A nerve theorem & graphical calculus for compact closed categories

AusCat 28/10/20

Sophie Raynor

A nerve theorem & graphical calculus for compact closed categories

- Last week: An iterated distributive law for circuit operads
- This week: The same for compact closed categories.

⇝ operadic characterisation of compact closed cats

Why?

eg

- decompositions of higher cobordisms
- model structures on Comp
  Piv
- explicit adjunctions to planar algebras
  pivotal categories

$$\begin{tikzcd}
P_A \arrow[r, shift left=2] \arrow[r, phantom, "\perp"] \arrow[d, shift left=2] \arrow[d, phantom, "\vdash"] & sP_{IV} \arrow[l, shift left=2] \arrow[d, shift right=2] \arrow[d, phantom, "\dashv"] \\
CO \arrow[u, shift left=2] \arrow[r, shift left=2] \arrow[r, phantom, "\perp"] & s\,Comp \arrow[l, shift left=2] \arrow[u, shift right=2]
\end{tikzcd}$$

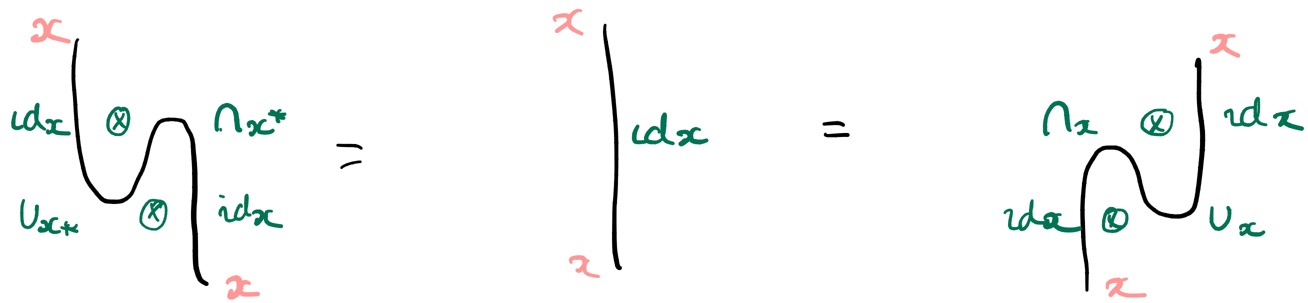
Structure.

- Recap on compact closed categories & circuit operads
  - definitions
  - construction of the monads & dist laws
- Distributive laws for compact closed categories
  - more monads
  - distributive law.

Compact Closed Categories

A compact closed category is a symm. mon. cat $(X, \otimes, I)$ such that

- each object $x \in X$ has a dual $x^* \in X$:
  - there are distinguished morphisms
    - $\cap_x : \quad I \longrightarrow x \otimes x^*$
    - $\cup^x : \quad x^* \otimes x \longrightarrow I$ satisfying



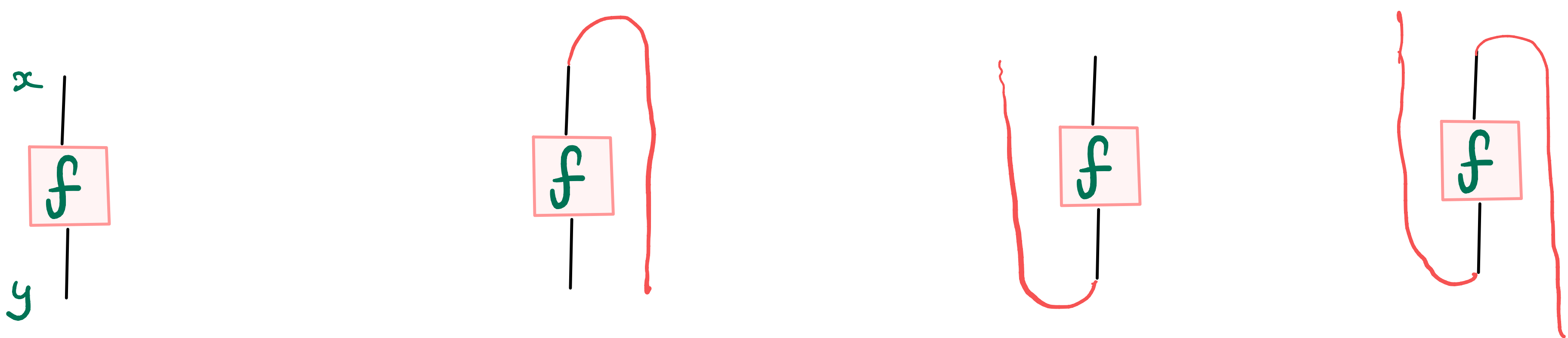
Internal homs $[x,y] = y x^*$

X Compact closed with objects x, y

$$X(x,y) \cong X(I, yx^*) \cong X(y^*x, I) \cong X(y^*, x^*)$$

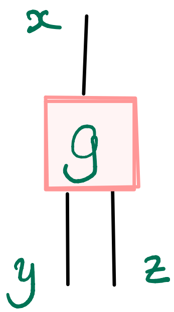
$$f \longmapsto \llcorner f \lrcorner \longmapsto \ulcorner f \urcorner \longmapsto f^*$$

name　　　　coname

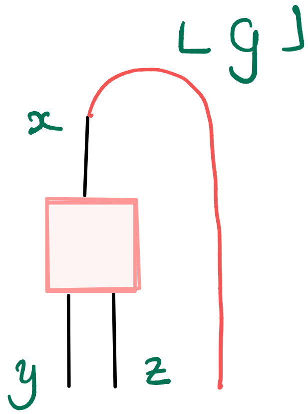


coevaluation　　　　evaluation

So, $\forall x, y, z$



$\cong$



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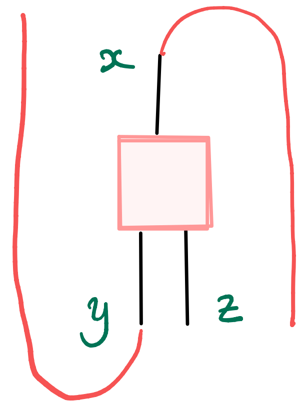

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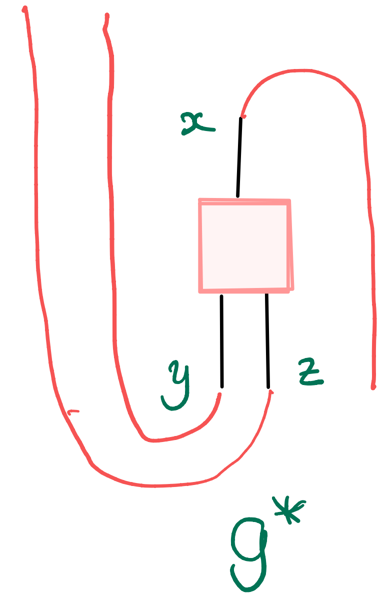
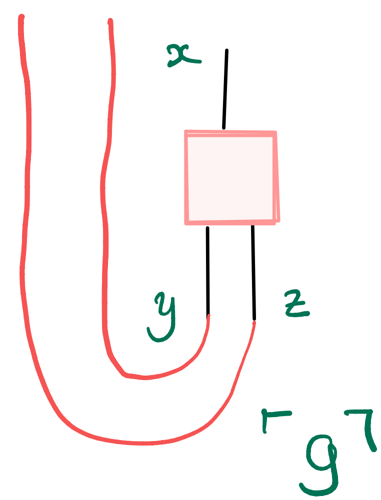
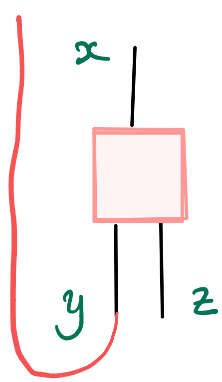


$\cong$



$\cong$

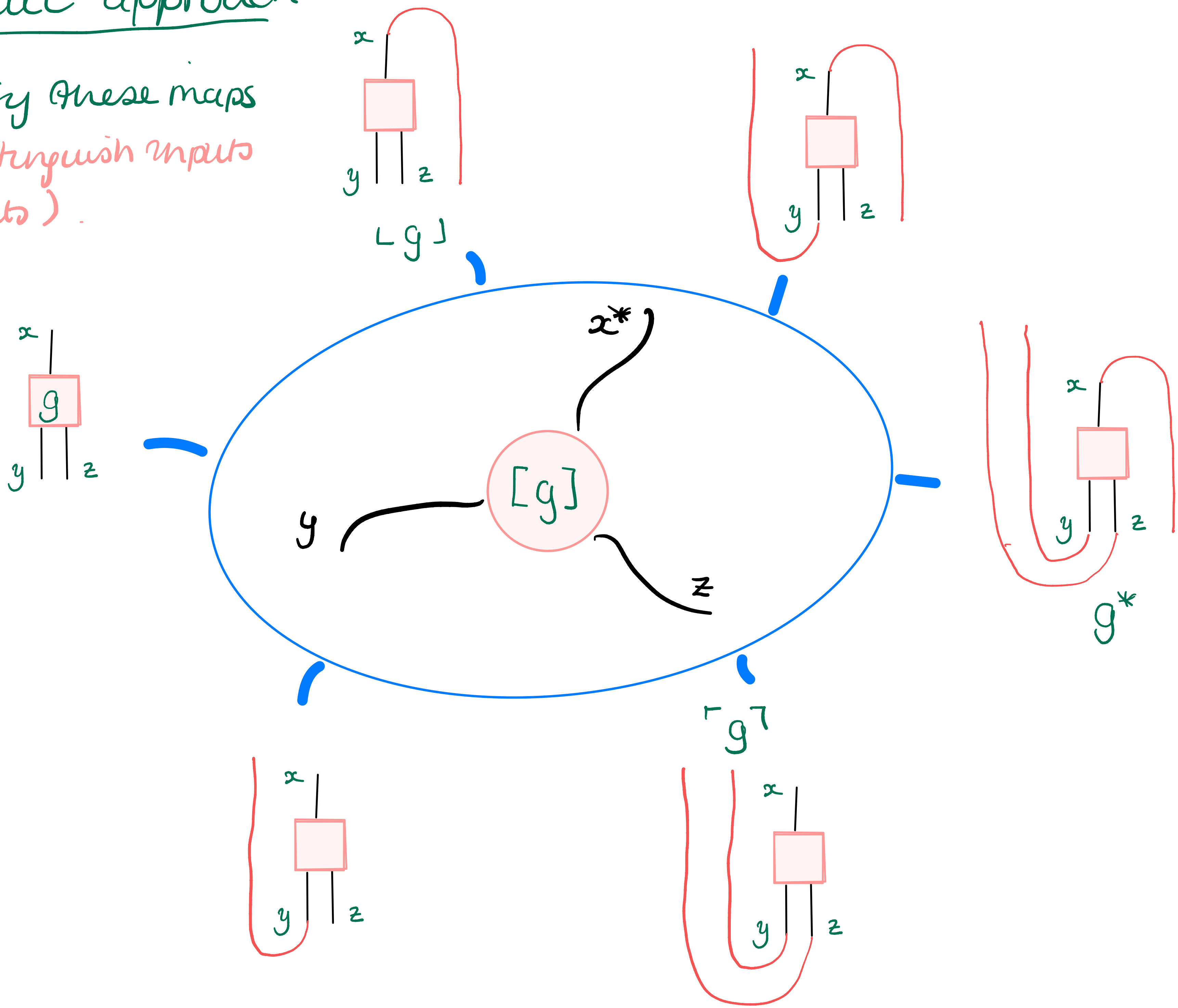


$\cong$

as well as all the symmetry isomorphisms....

Operadic approach

Identify these maps
(don't distinguish inputs & outputs).

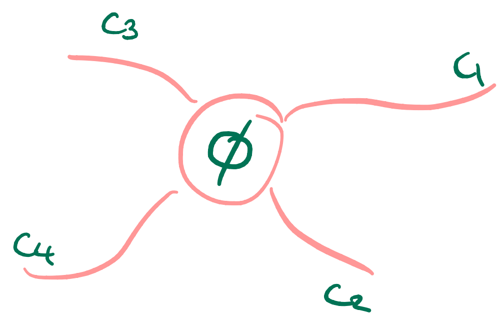


Circuit operads recap

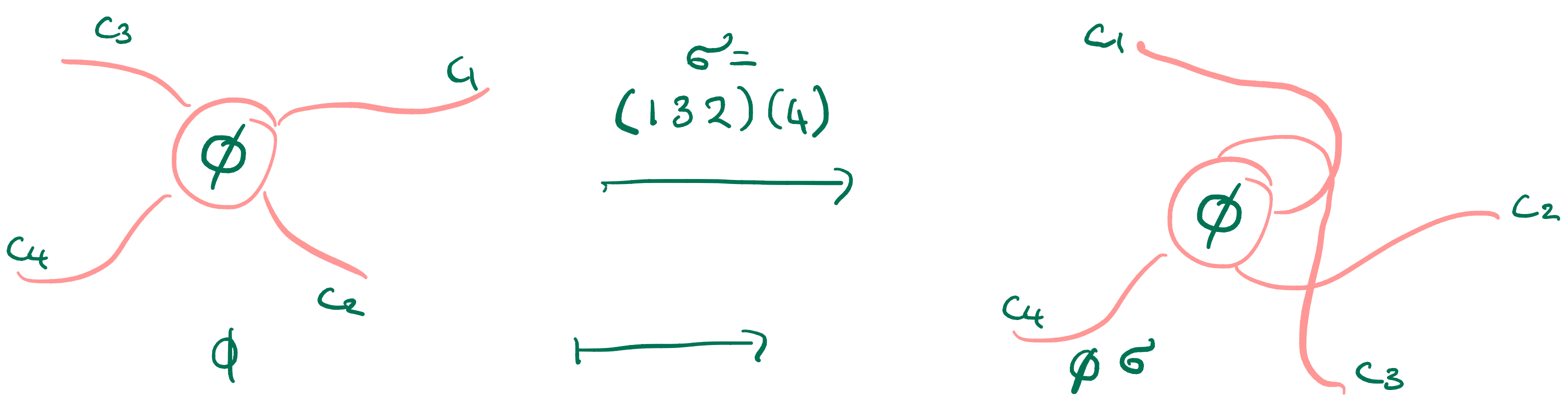
Set $\mathcal{C}$ involution $\omega : \mathcal{C} \to \mathcal{C}$, $\omega^2 = \mathrm{id}$.

A $(\mathcal{C}, \omega)$-coloured circuit operad $(S, \boxtimes, \xi, \varepsilon)$ consists of

- for all $n \in \mathbb{N}$ and $\underline{c} = (c_1, \dots, c_n) \in \mathcal{C}^n$,
  - a set $S_{\underline{c}}$



  - action of Σ_n on $S_n \overset{\text{def}}{=} \coprod_{\underline{c} \in \mathcal{C}^n} S_{\underline{c}}$



& ...

Circuit operads recap

Set $\mathcal{C}$ involution $\omega : \mathcal{C} \to \mathcal{C}$, $\omega^2 = id$.

A $(\mathcal{C},\omega)$-coloured circuit operad $(S, \boxtimes, \xi, \varepsilon)$ consists of

- Sets $S_{\underline{c}}$ $\forall \underline{c} \in \mathcal{C}^n$, $n \in \mathbb{N}_{\geq 0}$
- Σ_n action on $S_n = \coprod_{\underline{c} \in \mathcal{C}^n} S_{\underline{c}}$, $n \in \mathbb{N}$
- External product $(\boxtimes, \iota)$:

$$\forall \underline{c} \in \mathcal{C}^m, \quad \underline{d} \in \mathcal{C}^n \qquad \boxtimes : S_{\underline{c}} \times S_{\underline{d}} \longrightarrow S_{\underline{c}\,\underline{d}}$$

$\underline{c} = (c_1, \dots, c_m)$ $\quad$ $\underline{d} = (d_1, \dots, d_n)$ $\quad$ $\underline{c}\underline{d} = (c_1, \dots, c_m, d_1, \dots, d_n)$

external unit

$$\iota \in S_0 \; : \; \phi \boxtimes \iota = \iota \boxtimes \phi = \phi, \quad \forall \phi$$

& ...

