

# Impact of voting rules and collusion in the Eurovision Song Contest

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## Abstract

The Eurovision Song Contest is an annual event which has gone through multiple voting rules over its 63-year history. This thesis examines the effects these voting rules have had on the outcome by applying the different rules used throughout the history of the contest on the data of contests which did not use those rules. Furthermore, a solver was programmed which tries to find a custom voting rule which makes a certain country win in a certain iteration of the contest. This solver was used to find the amount of countries which could be made to win with a custom rule in the time period from 1975 to 2015. Finally, this thesis tries to verify claims of collusion between certain countries that other authors have made. By removing the ballots of colluding countries, recomputing the result and comparing to the original result, the effect of collusion during a certain time period can be measured. The results show that collusion does not benefit colluding countries in most cases. This is likely a result of the method used to conclude collusion: it only examines the rankings of each country and does not take into account other factors linked to bias in the contest by earlier research.

## 1 Introduction

The Eurovision Song Contest (ESC) is an annual international song competition, in which each participating country submits an original song to be performed on television and radio, then casts votes for the other countries' songs to determine a winner. The first edition, in which seven countries participated, took place in 1956, while last year's ESC had a total of 41 countries participating.

Over time, the voting rules of the contest have been changed, to accommodate for the extra participants and to give the viewers a voice in the outcome of the contest in the form of a televote. The number of points each country can hand out has changed, as well as the number of countries a country can award points to, the number of points that each ranked country receives, and the amount of influence the jury vote and the televote have on the final ranking a country hands in. These changes have almost certainly influenced the results of ESCs, but there has been no research as to how much these rule changes have influenced results. This thesis will delve into the precise details of the voting rules in the ESC and determine how the rules have influenced the outcomes.

There has been extensive research in the area of voting biases in the ESC. Multiple studies have concluded that there is collusion between countries (Gatherer 2006; Mantzaris, Rein, and Hopkins 2018), most notably countries that share a border and countries that speak the same language. Gatherer (2006) concluded that the winner of two ESCs (2003 and 2005) had been changed as a result of collusion. Not only the top placing countries in the contest have an advantage because of collusion, though. Countries that place lower on average will feel the impact of an extra first-place score much more so than a country placing in the upper echelon of the rankings. This study will try to find out how collusion influences the final placing of all colluding countries.

The study of voting rules falls within the field of social choice theory, which concerns itself with the aggregation of individual preferences to reach a satisfactory collective decision (Suzumura, Sen, Arrow, et al. 2002). As this thesis lays out algorithms which reason about voting rules, it falls under the more specific field of computational social choice, which overlaps with the field of artificial intelligence (Chevaletre et al. 2007; Chevaletre et al. 2008; Brandt et al. 2016). The most notable subfield of AI in which social choice theory plays a role is that of multi-agent systems, which concerns itself with the study of systems in which multiple intelligent agents interact (Endriss 2011).

The layout of this thesis is as follows. First, the different rules of the ESC will be covered in an informal manner. Afterwards, a more abstract framework will be defined, in which all rules used in the ESC will fit. With this model in place, the first question of this thesis will be answered: How did the different rules influence the outcome of each ESC? As a follow-up, this study will try to find if a certain country can be made the winner of a contest by means of a custom rule. Lastly, the impact of collusion on the final placing of colluding countries will be measured. All code used to construct the results in this thesis can be found at [https://github.com/ScrubMasterLenny/ESC\\_voting\\_thesis](https://github.com/ScrubMasterLenny/ESC_voting_thesis).

## 2 Voting in the ESC

In an ESC, participants from each country first perform their song. After each country has performed, voting begins. Each country submits a ranking, which is either constructed by a jury, by tallying a televote, or by a combination of both these options. A televote allows members of the public to vote. Each country has a unique phone number its citizens can SMS or call to cast a vote for any country except their own. Alternatively, voters can also use the official Eurovision app to cast their vote (European Broadcasting Union 2020). After the televoting period is over, votes are tallied and a ranking is constructed by each country. To determine a winner, a voting rule is used.

From 2004 and onwards, at least one semi-final was conducted to thin out the field for the final (consult Wikipedia 2020, for an overview of all used rules with references to official sources). This thesis will not study any of the semi-final results, since the semi-finals generate less public interest than the finals.

A *voting rule* is a function which takes as input a number of ballots, and outputs a winner. A ballot consists of a collection of individual preferences, and each country can hand in one ballot.

To aggregate ESC rankings, different voting rules have been used over the years. These rules can be roughly divided into two categories: *scoring rules* and *cumulative voting rules*.

### Scoring rules

A scoring rule is defined by a *scoring vector*, which is a vector as long as the ranking a voter submits. The country ranked  $x^{\text{th}}$  receives the score defined at the  $x^{\text{th}}$  place in the scoring vector (Pacuit 2019). Note that scoring rules only handle strict linear orders. To compute the total score for each country, sum over the rankings of all countries and add the number of points specified by each ranking. The country which receives the most points wins.

An example of a scoring rule is the *Borda count*, which is a scoring rule where each voter ranks all  $n$  candidates in order of preference. The candidate ranked first receives  $n - 1$  points, while the candidate ranked second receives  $n - 2$  points, and so forth until the last candidate, who receives  $n - n = 0$  points (Pacuit 2019). For the Borda count, the scoring vector is defined as

$$w = (n - 1, n - 2, \dots, n - n).$$

The first voting rule, used in 1956, was a scoring rule. It saw each country award two points to their favourite song and zero points to the others. This is equivalent to a *plurality rule*, which is the voting system used to elect members of the House of Representatives in the Netherlands and other countries. The scoring vector for the plurality rule is defined as  $w = (1, 0, \dots, 0)$  (or in this case,  $w = (2, 0, \dots, 0)$ , but the difference does not matter). There is no data available for this contest as the ballots were kept secret. Not even the final score of each country is publicly known.

In 1962 and 1963, two different scoring rules were used. The scoring vectors for these rules are:

$$1962 : w = (3, 2, 1, 0, \dots, 0);$$

$$1963 : w = (5, 4, 3, 2, 1, 0, \dots, 0).$$

From 1975 until today, the voting rule has been a scoring rule. Although this rule has gone through some iterations, it has largely remained the same. The scoring vector for this rule, which will be called the *1975 scoring vector*, is

$$w = (12, 10, 8, 7, 6, 5, 4, 3, 2, 1, 0, \dots, 0).$$

The first adjustment to this voting rule was introduced in 1997: instead of using a jury to decide their ranking, some countries started using a *televoting system*. The televoting system works as follows: Each phone number in a country can message or call a special phone number and cast a vote on a song. Alternatively, voters can also use the official Eurovision app to cast their vote (European Broadcasting Union 2020). When televoting is finished, all countries tally their votes and construct a top ten ranking. All countries used a televoting system the following year, only resorting to a jury in case of a telephone system failure.

