INDEFINITE INTEGRATION

If $\int \frac{d\theta}{\cos^2\theta\,(\tan 2\theta + \sec 2\theta)} = \lambda \tan\theta + 2\log_e |f(\theta)| + C$

where C is a constant of integration, then the ordered pair (λ, f(θ)) is equal to:

A (1, 1 + tanθ)

B (–1, 1 – tanθ)

C (–1, 1 + tanθ)

D (1, 1 – tanθ)

Solution

$$\int \frac{\sec^2\theta}{\dfrac{1+\tan^2\theta}{1-\tan^2\theta}+\dfrac{2\tan\theta}{1-\tan^2\theta}}\,d\theta$$

$$= \int \frac{\sec^2\theta\left(1-\tan^2\theta\right)}{(1+\tan\theta)^2}\,d\theta$$

$$= \int \frac{\sec^2\theta(1-\tan\theta)}{1+\tan\theta}\,d\theta$$

$\tan\theta = t \Rightarrow \sec^2\theta\, d\theta = dt$

$$= \int\left(\frac{1-t}{1+t}\right)dt = \int\left(-1+\frac{2}{1+t}\right)dt$$

$= -t + 2\log(1+t) + C$

$= -\tan\theta + 2\log(1+\tan\theta) + C$

$\Rightarrow \qquad \lambda = -1$ and $f(\theta) = 1 + \tan\theta$

If f'(x) = $\tan^{-1}$(secx + tanx), $-\frac{\pi}{2} < x < \frac{\pi}{2}$ and f(0) = 0, then f(1) is equal to :

A ¼

B $\frac{\pi + 2}{4}$

C $\frac{\pi - 1}{4}$

D $\frac{\pi + 1}{4}$

Let's Solve

If f'(x) = $\tan^{-1}$(secx + tanx), $-\frac{\pi}{2} < x < \frac{\pi}{2}$ and f(0) = 0, then f(1) is equal to :

A ¼

B $\frac{\pi + 2}{4}$

C $\frac{\pi - 1}{4}$

D $\frac{\pi + 1}{4}$

Let's Solve

If $f'(x) = \tan^{-1}(\sec x + \tan x)$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$ and $f(0) = 0$, then $f(1)$ is equal to :

SOLUTION:

$$f'(x) = \tan^{-1}(\sec x + \tan x) = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right) = \tan^{-1}\left(\frac{1-\cos\left(\frac{\pi}{2}+x\right)}{\sin\left(\frac{\pi}{2}+x\right)}\right) = \tan^{-1}\left(\frac{2\sin^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\sin\left(\frac{\pi}{4}+\frac{x}{2}\right)\cos\left(\frac{\pi}{4}+\frac{x}{2}\right)}\right)$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right) = \frac{\pi}{4}+\frac{x}{2}$$

$$(f'(x))dx = \frac{\pi}{4}+\frac{x}{2}dx$$

$$f(x) = \frac{\pi}{4}x + \frac{x^2}{4} + c$$

$$f(0) = c = 0 \Rightarrow \quad f(x) = \frac{\pi}{4}x + \frac{x^2}{4}$$

$$\text{So } f(1) = \frac{\pi+1}{4}$$

The integral $\int \frac{dx}{(x+4)^{8/7}(x-3)^{6/7}}$ is equal to :

A $\left(\frac{x-3}{x+4}\right)^{1/7}+C$

B $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{-13/7}+C$

C $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{3/7}+C$

D $-\left(\frac{x-3}{x+4}\right)^{-1/7}+C$

The integral $\int \frac{dx}{(x+4)^{8/7}(x-3)^{6/7}}$ is equal to :

A $\left(\frac{x-3}{x+4}\right)^{1/7} + C$

B $-\frac{1}{13}\left(\frac{x-3}{x+4}\right)^{-13/7} + C$

C $\frac{1}{2}\left(\frac{x-3}{x+4}\right)^{3/7} + C$

D $-\left(\frac{x-3}{x+4}\right)^{-1/7} + C$

The integral $\int \frac{dx}{(x+4)^{8/7}(x-3)^{6/7}}$ is equal to :

SOLUTION:

$$\int\left(\frac{x-3}{x+4}\right)^{\frac{-6}{7}}\frac{1}{(x+4)^2}dx$$

Let $\frac{x-3}{x+4}=t^7$,

$$\frac{7}{(x+4)^2}dx=7t^6dt$$

$$\int t^{-6}t^6dt=t+c$$

Let's Solve

If $\int \frac{\cos x \; dx}{\sin^3 x\left(1+\sin^6 x\right)^{2/3}} = f(x)\left(1+\sin^6 x\right)^{1/\lambda}+c$ where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to :

A 2

B -9/8

C 9/8

D -2

Let's Solve

If $\int \frac{\cos x \ dx}{\sin^3 x\left(1+\sin^6 x\right)^{2/3}} = f(x)\left(1+\sin^6 x\right)^{1/\lambda} + c$ where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to :

A 2

B -9/8

C 9/8

D -2

If $\int \frac{\cos x\ dx}{\sin^3 x\left(1+\sin^6 x\right)^{2/3}} = f(x)\left(1+\sin^6 x\right)^{1/\lambda} + c$ where c is a constant of integration, then $\lambda f\left(\frac{\pi}{3}\right)$ is equal to :

SOLUTION:

