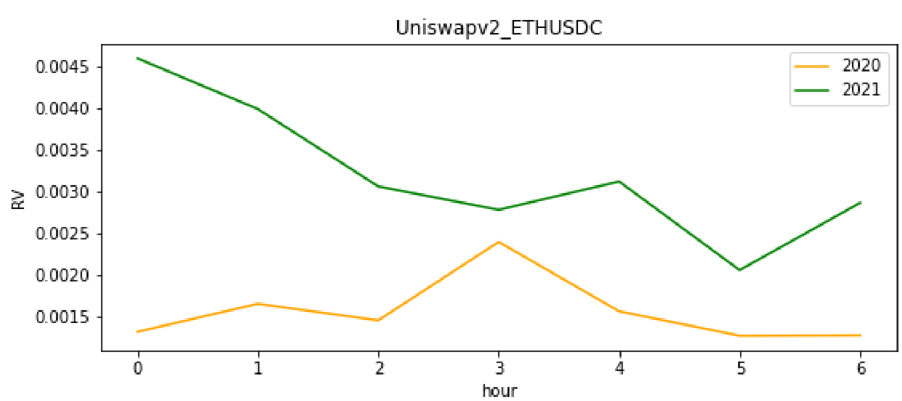
where $y = 2019, 2020, 2021$ and $i = 0, \ldots, 6$, 0 for Monday and 6 for Sunday. W is the number of weeks for y. $RV_{y,7w+i}^{day}$ is a realied variance of the i-th day of the w-th week for a year y.

$\overline{RV}_{y,i}^{dow}$ is a simple average of realized variance for the i-th day of the week. The following plots show$\overline{RV}_{y,i}^{dow}$ for each year. i is on x-axis and each line represents $\overline{RV}_{y,i}^{dow}$ for the same y.


Binance_BTCUSDT
RV
day of the week
2019
2020
2021


Binance_ETHUSDC
RV
day of the week
2019
2020
2021


Coinbase_ETHUSD
RV
day of the week
2019
2020
2021


Uniswapv2_ETHUSDC
RV
hour
2020
2021

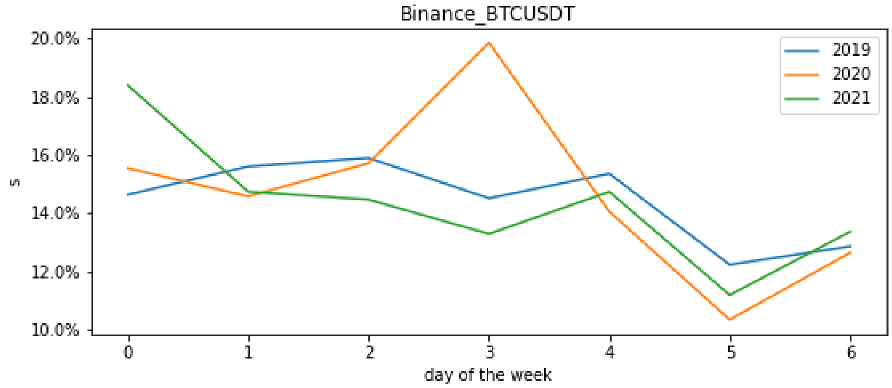
Prices in centralized exchanges show similar volatility patterns. The RV of Uniswap_v2 shows similar patterns but the different level. That is because market microstructure noise such as bid-asks bounce or tick size impact 1-minute RV of centralized market, which does not happen in the AMM based decentralized exchanges.

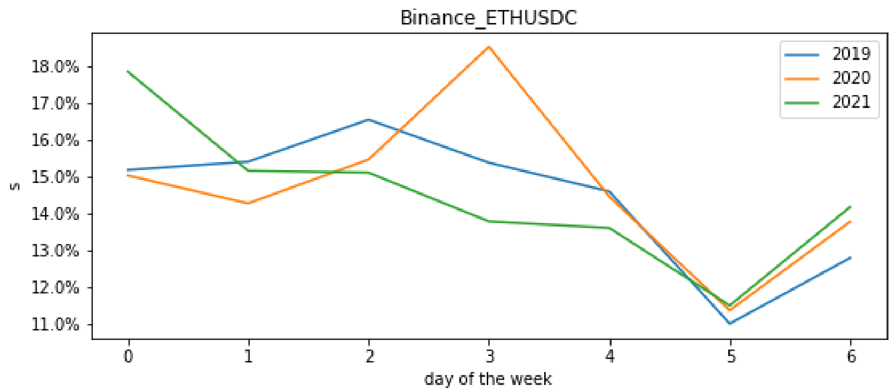
The level of volatility is high in 2021 and low in the other years. This makes comparison hard. We can adjust this level by the following statistics that represents seasonality.

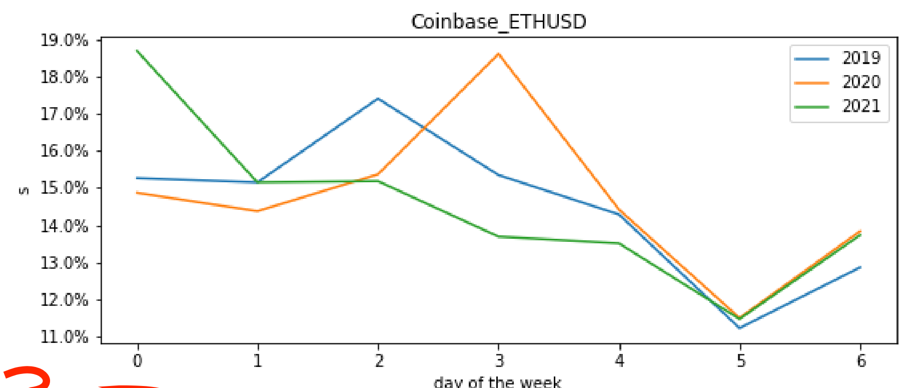
$$s_{y,i}^{dow} = \frac{100}{W} \sum_{w=0}^{W-1} \frac{RV_{y,7w+i}^{day}}{\sum_{j=0}^{6} RV_{y,7w+i-j}^{day}}$$

$\sum_{j=0}^{6} RV_{y,7w+i-j}^{day}$ is a realized variance for the past week, so $s_{y,i}^{dow}$ is an average percentage of weekly realized variance that daily realized variance for the i-th day of the week accounts for. Note that $\sum_{i=0}^{6} s_{y,i}^{day} \approx 100$ since weekly realized variance is the sum of the daily realized variances. Since $s_{y,i}^{dow}$ is bounded above by 1, it is more robust to outliers than $\overline{RV}_{y,i}^{dow}$.

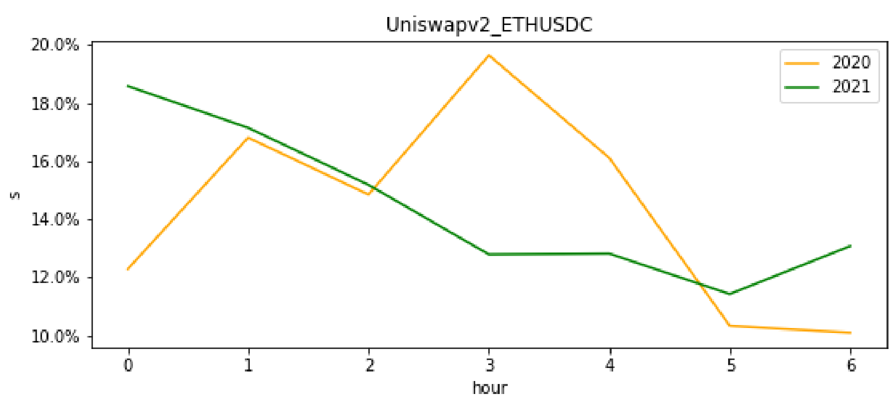
The following plots show $s_{y,i}^{dow}$. i is on the x-axis.