From 2000 to 2002, every country could decide whether they wanted to use the televoting system exclusively, or use both a televote and a jury to determine their ranking. This is called the *jury-televote 50/50 system* and it works as follows. Both the jury and the televoters construct a ranking. These two rankings are combined by awarding points to them using the 1975 scoring vector. This yields the final ranking submitted by the country, which is used to award points in the actual contest. In the case of a tie, the televote ranking takes precedence. An example can be found in Section 3.3.

The televoting system was reintroduced between 2003 and 2008, while in 2009 the jury–televote 50/50 system became mandatory again. In 2013, the procedure for the jury-televote 50/50 system was changed slightly: instead of using the 1975 scoring vector, a Borda count ( $w = (n - 1, n - 2, \dots, n - n)$ ) was now used to aggregate the jury- and televotes. The 1975 scoring vector was still used to hand out the points for a finalized ranking. In 2015 the final change was made to this voting rule: Each country now hands in two ballots, one containing a ranking constructed by televoters, and the other containing a ranking constructed by a jury. This means that the number of points handed out is effectively doubled.

### Cumulative voting rules

Cumulative voting rules are rules in which each voter has a fixed number of points she can distribute between candidates however she prefers (Pacuit 2019). To determine a candidates' score, sum over all the rankings and award the number of points specified by each ranking. The winner is the country with the most points. Cumulative voting rules have been used in a minority of ESCs.

The rule used from 1957 to 1961 was a cumulative voting rule: each country distributed ten points among their favourite songs. This rule has been phased out and brought back twice, between 1967 and 1970 and once more in 1974. Its usage in these seven contests makes it the second most used rule in ESC history.

### Other rules

From 1964 to 1966 a unique system was used, which falls between the two categories mentioned earlier. 9 points were available to distribute for each country, and they could be distributed in three different ways:

- a. 9 points could be awarded to one song;
- b. 6 and 3 points could be awarded to two songs;
- c. 5, 3 and 1 point(s) could be awarded to three different songs.

To obtain the score of a country, sum over all the rankings and hand out the appropriate number of points to each country. The winner is the country with the most points. The three scoring vectors specified by this system are:

$$1964 - 66_a : w = (9, 0, 0, 0, \dots, 0);$$

$$1964 - 66_b : w = (6, 3, 0, 0, \dots, 0);$$

$$1964 - 66_c : w = (5, 3, 1, 0, \dots, 0).$$

From 1971 to 1973 a system with two juries was used. Both juries could hand out 1 to 5 points to each song. This is a so-called *range voting system* (Pacuit 2019), and it is the first and the last time such a system was used in an ESC. To determine the score of a country, sum over all the scores awarded to that country. The country with the most points is the winner.

## 3 General framework

As stated in the previous section, multiple categories of voting rules have been used in the ESC. This thesis aims to extract voting data from contests and determine whether the outcome would have been different if another rule had been used. While some rules are easily applicable on the data of other ESCs, most are not. To apply those cumbersome rules to the data of other ESCs, a more abstract framework is required, which will be defined next.

### 3.1 Model

An *agent* is an entity that can cast a vote in a ballot. In this case, agents are equivalent to countries. A *candidate* or *alternative* is an entity that can be voted on in a ballot. In an ESC, agents and candidates are both countries; we can combine these definitions. The only difference between the set of agents and the set of candidates is that each agent does not have itself in their set of candidates, as a country cannot hand out points to itself.

For a country  $c$  in the set of all participating countries  $C$  with  $|C| = n$ , we define the preference relation  $x \succeq_c y \forall x, y \in C \setminus \{c\}$  as “candidate  $x$  is preferred as much as or more than candidate  $y$  by agent  $c$ ”.

If  $x \succeq_c y$  and  $y \not\succeq_c x$  then  $x$  is strictly preferred to  $y$  by agent  $c$  and we write  $x \succ_c y$ .  $\succ_c$  is a *linear order* over  $C \setminus \{c\}$ . If  $x \succeq_c y$  and  $y \succeq_c x$ ,  $x$  and  $y$  are equally preferred and we write  $x \sim_c y$ .

The set of all possible linear orders over  $C$  is denoted by  $\mathcal{L}(C)$ . The linear order on the ballot of country  $c$  should not contain  $c$  itself, as it is not allowed to vote for one’s own country. Thus, the set of all possible rankings country  $c$  can put on their ballot is denoted by  $\mathcal{L}(C \setminus \{c\})$ . A profile  $P = (\succeq_1, \succeq_2, \dots, \succeq_n)$  specifies a ranking for each country  $c \in C$ , and  $\times_{c \in C} \mathcal{L}(C \setminus \{c\})$  denotes the set of all possible profiles involving countries in  $C$ .

To define an ESC ballot, more than just a ranking is needed. Some rules have multiple scoring vector options, and if such a rule is used, each country needs to choose which scoring vector it will apply on its ranking. To incorporate this choice into the model, a generalisation of the scoring rule concept is used in which all ESC rules fit. Each country  $c \in C$  can choose its scoring vector  $w_c$  from a set of scoring vectors  $\mathcal{W}$ , which differs per rule.

The set of possible scoring vectors  $\mathcal{W}$  is given by the rule  $F_{\mathcal{W}}$ . This rule is a function, taking  $n = |C|$  ballots as input and outputting a set of winners in  $2^C \setminus \{\emptyset\}$ , the power set of  $C$  minus the empty set. Each ballot submitted by an agent  $c \in C$  is defined by a ranking  $\succeq_c \in \mathcal{L}(C \setminus \{c\})$  and a scoring vector  $w_c \in \mathcal{W}$ . To summarize:

$$F_{\mathcal{W}} : \times_{c \in C} (\mathcal{L}(C \setminus \{c\}) \times \mathcal{W}) \rightarrow 2^C \setminus \{\emptyset\}.$$

With all the formalisation in place, the set of possible scoring vectors for each ESC will now be defined.

