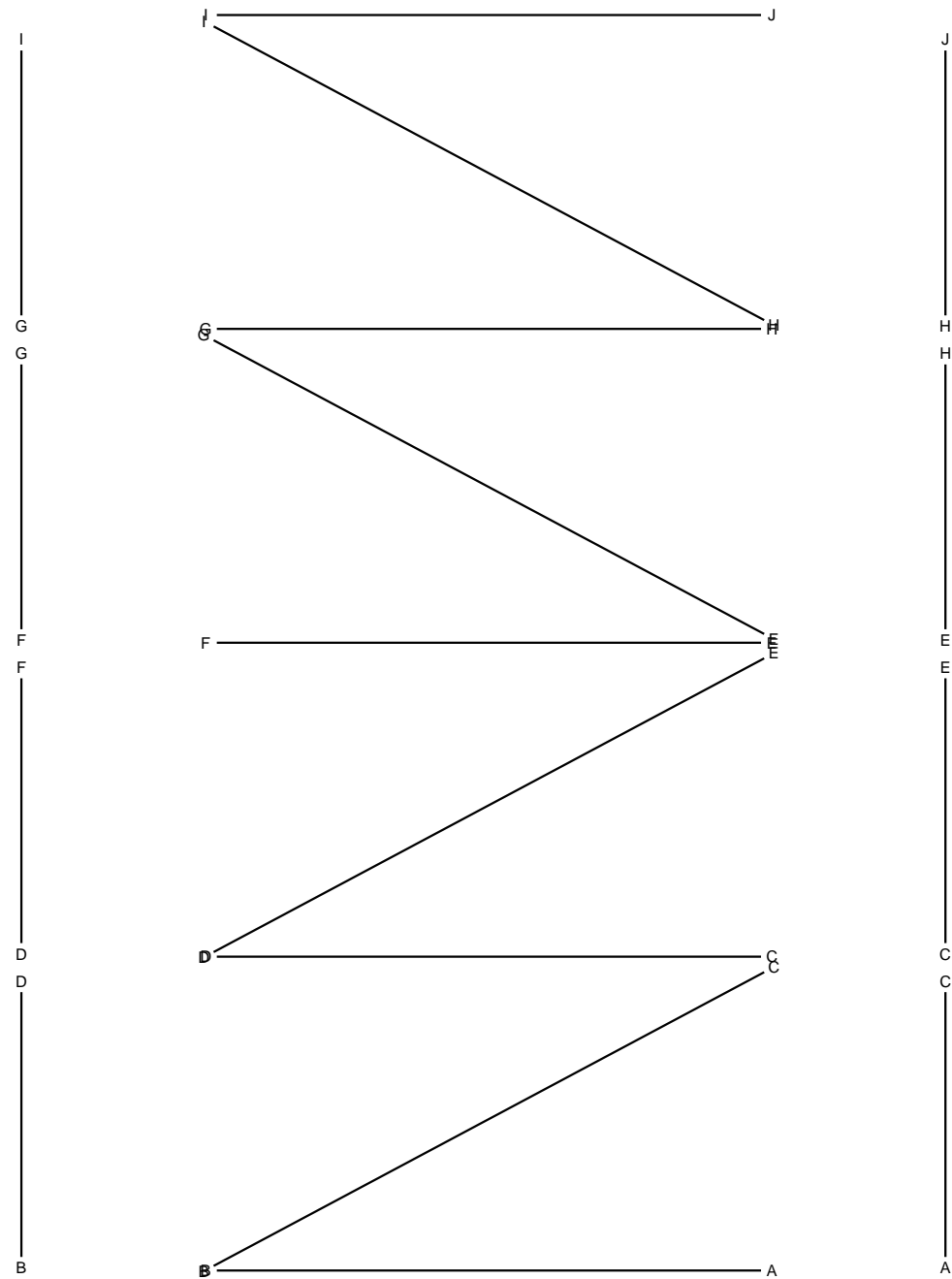
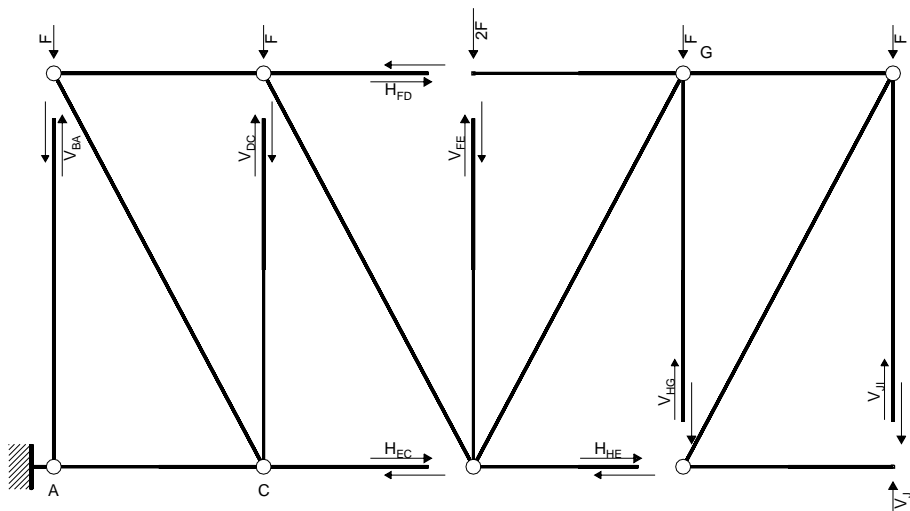