Binance_BTCUSDT
2019
2020
2021
s
20.0%
18.0%
16.0%
14.0%
12.0%
10.0%
0
1
2
3
4
5
6
day of the week


Binance_ETHUSDC
2019
2020
2021
s
18.0%
17.0%
16.0%
15.0%
14.0%
13.0%
12.0%
11.0%
0
1
2
3
4
5
6
day of the week


Coinbase_ETHUSD
2019
2020
2021
s
19.0%
18.0%
17.0%
16.0%
15.0%
14.0%
13.0%
12.0%
11.0%
0
1
2
3
4
5
6
day of the week

?


Uniswapv2_ETHUSDC
2020
2021
s
20.0%
18.0%
16.0%
14.0%
12.0%
10.0%
0
1
2
3
4
5
6
hour

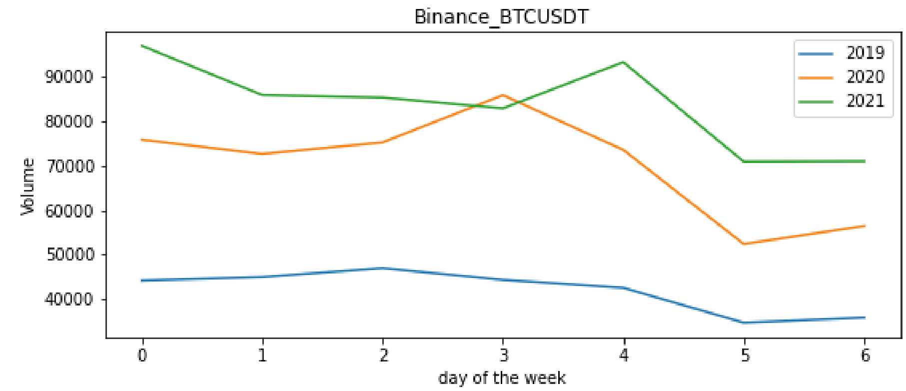
4

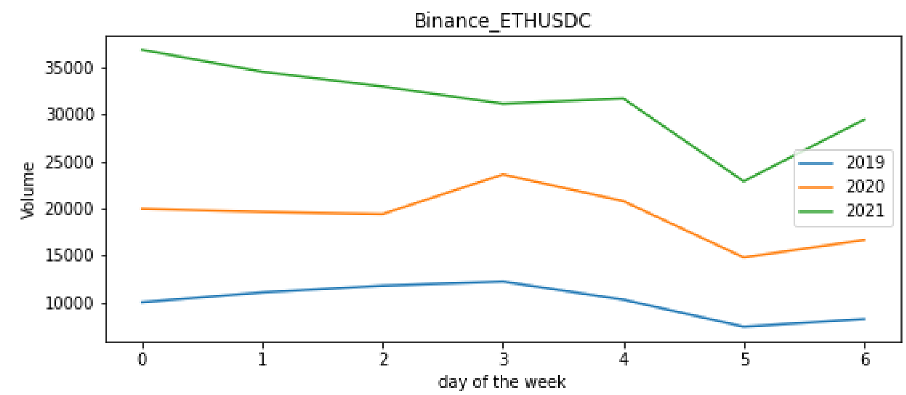
3.1.2 Volume

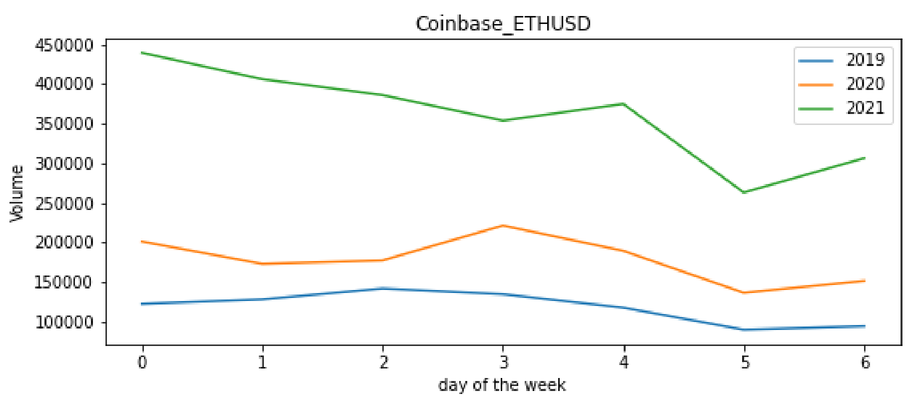
Similar to the realized volatility, we define the mean of the volume for a day of the week i given a year y as

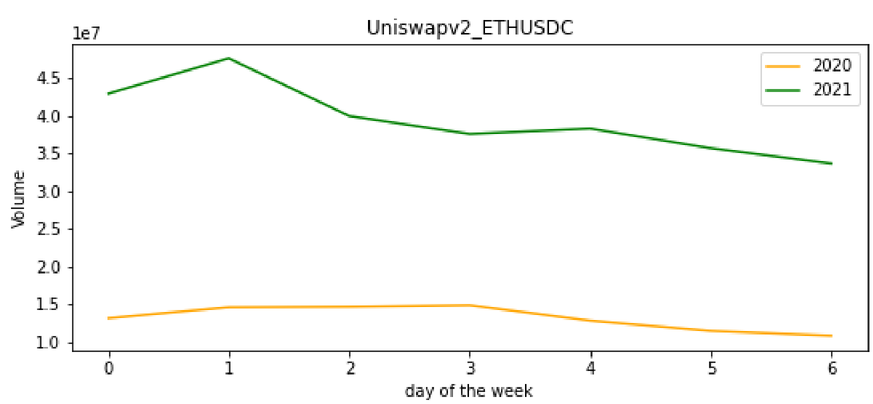
$$\overline{V}_{y,i}^{dow} = \frac{1}{W}\sum_{w=0}^{W-1} V_{y,7w+i}^{day}$$

where $y = 2019, 2020, 2021$ and $i = 0, \ldots, 6$. W is the number of weeks for y. $V_{y,7w+i}^{day}$ is a daily volume of the i-th day of the w-th week for a year y. $\overline{V}_{y,i}^{dow}$ is a simple average of volume for the i-th day of the week.