### Scoring rules

All scoring rules have only one possible scoring vector.

$$1962 : \mathcal{W} = \{(3, 2, 1, 0, \dots, 0)\}$$

$$1963 : \mathcal{W} = \{(5, 4, 3, 2, 1, 0, \dots, 0)\}$$

$$1975 - 2015 : \mathcal{W} = \{(12, 10, 8, 7, 6, 5, 4, 3, 2, 1, 0, \dots, 0)\}$$

### Other rules

In the cumulative voting rule used in 1957–1961, 1967–1970 and 1974, an agent has 10 points to distribute between candidates. The number ten can be partitioned in 42 ways, which means that there are 42 possible scoring vectors when using this rule:

$$\mathcal{W} = \left\{ w = (w_1, \dots, w_n) \in \{0, \dots, 10\}^n \mid \sum_{i=1}^n w_i = 10 \text{ and } w_i \geq w_{i+1} \text{ for all } i < n \right\}.$$

The rule used from 1964 to 1966, which is a combination of a scoring rule and a cumulative voting rule, has three possible scoring vectors:

$$\mathcal{W} = \{(9, 0, \dots, 0), (6, 3, 0, \dots, 0), (5, 3, 1, 0, \dots, 0)\}.$$

From 1971 to 1973, a range voting rule was used. Each contestant received 2 to 10 points, which amounts to  $9^{n-1}$  possible scoring vectors, where  $n$  is the number of countries participating.

$$\mathcal{W} = \{w = (w_1, \dots, w_n) \in \{2, \dots, 10\}^n \mid w_i \geq w_{i+1} \text{ for all } i < n\}$$

## 3.2 Implementation of the model

This section will discuss the implementation of the model in Section 3.1. The model has been implemented in a Jupyter Notebook running on Python 3.6. This Jupyter Notebook can be found at [https://github.com/ScrubMasterLenny/ESC\\_voting\\_thesis](https://github.com/ScrubMasterLenny/ESC_voting_thesis). The dataset has been graciously provided by Janne Spijkervet and is available at [https://github.com/Spijkervet/eurovision\\_dataset/releases](https://github.com/Spijkervet/eurovision_dataset/releases) (votes.csv).

The model has been implemented as an object-oriented class called Contest. A Contest object contains a list of Country objects, another object-oriented class, and a scoring vector, which is defined as a list. Each Country object contains the name of a country and a ranking. If the Contest uses a cumulative voting rule or a range voting rule, each Country object additionally contains the scoring vector chosen by this country.

For each year of the ESC covered in the dataset, a Contest object is created. For each country voting in each ESC's final, a Country object is created, and all these Country objects are put in the Country list of the Contest. Lastly, the scoring vector is added to the Contest object, or, if the Contest uses a cumulative voting rule or a range voting rule, each Country objects additionally contains a scoring vector.

### 3.3 Post-2015 rule

A question that comes to mind is that of the post-2015 rule. The rule update in 2015 resulted in two ballots being handed in by each country: one by the jury of that country and one by the people voting at home in that country. However, the model in Section 3.1 lets each country hand in only one ballot. To handle this discrepancy, two solutions are proposed.

The first solution lets each country hand in two ballots: one containing the jury ranking, and the other containing the televote ranking.

The second solutions makes countries combine the rankings from the jury and the televoters to obtain a single ranking. There are multiple methods by which this single ranking can be obtained. From 2004 to 2012, such a method was conceived and used in the ESC. For each country, the jury vote and the televote were first combined. Using this combined ranking, points were handed out. More formally, a rule  $F_w$  was used, which takes as input two rankings  $>_c$  and a scoring vector  $w$  for each country  $c \in C$ . It then outputs a single ranking  $>'_c$  by applying the scoring vector on the two rankings, combining the points, and interpreting a new ranking using this combined point total. Ties were broken by giving the higher ranking to the candidate with the highest televoting score. An example can be found below.

**Example 1** Take two rankings  $>_{c_{jury}}$  and  $>_{c_{tele}}$  by country  $c$ , and a scoring vector  $w$ , specified below.

$$\begin{aligned} >_{c_{jury}} &= (x >_c y >_c z) \\ >_{c_{tele}} &= (y >_c x >_c z) \\ w &= (3, 2, 1) \end{aligned}$$

Compute the total score for all countries on these rankings by applying the scoring vector  $w$  on each ranking.

$$total = \{(x, 5), (y, 5), (z, 2)\}$$

Lastly, the new ranking is interpreted. The candidate with the highest televoting score wins the tie.

$$>_{c_{combined}} = (y >_c x >_c z)$$

The first solution was used to compute the score when the original scoring vector ( $w = (12, 10, 8, 7, 6, 5, 4, 3, 2, 1, 0, \dots, 0)$ ) was applied to post-2015 ESCs. This solution was chosen because it corresponds to the system originally used to compute the score for these ESCs.

When applying a different scoring vector to a post-2015 ESC, the second solution was used. This was done because there was precedent for doing so: A majority of ESCs used the jury-televote 50/50 system from 1997 to 2015, which combines rankings in the exact same way. Thus, when a pre-1975 rule is applied to data from a post-2015 contest, the process from these jury-televote 50/50 ESCs is replicated.

### 3.4 Compatibility between ESC rules

Not all rules are directly applicable on the rankings extracted from ESCs using other rules. However, many of the scoring rules used in the history of the ESC can be applied to the rankings extracted from contests in which another scoring rule was originally used, because these extracted rankings are strict. The only requirement is that the new scoring vector is at most as long as the scoring vector originally used in this ESC. If the new scoring vector would be longer than the scoring vector originally used, a guess would have to be made as to which of the bottom ranked countries was preferred, which is bad practice. Table 1 maps out the compatibility between data and rules for scoring rule ESCs.

When looking at the rule used from 1964 to 1966, a combination of a scoring rule and a cumulative voting rule, these seems to be no good way to apply the data of these contests to other rules. However, out of the three possible point splits,  $(9, 0, \dots, 0)$  was not used, and  $(6, 3, 0, \dots, 0)$  was used only once. Therefore, simplifying this rule to a scoring rule with scoring vector  $w = (5, 3, 1, 0, \dots, 0)$  is logical, and makes the application of this rule to the data of other ESCs using scoring rules possible.

