

Svolgere l'analisi cinematica.

Riportare la soluzione su questo foglio.

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

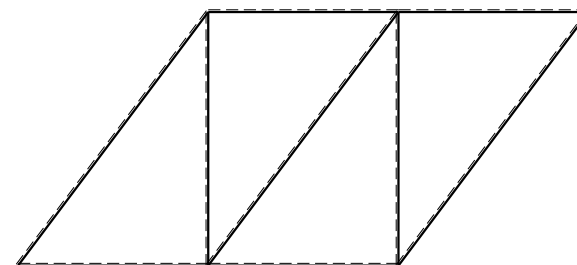
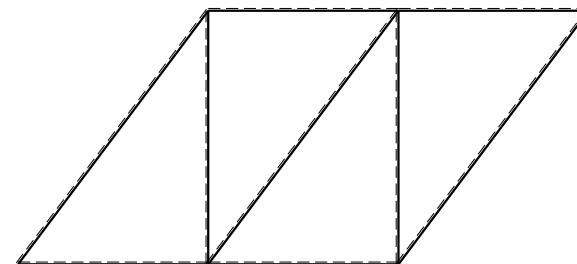
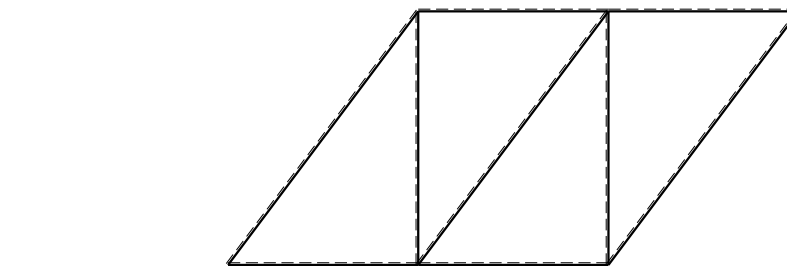
Tracciare i diagrammi delle azioni interne nelle aste.

Calcolare spostamento e rotazione di tutti i nodi.

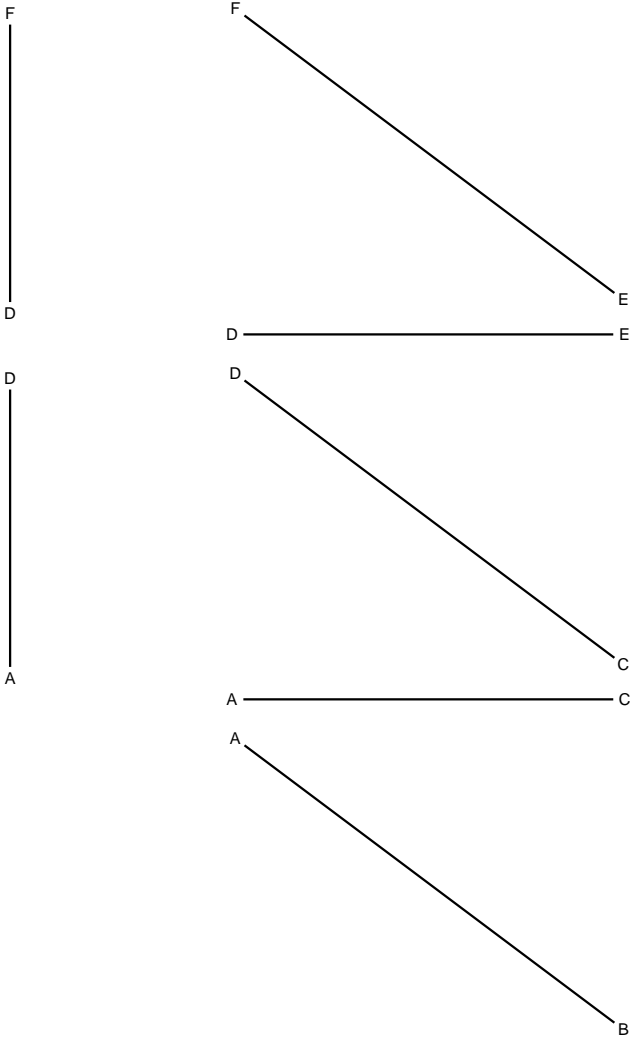
$A_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.

Allungamento termico assegnato  $\varepsilon$  su asta DC.

@ Adolfo Zavelani Rossi, Politecnico di Milano



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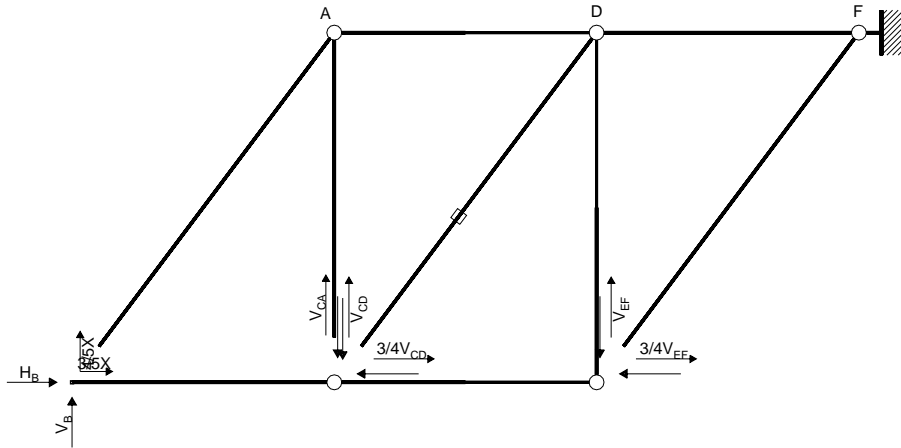


REAZIONI

$H_B =$	$V_B =$	$H_F =$	$V_F =$
$N_{AB} =$	$N_{CB} =$	$N_{AC} =$	$N_{DA} =$
$N_{DC} =$	$N_{EC} =$	$N_{FE} =$	$N_{DE} =$
$N_{FD} =$			

SPOSTAMENTI NODALI

$u_A =$	$u_B =$	$u_C =$	$u_D =$	$u_E =$
$v_A =$	$v_B =$	$v_C =$	$v_D =$	$v_E =$
$u_F =$				
$v_F =$				



## EQUAZIONI DI EQUILIBRIO

Rotazione intorno a F: aste FD DA DC DE AB AC EC CB

$$4H_B b - 9V_B b = 24/5 X b$$

Rotazione intorno a D: aste DA AB AC

$$-3V_{CA}b = 12/5Xb$$

Rotazione intorno a D: aste DE EC CB

$$4H_B b - 6V_B b + 3V_{CA} b - 3V_{EF} b = 0$$

Rotazione intorno a E: aste EC CB

$$-6V_B b + 3V_{CA} b + 3V_{CD} b = 0$$

Rotazione intorno a C: aste CB

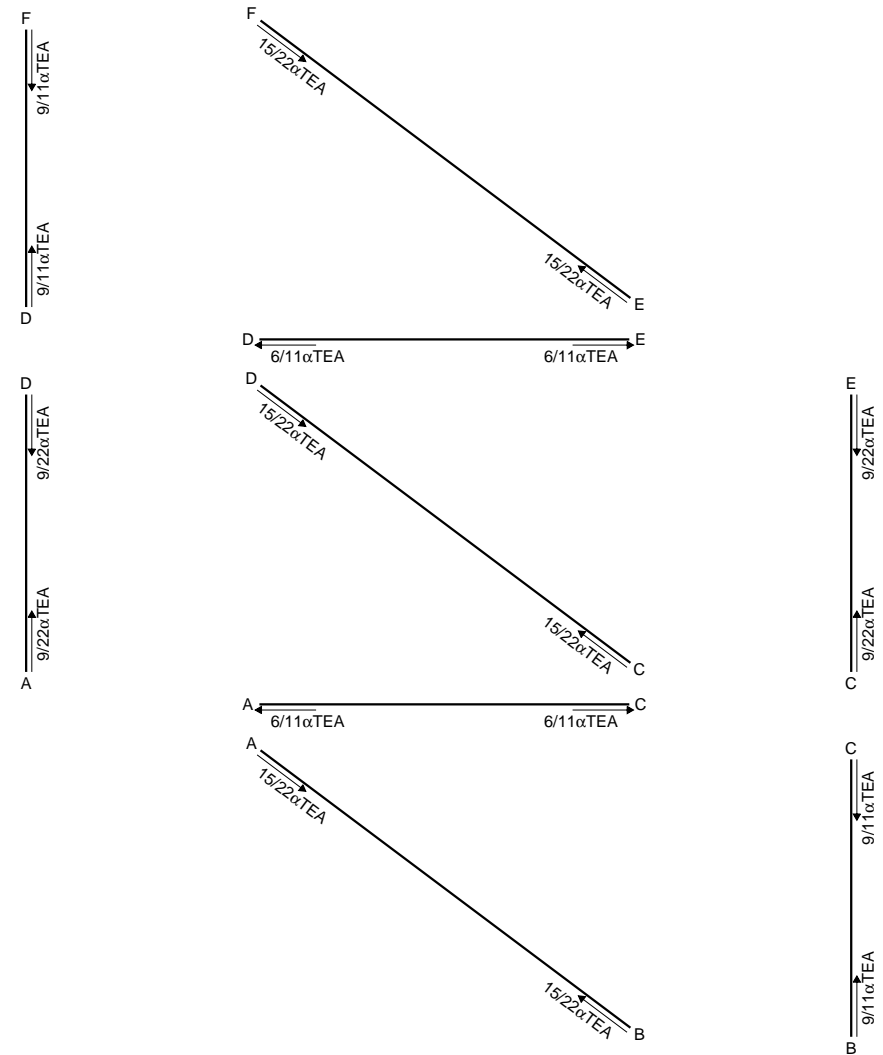
$$-3V_B b = 0$$

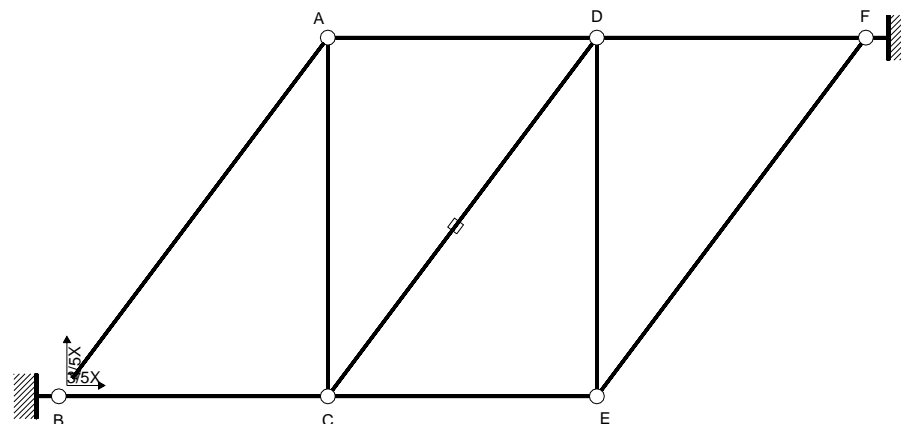
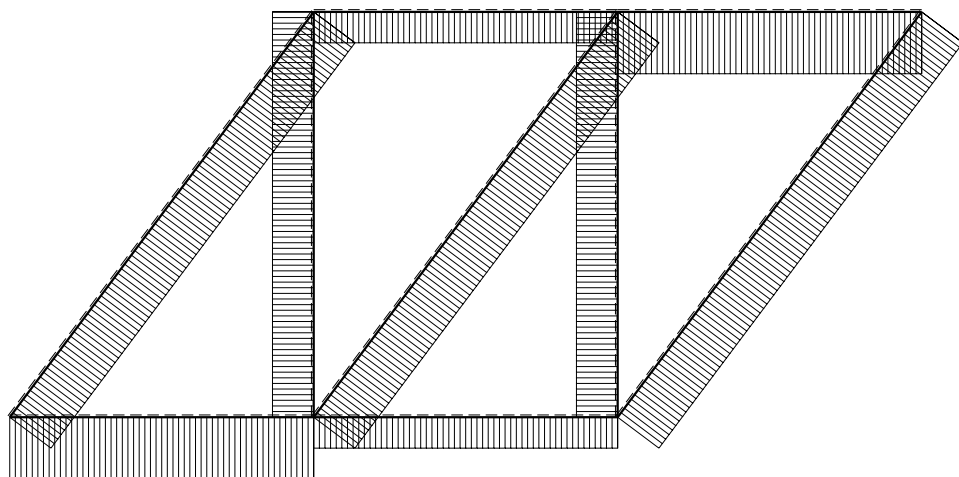
### Matrice di equilibrio

$$\begin{bmatrix} \mathbf{H_B b} & \mathbf{V_B b} & \mathbf{V_{CA} b} & \mathbf{V_{CD} b} & \mathbf{V_{EF} b} \end{bmatrix} \begin{bmatrix} \Phi_{FD} \\ \Phi_{DA} \\ \Phi_{DE} \\ \Phi_{EC} \\ \Phi_{CB} \end{bmatrix} = \begin{bmatrix} \mathbf{X_b} \\ 24/5 \\ 12/5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

### Soluzione del sistema

$$\begin{bmatrix} H_B b \\ V_{CA} b \\ V_B b \\ V_{CD} b \\ V_{EF} b \end{bmatrix} = \begin{bmatrix} 6/5 \\ -4/5 \\ 0 \\ 4/5 \\ 4/5 \end{bmatrix} \begin{bmatrix} X_b \end{bmatrix}$$





## REAZIONI IPERSTATICHE

$$X = V_{BA}$$

## CALCOLO DELLE REAZIONI IPERSTATICHE

$$L_{AB}^{XX} = N_{AB}^X N_{AB}^X I_{AB}/EA_{AB} = -1 \quad (-1) \quad 5 \quad 1/5 \quad Fb/EA = Fb/EA$$

$$L_{CB}^{XX} = N_{CB}^X N_{CB}^X I_{CB}/EA_{CB} = -6/5 \quad (-6/5) \quad 3 \quad 1/4 \quad Fb/EA = 27/25 \quad Fb/EA$$

$$L_{AC}^{XX} = N_{AC}^X N_{AC}^X I_{AC}/EA_{AC} = 4/5 \quad 4/5 \quad 4 \quad 1/4 \quad Fb/EA = 16/25 \quad Fb/EA$$

$$L_{DA}^{XX} = N_{DA}^X N_{DA}^X I_{DA}/EA_{DA} = -3/5 \quad (-3/5) \quad 3 \quad 1/3 \quad Fb/EA = 9/25 \quad Fb/EA$$

$$L_{DC}^{XX} = N_{DC}^X N_{DC}^X I_{DC}/EA_{DC} + N_{DC}^X \varepsilon_{DC} I_{DC} = -1 \quad (-1) \quad 5 \quad 1/3 \quad Fb/EA - 1 \quad 2 \quad 5 \quad Fb/EA = 5/3 \quad Fb/EA$$

$$L_{EC}^{XX} = N_{EC}^X N_{EC}^X I_{EC}/EA_{EC} = -3/5 \quad (-3/5) \quad 3 \quad 1/2 \quad Fb/EA = 27/50 \quad Fb/EA$$

$$L_{FE}^{XX} = N_{FE}^X N_{FE}^X I_{FE}/EA_{FE} = -1 \quad (-1) \quad 5 \quad 1/2 \quad Fb/EA = 5/2 \quad Fb/EA$$

$$L_{DE}^{XX} = N_{DE}^X N_{DE}^X I_{DE}/EA_{DE} = 4/5 \quad 4/5 \quad 4 \quad 1 \quad Fb/EA = 64/25 \quad Fb/EA$$

$$L_{FD}^{XX} = N_{FD}^X N_{FD}^X I_{FD}/EA_{FD} = -6/5 \quad (-6/5) \quad 3 \quad 1 \quad Fb/EA = 108/25 \quad Fb/EA$$

$$L_{DC}^{Xo} = N_{DC}^X N_{DC}^o I_{DC}/EA_{DC} + N_{DC}^X \varepsilon_{DC} I_{DC} = -1 \quad -1 \quad 2 \quad 5 \quad Fb/EA = -10 \quad Fb/EA$$