Svolgere l'analisi cinematica.  
Tracciare la deformata elastica.

Carichi e deformazioni date hanno verso efficace in disegno.  
Calcolare reazioni vincolari della struttura e delle aste.  
Tracciare i diagrammi delle azioni interne nelle aste.  
Calcolare spostamento e rotazione di tutti i nodi.  
 $A_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.24.05.07



REAZIONI					
$H_A =$	$V_A =$	$V_J =$			
$N_{AB} =$	$N_{CD} =$	$N_{EF} =$	$N_{GH} =$	$N_{IJ} =$	$N_{AC} =$
$N_{CE} =$	$N_{EH} =$	$N_{HJ} =$	$N_{BD} =$	$N_{DF} =$	$N_{FG} =$
$N_{GI} =$	$N_{BC} =$	$N_{DE} =$	$N_{EG} =$	$N_{HI} =$	
SPOSTAMENTI NODALI					
$u_A =$		$u_B =$		$u_C =$	
$v_A =$		$v_B =$		$v_C =$	
$u_D =$		$u_E =$		$u_F =$	
$v_D =$		$v_E =$		$v_F =$	
$u_G =$		$u_H =$		$u_I =$	
$v_G =$		$v_H =$		$v_I =$	
$u_{JJ} =$					
$v_J =$					



