Метод сеток для струны

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} - \frac{1.28}{(x + 0.6t + 1)^3}$$

$$0 \leq x \leq 1, 0 \leq t \leq 1$$

$$u(0,t) = \frac{1}{0.6t + 1}, u(1,t) = \frac{1}{0.6t + 2}$$

$$u(x,0) = \frac{1}{x + 1}, \frac{\partial u(x,0)}{\partial t} = \frac{-0.6}{(x + 1)^2}$$

a (generic function with 1 method)

- `a(x, t) = 1`

φ (generic function with 1 method)

- `φ(x, t) = -1.28 / (x + 0.6t + 1)^3`

γ₀ (generic function with 1 method)

- `γ₀(t) = 1 / (0.6t + 1)`

γ₁ (generic function with 1 method)

- `γ₁(t) = 1 / (0.6t + 2)`

α (generic function with 1 method)

- `α(x) = 1 / (x + 1)`

α₂ (generic function with 1 method)

- `α₂(x) = 2 / (x + 1)^3`

β (generic function with 1 method)

- `β(x) = -0.6 / (x + 1)^2`

Решение

```
h = 0.1
```

```
τ = 0.05
```

```
X =    [0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0]
```

```
T =
   [0.0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3, 0.35, 0.4, 0.45, 0.5, 0.55, 0.6, 0.65, 0.7, 0.75, 0.8
```

```
(11, 21)
```
- `M, N = length(X), length(T)`

```
U = 21×11 Matrix{Float64}:
    0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
    0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
    0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
    0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
    0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
    0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
    0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
     ⋮                             ⋮                             ⋮
    0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
    0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
    0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
    0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
    0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
    0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0
```
- `U = zeros(N, M)`

```
21×11 Matrix{Float64}:
 1.0        0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.5
 0.970874   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.492611
 0.943396   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.485437
 0.917431   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.478469
 0.892857   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.471698
 0.869565   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.465116
 0.847458   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.458716
 ⋮                                   ⋮                             ⋮
 0.689655   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.408163
 0.675676   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.403226
 0.662252   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.398406
 0.649351   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.393701
 0.636943   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.389105
 0.625      0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.0   0.384615
```
- `begin`
-     `U[:, 1] = γ₀.(T)`
-     `U[:, M] = γ₁.(T)`
-
-     `U`
- `end`

```
21×11 Matrix{Float64}:
 1.0       0.909091  0.833333  0.769231  …  0.588235  0.555556  0.526316  0.5
 0.970874  0.0       0.0       0.0          0.0       0.0       0.0       0.492611
 0.943396  0.0       0.0       0.0          0.0       0.0       0.0       0.485437
 0.917431  0.0       0.0       0.0          0.0       0.0       0.0       0.478469
 0.892857  0.0       0.0       0.0          0.0       0.0       0.0       0.471698
 0.869565  0.0       0.0       0.0          0.0       0.0       0.0       0.465116
 0.847458  0.0       0.0       0.0       …  0.0       0.0       0.0       0.458716
 ⋮                                       ⋱                                ⋮
 0.689655  0.0       0.0       0.0       …  0.0       0.0       0.0       0.408163
 0.675676  0.0       0.0       0.0          0.0       0.0       0.0       0.403226
 0.662252  0.0       0.0       0.0          0.0       0.0       0.0       0.398406
 0.649351  0.0       0.0       0.0          0.0       0.0       0.0       0.393701
 0.636943  0.0       0.0       0.0          0.0       0.0       0.0       0.389105
 0.625     0.0       0.0       0.0       …  0.0       0.0       0.0       0.384615
```

```julia
begin
    U[1, 2:(M-1)] = α.(X[2:(M-1)])
    U
end
```

```
21×11 Matrix{Float64}:
 1.0       0.909091  0.833333  0.769231  …  0.588235  0.555556  0.526316  0.5
 0.970874  0.884298  0.8125    0.751479     0.577855  0.546296  0.518006  0.492611
 0.943396  0.0       0.0       0.0          0.0       0.0       0.0       0.485437
 0.917431  0.0       0.0       0.0          0.0       0.0       0.0       0.478469
 0.892857  0.0       0.0       0.0          0.0       0.0       0.0       0.471698
 0.869565  0.0       0.0       0.0       …  0.0       0.0       0.0       0.465116
 0.847458  0.0       0.0       0.0          0.0       0.0       0.0       0.458716
 ⋮                                       ⋱                                ⋮
 0.689655  0.0       0.0       0.0       …  0.0       0.0       0.0       0.408163
 0.675676  0.0       0.0       0.0          0.0       0.0       0.0       0.403226
 0.662252  0.0       0.0       0.0          0.0       0.0       0.0       0.398406
 0.649351  0.0       0.0       0.0          0.0       0.0       0.0       0.393701
 0.636943  0.0       0.0       0.0          0.0       0.0       0.0       0.389105
 0.625     0.0       0.0       0.0       …  0.0       0.0       0.0       0.384615
```

```julia
begin
    U[2, 2:(M-1)] = α.(X[2:(M-1)]) +
        τ*β.(X[2:(M-1)])
        (τ^2)/2 * (a.(X[2:(M-1)], 0) .* α₂.(X[2:(M-1)]) + φ.(X[2:(M-1)], 0))
    U
end
```