Rule \ Data	1962	1963	1964-1966	1975-2012	2012-2015	2016-2019
1962	✓	✓	✓	✓	✓	✓
1963	✗	✓	✗	✓	✓	✓
1964-1966	✓	✓	✓	✓	✓	✓
1975-2012	✗	✗	✗	✓	✗	✓
2012-2015	✗	✗	✗	✗	✓	✗
2016-2019	✗	✗	✗	✗	✗	✓

Table 1: Compatibility of scoring rules with the data of ESCs using scoring rules

A ranking can be extracted from cumulative voting rule ESCs. However, almost every one of these rankings will contain ties as they are not required to be strict, which complicates the process of applying scoring rules to the data. Any ties will have to be resolved before applying a scoring rule on the data extracted from cumulative voting rule ESCs; the matter of tie breaking is discussed in subsection 3.5, but it has not been implemented because of time constraints.

Rankings extracted from ESCs using a range voting rule are guaranteed to have ties. A scoring rule can be applied on these rankings as long as all ties are broken first.

Applying a range voting rule on the data of a cumulative voting rule ESC is a problematic exercise. There is no clear way to convert a ranking from a rule that has a fixed number of points to distribute to a ranking which has a variable number of points distributed among candidates. Take for example the cumulative voting scoring vector  $w = (2, 2, 2, 2, 2, 0, \dots, 0)$ . To convert this vector to a range voting scoring vector, one would have to interpret how much more the songs which were given two points were preferred to the songs that received 0 points. Many range voting scoring vectors would be possible, from  $w = (3, 3, 3, 3, 3, 2, \dots, 2)$  to  $w = (10, 10, 10, 10, 10, 2, \dots, 2)$  and even  $w = (10, 10, 10, 10, 10, 9, \dots, 9)$ . As there is no easy way to do this, it has not been attempted in this thesis.

Converting a range voting scoring vector to a cumulative voting scoring vector runs into a similar problem. The range voting scoring vector  $w = (10, 9, 8, 7, 6, 5, 4, 3, 2, \dots, 2)$  cannot be converted to a cumulative voting scoring vector. This is because it has 9 distinct numbers. The cumulative voting rule has only ten points to distribute among candidates, and the scoring vector with the most distinct numbers (5) one can create with ten points is  $w = (4, 3, 2, 1, 0, \dots, 0)$ . There is simply no way to express the ranking  $\succeq_c$  resulting from interpreting the range voting scoring vector with the number of points provided by the cumulative voting rule.

### 3.5 Tie breaking

When extracting rankings from ESCs using a cumulative voting rule or a range voting rule, ties are an inevitability. To apply scoring rules to these rankings, a tie breaking mechanism is needed. Two mechanisms are proposed.

The first tie breaking mechanism equally divides the points in the scoring vector amongst candidates. More formally, if the contestants ranked  $x^{\text{th}}$  to  $x + i^{\text{th}}$  are equally preferred, then the scoring vector entries from  $x$  to  $x + i$  will be added up to a total  $t$ , and each contestant will receive a score equal to  $\frac{t}{i+1}$ . Note that this results in final scores having a decimal point.

The second tie breaking mechanism is randomisation. More formally, if the contestants ranked  $x^{\text{th}}$  to  $x + i^{\text{th}}$  are equally preferred, each contestant has a  $\frac{1}{i+1}$  chance of receiving each of the scores from  $x$  to  $x + i$  in the scoring vector. One can try all possible ways to break ties using this mechanism to see if the winner is affected by the randomness.

## 4 The impact of rules on the outcome

In this section, the influence of different rules on the outcome of a contest will be determined. The first subsection discusses the application of the different scoring rules used in the ESC on the rankings extracted from ESCs. The second subsection explores a thought experiment: If one could design a scoring rule and apply it on the rankings extracted from an ESC, could she make a specific country win?

### 4.1 Applying different rules on ESCs

The first question this thesis tries to answer relates to the different rules of the ESC. If one applies a rule to the rankings of an ESC which is not the rule that was originally used, does the winner change? The hypothesis is that the winner will not change unless the point difference between the highest placing countries is small.

#### 4.1.1 Method

To compute the final ranking for an ESC, iterate over all the Country objects in a Contest object. For each ranking in these Country objects, apply the scoring vector specified in the Contest object to it and add the result to the final ranking.

With Algorithm 1, results can be computed for an ESC. To change the scoring rule, we redefine the scoring vector in the Contest object. Note that this only allows scoring rule ESCs to be recomputed using a different rule, and not ESCs using a different rule.

**Data:** A Contest object

**Result:** A dictionary containing the final score of each country

result = dict(key = country, value = finalScore);

**for** Country in Contest **do**

    result[Country] = 0;

**end**

rule = Contest.getRule();

**for** Country in Contest **do**

    ranking = Country.getRanking();

    iter = 0;

**for** contestant in ranking **do**

**if** Country.getScoringVector() != None **then**

            // non-scoring rule ESC uses scoring vector in Country object

            result[contestant] += Country.getScoringVector[iter];

**else**

            // scoring rule ESC uses scoring vector in Contest object

            result[contestant] += rule[iter];

**end**

        iter += 1;

**end**

**end**

return result;

**Algorithm 1:** computeOutcome()

#### 4.1.2 Results and Discussion

This subsection will cover the results obtained by applying different rules to the data extracted from ESCs. The results can be found in Figure 1. Note that the time periods from 1957 to 1961 and from 1967 to 1974 were not covered because tie breaking was not implemented.

A black cell in Figure 1 indicates that the rule is not compatible with the data of this ESC. An entry with a gray background indicates that the country in the cell is the original winner of this contest. An entry with a green background indicates that the winner when using this rule is the same as when using the original rule. An entry with an orange background indicates that there were multiple winners when using this rule, and one of the winners is the winner when applying the original rule. An entry with a red background indicates that the winner when using this rule does not correspond to the original winner.

ISO 3166-1 alpha-2 country codes were used in Figure 1. To find the country which corresponds to a certain abbreviation, consult [https://en.wikipedia.org/wiki/List\\_of\\_ISO\\_3166\\_country\\_codes](https://en.wikipedia.org/wiki/List_of_ISO_3166_country_codes).