## Contributi nulli elementi

$$L_{AB}^{Xo} \quad L_{CB}^{Xo} \quad L_{AC}^{Xo} \quad L_{DA}^{Xo} \quad L_{EC}^{Xo} \quad L_{FE}^{Xo} \quad L_{DE}^{Xo} \quad L_{FD}^{Xo}$$

Contributi nulli nodi vincolati

$$L_B^{xx} \quad L_F^{xx} \quad L_B^{xo} \quad L_F^{xo}$$

Espressione risolvete

$$\left( L_{AB}^{xx} + L_{CB}^{xx} + L_{AC}^{xx} + L_{DA}^{xx} + L_{DC}^{xx} + L_{EC}^{xx} + L_{FE}^{xx} + L_{DE}^{xx} + L_{FD}^{xx} \right) X = - \left( L_{DC}^{xo} \right)$$

$$\left( 1 + 27/25 + 16/25 + 9/25 + 5/3 + 27/50 + 5/2 + 64/25 + 108/25 \right) X = \left( 10 \right) F$$

$$44/3 X = 10 F$$

Soluzione

$$X = 6/11 F$$

REAZIONI

$$H_B = 27/22\alpha TEA \quad V_B = 6/11\alpha TEA \quad H_F = -27/22\alpha TEA \quad V_F = -6/11\alpha TEA$$

$$N_{AB} = -15/22\alpha TEA \quad N_{CB} = -9/11\alpha TEA \quad N_{AC} = 6/11\alpha TEA \quad N_{DA} = -9/22\alpha TEA$$

$$N_{DC} = -15/22\alpha TEA \quad N_{EC} = -9/22\alpha TEA \quad N_{FE} = -15/22\alpha TEA \quad N_{DE} = 6/11\alpha TEA$$

$$N_{FD} = -9/11\alpha TEA$$

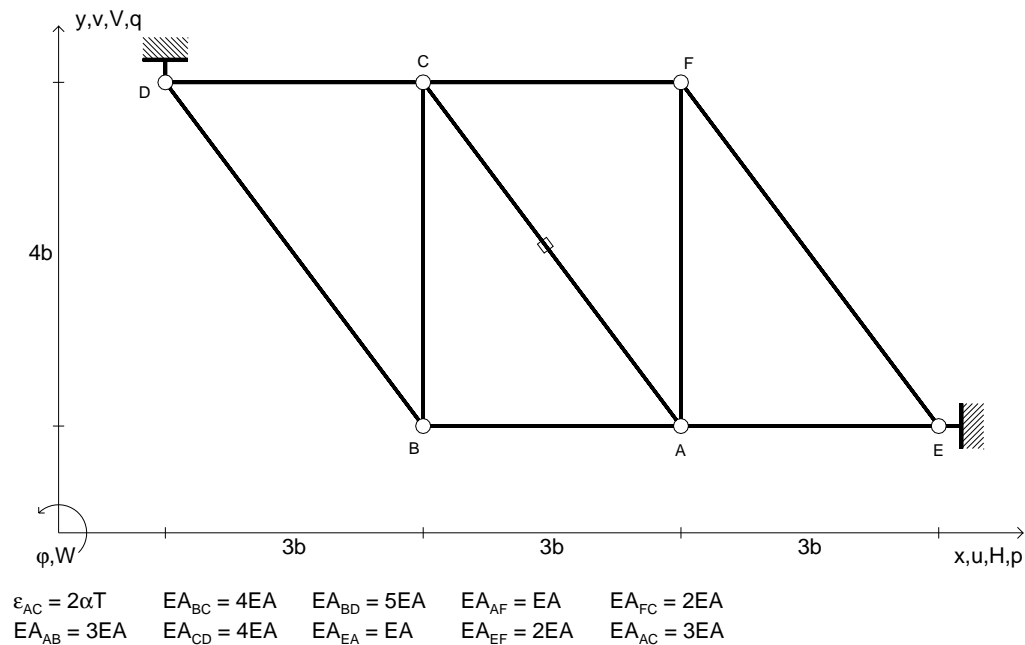
SPOSTAMENTI NODALI

$$\begin{array}{lllll} u_A = 63/22\alpha Tb & u_B = 0 & u_C = -27/44\alpha Tb & u_D = 27/11\alpha Tb & u_E = -27/22\alpha Tb \\ v_A = -3\alpha Tb & v_B = 0 & v_C = -39/11\alpha Tb & v_D = 921/176\alpha Tb & v_E = 537/176\alpha Tb \end{array}$$

$$u_F = 0$$

$$v_F = 0$$





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Calcolare spostamento e rotazione di tutti i nodi.

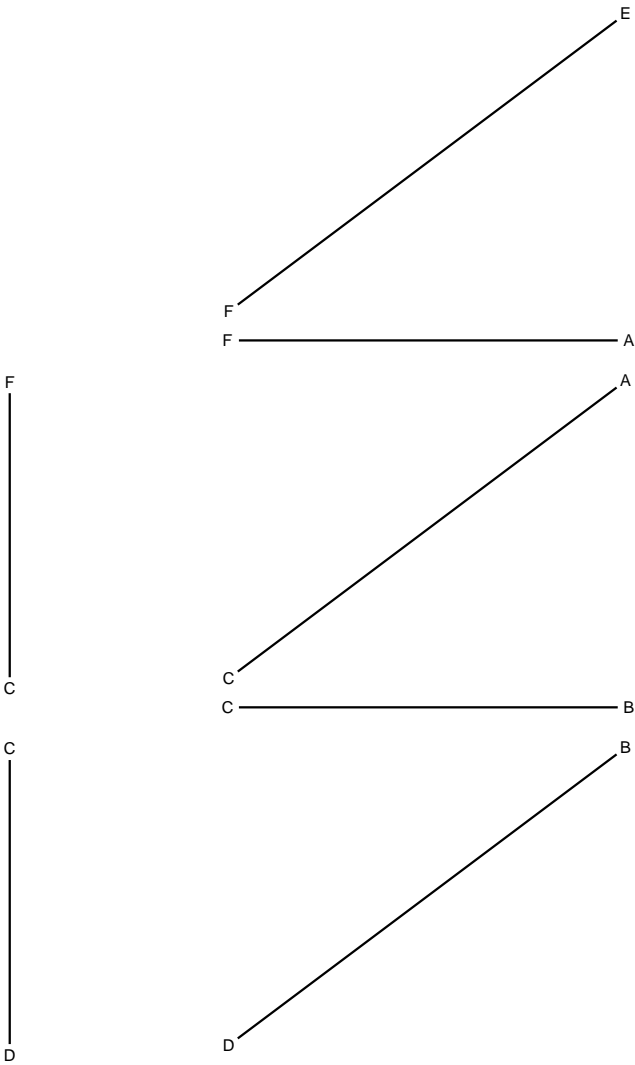
$A_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.

Allungamento termico assegnato  $\varepsilon$  su asta AC.

@ Adolfo Zavelani Rossi, Politecnico di Milano



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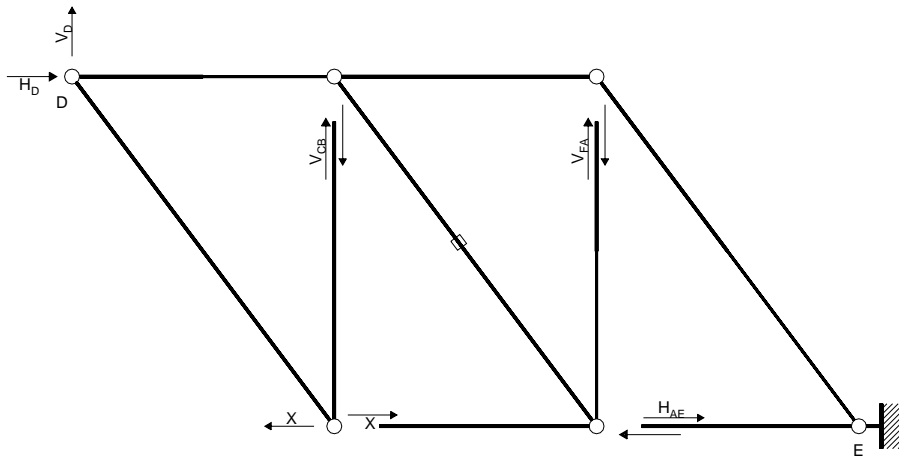
REAZIONI

$H_D =$	$V_D =$	$H_E =$	$V_E =$
$N_{AB} =$	$N_{BC} =$	$N_{CD} =$	$N_{BD} =$
$N_{EA} =$	$N_{AF} =$	$N_{EF} =$	$N_{FC} =$
$N_{AC} =$			

SPOSTAMENTI NODALI

$u_A =$	$u_B =$	$u_C =$	$u_D =$	$u_E =$
$v_A =$	$v_B =$	$v_C =$	$v_D =$	$v_E =$
$u_F =$				
$v_F =$				





## EQUAZIONI DI EQUILIBRIO

Rotazione intorno a E: aste EF FC CD CA AB DB AF BC

$$-4H_D b - 9V_D b = 0$$

Rotazione intorno a F: aste FC CD CA AB DB AF BC

$$-6V_D b - 4H_{AE} b = 0$$

Rotazione intorno a C: aste CD DB BC

$$-3V_D b = 4Xb$$

Rotazione intorno a C: aste CA AB AF

$$-4H_{AE}b + 3V_{FA}b = -4Xb$$

Rotazione intorno a D: aste DB BC

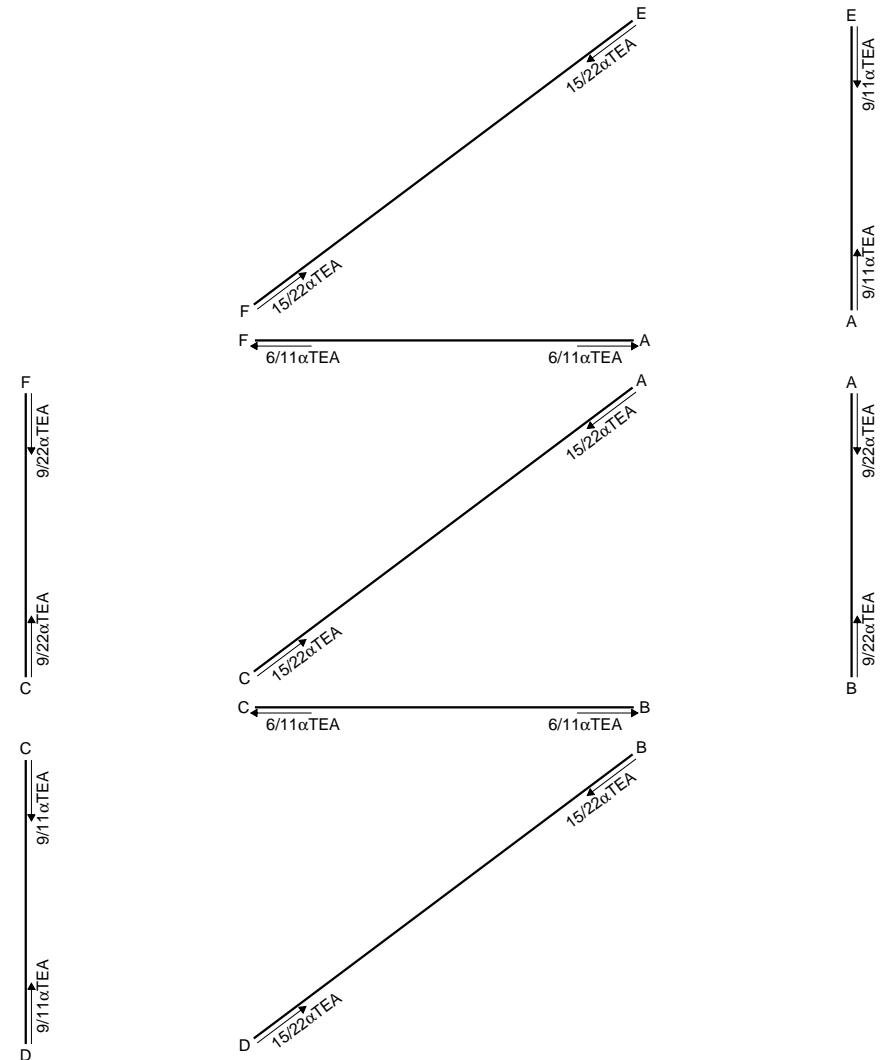
$$3V_{CB}b = 4Xb$$

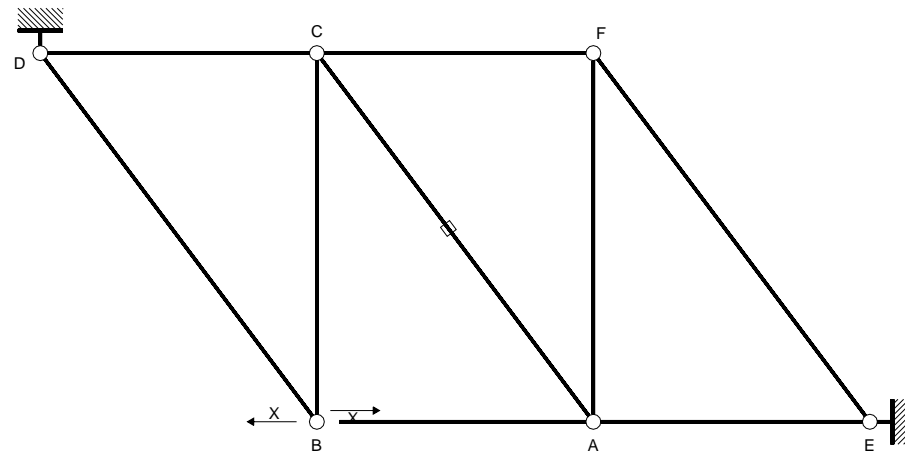
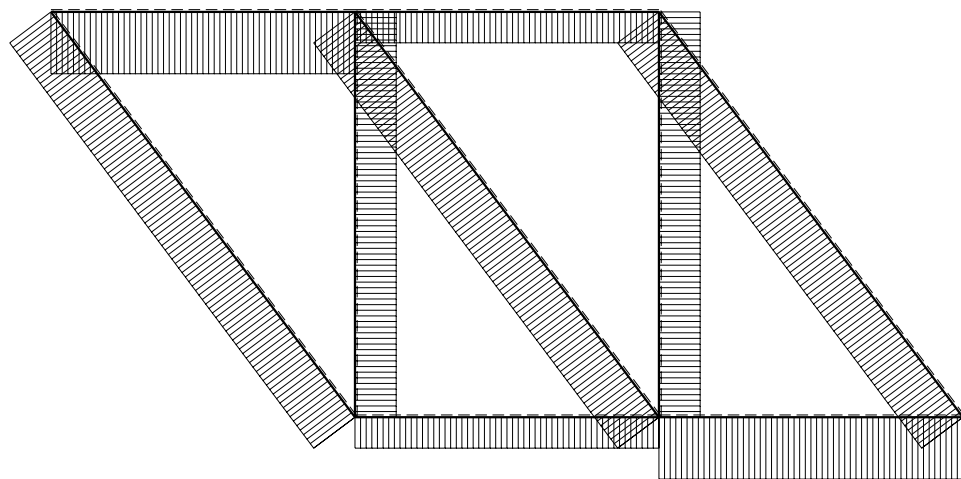
### Matrice di equilibrio

$$\begin{bmatrix} H_{D^b} & V_{D^b} & V_{CB^b} & H_{A^b} & V_{FA^b} \\ \Phi_{EF} & -4 & -9 & 0 & 0 \\ \Phi_{FC} & 0 & -6 & 0 & -4 \\ \Phi_{CD} & 0 & -3 & 0 & 0 \\ \Phi_{CA} & 0 & 0 & 0 & -4 \\ \Phi_{DB} & 0 & 0 & 3 & 0 \end{bmatrix} = \begin{bmatrix} X_b \\ 0 \\ 0 \\ 4 \\ -4 \\ 4 \end{bmatrix}$$