Binance_BTCUSDT
Volume
day of the week
2019
2020
2021


Binance_ETHUSDC
Volume
day of the week
2019
2020
2021


Coinbase_ETHUSD
Volume
day of the week
2019
2020
2021

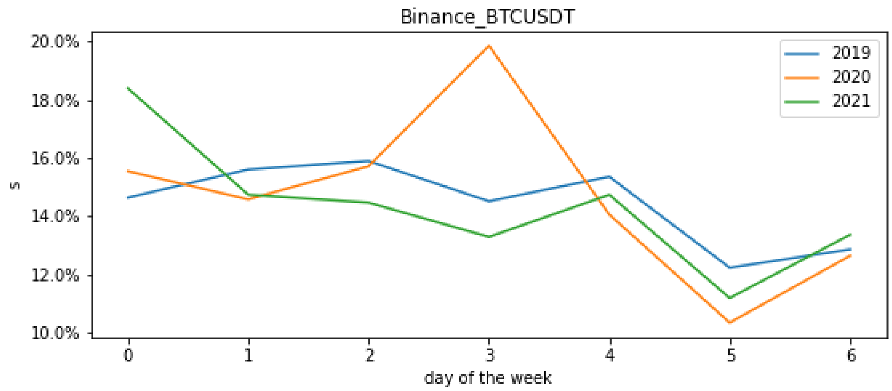

Uniswapv2_ETHUSDC
1e7
Volume
day of the week
2020
2021

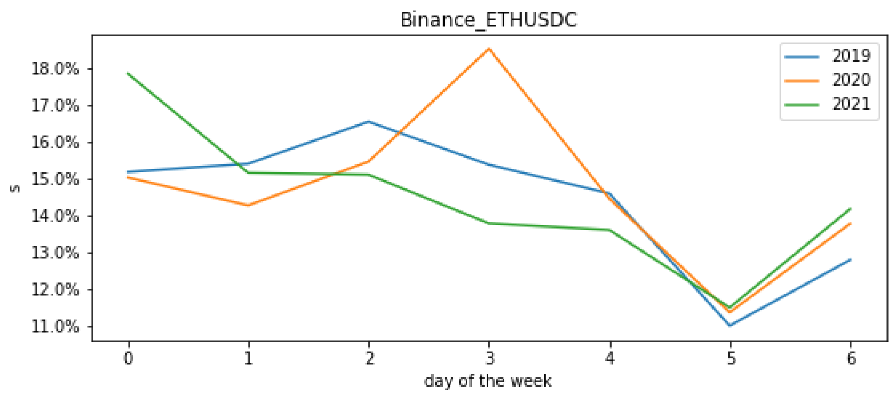
Volumes in centralized exchanges show similar patterns regardless of currency pairs. Uniswap_v2 has been rapidly growing over the past years, so its volume patterns are affected by the growth effect as well. Similar to the volatility analysis, we introduce seasonality statistics which is more robust to outliers.

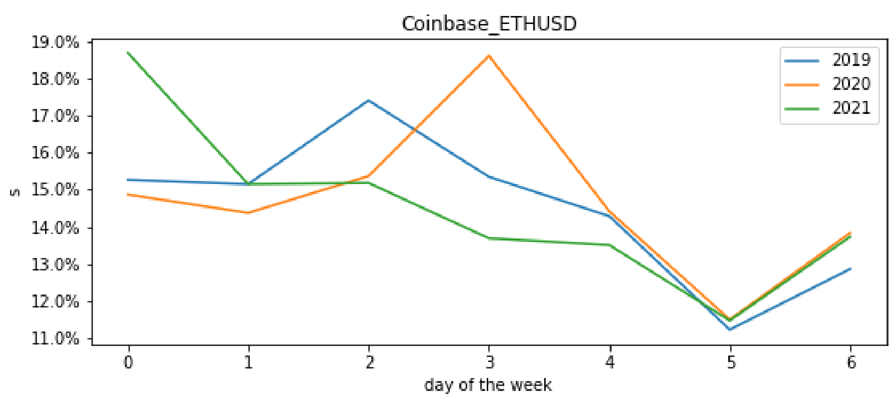
$$S^{dow}_{y,i} = \frac{100}{W} \sum_{w=0}^{W-1} \frac{V^{day}_{y,7w+i}}{\sum_{j=0}^{6} V^{day}_{y,7w+i-j}}$$

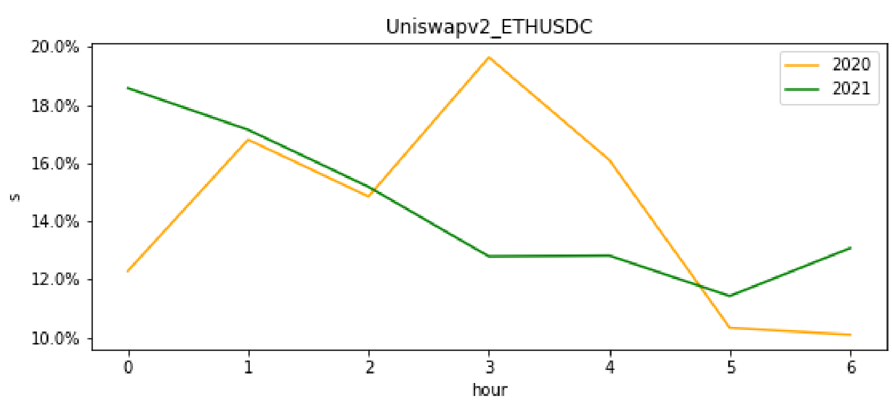
$\sum_{j=0}^{6} V^{day}_{y,7w+i-j}$ is the volume for the past week, so $S^{dow}_{y,i}$ is an average percentage of weekly volume that the i-th day of the week accounts for. Note that $\sum_{i=0}^{6} S^{dow}_{y,i} \approx 100$ since weekly realized variance is the sum of the daily realized variances.

The following plots show $S^{dow}_{y,i}$. i is on the x-axis.


Binance_BTCUSDT
20.0%
18.0%
16.0%
14.0%
12.0%
10.0%
s
0 1 2 3 4 5 6
day of the week
2019
2020
2021


Binance_ETHUSDC
18.0%
17.0%
16.0%
15.0%
14.0%
13.0%
12.0%
11.0%
s
0 1 2 3 4 5 6
day of the week
2019
2020
2021


Coinbase_ETHUSD
19.0%
18.0%
17.0%
16.0%
15.0%
14.0%
13.0%
12.0%
11.0%
s
0 1 2 3 4 5 6
day of the week
2019
2020
2021


Uniswapv2_ETHUSDC
20.0%
18.0%
16.0%
14.0%
12.0%
10.0%
s
0 1 2 3 4 5 6
hour
2020
2021

3.2 Intraday Pattern

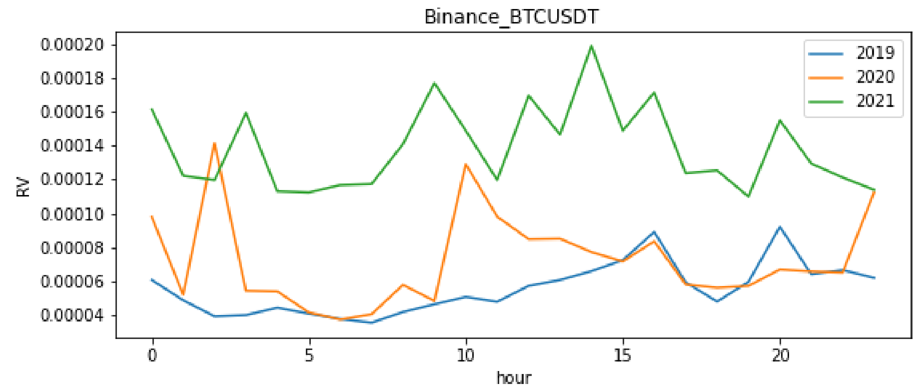
3.2.1 Volatility: 1minute RV

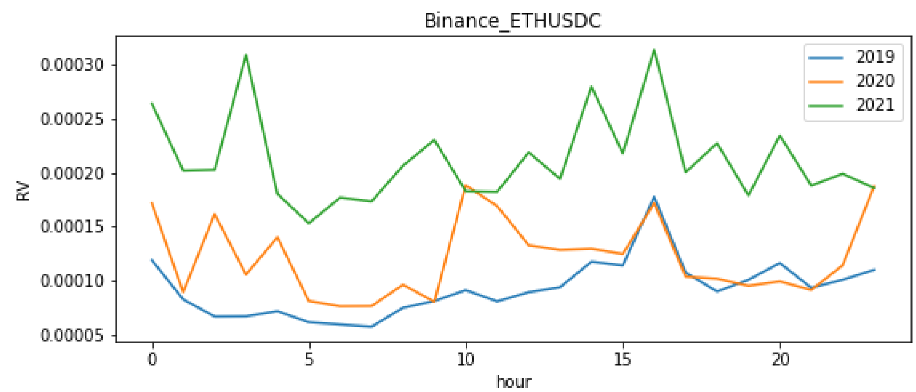
We define mean of the realized variance for a day of the week i given a year y as