$\sin x = t$

$\cos x\ dx = dx$

$$I = \int \frac{dt}{t^3\left(1+t^6\right)^{\frac{2}{3}}} = \int \frac{dt}{t^7\left(1+\frac{1}{t^6}\right)^{\frac{2}{3}}}$$

$$\text{Put } 1+\frac{1}{t^6} = r^3 \Rightarrow \frac{dt}{t^7} = \frac{-1}{2} r^2 dr$$

$$-\frac{1}{2}\int \frac{r^2\ dr}{r^2} = -\frac{1}{2}\ r + c = -\frac{1}{2}\left(\frac{\sin^6 x+1}{\sin^6 x}\right)^{\frac{1}{3}} + c = -\frac{1}{2\sin^2 x}\left(1+\sin^6 x\right)^{\frac{1}{3}} + c$$

$f(x) = -\frac{1}{2}\operatorname{cosec}^2 x$ and $\lambda = 3$

$$\lambda f\left(\frac{\pi}{3}\right) = -2$$

Let $\alpha \in \left(0, \frac{\pi}{2}\right)$ be fixed. If the integral

$\int \frac{\tan x + \tan \alpha}{\tan x - \tan \alpha} dx = A(x)\cos 2\alpha + B(x)\sin 2\alpha + C$ where C is a constant of integration, then the functions A(x) and B(x) are respectively:

A $x+\alpha$ and $\log_e\left|\sin(x+\alpha)\right|$

B $x-\alpha$ and $\log_e\left|\sin(x-\alpha)\right|$

C $x-\alpha$ and $\log_e\left|\cos(x-\alpha)\right|$

D $x+\alpha$ and $\log_e\left|\sin(x-\alpha)\right|$

Let $\alpha \in \left(0, \frac{\pi}{2}\right)$ be fixed. If the integral

$\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha} dx = A(x)\cos 2\alpha + B(x)\sin 2\alpha + C$ where C is a constant of integration, then the functions A(x) and B(x) are respectively:

A $x+\alpha$ and $\log_e\left|\sin(x+\alpha)\right|$

B $x-\alpha$ and $\log_e\left|\sin(x-\alpha)\right|$

C $x-\alpha$ and $\log_e\left|\cos(x-\alpha)\right|$

D $x+\alpha$ and $\log_e\left|\sin(x-\alpha)\right|$

Let $\alpha \in \left(0, \frac{\pi}{2}\right)$ be fixed. If the integral

$\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha} dx = A(x)\cos 2\alpha + B(x)\sin 2\alpha + C$ where C is a constant of integration, then the functions A(x) and B(x) are respectively:

Solution

Given integral $\int \frac{\tan x + \tan\alpha}{\tan x - \tan\alpha} dx = \int \frac{\sin(x+\alpha)}{\sin(x-\alpha)} dx$

Let $x - \alpha = t \Rightarrow dx = dt$

$$\int \frac{\sin(t+2\alpha)}{\sin t} dt = \int [\cos 2\alpha + \sin 2\alpha.\cot t]\, dt$$

$$= t\cos 2\alpha + \sin 2\alpha.\log|\sin t| + c$$

$$= (x-\alpha).\cos 2\alpha + \sin 2\alpha.\log|\sin(x-\alpha)| + c$$

The integral $\int \frac{2x^3 - 1}{x^4 + x} dx$ is equal to : (Here C is a constant of integration)

A $\frac{1}{2}\log_e \frac{|x^3+1|}{x^2} + C$

B $\frac{1}{2}\log_e \frac{(x^3+1)^2}{|x^3|} + C$

C $\log_e \left|\frac{x^3+1}{x}\right| + C$

D $\log_e \frac{|x^3+1|}{x^2} + C$

The integral $\int \frac{2x^3 - 1}{x^4 + x} dx$ is equal to : (Here C is a constant of integration)

A $\frac{1}{2}\log_e \frac{|x^3+1|}{x^2} + C$

B $\frac{1}{2}\log_e \frac{(x^3+1)^2}{|x^3|} + C$

C $\log_e \left|\frac{x^3+1}{x}\right| + C$

D $\log_e \frac{|x^3+1|}{x^2} + C$

The integral $\int \frac{2x^3 - 1}{x^4 + x}\, dx$ is equal to : (Here C is a constant of integration)

Solution

Given integral, $I = \int \frac{(2x^3 - 1)dx}{x^4 + x} = \int \frac{(2x - x^{-2})dx}{x^2 + x^{-1}}$

Put $x^2 + x^{-1} = u \Rightarrow (2x - x^{-2})dx = du$

$\Rightarrow I = \int \frac{du}{u} = \log|u| + c = \log\left|x^2 + x^{-1}\right| + c$

$= \log\left|\frac{x^3 + 1}{x}\right| + c$

If $\int x^5 e^{-x^2} dx = g(x) e^{-x^2} + c$, where c is a constant of integration, then $g(-1)$ is equal to:

A -1

B 1

C $-\frac{5}{2}$

D $-\frac{1}{2}$

If $\int x^5 e^{-x^2} dx = g(x) e^{-x^2} + c$, where c is a constant of integration, then $g(-1)$ is equal to:

A -1

B 1

C $-\frac{5}{2}$

D $-\frac{1}{2}$

Let's Solve

10-04-2019 - Evening Shift

If $\int x^5 e^{-x^2} dx = g(x)e^{-x^2} + c$, where c is a constant of integration, then $g(-1)$ is equal to:

$x \cdot x^4$

Solution

Let, $I = \int x^5 . e^{-x^2} dx$

Put $-x^2 = t \Rightarrow -2x\,dx = dt$

$$I = \int \frac{t^2 . e^t dt}{(-2)} = \frac{-1}{2} e^t \left(t^2 - 2t + 2\right) + c$$

$$\therefore g(x) = \frac{-1}{2}\left(x^4 + 2x^2 + 2\right)$$

$$\Rightarrow g(-1) = \frac{-5}{2}$$

$\int t^2 e^t dt$

$= t^2 e^t - \int [2t e^t] dt$

$t^2 e^t - 2 \int t e^t dt$

$t^2 e^t - 2 [t \cdot e^t - e^t]$

$e^t [t^2 - 2t + 2]$

Vedantu

If $\int e^{\sec x}\left(\sec x \tan x\, f(x)+\left(\sec x \tan x+\sec^{2} x\right)\right) dx=e^{\sec x} f(x)+C$, then a possible choice of $f(x)$ is:

A $\sec x+\tan x+C$

B $\sec x-\tan x-C$

C $\sec x+\tan x-C$

D $x \sec x+\tan x+C$

If $\int e^{\sec x}\left(\sec x \tan x\, f(x)+\left(\sec x \tan x+\sec^{2} x\right)\right) dx=e^{\sec x} f(x)+C$, then a possible choice of $f(x)$ is:

A $\sec x+\tan x+C$

B $\sec x-\tan x-C$

C $\sec x+\tan x-C$

D $x \sec x+\tan x+C$

If $\int e^{\sec x}\left(\sec x \tan x\, f(x)+\left(\sec x \tan x+\sec^2 x\right)\right)dx = e^{\sec x} f(x)+C$, then a possible choice of $f(x)$ is:

Solution

Given $\int e^{\sec x}\left(\sec x \tan x\, f(x)+\left(\sec x \tan x+\sec^2 x\right)\right)dx$

$$= e^{\sec x} f(x)+C \quad .....(1)$$

$$\because \int e^{g(x)}\left(\left(g'(x) f(x)\right)+f'(x)\right)dx = e^{g(x)} \times f(x)+C$$

On comparing above equation by equation (1)

$$f(x)=\int\left(\left(\sec x \tan x\right)+\sec^2 x\right)dx$$

$$\therefore f(x)=\sec x+\tan x+C$$

The integral $\int \sec^{\frac{2}{3}} x \operatorname{cosec}^{\frac{4}{3}} x \, dx$ is equal to:

A $-3\tan^{\frac{-1}{3}} x + C$

B $-\frac{3}{4}\tan^{\frac{-4}{3}} x + C$

C $-3\cot^{\frac{-1}{3}} x + C$

D $3\tan^{\frac{-1}{3}} x + C$

The integral $\int \sec^{\frac{2}{3}} x \operatorname{cosec}^{\frac{4}{3}} x \, dx$ is equal to:

A $-3\tan^{\frac{-1}{3}} x + C$

B $-\frac{3}{4}\tan^{\frac{-4}{3}} x + C$

C $-3\cot^{\frac{-1}{3}} x + C$

D $3\tan^{\frac{-1}{3}} x + C$

The integral $\int \sec^{\frac{2}{3}} x \operatorname{cosec}^{\frac{4}{3}} x \, dx$ is equal to:

Solution

$$I = \int \sec^{\frac{2}{3}} x . \operatorname{cosec}^{\frac{4}{3}} x \, dx$$

$$I = \int \frac{\sec^2 x dx}{\tan^{\frac{4}{3}} x}$$

Put tanx = z

$$\Rightarrow \sec^2 x dx = dz$$

$$\Rightarrow I = \int z^{-\frac{4}{3}} . \mathrm{dz} = \frac{z^{\frac{-1}{3}}}{\left(\frac{-1}{3}\right)} + C \Rightarrow I = -3(\tan x)^{\frac{-1}{3}} + C$$

If $\int \frac{dx}{x^3\left(1+x^6\right)^{2/3}} = xf(x)\left(1+x^6\right)^{\frac{1}{3}} + C$, where C is a constant of integration, then the function f(x) is equal to :

A $\frac{3}{x^2}$

B $-\frac{1}{6x^3}$

C $-\frac{1}{2x^2}$

D $-\frac{1}{2x^3}$

If $\int \frac{dx}{x^3\left(1+x^6\right)^{2/3}} = xf(x)\left(1+x^6\right)^{\frac{1}{3}} + C$, where C is a constant of integration, then the function f(x) is equal to :

A $\frac{3}{x^2}$

B $-\frac{1}{6x^3}$

C $-\frac{1}{2x^2}$

D $-\frac{1}{2x^3}$

If $\int \frac{dx}{x^3\left(1+x^6\right)^{2/3}} = xf(x)\left(1+x^6\right)^{\frac{1}{3}} + C$, where C is a constant of integration, then the function f(x) is equal to :

Solution

Let, $\int \frac{dx}{x^3\left(1+x^6\right)^{\frac{2}{3}}} = \int \frac{dx}{x^7\left(1+x^{-6}\right)^{\frac{2}{3}}}$

Put $1+x^{-6} = t^3$

$\Rightarrow -6x^{-7}\,dx = 3t^2 dt$

$\Rightarrow \frac{dx}{x^7} = \left(-\frac{1}{2}\right)t^2\,dt$

Now, $I = \int\left(-\frac{1}{2}\right)\frac{t^2 dt}{t^2} = -\frac{1}{2}t + C$

$= -\frac{1}{2}\left(1+x^{-6}\right)^{\frac{1}{3}} + C = -\frac{1}{2}\frac{\left(1+x^6\right)^{\frac{1}{3}}}{x^2} + C$

$= -\frac{1}{2x^3}x\left(1+x^6\right)^{\frac{1}{3}} + C$

Hence $f(x) = -\frac{1}{2x^3}$

$\int \frac{\sin \frac{5x}{2}}{\sin \frac{x}{2}} dx$ is equal to :
(where c is a constant of integration.)

