

Dato: pattino in A con molla long. // al pattino.

Svolgere l'analisi cinematica.

Risolvere con PLV e LE.

Tracciare la deformata elastica.

Riportare la soluzione su questo foglio.

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi delle azioni interne nelle aste.

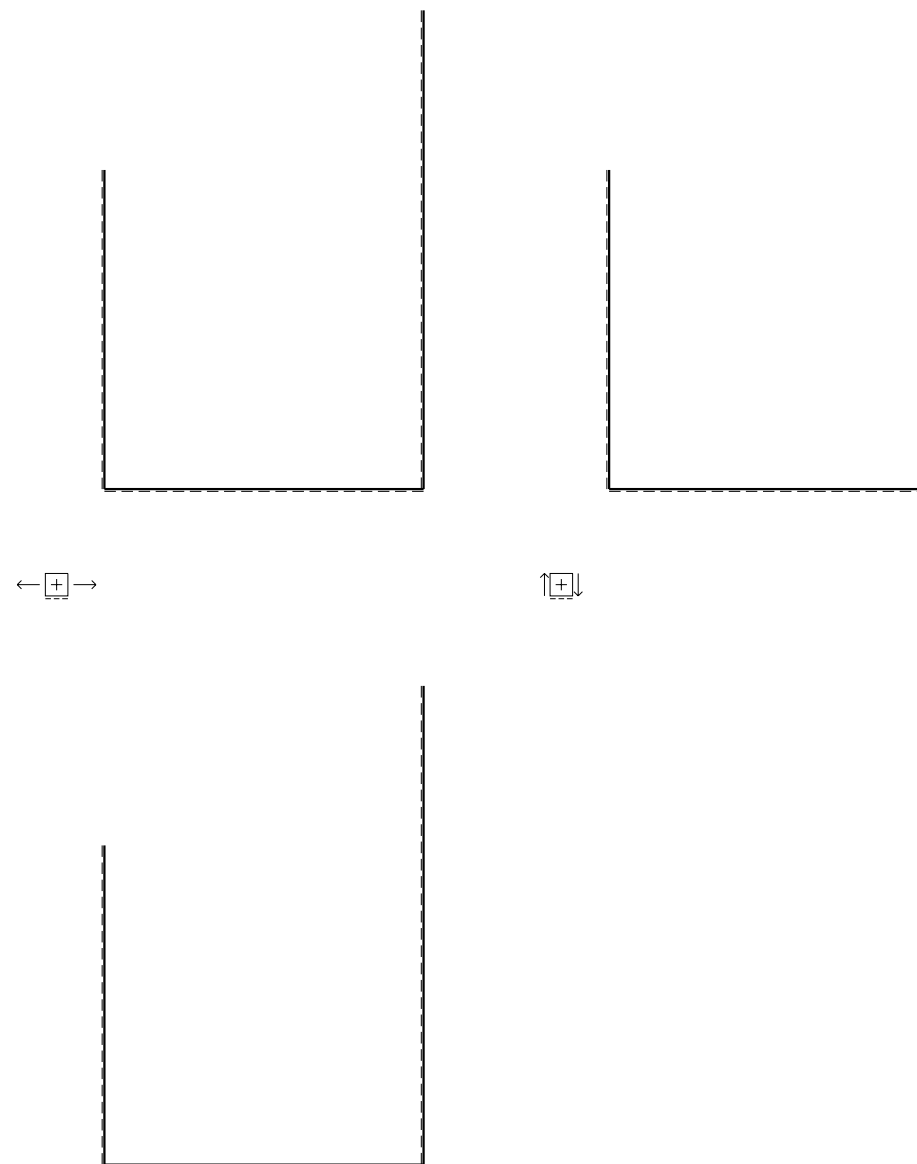
Esprimere la linea elastica delle aste.

Calcolare spostamento e rotazione di tutti i nodi.

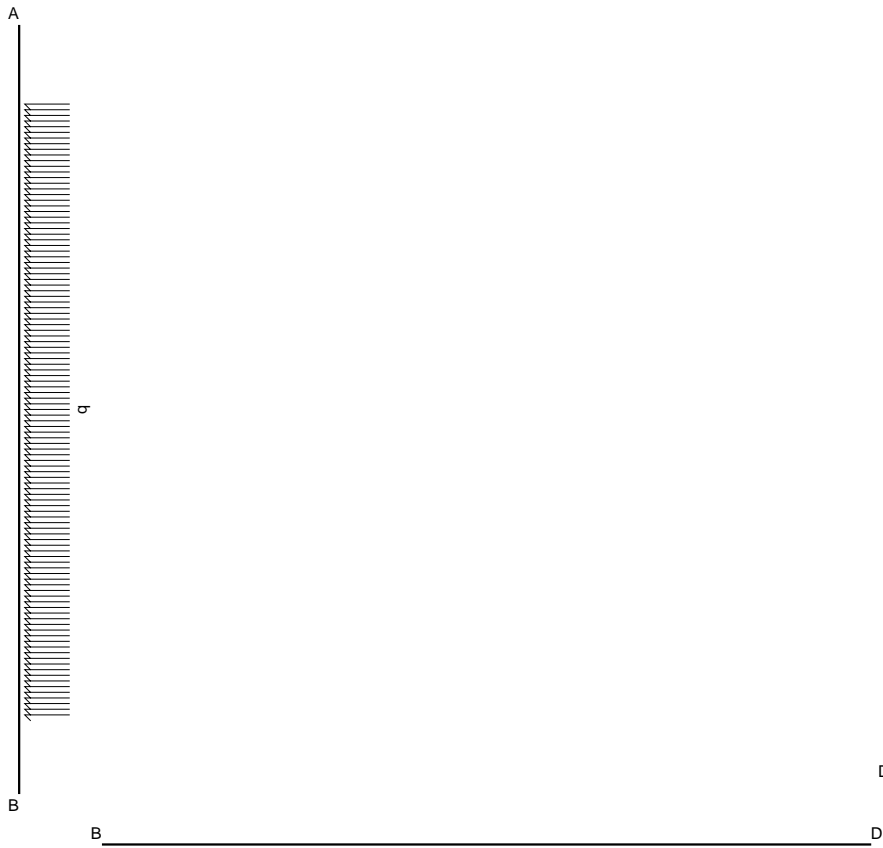
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta CD positiva se convessa a destra con inizio C.

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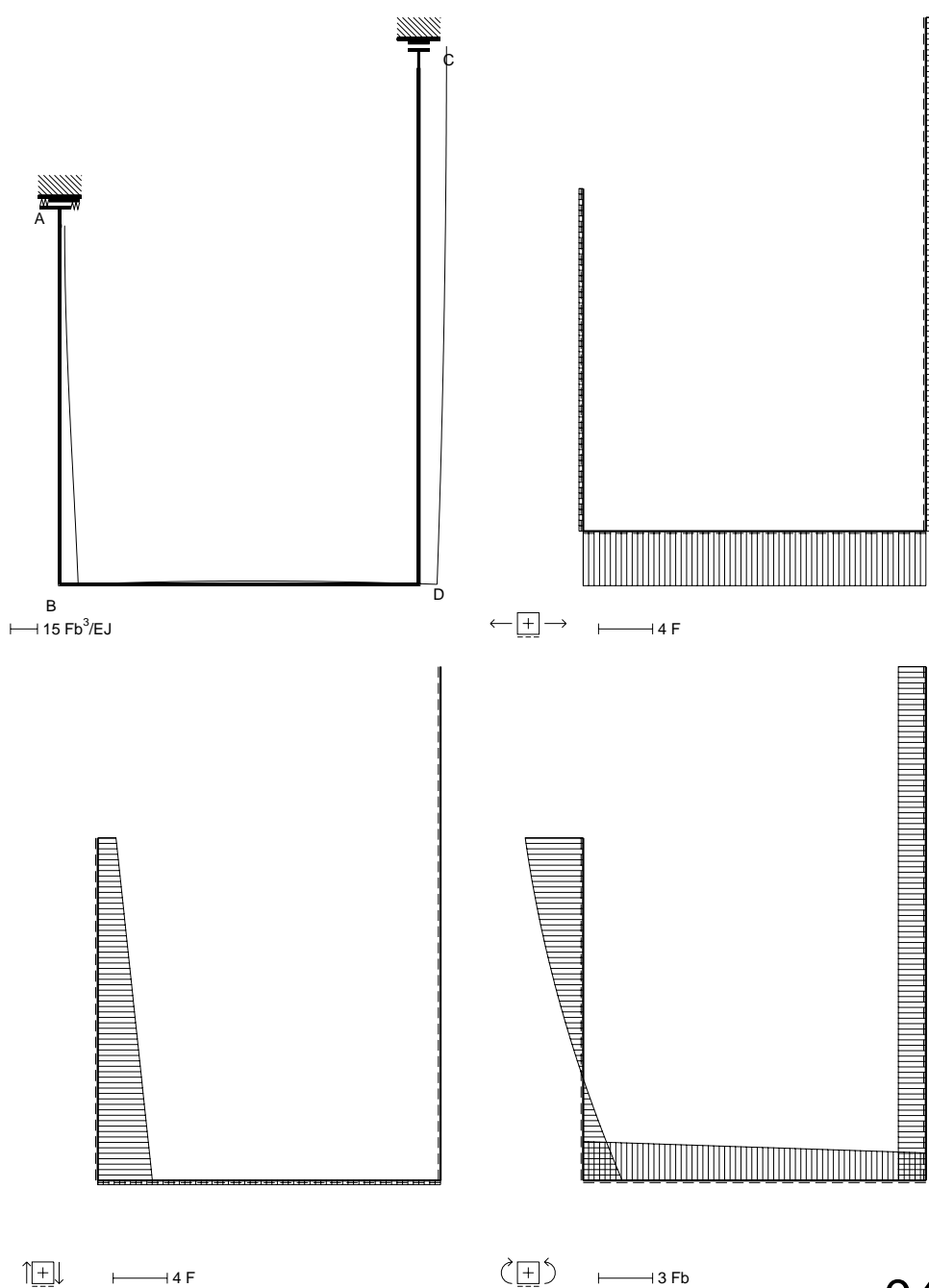
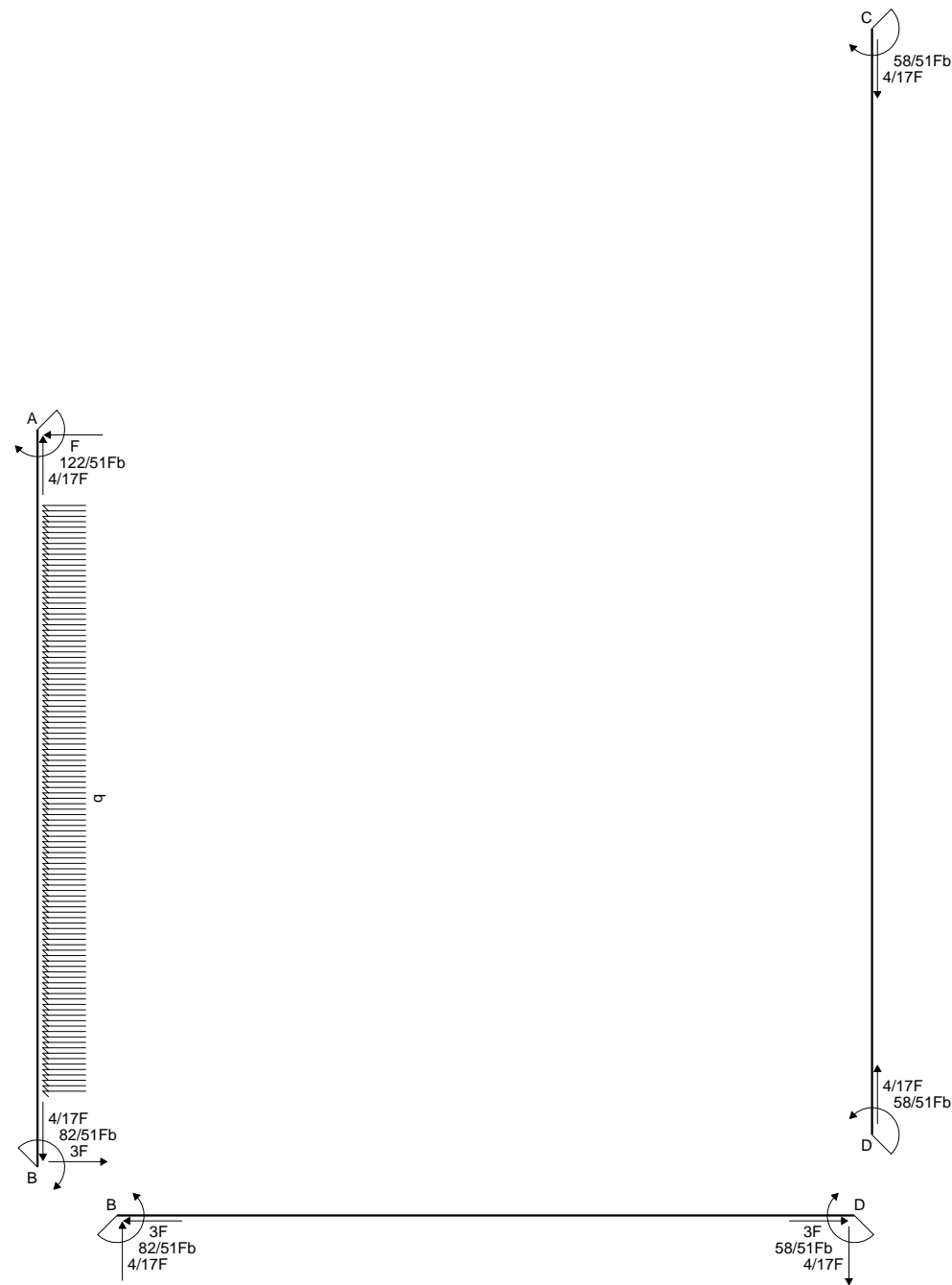


DEFORMATA (coordinate locali)

AB $y(x)EJ =$
CD $y(x)EJ =$
BD $y(x)EJ =$

SPOSTAMENTI NODALI

$u_A =$	$u_B =$	$u_{CCD} =$	$u_D =$
$v_A =$	$v_B =$	$v_C =$	$v_D =$
$\varphi_A =$	$\varphi_B =$	$\varphi_C =$	$\varphi_D =$



REAZIONI IPERSTATICHE

$$X = W_{DC} \quad Y = W_{BD}$$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{CD} K_{CD} φ_{BD} K_{BD}

Relazioni di congruenza

$$y'_{AB}(0) = 0$$

$$y'_{AB}(2b) - y'_{BD}(0) = 0$$

$$y'_{CD}(0) = 0$$

$$y'_{CD}(3b) - y'_{BD}(2b) = 0$$

$$y_{AB}(0) + H_A b^3/EJ = 0$$

$$y_{CD}(3b) - y_{AB}(2b) = 0$$

$$y_{BD}(0) = 0$$

$$y_{BD}(2b) = 0$$

$$M_{AB} = -Fx + 4Fb - 1/2qx^2 - Y$$

$$EJy = -Fx + 4Fb - 1/2qx^2 - Y$$

$$EJy = -1/2Fx^2 + 4Fbx - 1/6qx^3 - Y + EJ\varphi_{AB}$$

$$EJy = -1/6Fx^3 + 2Fbx^2 - 1/24qx^4 - Yb + EJ\varphi_{AB}x + EJK_{AB}$$

$$M_{CD} = Xb$$

$$EJy = -EJ\theta + 1/2Xb$$

$$EJy = -EJ\theta x + 1/2Xb + EJ\varphi_{CD}$$

$$EJy = -1/2EJ\theta x^2 + 1/2Xb + EJ\varphi_{CD}x + EJK_{CD}$$

$$M_{BD} = -1/2X + 1/2Y - Yb$$

$$EJy = -1/2X + 1/2Y - Yb$$

$$EJy = -1/2X + 1/2Y - Yb + EJ\varphi_{BD}$$

$$EJy = -1/2X + 1/2Y - Yb + EJ\varphi_{BD}x + EJK_{BD}$$

Condizioni al contorno

$$\begin{bmatrix} \varphi_{AB}b & K_{AB} & \varphi_{CD}b & K_{CD} & \varphi_{BD}b & K_{BD} & Xb^2/EJ & Yb^2/EJ \\ y'_{AB} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ y'_{BA} & 1 & 0 & 0 & -1 & 0 & 0 & -2 \\ y'_{CD} & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ y'_{DC} & 0 & 0 & 1 & 0 & -1 & 0 & 5/2 \\ y_{AB} & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ y_{DC} & -2 & -1 & 3 & 1 & 0 & 0 & 9/4 \\ y_{BD} & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ y_{DB} & 0 & 0 & 0 & 0 & 2 & 1 & -2/3 \end{bmatrix} = \begin{bmatrix} Fb^3/EJ & \alpha Tb \\ 0 & 0 \\ -14/3 & 0 \\ 0 & 0 \\ 0 & 3 \\ 1 & 0 \\ 6 & 9/2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Soluzione

$$\begin{bmatrix} \varphi_{AB}b \\ \varphi_{BD}b \\ \varphi_{CD}b \\ Xb^2/EJ \\ K_{AB} \\ K_{CD} \\ K_{BD} \\ Yb^2/EJ \end{bmatrix} = \begin{bmatrix} Fb^3/EJ \\ 0 \\ 74/51 \\ 0 \\ 58/51 \\ 1 \\ 292/51 \\ 0 \\ 82/51 \end{bmatrix}$$

DEFORMATA (coordinate locali)

$$AB \ y(x)EJ = Fb^3 + 61/51x^2Fb - 1/6x^3F - 1/24x^4q$$

$$BA \ y(x)EJ = 193/51Fb^3 - 74/51x^2Fb^2 - 41/51x^2Fb + 1/2x^3F - 1/24x^4q$$

$$CD \ y(x)EJ = 292/51Fb^3 - 11/51x^2Fb$$

$$DC \ y(x)EJ = 193/51Fb^3 + 22/17x^2Fb^2 - 11/51x^2Fb$$

$$BD \ y(x)EJ = 74/51x^2Fb^2 - 41/51x^2Fb + 2/51x^3F$$

$$DB \ y(x)EJ = 22/17x^2Fb^2 - 29/51x^2Fb - 2/51x^3F$$

SPOSTAMENTI NODALI

$$u_A = (Fb^3/EJ)$$

$$v_A = 0$$

$$\varphi_A = 0$$

$$u_B = 193/51(Fb^3/EJ)$$

$$v_B = 0$$

$$\varphi_B = 74/51(Fb^2/EJ)$$

$$u_{CCD} = 292/51(Fb^3/EJ)$$

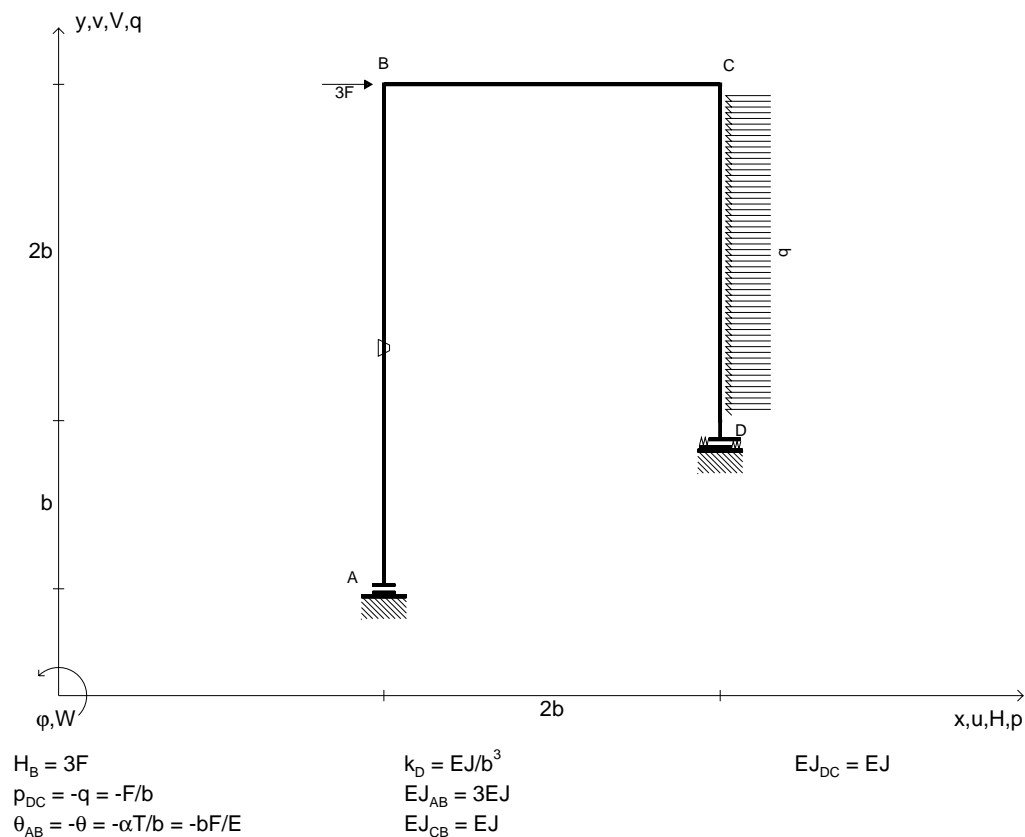
$$v_C = 0$$

$$\varphi_C = 0$$

$$u_D = 193/51(Fb^3/EJ)$$

$$v_D = 0$$

$$\varphi_D = -22/17(Fb^2/EJ)$$



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Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi delle azioni interne nelle aste.

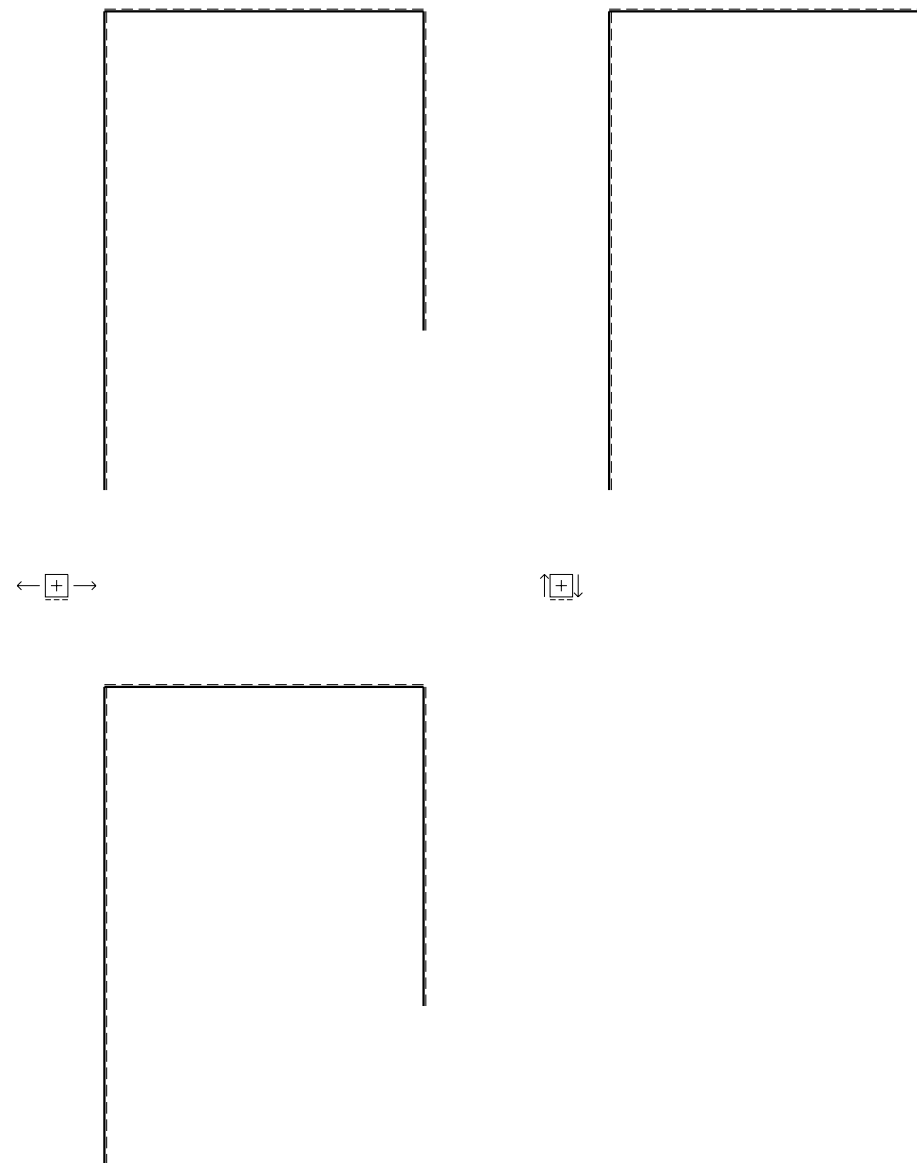
Esprimere la linea elastica delle aste.