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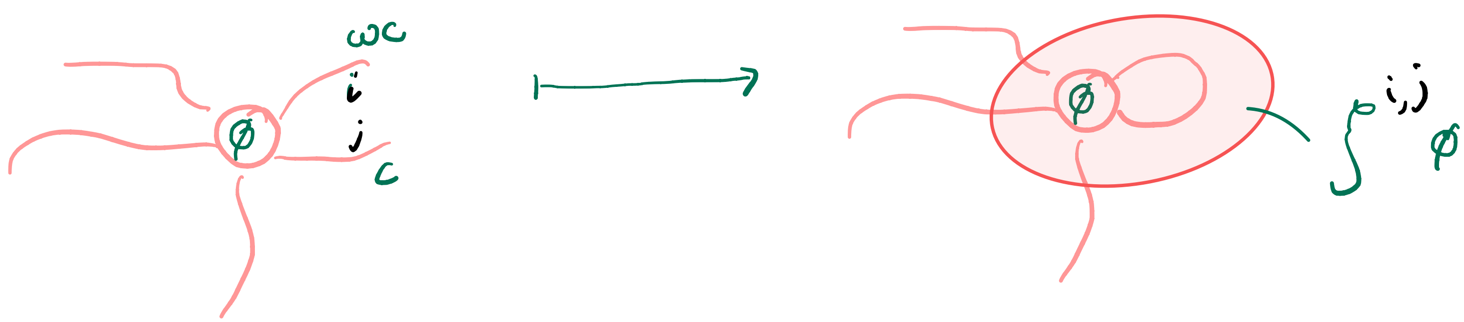
- Σ_n action on $S_n = \coprod_{\underline{c} \in \mathcal{C}^n} S_{\underline{c}}$, $n \in \mathbb{N}$
- $\boxtimes : S_{\underline{c}} \times S_{\underline{d}} \to S_{\underline{c}\underline{d}}$, $\forall \underline{c}, \underline{d}$
- $\iota \in S_0$ $\quad \phi \boxtimes \iota = \iota \boxtimes \phi = \phi \quad \forall \phi$
- Contraction ξ: $\forall \underline{c} = (c_1, \ldots, c_n) \in \mathcal{C}^n$, $\forall\, 1 \leq i < j \leq n$ s.t. $c_i = \omega c_j$

$$\xi^{i,j} : S_{\underline{c}} \longrightarrow S_{c_1, \ldots, c_{i-1}, c_{i+1}, \ldots, c_{j-1}, c_{j+1}, \ldots, c_n}$$



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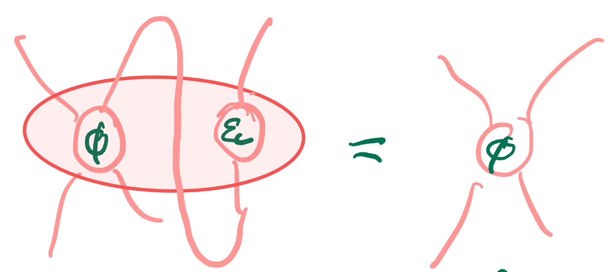
- $\boxtimes : S_{\underline{c}} \times S_{\underline{d}} \to S_{\underline{c}\,\underline{d}}$, $\forall \underline{c}, \underline{d}$
- $\iota \in S_0$ $\quad \phi \boxtimes \iota = \iota \boxtimes \phi = \phi \quad \forall \phi$
- $\xi^{ij} : S_{\underline{c}} \longrightarrow S_{\underline{c}/c_i, c_j}$
- "connected unit" $\varepsilon : \mathcal{C} \to S_2$, $c \longmapsto \varepsilon_c \in S_{(c, \omega c)}$
  - $\sigma_2 \, \varepsilon = \varepsilon \, \omega$ $\qquad \sigma_2 \in \Sigma_2$, $\sigma_2 \neq id$

  - $\forall \phi \in S_{\underline{c}, c}$ $\quad \xi(\phi \boxtimes \varepsilon_c) = \phi$



& ...

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- $\boxtimes : S_{\underline{c}} \times S_{\underline{d}} \to S_{\underline{c}\,\underline{d}}$, $\forall \underline{c}, \underline{d}$
- $\iota \in S_0$ $\quad \phi \boxtimes \iota = \iota \boxtimes \phi = \phi \quad \forall \phi$
- $\xi : S_{\underline{c}} \longrightarrow S_{\underline{c}/c_i, c_j}$ contraction
- $\varepsilon : \mathcal{C} \rightarrowtail S_2$, $c \mapsto \varepsilon_c \in S_{c, \omega c}$
  - $\sigma_2 \varepsilon = \varepsilon \omega$
  - $\xi_c (\phi \boxtimes \varepsilon) = \phi$

+ Axioms

- associative isos for $\boxtimes$
- compatibility of $\boxtimes, \xi$
- equivariance

- Previously:

A monadic adjunction

$$\mathcal{CO} \underset{\longleftarrow}{\overset{\longrightarrow}{\perp}} \mathcal{C}omp$$

Slogan:

"Small compact closed categories are circuit operads with

- (weak) monoid structure on $\mathcal{C}$

- isomorphisms

$$S_{c_1^1 \otimes \cdots \otimes c_{n_1}^1, \cdots, c_1^k \otimes \cdots \otimes c_{n_k}^k} \xleftarrow{\cong} S_{c_1^1, \cdots, c_{n_1}^1, c_1^2, \cdots, c_{n_k}^k}$$

"

A monadic adjunction $CO \underset{\longleftarrow}{\overset{\longrightarrow}{\perp}} Comp$

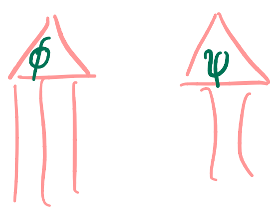
Right adjoint

Small compact closed category X w object monoid $\mathcal{M}$

$\longmapsto$ $(M, *)$-coloured circuit operad $\mathcal{S}^X$

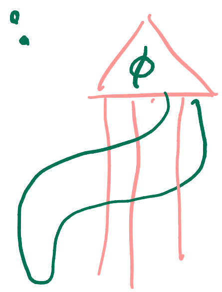
w $\mathcal{S}^X_{x_1, \dots, x_n} \overset{def}{=} \mathcal{M}(I, x_1 \otimes \cdots \otimes x_n)$

- $\phi \boxtimes \psi \overset{def}{=} \phi \otimes \psi$
- $\varepsilon : x \longmapsto \cap_x \in M(I, x x^*)$



$\iota = id_I$

- $\xi(\phi)$:



A monadic adjunction $\mathcal{CO} \underset{\longleftarrow}{\overset{\longrightarrow}{\bot}} \mathrm{Comp}$

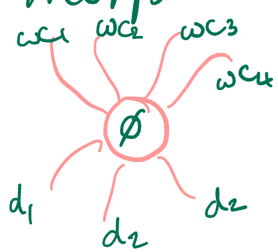
Left adjoint

$(\mathcal{C},\omega)$- coloured circuit operad $(S, \boxtimes, \xi, \varepsilon)$

$\longmapsto$ small compact closed category C^S

- objects in $M\mathcal{C}$ ← free monoid on $\mathcal{C}$
- dual $*$: $\underline{c} \longmapsto \underline{c}^*$

  $(c_1, \dots, c_n) \longmapsto (\omega c_n, \dots, \omega c_1)$
- morphisms $C^S(\underline{c}, \underline{d}) = S_{\underline{d}\,\underline{c}^*}$ $d_1,\dots,d_n,\omega c_m,\dots,\omega c_1$.



id, $\cap$, $\cup$ from ε

etc.

- Last week
  - $CO \simeq Alg(\mathbb{L}\mathbb{D}\mathbb{T})$ such that.

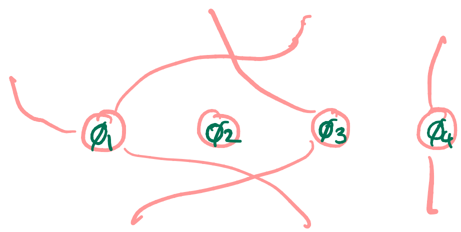
$\mathbb{L}, \mathbb{D}, \mathbb{T}$ monads on category GS of graphical species:

$$\boxed{GS \overset{def}{=} PS(\mathbb{S})}$$

$\mathbb{S}$

- objects
  - $n \in \mathbb{N}_{\geq 0}$
  - $\S$ — distinguished 'gluing or object object'
- morphisms
  - $\mathbb{S}(\S, \S) = \{id_\S, \tau\}$
  - $\mathbb{S}(\S, n) = \{ch_i, ch_i \circ \tau\}_{i=1}^{n}$
  - $\mathbb{S}(n, \S) = \emptyset \quad \forall n$
  - $\mathbb{S}(m,n) = \begin{cases} \emptyset, & m \neq n \\ \Sigma_n, & m = n \end{cases}$

- $\mathbb{L} = (L, \mu^{\mathbb{L}}, \eta^{\mathbb{L}})$ "free graded monoid monad on arities"
  - $LS_{\mathfrak{c}} = S_{\mathfrak{c}}$
  - $LS_n = \coprod_{k \in \mathbb{N}} \coprod_{\sum_{i=1}^{k} n_i = n} \prod_{i=1}^{k} S_{n_i}$

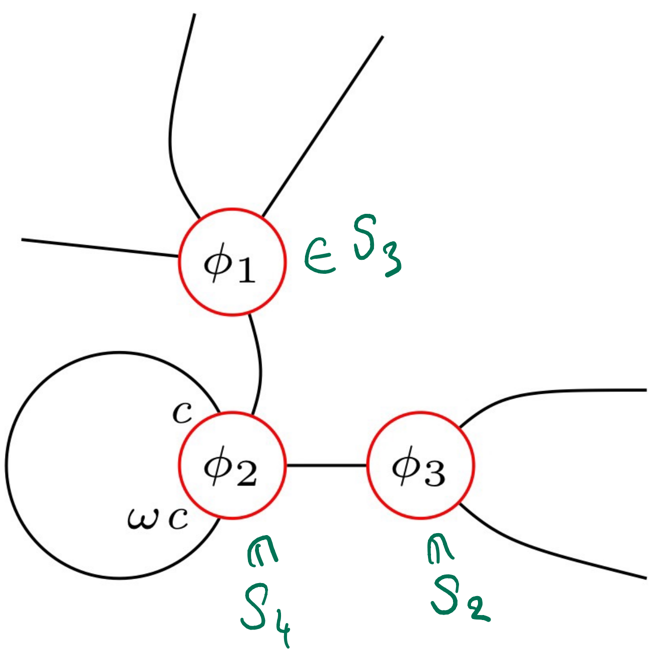


Algebras for $\mathbb{L}$ are graphical species with $(\boxtimes, \iota)$

- $\mathbb{D} = (D, \mu^{\mathbb{D}}, \eta^{\mathbb{D}})$
  - $DS_{\mathfrak{c}} = S_{\mathfrak{c}}$
  - $DS_n = \begin{cases} S_n & n \neq 0, n \neq 2 \\ S_0 \amalg \{o^{DS}_{\tilde{c}}\}_{\tilde{c} \in S_{\mathfrak{c}}/S(\tau)} & n = 0 \\ S_2 \amalg \{\epsilon^{DS}_{c}\}_{c \in S_{\mathfrak{c}}} & n = 2 \end{cases}$

Algebras for $\mathbb{D}$ are graphical species w/ unit $\epsilon : S_{\mathfrak{c}} \to S_2$ + choice of contractions of unit $o : S_{\mathfrak{c}} \to S_0$

- $\mathbb{T} = (T, \mu^{\mathbb{T}}, \eta^{\mathbb{T}})$
  - $TS_{\xi} = S_{\xi}$
  - $TS_n = \{$ connected graphs w n 'ports' 'decorated' by $S\} / \sim$

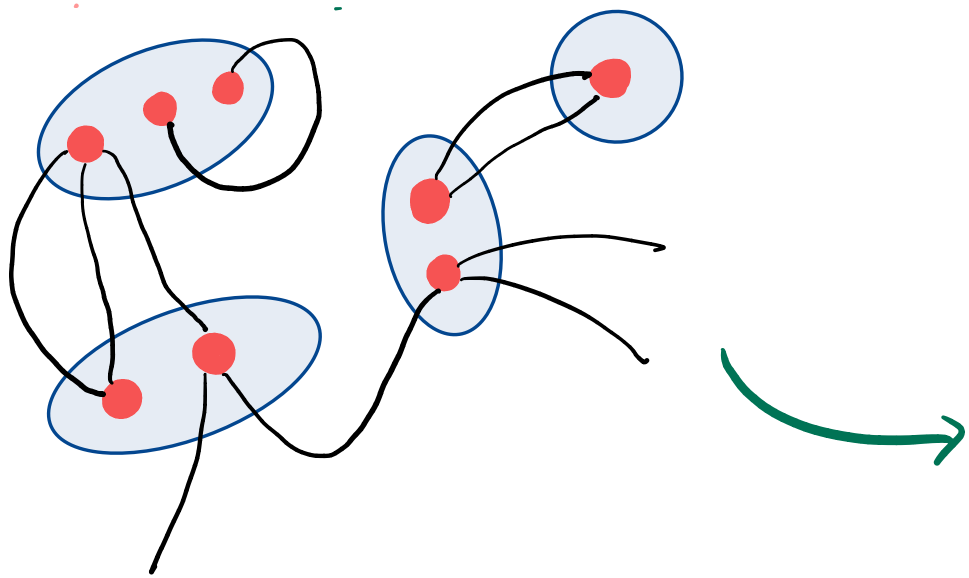


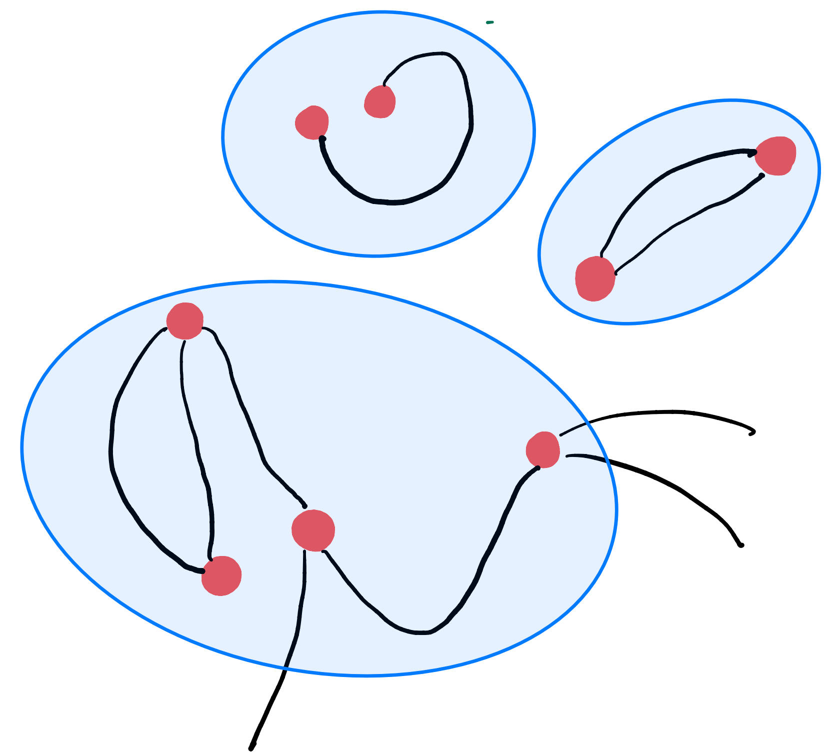
$\in TS_5$

Algebras for $\mathbb{T}$ are 'non-unital modular operads':

have well behaved contraction ξ.

Distributive law $TL \Rightarrow LT$





Monad so defined is equivalent to taking all graphs & not just connected graphs in $\underline{\text{defn}}$ of $\mathbb{T}$.

Algebras for $\mathbb{LT}$ are non-unital circuit operads.

Nerve for CO

Let GS_* be category of algebras for $\mathbb{D}$.

And let $\tilde{\mathbb{L}}\mathbb{T}_*$ be the composite monad on GS_* s.t

$$GS_*^{\tilde{\mathbb{L}}\mathbb{T}_*} \simeq GS^{\mathbb{L}\mathbb{D}\mathbb{T}} \simeq CO$$

objects are graphs →

$$\begin{tikzcd}
 & \equiv^{\times} \arrow[r, "\text{dense}", "\text{f.f}"'] & CO \arrow[r, "N", "\text{f.f}"'] \arrow[d, shift left, "u^{\tilde{\mathbb{L}}\mathbb{T}_*}"] & ps(\equiv^{\times}) \arrow[d] \\
\mathbb{S}_* \arrow[r, "\text{dense}", "\text{ff}"'] & Gr_* \arrow[u, "\text{b.o}"] \arrow[r, "\text{dense}", "\text{ff}"'] & GS_* \arrow[u, shift left, "f^{\tilde{\mathbb{L}}\mathbb{T}_*}"] \arrow[r, "\text{ff}"'] \arrow[d, shift left, "u^{\mathbb{D}}"] & ps(Gr_*) \arrow[d] \\
\mathbb{S} \arrow[u, "\text{b.o}"] \arrow[ur, dashed, "\text{dense}"] \arrow[r, "\text{dense}", "\text{f.f}"'] & Gr \arrow[u, "\text{b.o}"] \arrow[r, "\text{dense}", "\text{f.f}"'] & GS \arrow[u, shift left, "f^{\mathbb{D}}"] \arrow[r, "\text{f.f}"'] & ps(Gr)
\end{tikzcd}$$

The functor $N: CO \to ps(\equiv^{\times})$ is fully faithful.

