

# Numerical Simulation of Optical Wave Propagation

With examples in MATLAB<sup>®</sup>

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# Numerical Simulation of Optical Wave Propagation

With examples in MATLAB®

Jason D. Schmidt

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# Contents

- Preface ..... ix**
- Chapter 1 Foundations of Scalar Diffraction Theory ..... 1**
  - 1.1 Basics of Classical Electrodynamics ..... 1
    - 1.1.1 Sources of electric and magnetic fields ..... 2
    - 1.1.2 Electric and magnetic fields ..... 2
  - 1.2 Simple Traveling-Wave Solutions to Maxwell's Equations ..... 5
    - 1.2.1 Obtaining a wave equation ..... 5
    - 1.2.2 Simple traveling-wave fields ..... 7
  - 1.3 Scalar Diffraction Theory ..... 9
  - 1.4 Problems ..... 12
- Chapter 2 Digital Fourier Transforms ..... 15**
  - 2.1 Basics of Digital Fourier Transforms ..... 15
    - 2.1.1 Fourier transforms: from analytic to numerical ..... 15
    - 2.1.2 Inverse Fourier transforms: from analytic to numerical ..... 17
    - 2.1.3 Performing discrete Fourier transforms in software ..... 18
  - 2.2 Sampling Pure-Frequency Functions ..... 21
  - 2.3 Discrete vs Continuous Fourier Transforms ..... 23
  - 2.4 Alleviating Effects of Discretization ..... 26
  - 2.5 Three Case Studies in Transforming Signals ..... 30
    - 2.5.1 Sinc signals ..... 30
    - 2.5.2 Gaussian signals ..... 31
    - 2.5.3 Gaussian signals with quadratic phase ..... 33
  - 2.6 Two-Dimensional Discrete Fourier Transforms ..... 35
  - 2.7 Problems ..... 37
- Chapter 3 Simple Computations Using Fourier Transforms ..... 39**
  - 3.1 Convolution ..... 39
  - 3.2 Correlation ..... 43

3.3 Structure Functions.....	47
3.4 Derivatives .....	50
3.5 Problems .....	53
<b>Chapter 4 Fraunhofer Diffraction and Lenses .....</b>	<b>55</b>
4.1 Fraunhofer Diffraction .....	55
4.2 Fourier-Transforming Properties of Lenses .....	58
4.2.1 Object against the lens .....	59
4.2.2 Object before the lens .....	59
4.2.3 Object behind the lens .....	61
4.3 Problems .....	64
<b>Chapter 5 Imaging Systems and Aberrations.....</b>	<b>65</b>
5.1 Aberrations .....	65
5.1.1 Seidel aberrations .....	66
5.1.2 Zernike circle polynomials .....	66
5.1.2.1 Decomposition and mode removal .....	73
5.1.2.2 RMS wavefront aberration.....	75
5.2 Impulse Response and Transfer Function of Imaging Systems .....	77
5.2.1 Coherent imaging .....	77
5.2.2 Incoherent imaging .....	79
5.2.3 Strehl ratio .....	82
5.3 Problems .....	84
<b>Chapter 6 Fresnel Diffraction in Vacuum .....</b>	<b>87</b>
6.1 Different Forms of the Fresnel Diffraction Integral .....	88
6.2 Operator Notation .....	89
6.3 Fresnel-Integral Computation.....	90
6.3.1 One-step propagation.....	90
6.3.2 Two-step propagation .....	92
6.4 Angular-Spectrum Propagation .....	95
6.5 Simple Optical Systems.....	102
6.6 Point Sources .....	107
6.7 Problems .....	113
<b>Chapter 7 Sampling Requirements for Fresnel Diffraction .....</b>	<b>115</b>
7.1 Imposing a Band Limit.....	115
7.2 Propagation Geometry.....	117
7.3 Validity of Propagation Methods .....	120

7.3.1 Fresnel-integral propagation.....	120
7.3.1.1 One step, fixed observation-plane grid spacing .....	120
7.3.1.2 Avoiding aliasing .....	121
7.3.2 Angular-spectrum propagation .....	124
7.3.3 General guidelines .....	128
7.4 Problems.....	130

## **Chapter 8 Relaxed Sampling Constraints with Partial**

<b>Propagations.....</b>	<b>133</b>
8.1 Absorbing Boundaries.....	134
8.2 Two Partial Propagations .....	135
8.3 Arbitrary Number of Partial Propagations .....	138
8.4 Sampling for Multiple Partial Propagations.....	139
8.5 Problems.....	146

## **Chapter 9 Propagation through Atmospheric Turbulence ..... 149**

9.1 Split-Step Beam Propagation Method.....	149
9.2 Refractive Properties of Atmospheric Turbulence .....	150
9.2.1 Kolmogorov Theory of turbulence .....	152
9.2.2 Optical propagation through turbulence .....	156
9.2.3 Optical parameters of the atmosphere .....	157
9.2.4 Layered atmosphere model.....	164
9.2.5 Theory.....	164
9.3 Monte-Carlo Phase Screens.....	166
9.4 Sampling Constraints .....	172
9.5 Executing Properly Sampled Simulation.....	174
9.5.1 Determine propagation geometry and turbulence conditions.....	174
9.5.2 Analyze the sampling constraints .....	176
9.5.3 Perform a vacuum simulation.....	178
9.5.4 Perform the turbulent simulations .....	179
9.5.5 Verify the output.....	180
9.6 Conclusion.....	182
9.7 Problems.....	183