Calcolare spostamento e rotazione di tutti i nodi.

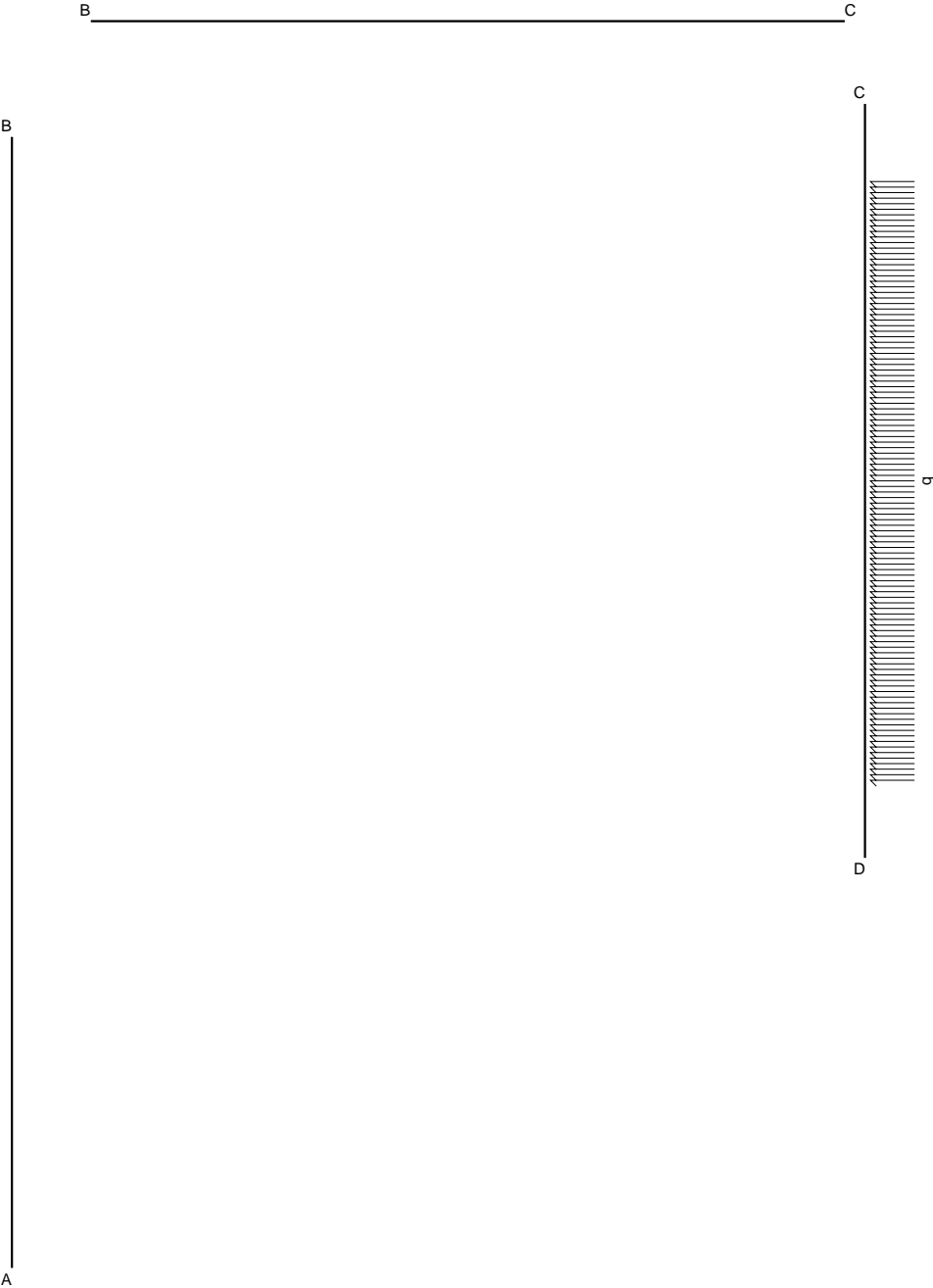
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta AB positiva se convessa a destra con inizio A.

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DEFORMATA (coordinate locali)

AB $y(x)EJ =$

CB $y(x)EJ =$

DC $y(x)EJ =$

SPOSTAMENTI NODALI

$u_{AAB} =$

$u_B =$

$u_C =$

$u_D =$

$v_A =$

$v_B =$

$v_C =$

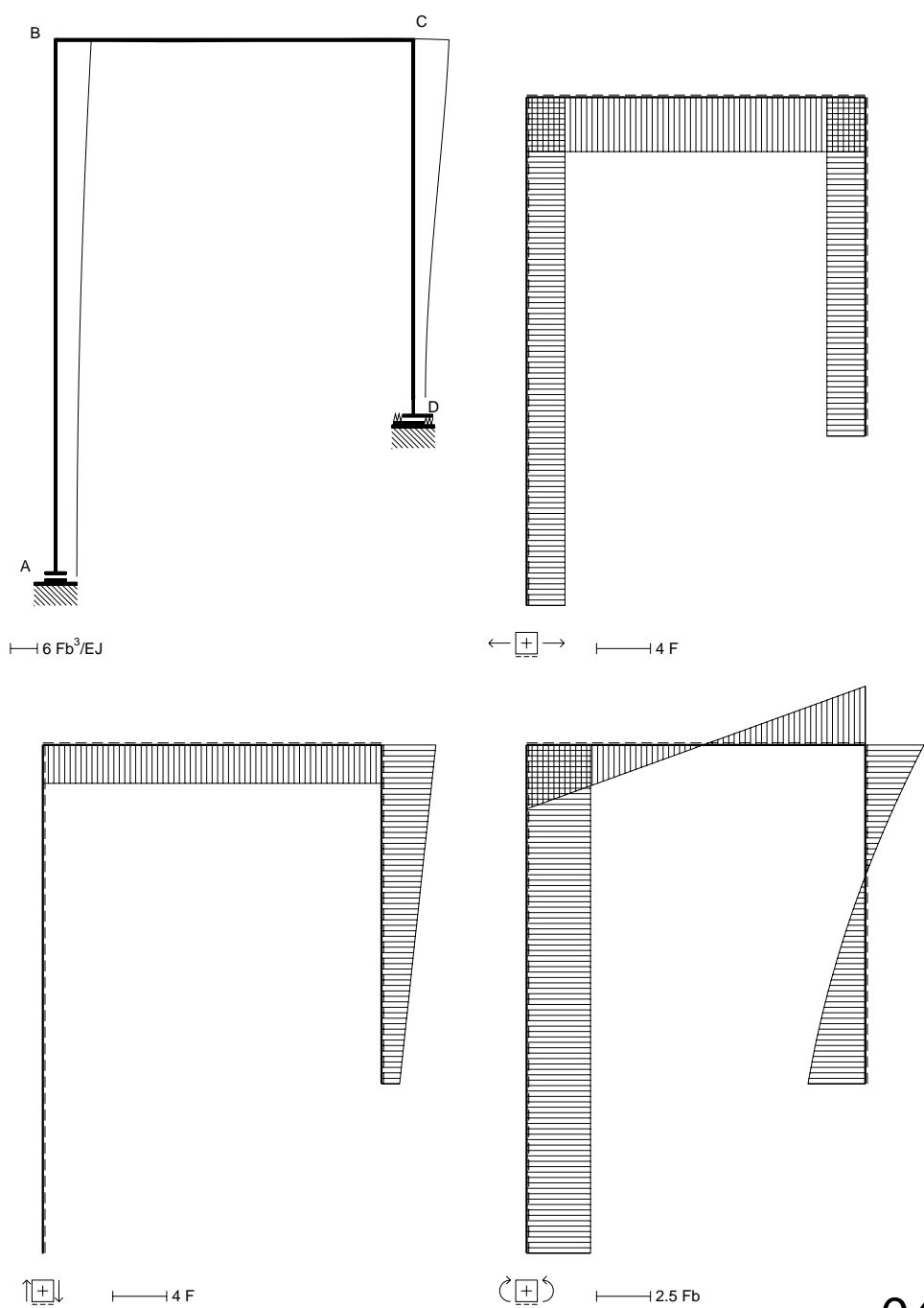
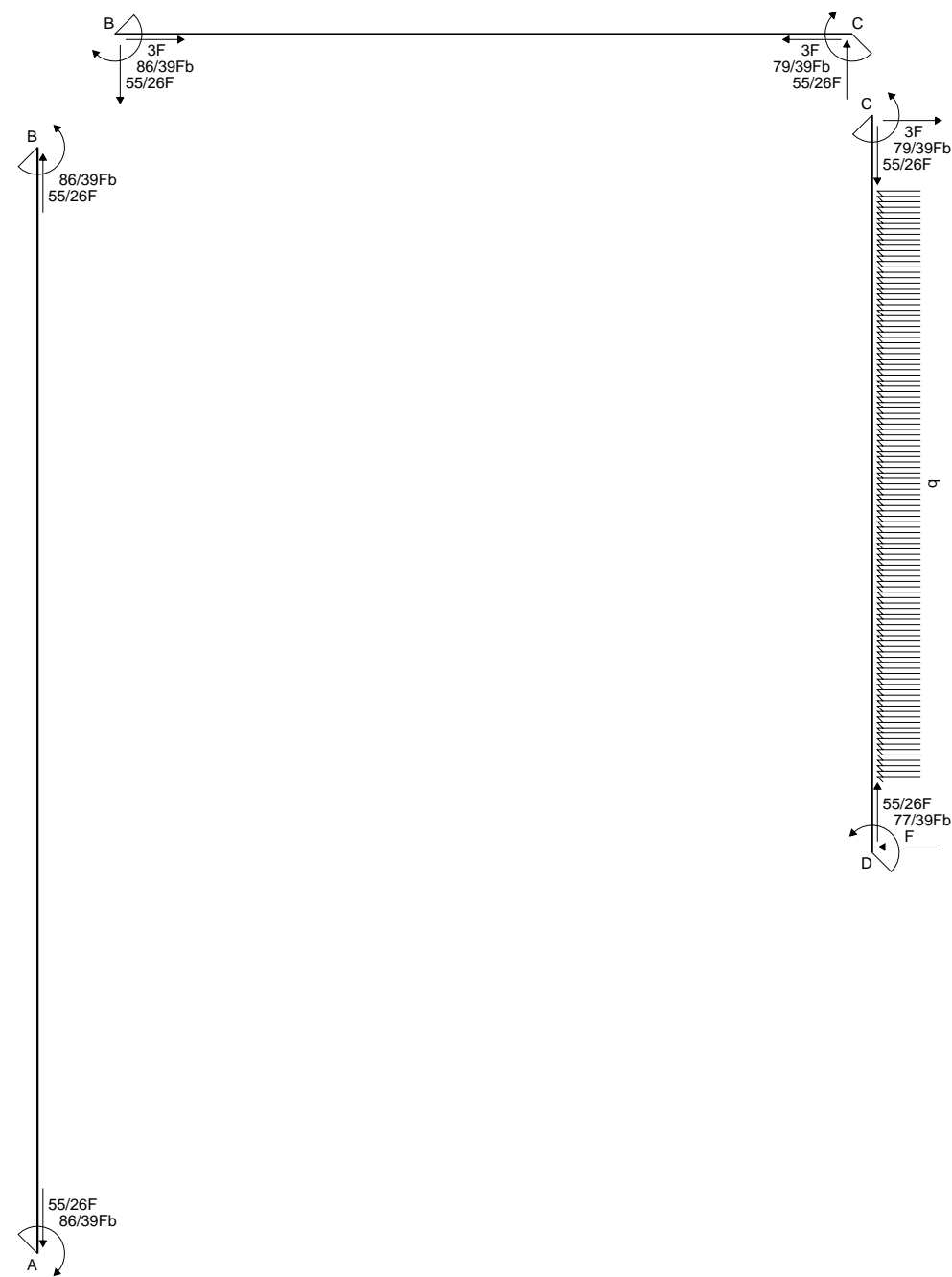
$v_D =$

$\varphi_A =$

$\varphi_B =$

$\varphi_C =$

$\varphi_D =$



REAZIONI IPERSTATICHE

$$X = W_{BA} \quad Y = W_{CB}$$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{CB} K_{CB} φ_{DC} K_{DC}

Relazioni di congruenza

$$\begin{aligned} y'_{AB}(0) &= 0 \\ y'_{AB}(3b) - y'_{CB}(2b) &= 0 \\ y'_{CB}(0) - y'_{DC}(2b) &= 0 \\ y'_{DC}(0) &= 0 \\ y_{CB}(0) &= 0 \\ y_{CB}(2b) &= 0 \\ y_{DC}(0) - H_D b^3/EJ &= 0 \\ y_{DC}(2b) - y_{AB}(3b) &= 0 \end{aligned}$$

$$\begin{aligned} M_{AB} &= X \\ EJy &= -EJ\theta + 1/3X \\ EJy &= -EJ\theta x + 1/3X + EJ\varphi_{AB} \\ EJy &= -1/2EJ\theta x^2 + 1/3Xb + EJ\varphi_{AB}x + EJK_{AB} \end{aligned}$$

$$\begin{aligned} M_{CB} &= -1/2X + 1/2Y - Yb \\ EJy &= -1/2X + 1/2Y - Yb \\ EJy &= -1/2X + 1/2Y - Yb + EJ\varphi_{CB} \\ EJy &= -1/2X + 1/2Y - Yb + EJ\varphi_{CB}x + EJK_{CB} \end{aligned}$$

$$\begin{aligned} M_{DC} &= Fx - 4Fb + 1/2qx^2 - Yb \\ EJy &= Fx - 4Fb + 1/2qx^2 - Yb \\ EJy &= 1/2Fx^2 - 4Fbx + 1/6qx^3 - Yb + EJ\varphi_{DC} \\ EJy &= 1/6Fx^3 - 2Fbx^2 + 1/24qx^4 - Yb + EJ\varphi_{DC}x + EJK_{DC} \end{aligned}$$

Condizioni al contorno

$$\begin{bmatrix} \varphi_{AB}b & K_{AB} & \varphi_{CB}b & K_{CB} & \varphi_{DC}b & K_{DC} & Xb^2/EJ & Yb^2/EJ \end{bmatrix} \begin{bmatrix} Fb^3/EJ & \alpha Tb \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 3 \\ -14/3 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ -1 & 0 \\ 6 & -9/2 \end{bmatrix}$$

Soluzione

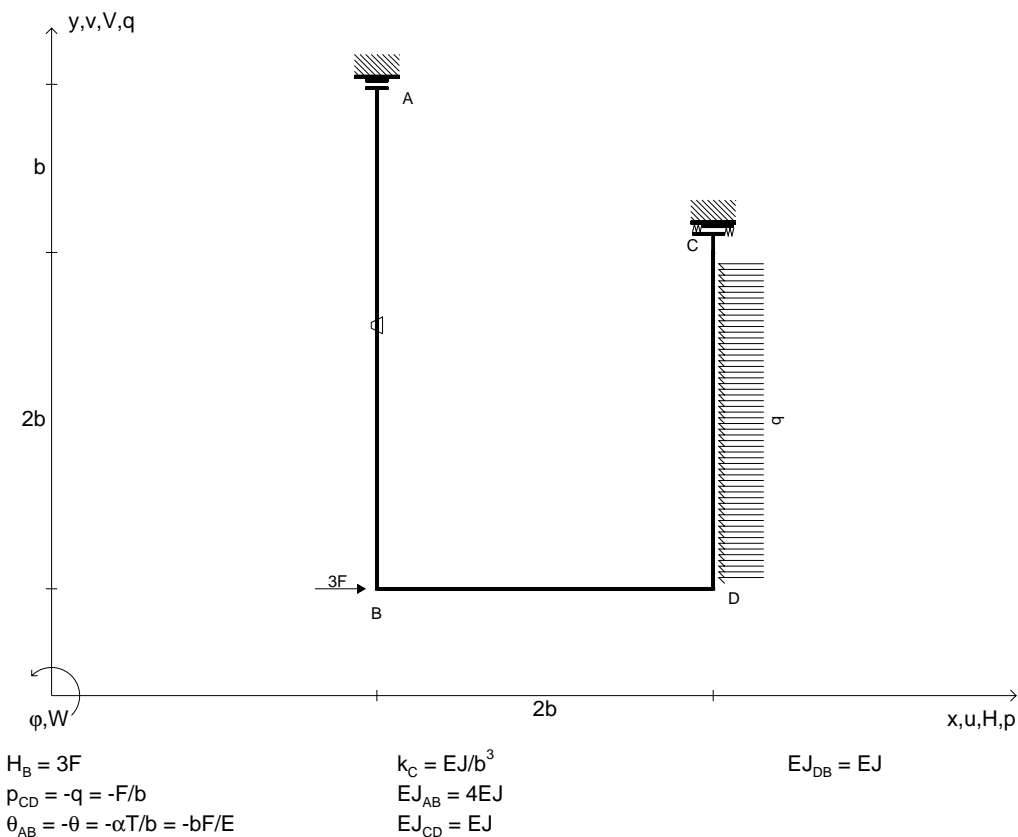
$$\begin{bmatrix} \varphi_{AB}b \\ \varphi_{CB}b \\ \varphi_{DC}b \\ Xb^2/EJ \\ K_{CB} \\ Yb^2/EJ \\ K_{DC} \\ K_{AB} \end{bmatrix} = \begin{bmatrix} 0 \\ -8/13 \\ 0 \\ 86/39 \\ 0 \\ -79/39 \\ -1 \\ -137/78 \end{bmatrix} \begin{bmatrix} Fb^3/EJ \end{bmatrix}$$

DEFORMATA (coordinate locali)

$$\begin{aligned} AB \ y(x)EJ &= -137/78Fb^3 - 31/234x^2Fb \\ BA \ y(x)EJ &= -115/39Fb^3 + 31/39xFb^2 - 31/234x^2Fb \\ CB \ y(x)EJ &= -8/13xFb^2 + 79/78x^2Fb - 55/156x^3F \\ BC \ y(x)EJ &= 31/39xFb^2 - 43/39x^2Fb + 55/156x^3F \\ DC \ y(x)EJ &= -Fb^3 - 77/78x^2Fb + 1/6x^3F + 1/24x^4q \\ CD \ y(x)EJ &= -115/39Fb^3 + 8/13xFb^2 + 79/78x^2Fb - 1/2x^3F + 1/24x^4q \end{aligned}$$

SPOSTAMENTI NODALI

$$\begin{aligned} u_{AAB} &= 137/78(Fb^3/EJ) & u_B &= 115/39(Fb^3/EJ) & u_C &= 115/39(Fb^3/EJ) & u_D &= (Fb^3/EJ) \\ v_A &= 0 & v_B &= 0 & v_C &= 0 & v_D &= 0 \\ \varphi_A &= 0 & \varphi_B &= -31/39(Fb^2/EJ) & \varphi_C &= -8/13(Fb^2/EJ) & \varphi_D &= 0 \end{aligned}$$



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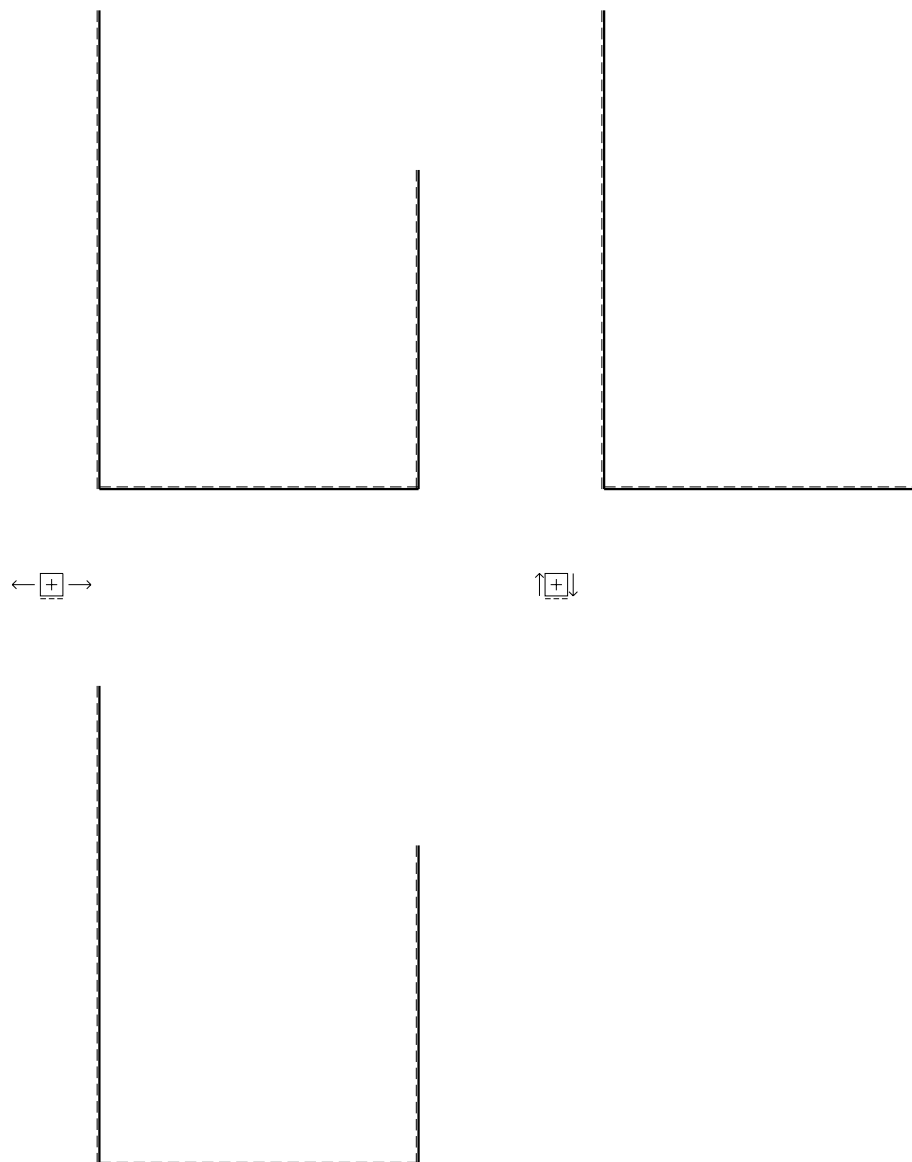
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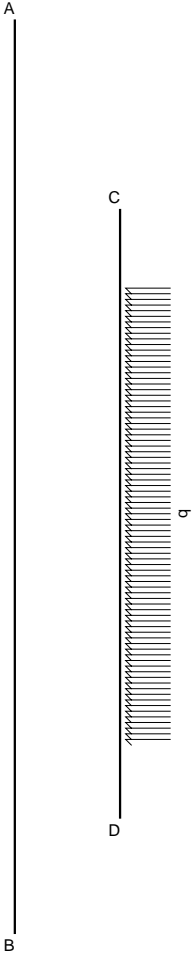
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Curvatura θ asta AB positiva se convessa a destra con inizio A.

