Math 211-01 Spring 2022

Final Exam (Take Home Part) Due by 8am on May 10, 2022 via Gradescope

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DIRECTIONS: You must work on this exam independently but you can ask your instructor for clarifications. Read the questions carefully and follow the instructions. Write your answers/solutions legibly in the space provided. Please use a copy of this document for your submission. I will not accept answers that are written in a different document.

1. Consider the following system of linear equations:

$$\begin{cases} x_1 - 4x_2 + 2x_3 &= -1 \\ 3x_2 + 5x_3 &= -3 \\ -2x_1 + 8x_2 - 4x_3 &= 2 \end{cases}$$

(a) (1 PT) Express the above system as a matrix-vector equation of the form $A\mathbf{x} = \mathbf{b}$.

$$\begin{bmatrix} 1 & -4 & 2 \\ 0 & 3 & 5 \\ -2 & 8 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$$

(b) (1 PT) What is the RREF of the augmented matrix of the above system? (You may use Matlab but your constants should be in the form of improper fractions.)

$$\left[\begin{array}{ccc|c} 1 & 0 & 26/3 & -5 \\ 0 & 1 & 5/3 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

(c) (1 PT) Describe the solution set of the above system parametrically.

$x_1 = -5 - \frac{26}{3}x_3$

$x_2 = -1 - \frac{5}{3}x_3$

$x_3 = x_3$

$\{(-5 - \frac{26}{3}x_3, -1 - 5/3\,x_3, x_3) \mid x_3 \in \mathbb{R}\}$

(d) (1 PT) Describe the solution set of the associated homogeneous equation $A\mathbf{x} = \mathbf{0}$ parametrically.

$\{(-\frac{26}{3}x_3, -5/3\,x_3, x_3) \mid x_3 \in \mathbb{R}\}$

2. Let

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 7 \\ 2 \\ 3 \\ 6 \\ 5 \\ 4 \end{bmatrix}.$$

(a) **(2 PTS) Find a least-square solution to $A\mathbf{x} = \mathbf{b}$. That is, find $\hat{\mathbf{x}}$ satisfying $A^T A\hat{\mathbf{x}} = A^T\mathbf{b}$. (You may use Matlab, but write the entries of your answer in improper fraction form.)**

Solution:

$$A^TA = \begin{bmatrix} 6 & 3 & 3 \\ 3 & 3 & 0 \\ 3 & 0 & 3 \end{bmatrix} = X \qquad [X \mid Y] = \begin{bmatrix} 6 & 3 & 3 & 27 \\ 3 & 3 & 0 & 12 \\ 3 & 0 & 3 & 15 \end{bmatrix} = B$$

$$A^Tb = \begin{bmatrix} 27 \\ 12 \\ 15 \end{bmatrix} = Y \qquad \text{rref}(B) = \begin{bmatrix} 1 & 0 & 1 & 5 \\ 0 & 1 & -1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad \hat{x} = \begin{bmatrix} 5 - x_3 \\ -1 + x_3 \\ x_3 \end{bmatrix}$$

Answer:

$$\begin{bmatrix} 5 \\ -1 \\ 0 \end{bmatrix}$$

(b) **(2 PTS) Compute the least square error $||A\hat{\mathbf{x}} - \mathbf{b}||$.**

Solution:

$$A\hat{x} - b = \begin{bmatrix} -3 \\ 2 \\ 1 \\ -1 \\ 0 \\ 1 \end{bmatrix} \qquad \sqrt{(-3)^2 + 2^2 + 1^2 + (-1)^2 + (1)^2} = 4$$

Answer:

4

3. Suppose $A = \begin{bmatrix} B & 0_{3\times2} \\ D & C \end{bmatrix}$, where $B = \begin{bmatrix} -1 & -1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 2 \\ 1 & 4 \end{bmatrix}$ and $D = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

(a) (1 PT) Find C^{-1}.

Answer:

$\begin{bmatrix} -2 & 1 \\ 1/2 & 0 \end{bmatrix}$

(b) (3 PTS) Find B^{-1} by row reducing $[B|I_3]$. Show your solution, indicating the row operations in each step.

