1. Proposed thermal blooming term $\Delta n_{thermal}$

$$\frac{\partial A_\ell(r)}{\partial z} = \frac{i}{2k^\ell}\Delta_\perp A_\ell(r) + ik_0^\ell \delta n_{\text{turb}}(r) A_\ell(r) - \frac{\alpha_{\text{loss}}^\ell}{2} A_\ell(r)$$

$$\downarrow$$

$$\frac{\partial A_\ell(r)}{\partial z} = \frac{i}{2k^\ell}\Delta_\perp A_\ell(r) + ik_0^\ell (\delta n_{\text{turb}}(r) + \Delta n_{\text{thermal}}) A_\ell(r) - \frac{\alpha_{\text{loss}}^\ell}{2} A_\ell(r)$$

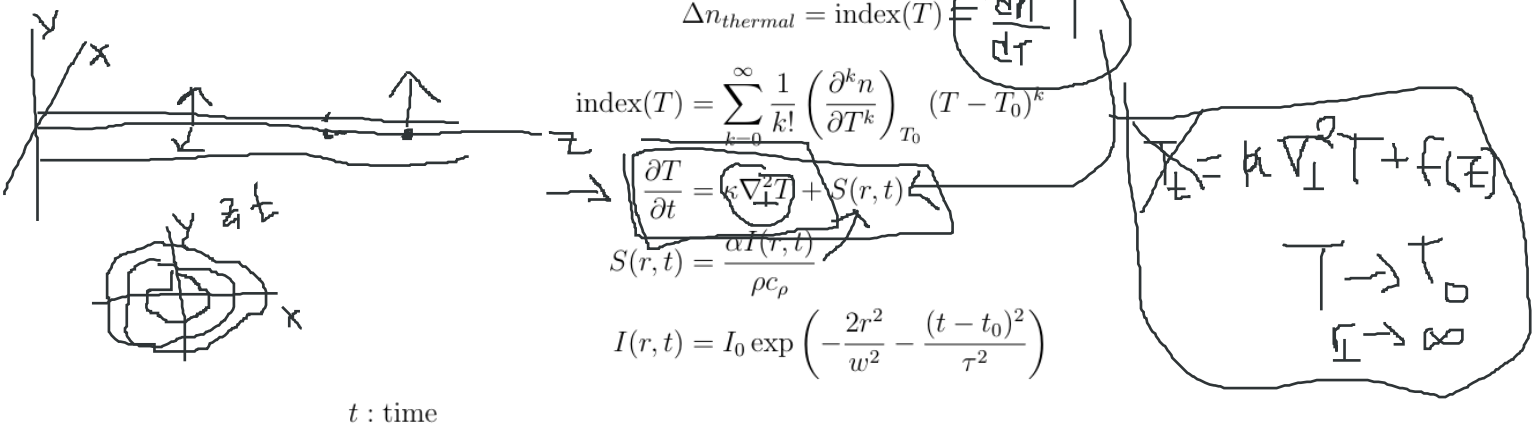
$$\Delta n_{thermal} = \text{index}(T)$$

$$\text{index}(T) = \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{\partial^k n}{\partial T^k}\right)_{T_0} (T - T_0)^k$$

$$\frac{\partial T}{\partial t} = \kappa \nabla_\perp^2 T + S(r,t)$$

$$S(r,t) = \frac{\alpha I(r,t)}{\rho c_\rho}$$

$$I(r,t) = I_0 \exp\left(-\frac{2r^2}{w^2} - \frac{(t-t_0)^2}{\tau^2}\right)$$



t : time

t_0 : starting time

r : radial distance from the beam's center

n : refractivity

T : temperature

T_0 : Reference temperature of the atmosphere at the ground level

$\frac{\partial^k n}{\partial T^k}$: kth derivative of the refractive index with respect to temperature

$\frac{\partial T}{\partial t}$: Temperature change per unit time

α : absorptivity (absorption per unit length)

κ : thermal Diffusivity of the medium

I : laser irradiance

$\alpha I(r,t)$: power absorbed by the medium per unit volume

$S(r,t)$: heat source term, rate at which the laser's energy is absorbed and converted into heat

ρ : mass density of the medium

c_ρ : specific heat at constant pressure

w : laser beam waist radius

τ : pulse duration

I_0 : peak intensity at the beam's center

This proposed model incorporates a thermal blooming effect, and a "Feedback" mechanism the models how changes in temperature overtime worsens the index of refraction.