		Year from which rankings were extracted																
		1962	1963	1964	1965	1966	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985	1986
Rule which was applied on the data	1962	FR	CH/DK	IT	LU	AT	NL	GB	GB	IL	IL	IE	CH	DE	LU	IE	NO	BE
	1963		DK				NL	GB	GB	IL	IL	IE	DE/GB/CH	DE	LU	IE/SE	NO	BE
	1964-1966	FR	CH/DK	IT	LU	AT	NL	GB	GB	IL	IL	IE	CH	DE	LU	IE	NO	BE
	1975-2012						NL	GB	FR	IL	IL	IE	GB	DE	LU	SE	NO	BE
	2012-2015																	
	2016-2019																	
	1987	1987	1988	1989	1990	1991	1992	1993	1994	1995	1996	1997	1998	1999	2000	2001	2002	2003
	1962	IE	GB	YU/GB	IT	SE	IE	IE	IE	NO	IE	GB	IL	IS	DK	EE	LV	TR
	1963	IE	CH	YU/GB	IT	SE	IE	IE	IE	NO	IE	GB	IL	SE	DK	EE	LV	TR
	1964-1966	IE	GB	GB	FR	SE	IE	IE	IE	NO	IE	GB	IL	IS	DK	EE	LV	TR
1975-2012	IE	CH	YU	IT	SE	IE	IE	IE	NO	IE	GB	IL	SE	DK	EE	LV	TR	
2012-2015																		
2016-2019																		
2004	2004	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018	2019		
1962	UA	GR	FI	RS	RU	NO	DE	AZ	SE	DK	AT	SE	UA	PT	IL	IT		
1963	UA	GR	FI	RS	RU	NO	DE	AZ	SE	DK	AT	SE	UA	PT	IL	NL		
1964-1966	UA	GR	FI	RS	RU	NO	DE	AZ	SE	DK	AT	SE	UA	PT	IL	IT		
1975-2012	UA	GR	FI	RS	RU	NO	DE	AZ	SE				UA	PT	IL	NL		
2012-2015										DK	AT	SE						
2016-2019													UA	PT	IL	NL		

Figure 1: Applications of different rules to the data of ESCs from 1962 to 1966 and from 1975 to 2019

In nine out of fifty years (18%), the application of at least one different rule results in a different winner.

The hypothesis attached to this question was that the winner will not change except when the point difference between the highest placing countries is small. In the nine years where using a different rule resulted in a different outcome, the first place score expressed as a ratio of the second place score was 1.069 on average, while this ratio in all other contests was 1.293 on average, which supports the hypothesis.

When a different rule gives a different winner during a certain year, eight out of nine times another different rule also gives a different winner. This further indicates that the outcome in these contests was close, and that the change of rule does not influence the winner of the contest a lot.

In eight out of nine cases, a change of rule only swaps the second- and first-placing countries (or ties them). The exception is the ESC in 1984: The United Kingdom, Germany and Switzerland share the first place when the 1963 rule is applied to the data. Interestingly, these three countries originally placed first, second and fourth. Switzerland also places first when applying the 1962 rule and the 1964-1966 rule to the data. This is likely because these rules award a much bigger share of the points to the country which is ranked first. Switzerland was ranked first by other countries the most out of any country in this contest: five times in total.

## 4.2 Making a certain country win

It seems that changes in ESC rules have influenced the final result in a minority of cases. An interesting thought experiment follows from these results: if one could design a rule, could the final result be influenced in such a way that country  $x$  is the winner? How many countries could one make a winner in each year? And what conditions should be satisfied to find such a rule? This section will discuss these questions.

### 4.2.1 Method

The aim is to find a scoring vector  $w = (w_1, \dots, w_n)$  with  $n = |C|$  such that a country  $x \in C$  places first in the final ranking. Each element of the scoring vector should be greater than the next element ( $w_i > w_{i+1}$ ), and all elements should be larger than zero. Lastly, every element should be an integer. The scope of this part of the research will be limited to scoring vectors where ten countries receive points, as this is the most common scoring vector size in the dataset.

To make country  $x$  win a contest, their final score  $s(x)$  has to be higher than every other country's score. This corresponds to a system of inequalities:

$$\forall c \in C \setminus \{x\} : s(x) > s(c).$$

To find an equation for the score of a country  $x \in C$ , all other countries' rankings are checked. If country  $x$  shows up in the ranking of a country, the score is added. If it does not show up, nothing is added. More formally, the score  $s(x)$  of a country  $x \in C$  is defined as follows:

$$s(x) = \sum_{c \in C \setminus \{x\}} w_{\text{rank}(x, \succeq_c)}.$$

Where  $\text{rank}(x, \succeq_c)$  is the placing of  $x$  in the ranking of  $c$ . When every country has its equation written down, a system of inequalities can be constructed.

**Example 2** Consider a fictional contest in which four countries participate:  $c_1$  to  $c_4$ . Below are the rankings each country has submitted.

$$\begin{aligned}\succeq_{c_1} &= (c_2 \succeq_{c_1} c_3 \succeq_{c_1} c_4) \\ \succeq_{c_2} &= (c_1 \succeq_{c_2} c_3 \succeq_{c_2} c_4) \\ \succeq_{c_3} &= (c_4 \succeq_{c_3} c_1 \succeq_{c_3} c_2) \\ \succeq_{c_4} &= (c_2 \succeq_{c_4} c_1 \succeq_{c_4} c_3)\end{aligned}$$

The score is computed for each country.

$$\begin{aligned}s(c_1) &= w_1 + 2 \times w_2 \\ s(c_2) &= 2 \times w_1 + w_3 \\ s(c_3) &= 2 \times w_2 + w_3 \\ s(c_4) &= w_1 + 2 \times w_3\end{aligned}$$

The first set of inequalities to be added concerns the scoring vector.  $w_1$ , the number of points for a first place ranking, should be greater than the number of points for a second place ranking, and so forth. Every element of the scoring vector should also be larger than zero, which can be accomplished by adding an inequality which states that the smallest element of the scoring vector is greater than zero. Suppose we want country  $c_2$  to win. Then three more inequalities have to be added, one for each other country which is participating.

$$\begin{aligned}w_1 &> w_2 > w_3 > 0 \\ s(c_2) &> s(c_1) \\ s(c_2) &> s(c_3) \\ s(c_2) &> s(c_4)\end{aligned}$$

Next, the equations constructed earlier are substituted for the scores of each country.

$$\begin{aligned}w_1 &> w_2 > w_3 > 0 \\ 2 \times w_1 + w_3 &> w_1 + 2 \times w_2 \\ 2 \times w_1 + w_3 &> 2 \times s_2 + w_3 \\ 2 \times w_1 + w_3 &> w_1 + 2 \times w_3\end{aligned}$$

To make  $c_2$  win, the most straightforward solution is  $w_1 = 4, w_2 = 2, w_3 = 1$ .

$$\begin{aligned}4 &> 2 > 1 > 0 \\ 2 \times 4 + 1 &= s(c_2) > s(c_1) = 4 + 2 \times 2 \\ 2 \times 4 + 1 &= s(c_2) > s(c_3) = 2 \times 2 + 1 \\ 2 \times 4 + 1 &= s(c_2) > s(c_4) = 4 + 2 \times 1\end{aligned}$$

To automate this process, the linear programming package PuLP was used (Mitchell, OSullivan, and Dunning 2011). Linear programming is a method to find the best outcome in a mathematical model consisting of linear relationships (Vanderbei et al. 2015). An upper bound of 10000 was applied on the elements of the scoring vector because of limited time and processing power. Algorithms 2, 3 and 4 illustrate the process in some detail and are written down below. For a complete overview, consult [https://github.com/ScrubMasterLenny/ESC\\_voting\\_thesis](https://github.com/ScrubMasterLenny/ESC_voting_thesis).