Answer:

$\begin{bmatrix} -1/3 & 2/3 & -1/3 \\ -1/3 & -1/3 & 2/3 \\ 1/3 & 1/3 & 1/3 \end{bmatrix}$

(c) (2 PTS) Compute $\det(A)$. Show your solution. Do not use Matlab.

Answer:

-6

(d) (3 PTS) Find the $\overset{\text{4th col}}{(4}, \overset{\text{3rd row}}{3)}$ entry of A^{-1} using Cramer's Rule. Show your solution. Do not use Matlab.

Answer:

$2/3$

c. $\det(C) = 0 - 2 = -2$

$\det(B) = 0 + 1 + 0 - (0 - 1 - 1) = 3$

$\det A = -2 \cdot 3 = -6$

d. $M = \begin{bmatrix} -1 & -1 & 1 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 4 \end{bmatrix} \xrightarrow{R_1 \to R_1 - R_2} \begin{bmatrix} -2 & -1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 2 \\ 1 & 1 & 1 & 4 \end{bmatrix}$

$R_2 \to R_4 - R_2$
$R_3 \to R_4 - R_3$ $\begin{bmatrix} -2 & -1 & 0 & 0 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 2 \\ 1 & 1 & 1 & 4 \end{bmatrix} \xrightarrow{R_2 \to R_2 - 2R_3}$

$\begin{array}{c} A \\ \begin{bmatrix} -2 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ 1 & 1 & 1 & 4 \end{bmatrix} \\ B \end{array}$ $\det(A) = -2$, $\det(B) = -2$, $\det M = 4$

$\frac{1}{-6} \cdot (-1)^{4+3} \cdot (4) = 2/3$

Solutions:
(Clearly indicate which item the solution is for.)

a) $\frac{1}{0-2} \begin{bmatrix} 4 & -2 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 1/2 & 0 \end{bmatrix}$

b) $\left[\begin{array}{ccc|ccc} -1 & -1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow{R_2 \to R_2 + R_1} \left[\begin{array}{ccc|ccc} -1 & -1 & 1 & 1 & 0 & 0 \\ 0 & -1 & 2 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R_1 \to -R_1]{R_3 \to R_3 + R_2} \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & -1 & 0 & 0 \\ 0 & -1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 3 & 1 & 1 & 1 \end{array}\right]$

$\xrightarrow[R_3 \to \frac{1}{3}R_3]{R_1 \to R_1 + R_2} \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 2 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1/3 & 1/3 & 1/3 \end{array}\right] \xrightarrow[R_2 \to -R_2 + 2R_3]{R_1 \to R_1 - R_3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -1/3 & 2/3 & -1/3 \\ 0 & 1 & 0 & -1/3 & -1/3 & 2/3 \\ 0 & 0 & 1 & 1/3 & 1/3 & 1/3 \end{array}\right]$

4. (You can use Matlab for the following items.) Let $T : \mathbb{R}^3 \longrightarrow \mathbb{R}^3$ be a linear transformation such that

$$T\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right) = \begin{bmatrix}4\\0\\2\end{bmatrix}, T\left(\begin{bmatrix}1\\2\\3\end{bmatrix}\right) = \begin{bmatrix}-1\\1\\1\end{bmatrix} \text{ and } T\left(\begin{bmatrix}1\\4\\9\end{bmatrix}\right) = \begin{bmatrix}0\\0\\0\end{bmatrix}.$$

(a) (3 PTS) Find the standard matrix rep. of T.

Answer:

$$\begin{bmatrix}14.5 & -16 & 5.5\\ -2.5 & 4 & -1.5\\ 3.5 & -2 & .5\end{bmatrix}$$

(b) (2 PTS) Find a basis for the $ker(T)$.

Answer:

$$\left\{\begin{bmatrix}1/9\\ 4/9\\ 1\end{bmatrix}\right\}$$

(c) (1 PT) Is T onto?