### Soluzione del sistema

$$\begin{bmatrix} H_D b \\ V_D b \\ H_{AE} b \\ V_{FA} b \\ V_{CB} b \end{bmatrix} = \begin{bmatrix} 3 \\ -4/3 \\ 2 \\ 4/3 \\ 4/3 \end{bmatrix} \begin{bmatrix} X_b \end{bmatrix}$$





REAZIONI IPERSTATICHE

$$X = H_{BA}$$

CALCOLO DELLE REAZIONI IPERSTATICHE

$$L_{AB}^{xx} = N_{AB}^x N_{AB}^x I_{AB}/EA_{AB} = -1 \quad (-1) \quad 3 \quad 1/3 \quad Fb/EA = Fb/EA$$

$$L_{BC}^{xx} = N_{BC}^x N_{BC}^x I_{BC}/EA_{BC} = 4/3 \quad 4/3 \quad 4 \quad 1/4 \quad Fb/EA = 16/9 \quad Fb/EA$$

$$L_{CD}^{xx} = N_{CD}^x N_{CD}^x I_{CD}/EA_{CD} = -2 \quad (-2) \quad 3 \quad 1/4 \quad Fb/EA = 3 \quad Fb/EA$$

$$L_{BD}^{xx} = N_{BD}^x N_{BD}^x I_{BD}/EA_{BD} = -5/3 \quad (-5/3) \quad 5 \quad 1/5 \quad Fb/EA = 25/9 \quad Fb/EA$$

$$L_{EA}^{xx} = N_{EA}^x N_{EA}^x I_{EA}/EA_{EA} = -2 \quad (-2) \quad 3 \quad 1 \quad Fb/EA = 12 \quad Fb/EA$$

$$L_{AF}^{xx} = N_{AF}^x N_{AF}^x I_{AF}/EA_{AF} = 4/3 \quad 4/3 \quad 4 \quad 1 \quad Fb/EA = 64/9 \quad Fb/EA$$

$$L_{EF}^{xx} = N_{EF}^x N_{EF}^x I_{EF}/EA_{EF} = -5/3 \quad (-5/3) \quad 5 \quad 1/2 \quad Fb/EA = 125/18 \quad Fb/EA$$

$$L_{FC}^{xx} = N_{FC}^x N_{FC}^x I_{FC}/EA_{FC} = -1 \quad (-1) \quad 3 \quad 1/2 \quad Fb/EA = 3/2 \quad Fb/EA$$

$$L_{AC}^{xx} = N_{AC}^x N_{AC}^x I_{AC}/EA_{AC} + N_{AC}^x \varepsilon_{AC} I_{AC} = -5/3 \quad (-5/3) \quad 5 \quad 1/3 \quad Fb/EA - 5/3 \quad 2 \quad 5 \quad Fb/EA = 125/27 \quad Fb/EA$$

$$L_{AC}^{x_0} = N_{AC}^x N_{AC}^0 I_{AC}/EA_{AC} + N_{AC}^x \varepsilon_{AC} I_{AC} = -5/3 - 5/3 \quad 2 \quad 5 \quad Fb/EA = -50/3 \quad Fb/EA$$

Contributi nulli elementi

$$L_{AB}^{X_0} \quad L_{BC}^{X_0} \quad L_{CD}^{X_0} \quad L_{BD}^{X_0} \quad L_{EA}^{X_0} \quad L_{AF}^{X_0} \quad L_{EF}^{X_0} \quad L_{FC}^{X_0}$$

Contributi nulli nodi vincolati

$$L_D^{XX} \quad L_E^{XX} \quad L_D^{X_0} \quad L_E^{X_0}$$

Espressione risolvante

$$\left( L_{AB}^{XX} + L_{BC}^{XX} + L_{CD}^{XX} + L_{BD}^{XX} + L_{EA}^{XX} + L_{AF}^{XX} + L_{EF}^{XX} + L_{FC}^{XX} + L_{AC}^{XX} \right) X = - \left( L_{AC}^{X_0} \right)$$

$$\left( 1 + 16/9 + 3 + 25/9 + 12 + 64/9 + 125/18 + 3/2 + 125/27 \right) X = \left( 50/3 \right) F$$

$$1100/27 X = 50/3 F$$

Soluzione

$$X = 9/22 F$$

REAZIONI

$$H_D = 27/22\alpha TEA \quad V_D = -6/11\alpha TEA \quad H_E = -27/22\alpha TEA \quad V_E = 6/11\alpha TEA$$

$$N_{AB} = -9/22\alpha TEA \quad N_{BC} = 6/11\alpha TEA \quad N_{CD} = -9/11\alpha TEA \quad N_{BD} = -15/22\alpha TEA$$

$$N_{EA} = -9/11\alpha TEA \quad N_{AF} = 6/11\alpha TEA \quad N_{EF} = -15/22\alpha TEA \quad N_{FC} = -9/22\alpha TEA$$

$$N_{AC} = -15/22\alpha TEA$$

SPOSTAMENTI NODALI

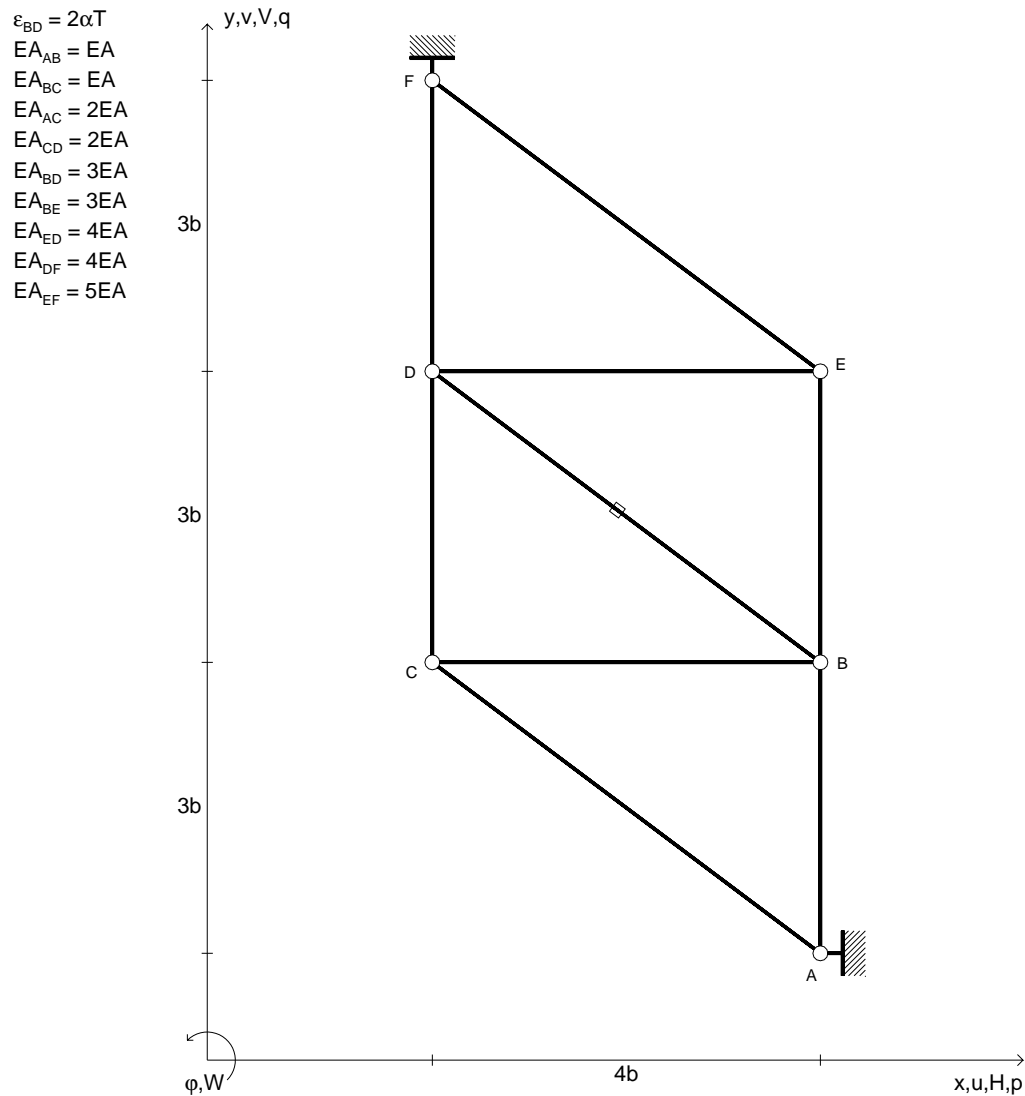
$$u_A = 27/11\alpha Tb \quad u_B = 63/22\alpha Tb \quad u_C = -27/44\alpha Tb \quad u_D = 0 \quad u_E = 0$$

$$v_A = -921/176\alpha Tb \quad v_B = 3\alpha Tb \quad v_C = 39/11\alpha Tb \quad v_D = 0 \quad v_E = 0$$

$$u_F = -27/22\alpha Tb$$

$$v_F = -537/176\alpha Tb$$





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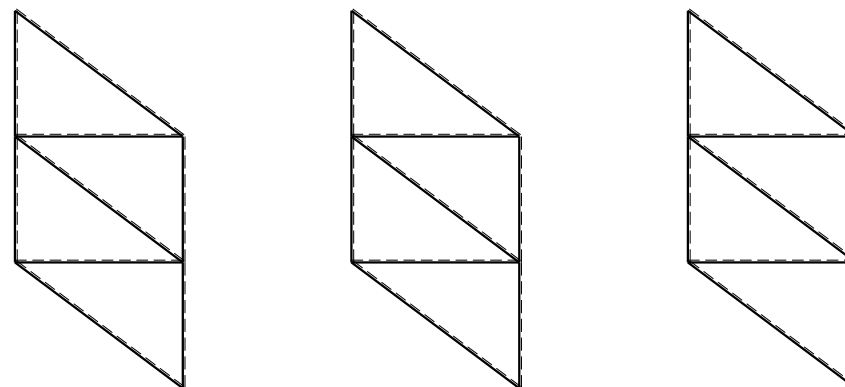
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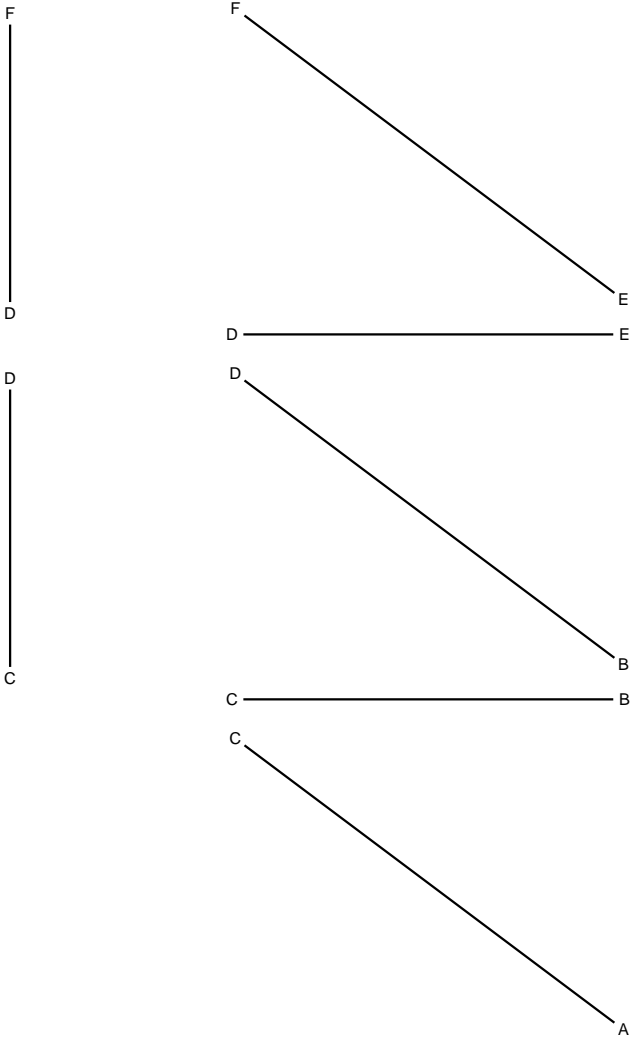
Calcolare spostamento e rotazione di tutti i nodi.

$A_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.

Allungamento termico assegnato  $\varepsilon$  su asta BD.