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B D

DEFORMATA (coordinate locali)

AB $y(x)EJ =$

CD $y(x)EJ =$

DB $y(x)EJ =$

SPOSTAMENTI NODALI

$u_{AAB} =$

$u_B =$

$u_C =$

$u_D =$

$v_A =$

$v_B =$

$v_C =$

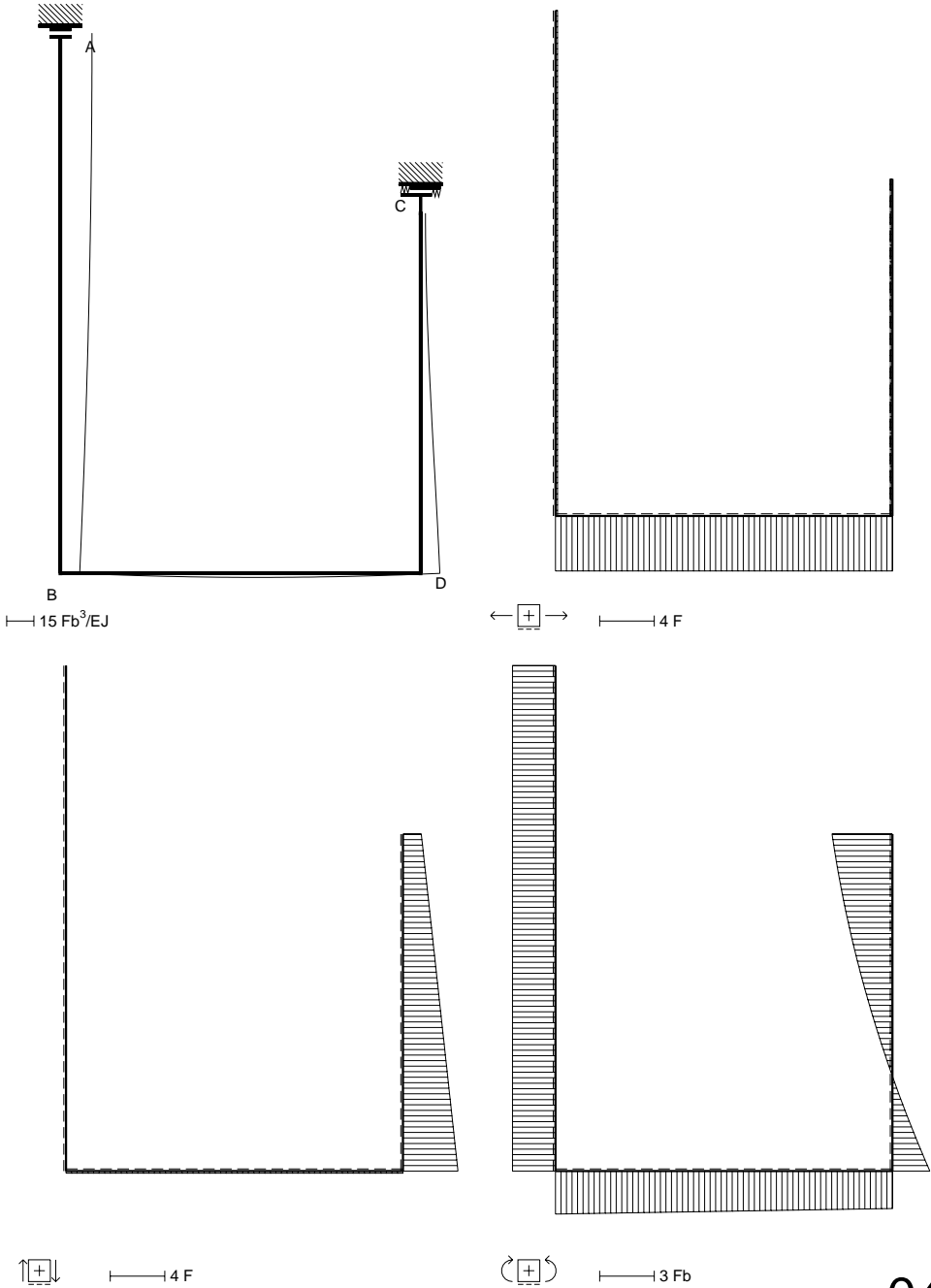
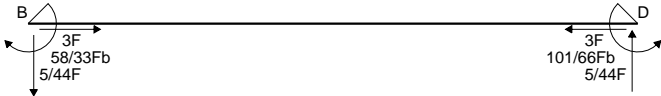
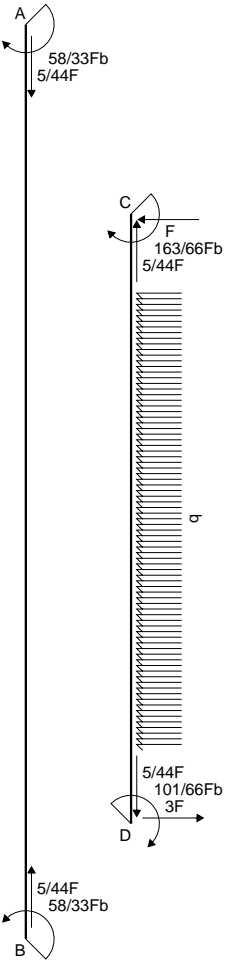
$v_D =$

$\varphi_A =$

$\varphi_B =$

$\varphi_C =$

$\varphi_D =$



REAZIONI IPERSTATICHE

$$X = W_{BA} \quad Y = W_{DB}$$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{CD} K_{CD} φ_{DB} K_{DB}

Relazioni di congruenza

$$y'_{AB}(0) = 0$$

$$y'_{AB}(3b) - y'_{DB}(2b) = 0$$

$$y'_{CD}(0) = 0$$

$$y'_{CD}(2b) - y'_{DB}(0) = 0$$

$$y_{CD}(0) + H_C b^3/EJ = 0$$

$$y_{CD}(2b) - y_{AB}(3b) = 0$$

$$y_{DB}(0) = 0$$

$$y_{DB}(2b) = 0$$

$$M_{AB} = X$$

$$EJy = -EJ\theta + 1/4X$$

$$EJy = -EJ\theta x + 1/4X + EJ\varphi_{AB}$$

$$EJy = -1/2EJ\theta x^2 + 1/4Xb + EJ\varphi_{AB}x + EJK_{AB}$$

$$M_{CD} = -Fx + 4Fb - 1/2qx^2 - Yb$$

$$EJy = -Fx + 4Fb - 1/2qx^2 - Yb$$

$$EJy = -1/2Fx^2 + 4Fbx - 1/6qx^3 - Yb + EJ\varphi_{CD}$$

$$EJy = -1/6Fx^3 + 2Fbx^2 - 1/24qx^4 - Yb + EJ\varphi_{CD}x + EJK_{CD}$$

$$M_{DB} = -1/2X + 1/2Y - Yb$$

$$EJy = -1/2X + 1/2Y - Yb$$

$$EJy = -1/2X + 1/2Y - Yb + EJ\varphi_{DB}$$

$$EJy = -1/2X + 1/2Y - Yb + EJ\varphi_{DB}x + EJK_{DB}$$

Condizioni al contorno

$$\begin{bmatrix} \varphi_{AB}b & K_{AB} & \varphi_{CD}b & K_{CD} & \varphi_{DB}b & K_{DB} & Xb^2/EJ & Yb^2/EJ \end{bmatrix} \begin{bmatrix} y'_{AB} \\ y'_{BA} \\ y'_{CD} \\ y'_{DC} \\ y_{CD} \\ y_{DC} \\ y_{DB} \\ y_{BD} \end{bmatrix} = \begin{bmatrix} Fb^3/EJ & \alpha Tb \end{bmatrix}$$

Soluzione

$$\begin{bmatrix} \varphi_{AB}b \\ \varphi_{DB}b \\ \varphi_{CD}b \\ Xb^2/EJ \\ K_{CD} \\ K_{AB} \\ K_{DB} \\ Yb^2/EJ \end{bmatrix} = \begin{bmatrix} Fb^3/EJ \\ 0 \\ 53/33 \\ 0 \\ 58/33 \\ 1 \\ 853/132 \\ 0 \\ 101/66 \end{bmatrix}$$

DEFORMATA (coordinate locali)

$$AB \ y(x)EJ = 853/132Fb^3 - 37/132x^2Fb$$

$$BA \ y(x)EJ = 130/33Fb^3 + 37/22x^2Fb^2 - 37/132x^2Fb$$

$$CD \ y(x)EJ = Fb^3 + 163/132x^2Fb - 1/6x^3F - 1/24x^4q$$

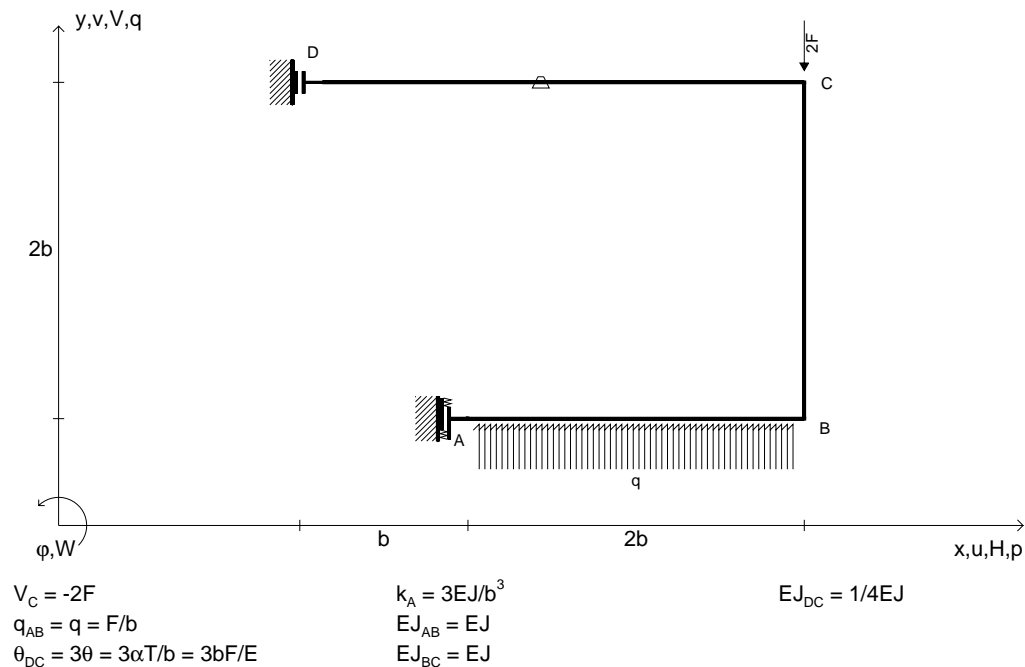
$$DC \ y(x)EJ = 130/33Fb^3 - 53/33x^2Fb^2 - 101/132x^2Fb + 1/2x^3F - 1/24x^4q$$

$$DB \ y(x)EJ = 53/33x^2Fb^2 - 101/132x^2Fb - 5/264x^3F$$

$$BD \ y(x)EJ = 37/22x^2Fb^2 - 29/33x^2Fb + 5/264x^3F$$

SPOSTAMENTI NODALI

$$\begin{array}{llll} u_{AAB} = 853/132(Fb^3/EJ) & u_B = 130/33(Fb^3/EJ) & u_C = (Fb^3/EJ) & u_D = 130/33(Fb^3/EJ) \\ v_A = 0 & v_B = 0 & v_C = 0 & v_D = 0 \\ \varphi_A = 0 & \varphi_B = -37/22(Fb^2/EJ) & \varphi_C = 0 & \varphi_D = 53/33(Fb^2/EJ) \end{array}$$



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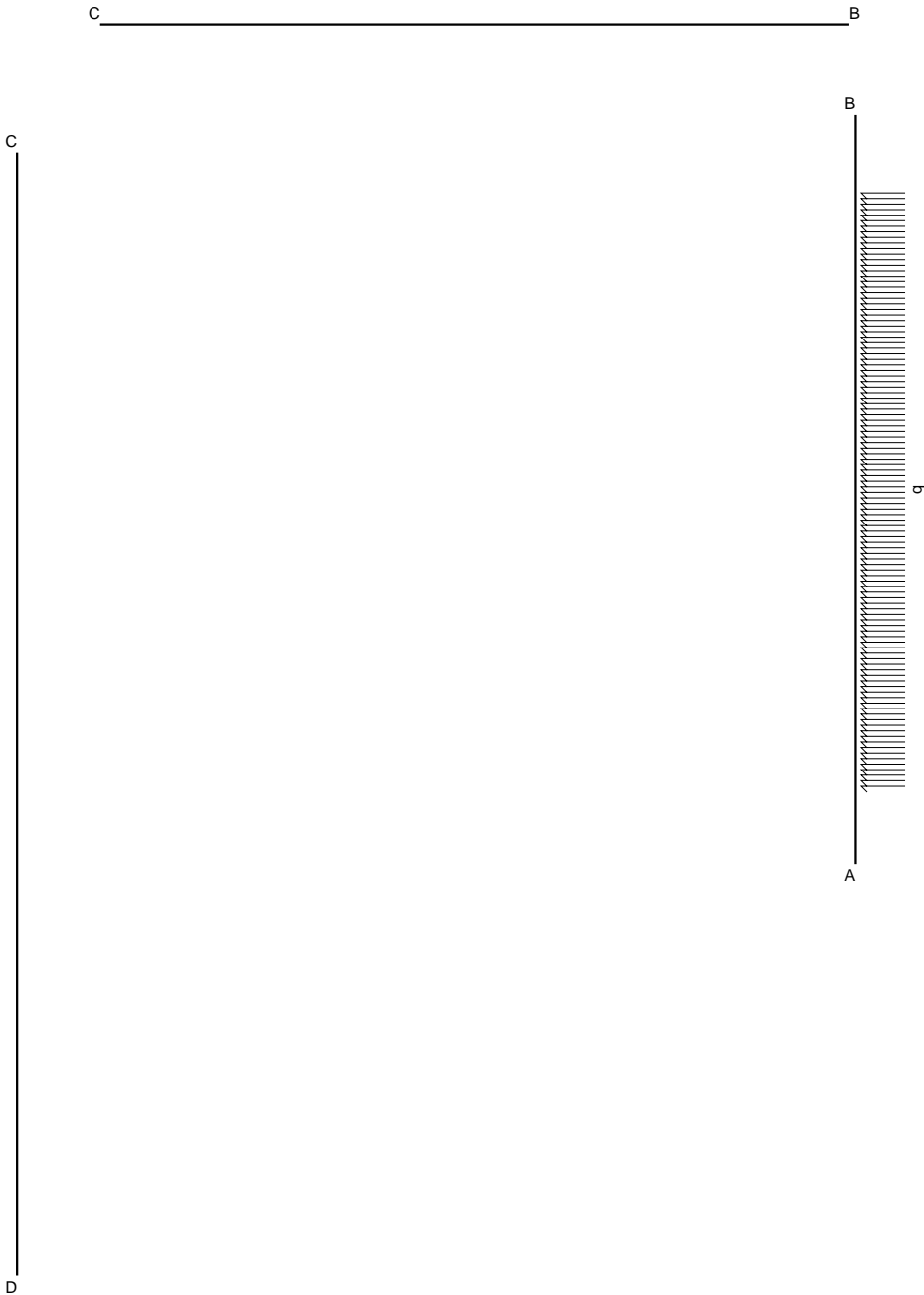
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta DC positiva se convessa a destra con inizio D.

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DEFORMATA (coordinate locali)

AB $y(x)EJ =$

BC $y(x)EJ =$

DC $y(x)EJ =$

SPOSTAMENTI NODALI

$u_A =$

$u_B =$

$u_C =$

$v_A =$

$v_B =$

$v_C =$

$\varphi_A =$

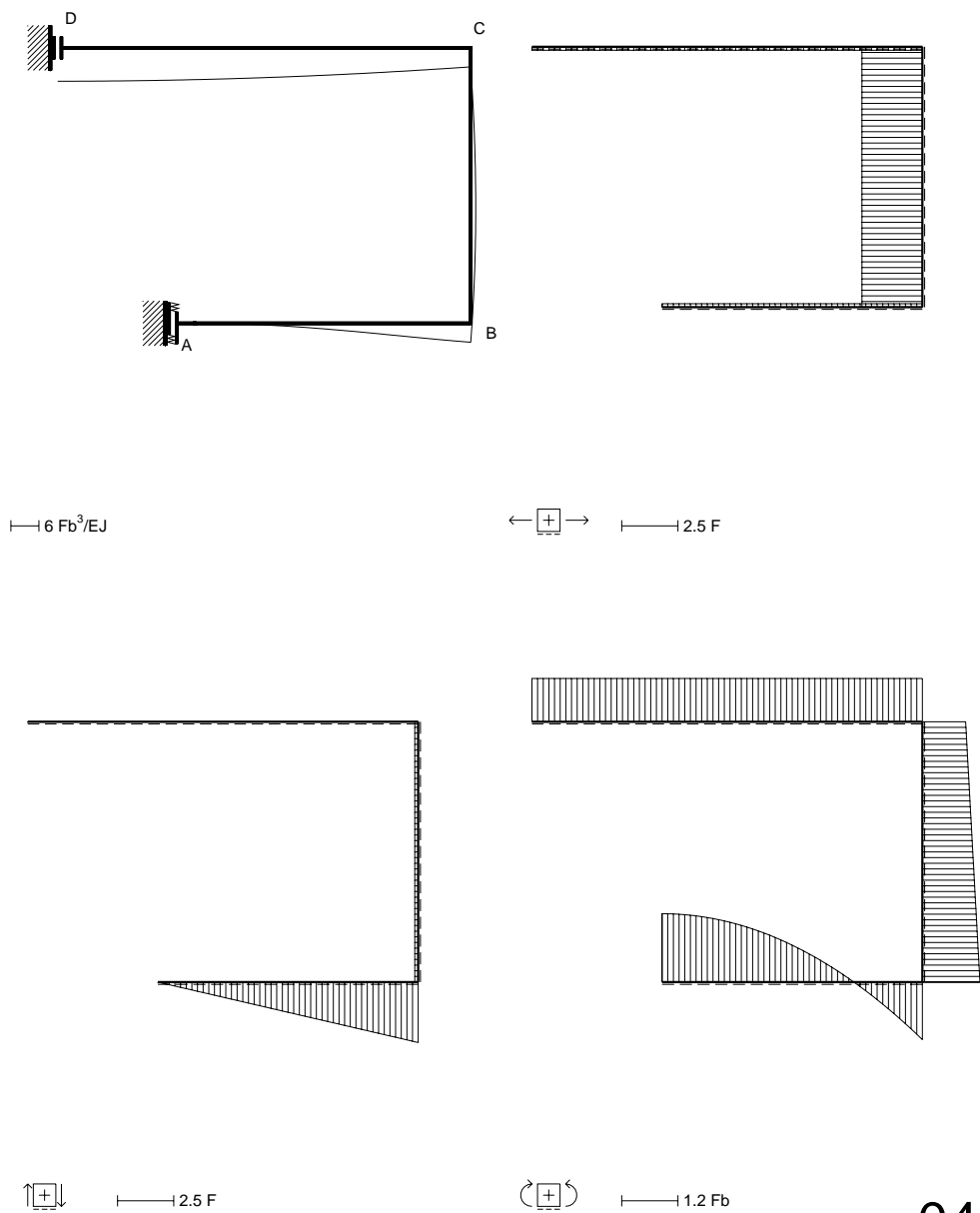
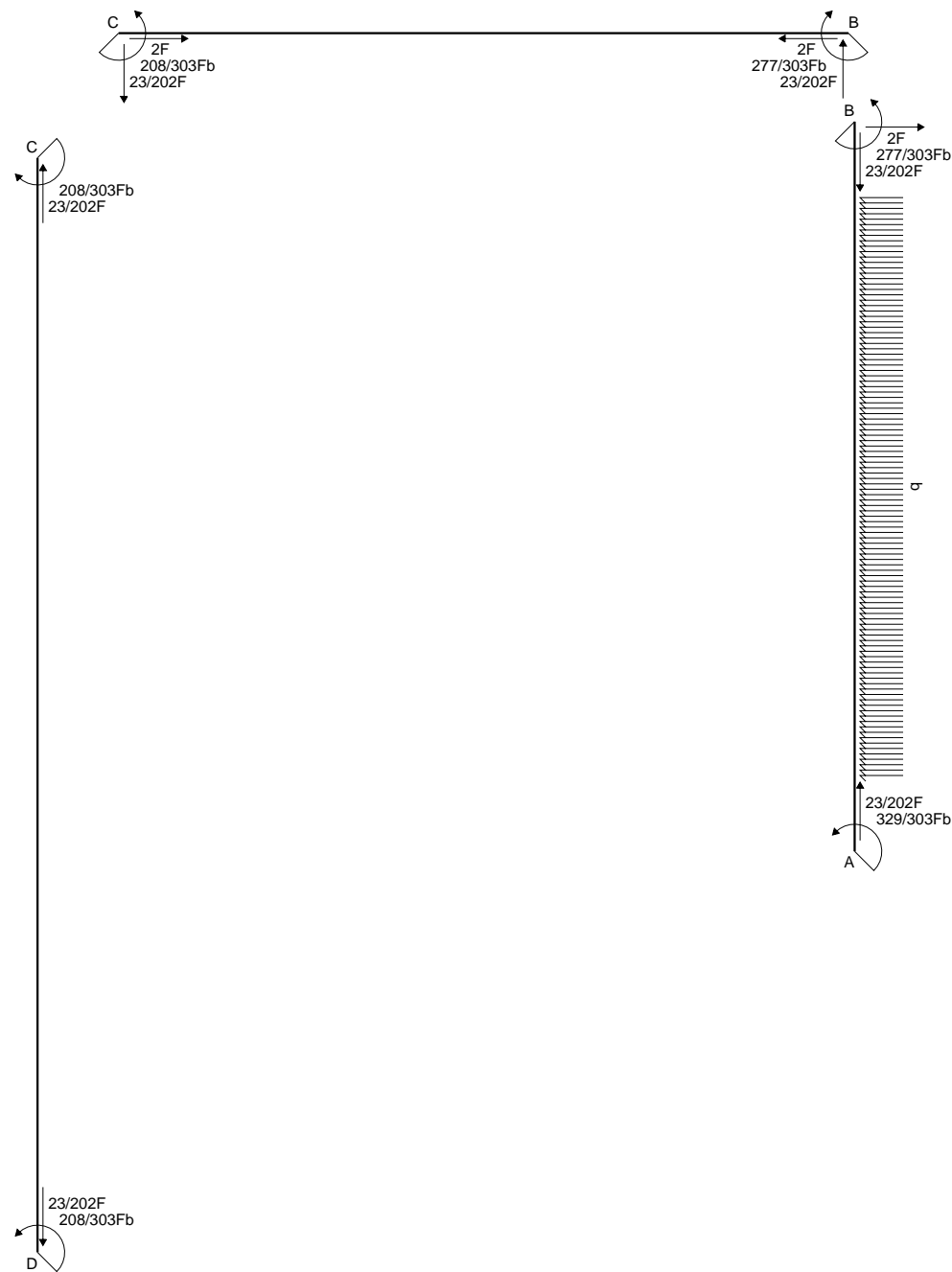
$\varphi_B =$

$\varphi_C =$

$u_D =$

$v_{DDC} =$

$\varphi_D =$



REAZIONI IPERSTATICHE

$$X = W_{BC} \quad Y = W_{CD}$$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{BC} K_{BC} φ_{DC} K_{DC}