## EQUAZIONI DI EQUILIBRIO

Rotazione intorno a A: aste AC CD CE CB BD DF DE EF EH EG GH GF GI IJ IH HJ

$$32V_j b = 96Fb$$

Rotazione intorno a C: aste CB BD DF DE EF EH EG GH GF GI IJ IH HJ

$$24V_j b + 8V_{BA} b = 48Fb$$

Rotazione intorno a B: aste BD DF DE EF EH EG GH GF GI IJ IH HJ

$$32V_j b - 8V_{DC} b - 15H_{EC} b = 96Fb$$

Rotazione intorno a D: aste DE EF EH EG GH GF GI IJ IH HJ

$$24V_j b - 15H_{EC} b = 56Fb$$

Rotazione intorno a E: aste EG GH GF GI IJ IH HJ

$$16V_j b + 15H_{FD} b = 24Fb$$

Rotazione intorno a G: aste GF

$$8V_{FE} b = -16Fb$$

Rotazione intorno a G: aste GI IJ IH HJ

$$8V_j b - 15H_{HE} b = 8Fb$$

Rotazione intorno a I: aste IH HJ

$$8V_{HG} b - 15H_{HE} b = 0$$

Rotazione intorno a H: aste HJ

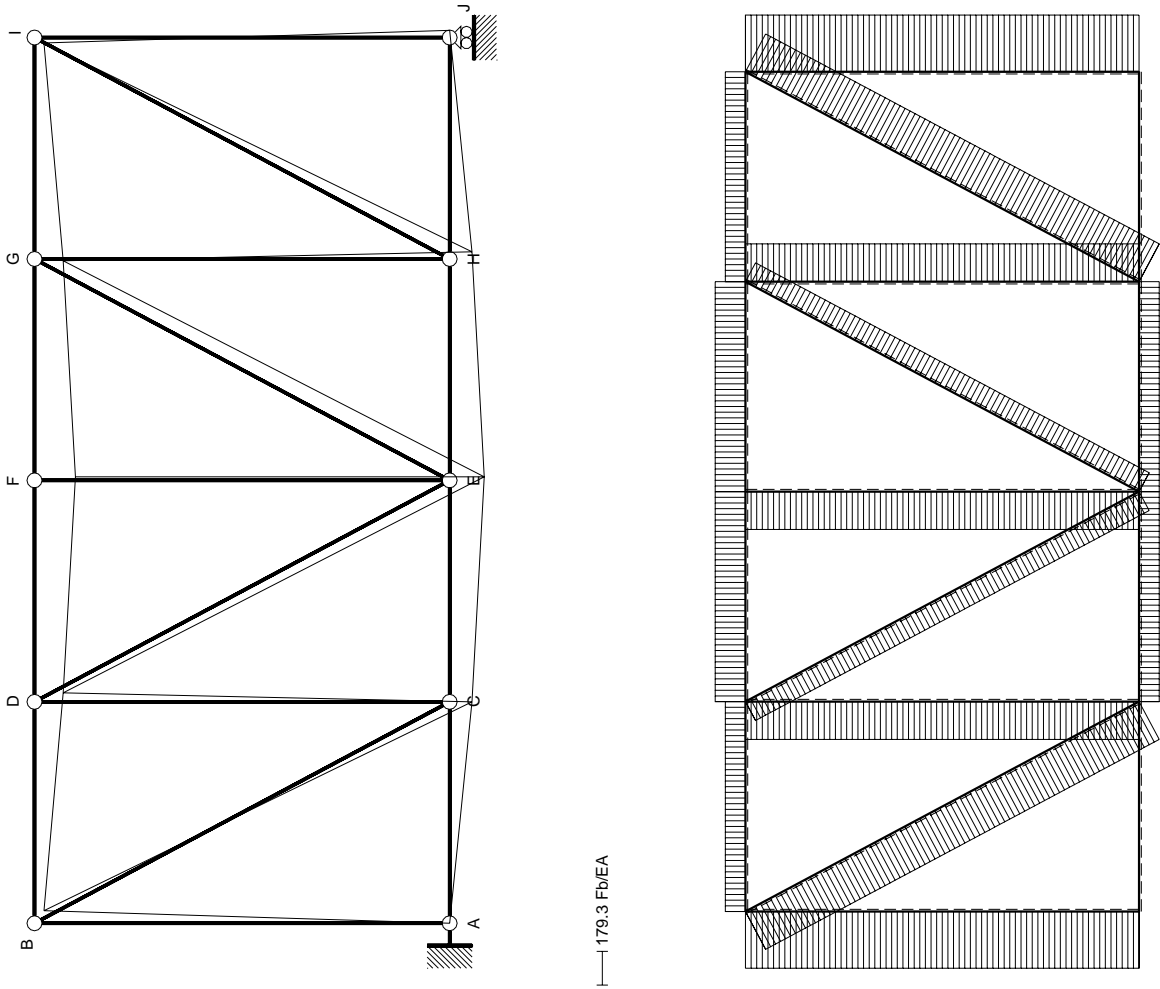
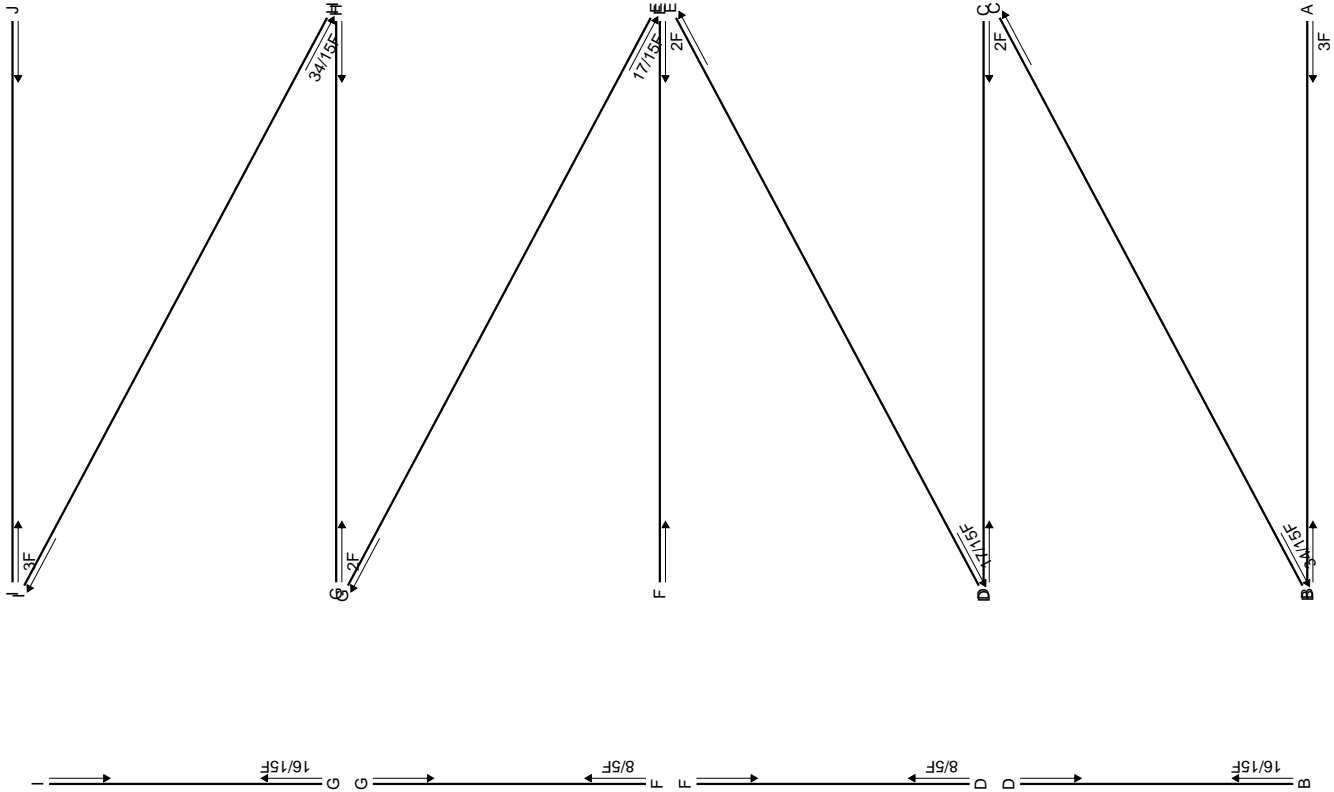
$$8V_j b - 8V_{JI} b = 0$$