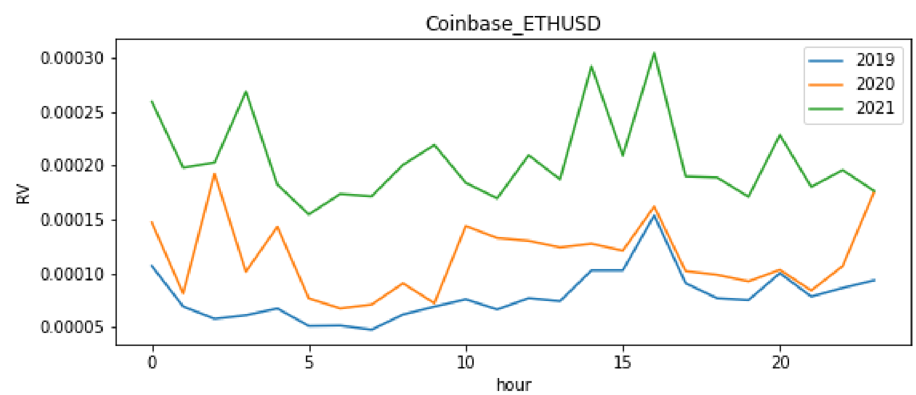
$$\overline{RV}_{y,i}^{hour} = \frac{1}{D}\sum_{d=0}^{D-1} RV_{y,24d+i}^{hour}$$

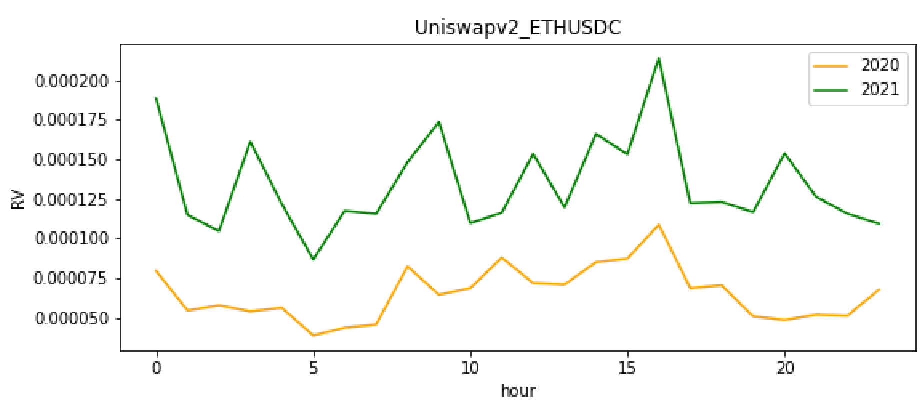
where $y = 2019, \ldots, 2021$ and $i = 0, \ldots, 23$ refers the hour of the day. D is the number of hours for y. $RV_{y,24d+i}^{hour}$ is a realied variance of the i-th hour of the d-th day for a year y.

$\overline{RV}_{y,i}^{hour}$ is a simple average of realized variance for the i-th hour of the day. The following plots show$\overline{RV}_{y,i}^{hour}$ for each year.


Binance_BTCUSDT
0.00020
0.00018
0.00016
0.00014
0.00012
0.00010
0.00008
0.00006
0.00004
RV
0
5
10
15
20
hour
2019
2020
2021


Binance_ETHUSDC
0.00030
0.00025
0.00020
0.00015
0.00010
0.00005
RV
0
5
10
15
20
hour
2019
2020
2021


Coinbase_ETHUSD
0.00030
0.00025
0.00020
0.00015
0.00010
0.00005
RV
0
5
10
15
20
hour
2019
2020
2021


Uniswapv2_ETHUSDC
0.000200
0.000175
0.000150
0.000125
0.000100
0.000075
0.000050
RV
0
5
10
15
20
hour
2020
2021

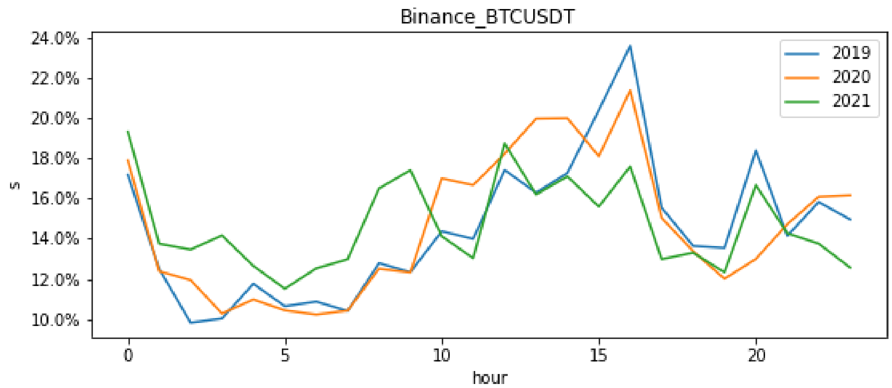
Similar to the weekly case, Uniswap and the centralized exchanges exhibit similar patterns.

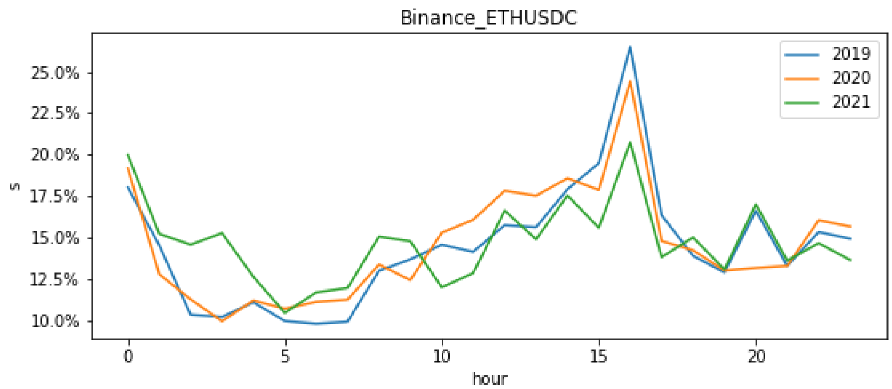
Seasonlity statistics for hourly volatility is

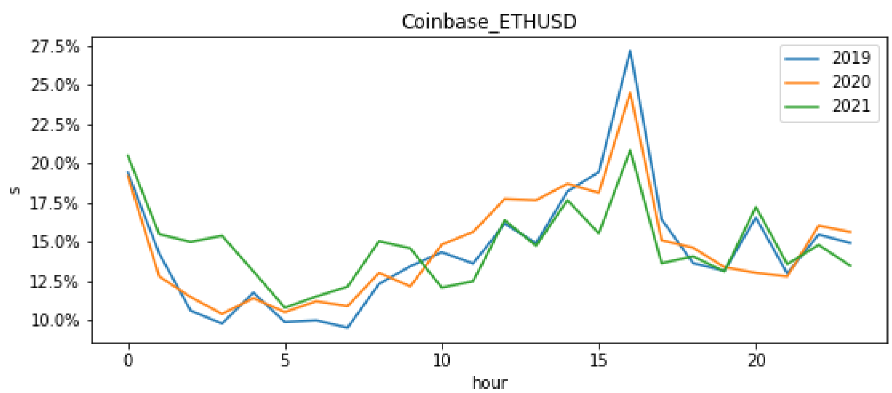
$$s^{hour}_{y,i} = \frac{100}{365} \sum_{d=0}^{365} \frac{RV^{hour}_{y,24d+i}}{\sum_{j=0}^{23} RV^{hour}_{y,24d+i-j}}$$