Relazioni di congruenza

$$y'_{AB}(0) = 0$$

$$y'_{AB}(2b) - y'_{BC}(0) = 0$$

$$y'_{BC}(2b) - y'_{DC}(3b) = 0$$

$$y'_{DC}(0) = 0$$

$$y_{AB}(0) + 1/3 V_A b^3 / EJ = 0$$

$$y_{BC}(0) = 0$$

$$y_{BC}(2b) = 0$$

$$y_{DC}(3b) - y_{AB}(2b) = 0$$

$$M_{AB} = -2Fb + 1/2 q x^2 - Xb$$

$$EJy = -2Fb + 1/2 q x^2 - Xb$$

$$EJy = -2Fbx + 1/6 q x^3 - Xb + EJ\varphi_{AB}$$

$$EJy = -Fbx^2 + 1/24 q x^4 - Xb + EJ\varphi_{AB}x + EJK_{AB}$$

$$M_{BC} = 1/2 X - Xb - 1/2 Y$$

$$EJy = 1/2 X - Xb - 1/2 Y$$

$$EJy = 1/2 X - Xb - 1/2 Y + EJ\varphi_{BC}$$

$$EJy = 1/2 X - Xb - 1/2 Y + EJ\varphi_{BC}x + EJK_{BC}$$

$$M_{DC} = Yb$$

$$EJy = 3EJ\theta + 4Yb$$

$$EJy = 3EJ\theta x + 4Yb + EJ\varphi_{DC}$$

$$EJy = 3/2 EJ\theta x^2 + 4Yb + EJ\varphi_{DC}x + EJK_{DC}$$

Condizioni al contorno

$$\begin{bmatrix} \varphi_{AB}b & K_{AB} & \varphi_{BC}b & K_{BC} & \varphi_{DC}b & K_{DC} & Xb^2/EJ & Yb^2/EJ \end{bmatrix} \begin{bmatrix} y'_{AB} \\ y'_{BA} \\ y'_{CB} \\ y'_{DC} \\ y_{AB} \\ y_{BC} \\ y_{CB} \\ y_{CD} \end{bmatrix} = \begin{bmatrix} Fb^3/EJ & \alpha Tb \end{bmatrix}$$

Soluzione

$$\begin{bmatrix} \varphi_{AB}b \\ \varphi_{BC}b \\ \varphi_{DC}b \\ Xb^2/EJ \\ K_{AB} \\ K_{BC} \\ Yb^2/EJ \\ K_{DC} \end{bmatrix} = \begin{bmatrix} Fb^3/EJ \\ 0 \\ -254/303 \\ 0 \\ -277/303 \\ 0 \\ 0 \\ -208/303 \\ -535/202 \end{bmatrix}$$

DEFORMATA (coordinate locali)

$$AB \ y(x)EJ = -329/606 x^2 Fb + 1/24 x^4 q$$

$$BA \ y(x)EJ = -152/101 Fb^3 + 254/303 x Fb^2 + 277/606 x^2 Fb - 1/3 x^3 F + 1/24 x^4 q$$

$$BC \ y(x)EJ = -254/303 x Fb^2 + 277/606 x^2 Fb - 23/1212 x^3 F$$

$$CB \ y(x)EJ = -77/101 x Fb^2 + 104/303 x^2 Fb + 23/1212 x^3 F$$

$$DC \ y(x)EJ = -535/202 Fb^3 + 77/606 x^2 Fb$$

$$CD \ y(x)EJ = -152/101 Fb^3 - 77/101 x Fb^2 + 77/606 x^2 Fb$$

SPOSTAMENTI NODALI

$$u_A = 0$$

$$v_A = 0$$

$$\varphi_A = 0$$

$$u_B = 0$$

$$v_B = -152/101 (Fb^3/EJ)$$

$$\varphi_B = -254/303 (Fb^2/EJ)$$

$$u_C = 0$$

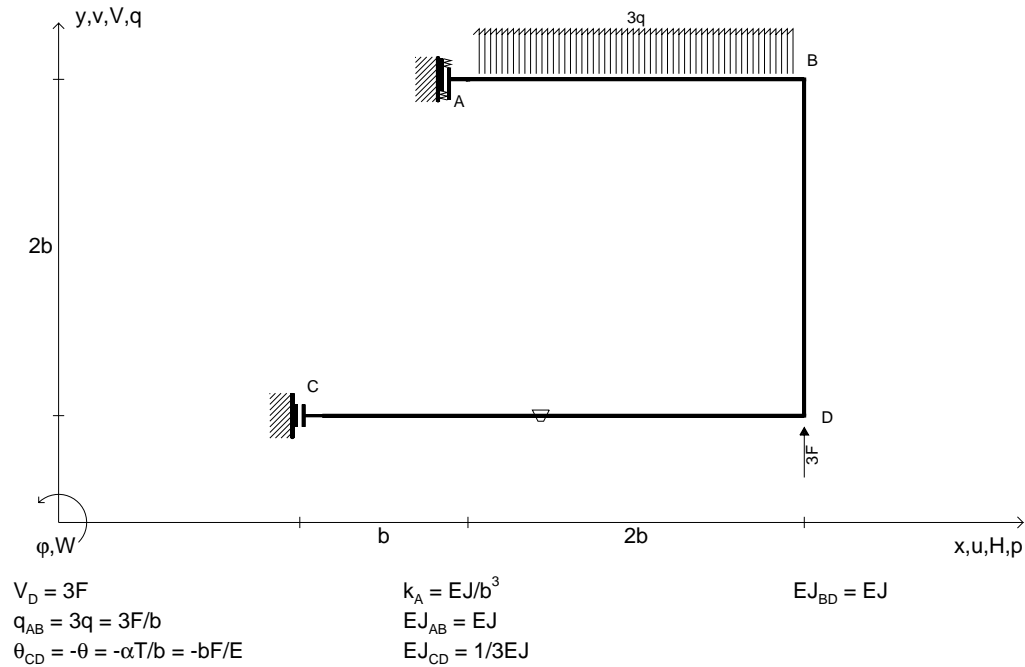
$$v_C = -152/101 (Fb^3/EJ)$$

$$\varphi_C = 77/101 (Fb^2/EJ)$$

$$u_D = 0$$

$$v_{DDC} = -535/202 (Fb^3/EJ)$$

$$\varphi_D = 0$$



Dato: pattino in A con molla long. // al pattino.

Svolgere l'analisi cinematica.

Risolvere con PLV e LE.

Tracciare la deformata elastica.

Riportare la soluzione su questo foglio.

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi delle azioni interne nelle aste.

Esprimere la linea elastica delle aste.

Calcolare spostamento e rotazione di tutti i nodi.

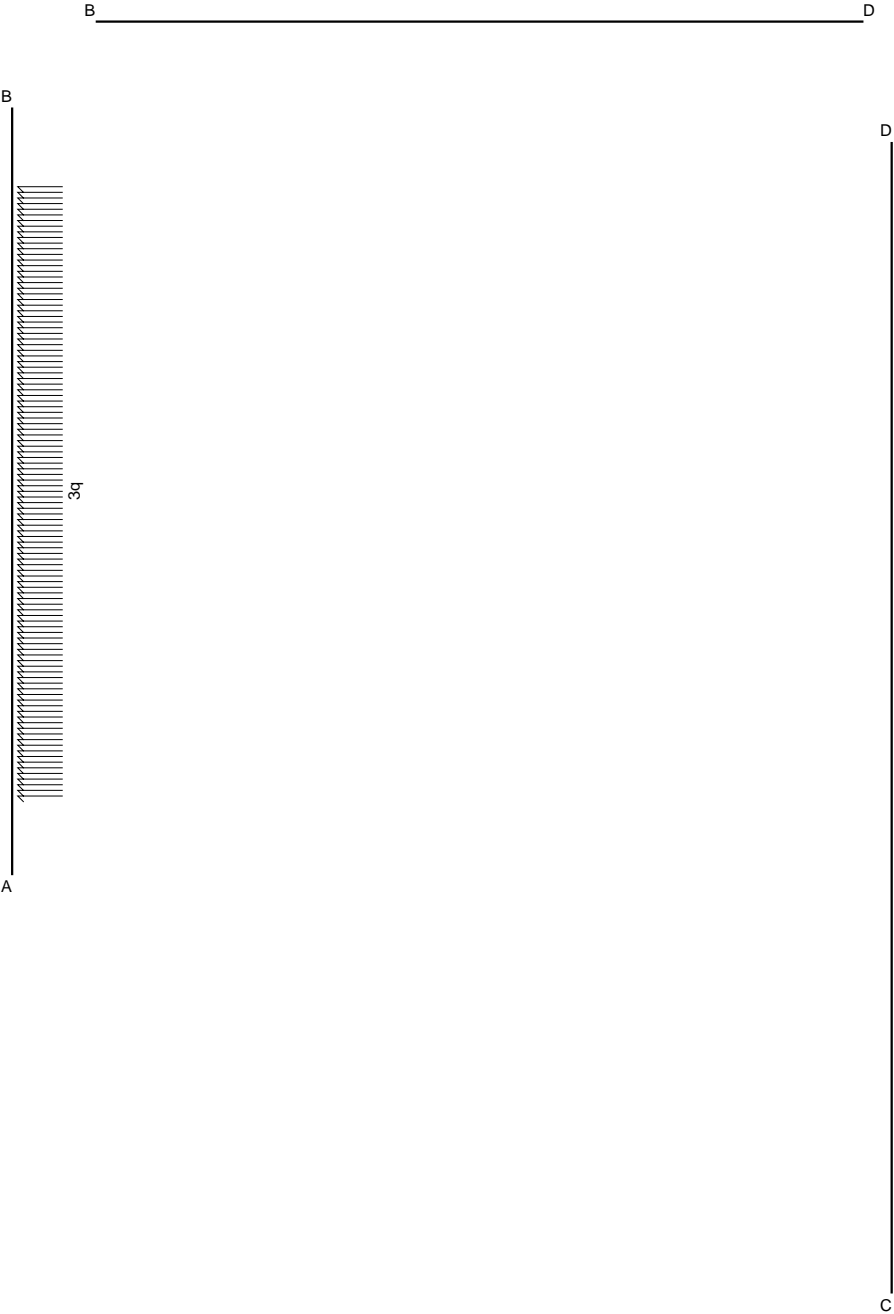
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta CD positiva se convessa a destra con inizio C.

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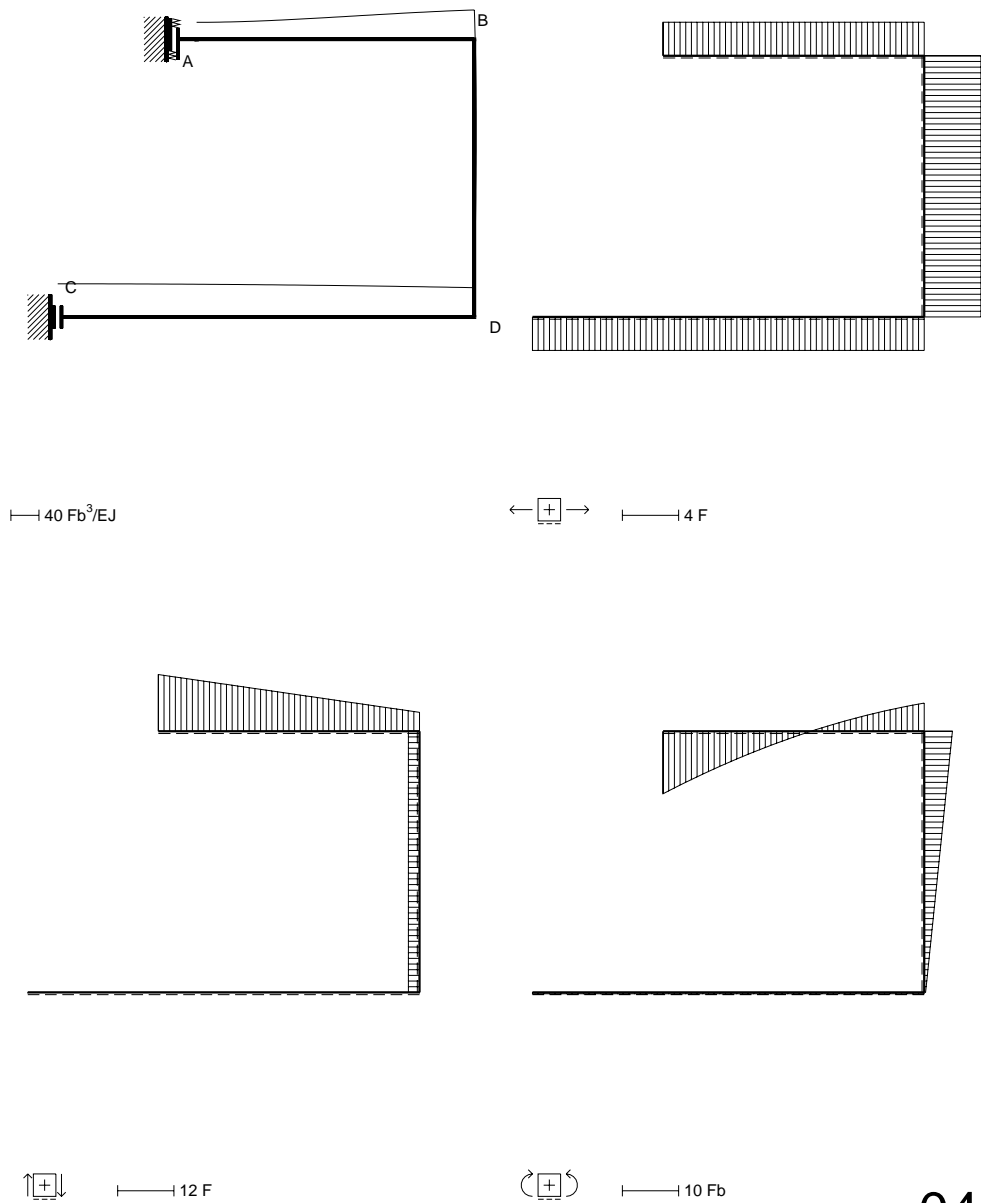
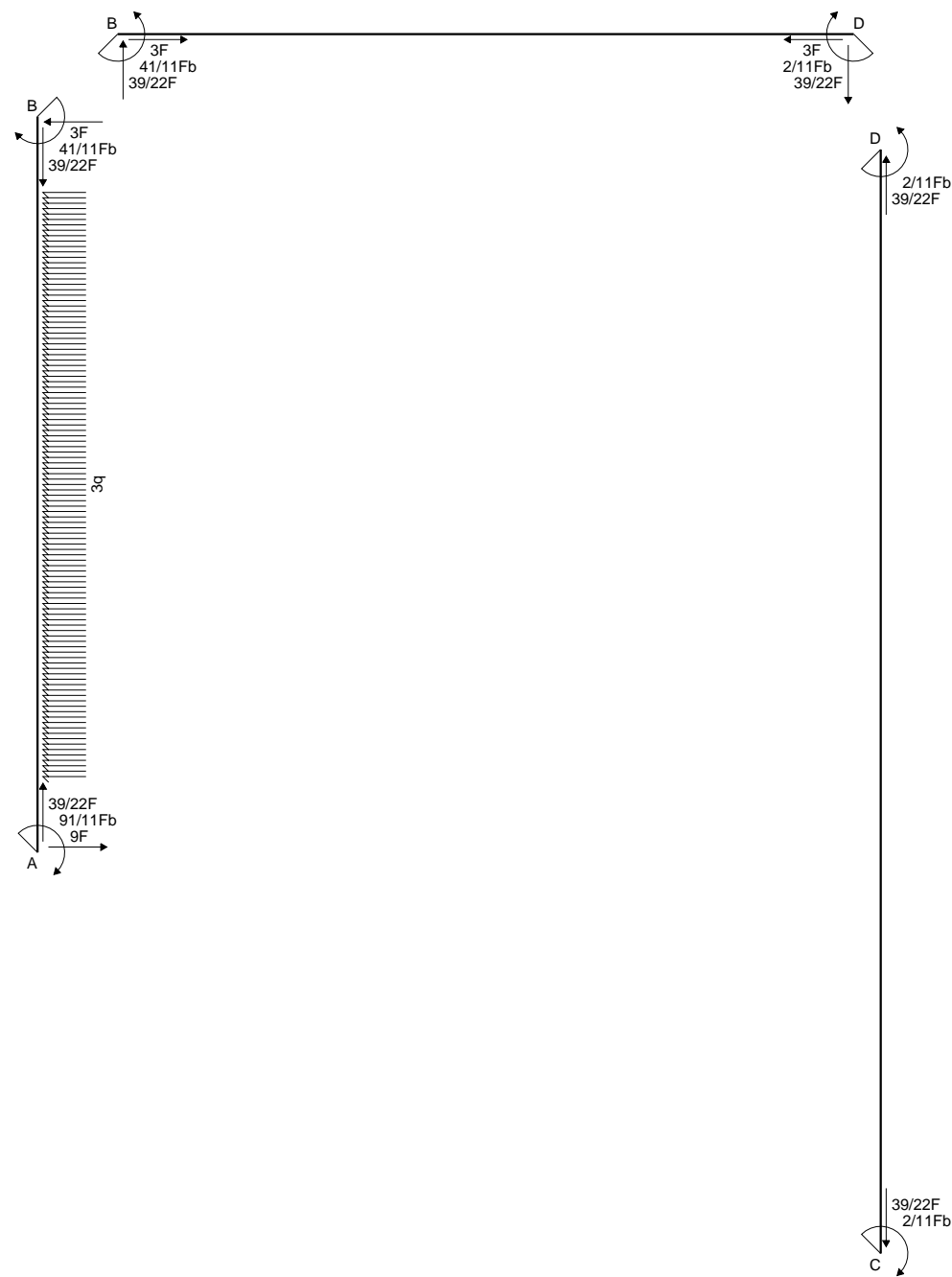


DEFORMATA (coordinate locali)

AB $y(x)EJ =$
CD $y(x)EJ =$
BD $y(x)EJ =$

SPOSTAMENTI NODALI

$u_A =$	$u_B =$	$u_C =$	$u_D =$
$v_A =$	$v_B =$	$v_{CCD} =$	$v_D =$
$\varphi_A =$	$\varphi_B =$	$\varphi_C =$	$\varphi_D =$



REAZIONI IPERSTATICHE

$$X = W_{DC} \quad Y = W_{BD}$$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{CD} K_{CD} φ_{BD} K_{BD}

Relazioni di congruenza

$$y'_{AB}(0) = 0$$

$$y'_{AB}(2b) - y'_{BD}(0) = 0$$

$$y'_{CD}(0) = 0$$

$$y'_{CD}(3b) - y'_{BD}(2b) = 0$$

$$y_{AB}(0) + V_A b^3/EJ = 0$$

$$y_{CD}(3b) - y_{AB}(2b) = 0$$

$$y_{BD}(0) = 0$$

$$y_{BD}(2b) = 0$$

$$M_{AB} = -9Fx + 12Fb + 3/2qx^2 - Yb$$

$$EJy = -9Fx + 12Fb + 3/2qx^2 - Yb$$

$$EJy = -9/2Fx^2 + 12Fbx + 1/2qx^3 - Yb + EJ\varphi_{AB}$$

$$EJy = -3/2Fx^3 + 6Fbx^2 + 1/8qx^4 - Yb + EJ\varphi_{AB}x + EJK_{AB}$$

$$M_{CD} = Xb$$

$$EJy = -EJ\theta + 3Xb$$

$$EJy = -EJ\theta x + 3Xb + EJ\varphi_{CD}$$

$$EJy = -1/2EJ\theta x^2 + 3Xb + EJ\varphi_{CD}x + EJK_{CD}$$

$$M_{BD} = -1/2X + 1/2Y - Yb$$

$$EJy = -1/2X + 1/2Y - Yb$$

$$EJy = -1/2X + 1/2Y - Yb + EJ\varphi_{BD}$$

$$EJy = -1/2X + 1/2Y - Yb + EJ\varphi_{BD}x + EJK_{BD}$$

Condizioni al contorno

$$\begin{bmatrix} \varphi_{AB}b & K_{AB} & \varphi_{CD}b & K_{CD} & \varphi_{BD}b & K_{BD} & Xb^2/EJ & Yb^2/EJ \end{bmatrix} \begin{bmatrix} y'_{AB} \\ y'_{BA} \\ y'_{CD} \\ y'_{DC} \\ y_{AB} \\ y_{DC} \\ y_{BD} \\ y_{DB} \end{bmatrix} = \begin{bmatrix} Fb^3/EJ & \alpha Tb \end{bmatrix}$$

Soluzione

$$\begin{bmatrix} \varphi_{AB}b \\ \varphi_{BD}b \\ \varphi_{CD}b \\ Xb^2/EJ \\ K_{AB} \\ K_{CD} \\ K_{BD} \\ Yb^2/EJ \end{bmatrix} = \begin{bmatrix} 0 \\ 28/11 \\ 0 \\ 2/11 \\ 9 \\ 387/22 \\ 0 \\ 41/11 \end{bmatrix} \begin{bmatrix} Fb^3/EJ \end{bmatrix}$$

DEFORMATA (coordinate locali)

$$AB \ y(x)EJ = 9Fb^3 + 91/22x^2Fb - 3/2x^3F + 1/8x^4q$$

$$BA \ y(x)EJ = 171/11Fb^3 - 28/11xFb^2 - 41/22x^2Fb + 1/2x^3F + 1/8x^4q$$

$$CD \ y(x)EJ = 387/22Fb^3 - 5/22x^2Fb$$

$$DC \ y(x)EJ = 171/11Fb^3 + 15/11xFb^2 - 5/22x^2Fb$$

$$BD \ y(x)EJ = 28/11xFb^2 - 41/22x^2Fb + 13/44x^3F$$

$$DB \ y(x)EJ = 15/11xFb^2 - 1/11x^2Fb - 13/44x^3F$$

SPOSTAMENTI NODALI

$$u_A = 0$$

$$v_A = 9(Fb^3/EJ)$$

$$\varphi_A = 0$$

$$u_B = 0$$

$$v_B = 171/11(Fb^3/EJ)$$

$$\varphi_B = 28/11(Fb^2/EJ)$$

$$u_C = 0$$

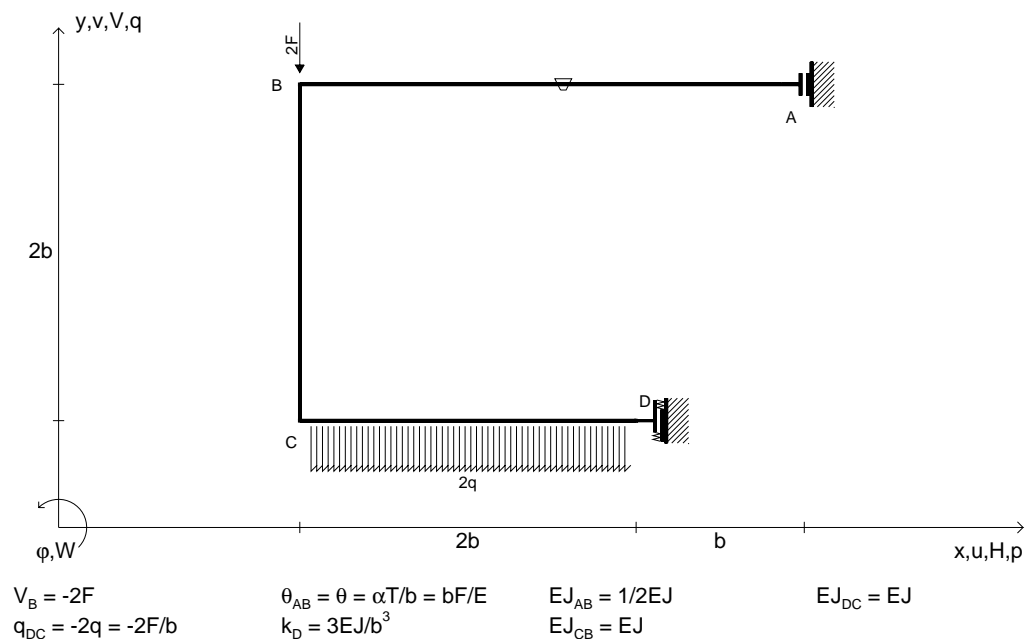
$$v_{CCD} = 387/22(Fb^3/EJ)$$

$$\varphi_C = 0$$

$$u_D = 0$$

$$v_D = 171/11(Fb^3/EJ)$$

$$\varphi_D = -15/11(Fb^2/EJ)$$



Dato: pattino in A con molla long. // al pattino.

Svolgere l'analisi cinematica.

Risolvere con PLV e LE.

Tracciare la deformata elastica.

Riportare la soluzione su questo foglio.