**Data:** ESCDictionary = a dictionary (key = year, value = Country object) with Contest objects, spanning a period of multiple years

**Result:** A dictionary (key= year, value = number of countries which is able to win using a custom rule)

```

result = dict(key = year, value = numberOfPossibleWinners);
for ESC in ESCDictionary do
    equations = dict();
    for country in ESCDictionary[ESC] do
        equation = constructEquation(ESCDictionary[ESC], country);
        equations[country] = equation;
    end
    for country in ESCDictionary[ESC] do
        inequalities = list( $w_1 > w_2, w_2 > w_3, \dots, w_{10} > 0$ );
        inequalities.append(makeCountryXWin(equations, country));
        if solve(inequalities) is True then
            result[ESC] += 1;
        end
    end
end
return result;

```

**Algorithm 2:** findNumberOfPossibleWinners()

**Data:** 1. ESC = A list of (country, ranking) tuples  
2. countryEquation = A country for which we want to construct an equation

**Result:** The score of this country, expressed in scoring vector variables

```

equation = string();
for (_, ranking) in ESC do
    if countryEquation in ranking then
        index = ranking.index(countryEquation) + 1;
        equation += "w" + string(index) + " + ";
    end
end
return equation[:-3];

```

**Algorithm 3:** constructEquation()

**Data:** 1. equations = a dictionary (key = country, value = equation)  
2. winningCountry = A country which we want to win

**Result:** A list of inequalities which need to be fulfilled to make winningCountry win

```

inequalities = list();
winningCountryEquation = equations[winningCountry];
for country in equations do
    if country = winningCountry then
        continue;
    end
    inequalities.append(winningCountryEquation + ">" + equations[country]);
end
return inequalities;

```

**Algorithm 4:** makeCountryXWin()

#### 4.2.2 Results and discussion

This subsection discusses the results obtained by running Algorithm 2 on the data of ESCs from 1975 to 2015. These results can be found in Figure 2 below.

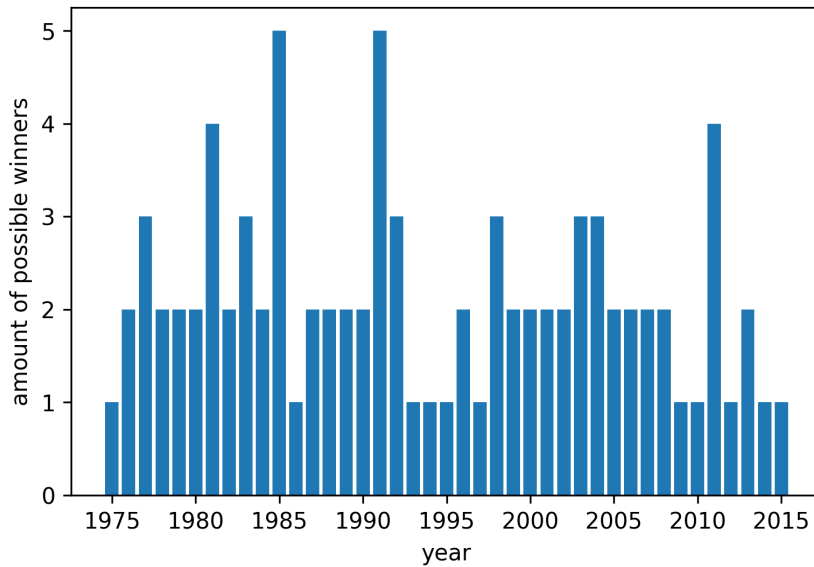


Figure 2: The number of countries which can be made to win with a custom scoring rule from 1975 to 2015

In every year where a rule change resulted in a different winner (see Section 4.1.2), a custom rule was found to make a different country win too. This is noteworthy because the scoring rules applied on the data of the ESCs from 1975 to 2015 in Section 4.1.2 had scoring vectors with a size smaller than ten, while the scoring vectors found by Algorithm 2 are all of size ten.

In five out of seven (71.4%) cases where other winners were found by applying different rules in Section 4.1.2, the number of winners found using the inequality solver was the same.

When looking at Figure 2, two outliers are visible: For the data of the ESCs in 1985 and in 1991, five winners were found by the inequality solver. The scoring vector with the highest point total out of the five scoring vectors found for the data of 1985 is  $w = (45, 41, 40, 31, 29, 28, 27, 26, 3, 1)$ , which results in Ireland winning. Ireland originally finished in sixth place in 1985. This also makes Ireland the only country which rose more than four places to become a winner by means of a custom rule. The scoring vector with the highest point total found for the data of 1991 is  $w = (36, 35, 34, 33, 32, 30, 29, 28, 27, 26)$ , which makes Spain the winner of the contest. Spain originally finished in fourth place.

The biggest scoring vector element found was 94, in the scoring vector  $w = (94, 10, 9, 8, 7, 6, 5, 4, 3, 2)$ . This scoring vector makes Armenia the winning country in the ESC of 2008. Armenia was originally ranked fourth in the final ranking.

## 5 The impact of collusion

Multiple studies have been conducted on the topic of collusion in the ESC. The most notable study, by Gatherer (2006), concluded that there are multiple so-called voting blocs: groups of countries that tend to rank each other higher than other candidates. Mantzaris, Rein, and Hopkins (2018) expanded on this research by including data from more ESCs and checking for one-way biases on top of checking for reciprocal relationships.