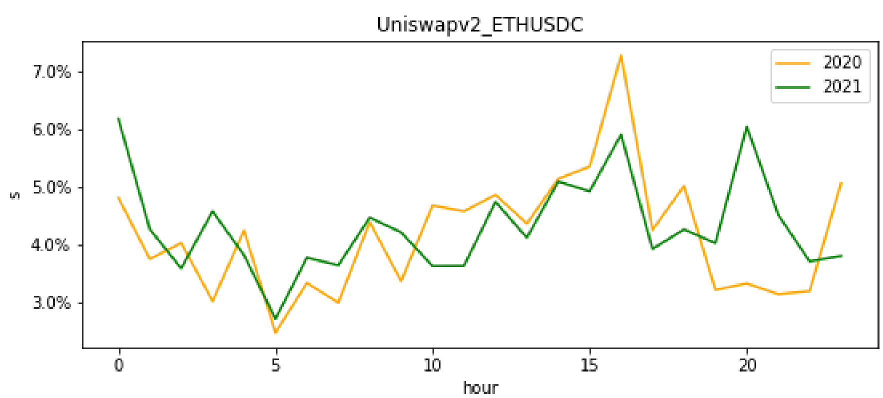
$\sum_{j=0}^{23} RV^{hour}_{y,24d+i-j}$ is a realized variance for the past day, so $s^{hour}_{y,i}$ is an average percentage of daily realized variance that the i-th hour of the day accounts for. Note that $\sum_{i=0}^{23} s^{id}_{y,i} \approx 100$ since daily realized variance is the sum of the hourly realized variances.

The following plots show $s^{hour}_{y,i}$. i is on the x-axis.


Binance_BTCUSDT
24.0%
22.0%
20.0%
18.0%
16.0%
14.0%
12.0%
10.0%
s
0
5
10
15
20
hour
2019
2020
2021


Binance_ETHUSDC
25.0%
22.5%
20.0%
17.5%
15.0%
12.5%
10.0%
s
0
5
10
15
20
hour
2019
2020
2021


Coinbase_ETHUSD
27.5%
25.0%
22.5%
20.0%
17.5%
15.0%
12.5%
10.0%
s
0
5
10
15
20
hour
2019
2020
2021


Uniswapv2_ETHUSDC
7.0%
6.0%
5.0%
4.0%
3.0%
s
0
5
10
15
20
hour
2020
2021

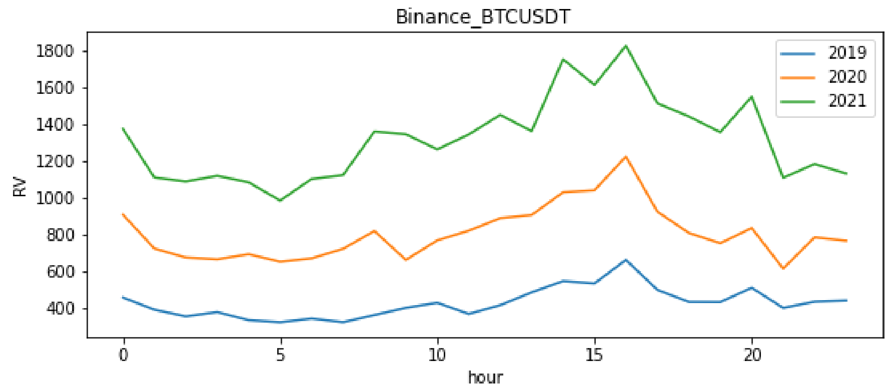
3.2.2 Volume

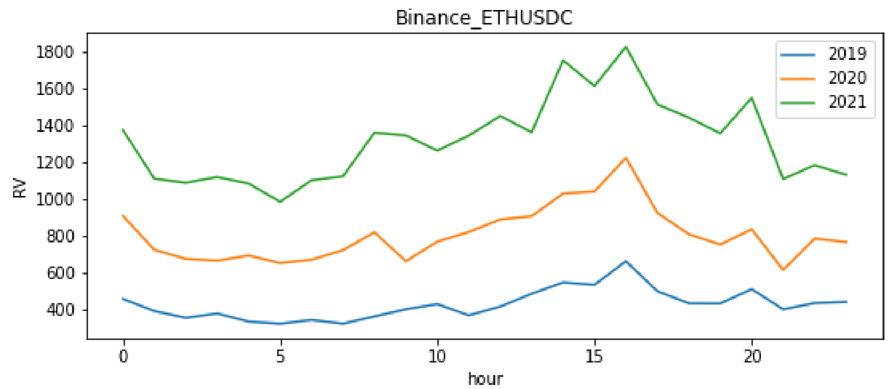
We define the mean of the volume for an hour of the day given a year y as

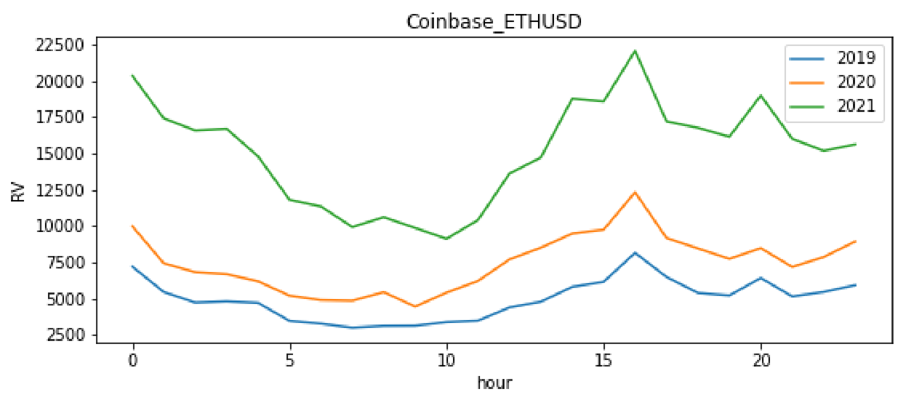
$$\overline{V}_{y,i}^{hour} = \frac{1}{D}\sum_{d=0}^{D-1} V_{y,24d+i}^{hour}$$

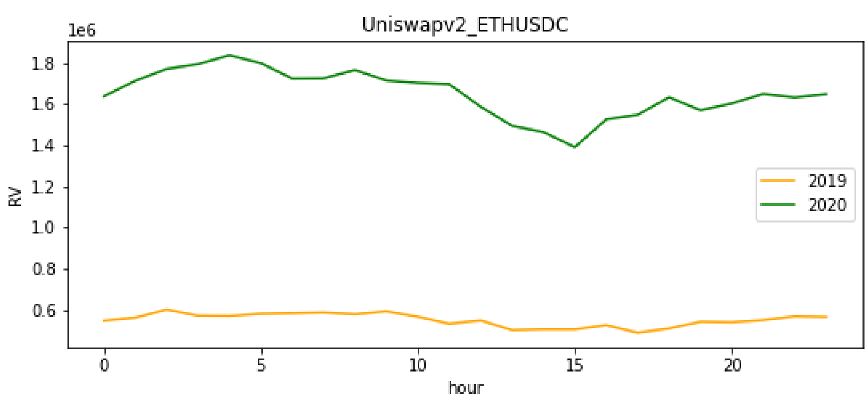
where $y = 2019, \ldots, 2021$ and $i = 0, \ldots, 23$ refers the hour of the day. D is the number of hours for y. $V_{y,24d+i}^{hour}$ is a volume of the i-th hour of the d-th day for a year y.