s = 0.25

```julia
s = τ^2 / h^2
```

```julia
for m in 2:(M-1), n in 2:(N-1)
    U[n+1, m] = s * U[n, m+1] + 2*(1-s)*U[n, m] + s*U[n, m-1] + τ^2 * φ(X[m], T[n])
end
```

```
21×11 Matrix{Float64}:
 1.0        0.909091    0.833333   …    0.555556    0.526316  0.5
 0.970874   0.884298    0.8125          0.546296    0.518006  0.492611
 0.943396   1.77007     1.62597         1.09289     1.03629   0.485437
 0.917431   2.88891     2.87988         1.92785     1.94859   0.478469
 0.892857   4.56082     5.04056         3.40136     3.52406   0.471698
 0.869565   7.06268     8.69965         6.00195     6.25397   0.465116
 0.847458  10.8098     14.8138          10.5921     10.9973   0.458716
 ⋮                                 ⋱                          ⋮
 0.689655  430.68     1208.7      …   1765.68     1709.31     0.408163
 0.675676  646.192    1920.71         3118.51     3005.49     0.403226
 0.662252  969.456    3042.62         5507.85     5287.96     0.398406
 0.649351 1454.35     4806.29         9727.51     9309.01     0.393701
 0.636943 2181.69     7573.03        17178.3     16395.5      0.389105
 0.625    3272.69    11905.0      …  30331.1     28887.9      0.384615
```

- U

	t	x=0.0	x=0.1	x=0.2	x=0.3	x=0.4	x=0.5	x=0.6	x=
1	0.0	1.0	0.909091	0.833333	0.769231	0.714286	0.666667	0.625	0.58
2	0.05	0.970874	0.884298	0.8125	0.751479	0.69898	0.653333	0.613281	0.57
3	0.1	0.943396	1.77007	1.62597	1.50373	1.39858	1.30717	1.22698	1.15
4	0.15	0.917431	2.88891	2.87988	2.66081	2.47277	2.30956	2.16656	2.04
5	0.2	0.892857	4.56082	5.04056	4.71	4.37339	4.08174	3.82657	3.60
6	0.25	0.869565	7.06268	8.69965	8.32402	7.73668	7.2152	6.75966	6.35
7	0.3	0.847458	10.8098	14.8138	14.6599	13.6852	12.7563	11.9427	11.2
8	0.35	0.826446	16.425	24.922	25.6923	24.1919	22.555	21.1025	19.8
9	0.4	0.806452	24.8427	41.4881	44.768	42.7102	39.8798	37.292	35.0
10	0.45	0.787402	37.4643	68.4417	77.5232	75.2565	70.4967	65.9075	61.8
	more								
21	1.0	0.625	3272.69	11905.0	22537.9	30333.7	33626.4	33647.6	3218