Essential image of N given by $P \in ps(\equiv^{\times})$ s.t

$$P(\mathcal{G}) \cong \lim_{(C,f)\in \mathbb{S}\downarrow G} P(C)$$

Now

Do the same for Comp!

Recall

"Small compact closed categories are circuit operads with

- (weak) monoid structure on $\mathcal{C}$
- isomorphisms

$$S_{c_1^1\otimes\cdots\otimes c_{n_1}^1,\ldots,c_1^k\otimes\cdots\otimes c_{n_k}^k} \xleftarrow{\cong} S_{c_1^1,\ldots,c_{n_1}^1,c_1^2,\ldots,c_{n_k}^k}$$
"

Another monad on GS !?! Seriously? Enough already!

$$\mathbb{M} = (M, \mu^{M}, \eta^{M})$$

$M\mathcal{C} \stackrel{def}{=} MS_{\mathcal{G}}$ is free monoid on $\mathcal{C} = S_{\mathcal{G}}$

$\omega^{M} \stackrel{def}{=} MS(\tau) \;:\; c_1 \cdots c_n \longmapsto \omega c_n, \ldots, \omega c_1$

$(\omega = S(\tau) : C^{\circlearrowleft})$

For $k \in \mathbb{N}$ $\quad \underline{c}^{j} \in \mathcal{C}^{n_k} \quad 1 \leq j \leq n_k$

$MS_{\underline{c}^1, \ldots, \underline{c}^k} = S_{\underline{c^1 \quad c^k}}$ where $\underline{c^1 \ldots c^k}$ is concatenation

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Distributive law $LM \Rightarrow ML$

Algebras for $\mathbb{ML}$

graphical species S s.t

- $(S_n)_{n \in \mathbb{N}}$ graded monoid
- $\mathcal{C}$ has monoid structure compatible w ω : $\omega(c\,d) = \omega d\, \omega c$
- If $\iota \in \mathcal{C}$ is monoidal unit then

$$S_{(\iota)} \cong S_0$$

$$\begin{tikzcd}
S_0 \ni \phi \arrow[r, "\eta"] \arrow[dr, equal] & \eta\phi \in MLS_{(-)} \arrow[d] \\
 & \phi \in S_\iota
\end{tikzcd}$$

Write

$$\mathbb{ML} = \mathbb{B} = (B, \mu^B, \eta^B)$$

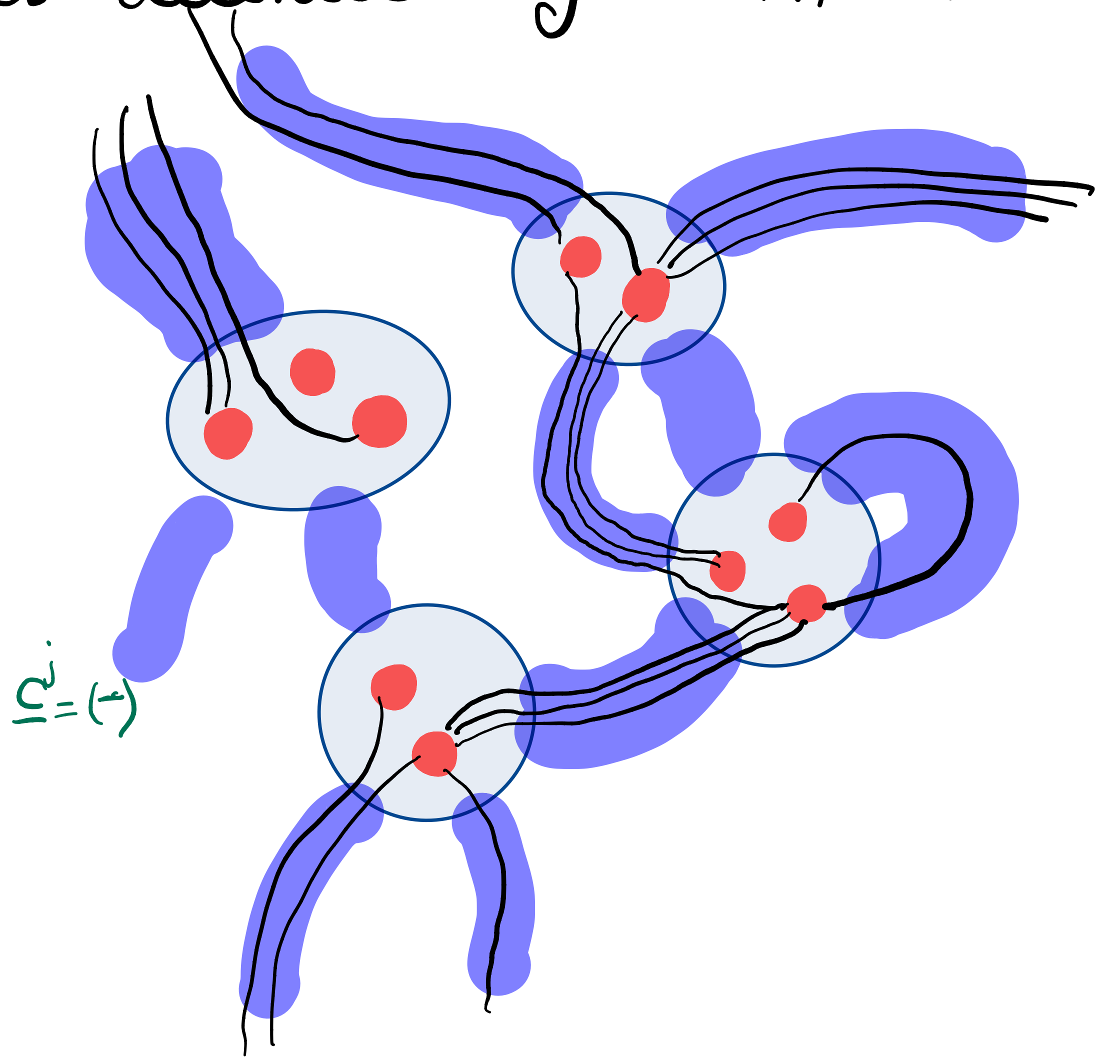
$TB \Rightarrow BT$

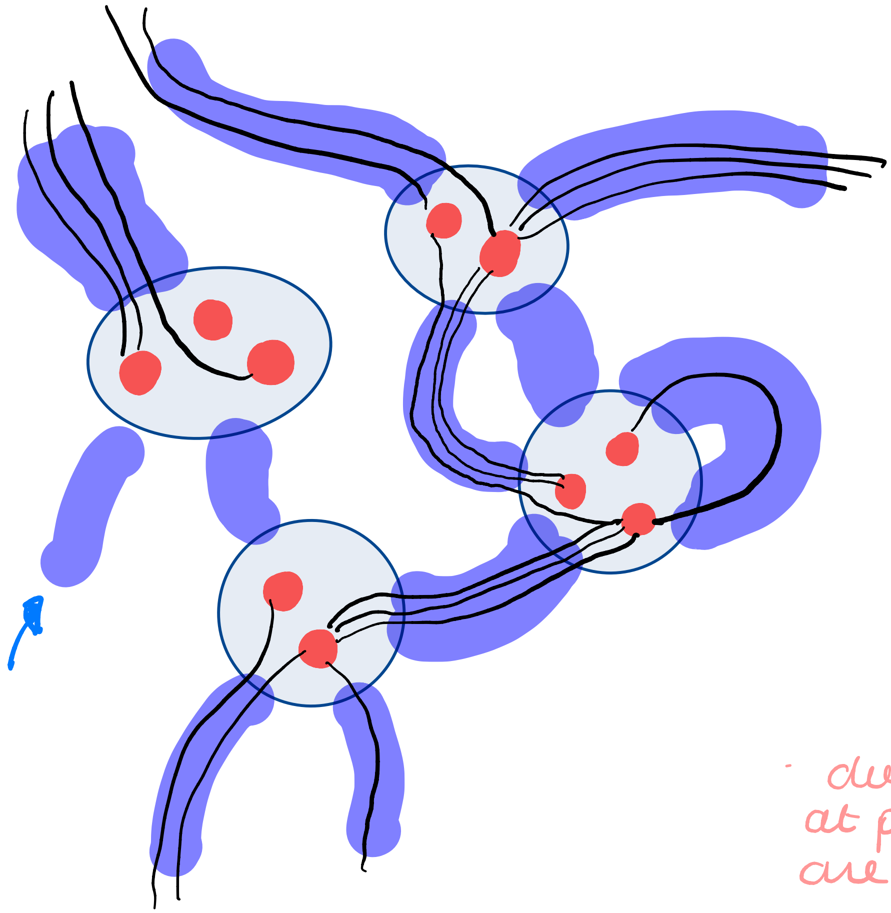
An element of $TBS_{\underline{c}^1,\dots,\underline{c}^k}$

connected graph with edges decorated by $\underline{c} \in Be$

vertices decorated by $(\phi_1,\dots,\phi_2) \in LS$.

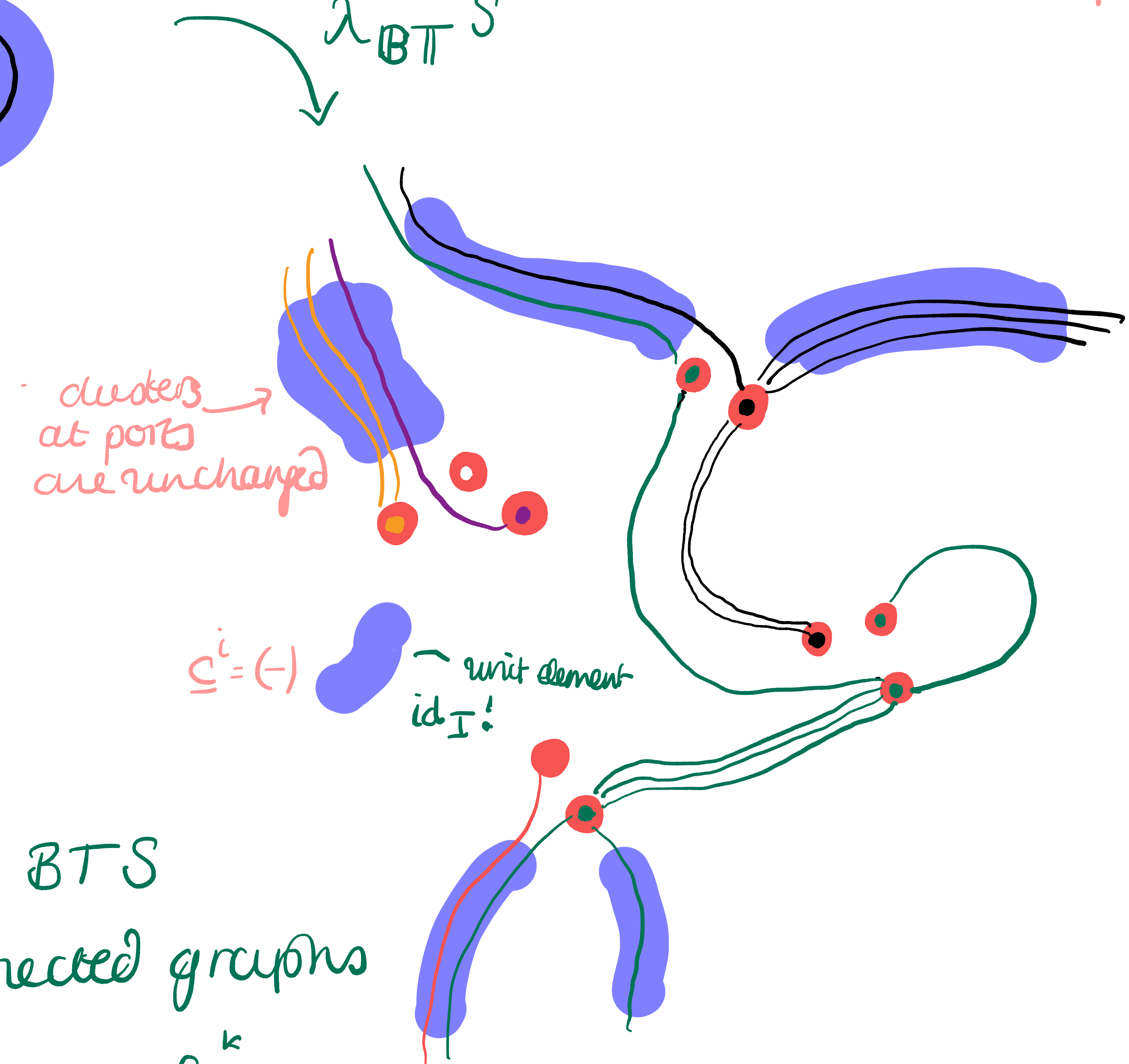
ports decorated by $\underline{c}^1,\dots,\underline{c}^k$





Step I. Delete clustering of inner edges

Step II Delete vertices and take connected comps.



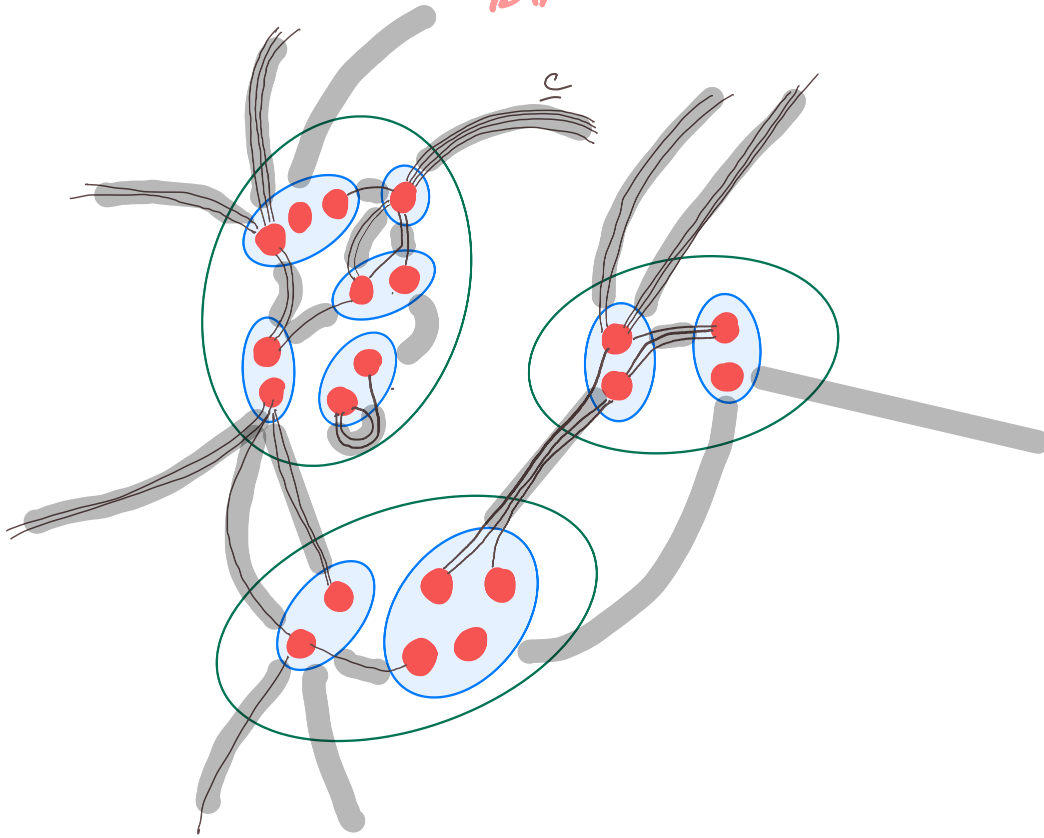
BTS

A disjoint union of connected graphs w partition of ports. $\underline{c}^1, \ldots, \underline{c}^k$

Verifying the axioms

1)