## Matrice di equilibrio

$$\begin{bmatrix} \varphi_{AC} & \varphi_{CB} & \varphi_{BD} & \varphi_{DE} & \varphi_{EG} & \varphi_{GF} & \varphi_{GI} & \varphi_{IH} & \varphi_{HJ} \end{bmatrix} \begin{bmatrix} V_j b & V_{BA} b & V_{DC} b & V_{FE} b & V_{HG} b & V_{JI} b & H_{EC} b & H_{HE} b & H_{FD} b \end{bmatrix} = \begin{bmatrix} 96 \\ 48 \\ 96 \\ 56 \\ 24 \\ -16 \\ 8 \\ 0 \\ 0 \end{bmatrix}$$

## Soluzione del sistema

$$\begin{bmatrix} V_j b \\ V_{BA} b \\ V_{DC} b \\ H_{EC} b \\ H_{FD} b \\ V_{FE} b \\ H_{HE} b \\ V_{HG} b \\ V_{JI} b \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ -2 \\ 16/15 \\ -8/5 \\ -2 \\ 16/15 \\ 2 \\ 3 \end{bmatrix}$$



REAZIONI

$H_A = 0 \quad V_A = 3F \quad V_J = 3F$

$N_{AB} = -3F \quad N_{CD} = -2F \quad N_{EF} = -2F \quad N_{GH} = -2F \quad N_{IJ} = -3F \quad N_{AC} = 0$

$N_{CE} = 16/15F \quad N_{EH} = 16/15F \quad N_{HJ} = 0 \quad N_{BD} = -16/15F \quad N_{DF} = -8/5F \quad N_{FG} = -8/5F$

$N_{GI} = -16/15F \quad N_{BC} = 34/15F \quad N_{DE} = 17/15F \quad N_{EG} = 17/15F \quad N_{HI} = 34/15F$

SPOSTAMENTI NODALI

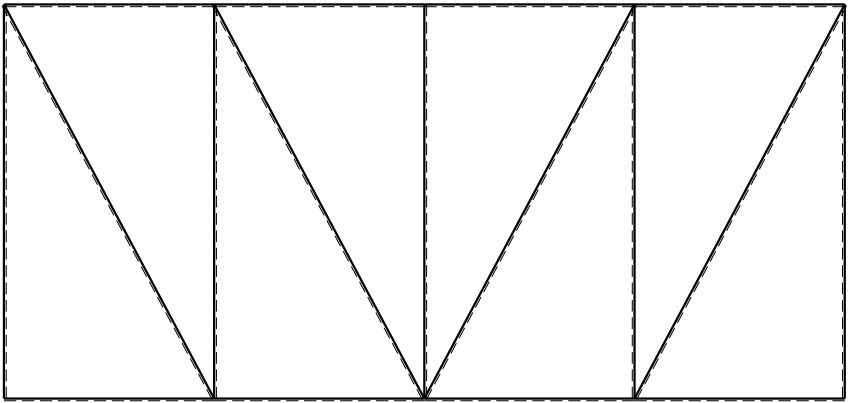
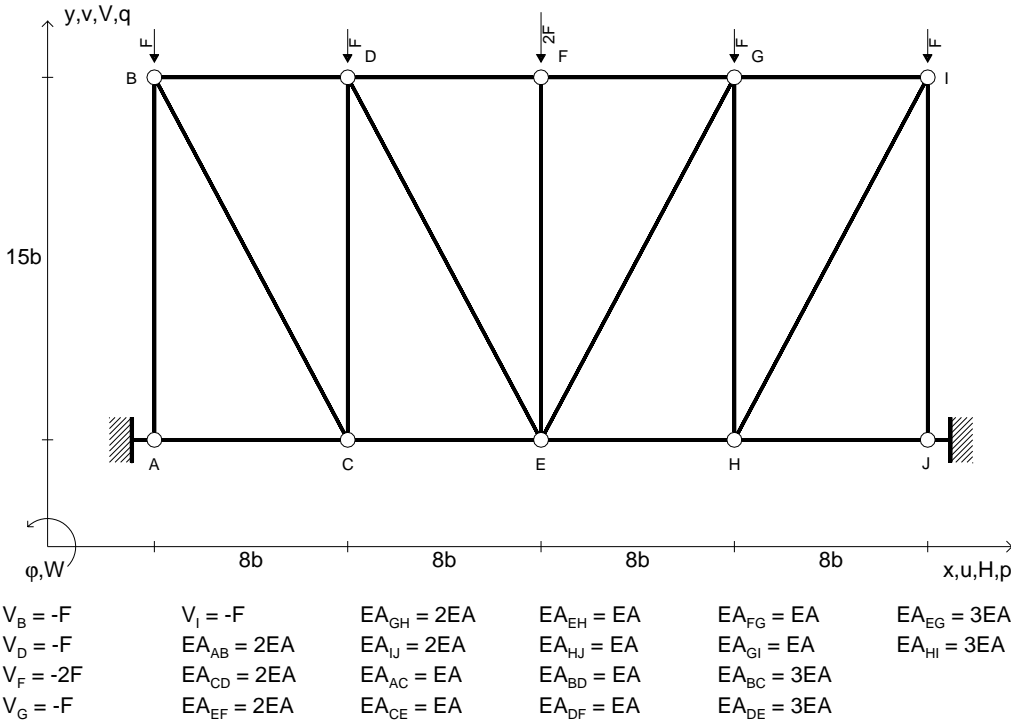
$u_A = 0 \quad u_B = 448/15(Fb/EA) \quad u_C = 0$   
 $v_A = 0 \quad v_B = -45/2(Fb/EA) \quad v_C = -71531/1350(Fb/EA)$

