```
Activating project at `~/.julia/environments/v1.8`
```

Метод сеток для Пуассона

$$\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = \frac{2}{1+y}\left(1 + \left(\frac{x+0.2}{1+y}\right)^2\right)$$

$$0 \le x, y \le 1$$

$$u(x,0) = (x+0.2)^2, u(x,1) = \frac{(x+0.2)^2}{2}$$

$$u(0,y) = \frac{0.2^2}{1+y}, u(1,y) = \frac{1.2^2}{1+y}$$

```
φ (generic function with 1 method)
```

```
φ(x, y) = 2 / (1+y) * (1 + ((x+0.2)/(1+y))^2)
```

```
h = 0.2
```

```
h = .2
```

```
X = 0.0:0.2:1.0
```

```
X = 0:h:1
```

```
Y = 0.0:0.2:1.0
```

```
Y = 0:h:1
```

```
α₀ (generic function with 1 method)
```

```
α₀(x) = (x+0.2)^2
```

```
α₁ (generic function with 1 method)
```

```
α₁(x) = 0.5 * (x+0.2)^2
```

```
β₀ (generic function with 1 method)
```

```
β₀(y) = 0.04 / (1+y)
```

```
β₁ (generic function with 1 method)
```

```
β₁(y) = 1.44 / (1+y)
```

```
U = 6×6 Matrix{Any}:
     #undef  #undef  #undef  #undef  #undef  #undef
     #undef  #undef  #undef  #undef  #undef  #undef
     #undef  #undef  #undef  #undef  #undef  #undef
     #undef  #undef  #undef  #undef  #undef  #undef
     #undef  #undef  #undef  #undef  #undef  #undef
     #undef  #undef  #undef  #undef  #undef  #undef
```

```
U = Matrix(undef, 6, 6)
```

```
6×6 Matrix{Any}:
   0.04    0.16    0.36    0.64    1.0     1.44
 #undef  #undef  #undef  #undef  #undef  #undef
 #undef  #undef  #undef  #undef  #undef  #undef
 #undef  #undef  #undef  #undef  #undef  #undef
 #undef  #undef  #undef  #undef  #undef  #undef
   0.02    0.08    0.18    0.32    0.5     0.72
```

```
begin
    U[1, :] = α₀.(X)
    U[end, :] = α₁.(X)
    U
end
```

```
6×6 Matrix{Any}:
 0.04       0.16    0.36    0.64    1.0   1.44
 0.0333333  #undef  #undef  #undef  #undef  1.2
 0.0285714  #undef  #undef  #undef  #undef  1.02857
 0.025      #undef  #undef  #undef  #undef  0.9
 0.0222222  #undef  #undef  #undef  #undef  0.8
 0.02         0.08    0.18    0.32    0.5   0.72
```

```
begin
    U[:, 1] = β₀.(Y)
    U[:, end] = β₁.(Y)
    U
end
```

```
σ = 1
```

```
σ = 1
```

Подбираем отображение для построения системы

```
mapping =
  Dict(CartesianIndex(5, 4) ⇒ CartesianIndex(4, 3), CartesianIndex(5, 2) ⇒ CartesianIndex
```

```
mapping = Dict(
    collect(CartesianIndices(U))[2:end-1, 2:end-1]
        .=>
    collect(CartesianIndices((4, 4)))
)
```

```
M = 4×4×5 Array{Int64, 3}:
[:, :, 1] =
 2   8  14  20
 3   9  15  21
 4  10  16  22
 5  11  17  23

[:, :, 2] =
  7  13  19  25
  8  14  20  26
  9  15  21  27
 10  16  22  28

[:, :, 3] =
  8  14  20  26
  9  15  21  27
 10  16  22  28
 11  17  23  29

[:, :, 4] =
  9  15  21  27
 10  16  22  28
 11  17  23  29
 12  18  24  30

[:, :, 5] =
 14  20  26  32
 15  21  27  33
 16  22  28  34
 17  23  29  35
```

```
M = cat([
    (collect(LinearIndices(U)).-6)[2:end-1, 2:end-1],
    (collect(LinearIndices(U)).-1)[2:end-1, 2:end-1],
    collect(LinearIndices(U))[2:end-1, 2:end-1],
    (collect(LinearIndices(U)).+1)[2:end-1, 2:end-1],
    (collect(LinearIndices(U)).+6)[2:end-1, 2:end-1]
]..., dims=3)
```

```
[2, 7, 8, 9, 14]
```

```
M[mapping[CartesianIndex((2, 2))], :]
```

Решаем систему

```
Φ =
 [0.0832, 0.0928, 0.1088, 0.1312, 0.16, 0.1952, 0.0685185, 0.0740741, 0.0833333, 0.0962963,
```

```
Φ = vcat([
    φ(x, y)
    for x in X, y in Y
]...) * h^2
```

```
36×36 Matrix{Float64}:
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  …  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  …  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 ⋮                        ⋮              ⋱              ⋮                        ⋮
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  …  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0     0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
 0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  …  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
```

```
begin
    A = zeros(36, 36)

    for key in collect(keys(mapping))
        A[M[mapping[key], :][3], M[mapping[key], :]] = [1, σ, -2*(1+σ), σ, 1]
    end

    A
end
```

```
[1, 1, -4, 1, 1]
```

```
[1, σ, -2*(1+σ), σ, 1]
```

```
ind =
 [2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27,
```

```
ind = vcat([
    M[mapping[key], :]
    for key in collect(keys(mapping))
]...) |> unique |> sort
```

```
[0.0, 0.0313279, 0.0542043, 0.0569848, 0.0363412, 0.0, 0.0313279, -0.0217875, -0.0423047,
```

```
begin
    u = zeros(6, 6)
    u[1:36] = A[ind, :] \ Φ[ind]
end
```

```
6×6 Matrix{Any}:
 0.04       0.16        0.36        0.64        1.0        1.44
 0.0333333  -0.0217875  -0.0334269  -0.0288685  -0.0126402  1.2
 0.0285714  -0.0423047  -0.070564   -0.0626069  -0.0271633  1.02857
 0.025      -0.0477383  -0.0762791  -0.0668004  -0.0292796  0.9
 0.0222222  -0.0330577  -0.044212   -0.036536   -0.0166805  0.8
 0.02        0.08        0.18        0.32        0.5        0.72
```

```
begin
    U[2:end-1, 2:end-1] = u[2:end-1, 2:end-1]
    U
end
```