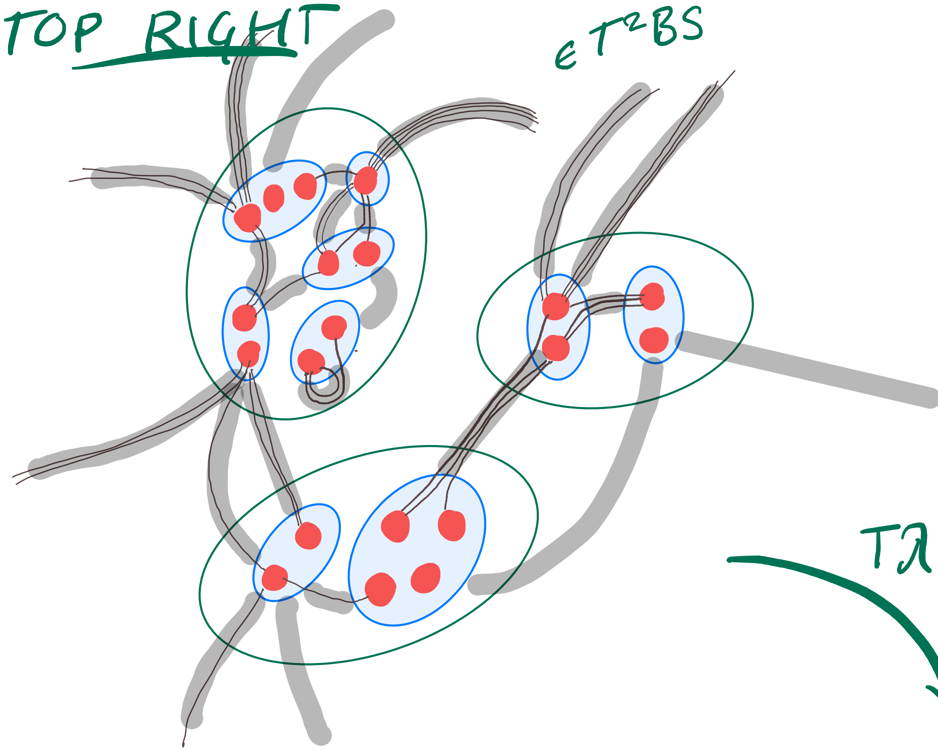
$$\begin{tikzcd}
T^2B \arrow[r, Rightarrow, "T\lambda_{BT}"] \arrow[d, Rightarrow, "\mu^T B"'] & TBT \arrow[r, Rightarrow, "\lambda_{BT}T"] \arrow[d, phantom, "\circlearrowright"] & BT^2 \arrow[d, Rightarrow, "B\mu^T"] \\
TB \arrow[rr, Rightarrow, "\lambda_{BT}"'] & & BT
\end{tikzcd}$$



$\in T^2BS$

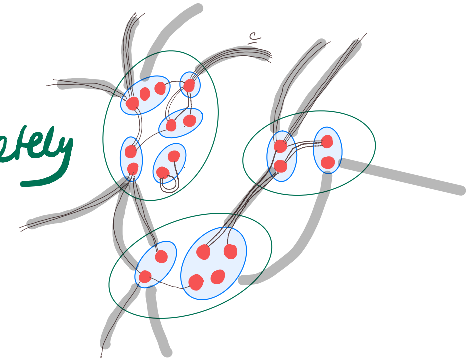
TOP RIGHT

$\in T^2BS$

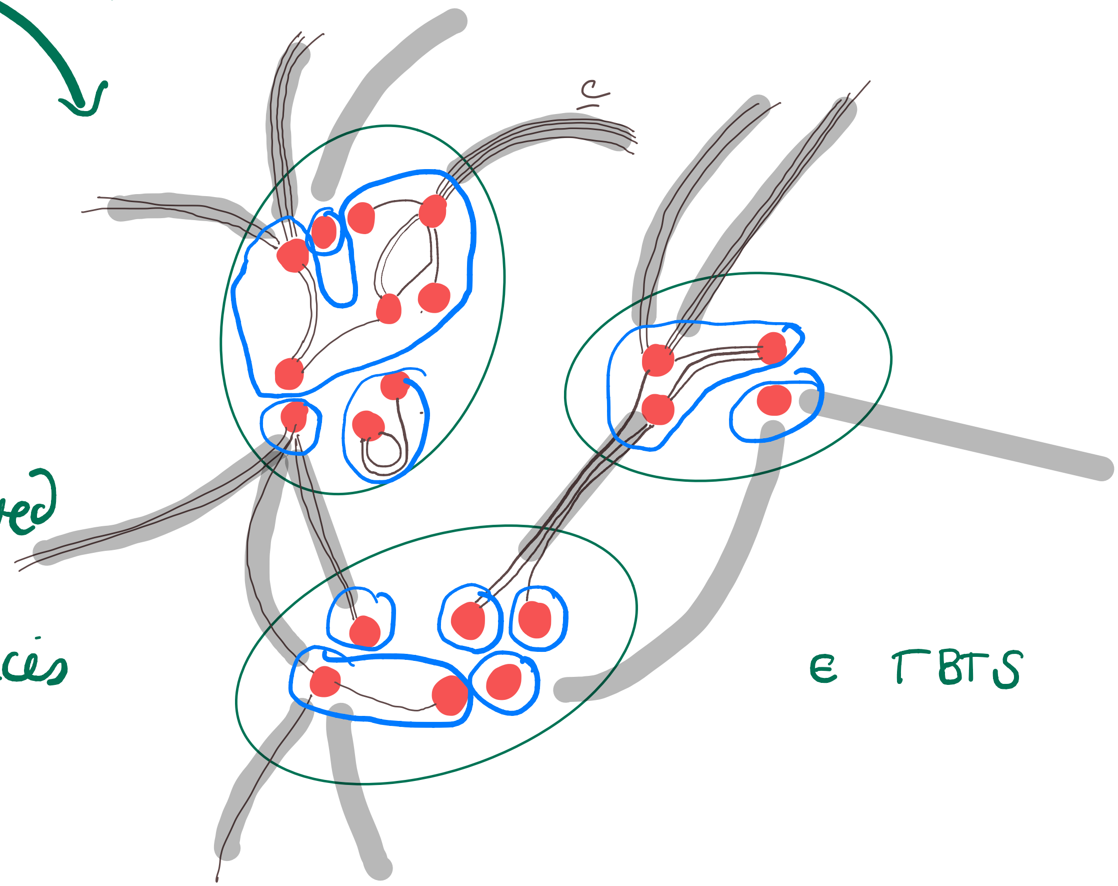


STEP I

delete clusters of edges completely inside green vertices



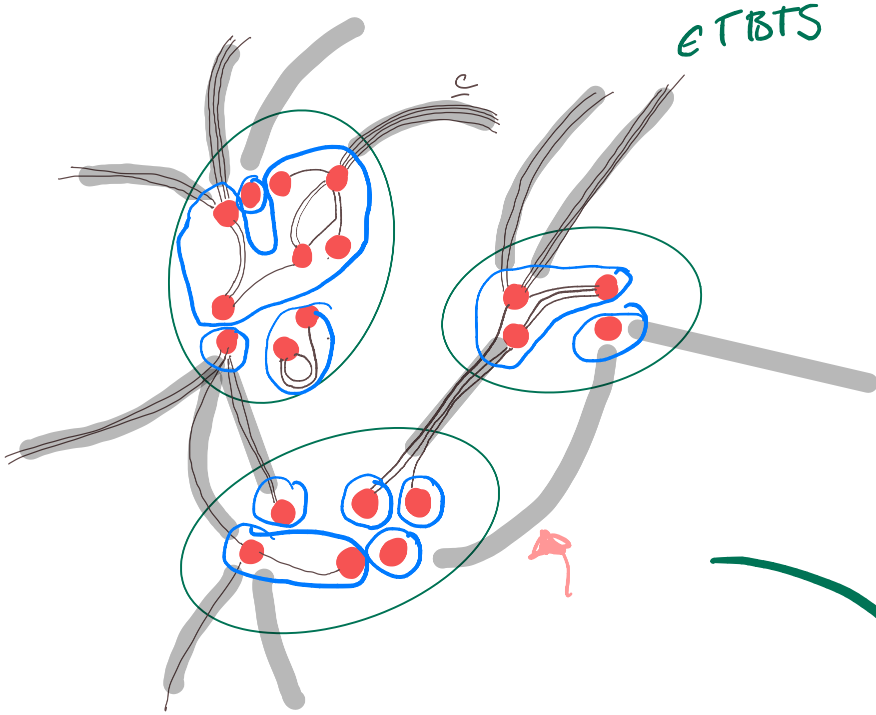
$T\lambda_{BT}$



$\in TBTS$

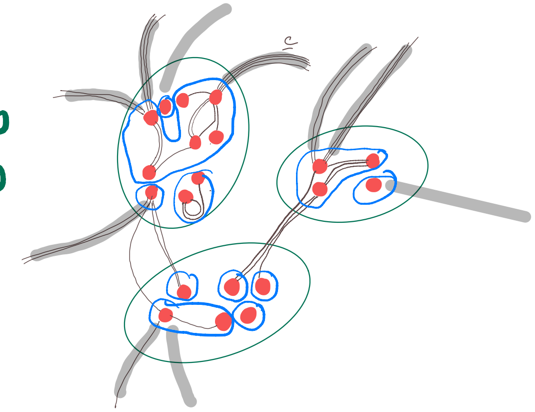
STEP II

Delete internal (blue) vertices and take connected components INSIDE Green vertices

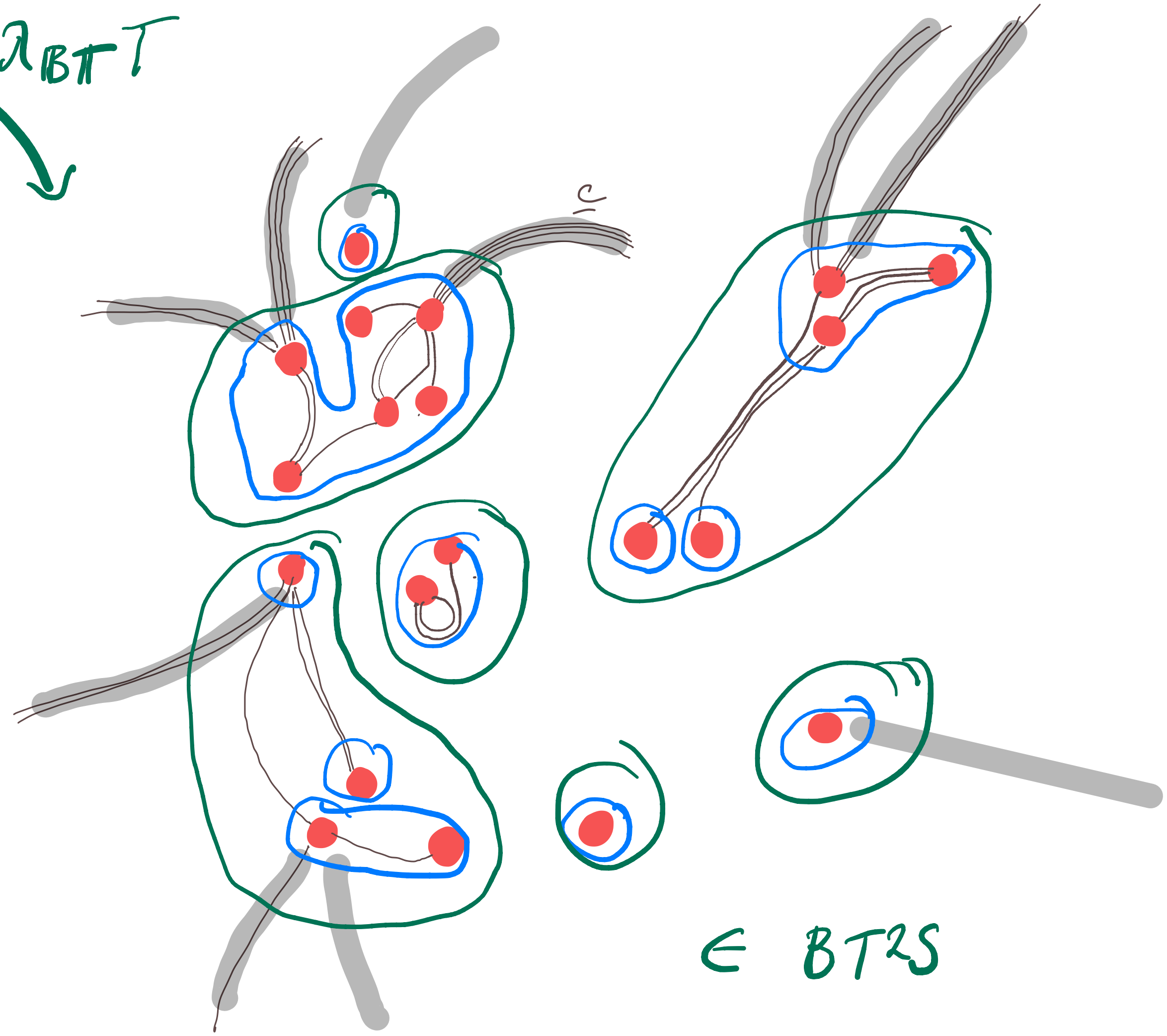


STEP I

delete clusters of inner edges

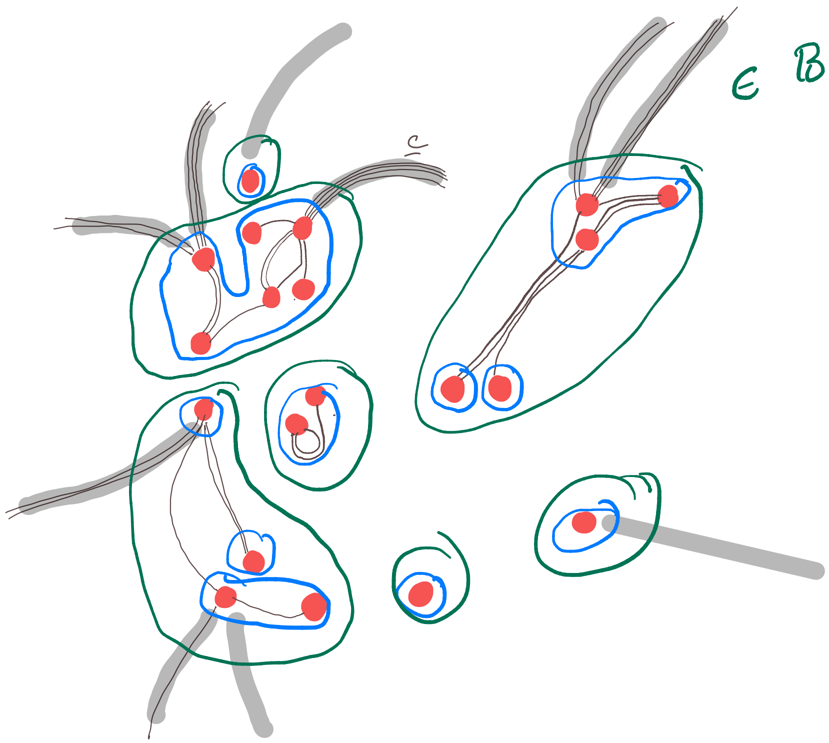


$\xrightarrow{\lambda_{BTT}}$



STEP II

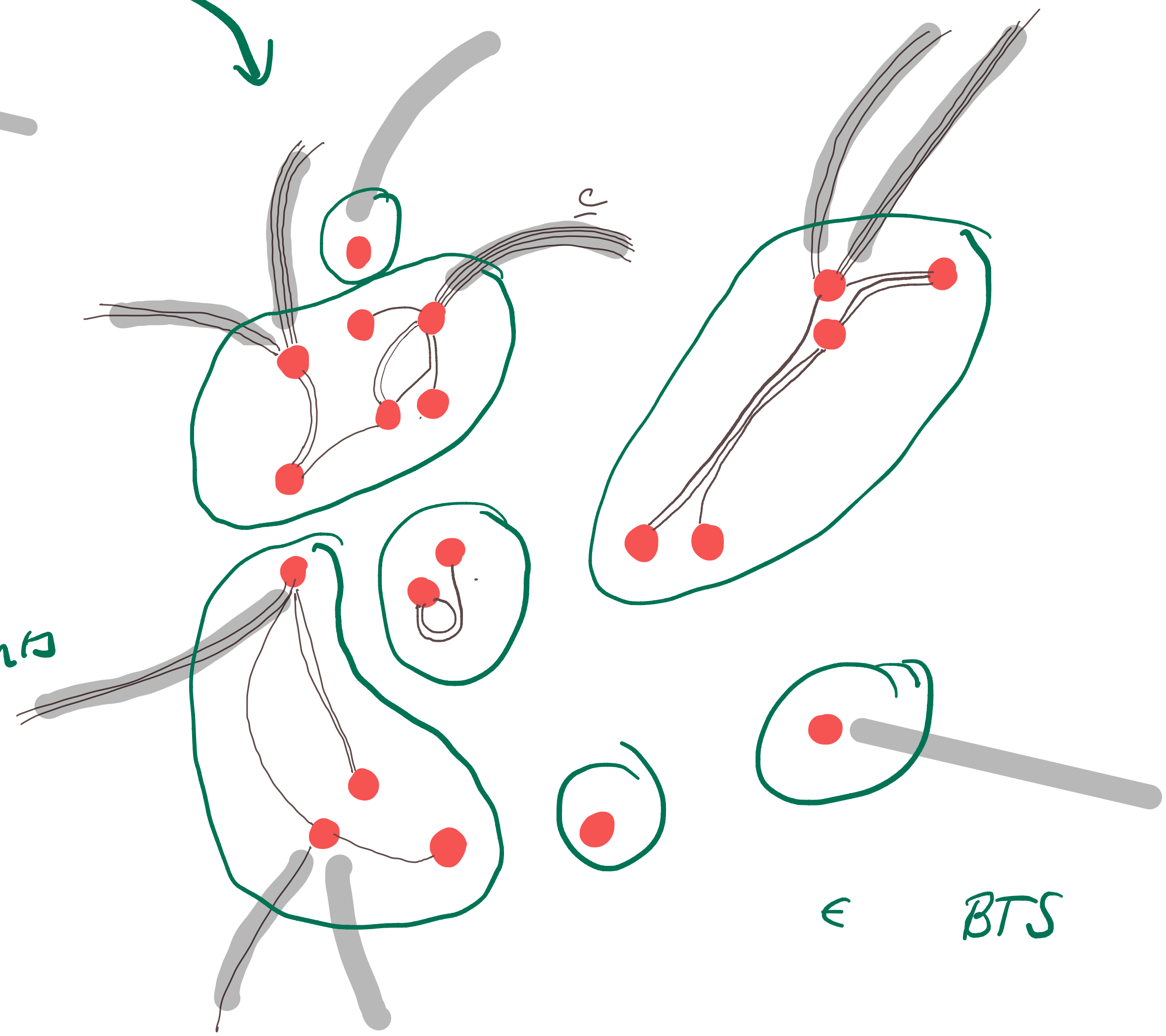
Delete green vertices and take connected components.