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi delle azioni interne nelle aste.

Esprimere la linea elastica delle aste.

Calcolare spostamento e rotazione di tutti i nodi.

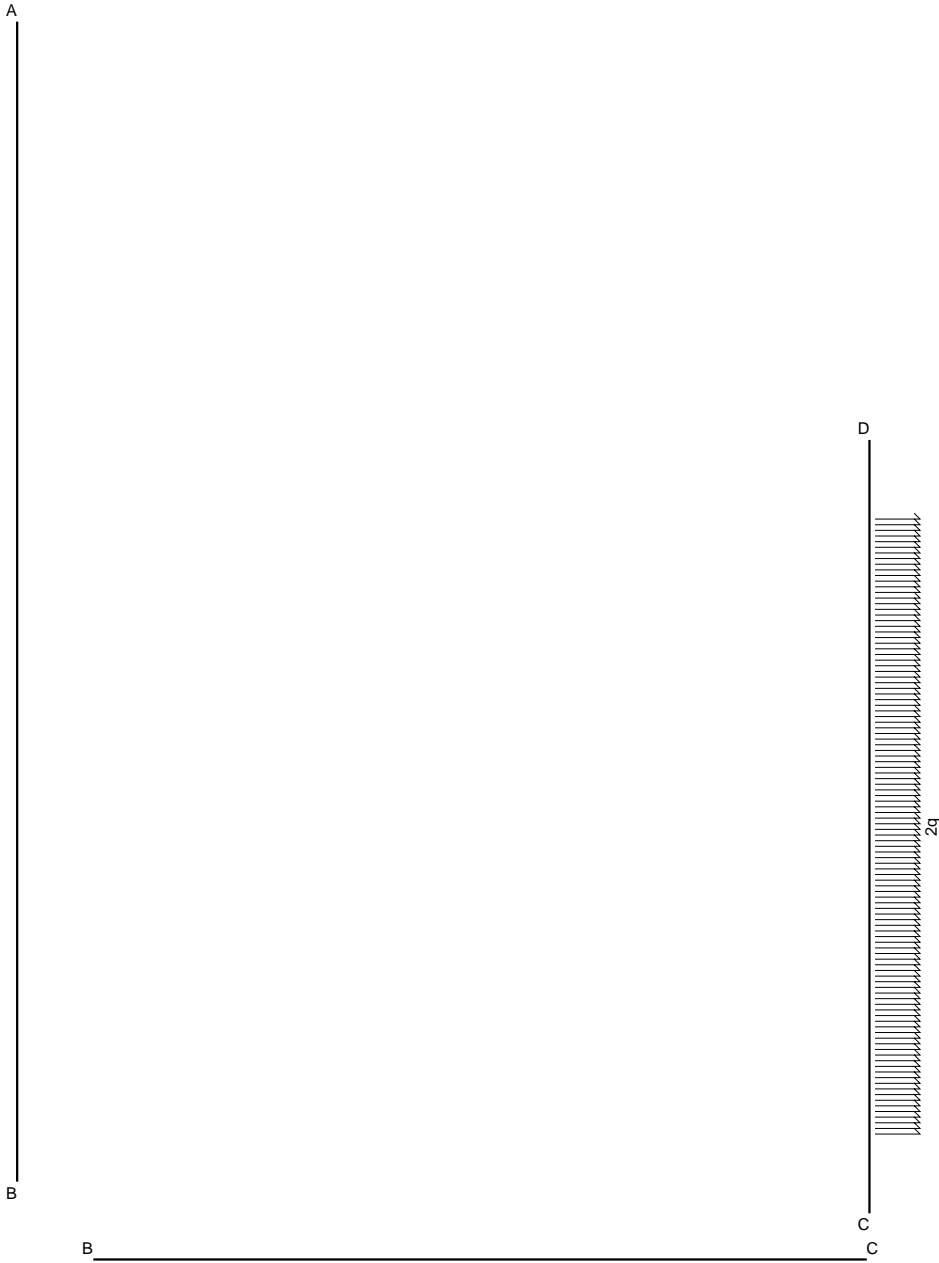
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta AB positiva se convessa a destra con inizio A.

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DEFORMATA (coordinate locali)

AB $y(x)EJ =$

CB $y(x)EJ =$

DC $y(x)EJ =$

SPOSTAMENTI NODALI

$u_A =$

$v_{AAB} =$

$\varphi_A =$

$u_B =$

$v_B =$

$\varphi_B =$

$u_C =$

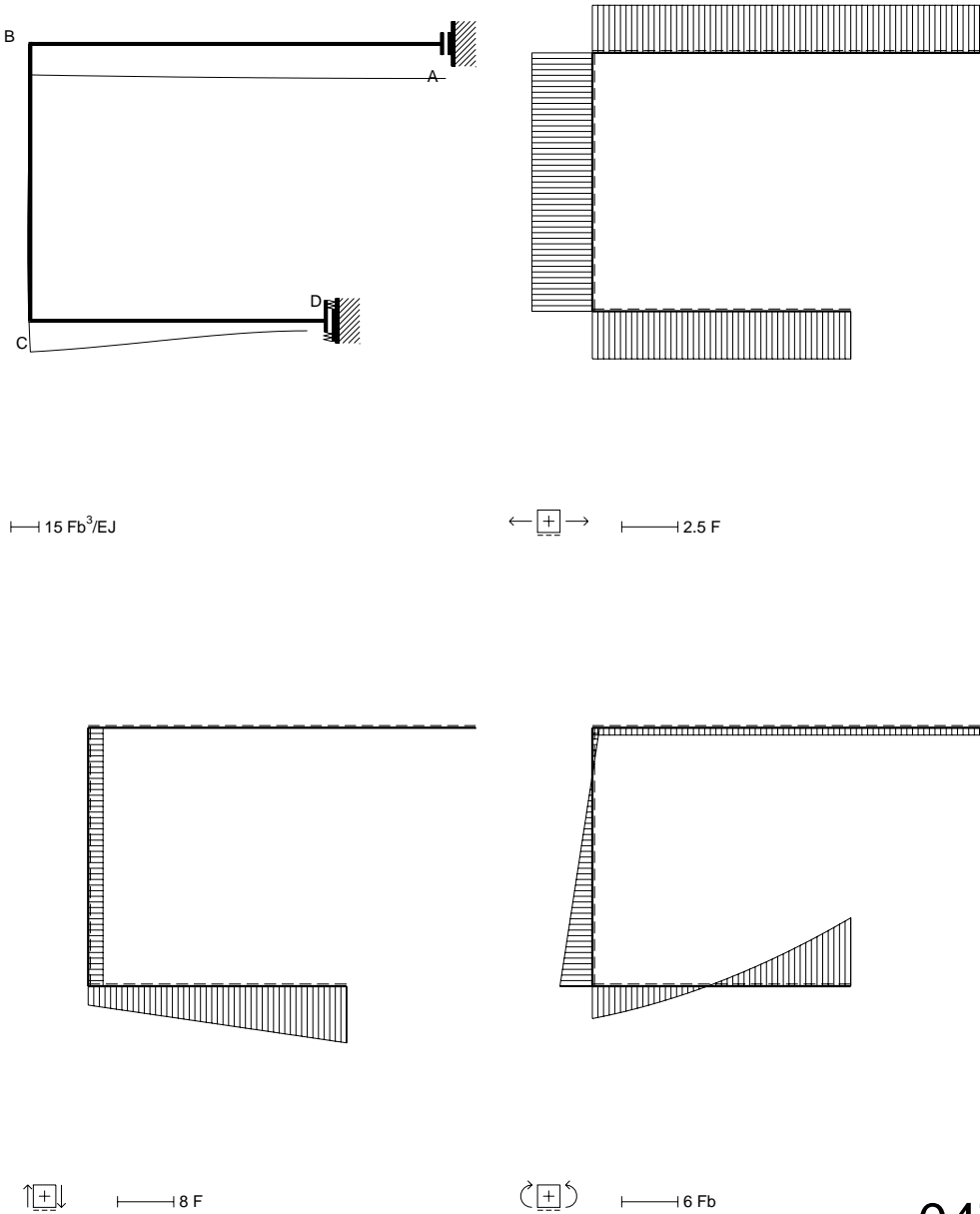
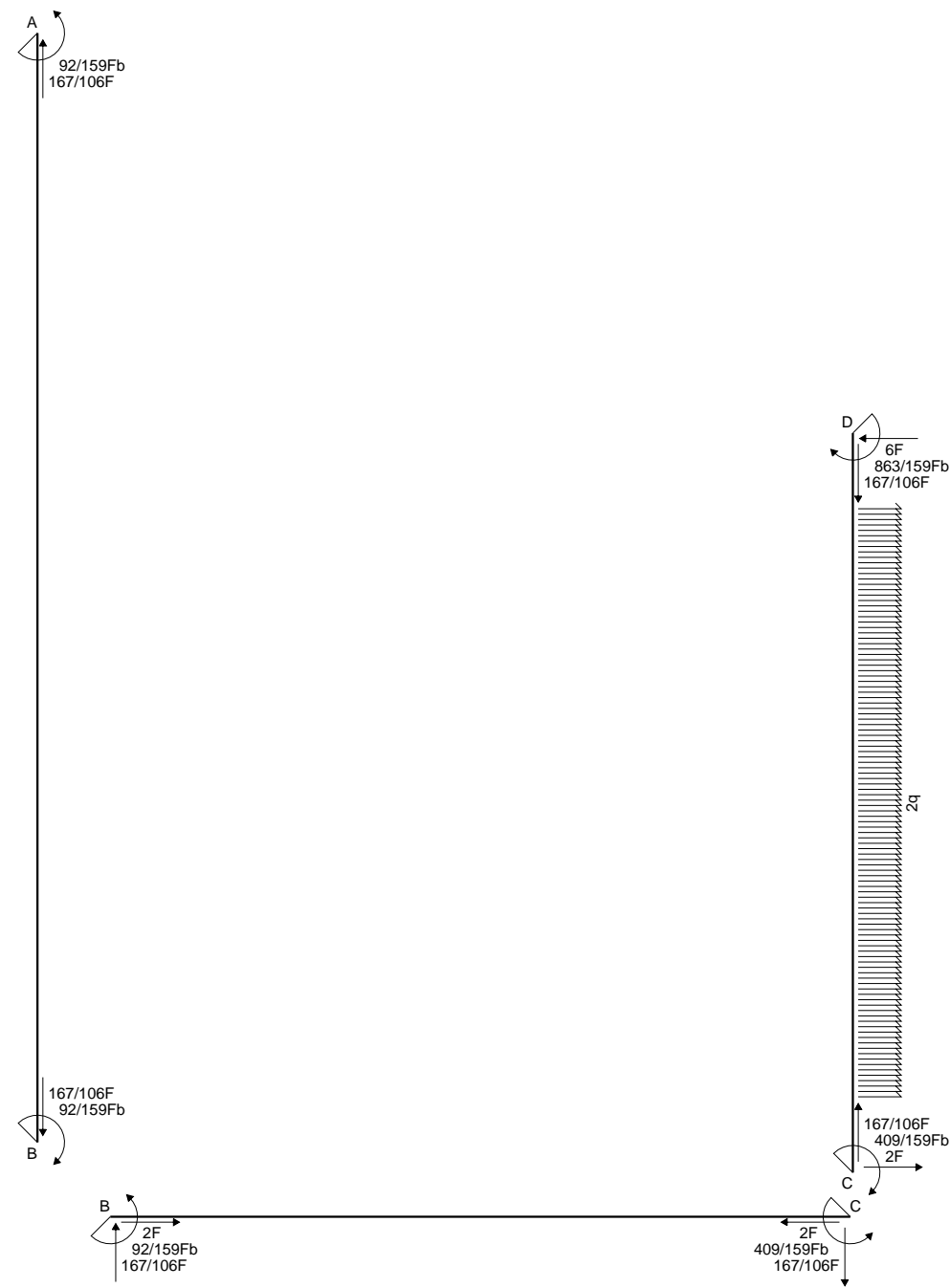
$v_C =$

$\varphi_C =$

$u_D =$

$v_D =$

$\varphi_D =$



REAZIONI IPERSTATICHE

$$X = W_{BA} \quad Y = W_{CB}$$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{CB} K_{CB} φ_{DC} K_{DC}

Relazioni di congruenza

$$\begin{aligned} y'_{AB}(0) &= 0 \\ y'_{AB}(3b) - y'_{CB}(2b) &= 0 \\ y'_{CB}(0) - y'_{DC}(2b) &= 0 \\ y'_{DC}(0) &= 0 \\ y_{CB}(0) &= 0 \\ y_{CB}(2b) &= 0 \\ y_{DC}(0) - 1/3V_D b^3/EJ &= 0 \\ y_{DC}(2b) - y_{AB}(3b) &= 0 \end{aligned}$$

$$\begin{aligned} M_{AB} &= X \\ EJy &= EJ\theta + 2X \\ EJy &= EJ\theta x + 2X + EJ\varphi_{AB} \\ EJy &= 1/2EJ\theta x^2 + 2Xb + EJ\varphi_{AB}x + EJK_{AB} \end{aligned}$$

$$\begin{aligned} M_{CB} &= -1/2X + 1/2Y - Yb \\ EJy &= -1/2X + 1/2Y - Yb \\ EJy &= -1/2X + 1/2Y - Yb + EJ\varphi_{CB} \\ EJy &= -1/2X + 1/2Y - Yb + EJ\varphi_{CB}x + EJK_{CB} \end{aligned}$$

$$\begin{aligned} M_{DC} &= -6Fx + 8Fb + qx^2 - Yb \\ EJy &= -6Fx + 8Fb + qx^2 - Yb \\ EJy &= -3Fx^2 + 8Fbx + 1/3qx^3 - Yb + EJ\varphi_{DC} \\ EJy &= -Fx^3 + 4Fbx^2 + 1/12qx^4 - Yb + EJ\varphi_{DC}x + EJK_{DC} \end{aligned}$$

Condizioni al contorno

$$\begin{bmatrix} \varphi_{AB}b & K_{AB} & \varphi_{CB}b & K_{CB} & \varphi_{DC}b & K_{DC} & Xb^2/EJ & Yb^2/EJ \end{bmatrix} \begin{bmatrix} y'_{AB} \\ y'_{BA} \\ y'_{CB} \\ y'_{DC} \\ y_{CB} \\ y_{BC} \\ y_{DC} \\ y_{CD} \end{bmatrix} = \begin{bmatrix} Fb^3/EJ & \alpha Tb \end{bmatrix}$$

Soluzione

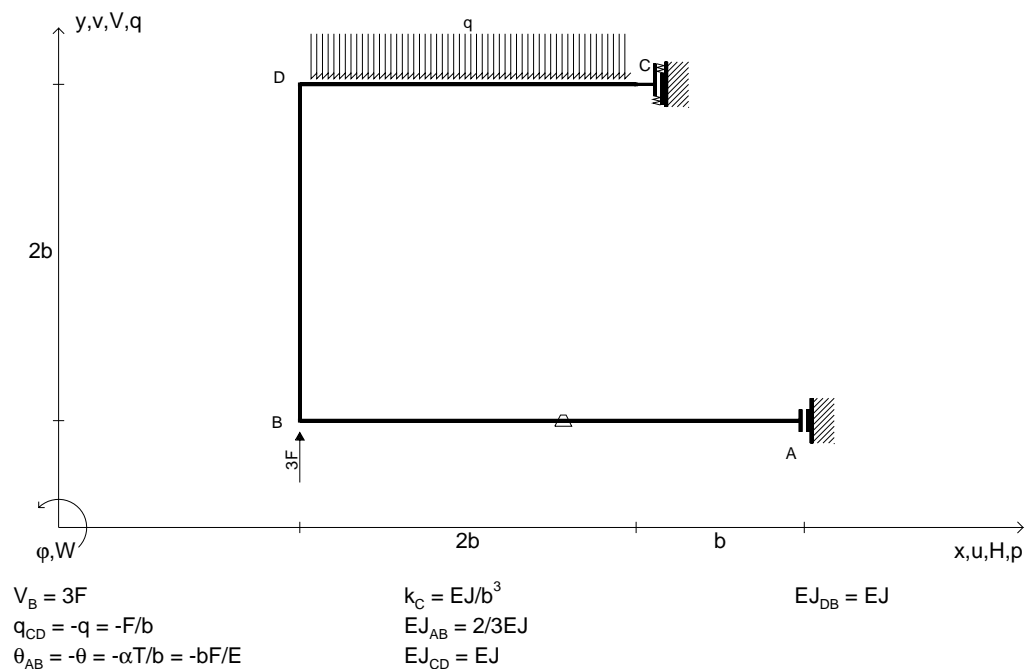
$$\begin{bmatrix} \varphi_{AB}b \\ \varphi_{CB}b \\ \varphi_{DC}b \\ Xb^2/EJ \\ K_{CB} \\ Yb^2/EJ \\ K_{DC} \\ K_{AB} \end{bmatrix} = \begin{bmatrix} 0 \\ 242/159 \\ 0 \\ -92/159 \\ 0 \\ 409/159 \\ 2 \\ 731/106 \end{bmatrix}$$

DEFORMATA (coordinate locali)

$$\begin{aligned} AB \quad y(x)EJ &= 731/106Fb^3 - 25/318x^2Fb \\ BA \quad y(x)EJ &= 328/53Fb^3 + 25/53xFb^2 - 25/318x^2Fb \\ CB \quad y(x)EJ &= 242/159xFb^2 - 409/318x^2Fb + 167/636x^3F \\ BC \quad y(x)EJ &= 25/53xFb^2 + 46/159x^2Fb - 167/636x^3F \\ DC \quad y(x)EJ &= 2Fb^3 + 863/318x^2Fb - x^3F + 1/12x^4q \\ CD \quad y(x)EJ &= 328/53Fb^3 - 242/159xFb^2 - 409/318x^2Fb + 1/3x^3F + 1/12x^4q \end{aligned}$$

SPOSTAMENTI NODALI

$$\begin{aligned} u_A &= 0 & u_B &= 0 & u_C &= 0 \\ v_{AAB} &= -731/106(Fb^3/EJ) & v_B &= -328/53(Fb^3/EJ) & v_C &= -328/53(Fb^3/EJ) \\ \varphi_A &= 0 & \varphi_B &= -25/53(Fb^2/EJ) & \varphi_C &= 242/159(Fb^2/EJ) \\ u_D &= 0 \\ v_D &= -2(Fb^3/EJ) \\ \varphi_D &= 0 \end{aligned}$$



Dato: pattino in A con molla long. // al pattino.

Svolgere l'analisi cinematica.

Risolvere con PLV e LE.

Tracciare la deformata elastica.

Riportare la soluzione su questo foglio.

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi delle azioni interne nelle aste.

Esprimere la linea elastica delle aste.

Calcolare spostamento e rotazione di tutti i nodi.

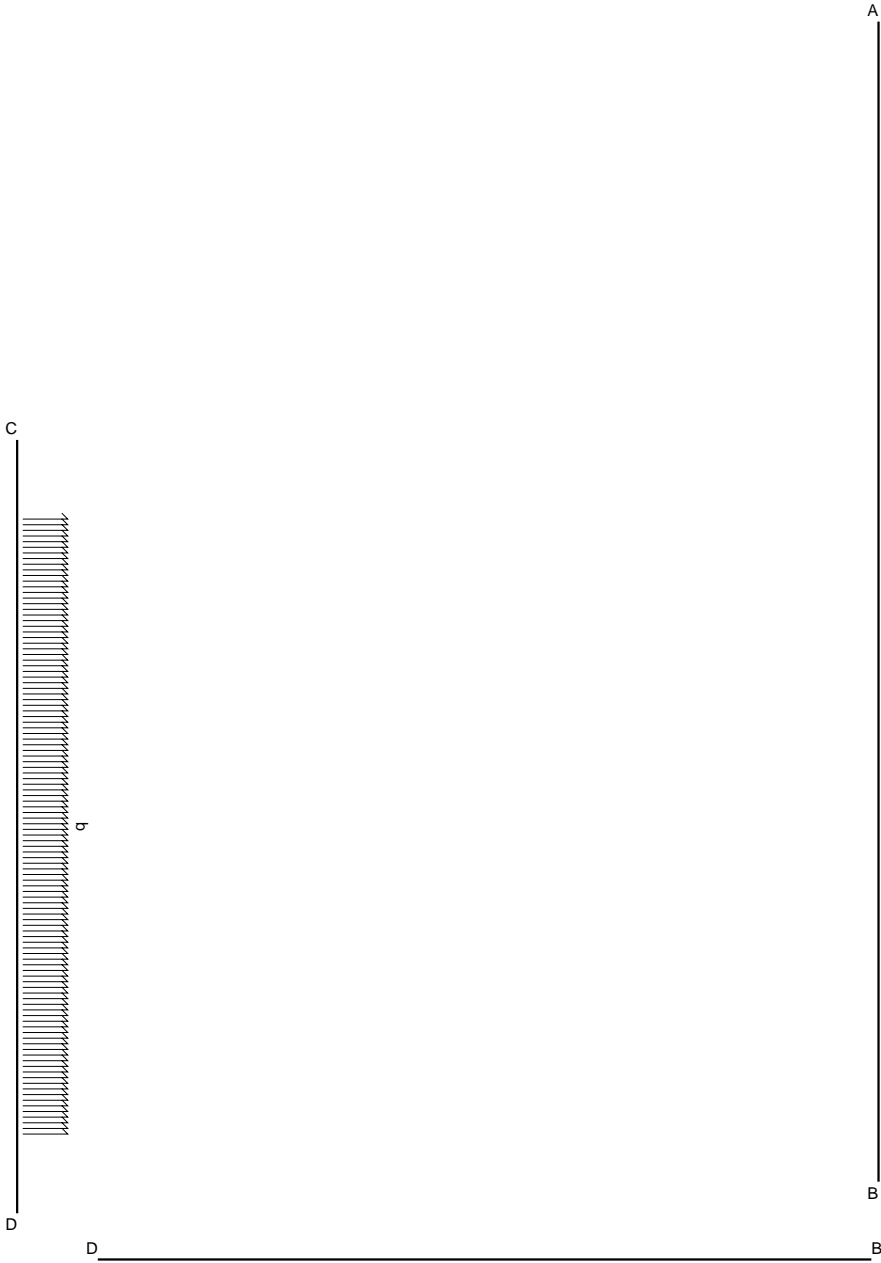
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta AB positiva se convessa a destra con inizio A.