$\overline{V}_{y,i}^{hour}$ is a simple average of hourly volume for the i-th hour of the day. The following plots show$\overline{V}_{y,i}^{hour}$ for each year. i is on x-axis and each line represents $\overline{V}_{y,i}^{hour}$for the same y.


Binance_BTCUSDT
2019
2020
2021
RV
hour


Binance_ETHUSDC
2019
2020
2021
RV
hour


Coinbase_ETHUSD
2019
2020
2021
RV
hour

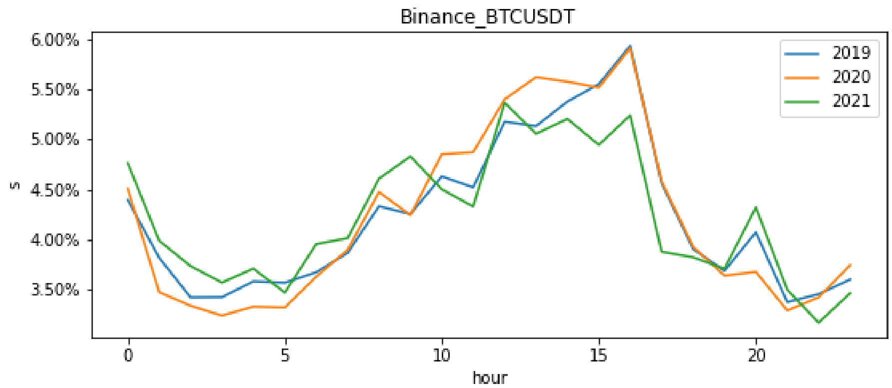

Uniswapv2_ETHUSDC
1e6
2019
2020
RV
hour

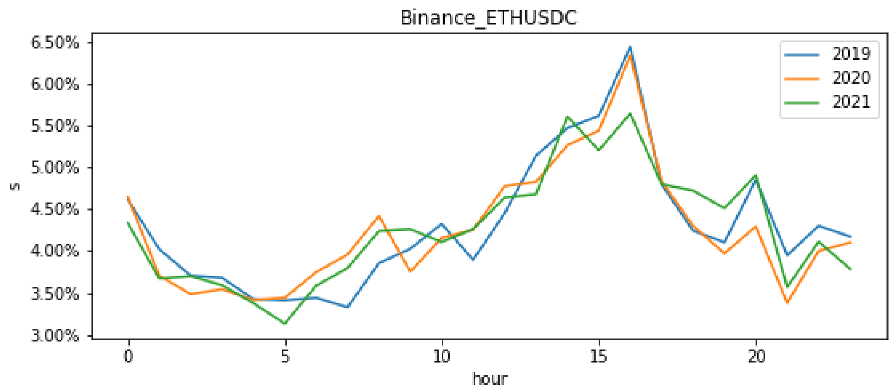
Volumes in centralized exchanges show similar patterns regardless of currency pairs. Since Uniswap_v2 has grown exponentially over the past years, it shows very different volatility patterns compared the others and it makes comparison hard. We can reduce this effect this by the following statistics.

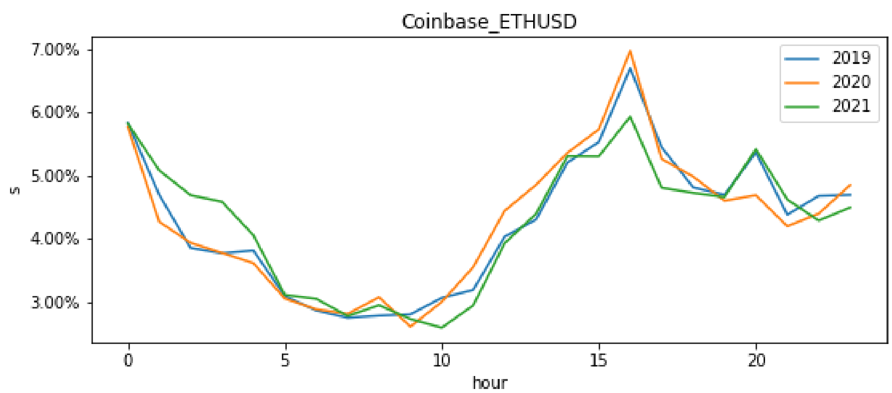
$$S_{y,i}^{hour} = \frac{100}{D} \sum_{d=0}^{D-1} \frac{V_{y,24d+i}^{hour}}{\sum_{j=0}^{23} V_{y,24d+i-j}^{hour}}$$

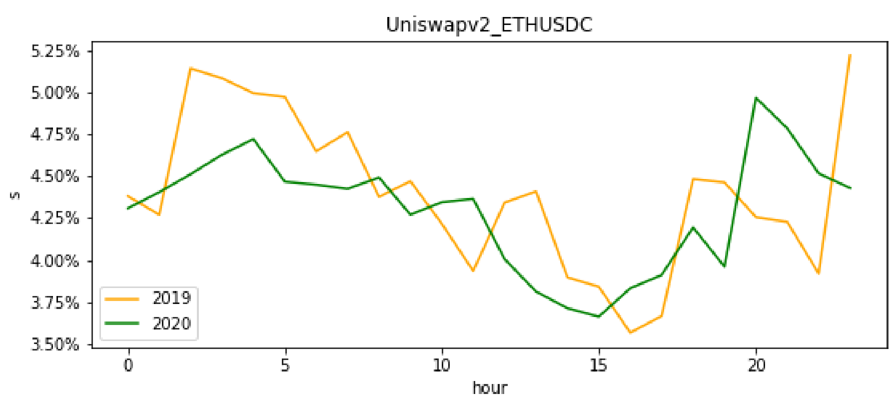
$\sum_{j=0}^{23} V_{y,24d+i-j}^{hour}$ is a realized variance for the past day, so $S_{y,i}^{hour}$ is an average percentage of daily volume that hourly volume of the i-th hour of the day accounts for. Note that $\sum_{i=0}^{23} S_{y,i}^{hour} \approx 100$

The following plots show $S_{y,i}^{hour}$. i is on the x-axis.


Binance_BTCUSDT
6.00%
5.50%
5.00%
4.50%
4.00%
3.50%
s
0
5
10
15
20
hour
2019
2020
2021


Binance_ETHUSDC
6.50%
6.00%
5.50%
5.00%
4.50%
4.00%
3.50%
3.00%
s
0
5
10
15
20
hour
2019
2020
2021


Coinbase_ETHUSD
7.00%
6.00%
5.00%
4.00%
3.00%
s
0
5
10
15
20
hour
2019
2020
2021


Uniswapv2_ETHUSDC
5.25%
5.00%
4.75%
4.50%
4.25%
4.00%
3.75%
3.50%
s
0
5
10
15
20
hour
2019
2020

The volume in Uniswap moves in the opposite direction to that of the centralized exchanges. It hits local maximum around 5 and decreases over time until 15. It is still open question what causes the difference. It may be caused by geographical component. For example, Chinese traders have limited access to the main centralized exchanges, so they may choose to use Uniswap instead.

