Centroid, Moment of Inertia, and Deflection Analysis of Structural Elements using MATLAB

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Contents

1 Introduction

In civil engineering, understanding key principles, such as the centroid, moment of inertia, and beam deflection is essential for analyzing and constructing safe and stable structures. This MATLAB project implements a live script to calculate the centroid, moment of inertia, and beam deflection of multiple structural elements widely encountered in structural engineering. This tool allows engineers to test diverse configurations and optimize the design of structures to minimize deflection, enhance stability, and ensure safety.

1.1 Centroid Calculation

The live script's centroid section computes the centroid of various 2D geometric shapes (e.g., rectangle, equilateral triangle, circle) by accepting the user's inputs of the particular shape's dimensions. This program extends to composite shapes (e.g., I-Beam, T-Section, Hollow Rectangle). It visualizes the shapes and illustrates the location of the centroid on a graph.

1.2 Moment of Inertia Calculation

The live script's moment of inertia section applies the Parallel Axis Theorem to calculate the moment of inertia for both individual and composite shapes. It plots the cross-sectional shapes and displays the computed moment of inertia.

1.3 Beam Deflection Calculation

The live script's beam deflection section calculates the beam deflection for simply supported beams with a concentrated point load. This application enables the user to input the length, cross-sectional geometry, and loading conditions to accurately plot the deflection along the deflection curve.

2 Definitions

The **centroid** is the geometric point defined as the arithmetic mean position of all the points in a shape. It is often referred to as the "center of mass" or the "center of gravity" of a geometric object [5].

The **moment of inertia** is the scalar property of a body in rotational motion which opposes the change in its rotational motion due to external forces and is expressed in kgm^2. It is the sum of the product of the mass of each particle and the square of the distance from the rotational axis [4].

The **beam deflection** is the movement of a beam from its original position due to the forces and loads being applied to the member. It is also known as displacement and can occur from externally applied loads, the weight of the structure itself, and the force of gravity [9].

A **simply support beam** spans a single distance between two supports. One end is supported by a pinned support, while the other end is supported by a roller support [8].

A **pinned support** is a common type of support and is most commonly compared to a hinge in civil engineering. A pinned support allows rotation to occur but no translation (i.e. it resists horizontal and vertical forces) [10].

A **rolled support** can resist a vertical force but not a horizontal force. A roller support or connection is free to move horizontally because there is no restriction [10].

The **Parallel Axis Theorem** states that the moment of inertia of a body about an axis parallel to the body passing through its center is equal to the sum of the moment of inertia of the body about the axis passing through its center and the product of the mass of the body times the square of the distance between the two axes [2].

A **I-Beam** is an iron or steel beam that is I-shaped in cross-section. They are utilized to construct bridges, parking garages, skyscrapers, hospitals, etc [6].

A **T-Section** is a reinforced concrete beam with a T-shaped cross section, used for efficient structural support in buildings and bridges [7].

A **Hollow Rectangle** is a rectangular-shaped steel tube with hollow cross-sections. They are essential structural steel components that provide structural support and load- bearing capacity in various construction applications. They are commonly used in the construction of frames, supports, columns, and beams where rectangular sections are required [11].

A **Point Load**, also known as a concentrated load, is an action applied to a specific point within a structure [3].

Young's Modulus, usually represented by E, is a constant characteristic that measures a material's capacity to deform elastically under tension or compression [1].

3 Methodology

3.1 Centroid Equations

There are multiple equations used to calculate the centroid of individual shapes.

- X and Y represent the length and width of the shape, respectively.
- X_c and Y_c represents the x and y coordinates for the location of the centroid.
- D represents the diameter of a circle's dimensions.
- A_i is the area of each of the individual composite shapes. Y_i is the distance from the centroid from the bottom of the structure.

Shape	**Centroid Equation** (X_c)	**Centroid Equation** (Y_c)
Rectangle	$X/2$	$Y/2$
Equilateral Triangle	$X/2$	$X \cdot \sqrt{3}/6$
Circle	$D/2$	$D/2$
I-Beam	X/2	$(\sum A_i \cdot Y_i)/(\sum A_i)$
T-Section	X/2	$(\sum A_i \cdot Y_i)/(\sum A_i)$
Hollow Rectangle	$X/2$	$Y/2$

3.2 Moment of Inertia Equations

There are multiple equations used to calculate the moment of inertia of individual shapes.