$u_D = 64/3(Fb/EA) \quad u_E = 128/15(Fb/EA) \quad u_F = 128/15(Fb/EA)$   
 $v_D = -91781/1350(Fb/EA) \quad v_E = -36941/450(Fb/EA) \quad v_F = -43691/450(Fb/EA)$

$u_G = -64/15(Fb/EA) \quad u_H = 256/15(Fb/EA) \quad u_I = -64/5(Fb/EA)$   
 $v_G = -91781/1350(Fb/EA) \quad v_H = -71531/1350(Fb/EA) \quad v_I = -45/2(Fb/EA)$

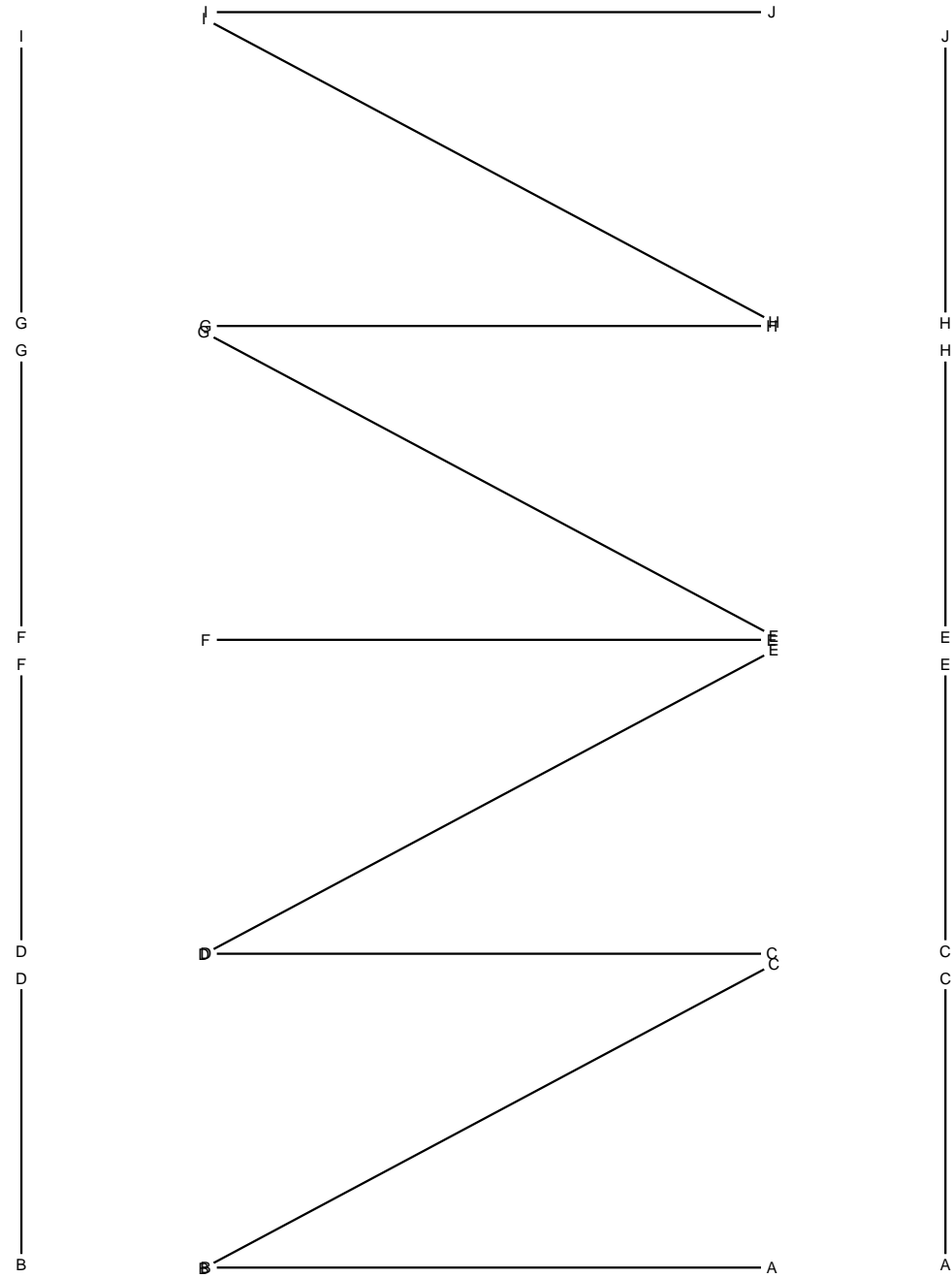
$u_{JJl} = 256/15(Fb/EA)$   
 $v_J = 0$





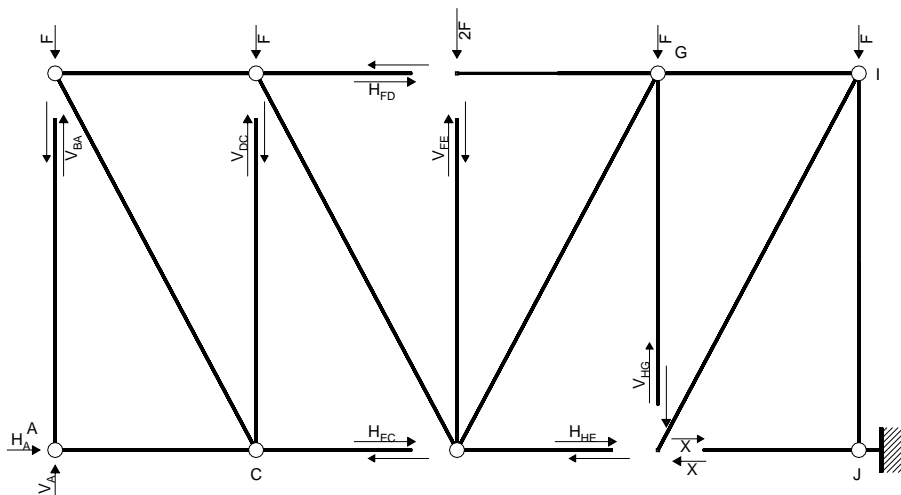
Svolgere l'analisi cinematica.  
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Calcolare spostamento e rotazione di tutti i nodi.  
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@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.24.05.07



REAZIONI					
$H_A =$	$V_A =$	$H_J =$	$V_J =$		
$N_{AB} =$	$N_{CD} =$	$N_{EF} =$	$N_{GH} =$	$N_{IJ} =$	$N_{AC} =$
$N_{CE} =$	$N_{EH} =$	$N_{HJ} =$	$N_{BD} =$	$N_{DF} =$	$N_{FG} =$
$N_{GI} =$	$N_{BC} =$	$N_{DE} =$	$N_{EG} =$	$N_{HI} =$	
SPOSTAMENTI NODALI					
$u_A =$		$u_B =$		$u_C =$	
$v_A =$		$v_B =$		$v_C =$	
$u_D =$		$u_E =$		$u_F =$	
$v_D =$		$v_E =$		$v_F =$	
$u_G =$		$u_H =$		$u_I =$	
$v_G =$		$v_H =$		$v_I =$	
$u_J =$					
$v_J =$					