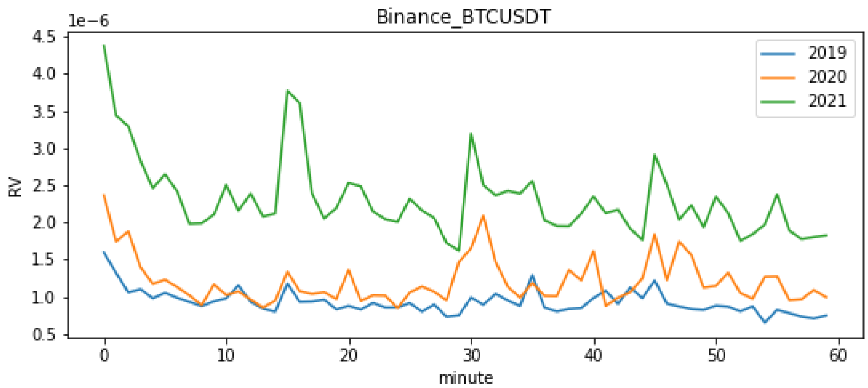
3.3 Intra-hour Pattern

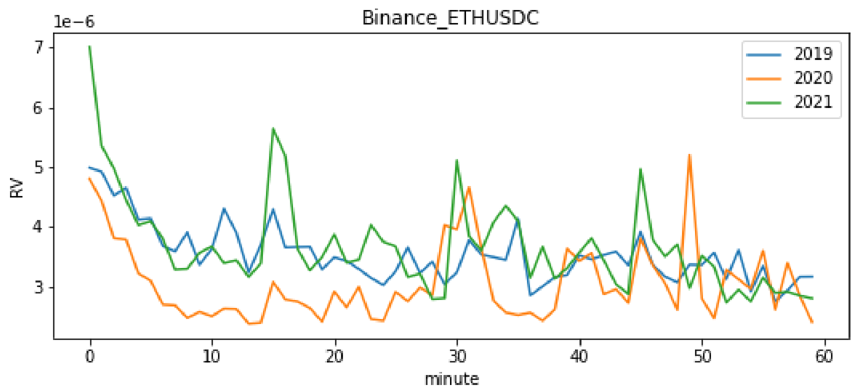
3.3.1 Volatility

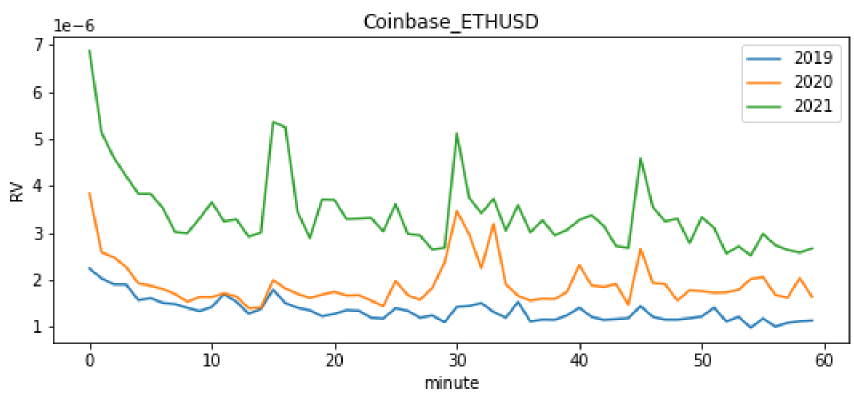
Similarly, we define

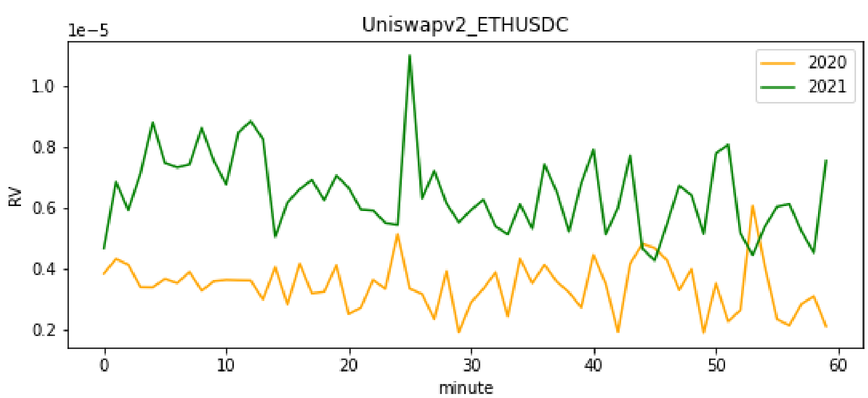
$$\overline{RV}_{y,i}^{min} = \frac{1}{H} \sum_{h=0}^{H-1} RV_{y,60h+i}^{1min}$$

for $i = 0, \ldots 59$ and H is the number of hours for year y, so $H = 24 \times 365$.