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DEFORMATA (coordinate locali)

AB $y(x)EJ =$

CD $y(x)EJ =$

DB $y(x)EJ =$

SPOSTAMENTI NODALI

$u_A =$

$u_B =$

$u_C =$

$u_D =$

$v_{AAB} =$

$v_B =$

$v_C =$

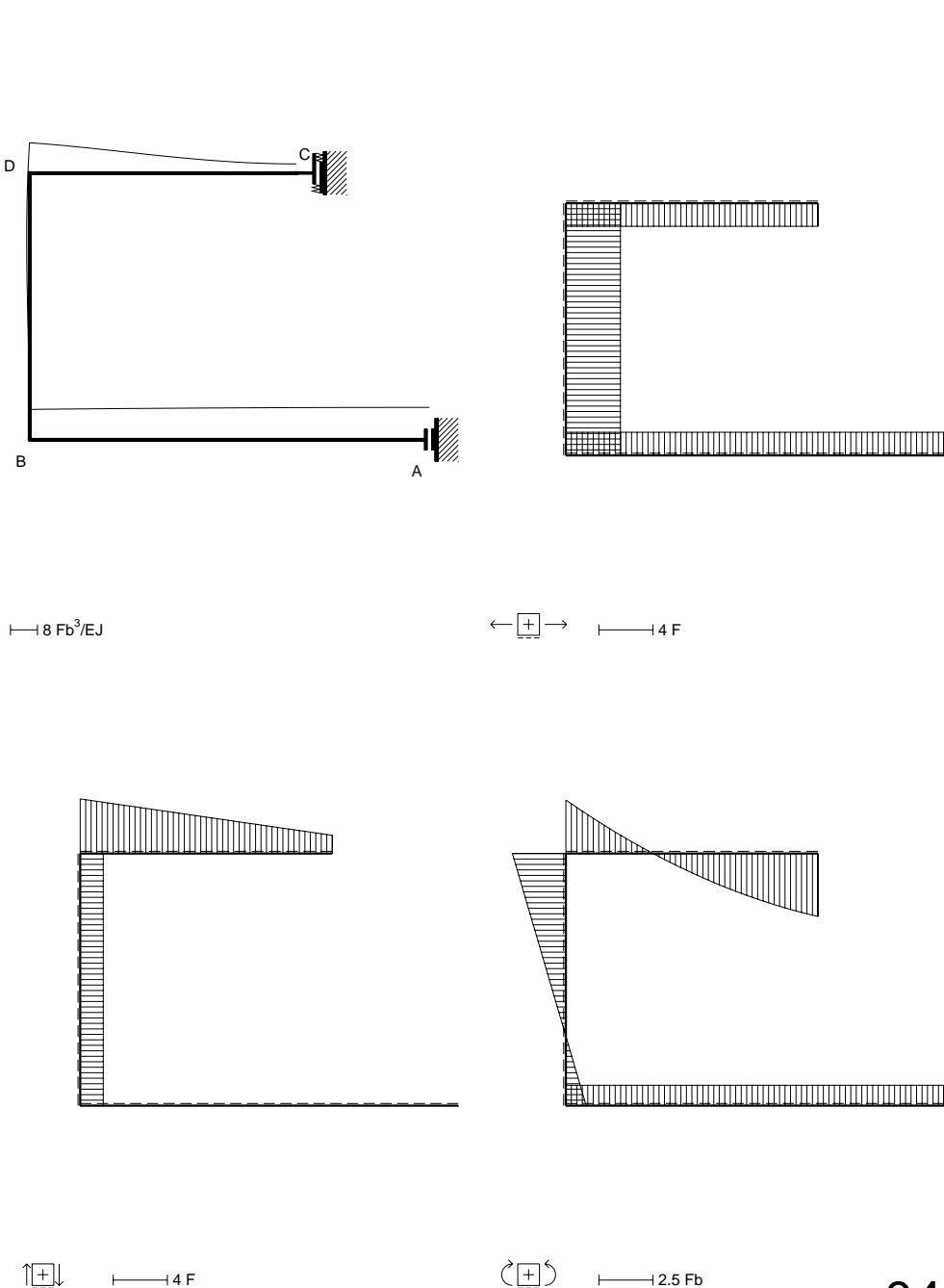
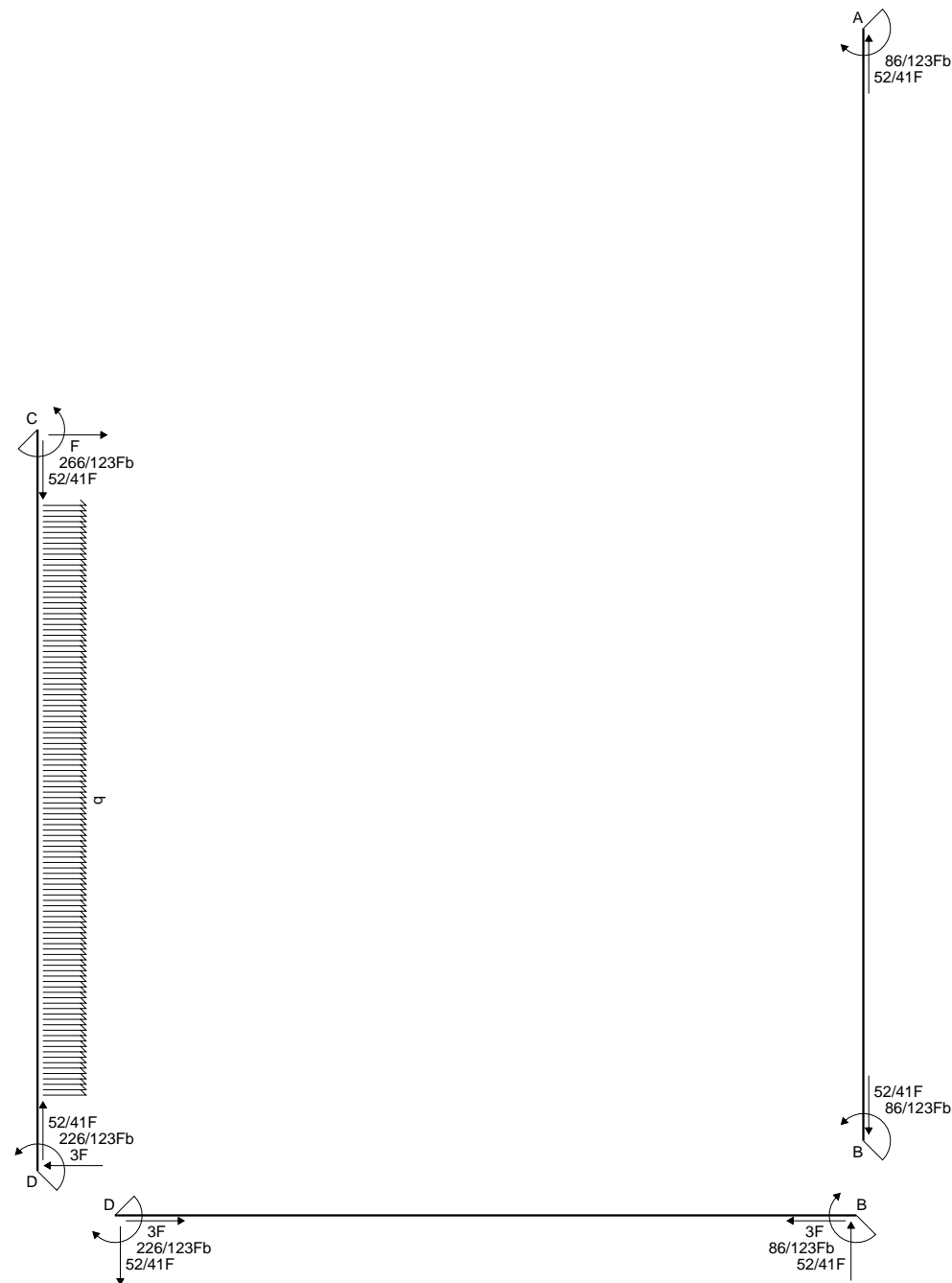
$v_D =$

$\varphi_A =$

$\varphi_B =$

$\varphi_C =$

$\varphi_D =$



REAZIONI IPERSTATICHE

$$X = W_{BA} \quad Y = W_{DB}$$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{CD} K_{CD} φ_{DB} K_{DB}

Relazioni di congruenza

$$\begin{aligned} y'_{AB}(0) &= 0 \\ y'_{AB}(3b) - y'_{DB}(2b) &= 0 \\ y'_{CD}(0) &= 0 \\ y'_{CD}(2b) - y'_{DB}(0) &= 0 \\ y_{CD}(0) - V_C b^3/EJ &= 0 \\ y_{CD}(2b) - y_{AB}(3b) &= 0 \\ y_{DB}(0) &= 0 \\ y_{DB}(2b) &= 0 \end{aligned}$$

$$\begin{aligned} M_{AB} &= X \\ EJy &= -EJ\theta + 3/2X \\ EJy &= -EJ\theta x + 3/2X + EJ\varphi_{AB} \\ EJy &= -1/2EJ\theta x^2 + 3/2Xb + EJ\varphi_{AB}x + EJK_{AB} \end{aligned}$$

$$\begin{aligned} M_{CD} &= Fx - 4Fb + 1/2qx^2 - Yb \\ EJy &= Fx - 4Fb + 1/2qx^2 - Yb \\ EJy &= 1/2Fx^2 - 4Fbx + 1/6qx^3 - Yb + EJ\varphi_{CD} \\ EJy &= 1/6Fx^3 - 2Fbx^2 + 1/24qx^4 - Yb + EJ\varphi_{CD}x + EJK_{CD} \end{aligned}$$

$$\begin{aligned} M_{DB} &= -1/2X + 1/2Y - Yb \\ EJy &= -1/2X + 1/2Y - Yb \\ EJy &= -1/2X + 1/2Y - Yb + EJ\varphi_{DB} \\ EJy &= -1/2X + 1/2Y - Yb + EJ\varphi_{DB}x + EJK_{DB} \end{aligned}$$

Condizioni al contorno

$$\begin{bmatrix} \varphi_{AB}b & K_{AB} & \varphi_{CD}b & K_{CD} & \varphi_{DB}b & K_{DB} & Xb^2/EJ & Yb^2/EJ \end{bmatrix} \begin{bmatrix} y'_{AB} \\ y'_{BA} \\ y'_{CD} \\ y'_{DC} \\ y_{CD} \\ y_{DC} \\ y_{DB} \\ y_{BD} \end{bmatrix} = \begin{bmatrix} Fb^3/EJ & \alpha Tb \end{bmatrix}$$

Soluzione

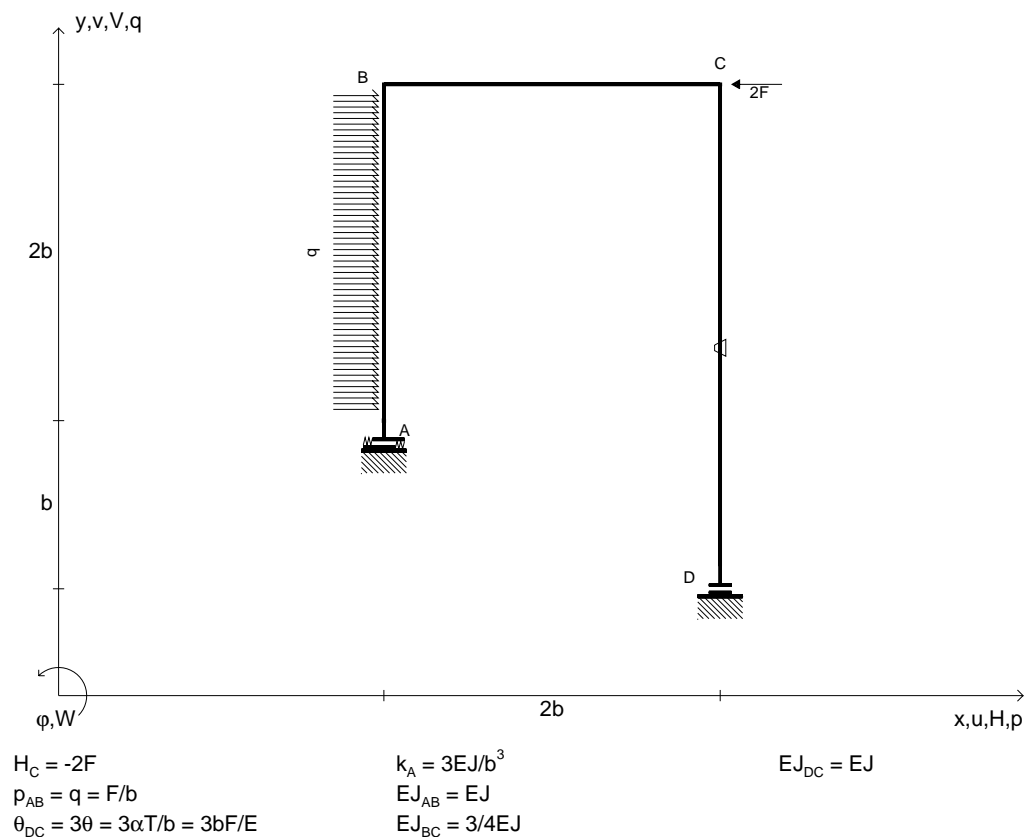
$$\begin{bmatrix} \varphi_{AB}b \\ \varphi_{DB}b \\ \varphi_{CD}b \\ Xb^2/EJ \\ K_{CD} \\ K_{AB} \\ K_{DB} \\ Yb^2/EJ \end{bmatrix} = \begin{bmatrix} Fb^3/EJ \\ 0 \\ -122/123 \\ 0 \\ 86/123 \\ -1 \\ -436/123 \\ 0 \\ -226/123 \end{bmatrix}$$

DEFORMATA (coordinate locali)

$$\begin{aligned} AB \ y(x)EJ &= -436/123Fb^3 + 1/41x^2Fb \\ BA \ y(x)EJ &= -409/123Fb^3 - 6/41xFb^2 + 1/41x^2Fb \\ CD \ y(x)EJ &= -Fb^3 - 133/123x^2Fb + 1/6x^3F + 1/24x^4q \\ DC \ y(x)EJ &= -409/123Fb^3 + 122/123xFb^2 + 113/123x^2Fb - 1/2x^3F + 1/24x^4q \\ DB \ y(x)EJ &= -122/123xFb^2 + 113/123x^2Fb - 26/123x^3F \\ BD \ y(x)EJ &= -6/41xFb^2 - 43/123x^2Fb + 26/123x^3F \end{aligned}$$

SPOSTAMENTI NODALI

$$\begin{aligned} u_A &= 0 & u_B &= 0 & u_C &= 0 & u_D &= 0 \\ v_{AAB} &= 436/123(Fb^3/EJ) & v_B &= 409/123(Fb^3/EJ) & v_C &= (Fb^3/EJ) & v_D &= 409/123(Fb^3/EJ) \\ \varphi_A &= 0 & \varphi_B &= 6/41(Fb^2/EJ) & \varphi_C &= 0 & \varphi_D &= -122/123(Fb^2/EJ) \end{aligned}$$



Dato: pattino in A con molla long. // al pattino.

Svolgere l'analisi cinematica.

Risolvere con PLV e LE.

Tracciare la deformata elastica.

Riportare la soluzione su questo foglio.

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi delle azioni interne nelle aste.

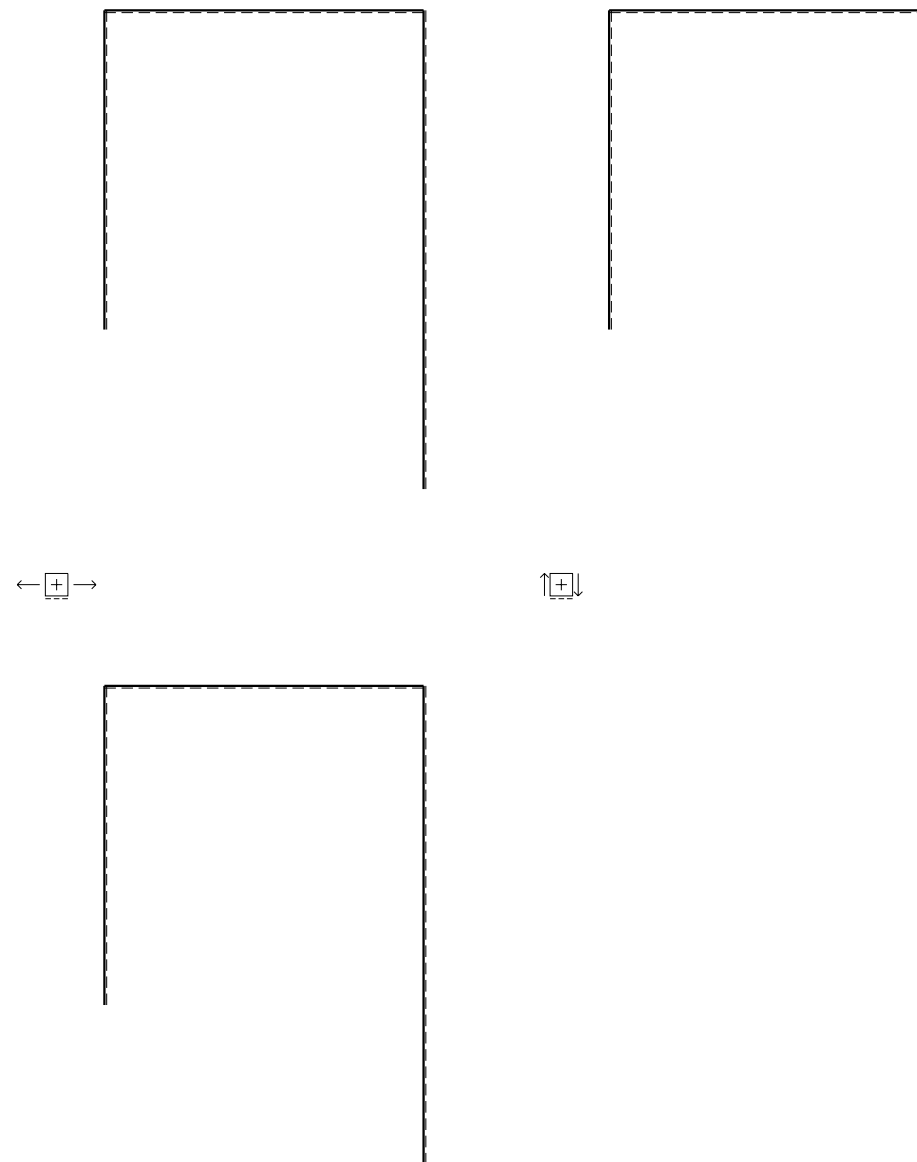
Esprimere la linea elastica delle aste.

Calcolare spostamento e rotazione di tutti i nodi.

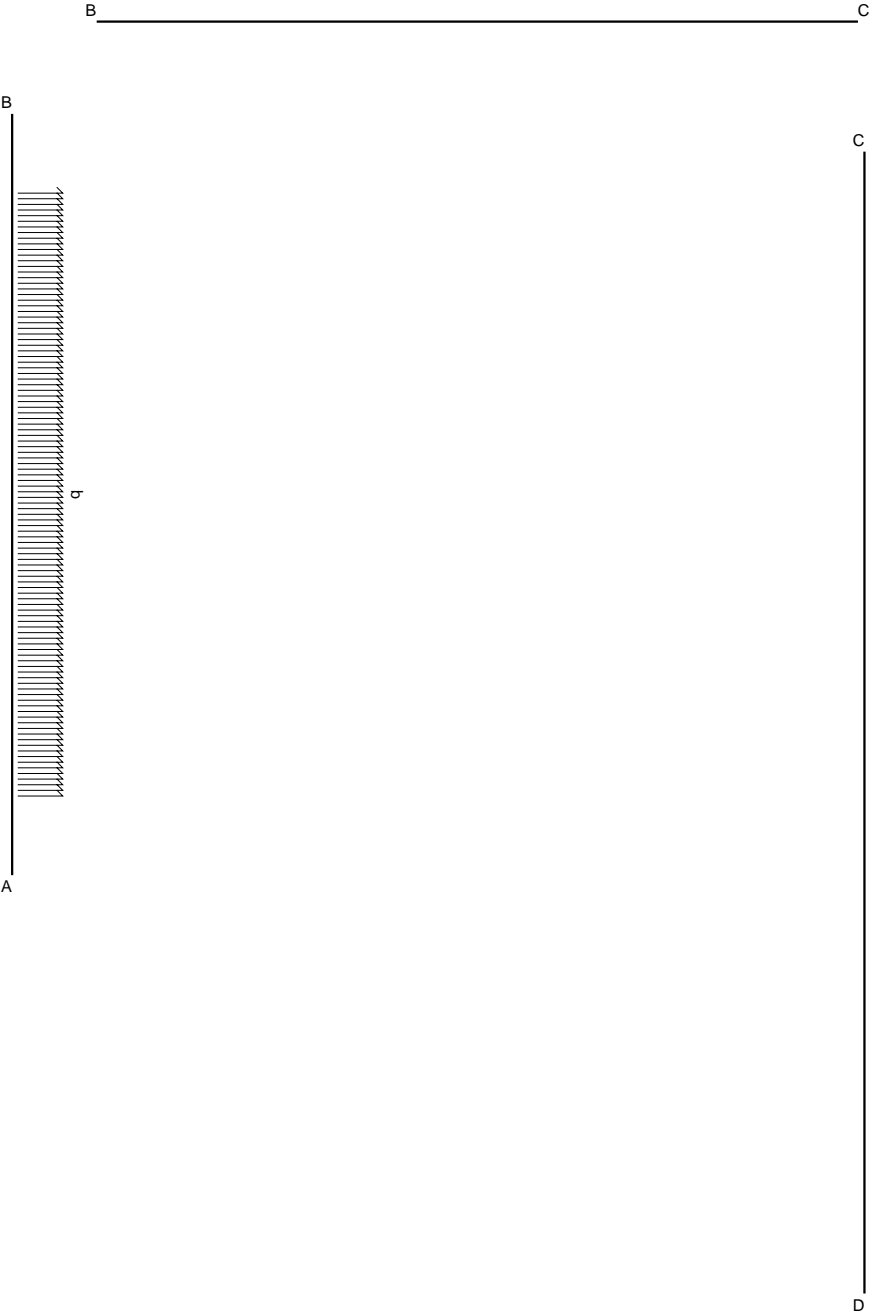
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta DC positiva se convessa a destra con inizio D.

