```
begin
    import Pkg;     Pkg.activate()

    using DataFrames
    using Plots
end
```

```
  Activating project at `~/.julia/environments/v1.8`
```

```
euler (generic function with 1 method)
```

```
function euler(s, a, b, x₀, T₀, S₀, h)
    x = [x₀]
    T = [T₀]
    S = [S₀]

    while x[end]+h < b
        push!(T, T[end] + h * S[end])
        push!(S, S[end] + h * s(x[end], T[end-1], S[end]))
        push!(x, x[end] + h)
    end

    return x, T, S
end
```

Метод вариации постоянных

$$y'' + \frac{1}{x}y' - 2y = x^2$$

$$0.5 \leq x \leq 1, y'(0.5) = -0.5, y'(1) = -1$$

```
(0, 0)
```

```
α₀, α₁ = 0, 0
```

```
(1, 1)
```

```
β₀, β₁ = 1, 1
```

```
(-0.5, -1.0)
```

```
γ₀, γ₁ = -.5, -1.
```

Z, Z_1, Z_2

$$\begin{cases} Z''(x) + \frac{1}{x}Z'(x) - 2Z(x) = x^2, 0.5 \le x \le 1 \\ Z(0.5) = 0, Z'(0.5) = 0 \end{cases}$$

$$\begin{cases} T'(x) = S(x) \\ S'(x) = -\frac{1}{x}S(x) + 2T(x) + x^2, 0.5 \le x \le 1 \\ T(0.5) = 0, S(0.5) = 0 \end{cases}$$

```
(
    1:  [0.5, 0.6, 0.7, 0.8, 0.9, 1.0]
    2:  [0.0, 0.0, 0.0025, 0.00818333, 0.0180048, 0.0331622]
    3:  [0.0, 0.025, 0.0568333, 0.0982143, 0.151574, 0.219334]
)
```

```
x, T, S = euler(
    (x, t, s) -> -s/x + 2*t + x^2,
    0.5, 1.,
    0.5,
    .0, .0,
    .1
)
```

$$\begin{cases} {Z_1}''(x) + \frac{1}{x}{Z_1}'(x) - 2Z_1(x) = 0, 0.5 \le x \le 1 \\ Z_1(0.5) = 0, {Z_1}'(0.5) = 1 \end{cases}$$

$$\begin{cases} {T_1}'(x) = S_1(x) \\ {S_1}'(x) = -\frac{1}{x}S_1(x) + 2T_1(x), 0.5 \le x \le 1 \\ T_1(0.5) = 0, S_1(0.5) = 1 \end{cases}$$

```
([0.0, 0.1, 0.18, 0.248667, 0.311124, 0.370747], [1.0, 0.8, 0.686667, 0.624571, 0.596233, 0
```

```
T₁, S₁ = euler(
    (x, t, s) -> -s/x + 2*t,
    0.5, 1.,
    0.5,
    .0, 1.,
    .1
)[2:3]
```

$$\begin{cases} {Z_2}''(x) + \frac{1}{x}{Z_2}'(x) - 2Z_2(x) = 0, 0.5 \le x \le 1 \\ Z_2(0.5) = 1, {Z_2}'(0.5) = 0 \end{cases}$$

$$\begin{cases} {T_2}'(x) = S_2(x) \\ {S_2}'(x) = -\frac{1}{x}S_2(x) + 2T_2(x), 0.5 \le x \le 1 \\ T_2(0.5) = 1, S_2(0.5) = 0 \end{cases}$$

```
(
    1:  [1.0, 1.0, 1.02, 1.05667, 1.1085, 1.17498]
    2:  [0.0, 0.2, 0.366667, 0.518286, 0.664833, 0.812662]
)
```

```
T₂, S₂ = euler(
    (x, t, s) -> -s/x + 2*t,
    0.5, 1.,
    0.5,
    1., .0,
    .1
)[2:3]
```

C_1, C_2

```
A = 2×3 Matrix{Float64}:
     1.0      0.0       -0.5
     0.59221  0.812662  -1.21933
```

```
A = [
    α₀*T₁[1] + β₀*S₁[1] α₀*T₂[1] + β₀*S₂[1] γ₀ - α₀*T[1] - β₀*S[1]
    α₁*T₁[end] + β₁*S₁[end] α₁*T₂[end] + β₁*S₂[end] γ₁ - α₁*T[end] - β₁*S[end]
]
```

```
[-0.5, -1.13605]
```

```
C₁, C₂ = A[:, [1,2]] \ A[:, 3]
```

yk

```
yₖ =  [-1.13605, -1.18605, -1.24628, -1.31658, -1.39687, -1.48705]
```

```
yₖ = C₁.*T₁ + C₂.*T₂ + T
```

	x_k	T	S	T_1	S_1	T_2	S_2	y_k
1	0.5	0.0	0.0	0.0	1.0	1.0	0.0	-1.13605
2	0.6	0.0	0.025	0.1	0.8	1.0	0.2	-1.18605
3	0.7	0.0025	0.0568333	0.18	0.686667	1.02	0.366667	-1.24628
4	0.8	0.00818333	0.0982143	0.248667	0.624571	1.05667	0.518286	-1.31658
5	0.9	0.0180048	0.151574	0.311124	0.596233	1.1085	0.664833	-1.39687
6	1.0	0.0331622	0.219334	0.370747	0.59221	1.17498	0.812662	-1.48705

```
DataFrame(
    xₖ=x,
    T=T, S=S,
    T₁=T₁, S₁=S₁,
    T₂=T₂, S₂=S₂,
    yₖ=yₖ
)
```