A $2x + \sin x + 2 \sin 2x + c$

B $x + 2 \sin x + 2 \sin 2x + c$

C $x + 2 \sin x + \sin 2x + c$

D $2x + \sin x + \sin 2x + c$

$\int \frac{\sin \frac{5x}{2}}{\sin \frac{x}{2}} dx$ is equal to :

(where c is a constant of integration.)

A $2x + \sin x + 2 \sin 2x + c$

B $x + 2 \sin x + 2 \sin 2x + c$

C $x + 2 \sin x + \sin 2x + c$

D $2x + \sin x + \sin 2x + c$

$\int \frac{\sin \frac{5x}{2}}{\sin \frac{x}{2}} dx$ is equal to :
(where c is a constant of integration.)

Solution

$$\int \frac{\sin\left(\frac{5x}{2}\right)}{\sin\left(\frac{x}{2}\right)} dx = \int \frac{2\cos\frac{x}{2}.\sin\frac{5x}{2}}{2\cos\frac{x}{2}.\sin\frac{x}{2}} dx$$

$$= \int \frac{\sin 3x + \sin 2x}{\sin x} dx$$

$$= \int \left(3 - 4\sin^2 x + 2\cos x\right) dx$$

$$\left[\because \sin 2x = 2\sin x \cos x \text{ and } \sin 3x = 3\sin x - 4\sin^3 x\right]$$

$$= \int \left(3 - 2\left(1 - \cos 2x\right) + 2\cos x\right) dx$$

$$= \int \left(1 + 2\cos x + 2\cos 2x\right) dx$$

$$= x + 2\sin x + \sin 2x + c$$

The integral $\int \frac{3x^{13}+2x^{11}}{\left(2x^4+3x^2+1\right)^4}dx$ is equal to

$\boxed{\rightarrow x^4}$ X

$\rightarrow x^2$

A $\frac{x^4}{6\left(2x^4+3x^2+1\right)^3}+C$

B $\frac{x^{12}}{6\left(2x^4+3x^2+1\right)^3}+C$

C $\frac{x^4}{\left(2x^4+3x^2+1\right)^3}+C$

D $\frac{x^{12}}{\left(2x^4+3x^2+1\right)^3}+C$

The integral $\int \frac{3x^{13}+2x^{11}}{\left(2x^4+3x^2+1\right)^4}dx$ is equal to

A $\frac{x^4}{6\left(2x^4+3x^2+1\right)^3}+C$

B $\frac{x^{12}}{6\left(2x^4+3x^2+1\right)^3}+C$

C $\frac{x^4}{\left(2x^4+3x^2+1\right)^3}+C$

D $\frac{x^{12}}{\left(2x^4+3x^2+1\right)^3}+C$

Let's Solve

The integral $\int \frac{3x^{13}+2x^{11}}{\left(2x^4+3x^2+1\right)^4}dx$ is equal to

Solution

$$I=\int \frac{3x^{13}+2x^{11}}{\left(2x^4+3x^2+1\right)^4}dx=\int \frac{3x^{13}+2x^{11}}{x^{16}\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^4}dx$$

$$I=\int \frac{\frac{3}{x^3}+\frac{2}{x^5}}{\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^4}dx$$

Let $2+\frac{3}{x^2}+\frac{1}{x^4}=t,\ -2\left(\frac{3}{x^3}+\frac{2}{x^5}\right)dx=dt$

Then, $I=\int \frac{-\frac{dt}{2}}{t^4}=-\frac{1}{2}\frac{t^{-4+1}}{-4+1}+C$

$$I=\frac{-1}{2}\times\frac{1}{(-3)}\frac{1}{\left(2+\frac{3}{x^2}+\frac{1}{x^4}\right)^3}+C$$