$\in BT^2S$

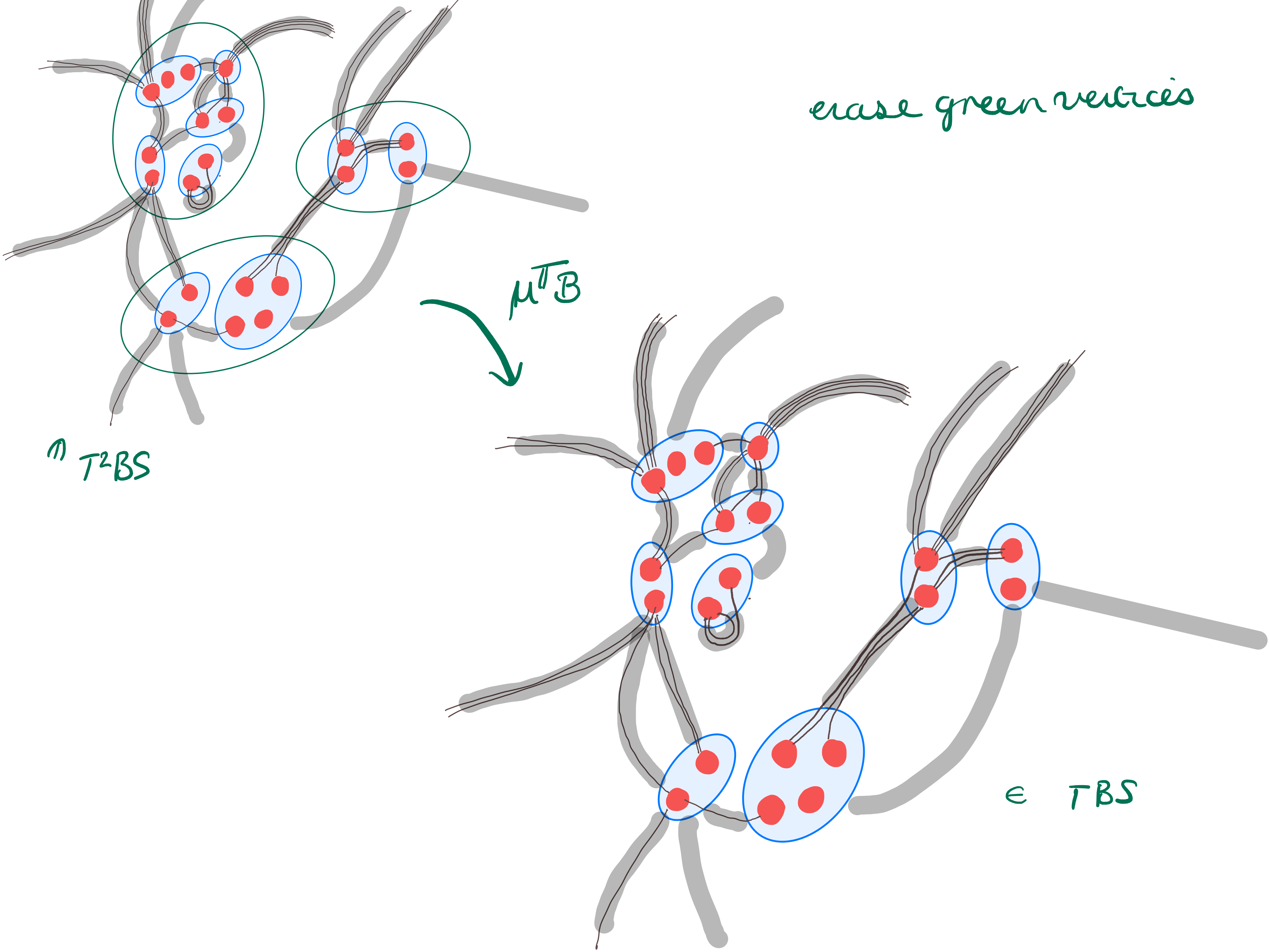
$\xrightarrow{B\mu^T S}$

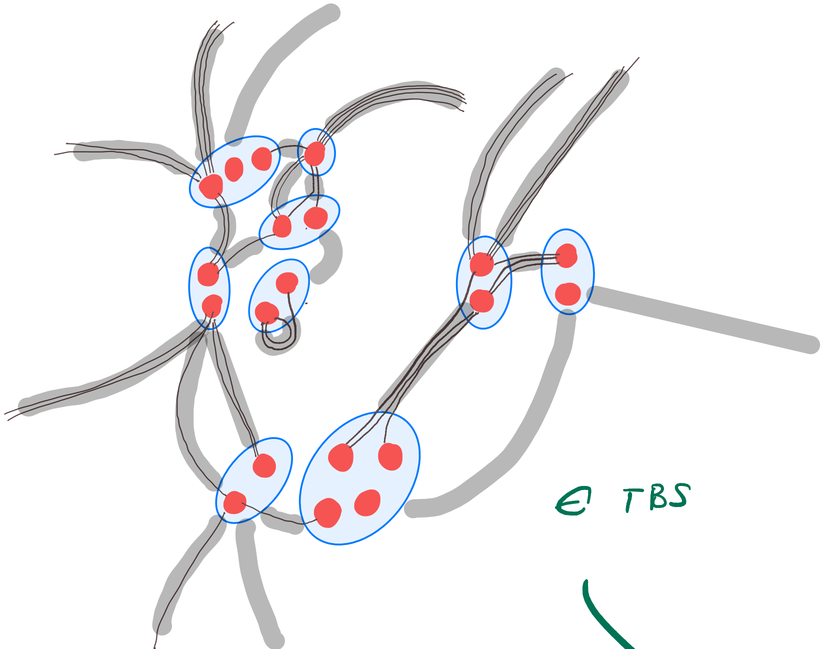
delete blue nesting inside green components



$\in BTS$

LELF-BOTTOM

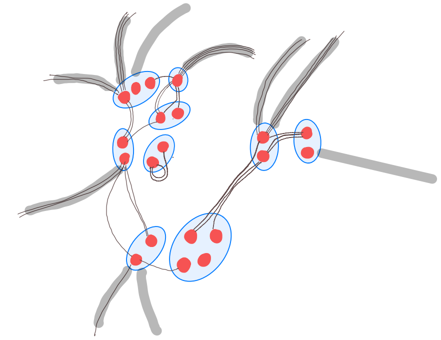




$\in$ TBS

STEP I

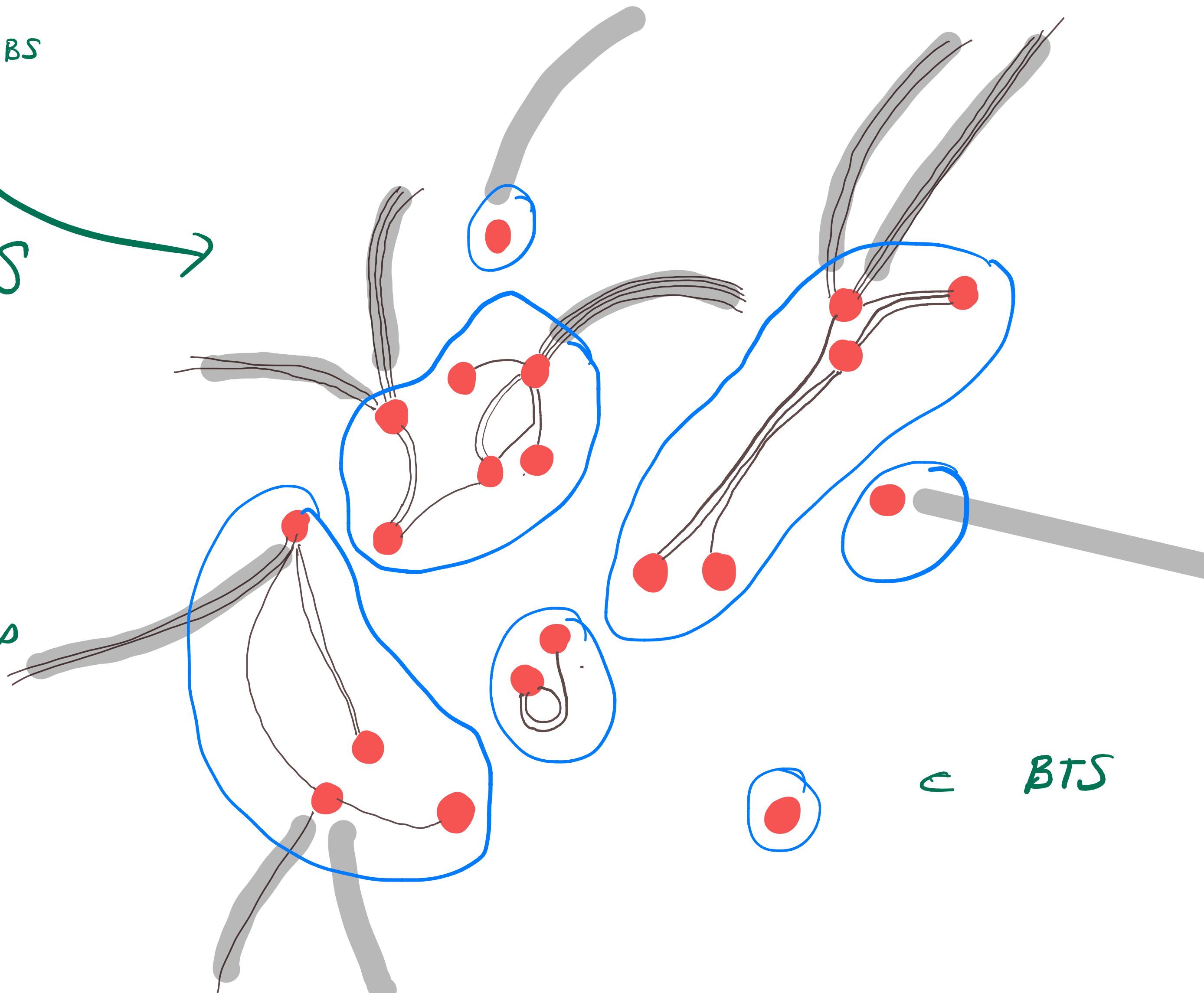
Erase clustering of inner edges.



λ_{BTS}

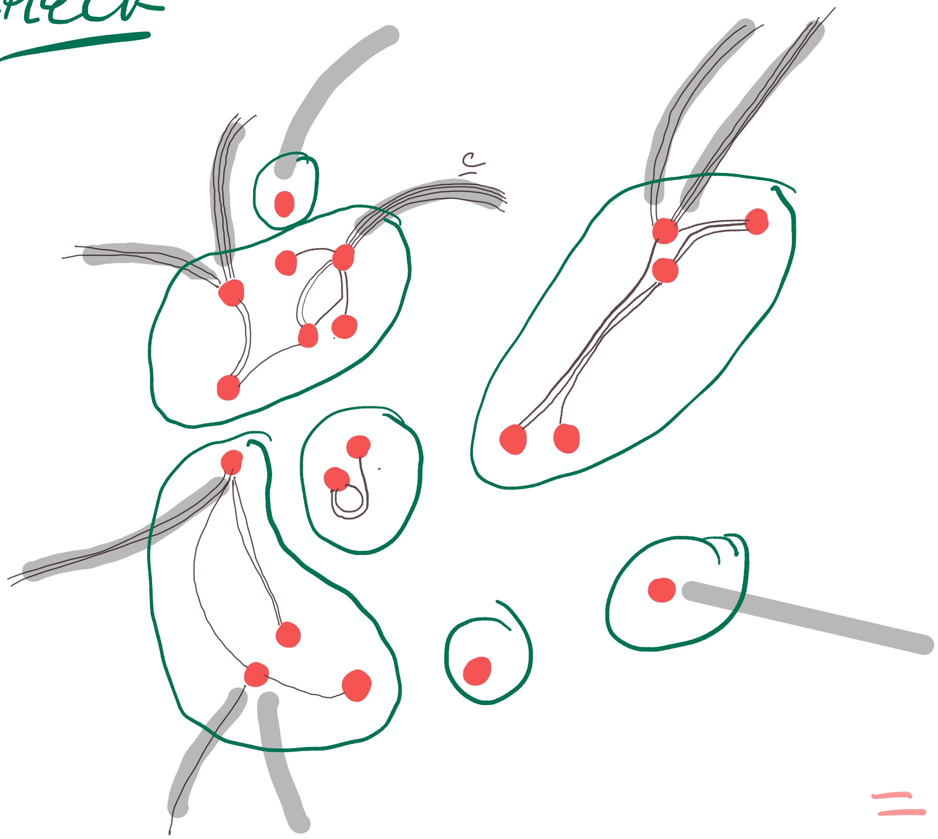
STEP II

erase blue vertices and take connected components.



