

# Метод "кривых" сеток для Пуассона

$$\frac{\partial^2 u(x, y)}{\partial x^2} + \frac{\partial^2 u(x, y)}{\partial y^2} = 1.91$$

$$x^2 + y^2 = 1$$

$$u_{\Gamma}(x, y) = 0.7x^2 + 0.3y^2 + 0.6$$

`h = 0.2`

- `h = 0.2`

`UΓ` (generic function with 1 method)

- `UΓ((x, y)) = 0.7*x^2 + 0.3*y^2 + 0.6`

## Область значения функции

`(-1.0:0.2:1.0, -1.0:0.2:1.0)`

- `begin`
- `X = -1:h:1`
- `Y = -1:h:1`
- 
- `X, Y`
- `end`

```

Γ = 11×11 Matrix{Union{Missing, Tuple{Float64, Float64}}}:
missing      missing      missing      ...      missing      missing      missing
missing      missing      (-0.8, -0.6)  (-0.8, 0.6)  missing      missing
missing      (-0.6, -0.8)  (-0.6, -0.6)  (-0.6, 0.6)  (-0.6, 0.8)  missing
missing      (-0.4, -0.8)  (-0.4, -0.6)  (-0.4, 0.6)  (-0.4, 0.8)  missing
missing      (-0.2, -0.8)  (-0.2, -0.6)  (-0.2, 0.6)  (-0.2, 0.8)  missing
(0.0, -1.0)  (0.0, -0.8)      (0.0, -0.6)  ...      (0.0, 0.6)   (0.0, 0.8)   (0.0, 1.0)
missing      (0.2, -0.8)      (0.2, -0.6)  (0.2, 0.6)   (0.2, 0.8)   missing
missing      (0.4, -0.8)      (0.4, -0.6)  (0.4, 0.6)   (0.4, 0.8)   missing
missing      (0.6, -0.8)      (0.6, -0.6)  (0.6, 0.6)   (0.6, 0.8)   missing
missing      missing      (0.8, -0.6)  (0.8, 0.6)   missing      missing
missing      missing      missing      ...      missing      missing      missing

```

```

• Γ = [
•     x^2 + y^2 ≤ 1 ? (x, y) : missing
•     for x in X, y in Y
• ]

```

Γ\_ind =

```
[CartesianIndex(6, 1), CartesianIndex(3, 2), CartesianIndex(4, 2), CartesianIndex(5, 2), (
```

```
• Γ_ind = collect(CartesianIndices(Γ))[:, !ismissing.(Γ)]
```

```

11×11 Matrix{Union{Missing, Tuple{Float64, Float64}}}:
missing      missing      missing      ...      missing      missing      missing
missing      missing      (-0.8, -0.6)  (-0.8, 0.6)  missing      missing
missing      (-0.6, -0.8)  missing      missing      (-0.6, 0.8)  missing
missing      (-0.4, -0.8)  missing      missing      (-0.4, 0.8)  missing
missing      missing      missing      missing      missing      missing
(0.0, -1.0)  missing      missing      ...      missing      missing      (0.0, 1.0)
missing      missing      missing      missing      missing      missing
missing      (0.4, -0.8)      missing      missing      (0.4, 0.8)   missing
missing      (0.6, -0.8)      missing      missing      (0.6, 0.8)   missing
missing      missing      (0.8, -0.6)  (0.8, 0.6)   missing      missing
missing      missing      missing      ...      missing      missing      missing

```

```

• begin
•     B = [
•         isapprox(x^2 + y^2, 1; atol=0.2) ? (x, y) : missing
•         for x in X, y in Y
•     ]
•
•     B_ind = collect(CartesianIndices(B))[:, !ismissing.(Γ) .&& !ismissing.(B)]
•     B[:, !ismissing.(Γ) .&& !ismissing.(B)] .= missing
•     B
• end

```

## Индексы точек внутренней области

```
[CartesianIndex(6, 1), CartesianIndex(3, 2), CartesianIndex(4, 2), CartesianIndex(8, 2), (
```

```
• B_ind
```

```
I_ind =
```

```
[CartesianIndex(5, 2), CartesianIndex(6, 2), CartesianIndex(7, 2), CartesianIndex(3, 3), CartesianIndex(4, 3), CartesianIndex(5, 3), CartesianIndex(6, 3), CartesianIndex(7, 3), CartesianIndex(8, 3), CartesianIndex(9, 3), CartesianIndex(10, 3)]
```

```
• I_ind = Γ_ind[.!(Γ_ind .∈ (B_ind,))]
```

```
true
```

```
• isdisjoint(B_ind, I_ind)
```