- B and D represent the base and depth of the shape, respectively.
- B and D represent the base and depth of the larger rectangle. While, B_1 and D_1 represent the base and depth of the smaller rectangle.
- H represents the height of the equilateral triangle, respectively.
- C represents the distance between the apex to one of the bottom vertexes of the equilateral triangle.
- R represents the radius of a circle.
- $\sum \bar{I}_i + (A_i \cdot D_i^2)$ where I_i is $(1/12)BH^3$, A_i is the area of each of the individual composite shapes
- D_i for I_{xx} is each of the individual composite shape's distance from the bottom of the structure to its centroid minus the centroid for the y-value.
- D_i for I_{yy} is is each of the individual composite shape's distance from the leftmost point of the structure to its centroid minus the centroid for the x-value.

Shape	Moment of Inertia Equation (I_{xx})	Moment of Inertia Equation (I_{yy})
Rectangle	$(BD^3)/12$	$(DB^3)/12$
Equilateral Triangle	$(BH^3)/36$	$(BH/36) \cdot (B^2 - BC - C^2)$
Circle	$(\pi \cdot (R^4))/4$	$(\pi \cdot (R^4))/4$
I-Beam	$\sum \bar{I}_i + (A_i \cdot D_i^2)$	$\sum \bar{I}_i + (A_i \cdot D_i^2)$
T-Section	$\sum \bar{I}_i + (A_i \cdot D_i^2)$	$\sum \bar{I}_i + (A_i \cdot D_i^2)$
Hollow Rectangle	$((BD^3)/12) - (B_1 D_1^3)/12$	$((DB^3)/12) - (D_1 B_1^3)/12$

3.3 Beam Deflection Equations

There are three equations used to calculate the beam deflection for simply support beams.

- θ_{max} represents the slope of the deflection
- P represents the force of an applied point load
- L represents the length of the beam
- E represents Young's Modulus, and I is the moment of inertia.
- X is any length along the beam as long as it is between 0 and $L/2$.
- v_{max} represents the displacement or deflection at a given point along the beam
- v is the plotted graph of the elastic curve

Beam	Slope	Deflection	Elastic Curve
Simply Supported Beam	$\theta_{max} = (-PL^2)/16EI$	$v_{max} = (-PL^3)/48EI$	$v = ((-Px)/48EI) \cdot (3\mathrm{L}^2 - 4X^2)$

4 Results

The MATLAB live script accurately computes the centroid, moment of inertia, and beam deflection for various shapes, dimensions, and initial conditions. The following graphs enable the user to experiment with various inputs, providing flexibility for structural engineers working on different structural systems.

4.1 Plotted Centroid Graphs

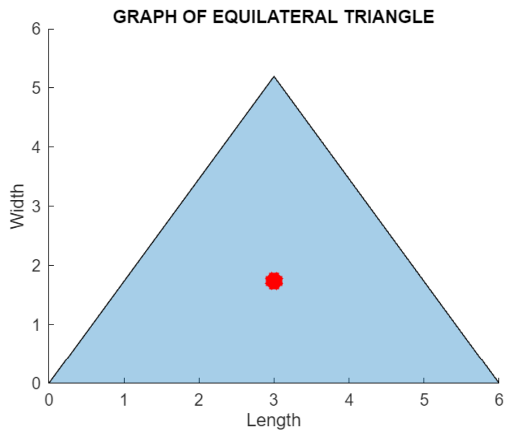

Figure 1: This equilateral triangle has a side length of 6 units and the centroid (red dot) is located at the coordinates (3,1.7321)

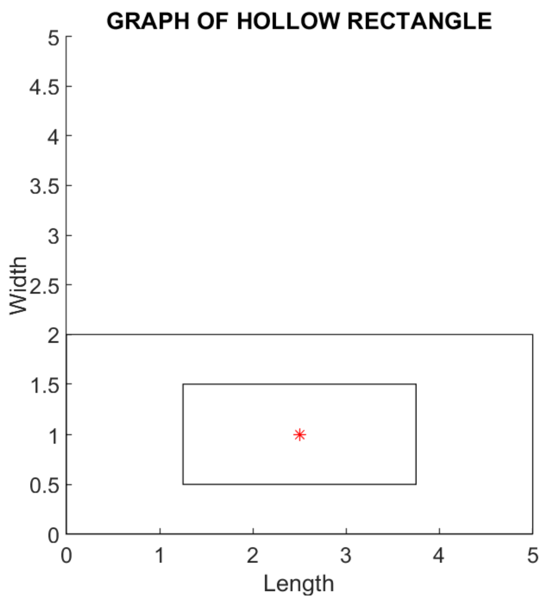

Figure 2: This hollow rectangle is composed of a larger and smaller rectangle. The larger rectangle has a length of 5 units and a width of 2 units. The smaller rectangle has a length of 2.5 units and a width of 1 unit. The centroid (red dot) is located at the coordinates (2.5,1)