## **Appendix A Function Definitions..... 185**

## **Appendix B MATLAB Code Listings ..... 187**

## **References ..... 189**

## **Index ..... 195**





# Preface

Diffraction is a very interesting and active area of optical research. Unfortunately, analytic solutions are rare in many practical problems, particularly when optical waves propagate through randomly fluctuating media. For many of these problems, researchers must resort to numerical solutions. Still, simulations in optical diffraction are challenging. Usually, these simulations take advantage of discrete Fourier transforms, which means using discretely spaced samples on a finite-sized grid. This leads to a few tradeoffs in speed and memory versus accuracy. Thus, the parameters of the sampling grids must be chosen very carefully. Some people seek to fully automate those choices, but this cannot be done automatically in every case. To determine grid properties, one must carefully consider computational speed, available computer memory, the Nyquist sampling criterion, geometry, accurate representation of source apertures, and impact on the propagated field's quantities of interest.

This book grew out of an independent study I did while I was a doctoral student at University of Dayton. The study was directed by LtCol Matthew Goda, then a professor at the Air Force Institute of Technology (AFIT). After the independent study was over, Goda then created a course at AFIT on wave-optics simulations. When I graduated, I became a professor at AFIT while Goda moved on to a new military assignment. When I began teaching the wave-optics simulation course, there was no book written to the level of detail required for a graduate course focused on wave-optics simulations and sampling requirements. The course was always taught out of the professor's notes, originally compiled by Goda. Compiling these notes was no small feat, and Goda did a tremendous job combining material from books on discrete Fourier transforms, optics journal articles and conference proceedings, technical reports from companies like the Optical Sciences Company and MZA Associates Corporation, and private communication with researchers.

Until this book, simulations have always been an afterthought in just a few books on image processing and nonlinear optics. Clearly there was a gap between the practical knowledge required to perform wave-optics simulations and the theoretical material covered in great Fourier-optics textbooks like those by Joseph Goodman and Jack Gaskill. I have heard professors across the U.S. talk about how they include material on simulations in their graduate Fourier-optics courses. I applaud them for that effort because it is challenging to teach students both the theory and practical simulation of Fourier optics in one course. However, if the stu-

dents are to become capable enough to write wave-optics simulations for thesis or dissertation research and beyond, they cannot get enough detail in a one-term Fourier-optics course. This is why AFIT has separate courses on Fourier optics and wave-optics simulations.

This book is intended for graduate students in programs like physics, electrical engineering, electro-optics, or optical science. The book gives all of the relevant equations from Fourier optics, but to fully understand and appreciate the material, it is important to have a thorough understanding of Fourier optics before reading this book.

I believe that part of the benefit of this book is the use of specific code examples, rather than just pseudo-code. However, the programming or scripting language for the examples needs to be one that is widely used and easy to understand by those who do not already use it. For those reasons, I have used MATLAB in all of the examples throughout this book. It is heavily used in engineering both at universities and research institutions. Further, it is easy to read because of its simple language and because many numerical algorithms, such as discrete Fourier transforms and convolution, are part of its basic library. If I used other languages like C, C++, FORTRAN, Java, and Python, I would need to pick a particular external library of numerical routines or write my own algorithms and include them in the book. I believe that using MATLAB in this book allows readers to focus on the wave propagation, rather than the most basic numerical algorithms like discrete Fourier transforms. Further, any user with access to the MATLAB interpreter can execute the code examples as shown. No additional libraries need to be acquired and installed. Moreover, my examples rarely use MATLAB's toolboxes, relying heavily on its basic functionality. Readers should note that the code examples used throughout the book are designed for conceptual simplicity, rather than optimized for speed or memory usage. I encourage readers to rework my MATLAB examples to achieve greater performance or even implement them in other languages.

I offer my thanks and appreciation to all those who have paved the way for this work, particularly Glenn Tyler, David Fried, and Phillip Roberts at the Optical Sciences Company and Steve Coy at MZA Associates Corporation. In 1982, Fried and Tyler wrote a technical report describing methods of simulating optical wave propagation and related sampling constraints. A few years later, Roberts wrote a follow-on report giving another clear, nicely detailed description of one-step, two-step, and angular spectrum propagation methods. More recently, Coy wrote a technical report that gives a very nice description of the relationship between sampling requirements propagation geometry. These reports formed the beginnings of Goda's notes and eventually this book.

Also, thanks to those who answered my questions about wave-optics simulations while I was a student at UD and then while I taught the wave-optics simulation course as a professor at AFIT: Jeffrey Barchers, Troy Rhoadarmer, Terry Brennan, and Don Link. These gentlemen are experienced and accomplished researchers

whose advice was very much appreciated. Additionally, thanks to Michael Havrilla for his help with the basic electrodynamics in Ch. 1.

Special thanks to Matthew Goda for his foundational work in the course and its notes. Without him, this book would not be possible. He made much of the material in this book accessible to dozens of students who went on to do great things for the U.S. Air Force. Finally, I'd like to thank all those students who helped find errors in the drafts of this book and whose inquisitive nature caused me to refine and add material along the way.

*Jason Schmidt*

*June, 2010*