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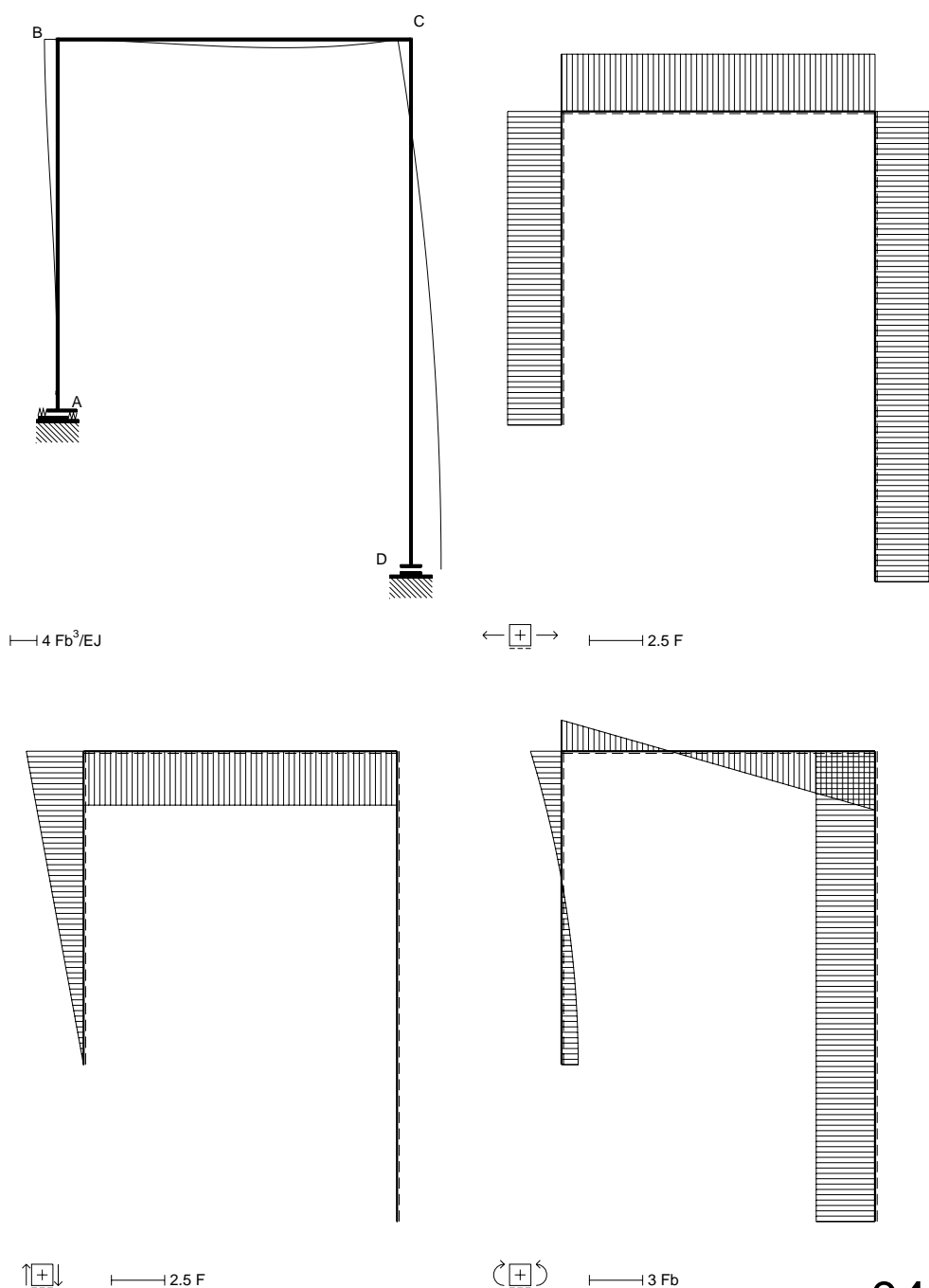
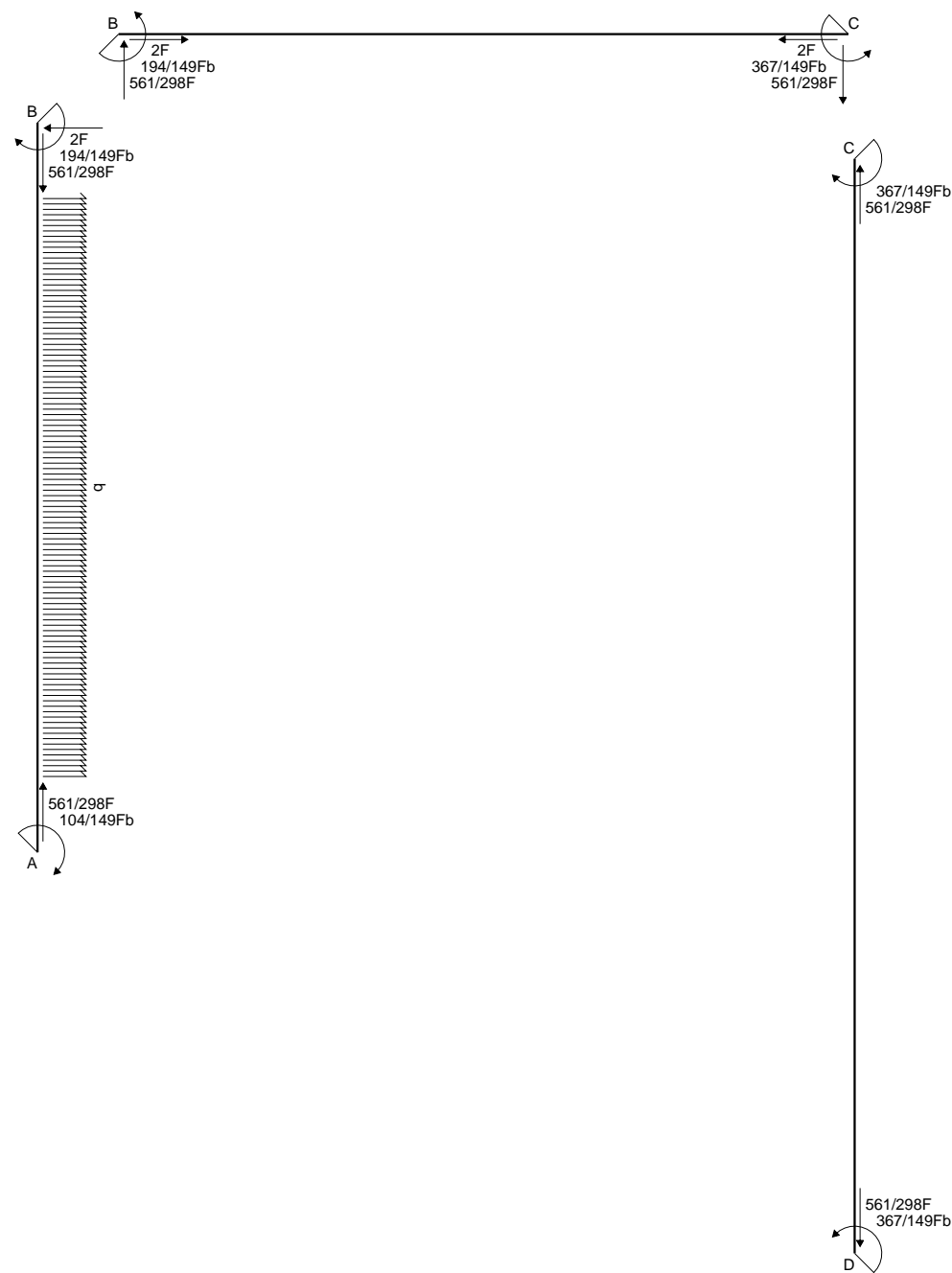


DEFORMATA (coordinate locali)

AB $y(x)EJ =$
BC $y(x)EJ =$
DC $y(x)EJ =$

SPOSTAMENTI NODALI

$u_A =$	$u_B =$	$u_C =$	$u_{DDC} =$
$v_A =$	$v_B =$	$v_C =$	$v_D =$
$\varphi_A =$	$\varphi_B =$	$\varphi_C =$	$\varphi_D =$



REAZIONI IPERSTATICHE

$$X = W_{BC} \quad Y = W_{CD}$$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{BC} K_{BC} φ_{DC} K_{DC}

Relazioni di congruenza

$$\begin{aligned} y'_{AB}(0) &= 0 \\ y'_{AB}(2b) - y'_{BC}(0) &= 0 \\ y'_{BC}(2b) - y'_{DC}(3b) &= 0 \\ y'_{DC}(0) &= 0 \\ y_{AB}(0) - 1/3H_A b^3/EJ &= 0 \\ y_{BC}(0) &= 0 \\ y_{BC}(2b) &= 0 \\ y_{DC}(3b) - y_{AB}(2b) &= 0 \end{aligned}$$

$$\begin{aligned} M_{AB} &= 2Fb - 1/2qx^2 - Xb \\ EJy &= 2Fb - 1/2qx^2 - Xb \\ EJy &= 2Fbx - 1/6qx^3 - Xb + EJ\varphi_{AB} \\ EJy &= Fbx^2 - 1/24qx^4 - Xb + EJ\varphi_{AB}x + EJK_{AB} \end{aligned}$$

$$\begin{aligned} M_{BC} &= 1/2X - Xb - 1/2Y \\ EJy &= 2/3X - 4/3Xb - 2/3Y \\ EJy &= 2/3X - 4/3Xb - 2/3Y + EJ\varphi_{BC} \\ EJy &= 2/3X - 4/3Xb - 2/3Y + EJ\varphi_{BC}x + EJK_{BC} \end{aligned}$$

$$\begin{aligned} M_{DC} &= Yb \\ EJy &= 3EJ\theta + Yb \\ EJy &= 3EJ\theta x + Yb + EJ\varphi_{DC} \\ EJy &= 3/2EJ\theta x^2 + Yb + EJ\varphi_{DC}x + EJK_{DC} \end{aligned}$$

Condizioni al contorno

$$\begin{bmatrix} \varphi_{AB}b & K_{AB} & \varphi_{BC}b & K_{BC} & \varphi_{DC}b & K_{DC} & Xb^2/EJ & Yb^2/EJ \end{bmatrix} \begin{bmatrix} y'_{AB} \\ y'_{BA} \\ y'_{CB} \\ y'_{DC} \\ y_{AB} \\ y_{BC} \\ y_{CB} \\ y_{CD} \end{bmatrix} = \begin{bmatrix} Fb^3/EJ & \alpha Tb \end{bmatrix}$$

Soluzione

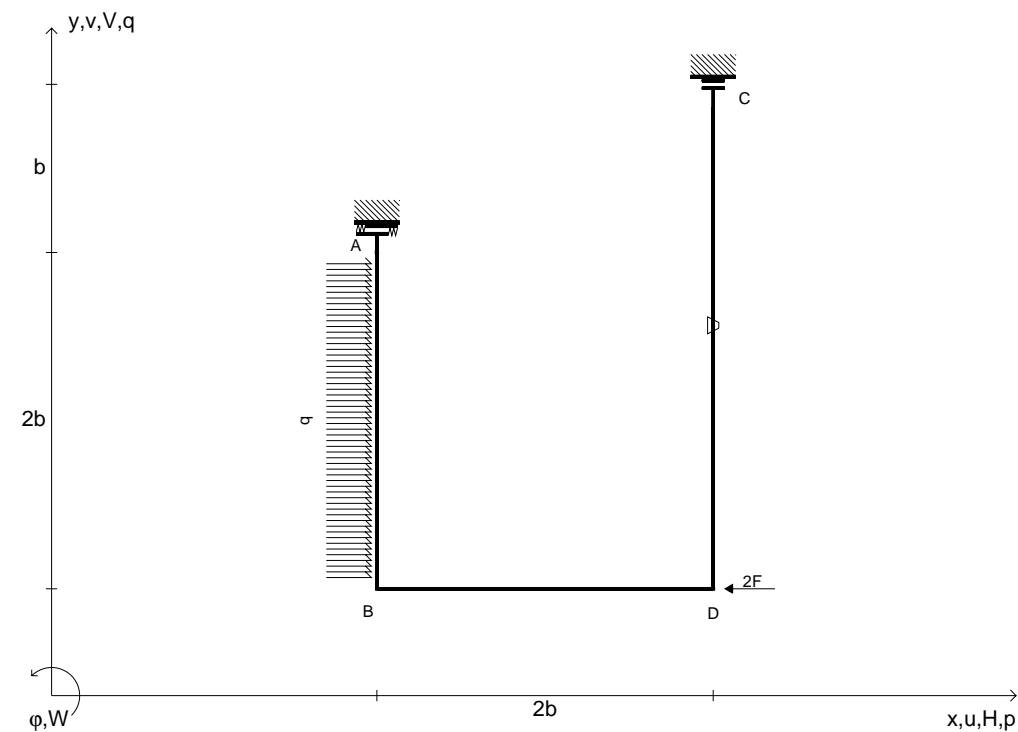
$$\begin{bmatrix} \varphi_{AB}b \\ \varphi_{BC}b \\ \varphi_{DC}b \\ Xb^2/EJ \\ K_{AB} \\ K_{BC} \\ Yb^2/EJ \\ K_{DC} \end{bmatrix} = \begin{bmatrix} Fb^3/EJ \\ 0 \\ 28/447 \\ 0 \\ 194/149 \\ 0 \\ 0 \\ -367/149 \\ -754/447 \end{bmatrix}$$

DEFORMATA (coordinate locali)

$$\begin{aligned} AB \ y(x)EJ &= 52/149x^2Fb - 1/24x^4q \\ BA \ y(x)EJ &= 326/447Fb^3 - 28/447xFb^2 - 97/149x^2Fb + 1/3x^3F - 1/24x^4q \\ BC \ y(x)EJ &= 28/447xFb^2 - 388/447x^2Fb + 187/447x^3F \\ CB \ y(x)EJ &= -240/149xFb^2 + 734/447x^2Fb - 187/447x^3F \\ DC \ y(x)EJ &= -754/447Fb^3 + 40/149x^2Fb \\ CD \ y(x)EJ &= 326/447Fb^3 - 240/149xFb^2 + 40/149x^2Fb \end{aligned}$$

SPOSTAMENTI NODALI

$$\begin{aligned} u_A &= 0 & u_B &= -326/447(Fb^3/EJ) & u_C &= -326/447(Fb^3/EJ) & u_{DDC} &= 754/447(Fb^3/EJ) \\ v_A &= 0 & v_B &= 0 & v_C &= 0 & v_D &= 0 \\ \varphi_A &= 0 & \varphi_B &= 28/447(Fb^2/EJ) & \varphi_C &= 240/149(Fb^2/EJ) & \varphi_D &= 0 \end{aligned}$$



$$H_D = -2F$$

$$p_{AB} = q = F/b$$

$$\theta_{CD} = 3\theta = 3\alpha T/b = 3bF/E$$

$$k_A = 3EJ/b^3$$

$$EJ_{AB} = EJ$$

$$EJ_{CD} = EJ$$

$$EJ_{BD} = EJ$$

Dato: pattino in A con molla long. // al pattino.

Svolgere l'analisi cinematica.

Risolvere con PLV e LE.

Tracciare la deformata elastica.

Riportare la soluzione su questo foglio.

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi delle azioni interne nelle aste.

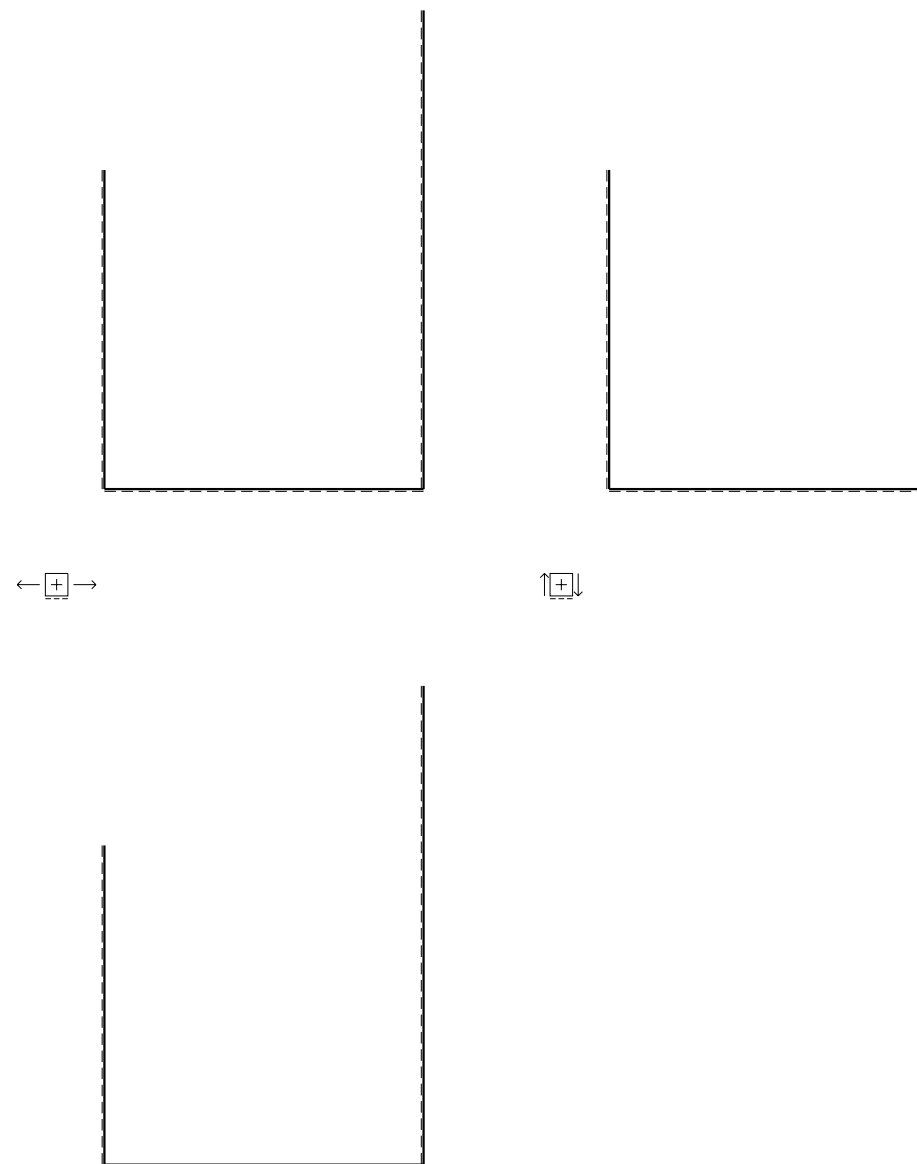
Esprimere la linea elastica delle aste.

Calcolare spostamento e rotazione di tutti i nodi.

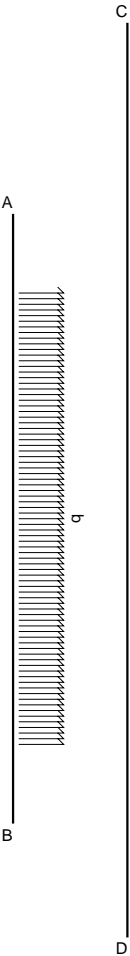
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta CD positiva se convessa a destra con inizio C.

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B _____ D

DEFORMATA (coordinate locali)

AB $y(x)EJ =$

CD $y(x)EJ =$

BD $y(x)EJ =$

SPOSTAMENTI NODALI

$u_A =$

$u_B =$

$u_{CCD} =$

$u_D =$

$v_A =$

$v_B =$

$v_C =$

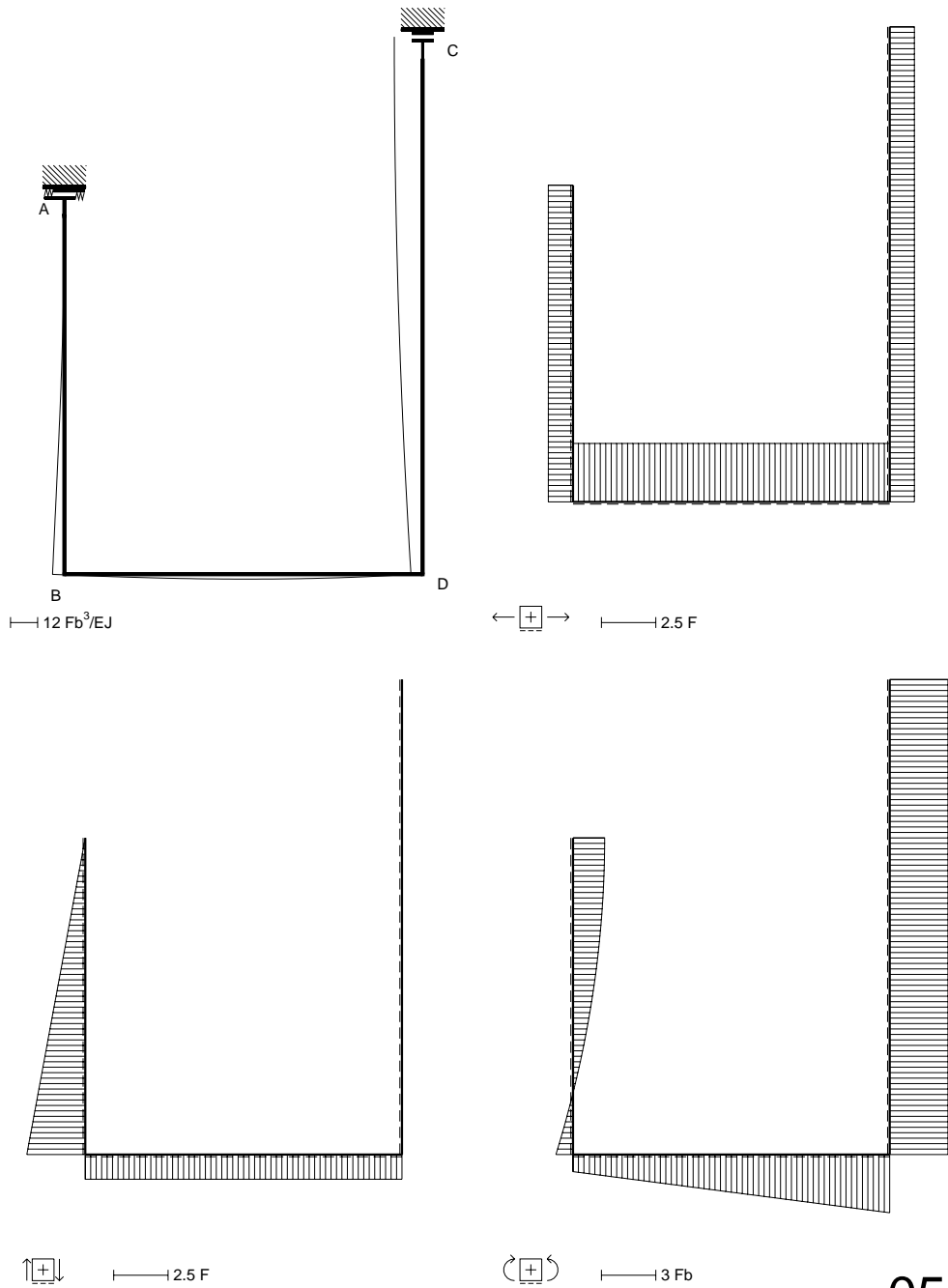
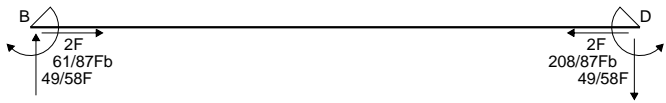
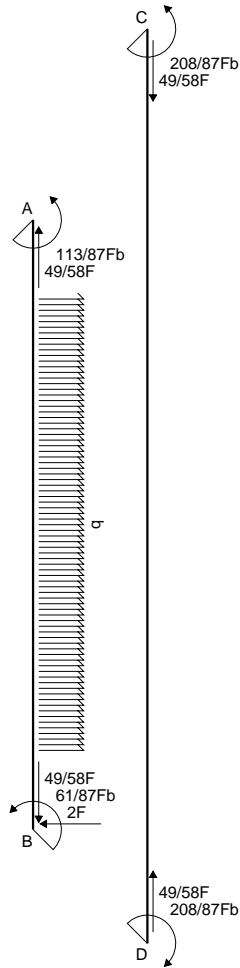
$v_D =$

$\varphi_A =$

$\varphi_B =$

$\varphi_C =$

$\varphi_D =$



REAZIONI IPERSTATICHE

$$X = W_{DC} \quad Y = W_{BD}$$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{CD} K_{CD} φ_{BD} K_{BD}

Relazioni di congruenza

$$y'_{AB}(0) = 0$$

$$y'_{AB}(2b) - y'_{BD}(0) = 0$$

$$y'_{CD}(0) = 0$$

$$y'_{CD}(3b) - y'_{BD}(2b) = 0$$

$$y_{AB}(0) + 1/3H_A b^3/EJ = 0$$

$$y_{CD}(3b) - y_{AB}(2b) = 0$$

$$y_{BD}(0) = 0$$

$$y_{BD}(2b) = 0$$

$$M_{AB} = -2Fb + 1/2qx^2 - Y$$

$$EJy = -2Fb + 1/2qx^2 - Y$$

$$EJy = -2Fbx + 1/6qx^3 - Y + EJ\varphi_{AB}$$

$$EJy = -Fbx^2 + 1/24qx^4 - Yb + EJ\varphi_{AB}x + EJK_{AB}$$

$$M_{CD} = Xb$$

$$EJy = 3EJ\theta + Xb$$

$$EJy = 3EJ\theta x + Xb + EJ\varphi_{CD}$$

$$EJy = 3/2EJ\theta x^2 + Xbx + EJ\varphi_{CD}x + EJK_{CD}$$

$$M_{BD} = -1/2X + 1/2Y - Yb$$

$$EJy = -1/2X + 1/2Y - Yb$$

$$EJy = -1/2X + 1/2Y - Yb + EJ\varphi_{BD}$$

$$EJy = -1/2X + 1/2Y - Yb + EJ\varphi_{BD}x + EJK_{BD}$$

Condizioni al contorno

$$\begin{bmatrix} \varphi_{AB}b & K_{AB} & \varphi_{CD}b & K_{CD} & \varphi_{BD}b & K_{BD} & Xb^2/EJ & Yb^2/EJ \end{bmatrix} \begin{bmatrix} y'_{AB} \\ y'_{BA} \\ y'_{CD} \\ y'_{DC} \\ y_{AB} \\ y_{DC} \\ y_{BD} \\ y_{DB} \end{bmatrix} = \begin{bmatrix} Fb^3/EJ & \alpha Tb \end{bmatrix}$$

Soluzione

$$\begin{bmatrix} \varphi_{AB}b \\ \varphi_{BD}b \\ \varphi_{CD}b \\ Xb^2/EJ \\ K_{AB} \\ K_{CD} \\ K_{BD} \\ Yb^2/EJ \end{bmatrix} = \begin{bmatrix} 0 \\ -110/87 \\ 0 \\ -208/87 \\ 0 \\ -271/58 \\ 0 \\ -61/87 \end{bmatrix} \begin{bmatrix} Fb^3/EJ \end{bmatrix}$$