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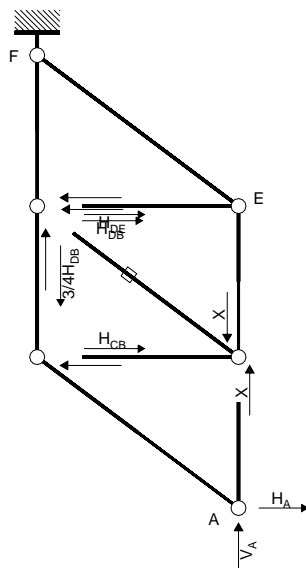


REAZIONI

$H_A =$	$V_A =$	$H_F =$	$V_F =$
$N_{AB} =$	$N_{BC} =$	$N_{AC} =$	$N_{CD} =$
$N_{BD} =$	$N_{BE} =$	$N_{ED} =$	$N_{DF} =$
$N_{EF} =$			

SPOSTAMENTI NODALI

$u_A =$	$u_B =$	$u_C =$	$u_D =$	$u_E =$
$v_A =$	$v_B =$	$v_C =$	$v_D =$	$v_E =$
$u_F =$				
$v_F =$				



## EQUAZIONI DI EQUILIBRIO

Rotazione intorno a F: aste FD DC CA AB

$$9H_A b + 4V_A b - 6H_{CB} b - 3H_{DB} b - 3H_{DE} b = -4Xb$$

Rotazione intorno a F: aste FE EB ED BC BD

$$6H_{CB} b + 3H_{DB} b + 3H_{DE} b = 4Xb$$

Rotazione intorno a D: aste DC CA AB

$$6H_A b + 4V_A b - 3H_{CB} b = -4Xb$$

Rotazione intorno a E: aste EB BC BD

$$3H_{CB} b + 3H_{DB} b = 0$$

Rotazione intorno a C: aste CA AB

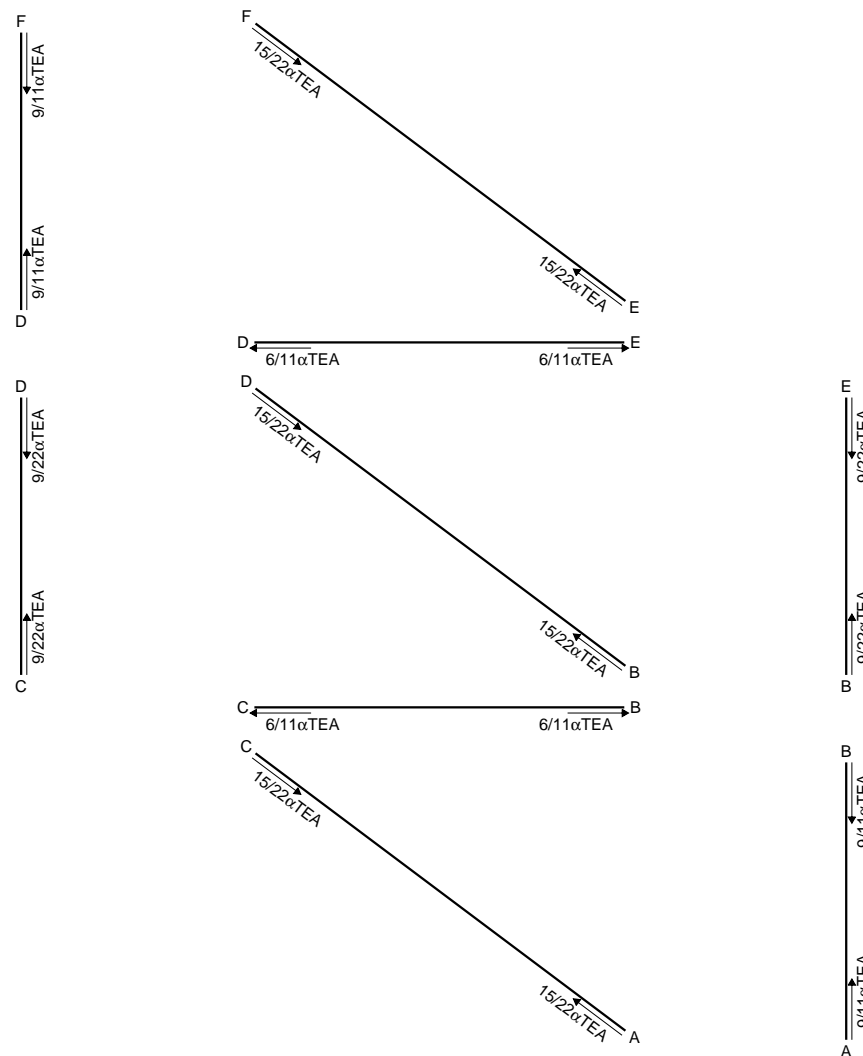
$$3H_A b + 4V_A b = -4Xb$$

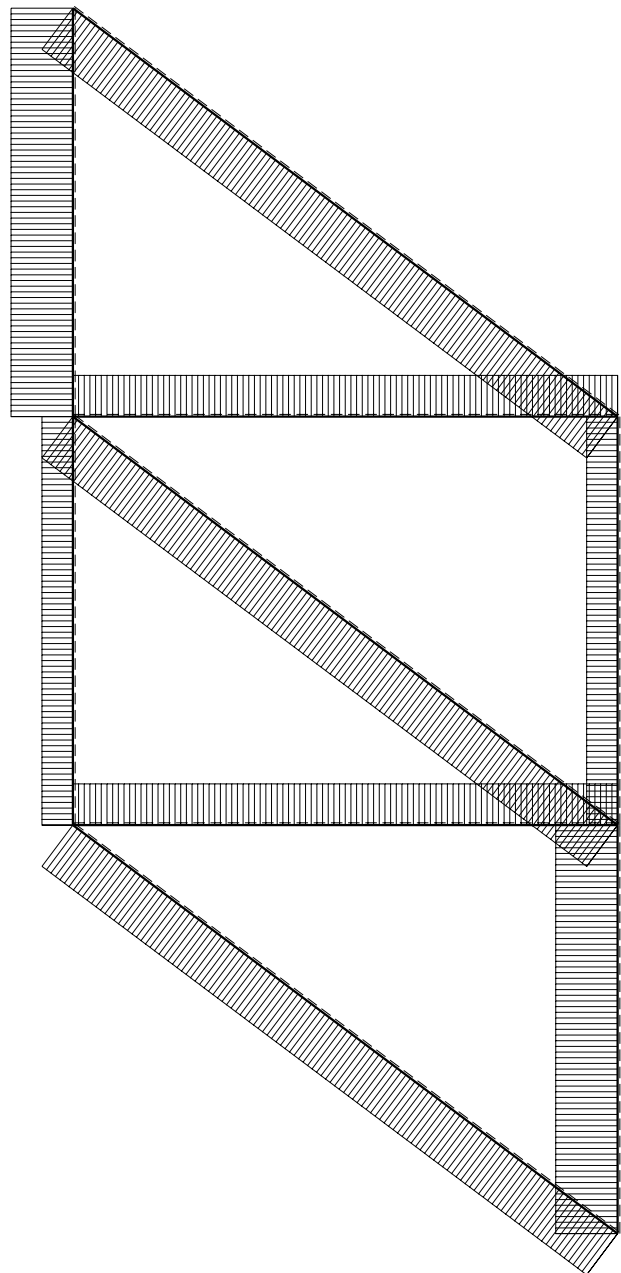
Matrice di equilibrio

$$\begin{bmatrix} \varphi_{FD} \\ \varphi_{FE} \\ \varphi_{DC} \\ \varphi_{EB} \\ \varphi_{CA} \end{bmatrix} \begin{bmatrix} H_A b & V_A b & H_{CB} b & H_{DB} b & H_{DE} b \\ 9 & 4 & -6 & -3 & -3 \\ 0 & 0 & 6 & 3 & 3 \\ 6 & 4 & -3 & 0 & 0 \\ 0 & 0 & 3 & 3 & 0 \\ 3 & 4 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} Xb \\ -4 \\ -4 \\ 0 \\ -4 \end{bmatrix}$$

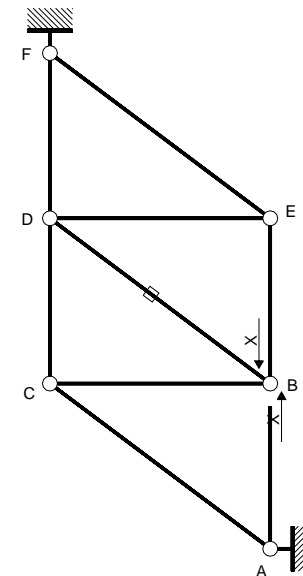
Soluzione del sistema

$$\begin{bmatrix} H_A b \\ H_{CB} b \\ V_A b \\ H_{DB} b \\ H_{DE} b \end{bmatrix} = \begin{bmatrix} 2/3 \\ 2/3 \\ -3/2 \\ -2/3 \\ 2/3 \end{bmatrix}$$





← ⊕ → | 1 αTEA



### REAZIONI IPERSTATICHE

$$X = V_{BA}$$

### CALCOLO DELLE REAZIONI IPERSTATICHE

$$L_{AB}^{xx} = N_{AB}^x N_{AB}^x I_{AB}/EA_{AB} = 1 \ 1 \ 3 \ 1 \ Fb/EA = 3 \ Fb/EA$$

$$L_{BC}^{xx} = N_{BC}^x N_{BC}^x I_{BC}/EA_{BC} = -2/3 \ (-2/3) \ 4 \ 1 \ Fb/EA = 16/9 \ Fb/EA$$

$$L_{AC}^{xx} = N_{AC}^x N_{AC}^x I_{AC}/EA_{AC} = 5/6 \ 5/6 \ 5 \ 1/2 \ Fb/EA = 125/72 \ Fb/EA$$

$$L_{CD}^{xx} = N_{CD}^x N_{CD}^x I_{CD}/EA_{CD} = 1/2 \ 1/2 \ 3 \ 1/2 \ Fb/EA = 3/8 \ Fb/EA$$

$$L_{BD}^{xx} = N_{BD}^x N_{BD}^x I_{BD}/EA_{BD} + N_{BD}^x \epsilon_{BD} I_{BD} = 5/6 \ 5/6 \ 5 \ 1/3 \ Fb/EA + 5/6 \ 2 \ 5 \ Fb/EA = 125/108 \ Fb/EA$$

$$L_{BE}^{xx} = N_{BE}^x N_{BE}^x I_{BE}/EA_{BE} = 1/2 \ 1/2 \ 3 \ 1/3 \ Fb/EA = 1/4 \ Fb/EA$$

$$L_{ED}^{xx} = N_{ED}^x N_{ED}^x I_{ED}/EA_{ED} = -2/3 \ (-2/3) \ 4 \ 1/4 \ Fb/EA = 4/9 \ Fb/EA$$

$$L_{DF}^{xx} = N_{DF}^x N_{DF}^x I_{DF}/EA_{DF} = 1 \ 1 \ 3 \ 1/4 \ Fb/EA = 3/4 \ Fb/EA$$

$$L_{EF}^{xx} = N_{EF}^x N_{EF}^x I_{EF}/EA_{EF} = 5/6 \ 5/6 \ 5 \ 1/5 \ Fb/EA = 25/36 \ Fb/EA$$



$$L_{BD}^{X_0} = N_{BD}^X N_{BD}^0 I_{BD}/EA_{BD} + N_{BD}^X \epsilon_{BD} I_{BD} = 5/6 \cdot 5/6 \cdot 2 \cdot 5 \cdot Fb/EA = 25/3 \cdot Fb/EA$$

Contributi nulli elementi

$$L_{AB}^{X_0} \quad L_{BC}^{X_0} \quad L_{AC}^{X_0} \quad L_{CD}^{X_0} \quad L_{BE}^{X_0} \quad L_{ED}^{X_0} \quad L_{DF}^{X_0} \quad L_{EF}^{X_0}$$

Contributi nulli nodi vincolati

$$L_A^{XX} \quad L_F^{XX} \quad L_A^{X_0} \quad L_F^{X_0}$$

Espressione risolvete

$$\left( L_{AB}^{XX} + L_{BC}^{XX} + L_{AC}^{XX} + L_{CD}^{XX} + L_{BD}^{XX} + L_{BE}^{XX} + L_{ED}^{XX} + L_{DF}^{XX} + L_{EF}^{XX} \right) X = - \left( L_{BD}^{X_0} \right)$$

$$\left( 3 + 16/9 + 125/72 + 3/8 + 125/108 + 1/4 + 4/9 + 3/4 + 25/36 \right) X = \left( - 25/3 \right) F$$

$$275/27 X = - 25/3 F$$

Soluzione

$$X = -9/11 F$$

REAZIONI

$$H_A = -6/11\alpha TEA \quad V_A = 27/22\alpha TEA \quad H_F = 6/11\alpha TEA \quad V_F = -27/22\alpha TEA$$

$$N_{AB} = -9/11\alpha TEA \quad N_{BC} = 6/11\alpha TEA \quad N_{AC} = -15/22\alpha TEA \quad N_{CD} = -9/22\alpha TEA$$

$$N_{BD} = -15/22\alpha TEA \quad N_{BE} = -9/22\alpha TEA \quad N_{ED} = 6/11\alpha TEA \quad N_{DF} = -9/11\alpha TEA$$

$$N_{EF} = -15/22\alpha TEA$$

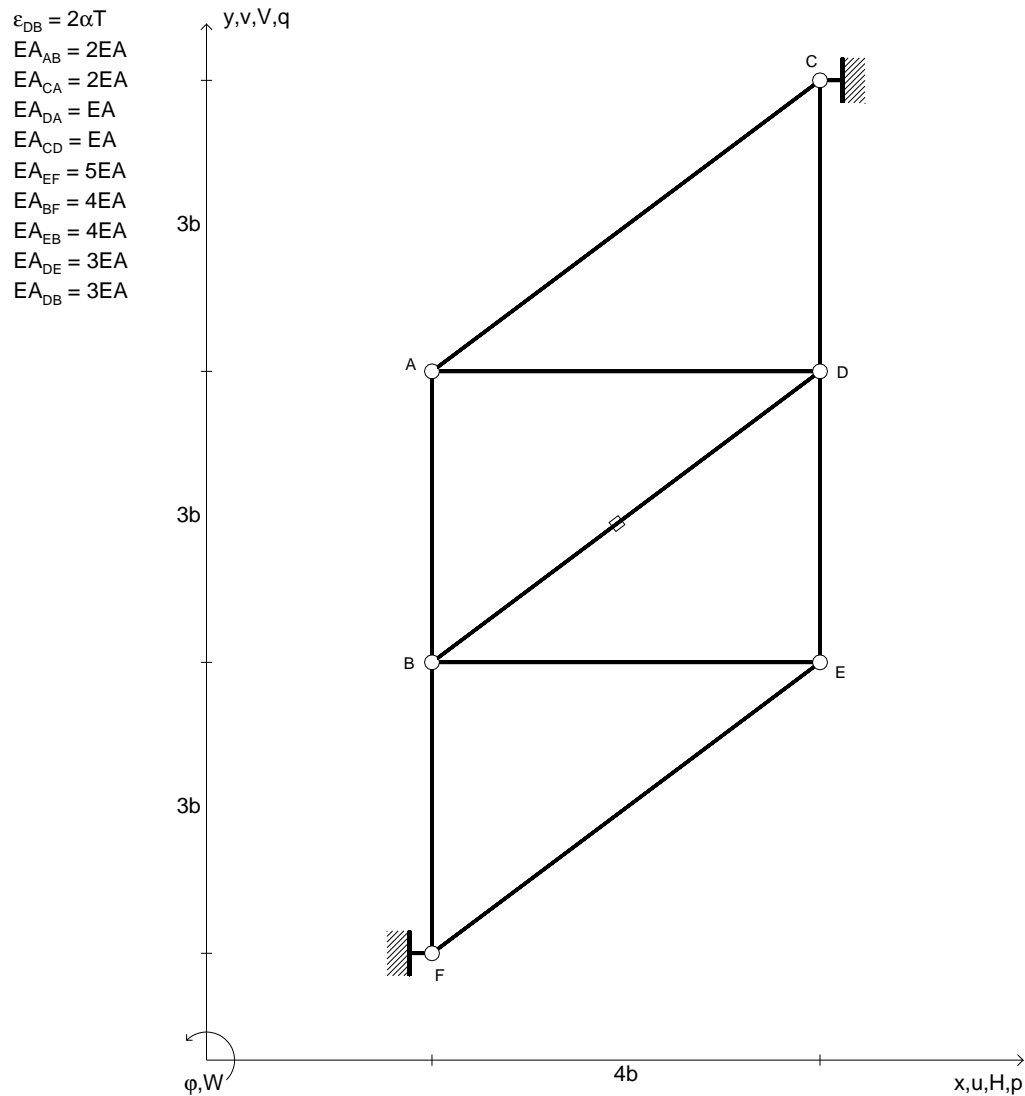
SPOSTAMENTI NODALI

$$\begin{array}{lllll} u_A = 0 & u_B = 921/176\alpha Tb & u_C = 537/176\alpha Tb & u_D = -39/11\alpha Tb & u_E = -3\alpha Tb \\ v_A = 0 & v_B = -27/11\alpha Tb & v_C = 27/22\alpha Tb & v_D = 27/44\alpha Tb & v_E = -63/22\alpha Tb \end{array}$$

$$u_F = 0$$

$$v_F = 0$$





Svolgere l'analisi cinematica.