Answer:

No

Solutions: (Clearly indicate which item the solution is for.)

a)

$$\left[\begin{array}{ccc|c} t_{11} & t_{12} & t_{13} & 4\\ t_{11} & 2t_{12} & 3t_{13} & -1\\ t_{11} & 4t_{12} & 9t_{13} & 0\end{array}\right] \xrightarrow{RREF} \begin{bmatrix}1 & 0 & 0 & 14.5\\ 0 & 1 & 0 & -16\\ 0 & 0 & 1 & 5.5\end{bmatrix}$$

$$\left[\begin{array}{ccc|c} t_{21} & t_{22} & t_{23} & 0\\ t_{21} & 2t_{22} & 3t_{23} & 1\\ t_{21} & 4t_{22} & 9t_{23} & 0\end{array}\right] \xrightarrow{RREF} \begin{bmatrix}1 & 0 & 0 & -2.5\\ 0 & 1 & 0 & 4\\ 0 & 0 & 1 & -1.5\end{bmatrix}$$

$$\left[\begin{array}{ccc|c} t_{31} & t_{32} & t_{33} & 2\\ t_{31} & 2t_{32} & 3t_{33} & 1\\ t_{31} & 4t_{32} & 9t_{33} & 0\end{array}\right] \xrightarrow{RREF} \begin{bmatrix}1 & 0 & 0 & 3.5\\ 0 & 1 & 0 & -2\\ 0 & 0 & 1 & .5\end{bmatrix}$$

b)

$$A = \begin{bmatrix}14.5 & -16 & 5.5\\ -2.5 & 4 & -1.5\\ 3.5 & -2 & .5\end{bmatrix}$$

$$rref(A) = \begin{bmatrix}1 & 0 & -1/9\\ 0 & 1 & -4/9\\ 0 & 0 & 0\end{bmatrix}$$

$x_1 = 1/9\, x_3$
$x_2 = 4/9\, x_3$
$x_3 = x_3$

C, there is not a pivot in every row of rref(A), so No.

5. (You can use Matlab for the following items.) Let

$$\mathbf{p}(t) = t^2 + t + 1, \qquad \mathbf{q}(t) = 3t - 2, \qquad \mathbf{r}(t) = 2 - 5t^2,$$

and $T : \mathbb{R}^2 \longrightarrow \mathbb{P}_2[t]$ such that

$$T\left(\begin{bmatrix} a \\ b \end{bmatrix}\right) = (2a + 3b)\mathbf{p} + (a - b)\mathbf{q} + (3a + 2b)\mathbf{r}$$

(a) (2 PTS) Find the matrix of T relative to the bases $\{\mathbf{e}_1, \mathbf{e}_2\}$ and $\{\mathbf{p}, \mathbf{q}, \mathbf{r}\}$.

Answer:

$$\begin{bmatrix} 6 & 9 \\ 5 & 0 \\ -13 & -7 \end{bmatrix}$$

(b) (2 PTS) Find a basis for the $ran(T)$.

Answer:

$$\begin{bmatrix} 6 & 9 \\ 5 & 0 \\ -13 & -7 \end{bmatrix}$$

(c) (1 PT) Is T one-one?

Answer:

Yes

Solutions: (Clearly indicate which item the solution is for.)

a)

$$M = \left[\left[T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right)\right]_{pqr} \quad \left[T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right)\right]_{pqr} \right]$$

$$T\left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right) = 2p + q + 3r = 2t^2 + 2t + 2 + 3t - 2 + 6 - 15t^2 = -13t^2 + 5t + 6$$

$$T\left(\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right) = 3p - q + 2r = 3t^2 + 3t + 3 - 3t + 2 + 4 - 10t^2 = -7t^2 + 9$$

$$M = \begin{bmatrix} 6 & 9 \\ 5 & 0 \\ -13 & -7 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

c) $\text{rref}(M) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

b. $\text{rref}(M) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ hence the col. are linearly independent.

6. Let $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

(a) (2 PTS) Find the characteristic polynomial of A. Show your complete solution. (Do not use Matlab.)

Answer:

$t^3 - 3t - 2$

(b) (2 PTS) Determine the eigenvalues of A and their algebraic multiplicities.

Answer:

λ	AM
-1	2
2	1

(c) (3 PTS) Find a basis for each eigenspace and determine the geometric multiplicity of each eigenvalue. (You may use Matlab.)