## Решение

```
11×11 Matrix{Float64}:
```

```
Inf  Inf  Inf  Inf  Inf  1.3  Inf  Inf  Inf  Inf  Inf
Inf  Inf  1.156  1.096  Inf  Inf  Inf  1.096  1.156  Inf  Inf
Inf  1.044  Inf  Inf  Inf  Inf  Inf  Inf  Inf  1.044  Inf
Inf  0.904  Inf  Inf  Inf  Inf  Inf  Inf  Inf  0.904  Inf
Inf  Inf  Inf  Inf  Inf  Inf  Inf  Inf  Inf  Inf  Inf
0.9  Inf  Inf  Inf  Inf  Inf  Inf  Inf  Inf  Inf  0.9
Inf  Inf  Inf  Inf  Inf  Inf  Inf  Inf  Inf  Inf  Inf
Inf  0.904  Inf  Inf  Inf  Inf  Inf  Inf  Inf  0.904  Inf
Inf  1.044  Inf  Inf  Inf  Inf  Inf  Inf  Inf  1.044  Inf
Inf  Inf  1.156  1.096  Inf  Inf  Inf  1.096  1.156  Inf  Inf
Inf  Inf  Inf  Inf  Inf  1.3  Inf  Inf  Inf  Inf  Inf
```

```
• begin
•   U = fill(Inf, size(Γ))
•   U[B_ind] = UΓ.(B[B_ind])
•
•   U
• end
```

```
view(::Matrix{Float64}, CartesianIndex{2}[CartesianIndex(5, 2), CartesianIndex(6, 2), CartesianIndex(7, 2), CartesianIndex(3, 3), CartesianIndex(4, 3), CartesianIndex(5, 3), CartesianIndex(6, 3), CartesianIndex(7, 3), CartesianIndex(8, 3), CartesianIndex(9, 3), CartesianIndex(10, 3)])
```

```
• U[I_ind] .= 0
```

```
11×11 Matrix{Float64}:
```

```
Inf  Inf  Inf  Inf  Inf  1.3  Inf  Inf  Inf  Inf  Inf
Inf  Inf  1.156  1.096  0.0  0.0  0.0  1.096  1.156  Inf  Inf
Inf  1.044  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.044  Inf
Inf  0.904  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.904  Inf
Inf  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  Inf
0.9  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.9
Inf  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  Inf
Inf  0.904  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.904  Inf
Inf  1.044  0.0  0.0  0.0  0.0  0.0  0.0  0.0  1.044  Inf
Inf  Inf  1.156  1.096  0.0  0.0  0.0  1.096  1.156  Inf  Inf
Inf  Inf  Inf  Inf  Inf  1.3  Inf  Inf  Inf  Inf  Inf
```

```
• U
```

```
[CartesianIndex(5, 2), CartesianIndex(6, 2), CartesianIndex(7, 2), CartesianIndex(3, 3), CartesianIndex(4, 3), CartesianIndex(5, 3), CartesianIndex(6, 3), CartesianIndex(7, 3), CartesianIndex(8, 3), CartesianIndex(9, 3), CartesianIndex(10, 3)]
```

```
• collect(CartesianIndices(U))[I_ind]
```

Dict(CartesianIndex(8, 4)  $\Rightarrow$  [35, 40, 41, 42, 47], CartesianIndex(4, 7)  $\Rightarrow$  [64, 69, 70, 71

```
• begin
•     M = hcat([
•         (collect(LinearIndices(U)).-6)[I_ind],
•         (collect(LinearIndices(U)).-1)[I_ind],
•         collect(LinearIndices(U))[I_ind],
•         (collect(LinearIndices(U)).+1)[I_ind],
•         (collect(LinearIndices(U)).+6)[I_ind]
•     ])
•
•     mapping = Dict(
•         collect(CartesianIndices(U))[I_ind] .=> [
•             M[i, :]
•             for i in 1:size(M, 1)
•         ]
•     )
• end
```

$\Phi$  =

[0.0764, 0.0764, 0.0764, 0.0764, 0.0764, 0.0764, 0.0764, 0.0764, 0.0764, 0.0764, 0.0764, 0.

```
•  $\Phi$  = fill(1.91 * h^2, 121)
```

121x121 Matrix{Float64}:

```
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
⋮      ⋮      ⋮      ⋮      ⋮      ⋮      ⋮      ⋮      ⋮      ⋮      ⋮      ⋮      ⋮      ⋮
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0  ...  0.0  0.0  0.0  0.0  0.0  0.0  0.0  0.0
```

```
• begin
•     A = zeros(121, 121)
•
•     for ind in I_ind
•         A[mapping[ind][3], mapping[ind]] = [1, 1, -4, 1, 1]
•     end
•
•     A
• end
```

S\_ind =

[10, 11, 12, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, more ,10:

```
• # индексы точек, где будем решать систему
• S_ind = unique(sort(vcat(last.(collect(mapping))...)))
```

[0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.00639184, 0.00790038, 0.00472398, 0.0, 0.0,

```
• begin
•     u = zeros(11, 11)
•     u[1:121] = A[S_ind, :] \ Q[S_ind]
• end
```

[-0.017667, -0.0204857, -0.0109956, -0.0174773, -0.0175655, -0.0197655, -0.0118037, -0.005

```
• U[I_ind] = u[I_ind]
```

	X=-0.2	X=0.0	X=0.2	X=0.4	X=0.6	X=0.8	X=1.0
1	Inf	1.3	Inf	Inf	Inf	Inf	Inf
2	-0.0272493	-0.0245065	-0.0273693	1.096	1.156	Inf	Inf
3	-0.0622965	-0.0768238	-0.0739077	-0.0242773	-0.0241233	1.044	Inf
4	-0.0442	-0.0639858	-0.0655791	-0.0274289	-0.0222745	0.904	Inf
5	0.0135432	-0.0043052	-0.00793564	-0.0207651	-0.00530893	-0.0109956	Inf
6	0.0186995	0.0234151	0.0186995	0.00719243	-0.0118037	-0.0204857	0.9
7	-0.00793564	-0.0043052	0.0135432	-0.00776942	-0.0197655	-0.017667	Inf
8	-0.0655791	-0.0639858	-0.0442	-0.0143432	-0.0175655	0.904	Inf
9	-0.0739077	-0.0768238	-0.0622965	-0.015818	-0.0174773	1.044	Inf
10	-0.0273693	-0.0245065	-0.0272493	1.096	1.156	Inf	Inf
11	Inf	1.3	Inf	Inf	Inf	Inf	Inf

```
• DataFrame(
•     [Y U],
•     ["Y"; "X=".*string(X)]
• )
```