Riportare la soluzione su questo foglio.

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

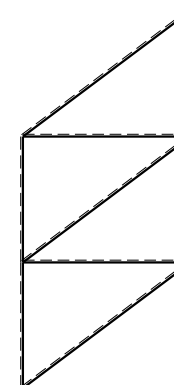
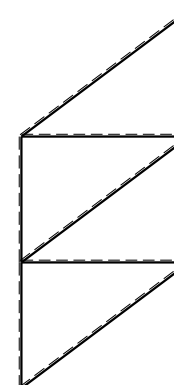
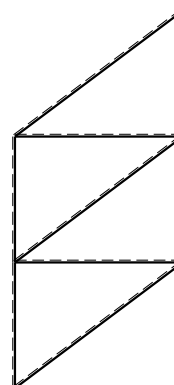
Tracciare i diagrammi delle azioni interne nelle aste.

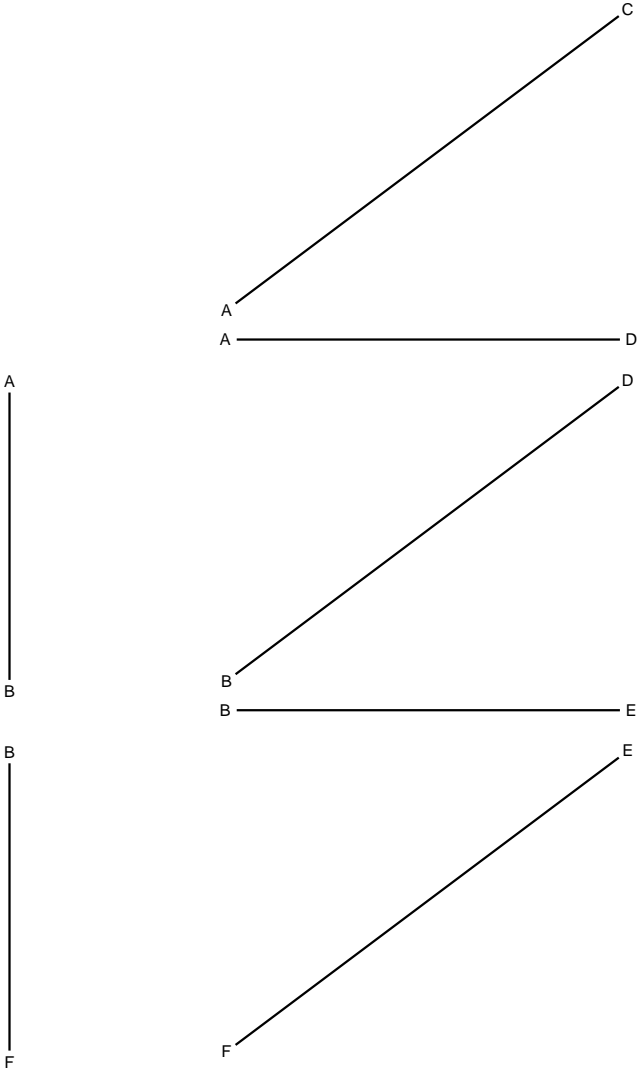
Calcolare spostamento e rotazione di tutti i nodi.

$A_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.

Allungamento termico assegnato  $\varepsilon$  su asta DB.

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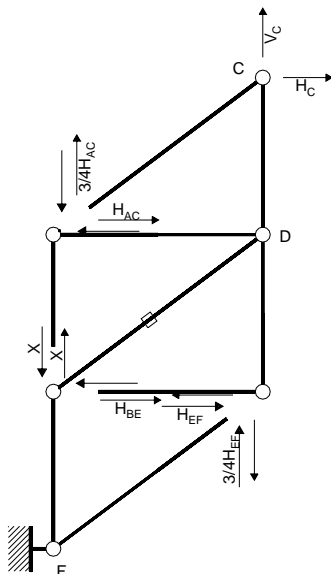


REAZIONI

$H_C =$	$V_C =$	$H_F =$	$V_F =$
$N_{AB} =$	$N_{CA} =$	$N_{DA} =$	$N_{CD} =$
$N_{EF} =$	$N_{BF} =$	$N_{EB} =$	$N_{DE} =$
$N_{DB} =$			

SPOSTAMENTI NODALI

$u_A =$	$u_B =$	$u_C =$	$u_D =$	$u_E =$
$v_A =$	$v_B =$	$v_C =$	$v_D =$	$v_E =$
$u_F =$				
$v_F =$				



## EQUAZIONI DI EQUILIBRIO

Rotazione intorno a F: aste FB BD DA DC DE AB CA EB

$$-9H_Cb + 4V_Cb = 0$$

Rotazione intorno a B: aste BD DA DC DE AB CA EB

$$-6H_Cb + 4V_Cb - 3H_{EF}b = 0$$

Rotazione intorno a D: aste DA AB

$$3H_{AC}b = 4Xb$$

Rotazione intorno a D: aste DC CA

$$-3H_Cb - 3H_{AC}b = 0$$

Rotazione intorno a D: aste DE EB

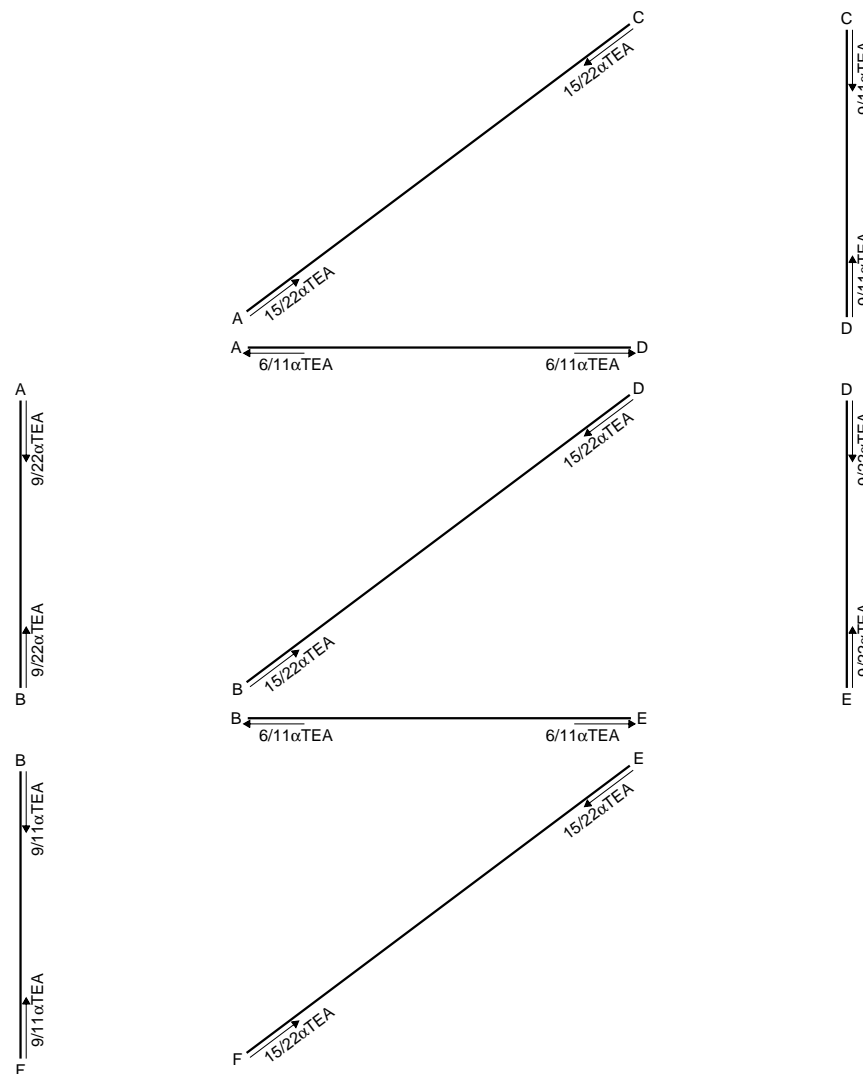
$$-3H_{EF}b + 3H_{BE}b = 0$$

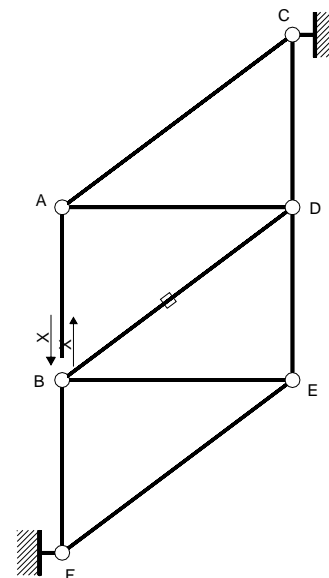
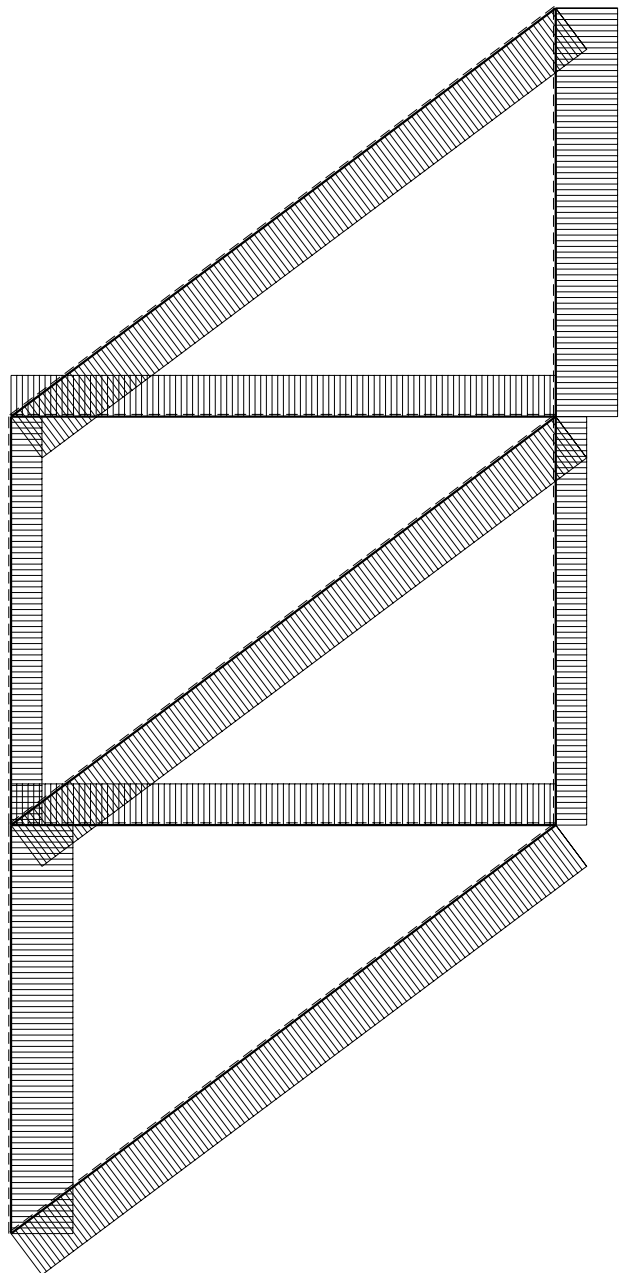
Matrice di equilibrio

$$\begin{bmatrix} \phi_{FB} \\ \phi_{BD} \\ \phi_{DA} \\ \phi_{DC} \\ \phi_{DE} \end{bmatrix} \begin{bmatrix} H_Cb & V_Cb & H_{AC}b & H_{EF}b & H_{BE}b \end{bmatrix} = \begin{bmatrix} Xb \\ 0 \\ 4 \\ 0 \\ 0 \end{bmatrix}$$

Soluzione del sistema

$$\begin{bmatrix} H_Cb \\ V_Cb \\ H_{AC}b \\ H_{EF}b \\ H_{BE}b \end{bmatrix} = \begin{bmatrix} -4/3 \\ -3 \\ 4/3 \\ -4/3 \\ -4/3 \end{bmatrix}$$