Answer:

$\lambda = -1$ $\quad \left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$

$\lambda = 2$ $\quad \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$

Solutions: (Clearly indicate which item the solution is for.)

a) $\det(tI_3 - A) = \begin{vmatrix} t & -1 & -1 \\ -1 & t & -1 \\ -1 & -1 & t \end{vmatrix} = t^3 - 2 - (t + t + t) = t^3 - 3t - 2$

b) $t^3 - 3t - 2 = (t+1)^2(t-2)$

$t = -1, 2$

$\lambda = 2$ $\quad Q = \lambda I - A = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$

$\mathrm{rref}(Q) = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad x_1 = x_3$, $x_2 = x_3$, $x_3 = x_3$

$x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

c)

$\lambda = -1$

$M = \lambda I - A = \begin{bmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{bmatrix}$

$\mathrm{rref}(M) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ $\quad x_1 = -x_2 - x_3$, $x_2 = x_2$, $x_3 = x_3$

$\begin{bmatrix} -x_2 - x_3 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

7. Let $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \\ 3 & 3 & 3 \end{bmatrix}$. (You may use Matlab to answer the following.)

(a) (1 PT) Find a basis for $Col(A)$

Answer:

$\text{Span}\left\{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right\}$

(b) (1 PT) Find a basis for $Nul(A)$

Answer:

$\left\{\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\right\}$

(c) (2 PTS) Find a basis for $(Col(A))^{\perp}$

Answer:

$\left\{\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}\right\}$

(d) (2 PTS) Find a basis for $(Nul(A))^{\perp}$.

Answer:

$\text{Span}\left\{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right\}$

Solutions: (Clearly indicate which item the solution is for.)

$rref(A) = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$rref(A^T) = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

a). $Col(A) = \text{span}\left\{\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}\right\}$

b). $Nul(A)$: $x_1 = -x_2 - x_3$

$x_2 = x_2$

$x_3 = x_3$

$Nul(A) = \left\{\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}\right\}$

c) $Nul(A^T)$: $x_1 = -2x_2 - 3x_3$

$x_2 = x_2$

$x_3 = x_3$

$Col(A)^{\perp} = Nul(A^T) = \left\{\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}\right\}$

d $Col(A^T) = \text{Span}\left\{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right\}$ $\quad Col\,A^T = (Nul(A))^{\perp}$

Answer:

8. Let $X = \begin{bmatrix} 4 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix}$ and set $S = \{A \in M_{3\times 3} \mid AX = XA\}$.

(a) (2 PTS) Use the subspace test to show that S is a subspace of $M_{3\times 3}$.

Subspace Test:

1). let $B = 0_{3\times 3}$

$BX = XB$, is true

therefore $0_{3\times 3} \in S$

A must b in the form $\begin{bmatrix} a+b & a & 0 \\ a & b & 0 \\ 0 & 0 & c \end{bmatrix}$

2) $A_1 = \begin{bmatrix} a+b & a & 0 \\ a & b & 0 \\ 0 & 0 & j \end{bmatrix}$

$A_2 = \begin{bmatrix} c+d & c & 0 \\ c & d & 0 \\ 0 & 0 & h \end{bmatrix}$

$A_1 + A_2 = \begin{bmatrix} (a+c)+(b+d) & a+c & 0 \\ a+c & b+d & 0 \\ 0 & 0 & j+h \end{bmatrix}$ same form ✓

$x \in \mathbb{R}$

$xA_1 = \begin{bmatrix} ax+bx & ax & 0 \\ ax & bx & 0 \\ 0 & 0 & jx \end{bmatrix}$ - still in form ✓

(b) (3 PTS) Determine the dimension of S.