$$I=\frac{1}{6}\frac{x^{12}}{\left(2x^4+3x^2+1\right)^3}+C$$

Let's Solve

The integral $\int \cos(\log_e x)\,dx$ is equal to

A $\frac{x}{2}\left[\sin(\log_e x) - \cos(\log_e x)\right] + C$

B $x\left[\cos(\log_e x) + \sin(\log_e x)\right] + C$

C $\frac{x}{2}\left[\cos(\log_e x) + \sin(\log_e x)\right] + C$

D $x\left[\cos(\log_e x) - \sin(\log_e x)\right] + C$

$\ln x = t \Rightarrow x = e^t$ $\quad I = \int e^t \cos(t)\, dt$

$\frac{1}{x}\,dx = dt$

$I = (\cos t)\, e^t + \int [\sin t\; e^t]\, dt$

$dx = x\, dt$

$(\sin t)\, e^t - \int [e^t \cos t]\, dt$

$dx = e^t\, dt$

The integral $\int \cos(\log_e x)dx$ is equal to

A $\frac{x}{2}\left[\sin(\log_e x) - \cos(\log_e x)\right] + C$

B $x\left[\cos(\log_e x) + \sin(\log_e x)\right] + C$

C $\frac{x}{2}\left[\cos(\log_e x) + \sin(\log_e x)\right] + C$

D $x\left[\cos(\log_e x) - \sin(\log_e x)\right] + C$

The integral $\int \cos(\log_e x)dx$ is equal to

Solution

Let the integral, $I = \int \cos(\ln x)dx$

$$\Rightarrow I = \cos(\ln x)x - \int \frac{-\sin(\ln x)}{x}.x.dx$$

$$= x\cos(\ln x) + \int \sin(\ln x)dx$$

$$= x\cos(\ln x) + \sin(\ln x).x - I$$

$$\Rightarrow 2I = x(\cos(\ln x) + \sin(\ln x)) + C$$

$$\Rightarrow I = \frac{x}{2}[\cos(\ln x) + \sin(\ln x)] + C$$

If $\int \frac{x+1}{\sqrt{2x-1}}dx = f(x)\sqrt{2x-1}+c$, where C is a constant of integration, then f(x) is equal to :

A $\frac{1}{3}(x+1)$ **B** $\frac{2}{3}(x+2)$ **C** $\frac{2}{3}(x-4)$ **D** $\frac{1}{3}(x+4)$

$2x-1=t^2 \Rightarrow x=\frac{t^2+1}{2}$

$\cancel{2}dx=\cancel{2}t\,dt$

$=\int \frac{\left(\frac{t^2+1}{2}\right)+1}{\cancel{t}}\cancel{t}\,dt = \int \frac{t^2+3}{2}dt$

$=\frac{1}{2}\left[\frac{t^3}{3}+3t\right]$

$=\frac{1}{6}\left[t^3+9t\right]$

$\frac{1}{6}t\left[t^2+9\right]$

$\frac{1}{6}\sqrt{2x-1}\,[2x-1+9]$

$\frac{1}{6}\sqrt{2x-1}\,[x+4]$

$\frac{1}{3}(x+4)$

If $\int \frac{x+1}{\sqrt{2x-1}}dx = f(x)\sqrt{2x-1}+c,$ where C is a constant of integration, then f(x) is equal to :

A $\frac{1}{3}(x+1)$

B $\frac{2}{3}(x+2)$

C $\frac{2}{3}(x-4)$

D $\frac{1}{3}(x+4)$

If $\int \frac{x+1}{\sqrt{2x-1}} dx = f(x)\sqrt{2x-1} + c,$ where C is a constant of integration, then f(x) is equal to :

Solution

Let,

$$I = \int \frac{x+1}{\sqrt{2x-1}} dx$$

Put $\sqrt{2x-1} = t$

$2x-1 = t^2 \Rightarrow dx = t\,dt$

$$I = \int \frac{(t^2+3)}{2} dt = \frac{t^3}{6} + \frac{3t}{2} + C = \frac{(2x-1)^{\frac{3}{2}}}{6} + \frac{3}{2}(2x-1)^{\frac{1}{2}} + C$$

$$= \sqrt{2x-1}\left(\frac{x+4}{3}\right) + C$$

$$= f(x).\sqrt{2x-1} + C$$

Hence, $f(x) = \frac{x+4}{3}$

Let's Solve

$A(x) = \frac{-1}{3x^3}$

$\boxed{m = 3}$

If $\int \frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$, for a suitable chosen integer m and a function A(x), where C is a constant of integration, then $\boxed{(A(x))^m}$ equals :

$\left(-\frac{1}{3x^3}\right)^3$

A $\frac{-1}{27x^9}$

B $\frac{-1}{3x^3}$

C $\frac{1}{27x^6}$

$\int \frac{x\sqrt{x^{-2}-1}}{x^4}dx = \int \frac{\sqrt{x^{-2}-1}}{x^3}dx$

$x^{-2} - 1 = t$ $\quad -\frac{1}{2}\int \sqrt{t}\,dt$

$-2\underbrace{x^{-3}dx} = dt$ $\quad -\frac{1}{2}\frac{t^{3/2}}{\frac{3}{2}} + c$

$\frac{dx}{x^3} = -\frac{1}{2}dt$ $\quad -\frac{1}{3}\left[\sqrt{x^{-2}-1}\right]^3$

$\left[\left(-\frac{1}{3}\right)\sqrt{\frac{1}{x^2}-1}\right]^3$

$\left(-\frac{1}{3x^3}\left(\sqrt{1-x^2}\right)\right)^3$

If $\int \frac{\sqrt{1-x^2}}{x^4}dx = A(x)\left(\sqrt{1-x^2}\right)^m + C$, for a suitable chosen integer m and a function A(x), where C is a constant of integration, then $(A(x))^m$ equals :

A $\frac{-1}{27x^9}$ B $\frac{-1}{3x^3}$ C $\frac{1}{27x^6}$ D $\frac{1}{9x^4}$

$$\left[\sqrt{x^{-2}-1}\right]^3$$

$$\left[\sqrt{\frac{1}{x^2}-1}\right]^3$$

$$\left[\frac{\sqrt{1-x^2}}{x}\right]^3 = \frac{1}{x^3}\left(\sqrt{1-x^2}\right)^3$$

If $\int \frac{\sqrt{1-x^2}}{x^4} dx = A(x)\left(\sqrt{1-x^2}\right)^m + C,$ for a suitable chosen integer m and a function A(x), where C is a constant of integration, then $(A(x))^m$ equals :

Solution

$$A(x)\left(\sqrt{1-x^2}\right)^m + C = \int \frac{\sqrt{1-x^2}}{x^4} dx = \int \frac{\sqrt{\frac{1}{x^2}-1}}{x^3} dx$$