Gatherer (2006) concluded that the winner of the contest had been influenced by one of these voting blocs in 2003 and 2005. However, while the impact on the winner might make for a great headline, the placings of the countries at the top of the ranking are often least influenced by this collusion. This is because the point distribution in ESCs follows a somewhat quadratic curve (see Figure 3); the lower placing countries are closer to each other in terms of points than the higher placing countries. More informally, an extra twelve-point rating might or might not propel the second place finisher to first, but it will almost definitely give the last placing country a big boost in the final ranking. In this section, the impact of collusion on the final scores of each country will be measured.

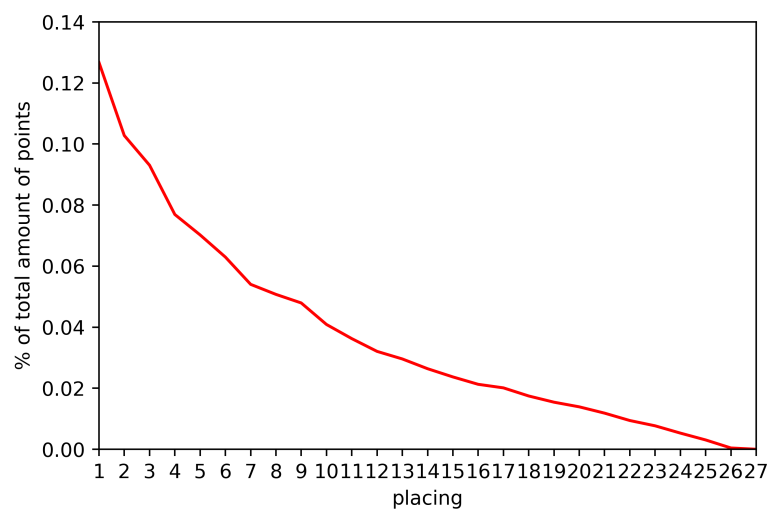


Figure 3: The average point distribution from 2000 to 2016

### 5.1 Method

To conclude collusion, Mantzaris, Rein, and Hopkins (2018) used an algorithm designed by Gatherer (2006). This algorithm looks at multiple contests in a time period. For each of these contests, it computes the average number of points a country will hand out to another country (the number of points a country can hand out, divided by the number of countries participating minus one), and it adds these averages up. For each pair of countries, it then computes the average score one country of the pair handed out to the other country in all these contests, and vice versa, and it adds these up. If each country of a pair handed out 105% or more of the average points to the other country, Gatherer (2006) says these countries are colluding. It should be noted that no justification is given for this threshold in the research of Gatherer (2006) and Mantzaris, Rein, and Hopkins (2018).

To measure the impact of collusion on the placing of countries, the ballots of colluding countries are removed from multiple contest during a certain period. If there is no collusion going on, each one of these ballots should evenly distribute points across all candidates, and upon recomputing the final ranking for all contests, each country should on average have the same final placing as before in each contest. Note that this is not possible for a single contest, because the number of contestants a country can hand out points to is always less than the number of candidates in a contest, which means one ballot can never evenly distribute all points across all candidates. Thus, it is required to look at multiple ESCs in a time period to come to a satisfactory conclusion.

The hypothesis of this study is that colluding countries will move down in the recomputed rankings on average. In other words: colluding countries attain a higher final ranking because of their collusion.

**Data:** 1. A dictionary (key = year, value = list of (country, ranking) tuples) with ESC results, spanning a period of multiple years.

2. country1, country2 = two countries which colluded during this time period

**Result:** The average change of placing for the two countries, resulting from removing their ballots from the contests

```
result = dict(key=country, value=avgChangeInPlacing);
for ESC in ESCDictionary do
    oldOutcome = computeOutcome;
    remove(country1, country2);
    newOutcome = computeOutcome;
    changeInPlacing = compareOutcomes(newOutcome, oldOutcome);
    for country in changeInPlacing do
        result[country] += changeInPlacing[country];
    end
    for country in result do
        result[country] = result[country] / participationCount(ESCDictionary, country);
    end
end
return result;
```

**Algorithm 5:** computeEffectOfCollusion()

**Data:** Two outcomes

**Result:** The change of placing for each country from the first outcome to the second

```
result = list();
for country in outcome(s) do
    difference = oldOutcome(country) - newOutcome(country);
    result.append(country, difference);
end
return result;
```

**Algorithm 6:** compareOutcomes()

**Data:** 1. A dictionary with ESCs, spanning a period of multiple years

2. A country

**Result:** The number of times the country participated in this period

```
result = 0;
for ESC in ESCDictionary do
    if country is in ESCDictionary[ESC] then
        result += 1;
    end
end
return result;
```

**Algorithm 7:** participationCount()

## 5.2 Results and discussion

This subsection discusses the results obtained by running Algorithm 5 on the data of ESCs from 1960 to 2016. These results can be found in Figures 4, 5, 6 and 7 below.

The intervals which were chosen in these figures correspond to those in the paper of Mantzaris, Rein, and Hopkins (2018). A black cell indicates that collusion cannot be concluded because the row and column contain the same country. An entry with a gray background indicates that collusion was not concluded by Mantzaris, Rein, and Hopkins (2018). An entry with a green background indicates that the collusion resulted in a higher ranking on average for the country in that row. An entry with a red background indicates that the collusion resulted in a lower ranking on average for the country in that row.

These figures should be read as follows: The country in the row gained/lost [table entry] number of places on average by colluding with the country in the column. For example: in Figure 4, France lost 0.3684 ranks on average by colluding with Monaco in the time period from 1960 to 1979.

	France	Ireland	Italy	Luxembourg	Monaco	Spain	UK
France					-0.3684		-0.4736
Ireland				-0.7333			
Italy						-0.421	
Luxembourg		-0.1333					-0.5
Monaco	-0.1052						
Spain			-0.4736				
UK	-0.5789			-0.15			

Figure 4: The effect of collusion from 1960 to 1979

From 1960 to 1979 every colluding country is, on average, at a disadvantage from colluding with each other. These countries moved down 0.3937 places on average as a result of their collusion. The number of points handed out in total is the lowest during this period, and the number of countries participating in these contests is low: fifteen on average. The removal of one ballot might have a larger effect on the final result of countries other than the colluding partner compared to later ESCs, which had more points being handed out per country, and more entrants in total.

Furthermore, the countries in Figure 4 finished in place 6.3 on average during this time period. This is higher than the average for all countries during this time period, which is 7.5. One could suggest that this higher average is a result of these countries colluding, but this doesn't add up with the results in Figure 4, which show that these countries actually move down because of colluding. One might hypothesize that well-performing countries are being marked as colluders unfairly.