## EQUAZIONI DI EQUILIBRIO

Rotazione intorno a J: aste JI IG IH GH GF GE EF EH ED DB DF BC CD CA CE AB

$$-32V_A b = -96Fb$$

Rotazione intorno a I: aste IG GH GF GE EF EH ED DB DF BC CD CA CE AB

$$15H_A b - 32V_A b - 8V_{HG} b + 15H_{HF} b = -96Fb$$

Rotazione intorno a I: aste IH

$$8V_{HG}b - 15H_{HF}b = 15Xb$$

Rotazione intorno a G: aste GF

$$8V_{FEb} = -16Fb$$

Rotazione intorno a G: aste GE EF EH ED DB DF BC CD CA CE AB

$$15H_A b - 24V_A b - 8V_{FF} b + 15H_{HF} b = -40Fb$$

Rotazione intorno a E: aste ED DB DF BC CD CA CE AB

$$-16V_A b - 15H_{FD} b = -24Fb$$

Rotazione intorno a D: aste DB BC CD CA CE AB

$$15H_A b - 8V_A b + 15H_{EC} b = -8Fb$$

Rotazione intorno a B: aste BC CD CA CE AB

$$15H_A b + 8V_{DC} b + 15H_{EC} b = 0$$

Rotazione intorno a C: aste CA AB

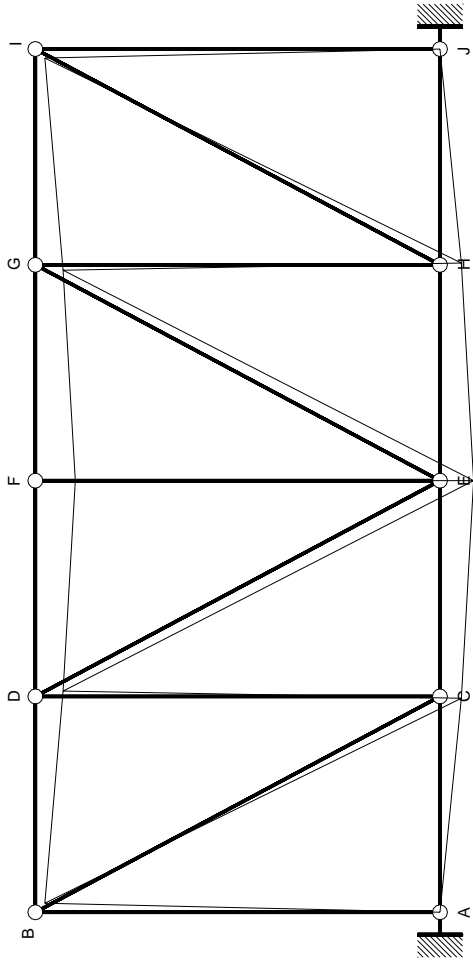
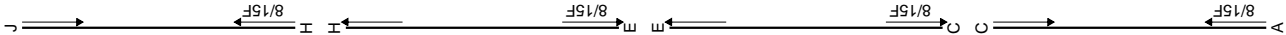
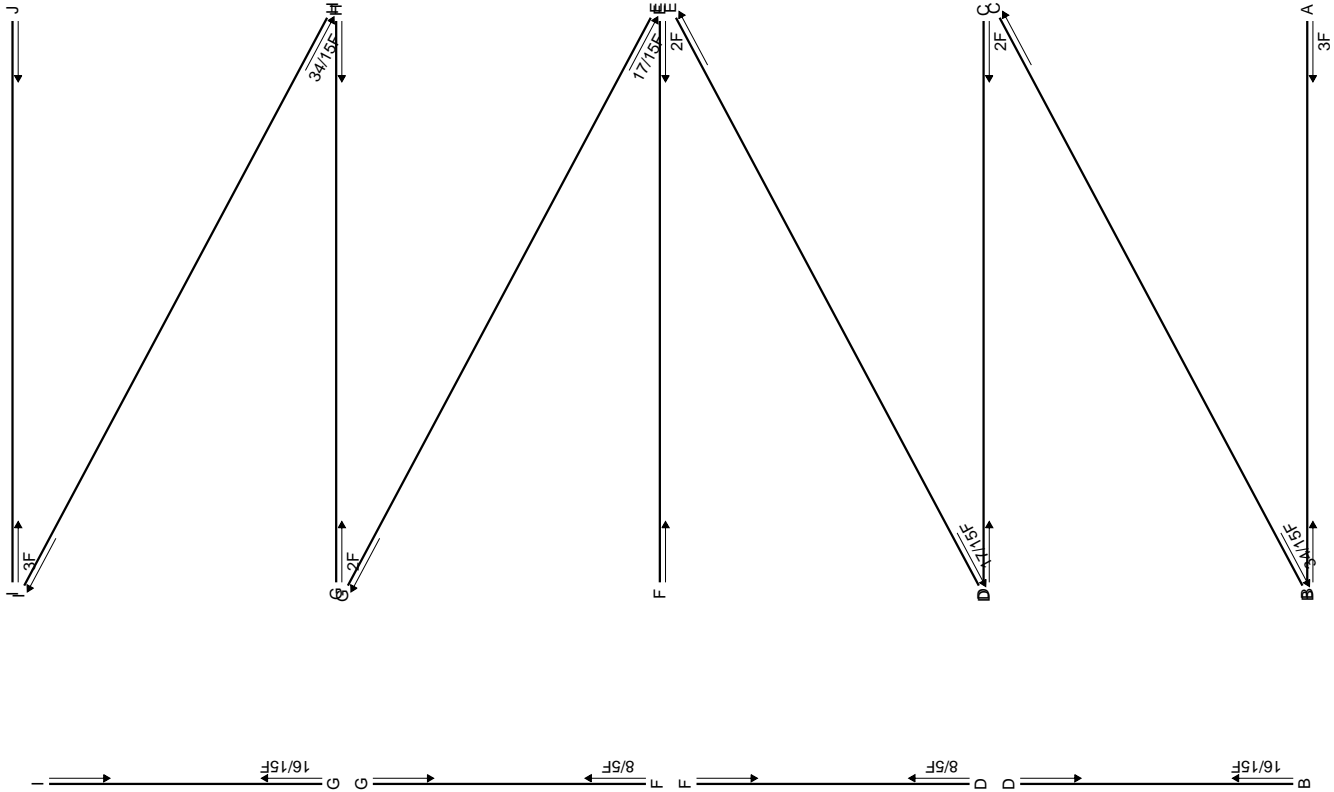
$$-8V_A b - 8V_{BA} b = 0$$