Solution:

$$\begin{bmatrix} 4 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}\begin{bmatrix} 4 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix} \Rightarrow \begin{bmatrix} 4a+2d & 4b+2e & 4c+2f \\ 2a+2d & 2b+2e & 2c+2f \\ -3g & -3h & -3i \end{bmatrix} =$$

$$\begin{bmatrix} 4a+2b & 2a+2b & -3c \\ 4d+2e & 2d+2e & -3f \\ 4g+2h & 2g+2h & -3i \end{bmatrix}$$

$4a+2b = 4a+2d$

$2b = 2d$

$4d+2e = 2a+2d$

$2b+2e = 2d+2e$ $\quad 2b = 2d$

$2a+2b = 4b+2e$

$2d+2e = 2a$

$2b+2e = 2a$

$-3c = 4c+2f$

$-3f = 2c+2f$

$-7c = 2f$

$-5f = 2c$

$f = -\frac{7}{2}c$ $\quad f=0$ $\quad c=0$

$\begin{bmatrix} b+e & b & 0 \\ b & e & 0 \\ 0 & 0 & i \end{bmatrix}$

$b\begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + e\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + i\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Answer: 3

9. Let $S = \left\{ \begin{array}{ccc} v_1 & v_2 & v_3 \\ \begin{bmatrix}1\\2\\3\\4\end{bmatrix}, & \begin{bmatrix}1\\-1\\-1\\1\end{bmatrix}, & \begin{bmatrix}-1\\3\\-3\\1\end{bmatrix} \end{array} \right\}$ and $W = Span(S)$.

(a) (1 PT) Is S an orthogonal set?

Solution:

$\langle v_1, v_2 \rangle = 1 - 2 - 3 + 4 = 0$ ✓

$\langle v_1, v_3 \rangle = -1 + 6 - 9 + 4 = 0$ ✓

$\langle v_2, v_3 \rangle = -1 - 3 + 3 + 1 = 0$ ✓

Answer: Yes

(b) (3 PTS) Find $\text{proj}_W \begin{bmatrix}6\\-2\\3\\1\end{bmatrix}$. Show your complete solution.

Solution:

$x = \begin{bmatrix}6\\-2\\3\\1\end{bmatrix}$

$\begin{bmatrix}-1\\3\\-3\\1\end{bmatrix}\begin{bmatrix}6\\-2\\3\\1\end{bmatrix} = \lambda\begin{bmatrix}1\\2\\3\\4\end{bmatrix}\cdot\begin{bmatrix}-1\\3\\-3\\1\end{bmatrix} + B\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix}\begin{bmatrix}-1\\3\\-3\\1\end{bmatrix} + \gamma\begin{bmatrix}-1\\3\\-3\\1\end{bmatrix}\cdot\begin{bmatrix}-1\\3\\-3\\1\end{bmatrix}$

$x = \lambda\begin{bmatrix}1\\2\\3\\4\end{bmatrix} + B\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix} + \gamma\begin{bmatrix}-1\\3\\-3\\1\end{bmatrix}$

$-20 = \gamma 20$

$\gamma = -1$

$\begin{bmatrix}1\\2\\3\\4\end{bmatrix}\cdot\begin{bmatrix}6\\-2\\3\\1\end{bmatrix} = \lambda\begin{bmatrix}1\\2\\3\\4\end{bmatrix}\cdot\begin{bmatrix}1\\2\\3\\4\end{bmatrix} + B\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix}\cdot\begin{bmatrix}1\\2\\3\\4\end{bmatrix} + \gamma\begin{bmatrix}-1\\3\\-3\\1\end{bmatrix}\cdot\begin{bmatrix}1\\2\\3\\4\end{bmatrix}$

$15 = 30\lambda + 0B$

$\lambda = 1/2$

$\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix}\cdot\begin{bmatrix}6\\-2\\3\\1\end{bmatrix} = \lambda\begin{bmatrix}1\\2\\3\\4\end{bmatrix}\cdot\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix} + B\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix}\cdot\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix} + \gamma\begin{bmatrix}-1\\3\\-3\\1\end{bmatrix}\cdot\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix}$

$6 = 0 + 4B + 0$

$B = \frac{3}{2}$

$\frac{1}{2}\begin{bmatrix}1\\2\\3\\4\end{bmatrix} + \frac{3}{2}\begin{bmatrix}1\\-1\\-1\\1\end{bmatrix} + \begin{bmatrix}-1\\3\\-3\\1\end{bmatrix} = \begin{bmatrix}1\\-7/2\\-3\\5/2\end{bmatrix}$

Answer: $\begin{bmatrix}1\\-7/2\\-3\\5/2\end{bmatrix}$