(I)

$$y'' = -g + \frac{1}{M}\beta e^{-\frac{y}{\alpha}}(y')^2$$

$y(t=0) = 130.000\ m \qquad y'(t=0) = V_0 = -5555\ m/s$

Cambio variables:

$y' = u \Rightarrow u' = -g + \frac{1}{M}\beta e^{-\frac{y}{\alpha}} u^2$

$y'' = u'$

con $y(t=0) = 130.000$ y $u(t=0) = -5555$

el nuevo sist. a resolver:

$$F_{y,t} = \begin{bmatrix} u \\ u' \end{bmatrix} = \begin{bmatrix} u \\ -g + \frac{1}{M}\beta e^{-\frac{y}{\alpha}} u^2 \end{bmatrix},$$

Condiciones iniciales: $\begin{bmatrix} 130.000 \\ -5555 \end{bmatrix} = \begin{bmatrix} y \\ y' \end{bmatrix} = \begin{bmatrix} y \\ u \end{bmatrix} = u_0$

RK2:

$$q_{1u} = hF(u_n, t_n)$$
$$q_{2u} = hF(u_n + q_{1u}, t_{n+1})$$
$$u_{n+1} = u_n + \frac{1}{2}(q_{1u} + q_{2u})$$

* $u \neq u$

1ª iteración $t = 0$

$$g_{1u} = F(u_0, t_0) \cdot h = F\left(\begin{bmatrix} 130.000 \\ -5555 \end{bmatrix}, 0\right) \cdot h$$

(y: $130.000$, $-5555$; $t$: $0$; $u$)

$$= \begin{bmatrix} -5555 \\ -g + \frac{1}{M} \beta e^{\frac{-130.000}{\alpha}} \cdot (-5555)^2 \end{bmatrix} \cdot \underbrace{h}_{0.01}$$

$$g_{2u} = h \cdot F(u_0 + g_{1u}, t_0 + h) =$$

$$h \cdot F\left(\begin{bmatrix} 130.000 \\ -5555 \end{bmatrix} + h \begin{bmatrix} (-5555) \\ -g + \frac{1}{M} \beta e^{\frac{-130.000}{\alpha}} \cdot (-5555)^2 \end{bmatrix}, 0 + h\right)$$

$$u_1 = \begin{bmatrix} 130.000 \\ -5555 \end{bmatrix} + \frac{1}{2}\left(g_{1u} + g_{2u}\right)$$

$q_{1u}$ :

$\alpha = 150.000$

$\beta = 0,0192$

$g = 3,72 \ m/s^2$

$M = 1000 \ Kg$

$$q_{1u} = \begin{bmatrix} -55,55 \\ 2,453 \end{bmatrix}$$

$$q_{2u} = 0.01 \cdot F\left(\begin{bmatrix} 129.994,45 \\ -5552,5467 \end{bmatrix}, 0,01\right)$$

(↑ $y$ : $129.994,45$; ↑ $t$ : $0,01$; ↘ $u$ : $-5552,5467$)

$$\begin{bmatrix} -5552,5467 \\ -g + \frac{1}{M}\beta e^{\frac{-129.994,45}{\alpha}} \cdot (-5552,5467)^2 \end{bmatrix} \cdot 0,01 =$$

$$\begin{bmatrix} -55,5254 \\ 2.4519 \end{bmatrix}$$

$$u_1 = \overbrace{\begin{bmatrix} 130.000 \\ -5555 \end{bmatrix}}^{u_0} + \frac{1}{2}\begin{bmatrix} \dots \end{bmatrix} = \begin{bmatrix} 129994,46 \\ -5552,5474 \end{bmatrix}$$

($y$ ↑ : $129994,46$; $u = y'$ ↓ : $-5552,5474$)