Auctioning with leveled commitment

Motivation:

When access to goods/resources is limited and demand exceeds supply, the need to distribute these resources in the most optimal way arises. One way to organize the process of resource distribution is to organize an auction, where the one who values the resource the most is going to win paying a high price. This way both the buyer and the seller will be satisfied.

Different types of auction mechanisms exist, and each of them has certain advantages and disadvantages. An "ideal" auction mechanism should maximize profits of sellers and buyers, maintain price stability, and be "fair" (this typically refers to the buyers being the best-off when they bid their true valuations) and non-biased.

Currently, e-auctions are quite wide-spread – the prominent examples are eBay and Google AdWords platforms. But these are only a tip of an iceberg, similar auctions take place in e.g. energy and financial markets, as well as in various tenders.

In "pure" auctioning buyers are not allowed to decommit from the deal made in an auction. To make the auctioning process more flexible, we might want to allow the buyers to break a previous deal, paying a penalty to the seller, in favor of a more profitable new deal. Auctions of that type are called auctions of leveled commitment. In this assignment we are going to look at both "pure" and leveled commitment auctioning.

Description:

Consider a modified second-price sealed-bid (Vickrey) auction scenario. On one side of the auction process are K sellers. The sellers make profit from the bids paid to them for the sold items. On the other side of the process are $N > K$ buyers, which are trying to purchase the items on sale and maximize their profit by reselling the items at a higher value.

In this scenario auctions are organized in R rounds. In every round each seller $k \in K$ organizes one auction. Thus, the total number of auctions in a simulation is $R \times K$. Order in which the actions take place within each round is always random, and is not known to the auction participants beforehand. In every round each buyer $n \in N$ can be the winner of only one auction. Therefore, in "pure" auctioning, buyers, which win an auction in a round, cannot participate in further auctions of the round, whereas in auctioning of leveled commitment – all N buyers participate in each auction of every round.

In every auction seller k sets a random starting price for an item it is selling, which is denoted as $S_k \in \{0, S_{\max}\}$, where $S_{\max}$ – is a universal maximum starting price, set once per simulation. Then, each buyer places its bid. For buyer n the bid for the item sold by seller k is defined as $B_{n,k} = \alpha_{n,k} S_k$, for $\alpha_n \geq 1$, where $\alpha_{n,k}$ – is a unique bidding factor of buyer n purchasing the item from seller k (see Bidding Strategy section for details). Once all the bids are placed, a market price is computed as an average of the placed bids $\varXi_k = \sum_{n=1 \dots N} \frac{B_{n,k}}{N}$. Note that the market price is known to participants of the auction immediately after the results of the auction are announced.

All the buyers, which bid was higher than the market price $\varXi_k$, cannot win because in this case they are going to make losses, which violates their rationality. According to the Vickrey mechanism, a buyer with the highest bid (closest to $\varXi_k$) is the winner of the auction, which then pays the second-highest bid to the seller.

In "pure" auctioning the profits of the sellers and buyers are as follows:

$$
\begin{aligned}
p_{s_k} &= \sum b_i \\
p_{b_n} &= \sum (\varXi_i - b_i) ,
\end{aligned}
\tag{1}
$$

where p_{s_k} – is the profit of the k-th seller, as a sum of the collected second bids b_i from i auctions organized; and p_{b_n} – profit of the n-th buyer, as a sum of differences between the market prices $\varXi_i$ and the paid second bids b_i from i won auctions (note, that values of i do not necessarily refer to the same auctions for buyers and sellers). Note that in case of only one (winning) bid is below market price, second highest bid is calculated as the average between the winning bid and the auction's starting price.

Till now we looked only at auctions without taking into account leveled commitment. If we do take it into account however, the auction procedure will be slightly different. In that case if a buyer, which already won an auction in a round, wins another auction in the same round, the buyer must decommit from the auction result which brings it the least profit. In this case, for each purchased item buyer n will pay to seller k (which sold the item) an annulling penalty fee $f_{n,k} = \varepsilon B_{n,k}$, where ε – is a penalty factor set once per simulation, and $B_{n,k}$ – (second-highest) bid paid by buyer n to seller k. Once seller k receives from buyer n the annulling request together with the returned item, the seller refunds the buyer with the amount of paid bid minus the amount of fee $(B_{n,k} - f_{n,k})$.

In auctioning with leveled commitment, the profits are computed taking into account the penalties:

$$
\begin{aligned}
p_{s_k} &= \sum b_i + \sum f_j \\
p_{b_n} &= \sum (\varXi_i - b_i) - \sum f_j ,
\end{aligned}
\tag{2}
$$

where j – denotes the auctions, results of which were annulled (again note, that values of i and j do not necessarily refer to the same auctions for buyers and sellers).