$\in$ BTS

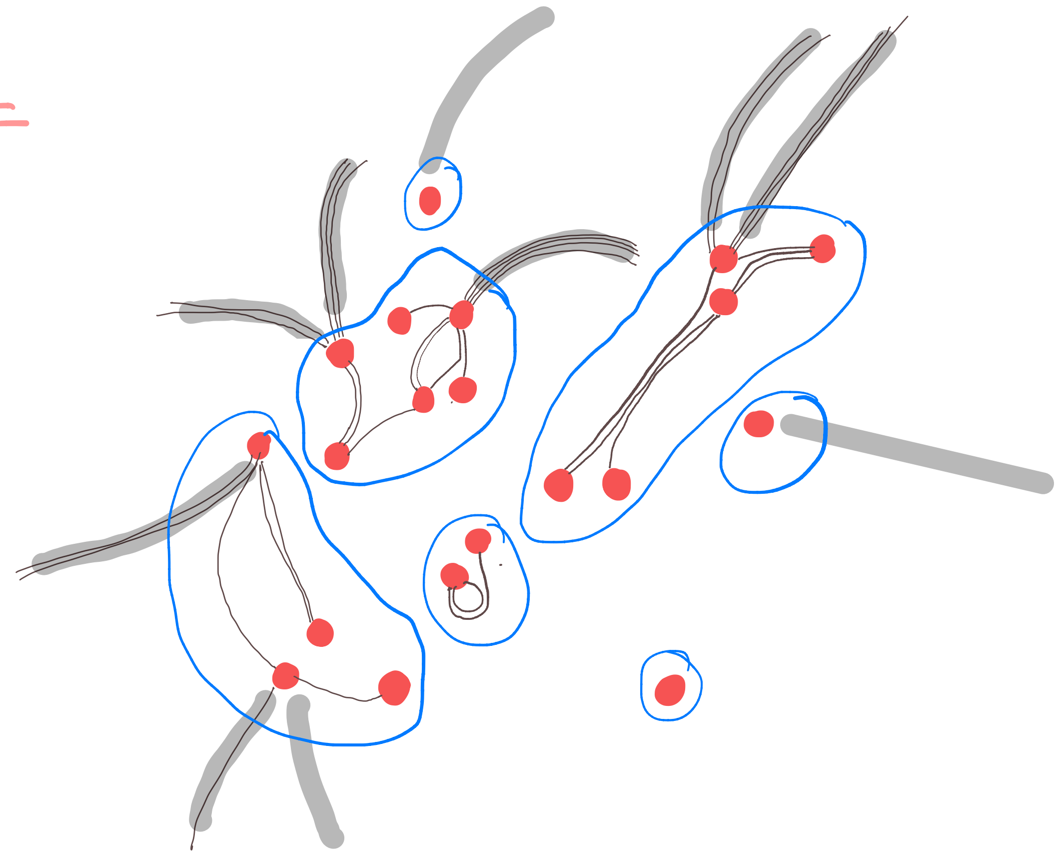
Check



Top-right

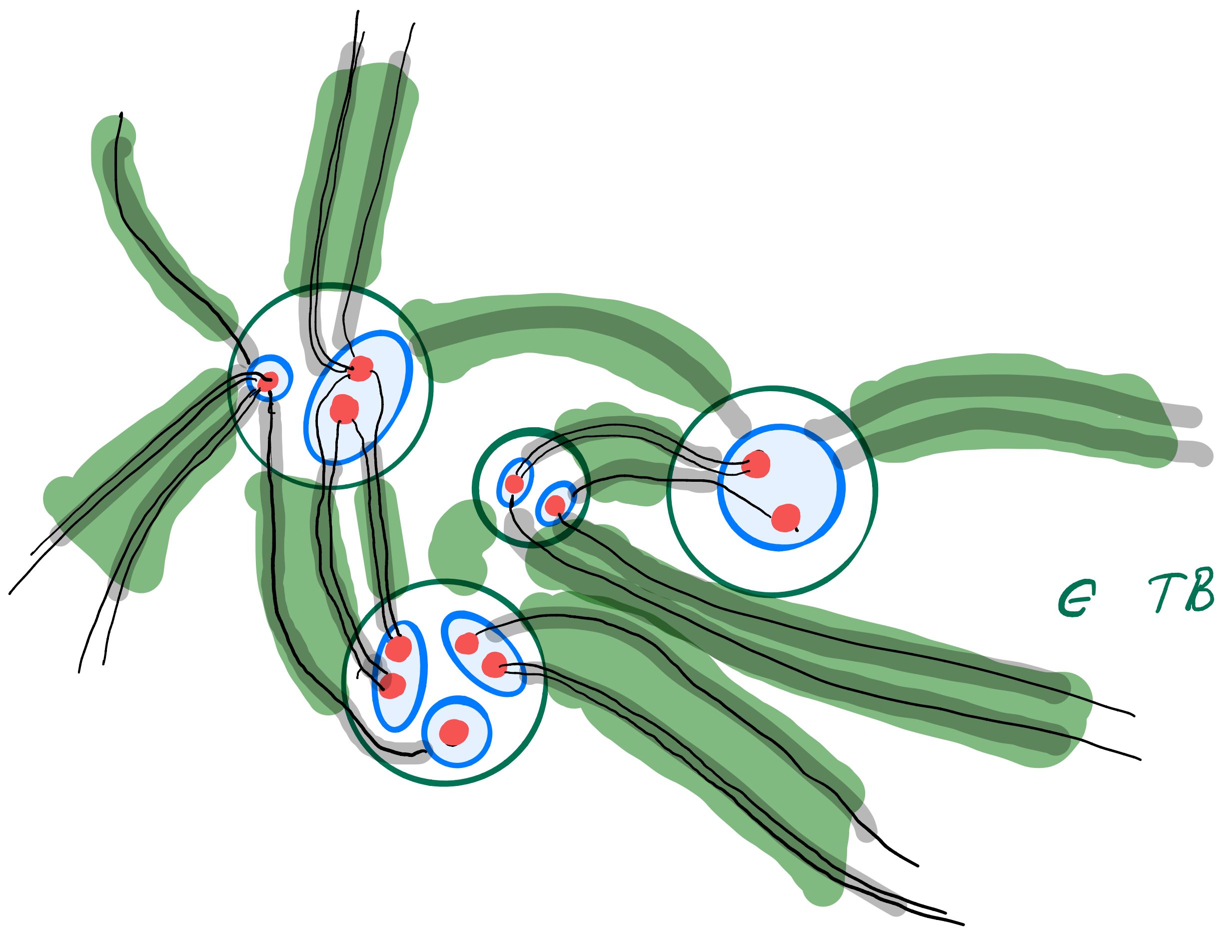
=

left-bottom



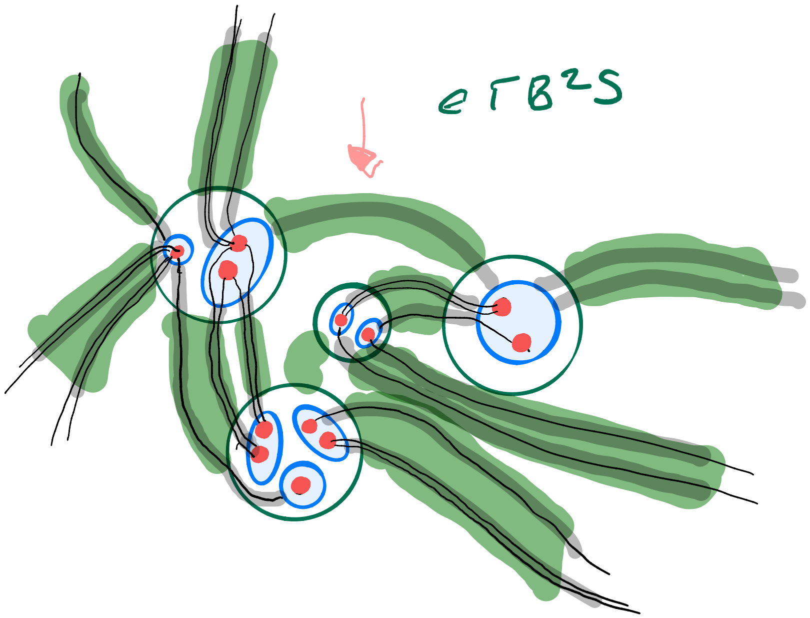
2)

$$\begin{tikzcd}
TB^2 \arrow[r, Rightarrow, "\lambda_{BT}B"] \arrow[d, Rightarrow, "T\mu^B"'] & BTB \arrow[r, Rightarrow, "B\lambda_{BT}"] \arrow[d, phantom, "\circlearrowright"] & B^2T \arrow[d, Rightarrow, "\mu^B T"] \\
TB \arrow[rr, Rightarrow, "\lambda_{BT}"'] & & BT
\end{tikzcd}$$



$\in TB^2S$

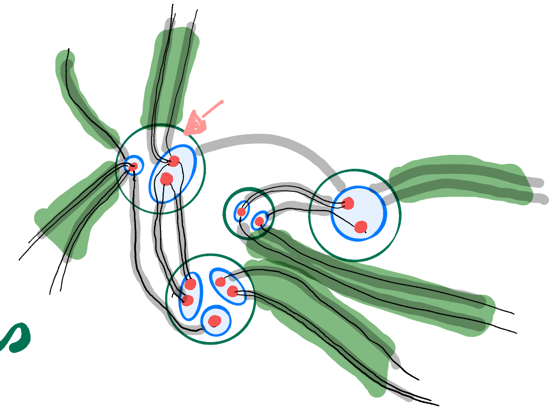
TOP RIGHT



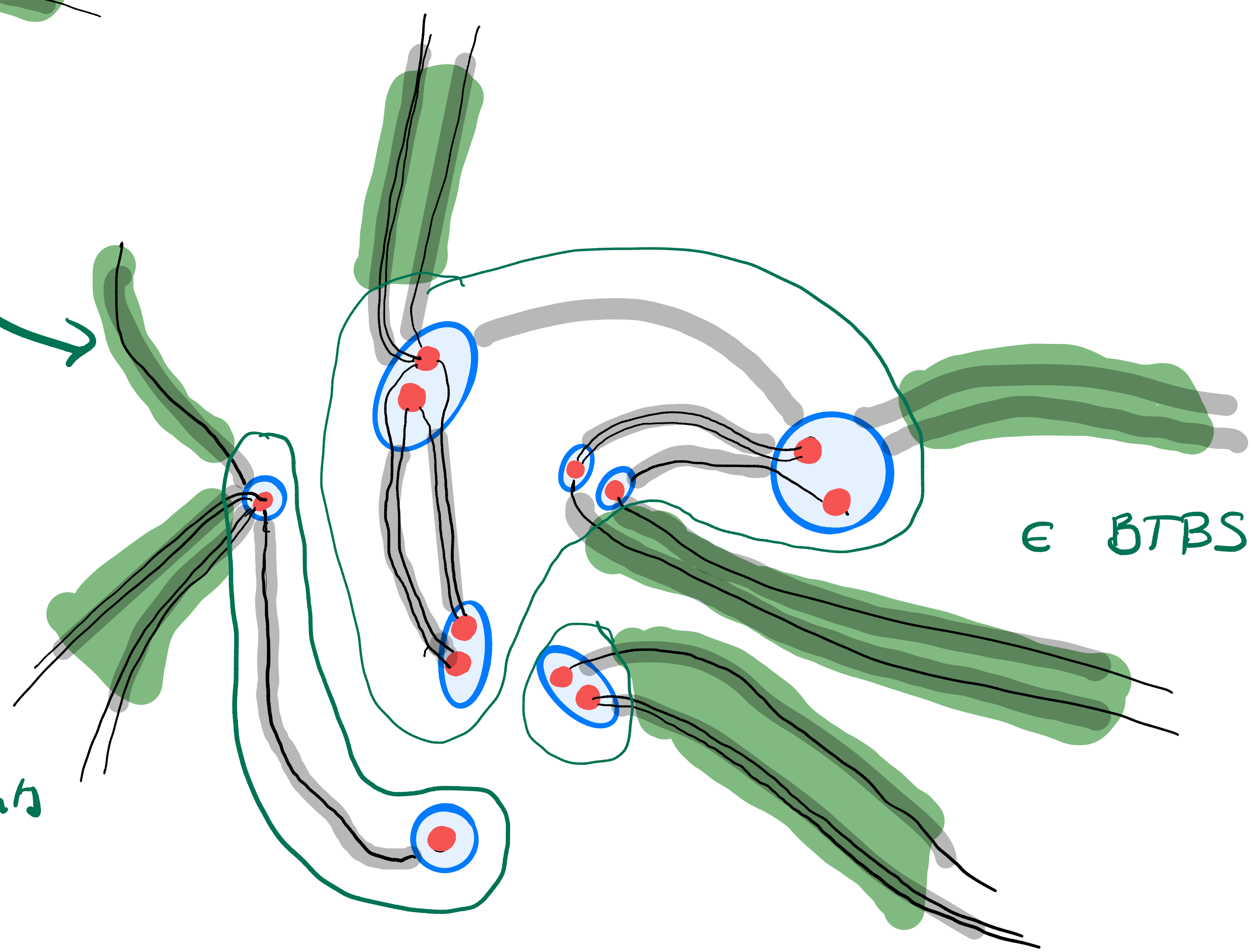
$\in TB^2S$

STEP I

Erase (green) clustering of inner edges



$\lambda_{BTT} B$

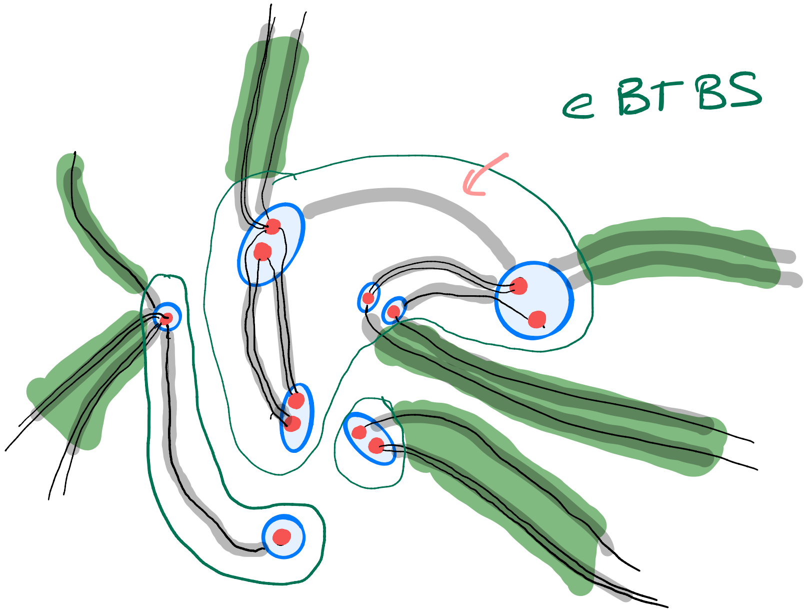


$\in$ BTBS

STEP 2

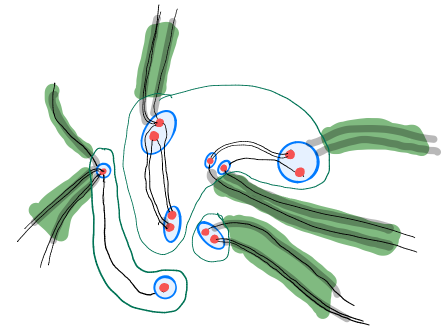
Erase green vertices & take connected components