Binance_BTCUSDT
1e−6
RV
minute
2019
2020
2021

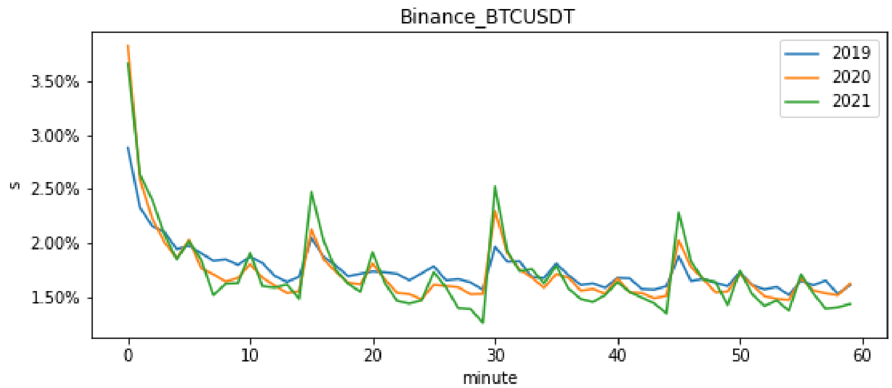

Binance_ETHUSDC
1e−6
RV
minute
2019
2020
2021

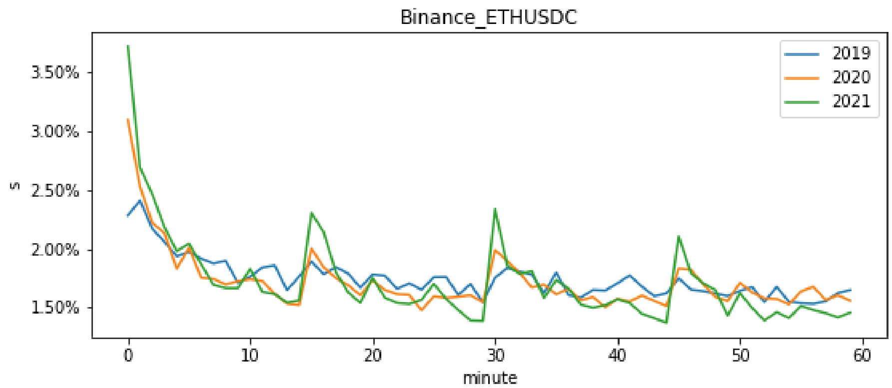

Coinbase_ETHUSD
1e−6
RV
minute
2019
2020
2021

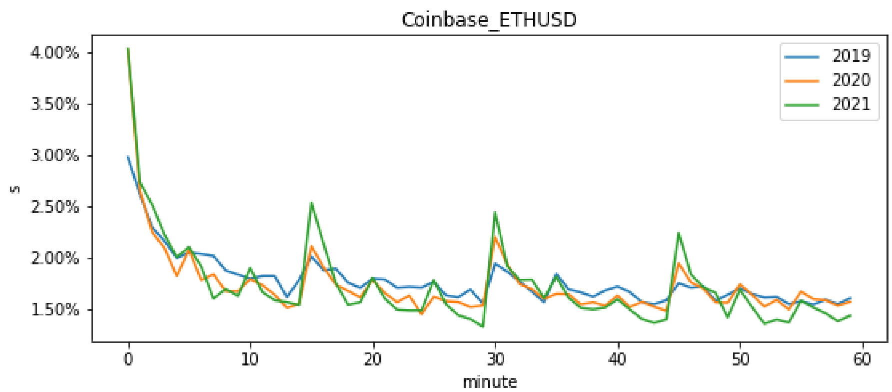

Uniswapv2_ETHUSDC
1e−5
RV
minute
2020
2021

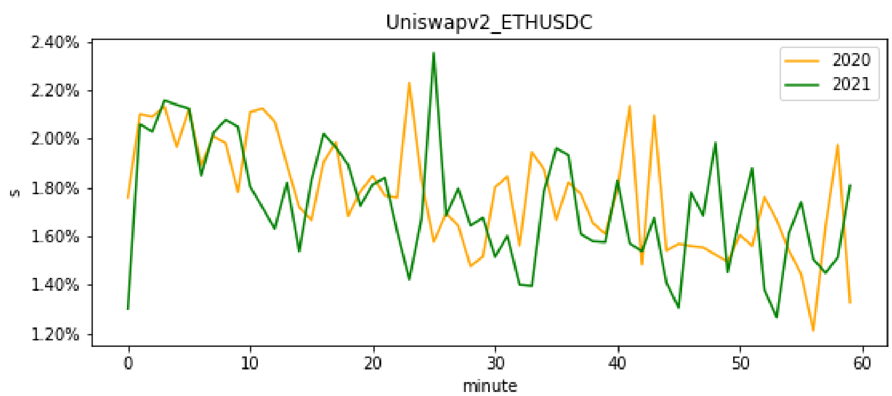
Seasonlity statistics

$$s_{y,i}^{min} = \frac{100}{H} \sum_{h=0}^{H} \frac{RV_{y,60h+i}^{min}}{\sum_{j=0}^{59} RV_{y,60h+i-j}^{min}}$$





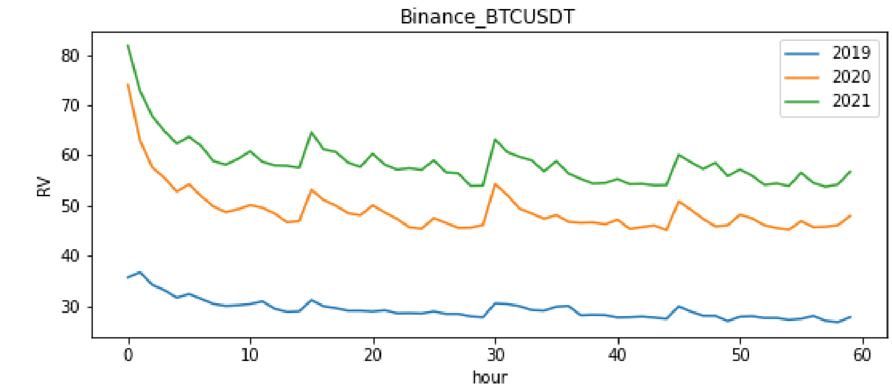


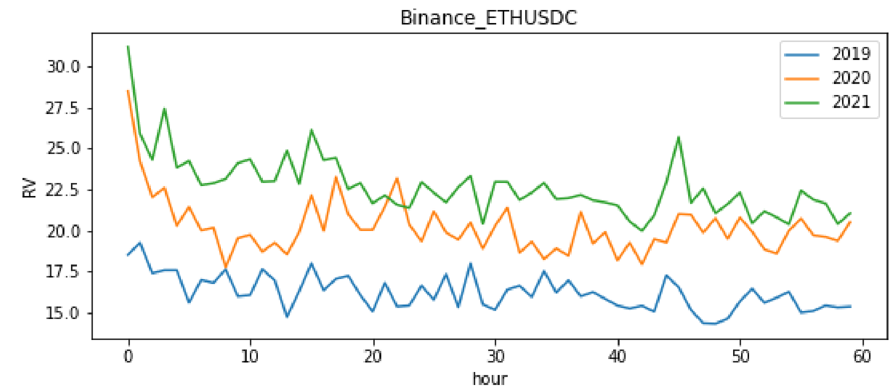


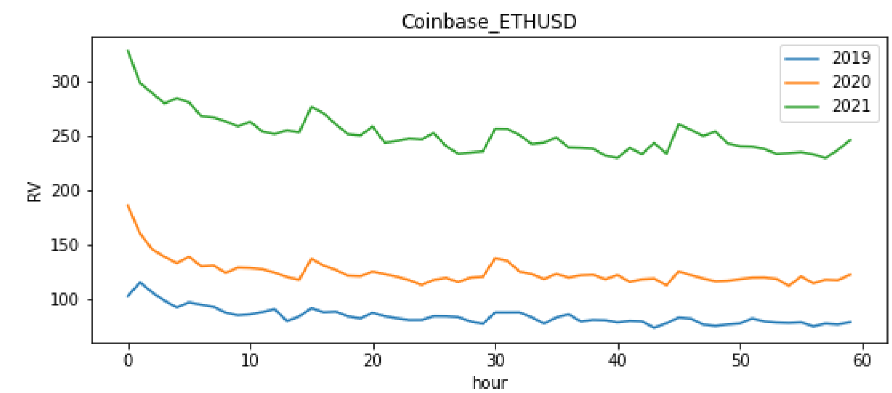
In the centralized exchanges, the volatility sharply increases at 0,15,30,45 minutes and this pattern has been intensified over the periods. However, Uniswap does not show this pattern.

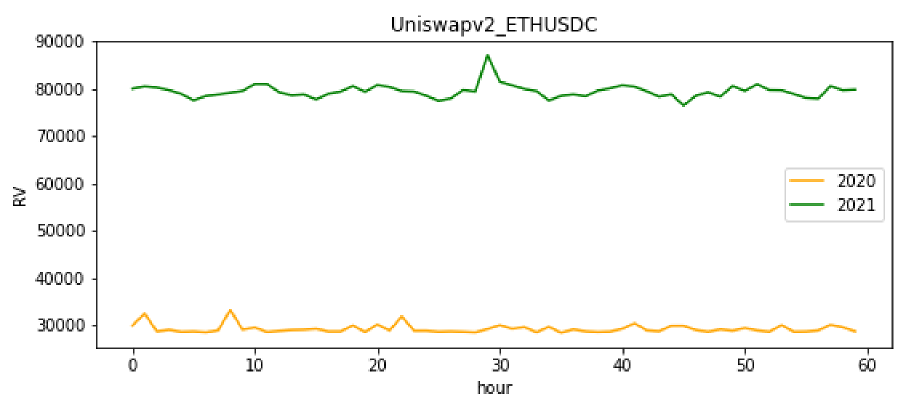
3.3.2 Volume

$$\overline{V}^{min}_{y,i} = \frac{1}{H}\sum_{h=0}^{H-1} V^{min}_{y,60h+i}$$


Binance_BTCUSDT
RV
hour
2019
2020
2021


Binance_ETHUSDC
RV
hour
2019
2020
2021


Coinbase_ETHUSD
RV
hour
2019
2020
2021


Uniswapv2_ETHUSDC
RV
hour
2020
2021

$$S_{y,i}^{min} = \frac{100}{H} \sum_{h=0}^{H-1} \frac{V_{y,60h+i}^{min}}{\sum_{j=0}^{59} V_{y,60h+i-j}^{min}}$$