## REAZIONI IPERSTATICHE

$$X = V_{BA}$$

### CALCOLO DELLE REAZIONI IPERSTATICHE

$$L_{AB}^{XX} = N_{AB}^X N_{AB}^X I_{AB}/EA_{AB} = -1 \quad (-1) \quad 3 \quad 1/2 \quad Fb/EA = 3/2 \quad Fb/EA$$

$$L_{CA}^{XX} = N_{CA}^X N_{CA}^X I_{CA}/EA_{CA} = -5/3 \quad (-5/3) \quad 5 \quad 1/2 \quad Fb/EA = 125/18 \quad Fb/EA$$

$$L_{DA}^{xx} = N_{DA}^x N_{DA}^x I_{DA}/EA_{DA} = \begin{bmatrix} 4/3 & 4/3 & 4 & 1 \end{bmatrix} \text{ Fb/EA} = 64/9 \text{ Fb/EA}$$

$$L_{CD}^{xx} = N_{CD}^x N_{CD}^x I_{CD}/EA_{CD} = -2 \quad (-2) \quad 3 \quad 1 \quad Fb/EA = 12 \quad Fb/EA$$

$$L_{EE}^{xx} = N_{EE}^x N_{EE}^x I_{EE}/EA_{EE} = -5/3 \quad (-5/3) \quad 5 \quad 1/5 \quad Fb/EA = 25/9 \quad Fb/EA$$

$$L_{BF}^{XX} = N_{BF}^X N_{BF}^X I_{BF}/EA_{BF} = -2 \quad (-2) \quad 3 \quad 1/4 \quad Fb/EA = 3 \quad Fb/EA$$

$$L_{EB}^{xx} = N_{EB}^x N_{EB}^x I_{EB}/EA_{EB} = 4/3 \quad 4/3 \quad 4 \quad 1/4 \quad Fb/EA = 16/9 \quad Fb/EA$$

$$L_{DE}^{XX} = N_{DE}^X N_{DE}^X I_{DE}/EA_{DE} = -1 \quad (-1) \quad 3 \quad 1/3 \quad Fb/EA = Fb/EA$$

$$L_{DB}^{xx} = N_{DB}^x N_{DB}^x I_{DB}/EA_{DB} + N_{DB}^x \varepsilon_{DB} I_{DB} = -5/3 \quad (-5/3) \quad 5 \quad 1/3 \quad Fb/EA - 5/3 \quad 2 \quad 5 \quad Fb/EA = 125/27 \quad Fb/EA$$

$$L_{DB}^{X_0} = N_{DB}^X N_{DB}^0 I_{DB}/EA_{DB} + N_{DB}^X \epsilon_{DB} I_{DB} = -5/3 - 5/3 \cdot 2 \cdot 5 \text{ Fb}/EA = -50/3 \text{ Fb}/EA$$

Contributi nulli elementi

$$L_{AB}^{X_0} \quad L_{CA}^{X_0} \quad L_{DA}^{X_0} \quad L_{CD}^{X_0} \quad L_{EF}^{X_0} \quad L_{BF}^{X_0} \quad L_{EB}^{X_0} \quad L_{DE}^{X_0}$$

Contributi nulli nodi vincolati

$$L_C^{XX} \quad L_F^{XX} \quad L_C^{X_0} \quad L_F^{X_0}$$

Espressione risolvante

$$\left( L_{AB}^{XX} + L_{CA}^{XX} + L_{DA}^{XX} + L_{CD}^{XX} + L_{EF}^{XX} + L_{BF}^{XX} + L_{EB}^{XX} + L_{DE}^{XX} + L_{DB}^{XX} \right) X = - \left( L_{DB}^{X_0} \right)$$

$$\left( 3/2 + 125/18 + 64/9 + 12 + 25/9 + 3 + 16/9 + 1 + 125/27 \right) X = \left( 50/3 \right) F$$

$$1100/27 X = 50/3 F$$

Soluzione

$$X = 9/22 F$$

REAZIONI

$$H_C = -6/11\alpha TEA \quad V_C = -27/22\alpha TEA \quad H_F = 6/11\alpha TEA \quad V_F = 27/22\alpha TEA$$

$$N_{AB} = -9/22\alpha TEA \quad N_{CA} = -15/22\alpha TEA \quad N_{DA} = 6/11\alpha TEA \quad N_{CD} = -9/11\alpha TEA$$

$$N_{EF} = -15/22\alpha TEA \quad N_{BF} = -9/11\alpha TEA \quad N_{EB} = 6/11\alpha TEA \quad N_{DE} = -9/22\alpha TEA$$

$$N_{DB} = -15/22\alpha TEA$$

SPOSTAMENTI NODALI

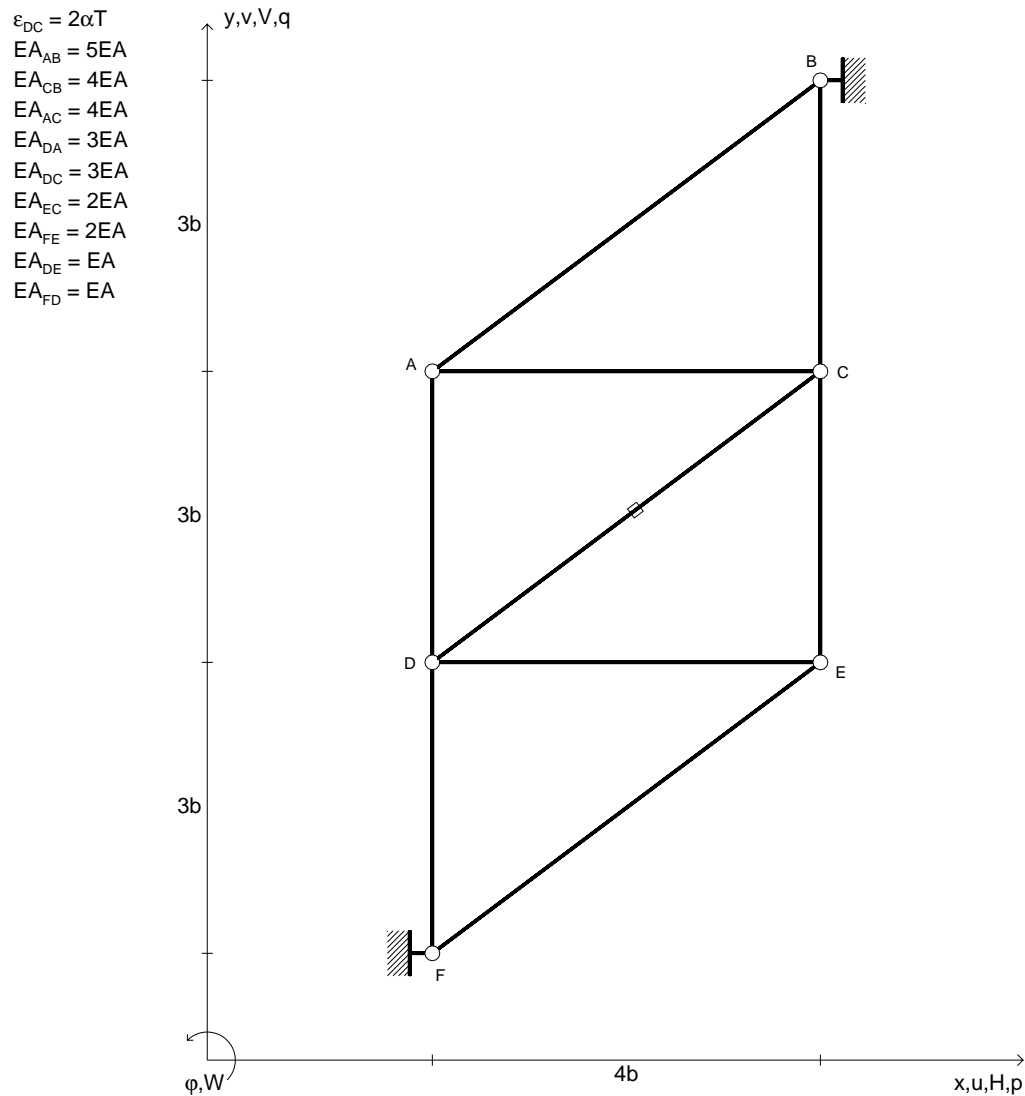
$$\begin{array}{lllll} u_A = 537/176\alpha Tb & u_B = -39/11\alpha Tb & u_C = 0 & u_D = 921/176\alpha Tb & u_E = -3\alpha Tb \\ v_A = -27/22\alpha Tb & v_B = -27/44\alpha Tb & v_C = 0 & v_D = 27/11\alpha Tb & v_E = 63/22\alpha Tb \end{array}$$

$$u_F = 0$$

$$v_F = 0$$







Svolgere l'analisi cinematica.

Riportare la soluzione su questo foglio.

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

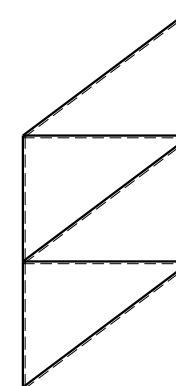
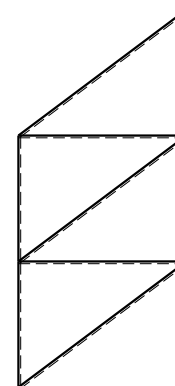
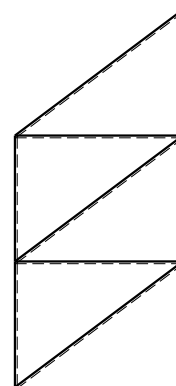
Tracciare i diagrammi delle azioni interne nelle aste.

Calcolare spostamento e rotazione di tutti i nodi.

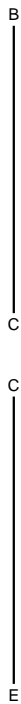
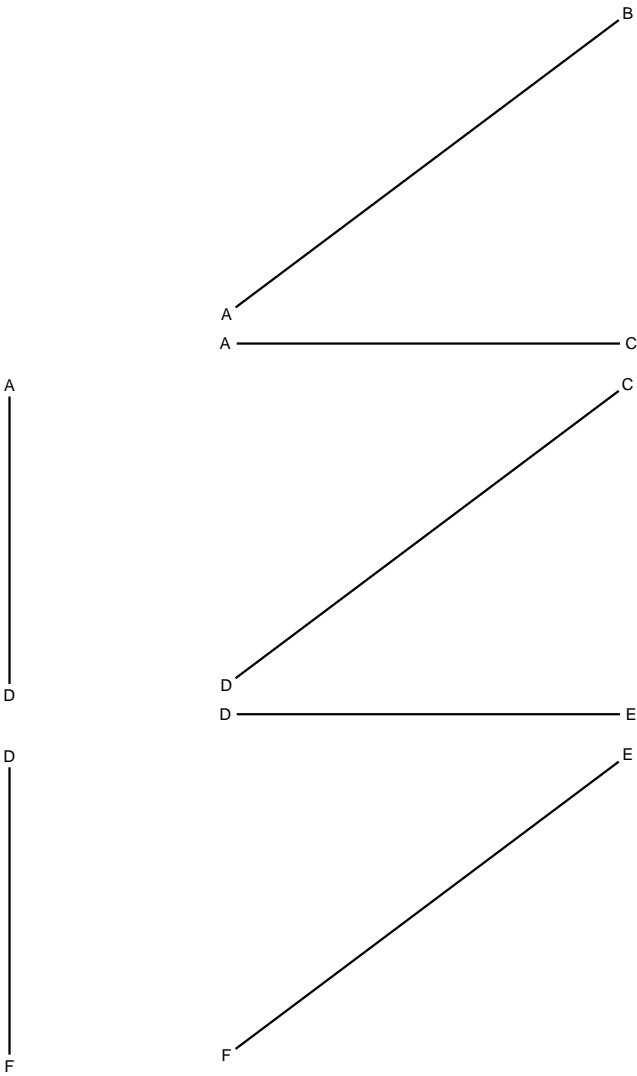
$A_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.

Allungamento termico assegnato  $\varepsilon$  su asta DC.

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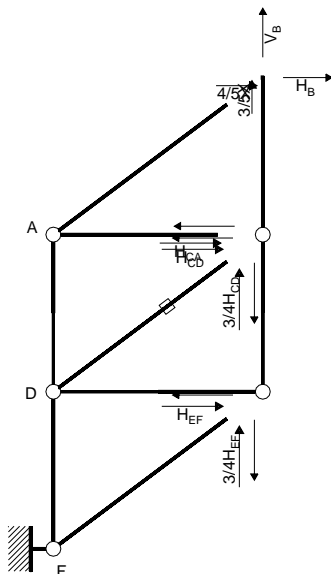


REAZIONI

$H_B =$	$V_B =$	$H_F =$	$V_F =$
$N_{AB} =$	$N_{CB} =$	$N_{AC} =$	$N_{DA} =$
$N_{DC} =$	$N_{EC} =$	$N_{FE} =$	$N_{DE} =$
$N_{FD} =$			

SPOSTAMENTI NODALI

$u_A =$	$u_B =$	$u_C =$	$u_D =$	$u_E =$
$v_A =$	$v_B =$	$v_C =$	$v_D =$	$v_E =$
$u_F =$				
$v_F =$				