$\in BTBS$

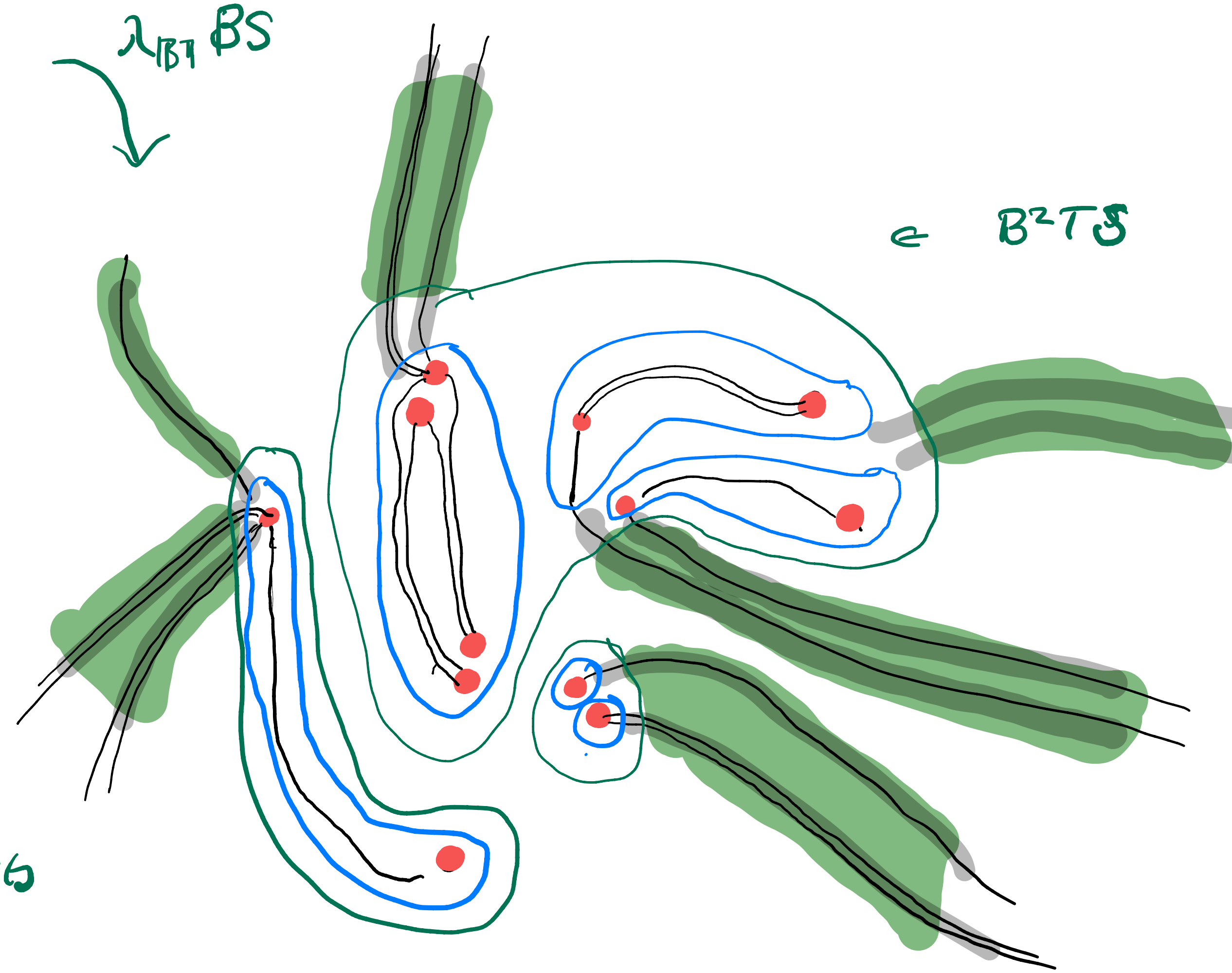


STEP I

erase clustering of edges inside green components

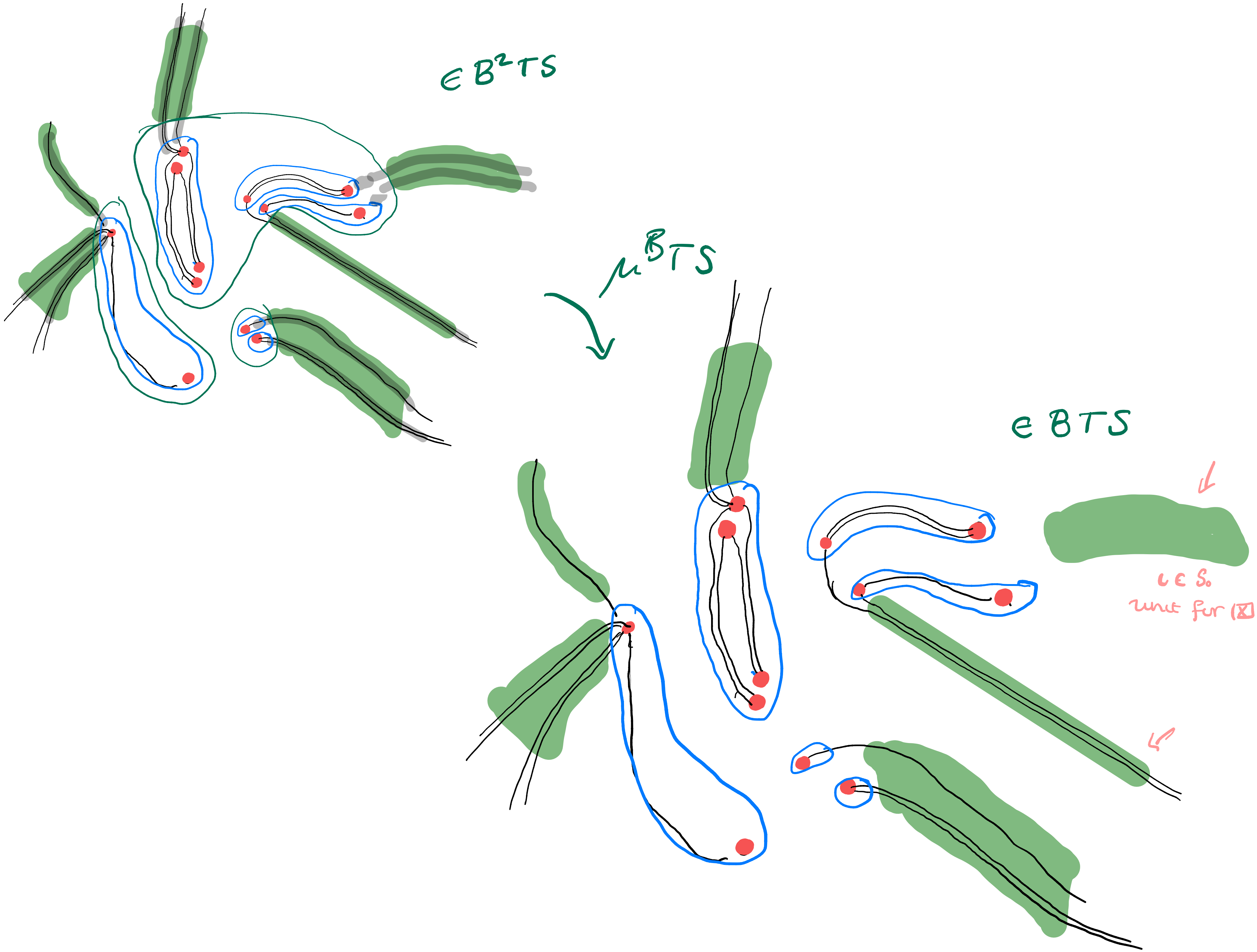


$\lambda_{BT} BS$



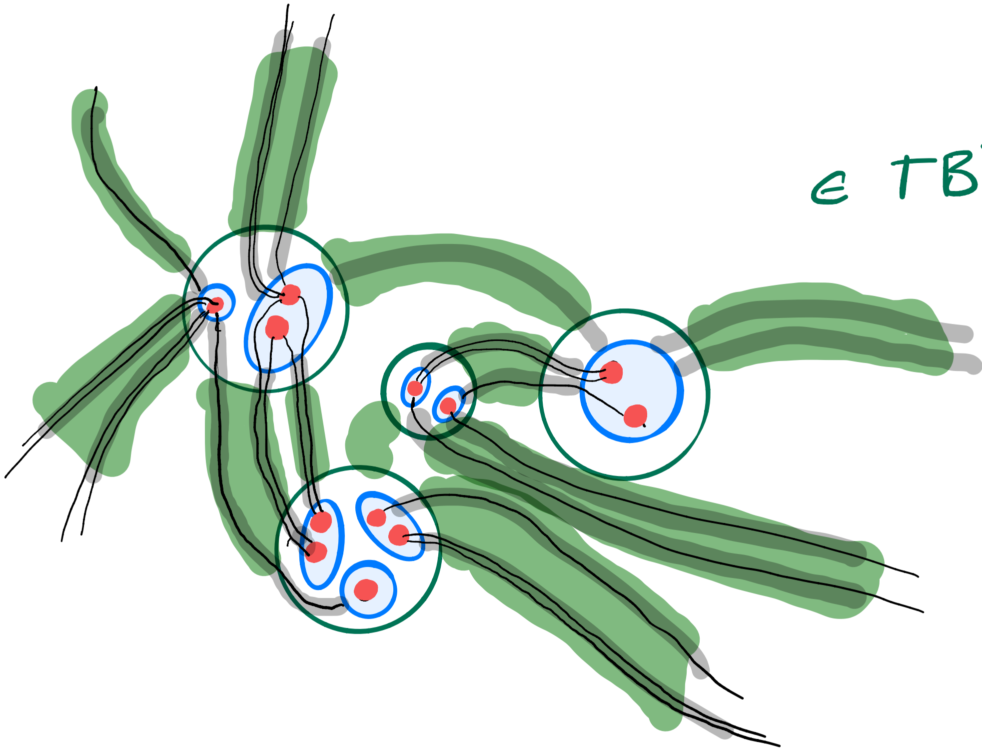
$\in B^2TS$

STEP II erase blue vertices and take connected components inside green components



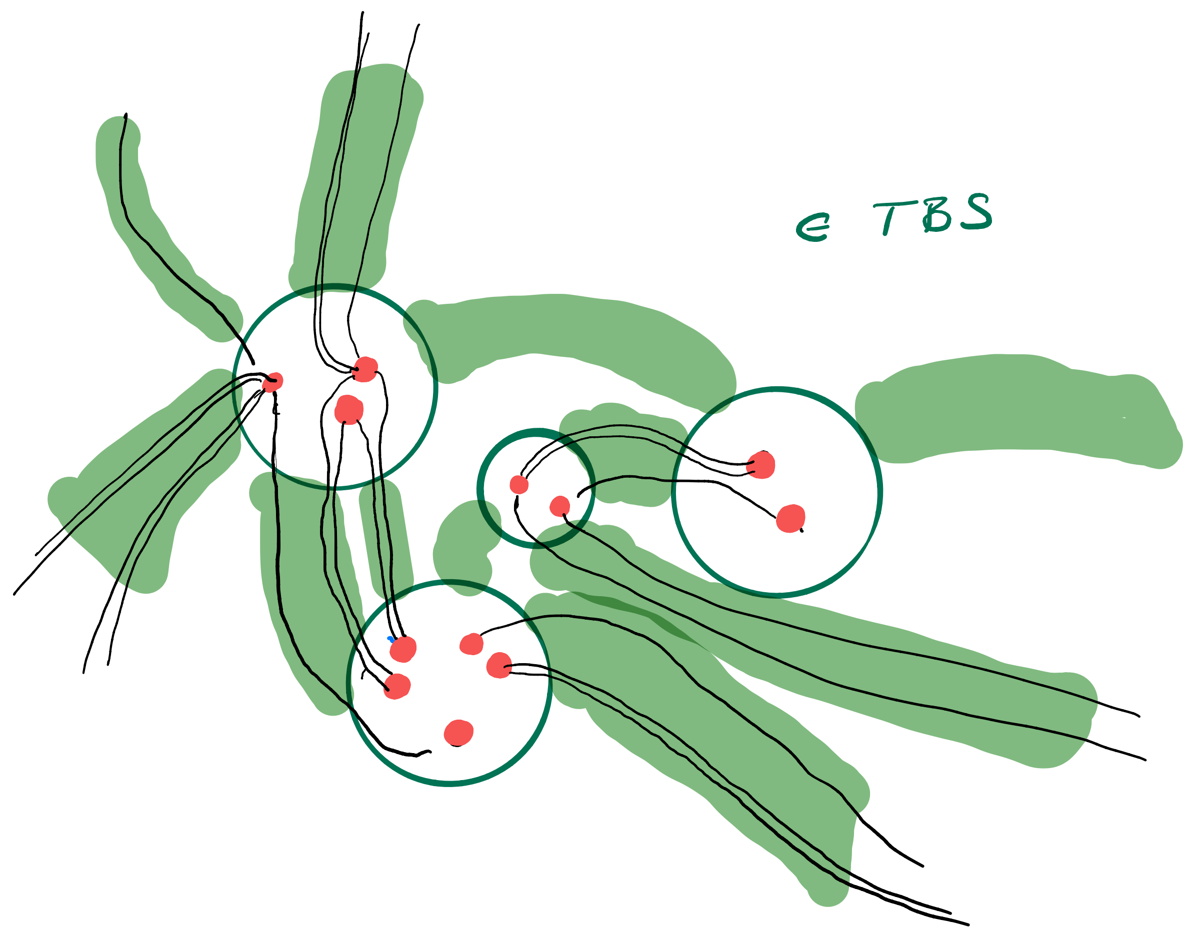
∈ B²TS
μ^B TS
∈ BTS
ι ∈ S₀
unit for (⊠

LEFT BOTTOM



$\in TB^2S$

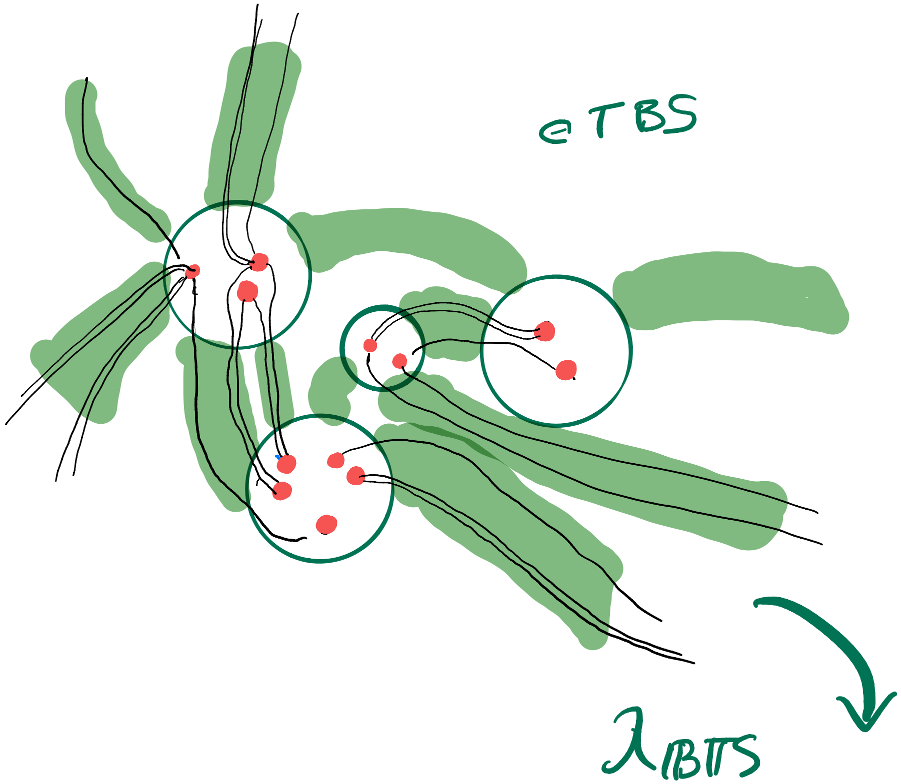
$T_{\mu}^{B}S$

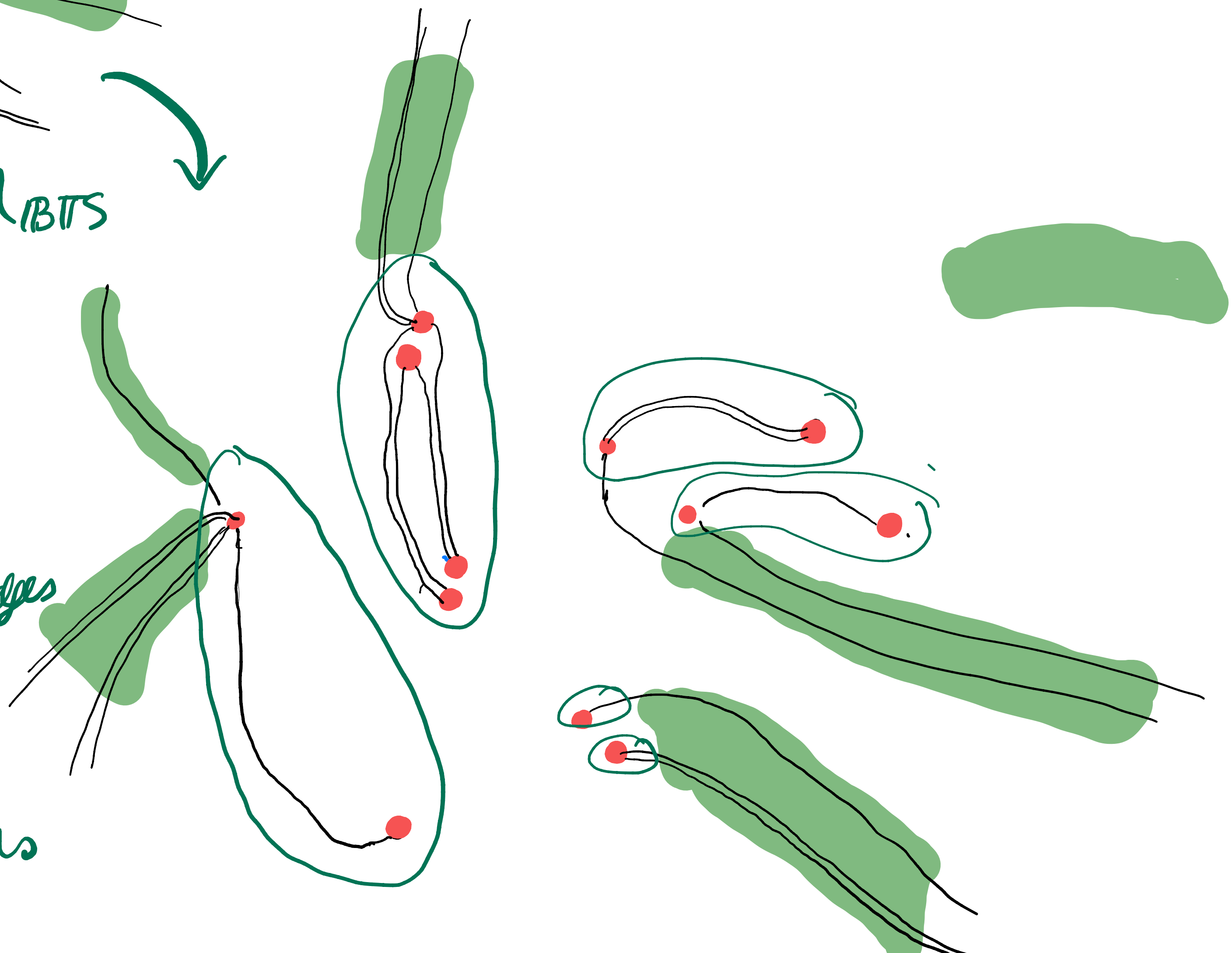


$\in TBS$

erase • grey clustered edges

• blue clustered vertices

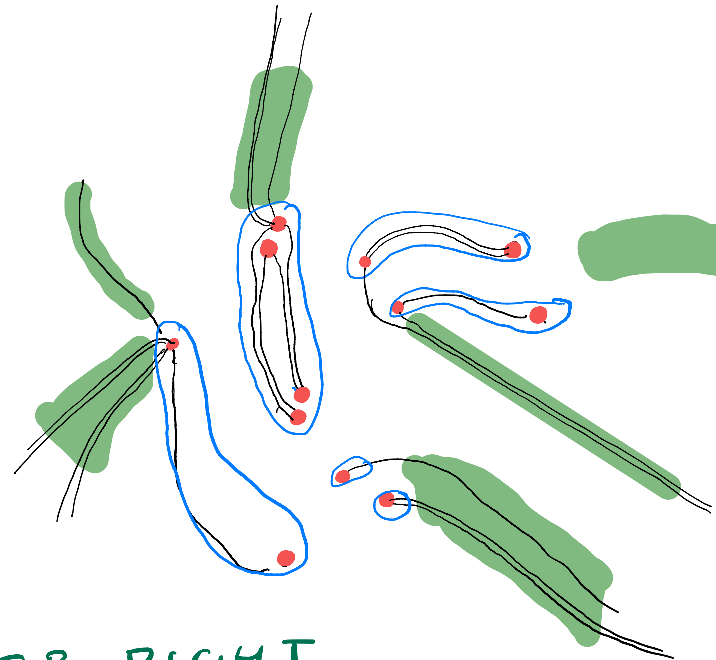




- erase green clusters of inner edges and green vertices.
- take connected components