DEFORMATA (coordinate locali)

$$AB \ y(x)EJ = -113/174x^2Fb + 1/24x^4q$$

$$BA \ y(x)EJ = -56/29Fb^3 + 110/87xFb^2 + 61/174x^2Fb - 1/3x^3F + 1/24x^4q$$

$$CD \ y(x)EJ = -271/58Fb^3 + 53/174x^2Fb$$

$$DC \ y(x)EJ = -56/29Fb^3 - 53/29xFb^2 + 53/174x^2Fb$$

$$BD \ y(x)EJ = -110/87xFb^2 + 61/174x^2Fb + 49/348x^3F$$

$$DB \ y(x)EJ = -53/29xFb^2 + 104/87x^2Fb - 49/348x^3F$$

SPOSTAMENTI NODALI

$$u_A = 0$$

$$u_B = -56/29(Fb^3/EJ)$$

$$u_{CCD} = -271/58(Fb^3/EJ)$$

$$u_D = -56/29(Fb^3/EJ)$$

$$v_A = 0$$

$$v_B = 0$$

$$v_C = 0$$

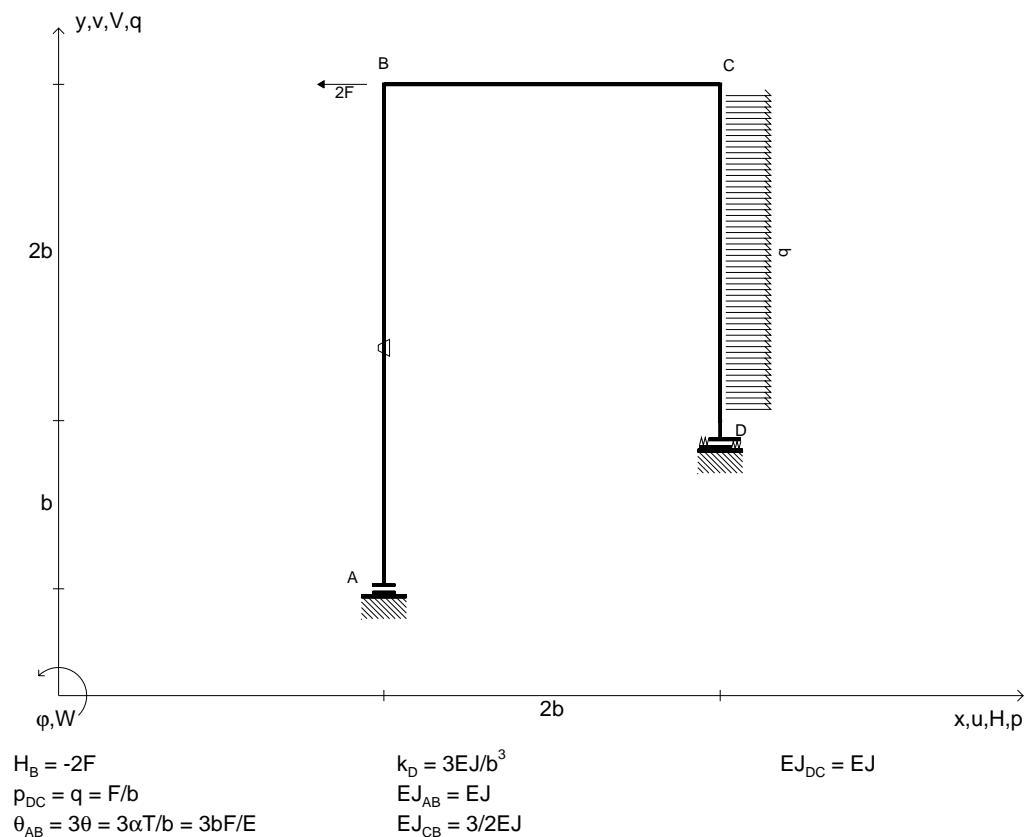
$$v_D = 0$$

$$\varphi_A = 0$$

$$\varphi_B = -110/87(Fb^2/EJ)$$

$$\varphi_C = 0$$

$$\varphi_D = 53/29(Fb^2/EJ)$$



Dato: pattino in A con molla long. // al pattino.

Svolgere l'analisi cinematica.

Risolvere con PLV e LE.

Tracciare la deformata elastica.

Riportare la soluzione su questo foglio.

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi delle azioni interne nelle aste.

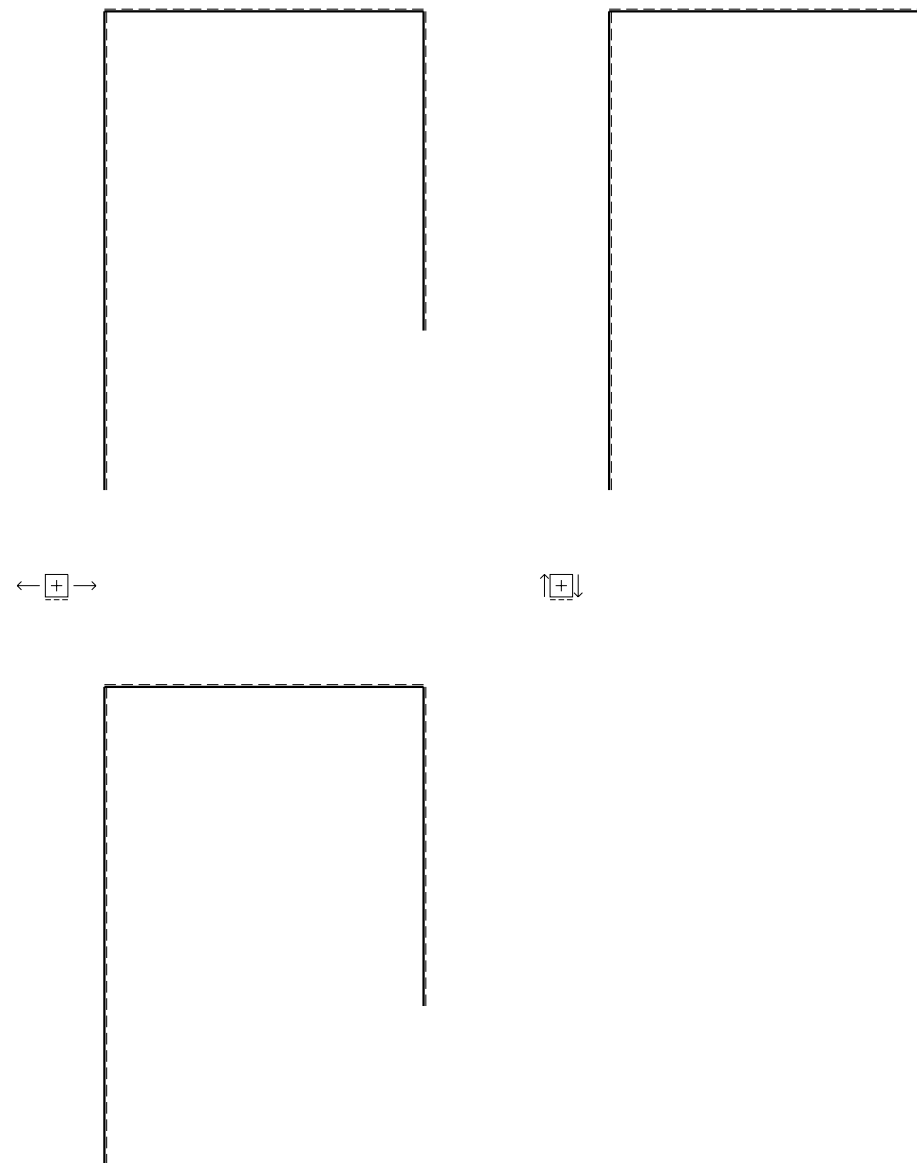
Esprimere la linea elastica delle aste.

Calcolare spostamento e rotazione di tutti i nodi.

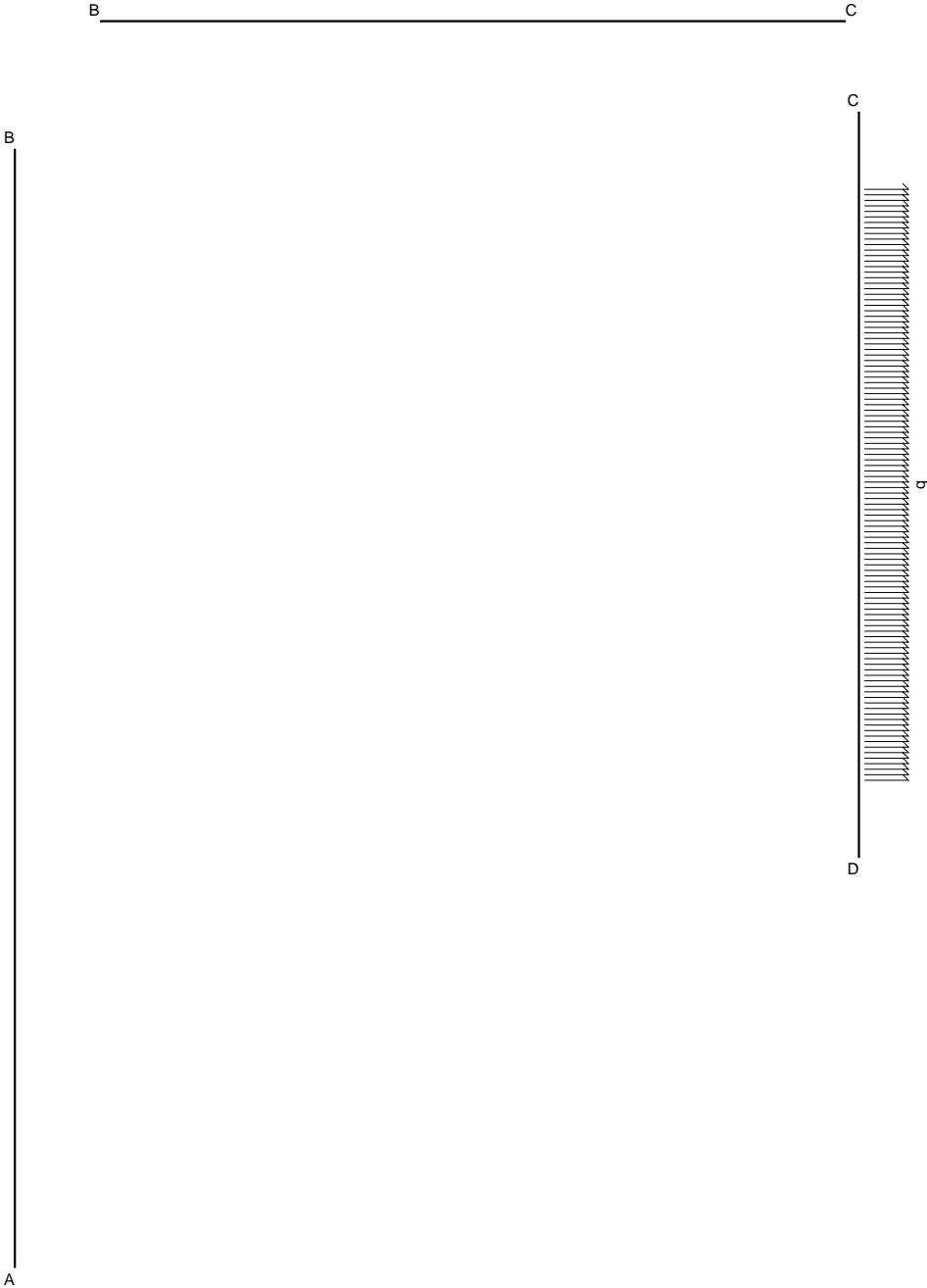
$J_{YZ} - x_{YZ} - \theta_{YZ}$ riferimento locale asta YZ con origine in Y.

Curvatura θ asta AB positiva se convessa a destra con inizio A.

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DEFORMATA (coordinate locali)

AB $y(x)EJ =$

CB $y(x)EJ =$

DC $y(x)EJ =$

SPOSTAMENTI NODALI

$u_{AAB} =$

$u_B =$

$u_C =$

$u_D =$

$v_A =$

$v_B =$

$v_C =$

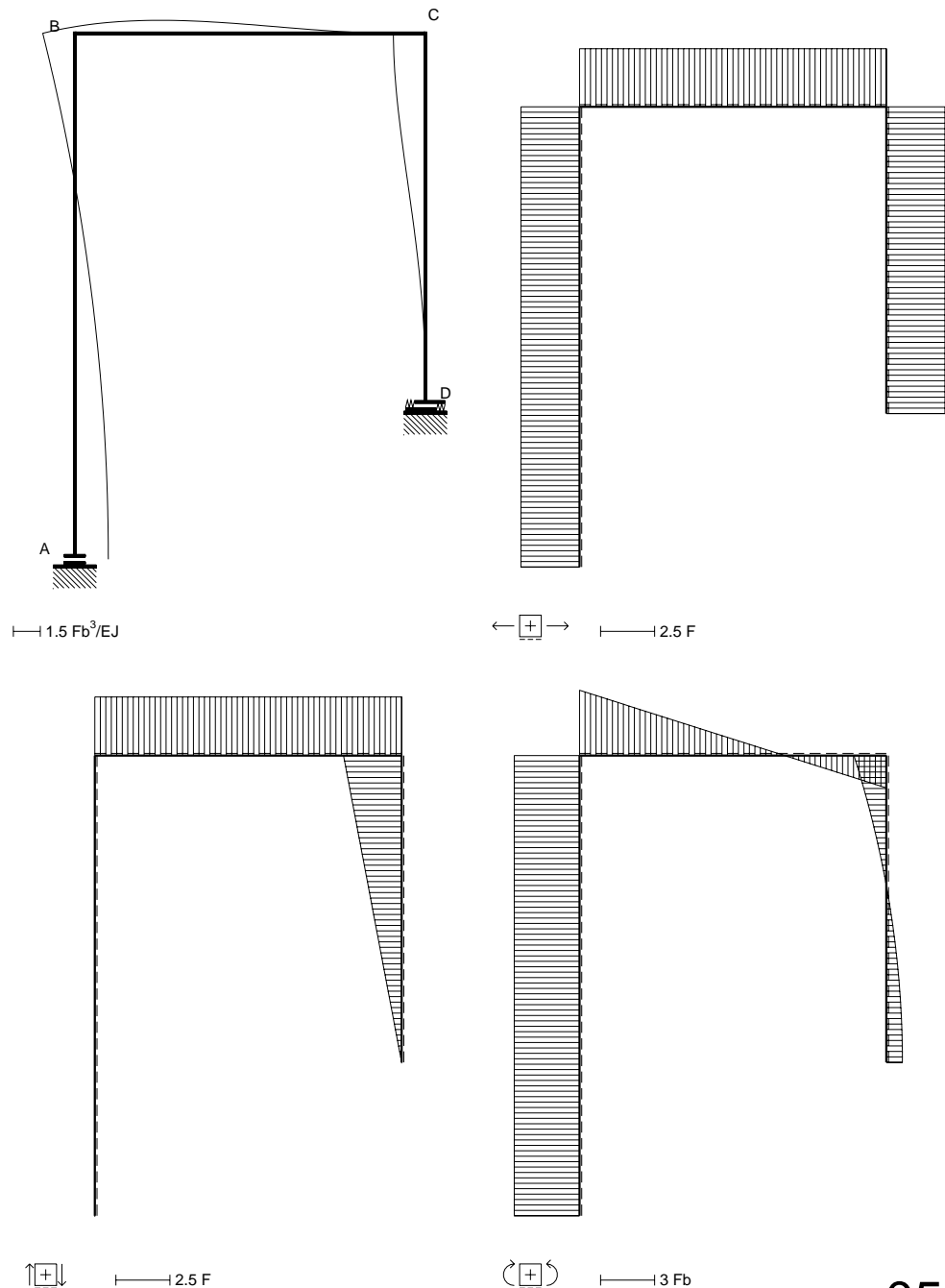
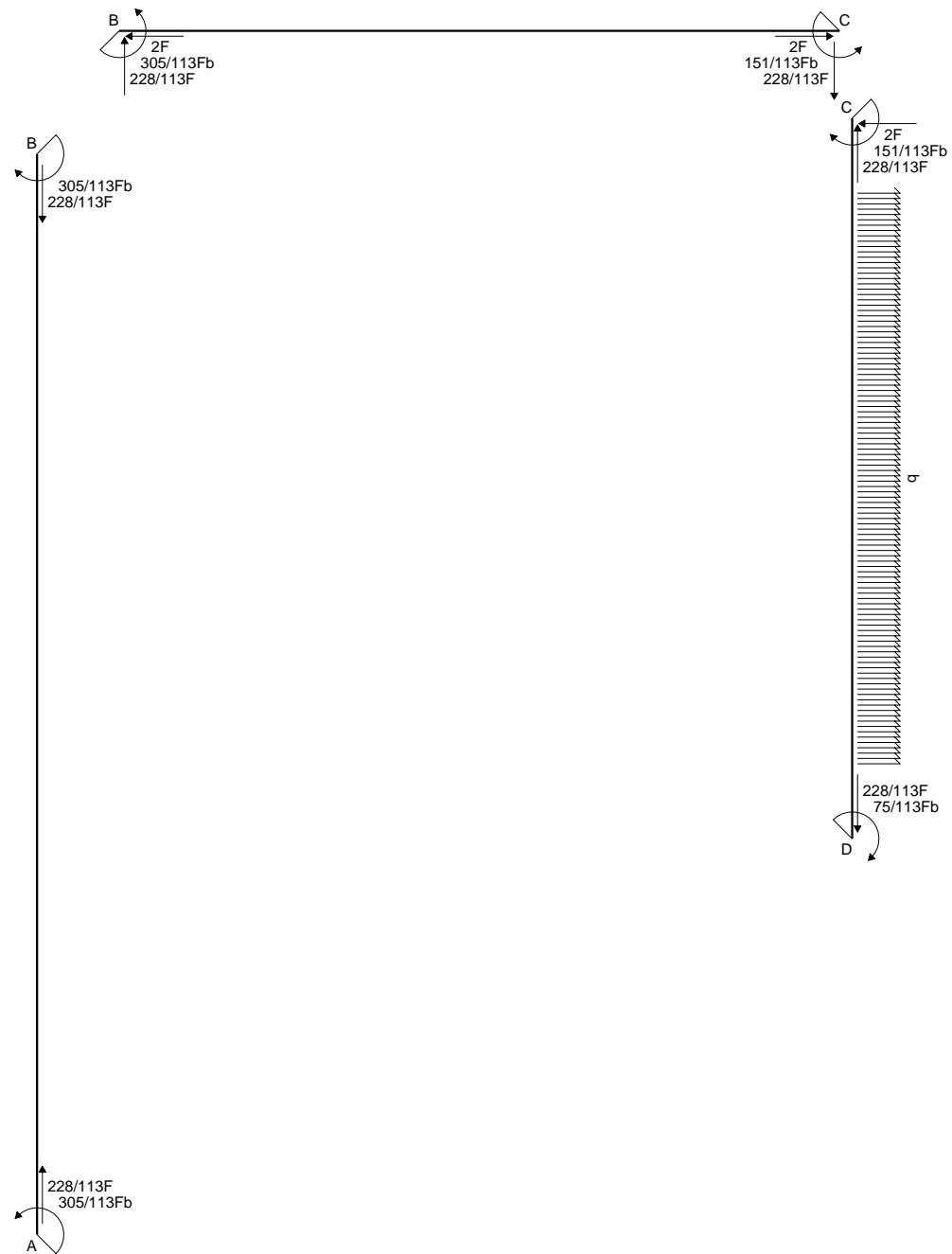
$v_D =$

$\varphi_A =$

$\varphi_B =$

$\varphi_C =$

$\varphi_D =$



REAZIONI IPERSTATICHE

$$X = W_{BA} \quad Y = W_{CB}$$

DETERMINAZIONE DELLA DEFORMATA ELASTICA

Costanti di integrazione: φ_{AB} K_{AB} φ_{CB} K_{CB} φ_{DC} K_{DC}

Relazioni di congruenza

$$\begin{aligned} y'_{AB}(0) &= 0 \\ y'_{AB}(3b) - y'_{CB}(2b) &= 0 \\ y'_{CB}(0) - y'_{DC}(2b) &= 0 \\ y'_{DC}(0) &= 0 \\ y_{CB}(0) &= 0 \\ y_{CB}(2b) &= 0 \\ y_{DC}(0) - 1/3HDb^3/EJ &= 0 \\ y_{DC}(2b) - y_{AB}(3b) &= 0 \end{aligned}$$

$$\begin{aligned} M_{AB} &= Xb \\ EJy &= 3EJ\theta + Xb \\ EJy &= 3EJ\theta x + Xb + EJ\varphi_{AB} \\ EJy &= 3/2EJ\theta x^2 + Xb + EJ\varphi_{AB}x + EJK_{AB} \end{aligned}$$

$$\begin{aligned} M_{CB} &= -1/2X + 1/2Y - Yb \\ EJy &= -1/3X + 1/3Y - 2/3Yb \\ EJy &= -1/3X + 1/3Y - 2/3Yb + EJ\varphi_{CB} \\ EJy &= -1/3X + 1/3Y - 2/3Yb + EJ\varphi_{CB}x + EJK_{CB} \end{aligned}$$

$$\begin{aligned} M_{DC} &= 2Fb - 1/2qx^2 - Yb \\ EJy &= 2Fb - 1/2qx^2 - Yb \\ EJy &= 2Fbx - 1/6qx^3 - Yb + EJ\varphi_{DC} \\ EJy &= Fbx^2 - 1/24qx^4 - Yb + EJ\varphi_{DC}x + EJK_{DC} \end{aligned}$$

Condizioni al contorno

$$\begin{bmatrix} \varphi_{AB}b & K_{AB} & \varphi_{CB}b & K_{CB} & \varphi_{DC}b & K_{DC} & Xb^2/EJ & Yb^2/EJ \end{bmatrix} \begin{bmatrix} y'_{AB} \\ y'_{BA} \\ y'_{CB} \\ y'_{DC} \\ y_{CB} \\ y_{BC} \\ y_{DC} \\ y_{CD} \end{bmatrix} = \begin{bmatrix} Fb^3/EJ & \alpha Tb \end{bmatrix}$$

Soluzione

$$\begin{bmatrix} \varphi_{AB}b \\ \varphi_{CB}b \\ \varphi_{DC}b \\ Xb^2/EJ \\ K_{CB} \\ Yb^2/EJ \\ K_{DC} \\ K_{AB} \end{bmatrix} = \begin{bmatrix} Fb^3/EJ \\ 0 \\ -2/339 \\ 0 \\ -305/113 \\ 0 \\ 151/113 \\ 0 \\ -235/339 \end{bmatrix}$$

DEFORMATA (coordinate locali)

$$\begin{aligned} AB \ y(x)EJ &= -235/339Fb^3 + 17/113x^2Fb \\ BA \ y(x)EJ &= 224/339Fb^3 - 102/113xFb^2 + 17/113x^2Fb \\ CB \ y(x)EJ &= -2/339xFb^2 - 151/339x^2Fb + 76/339x^3F \\ BC \ y(x)EJ &= -102/113xFb^2 + 305/339x^2Fb - 76/339x^3F \\ DC \ y(x)EJ &= 75/226x^2Fb - 1/24x^4q \\ CD \ y(x)EJ &= 224/339Fb^3 + 2/339xFb^2 - 151/226x^2Fb + 1/3x^3F - 1/24x^4q \end{aligned}$$

SPOSTAMENTI NODALI

$$\begin{aligned} u_{AAB} &= 235/339(Fb^3/EJ) & u_B &= -224/339(Fb^3/EJ) & u_C &= -224/339(Fb^3/EJ) & u_D &= 0 \\ v_A &= 0 & v_B &= 0 & v_C &= 0 & v_D &= 0 \\ \varphi_A &= 0 & \varphi_B &= 102/113(Fb^2/EJ) & \varphi_C &= -2/339(Fb^2/EJ) & \varphi_D &= 0 \end{aligned}$$