4.2 Plotted Moment of Inertia Graphs

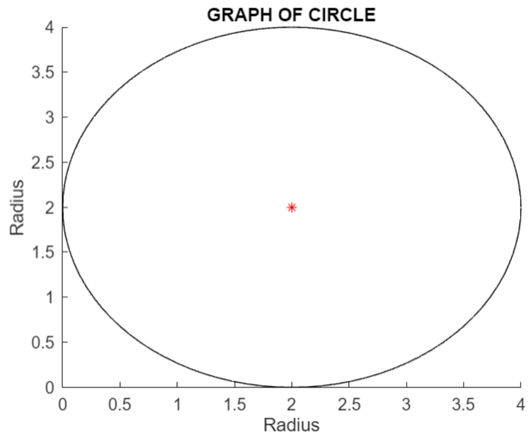

Figure 3: This circle has a diameter of 4 units or a radius of 2 units. The moment of inertia is located at the coordinates (12.5664, 12.5664)

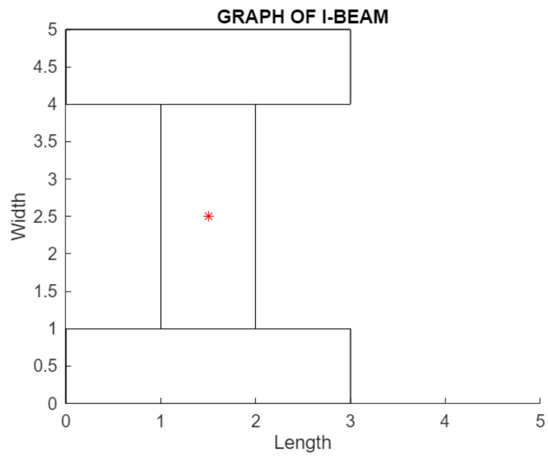

Figure 4: This I-Beam is composed of three individual rectangles. The two horizontal beams have the same length and width. In this case, The length of the two horizontal beams is 3 units and the width is 1 unit. The length of the vertical beam is 1 unit and the width is 3 units. The moment of inertia is located at the coordinates (26.75, 4.75)

4.3 Plotted Beam Deflection Graphs

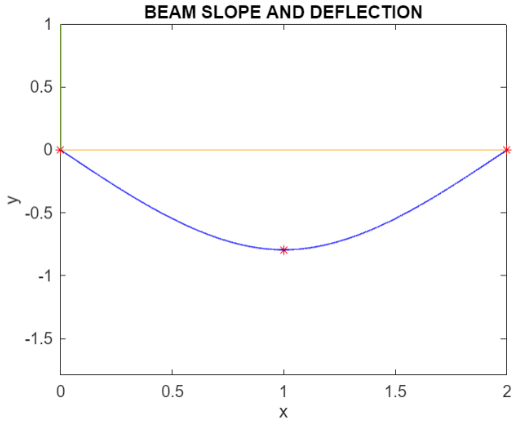

Figure 5: This beam deflection is caused by the point load force applied to the simply support beam. In this case, the length of the support beam is 2 units, the width is 1 unit, the point load is 1000 Newtons, and Young's Modulus or E is 210 GPa.

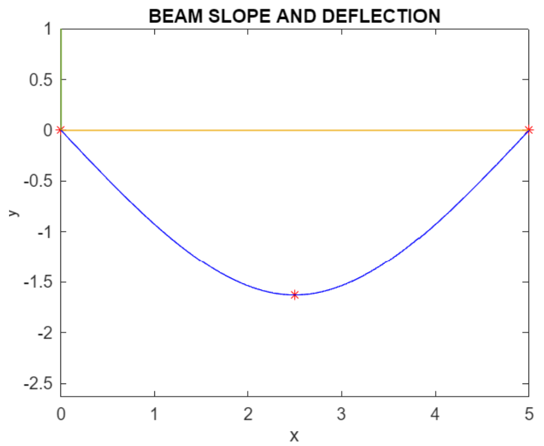

Figure 6: This beam deflection is caused by the point load force applied to the simply support beam. In this case, the length of the support beam is 5 units, the width is 2 units, the point load is 500 Newtons, and Young's Modulus is 800 GPa.

5 Conclusion

The MATLAB live script precisely calculates the centroid, moment of inertia, and beam deflection of various shapes and offers a helpful and flexible solution for engineers. The script not only efficiently completes the analysis process but also ensures high accuracy in determining the behavior of several structural elements. The integration of plotted graphs, making these complex concepts more user-friendly. Ultimately, this project highlights the importance and capabilities of MATLAB as a platform for solving fundamental engineering challenges and serves as a valuable resource for both students and professionals in the structural engineering field.

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