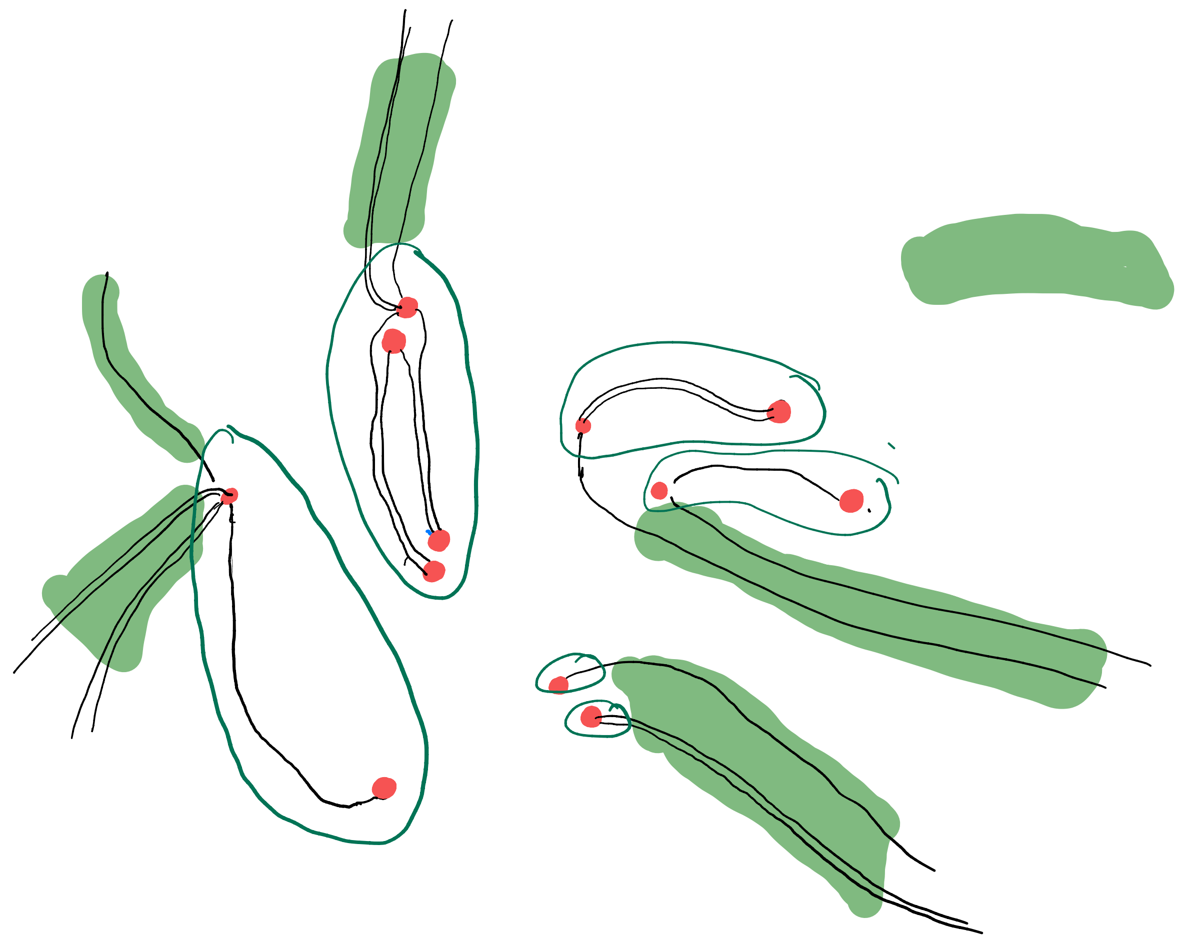
CHECK

other axons are easier to check

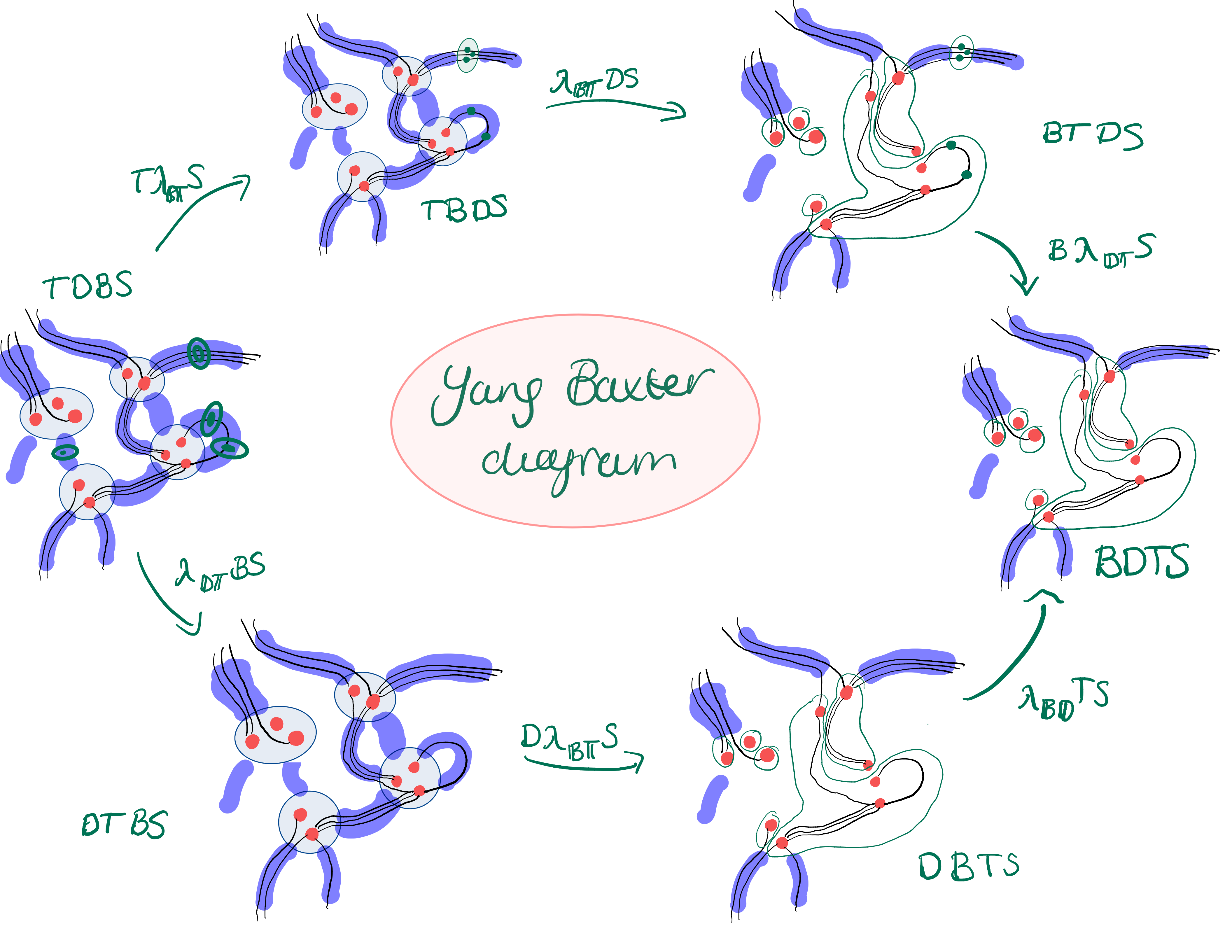


TOP RIGHT

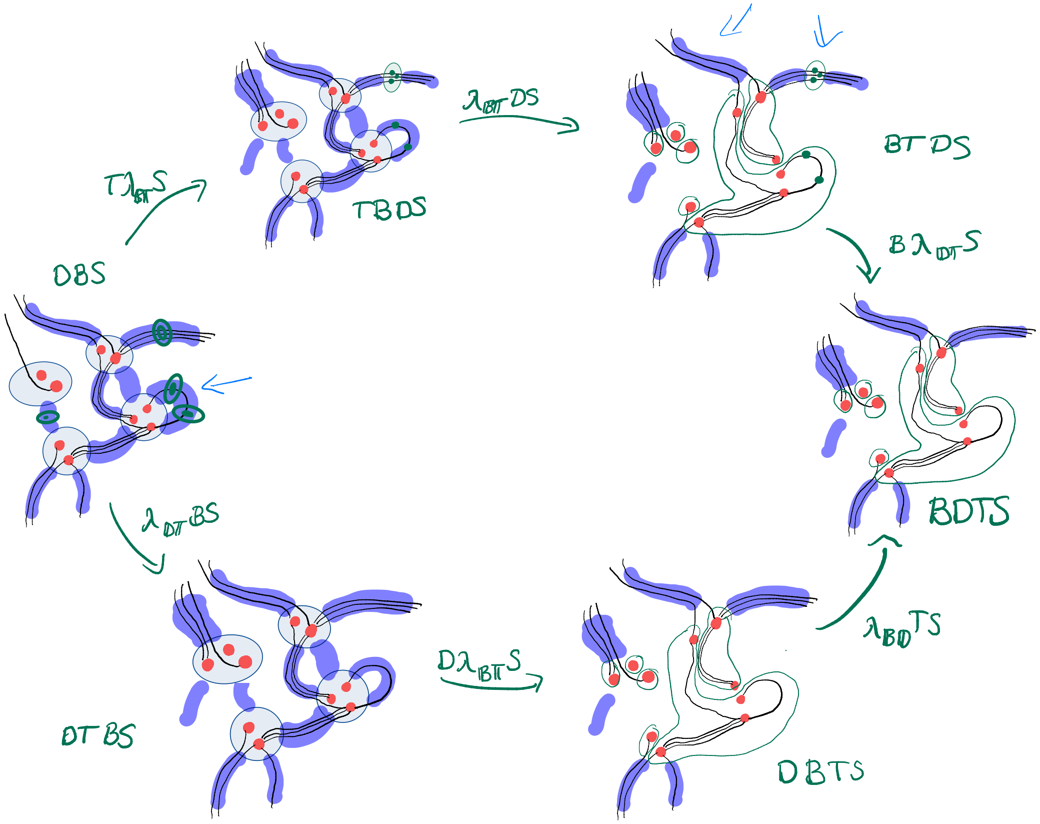
=



BOTTOM LEFT



Yang Baxter diagram
TBDS
BTDS
BDTS
DBTS
DTBS
TDBS
$T\lambda_{BT}S$
$\lambda_{BT}DS$
$B\lambda_{DT}S$
$\lambda_{BD}TS$
$D\lambda_{BT}S$
$\lambda_{DT}BS$



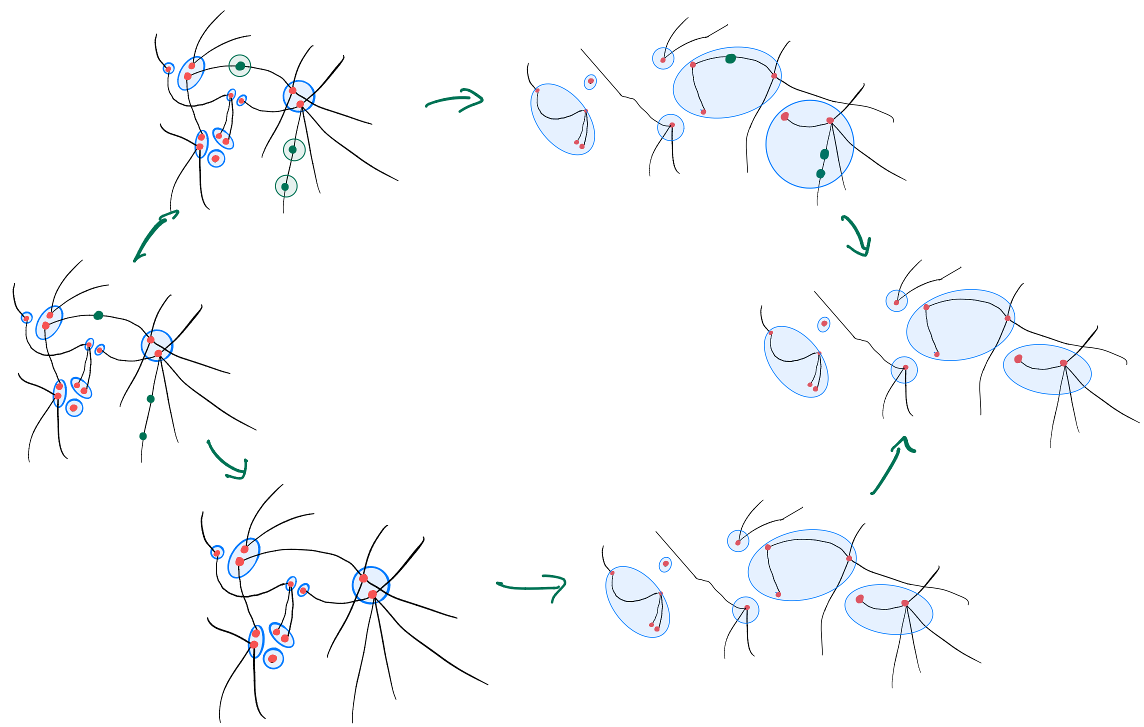
OBSERVATION

ML

//

BDT laws interact just as LDT laws

"Can forget product of colours except at ports"



THM Algebras for $\mathbb{BDT}$ are small compact closed categories

PF.

- Algebras for $\overset{\mathbb{ML}}{\overset{\shortparallel}{\mathbb{BDT}}}$ are circuit operads w/ monoid structure on colours $(\mathcal{C}, \otimes, \iota)$ and isos $S_{\underline{c^1 \ldots c^k}} \xrightarrow{\cong} S_{(c^1_1 \otimes \ldots \otimes c^1_{n_1}), \ldots, (c^k_1 \otimes \ldots \otimes c^k_{n_k})}$
- By distributive law for $\mathbb{B} = \mathbb{ML}$

$$S_0 \cong S_\iota$$

Nerve theorem

$$
\begin{tikzcd}
 & \equiv^{\otimes} \arrow[r, "\text{dense}", "f.f"'] & \mathrm{Comp} \arrow[r, "N^{\otimes}", "f.f"'] \arrow[d, shift left, "U^{\tilde{M}}"] & ps(\equiv^{\otimes}) \arrow[d] \\
 & \equiv^{\times} \arrow[u, "b.o"] \arrow[r, "\text{dense}", "f.f"'] & CO \arrow[u, shift left, "F^{\tilde{M}}"] \arrow[r, "N", "f.f"'] \arrow[d, shift left, "U^{\mathbb{L}T_*}"] & ps(\equiv^{\times}) \arrow[d] \\
\mathbb{S}_* \arrow[r, "\text{dense}", "ff"'] & Gr_* \arrow[u, "b.o"] \arrow[r, "\text{dense}", "ff"'] & GS_* \arrow[u, shift left, "F^{\mathbb{L}T_*}"] \arrow[r, "ff"'] \arrow[d, shift left, "U^{D}"] & ps(Gr_*) \arrow[d] \\
\mathbb{S} \arrow[u, "b.o"] \arrow[ur, dashed, "\text{dense}"] \arrow[r, "\text{dense}", "f.f"'] & Gr \arrow[u, "b.o"] \arrow[r, "\text{dense}", "f.f"'] & GS \arrow[u, shift left, "F^{D}"] \arrow[r, "f.f"'] & ps(Gr)
\end{tikzcd}
$$

The functor $N^{\otimes}: \mathrm{Comp} \to ps(\equiv^{\otimes})$ is fully faithful.

Essential image of N given by $P \in ps(\equiv^{\otimes})$ s.

$$P(\mathcal{G}) \cong \lim_{(C,f) \in \mathbb{S} \downarrow \mathcal{G}} P(C)$$

Thank
you!