## EQUAZIONI DI EQUILIBRIO

Rotazione intorno a F: aste FD DA DC DE AB AC EC CB

$$-9H_B b + 4V_B b = 24/5 X_b$$

Rotazione intorno a D: aste DA AB AC

$$-3H_{CA} b = 12/5 X_b$$

Rotazione intorno a D: aste DE EC CB

$$-6H_B b + 4V_B b + 3H_{CA} b - 3H_{EF} b = 0$$

Rotazione intorno a E: aste EC CB

$$-6H_B b + 3H_{CA} b + 3H_{CD} b = 0$$

Rotazione intorno a C: aste CB

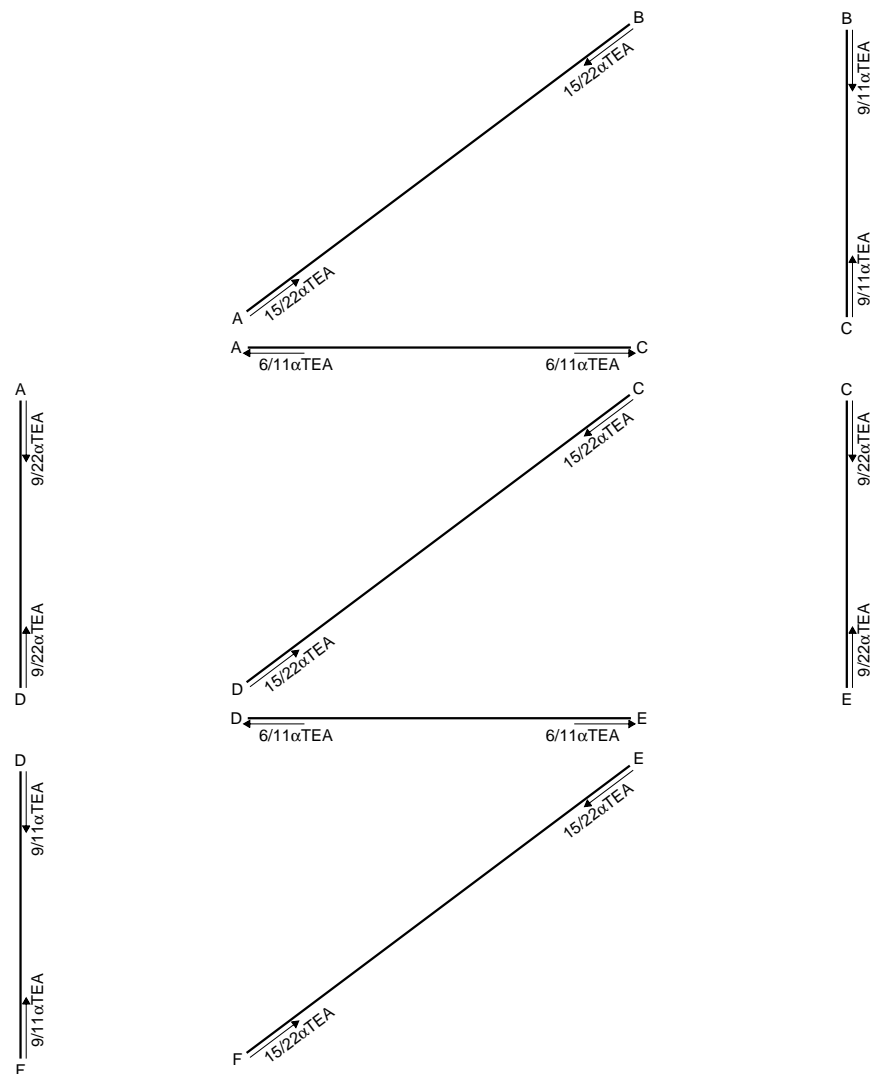
$$-3H_B b = 0$$

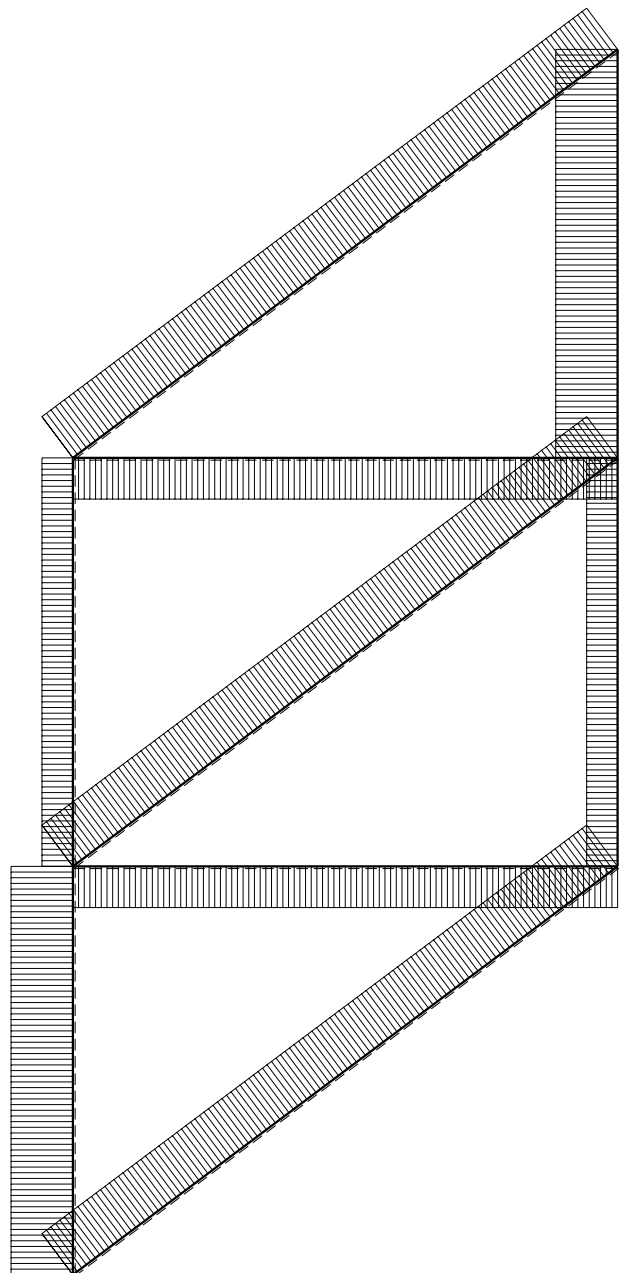
Matrice di equilibrio

$$\begin{bmatrix} \phi_{FD} \\ \phi_{DA} \\ \phi_{DE} \\ \phi_{EC} \\ \phi_{CB} \end{bmatrix} \begin{bmatrix} H_B b & V_B b & H_{CA} b & H_{CD} b & H_{EF} b \end{bmatrix} = \begin{bmatrix} X_b \\ 24/5 \\ 12/5 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

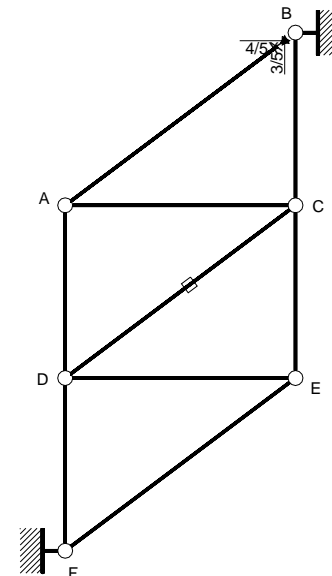
Soluzione del sistema

$$\begin{bmatrix} H_B b \\ H_{CA} b \\ V_B b \\ H_{CD} b \\ H_{EF} b \end{bmatrix} = \begin{bmatrix} X_b \\ 0 \\ -4/5 \\ 6/5 \\ 4/5 \end{bmatrix}$$





← ⊕ → | 1 αTEA



### REAZIONI IPERSTATICHE

$$X = H_{BA}$$

### CALCOLO DELLE REAZIONI IPERSTATICHE

$$L_{AB}^{xx} = N_{AB}^x N_{AB}^x I_{AB}/EA_{AB} = 1 \cdot 1 \cdot 5 \cdot 1/5 Fb/EA = Fb/EA$$

$$L_{CB}^{xx} = N_{CB}^x N_{CB}^x I_{CB}/EA_{CB} = 6/5 \cdot 6/5 \cdot 3 \cdot 1/4 Fb/EA = 27/25 Fb/EA$$

$$L_{AC}^{xx} = N_{AC}^x N_{AC}^x I_{AC}/EA_{AC} = -4/5 \cdot (-4/5) \cdot 4 \cdot 1/4 Fb/EA = 16/25 Fb/EA$$

$$L_{DA}^{xx} = N_{DA}^x N_{DA}^x I_{DA}/EA_{DA} = 3/5 \cdot 3/5 \cdot 3 \cdot 1/3 Fb/EA = 9/25 Fb/EA$$

$$L_{DC}^{xx} = N_{DC}^x N_{DC}^x I_{DC}/EA_{DC} + N_{DC}^x \epsilon_{DC} I_{DC} = 1 \cdot 1 \cdot 5 \cdot 1/3 Fb/EA + 1 \cdot 2 \cdot 5 Fb/EA = 5/3 Fb/EA$$

$$L_{EC}^{xx} = N_{EC}^x N_{EC}^x I_{EC}/EA_{EC} = 3/5 \cdot 3/5 \cdot 3 \cdot 1/2 Fb/EA = 27/50 Fb/EA$$

$$L_{FE}^{xx} = N_{FE}^x N_{FE}^x I_{FE}/EA_{FE} = 1 \cdot 1 \cdot 5 \cdot 1/2 Fb/EA = 5/2 Fb/EA$$

$$L_{DE}^{xx} = N_{DE}^x N_{DE}^x I_{DE}/EA_{DE} = -4/5 \cdot (-4/5) \cdot 4 \cdot 1 Fb/EA = 64/25 Fb/EA$$

$$L_{FD}^{xx} = N_{FD}^x N_{FD}^x I_{FD}/EA_{FD} = 6/5 \cdot 6/5 \cdot 3 \cdot 1 Fb/EA = 108/25 Fb/EA$$

$$L_{DC}^{X_0} = N_{DC}^X N_{DC}^0 I_{DC}/EA_{DC} + N_{DC}^X \varepsilon_{DC} I_{DC} = 1 \cdot 1 \cdot 2 \cdot 5 \cdot Fb/EA = 10 \cdot Fb/EA$$

Contributi nulli elementi

$$L_{AB}^{X_0} \quad L_{CB}^{X_0} \quad L_{AC}^{X_0} \quad L_{DA}^{X_0} \quad L_{EC}^{X_0} \quad L_{FE}^{X_0} \quad L_{DE}^{X_0} \quad L_{FD}^{X_0}$$

Contributi nulli nodi vincolati

$$L_B^{XX} \quad L_F^{XX} \quad L_B^{X_0} \quad L_F^{X_0}$$

Espressione risolvete

$$\left( L_{AB}^{XX} + L_{CB}^{XX} + L_{AC}^{XX} + L_{DA}^{XX} + L_{DC}^{XX} + L_{EC}^{XX} + L_{FE}^{XX} + L_{DE}^{XX} + L_{FD}^{XX} \right) X = - \left( L_{DC}^{X_0} \right)$$

$$\left( 1 + 27/25 + 16/25 + 9/25 + 5/3 + 27/50 + 5/2 + 64/25 + 108/25 \right) X = \left( -10 \right) F$$

$$44/3 X = -10 F$$

Soluzione

$$X = -6/11 F$$

REAZIONI

$$H_B = -6/11\alpha TEA \quad V_B = -27/22\alpha TEA \quad H_F = 6/11\alpha TEA \quad V_F = 27/22\alpha TEA$$

$$N_{AB} = -15/22\alpha TEA \quad N_{CB} = -9/11\alpha TEA \quad N_{AC} = 6/11\alpha TEA \quad N_{DA} = -9/22\alpha TEA$$

$$N_{DC} = -15/22\alpha TEA \quad N_{EC} = -9/22\alpha TEA \quad N_{FE} = -15/22\alpha TEA \quad N_{DE} = 6/11\alpha TEA$$

$$N_{FD} = -9/11\alpha TEA$$

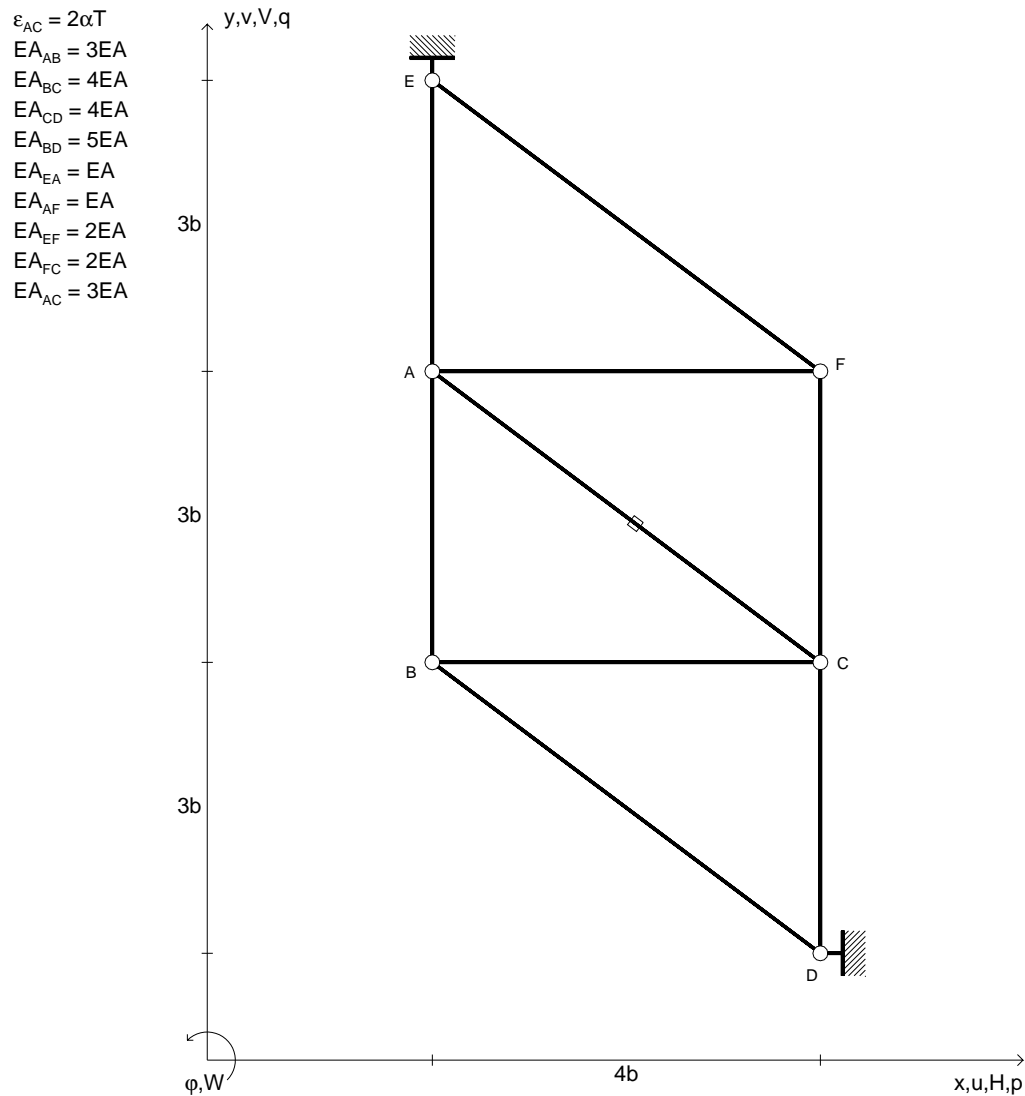
SPOSTAMENTI NODALI

$$\begin{array}{lllll} u_A = 3\alpha Tb & u_B = 0 & u_C = 39/11\alpha Tb & u_D = -921/176\alpha Tb & u_E = -537/176\alpha Tb \\ v_A = -63/22\alpha Tb & v_B = 0 & v_C = 27/44\alpha Tb & v_D = -27/11\alpha Tb & v_E = 27/22\alpha Tb \end{array}$$

$$u_F = 0$$

$$v_F = 0$$





Svolgere l'analisi cinematica.

Riportare la soluzione su questo foglio.

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

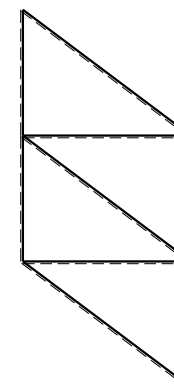
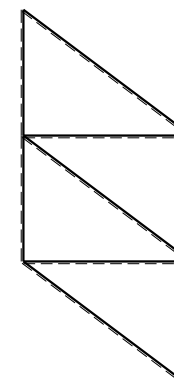
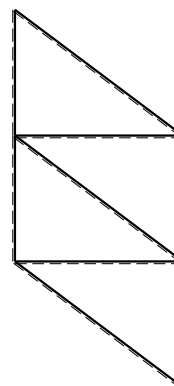
Tracciare i diagrammi delle azioni interne nelle aste.