Bidding Strategy:

The following is an example of the bidding strategy buyers can utilize. Note there are alternatives, which you might want to explore yourself.

When a buyer n calculates its bid for an item offered by seller k it needs to select value of a bidding factor $\alpha_{n,k}$. Initially, value of $\alpha_{n,k}$ can be set arbitrary. After each auction over items offered by seller k where buyer n participated, value of the bidding factor is updated, taking into account values of bid $B_{n,k}$ and market price $\varXi_k$ from the previous auction:

$$\hat{\alpha}_{n,k} = \begin{cases} \underline{\Delta}_n \alpha_{n,k}, \text{if } (B_{n,k} \text{ won}) \vee (B_{n,k} \geq \Xi_k\ ) \\ \overline{\Delta}_n \alpha_{n,k}, \text{otherwise,} \end{cases} \tag{3}$$

where $\underline{\Delta}_n$ – a bid decrease factor of buyer n ($\underline{\Delta}_n \leq 1$), and $\overline{\Delta}_n$ – is a bid increase factor of buyer n ($\overline{\Delta}_n \geq 1$). The bid factors are set per buyer once per simulation. This way the bids of the buyer n are going to adapt to results of auctions organized by seller k.

In case if the same buyer n has already purchased some other item with market price Ξ_x from another seller $x \in K$ in the same round and paid the seller x value of the second-highest bid $b_{n,x}$, the bid calculation for the new auction with seller k becomes more involved. In this case buyer n also has to take into account the loss of secured profit from the previous purchase and the amount of penalty $f_{n,x}$ to be paid to seller x for the annulling of the previous purchase. This way the bid calculation turns into:

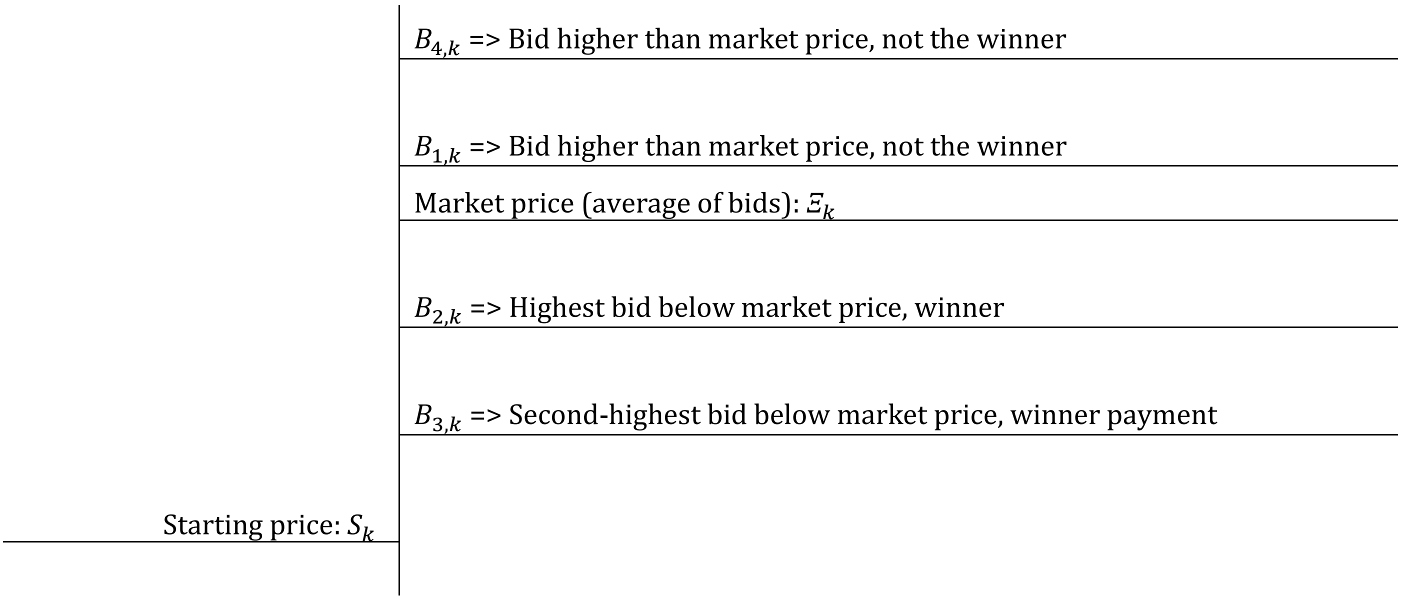
$$B_{n,k} = \alpha_{n,k} S_k\ - \left((\Xi_x\ - b_{n,x}) + f_{n,x} \right). \tag{4}$$

