Assignment 1 KAA306 Graph Theory with Applications

Scott Whitemore

1.

1.3.1 (a) An edge must always be incident with two and only two vertices by definition. The row and column sums are the total number of objects incident with the object identified by the row and column index, respectively. Therefore, the column sums, being the total number of vertices incident with any single edge, must equal two.

(b) Without the restriction of the graph being *simple* the column sums of the adjacency matrix could be **anything**. If we constrain the class of graphs under consideration to simple graphs, then the column sums of the adjacency matrix represent the total number of vertices that connect by one edge to the particular vertex identified by the matrix index. Since there can only be one edge between any distinct vertex pair, there is a maximum possible number of adjacencies equal to the total number vertices in the graph minus one.

1.6.8 (a) Consider a single component graph. If one removes an edge from this graph there are two possible scenarios:

1. The edge is removed, but there is at least one other edge connecting any two disjoint subgraphs – ω(G) does not change.

2. The edge was the only connection between two disjoint subgraphs – the subgraphs become disconected and ω(G) increases by one. Since any edge can only connect two vertices, it is not possible for the removal of an edge to increase ω(G) by more than one.

Any multi-component graph will act the same, as we can only take a single edge away from a single component at a time.

DRAW PICTURES

(b) Removing a vertex can have considerably larger effect than removing an edge. Consider the following two cases that break the equality:

1. We remove a vertex that is incident with no edges. In this case, the vertex makes up an entire disconnected component of its own, and so removing it removes a component; i.e. ω(G-v)  ω(G)

2. We remove a vertex that is the only vertex shared by 3 different subgraphs:

Let v be the vertex as described and be a subgraph containing v, .

Then we have 

If we remove v then we have 

So we went from ω(G) = 1 to ω(G) = 3. Clearly ω(G-v)  ω(G)+1

1.6.14 triangle

1.7.2 at some point there has to be a cycle, reduce to three nodes and consider