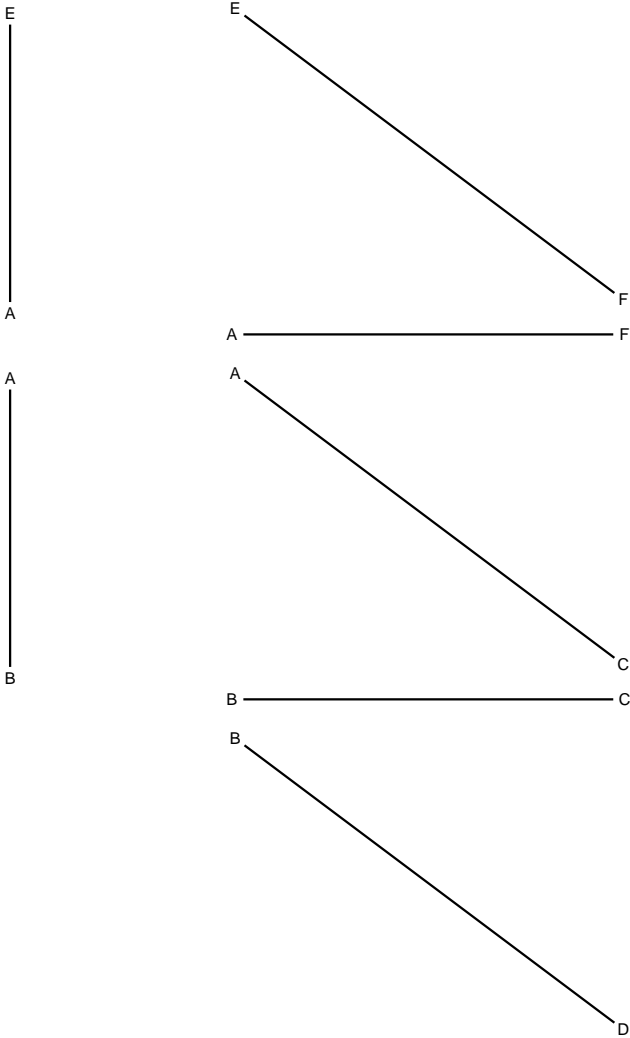
Calcolare spostamento e rotazione di tutti i nodi.

$A_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.

Allungamento termico assegnato  $\varepsilon$  su asta AC.

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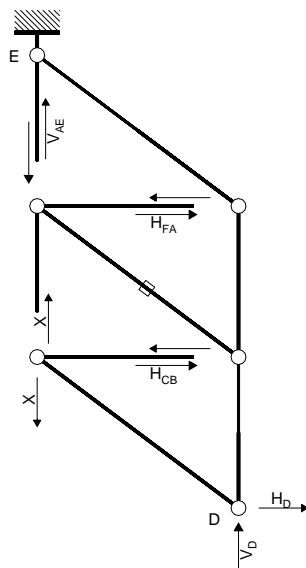
REAZIONI

$H_D =$	$V_D =$	$H_E =$	$V_E =$
$N_{AB} =$	$N_{BC} =$	$N_{CD} =$	$N_{BD} =$
$N_{EA} =$	$N_{AF} =$	$N_{EF} =$	$N_{FC} =$
$N_{AC} =$			

SPOSTAMENTI NODALI

$u_A =$	$u_B =$	$u_C =$	$u_D =$	$u_E =$
$v_A =$	$v_B =$	$v_C =$	$v_D =$	$v_E =$
$u_F =$				
$v_F =$				





## EQUAZIONI DI EQUILIBRIO

Rotazione intorno a E: aste EF FC CD CA AB DB AF BC

$$9H_D b + 4V_D b = 0$$

Rotazione intorno a F: aste FC CD CA AB DB AF BC

$$6H_D b + 4V_{AE} b = 0$$

Rotazione intorno a C: aste CD DB BC

$$3H_D b = -4Xb$$

Rotazione intorno a C: aste CA AB AF

$$4V_{AE} b - 3H_{FA} b = 4Xb$$

Rotazione intorno a D: aste DB BC

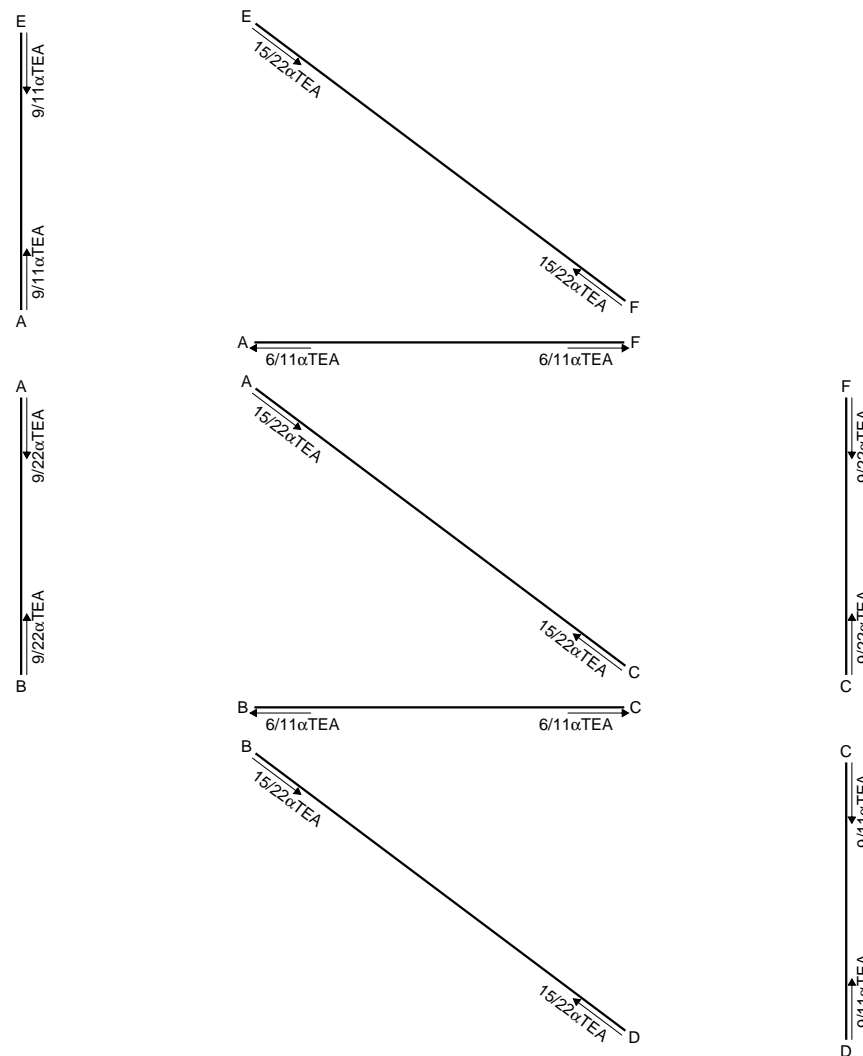
$$-3H_{CB} b = -4Xb$$

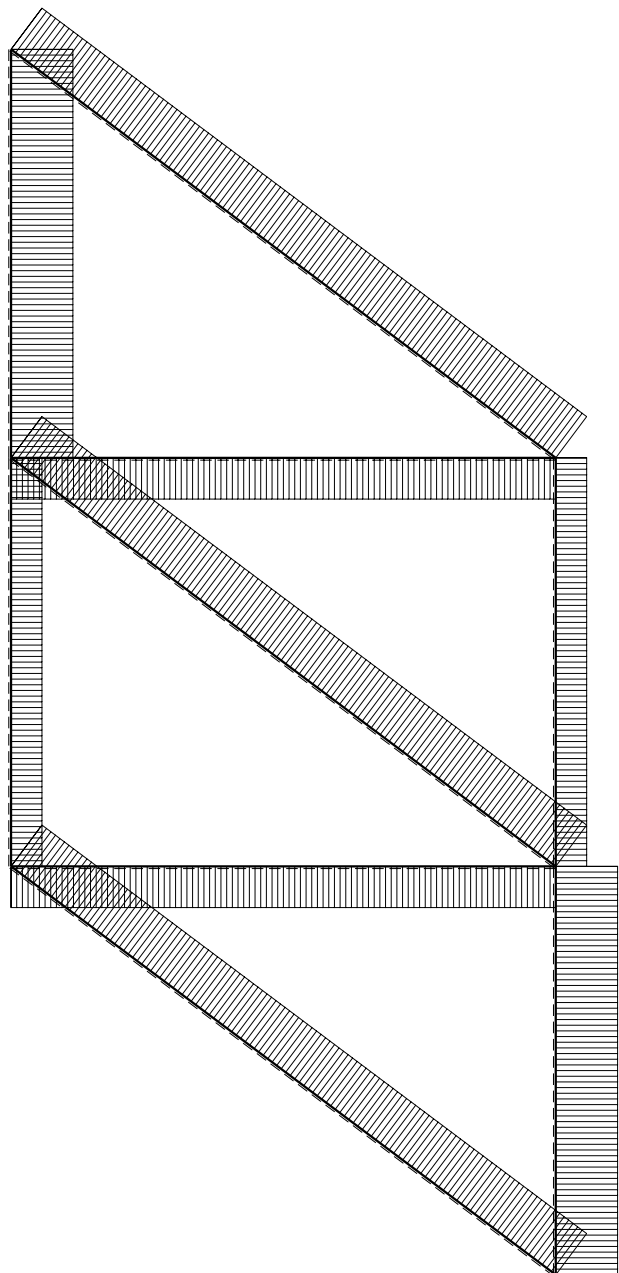
Matrice di equilibrio

$$\begin{bmatrix} \varphi_{EF} \\ \varphi_{FC} \\ \varphi_{CD} \\ \varphi_{CA} \\ \varphi_{DB} \end{bmatrix} \begin{bmatrix} H_D b & V_D b & H_{CB} b & V_{AE} b & H_{FA} b \end{bmatrix} = \begin{bmatrix} Xb \\ 0 \\ -4 \\ 4 \\ -4 \end{bmatrix}$$

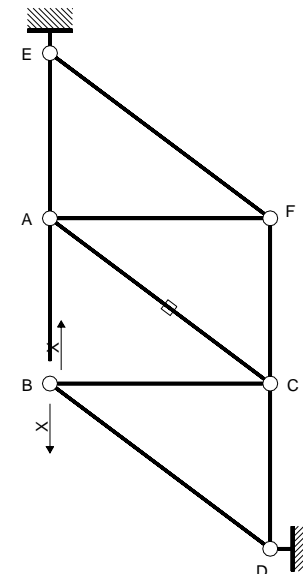
Soluzione del sistema

$$\begin{bmatrix} H_D b \\ V_D b \\ V_{AE} b \\ H_{FA} b \\ H_{CB} b \end{bmatrix} = \begin{bmatrix} Xb \\ -4/3 \\ 2 \\ 4/3 \\ 4/3 \end{bmatrix}$$





← ⊕ → | 1 αTEA



## REAZIONI IPERSTATICHE

$$X = V_{BA}$$

## CALCOLO DELLE REAZIONI IPERSTATICHE

$$L_{AB}^{xx} = N_{AB}^x N_{AB}^x I_{AB}/EA_{AB} = -1 \cdot (-1) \cdot 3 \cdot 1/3 Fb/EA = Fb/EA$$

$$L_{BC}^{xx} = N_{BC}^x N_{BC}^x I_{BC}/EA_{BC} = 4/3 \cdot 4/3 \cdot 4 \cdot 1/4 Fb/EA = 16/9 Fb/EA$$

$$L_{CD}^{xx} = N_{CD}^x N_{CD}^x I_{CD}/EA_{CD} = -2 \cdot (-2) \cdot 3 \cdot 1/4 Fb/EA = 3 Fb/EA$$

$$L_{BD}^{xx} = N_{BD}^x N_{BD}^x I_{BD}/EA_{BD} = -5/3 \cdot (-5/3) \cdot 5 \cdot 1/5 Fb/EA = 25/9 Fb/EA$$

$$L_{EA}^{xx} = N_{EA}^x N_{EA}^x I_{EA}/EA_{EA} = -2 \cdot (-2) \cdot 3 \cdot 1 Fb/EA = 12 Fb/EA$$

$$L_{AF}^{xx} = N_{AF}^x N_{AF}^x I_{AF}/EA_{AF} = 4/3 \cdot 4/3 \cdot 4 \cdot 1 Fb/EA = 64/9 Fb/EA$$

$$L_{EF}^{xx} = N_{EF}^x N_{EF}^x I_{EF}/EA_{EF} = -5/3 \cdot (-5/3) \cdot 5 \cdot 1/2 Fb/EA = 125/18 Fb/EA$$

$$L_{FC}^{xx} = N_{FC}^x N_{FC}^x I_{FC}/EA_{FC} = -1 \cdot (-1) \cdot 3 \cdot 1/2 Fb/EA = 3/2 Fb/EA$$

$$L_{AC}^{xx} = N_{AC}^x N_{AC}^x I_{AC}/EA_{AC} + N_{AC}^x \epsilon_{AC} I_{AC} = -5/3 \cdot (-5/3) \cdot 5 \cdot 1/3 Fb/EA - 5/3 \cdot 2 \cdot 5 Fb/EA = 125/27 Fb/EA$$

$$L_{AC}^{X_0} = N_{AC}^X N_{AC}^0 I_{AC}/EA_{AC} + N_{AC}^X \epsilon_{AC} I_{AC} = -5/3 - 5/3 \cdot 2 \cdot 5 \text{ Fb}/EA = -50/3 \text{ Fb}/EA$$

Contributi nulli elementi

$$L_{AB}^{X_0} \quad L_{BC}^{X_0} \quad L_{CD}^{X_0} \quad L_{BD}^{X_0} \quad L_{EA}^{X_0} \quad L_{AF}^{X_0} \quad L_{EF}^{X_0} \quad L_{FC}^{X_0}$$

Contributi nulli nodi vincolati

$$L_D^{XX} \quad L_E^{XX} \quad L_D^{X_0} \quad L_E^{X_0}$$

Espressione risolvete

$$\left( L_{AB}^{XX} + L_{BC}^{XX} + L_{CD}^{XX} + L_{BD}^{XX} + L_{EA}^{XX} + L_{AF}^{XX} + L_{EF}^{XX} + L_{FC}^{XX} + L_{AC}^{XX} \right) X = - \left( L_{AC}^{X_0} \right)$$

$$\left( 1 + 16/9 + 3 + 25/9 + 12 + 64/9 + 125/18 + 3/2 + 125/27 \right) X = \left( 50/3 \right) F$$

$$1100/27 X = 50/3 F$$

Soluzione

$$X = 9/22 F$$

REAZIONI

$$H_D = -6/11\alpha TEA \quad V_D = 27/22\alpha TEA \quad H_E = 6/11\alpha TEA \quad V_E = -27/22\alpha TEA$$

$$N_{AB} = -9/22\alpha TEA \quad N_{BC} = 6/11\alpha TEA \quad N_{CD} = -9/11\alpha TEA \quad N_{BD} = -15/22\alpha TEA$$

$$N_{EA} = -9/11\alpha TEA \quad N_{AF} = 6/11\alpha TEA \quad N_{EF} = -15/22\alpha TEA \quad N_{FC} = -9/22\alpha TEA$$

$$N_{AC} = -15/22\alpha TEA$$

SPOSTAMENTI NODALI

$$\begin{array}{lllll} u_A = -921/176\alpha Tb & u_B = 3\alpha Tb & u_C = 39/11\alpha Tb & u_D = 0 & u_E = 0 \\ v_A = 27/11\alpha Tb & v_B = 63/22\alpha Tb & v_C = -27/44\alpha Tb & v_D = 0 & v_E = 0 \end{array}$$

$$u_F = -537/176\alpha Tb$$

$$v_F = -27/22\alpha Tb$$