Another reason for this unexpected result might be the threshold used to conclude collusion by Gatherer (2006) and Mantzaris, Rein, and Hopkins (2018) in the algorithm described in Section 5.1. It could be the case that this threshold needs to be higher in order to conclude that collusion has taken place.

	Croatia	Cyprus	Denmark	France	Germany	Greece	Iceland	Ireland	Malta	Sweden	Switzerland	UK
Croatia									0.85			
Cyprus						0.9285						
Denmark							-0.1818			0.2941		
France							-0.2777					
Germany								-0.2777		-0.3684		
Greece		1.357										
Iceland			-0.1818							0.0769		
Ireland				-0.4444	-0.2777				-0.2222	-0.1578		
Malta	-0.1428							-0.5555				
Sweden		0.0588			-0.1578		-0.0769	-0.2631				-0.3
Switzerland												0.0555
UK										-0.35	-0.2777	

Figure 5: The effect of collusion from 1980 to 1999

Figure 5 also shows most countries moving down in the rankings because of colluding. Only in 7 out of 24 instances (29.2%) do countries move up in this 20-year period as a result of their collusion, and only in four instances did colluding partners actually move up more than 0.1 spots on average. All colluding countries together moved down 0.0371 places on average. The rule used is the same for all contests during this period. Televoting was used twice in this period, while jury voting was used eighteen times.

Compared to Figure 4, the results in Figure 5 diverge less from the hypothesis. However, they do not agree with the hypothesis either. The average placing for the countries in Figure 5 during this time period is 9.4455, while the average placing for all countries during this time period is 10.9.

	Armenia	Azerbaijan	Bos-Herz	Croatia	Georgia	N-Macedonia	Russia	Serbia	Turkey	Ukraine
Armenia					0.4285		0.3333			
Azerbaijan							0.375		0.8	0.285
Bos-Herz				0.5833				0.5	0.5	
Croatia			0.5			-0.25				
Georgia	0.2857									
N-Macedonia				0.4166						
Russia	0.2222	0.125								0.25
Serbia			0.5							
Turkey		0.6	-0.0833							
Ukraine		0.1428					0.25			

Figure 6: The effect of collusion on eastern European countries from 2000 to 2015

	Albania	Cyprus	Denmark	Finland	Greece	Iceland	Ireland	Moldova	Norway	Romania	Sweden	UK
Albania					0.25							
Cyprus					0.923							
Denmark						0.6428			0.1428		0.3333	
Finland											0.2857	
Greece	0.1666	0.8461										
Iceland			0.2857						0.4			
Ireland												0.5333
Moldova										0.8181		
Norway			-0.1428			-0.2					-0.1333	
Romania							1.3636					
Sweden			0.3333	0.4285				0.4666				
UK						0.8666						

Figure 7: The effect of collusion on other countries from 2000 to 2015

Figures 6 and 7 show a much more logical result: in 35 out of 40 (87.5%) instances, countries move up in the rankings as a result of their collusion. On average, each colluding country moved up 0.3843 spots in the rankings as a result of their collusion. Although the voting rule has not changed from 1980 to 2015, the number of times televoting was used increased in this period. Televoting was mandatory during this period in thirteen years, while in the period from 1980 to 1999 it was mandatory only twice. This suggests that the public is, on average, more biased than juries.

The average placing for the countries in Figures 6 and 7 during this time period is 11.7735, while the average for all countries is 12.4375. The ratio between these two numbers (1.05) is significantly lower compared to the ratios of the numbers mentioned in the discussions of Figure 4 (1.19) and 5 (1.15). This, along with the results in Figure 6 and 7 supports the hypothesis that the collusion concluded in the periods from 1960 to 1979 and from 1980 to 1999 is a result of bias against well-performing countries.

The results of this section are inconclusive. The hypothesis stated that colluding countries will move down in the recomputed rankings on average; however, this is not the case for most of the countries in two of the three time periods collusion was measured in. The hypothesis that the collusion in the periods from 1960 to 1979 and from 1980 to 1999 is a result of bias against well-performing countries has been brought up, but one would expect this bias to also show up in the other period.

Other studies have found that bias of countries towards each other is not necessarily a symptom of collusion. Rather, bias is the result of multiple factors, including when the country performs during the contest, the host country, the gender of a performer, and more (Haan, Dijkstra, and Dijkstra 2005; Spierdijk and Vellekoop 2006). This suggests that a results-based analysis is not enough to conclude collusion: a more extensive model which takes into account factors such as these might give more conclusive results.

## 6 Conclusion

In this thesis, a model has been outlined in which all rules which have been used in the history of the ESC fit. All scoring rules used in the history of the ESC have been applied on the data of ESCs using other rules, which showed that rule changes in the ESC have not influenced the winner often. A solver was programmed which tries to find a scoring rule such that a certain country wins the contest, and this solver was used to find as many possible winners for the relevant data. Lastly, an algorithm was created which measures the impact of collusion on ESCs. This algorithm was used to verify the results of Gatherer (2006) and Mantzaris, Rein, and Hopkins (2018) which found that multiple countries were colluding during certain time periods. The results showed that these accusations of collusion were not verifiable during most time periods.

The model in this thesis fits within the field of social choice theory, and can be used in further research concerning voting rules in the ESC. The application of different rules on the data extracted from ESCs which used a scoring rule was a success, but data from ESCs which did not use scoring rules was not used. The solver which was designed did provide correct results, but it has restrictions: only scoring vectors of size 10 can be found, and there is an upper bound on the variables the solver finds. The algorithm used to detect the effects of collusion did not provide results which were in line with the hypothesis. This is likely a result of the method used by Gatherer (2006) and Mantzaris, Rein, and Hopkins (2018) to conclude collusion, which does not take other factors linked to bias into account (Haan, Dijkstra, and Dijkstra 2005; Spierdijk and Vellekoop 2006), and also seems to have a bias against well-performing countries.



Future work could expand on this thesis by correcting the shortcomings mentioned in the previous paragraph. Implementing a tie breaking system enables scoring rules to be applied on the data of ESCs which did not use a scoring rule, and with some programming adjustments, the solver should be able to find scoring vectors of different sizes. The upper bound on the variables which are solved for could also be heightened, but this is unlikely to fetch many new results. The area which is hardest to expand on is the detection of collusion through results analysis. Without extensive voting data like televote tallies and complete jury rankings, it is very difficult to find conclusive proof through results analysis alone.

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