Let $\frac{1}{x^2} - 1 = u^2$

$$\Rightarrow -\frac{2}{x^3} = \frac{2u\,du}{dx}$$

$$\frac{dx}{x^3} = -u\,du$$

$$A(x)\left(\sqrt{1-x^2}\right)^m + C = \int -u^2 du = -\frac{u^3}{3} + C$$

$$= -\frac{1}{3}\left(\frac{1}{x^2} - 1\right)^{\frac{3}{2}} + C$$

$$= -\frac{1}{3}\frac{1}{x^3}\left(1-x^2\right)^{\frac{3}{2}} + C$$

$$= \frac{-1}{3x^3}\left(\sqrt{1-x^2}\right)^3 + C$$

Compare both sides,

$$\Rightarrow \mathrm{A}(x) = -\frac{1}{3x^3} \text{ and } m = 3$$

$$\Rightarrow (A(x))^3 = \frac{-1}{27x^9}$$

If $\int x^5 e^{-4x^3}\,dx = \frac{1}{48} e^{-4x^3} f(x) + C$, where C is a constant of integration, then f(x) is equal to :

$-4x^3 = t$

A $-2x^3 - 1$

B $-4x^3 - 1$

C $-2x^3 + 1$

D $2x^3 + 1$

If $\int x^5 e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + C$, where C is a constant of integration, then f(x) is equal to :

A $-2x^3 - 1$

B $-4x^3 - 1$

C $-2x^3 + 1$

D $2x^3 + 1$

Let's Solve

10-01-2019 - Evening Shift

If $\int x^5 e^{-4x^3} dx = \frac{1}{48} e^{-4x^3} f(x) + C$, where C is a constant of integration, then f(x) is equal to :

Solution

$$I = \int x^5 e^{-4x^3} dx$$

Put $-4x^3 = \theta$

$$\Rightarrow -12x^2 dx = d\theta$$

$$\Rightarrow x^2 dx = -\frac{d\theta}{12}$$

$$I = \int \frac{1}{48} \theta e^{\theta} d\theta = \frac{1}{48}\left[\theta e^{\theta} - e^{\theta}\right] + C$$

$$I = \frac{1}{48} e^{-4x^3}\left(-4x^3 - 1\right) + C$$

Then, by comparison

$$f(x) = -4x^3 - 1$$

Let's Solve

(2) → 25 - 30

2 hrs → 15 - 20

Let $n \geq 2$ be a natural number and $0 < \theta < \frac{\pi}{2}$ then $\int \frac{(\sin^n \theta + \sin \theta)^{\frac{1}{n}} \cos \theta}{\sin^{n+1} \theta} d\theta$ is

$\sin\theta = t \Rightarrow \cos\theta \, d\theta = dt$

A $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

B $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

C $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$

D $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1}\theta}\right)^{\frac{n+1}{n}}+C$

$$\int \frac{(t^n + t)^{1/n}}{t^{n+1}} dt$$

$$\int \frac{t\,[1 + t^{1-n}]^{1/n}}{t^{n+1}} dt$$

$$\int \frac{[1 + t^{1-n}]^{1/n}}{t^n} dt$$

Let $n \geq 2$ be a natural number and $0 < \theta < \frac{\pi}{2}$ then $\int \frac{\left(\sin^n \theta + \sin \theta\right)^{\frac{1}{n}} \cos \theta}{\sin^{n+1} \theta} d\theta$ is

A $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$

B $\frac{n}{n^2+1}\left(1-\frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$

C $\frac{n}{n^2-1}\left(1+\frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + C$

D $\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n+1} \theta}\right)^{\frac{n+1}{n}} + C$

Let $n \geq 2$ be a natural number and $0 < \theta < \frac{\pi}{2}$ then $\int \frac{\left(\sin^n \theta + \sin \theta\right)^{\frac{1}{n}} \cos \theta}{\sin^{n+1} \theta} d\theta$ is

Solution

Let, $I = \int \frac{\left(\sin^n \theta + \sin \theta\right)^{\frac{1}{n}} \cos \theta}{\sin^{n+1} \theta} d\theta$

Let $\sin \theta = u$

$\Rightarrow \cos \theta \, d\theta = du$

$$\therefore I = \int \frac{\left(u^n - u\right)^{\frac{1}{n}}}{u^{n+1}} du$$

$$= \int \frac{\left(1 - \frac{1}{u^{n-1}}\right)^{\frac{1}{n}}}{u^n} du = \int u^{-n} \left(1 - u^{1-n}\right)^{\frac{1}{n}} du$$

JEE Mains

Solve 20 Problems

Let $1-u^{1-n}=v$

$$\Rightarrow -(1-n)u^{-n}\,du=dv$$

$$\Rightarrow u^{-n}\,du=\frac{dv}{n-1}$$

$$\therefore I=\int v^{\frac{1}{n}}\cdot\frac{dv}{n-1}=\frac{1}{n-1}\cdot\frac{v^{\frac{1}{n}+1}}{\frac{1}{n}+1}+C$$

$$=\frac{n}{n^2-1}v^{\frac{n+1}{n}}+C=\frac{n}{n^2-1}\left(1-\frac{1}{u^{n-1}}\right)^{\frac{n+1}{n}}+C$$

$$=\frac{n}{n^2-1}\left(1-\frac{1}{\sin^{n-1}\theta}\right)^{\frac{n+1}{n}}+C$$

A

25 25

30

Let's Solve

If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)} dx, \quad (x \geq 0)$, and f(0) = 0, then the value of f(1) is