This formulation ensures that the buyer is eager to annul a result of another auction won in the same round if and only if the potential profit from purchase of the new item is going to be higher than the already secured profit from the purchased item plus annulling fee to be paid. Note that the potential profit amount is guaranteed to be higher (Δp_{b_n}>0 $\Leftrightarrow \hat{p}_{b_n} - p_{b_n}$>0 $\Leftrightarrow \hat{p}_{b_n}$>$p_{b_n}$) if and only if the difference between market prices of the new auction with seller k and the previous auction with seller k ($\Delta\Xi_k\ = \hat{\Xi}_k\ - \Xi_k\ $) is greater than a difference between the second-highest bids in the two auctions with seller k ($\Delta b_{n,k} = \hat{b}_{n,k} - b_{n,k}$):

$$\Delta p_{b_n} > 0 \text{ iff } \Delta\Xi_k\ > \Delta b_{n,k}. \tag{5}$$

If the condition above does not hold, buyer n can obtain profit lower than the amount of secured profit plus fee to be paid – in this case the buyer will have to make a rational choice on which of the two items to keep, and a result of which of the two auctions to annul.

Below is a graphical representation of the description above on a price axis, price ascends from the bottom:



Here 4 buyers placed their bids for an item in the auction of seller k. The market price Ξ_k was calculated. Bids of buyers 4 and 1 were higher than the market price, thus these buyers did not win. Bid of the buyer 2 was the highest bid below market price, thus buyer 2 is the winner. The second highest bid was from buyer 3, this bid is paid by the buyer 2 to the seller.

Goal:

Your task is to implement a model of the described auctioning scenario. The simulation needs the following input:

- Number of sellers K
- Number of buyers N
- Number of auction rounds R
- Universal maximum starting price $S_{\max}$
- Penalty factor ε
- Indication whether "pure" or leveled commitment auctioning is used
- Any other parameters utilized by the chosen bidding strategy (make sure to include explanations)

The simulation in turn should generate the following output:

- Statistics of market price Ξ_k development across rounds
- Sellers' profits p_{s_k}
- Buyers' profits p_{b_n}

Remarks:

Buyers are assumed to have unlimited resources. What we look at are the profits made.

Any random number is assumed to be generated from the uniform distribution, same for all affected entities (buyers/sellers/items).

Organization:

This assignment is performed in groups of **maximum** 4 students. The suggested timeline is as follows:

Week #	Key Tasks	Deliverable
1	Familiarize yourself with the task. Apply the described auctioning mechanism in basic scenarios without leveled commitment (how does the bidding strategy evolve, what is needed to win the auction, when is the profit maximized?).	
2	Implement a first version of the auction simulation and improve iteratively, include leveled commitment scenario. Start drafting a report.	
3	Run experiments by systematically varying the corresponding parameters: • Experiment with values of number of sellers K, number of buyers N (Note: for K and N start with small values (e.g. $K = 1$, $N = 2$) and make little increments of e.g. 1), and number of rounds R • Experiment with values of penalty factor ε • Experiment with values of bidding factors $\alpha_{n,k}$ • Experiment with values of bid in-/decrease factors $\overline{\Delta}_n$ and $\underline{\Delta}_n$ Work on the report.	
4	Finalize implementation, perform final experiments and complete the report.	Final Product; Report

In the scope of this lab assignment you are required to:

- reason about your choices: research on what techniques/methods/approaches/etc. are suitable for solving a given key task of the week, compare them and select the preferred one – be clear why this approach was preferred by you;
- conduct experiments in a systematic way: perform simulations and tests in order to validate your solution and evaluate its performance; and
- analyze the obtained results (What can be seen from the results? Are the results as expected? If not, then what might be the reasons?).

Deliverables:

All deliverables are submitted by a group as a solution to the corresponding assignment on Student Portal:

1. **Final Product**: software binaries (+ source code) or executable scripts in any programming language. Please create a structured .zip file with your code. Please make sure that the interface of your software accepts input and generates output in accordance with the specifications given in the Goal section.
2. **Report** describing your solution and experiments. Discuss the achieved results and draw conclusions (What can you deduce from your experiments with different parameter values? What is combination of the parameters in your opinion comes closest to the "ideal" auction? How realistic is the developed model? In which ways can it be improved?). Motivate and briefly explain any choices you made. More details about the report formatting and structure can be found in "Lab Task Assessment" document on Student Portal.

Grading:

Grade of this lab assignment corresponds to 1.25 points of the final course grade. The grade is given on a 0 to 10 scale, and is awarded to a group and thus applies to all its members.

The grade is a sum of the following components:

- 20% – Final Product
- 80% – Report