A $-\frac{1}{2}$

B $-\frac{1}{4}$

C $\frac{1}{2}$

D $\frac{1}{4}$

Let's Solve

If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)} dx, \quad (x \geq 0),$ and f(0) = 0, then the value of f(1) is

A $-\frac{1}{2}$

B $-\frac{1}{4}$

C $\frac{1}{2}$

D $\frac{1}{4}$

If $f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)} dx, \quad (x \geq 0),$ and f(0) = 0, then the value of f(1) is

Solution

$$f(x) = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)} dx, (x \geq 0)$$

$$= \int \frac{5x^8 + 7x^6}{x^{14}(x^{-5} + x^{-7} + 2)^2} dx$$

$$= \int \frac{5x^{-6} + 7x^{-8}}{(2 + x^{-7} + x^{-5})^2} dx$$

Let $2 + x^{-7} + x^{-5} = t$

$$\Rightarrow (-7x^{-8} - 5x^{-6}) dx = dt$$

$$\Rightarrow f(x) = \int \frac{-dt}{t^2} = \int -t^{-2} dt = t^{-1} + c$$

$$\Rightarrow f(x) = \frac{1}{2 + x^{-7} + x^{-5}} + c, f(0) = 0 \Rightarrow c = 0$$

$$\therefore f(1) = \frac{1}{4}$$

For $x^2 \neq n\pi$, $n \in N$ (the set of natural numbers), the integral is

$$\int x\sqrt{\frac{2\sin\left(x^2-1\right)-\sin 2\left(x^2-1\right)}{2\sin\left(x^2-1\right)+\sin 2\left(x^2-1\right)}}dx$$ equal to:

A $\log_e\left|\frac{1}{2}\sec^2\left(x^2-1\right)\right|+c$

B $\frac{1}{2}\log_e\left|\sec\left(x^2-1\right)\right|+c$

C $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

D $2\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right|-c$

For $x^2 \neq n\pi$, $n \in N$ (the set of natural numbers), the integral is

$\int x\sqrt{\frac{2\sin(x^2-1)-\sin 2(x^2-1)}{2\sin(x^2-1)+\sin 2(x^2-1)}}dx$ equal to:

A $\log_e\left|\frac{1}{2}\sec^2(x^2-1)\right|+c$

B $\frac{1}{2}\log_e\left|\sec(x^2-1)\right|+c$

C $\frac{1}{2}\log_e\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$

D $2\log_e\left|\sec\left(\frac{x^2-1}{2}\right)\right|-c$

For $x^2 \neq n\pi$, n ∈ N (the set of natural numbers), the integral is

$$\int x\sqrt{\frac{2\sin\left(x^2-1\right)-\sin 2\left(x^2-1\right)}{2\sin\left(x^2-1\right)+\sin 2\left(x^2-1\right)}}dx$$ equal to:

Solution

Consider the given integral

$$I=\int x\sqrt{\frac{2\sin\left(x^2-1\right)-2\sin\left(x^2-1\right)\cos\left(x^2-1\right)}{2\sin\left(x^2-1\right)+2\sin\left(x^2-1\right)\cos\left(x^2-1\right)}}dx$$

$$\left(\because\ \sin 2\theta = 2\sin\theta\ \cos\theta\right)$$

$$\Rightarrow\quad I=\int x\sqrt{\frac{1-\cos\left(x^2-1\right)}{1+\cos\left(x^2-1\right)}}dx \quad\Rightarrow\quad I=\int x\left|\tan\left(\frac{x^2-1}{2}\right)\right|dx$$

$$\text{Now let } \frac{x^2-1}{2}=t \quad\Rightarrow\quad \frac{2x}{2}dx=dt \quad\therefore\quad I=\int|\tan(t)|\,dt=\ln|\sec t|+C$$

$$or\quad I=\ln\left|\sec\left(\frac{x^2-1}{2}\right)\right|+c=\frac{1}{2}\ln\left|\sec^2\left(\frac{x^2-1}{2}\right)\right|+c$$

The integral $\int \frac{\sin^2 x \cos^2 x}{\left(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x\right)^2} dx$ is equal to

A $\frac{-1}{1+\cot^3 x} + C$

B $\frac{1}{3\left(1+\tan^3 x\right)} + C$

C $\frac{-1}{3\left(1+\tan^3 x\right)} + C$

D $\frac{1}{1+\cot^3 x} + C$

The integral $\int \frac{\sin^2 x \cos^2 x}{\left(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x\right)^2} dx$ is equal to

A $\frac{-1}{1+\cot^3 x} + C$

B $\frac{1}{3\left(1+\tan^3 x\right)} + C$

C $\frac{-1}{3\left(1+\tan^3 x\right)} + C$

D $\frac{1}{1+\cot^3 x} + C$

The integral $\int \frac{\sin^2 x \cos^2 x}{\left(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x\right)^2} dx$ is equal to

Solution:

$$I = \int \frac{\left(\tan^2 x\right)\cdot\left(\sec x\right)^6 dx}{\left(\left(\tan x\right)^5 + \left(\tan x\right)^2 + \left(\tan x\right)^3 + 1\right)^2}$$

Let $\tan x = t$

$$I = \int \frac{t^2\left(1+t^2\right)^2 dt}{\left(t^2+1\right)^2\left(t^2+1\right)^2}$$

Now $t^3 + 1 = Z$

$$I = \int \frac{\frac{1}{3} dZ}{Z^2} \quad \Rightarrow I = -\frac{1}{3\left(1+t^3\right)} + C$$

$$I = \frac{1}{3\left(1+\tan^3 x\right)} + C$$

If $\int \frac{\tan x}{1+\tan x+\tan^2 x} dx = x - \frac{K}{\sqrt{A}} \tan^{-1}\left(\frac{K\tan x+1}{\sqrt{A}}\right) + C,$

(C is a constant of integration), then the ordered pair (K, A) is equal to :

A (2, 1)

B (-2, 3)

C (2, 3)

D (-2, 1)

Let's Solve

JEE_2018_FT-5

If $\int \frac{\tan x}{1+\tan x+\tan^2 x} dx = x - \frac{K}{\sqrt{A}} \tan^{-1}\left(\frac{K\tan x+1}{\sqrt{A}}\right) + C,$

(C is a constant of integration), then the ordered pair (K, A) is equal to :

A (2, 1)

B (-2, 3)

C (2, 3)

D (-2, 1)

If $\int \frac{\tan x}{1+\tan x+\tan^2 x} dx = x - \frac{K}{\sqrt{A}} \tan^{-1}\left(\frac{K \tan x+1}{\sqrt{A}}\right) + C,$

(C is a constant of integration), then the ordered pair (K, A) is equal to :

Solution:

Let tan x = t

$\Rightarrow$ dt = $\sec^2 x$ dx = $(1 + \tan^2 x)$ dx = $(1 + t^2)$ dx

$$\Rightarrow \quad dx = \frac{dt}{1+t^2}$$

$$\therefore \quad \int \frac{\tan x}{\tan^2 x + \tan x + 1} dx = \int \frac{t}{(1+t^2)(1+t+t^2)} dt$$

$$= \int \left(\frac{1}{1+t^2} - \frac{1}{1+t+t^2}\right) dt$$

$$= \tan^{-1} t - \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2t+1}{\sqrt{3}}\right)$$

$$= x - \frac{2}{\sqrt{3}} \tan^{-1}\left(\frac{2 \tan x + 1}{\sqrt{3}}\right)$$

Clearly then (K, A) = (2, 3).

If $\int \frac{2x+5}{\sqrt{7-6x-x^2}}dx = A\sqrt{7-6x-x^2} + B\sin^{-1}\left(\frac{x+3}{4}\right) + C$ (where C is a constant of integration), then the ordered pair (A, B) is equal to :

A (2, 1)

B (-2, -1)

C (-2, 1)

D (2,-1)

Let's Solve

If $\int \frac{2x+5}{\sqrt{7-6x-x^2}}dx = A\sqrt{7-6x-x^2} + B\sin^{-1}\left(\frac{x+3}{4}\right) + C$ (where C is a constant of integration), then the ordered pair (A, B) is equal to :

A (2, 1)

B (-2, -1)

C (-2, 1)

D (2,-1)

Let's Solve

If $\int \frac{2x+5}{\sqrt{7-6x-x^2}}dx = A\sqrt{7-6x-x^2} + B\sin^{-1}\left(\frac{x+3}{4}\right)+C$ (where C is a constant of integration), then the ordered pair (A, B) is equal to :

Solution

Note that $7 - 6x - x^2 = 16 - (x+3)^2$ and $\frac{d}{dx}\left(7-6x-x^2\right) = -2x-6$

So, we have

$$\int \frac{2x+5}{\sqrt{7-6x-x^2}}dx = \int \frac{2x+6}{\sqrt{7-6x-x^2}}dx - \int \frac{1}{\sqrt{16-(x-3)^2}}dx$$

$$= -2\sqrt{7-6x-x^2} - \sin^{-1}\left(\frac{x+3}{4}\right)+C$$

So, we have A =- 2, B =- 1

If $f\left(\frac{x-4}{x+2}\right) = 2x+1, (x \in R - \{1, -2\})$, then $\int f(x)\,dx$ is equal to

A $12\ \log_e|1-x|+3\,x+C$

B $-12\ \log_e|1-x|-3x+C$

C $12\log_e|1-x|-3x+C$

D $-12\log_e|1-x|+3x+C$

If $f\left(\frac{x-4}{x+2}\right) = 2x + 1, (x \in R - \{1, -2\})$, then $\int f(x)\, dx$ is equal to

A $12 \log_e |1-x| + 3x + C$

B $-12 \log_e |1-x| - 3x + C$

C $12 \log_e |1-x| - 3x + C$

D $-12 \log_e |1-x| + 3x + C$

Let's Solve

If $f\left(\frac{x-4}{x+2}\right) = 2x+1, (x \in R - \{1, -2\})$, then $\int f(x)\, dx$ is equal to

SOLUTION:

$$\frac{x-4}{x+2} = y \Rightarrow \quad x-4 = yx+2y \Rightarrow x(1-y) = 2y+4 \Rightarrow x = \frac{2y+4}{1-y}$$

This gives us $f(y) = 2\left(\frac{2y+4}{1-y}\right)+1$

So, we have $f(x) = 2\left(\frac{2x+4}{1-x}\right)+1 = \frac{3x+9}{1-x} = -3\left(\frac{x-1+4}{x-1}\right) = -3 - \frac{12}{x-1}$

Thus $\int f(x)dx = -12\log_e|1-x| - 3x + c$