### Matrice di equilibrio

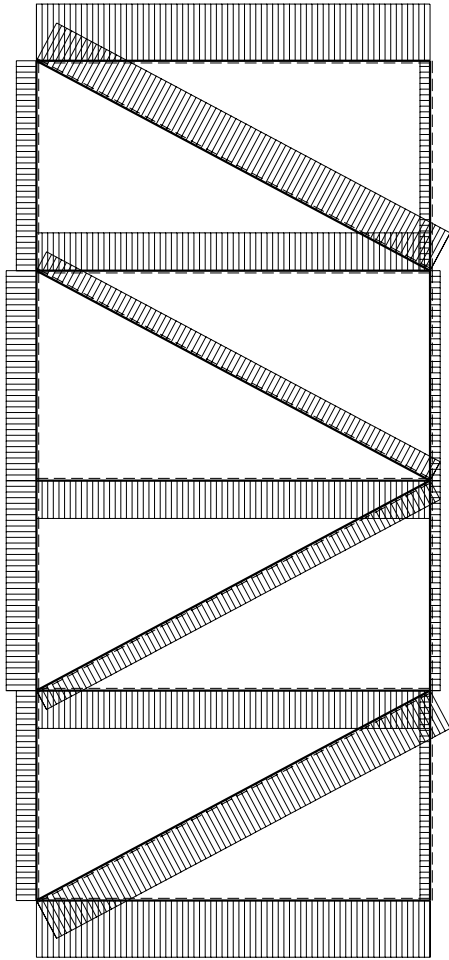
$$\begin{bmatrix} \Phi_{JI} \\ \Phi_{IG} \\ \Phi_{IH} \\ \Phi_{GF} \\ \Phi_{GE} \\ \Phi_{ED} \\ \Phi_{DB} \\ \Phi_{BC} \\ \Phi_{CA} \end{bmatrix} \begin{bmatrix} H_{Ab} & V_{Ab} & V_{BA} & V_{DC} & V_{FE} & V_{HG} & H_{EC} & H_{HE} & H_{FD} \end{bmatrix} = \begin{bmatrix} X_b & F_b \end{bmatrix} \begin{bmatrix} 0 & -96 \\ 0 & -96 \\ 15 & 0 \\ 0 & -16 \\ 0 & -40 \\ 0 & -24 \\ 0 & -8 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

### Soluzione del sistema

$$\begin{bmatrix} V_{Ab} \\ H_{Ab} \\ V_{HGb} \\ V_{FEb} \\ H_{HEb} \\ H_{FDb} \\ H_{ECb} \\ V_{DCb} \\ V_{RAb} \end{bmatrix} = \begin{bmatrix} Xb & Fb \\ 0 & 3 \\ 1 & 0 \\ 0 & 2 \\ 0 & -2 \\ -1 & 16/15 \\ 0 & -8/5 \\ -1 & 16/15 \\ 0 & -2 \\ 0 & -3 \end{bmatrix}$$



$179.3 F_b/Ea$



$\left[ \begin{array}{c} + \\ - \end{array} \right] \rightarrow \left[ \begin{array}{c} + \\ - \end{array} \right] 4F$

REAZIONI

$H_A = 8/15F$      $V_A = 3F$      $H_J = -8/15F$      $V_J = 3F$

$N_{AB} = -3F$      $N_{CD} = -2F$      $N_{EF} = -2F$      $N_{GH} = -2F$      $N_{IJ} = -3F$      $N_{AC} = -8/15F$

$N_{CE} = 8/15F$      $N_{EH} = 8/15F$      $N_{HJ} = -8/15F$      $N_{BD} = -16/15F$      $N_{DF} = -8/5F$      $N_{FG} = -8/5F$

$N_{GI} = -16/15F$      $N_{BC} = 34/15F$      $N_{DE} = 17/15F$      $N_{EG} = 17/15F$      $N_{HI} = 34/15F$

SPOSTAMENTI NODALI

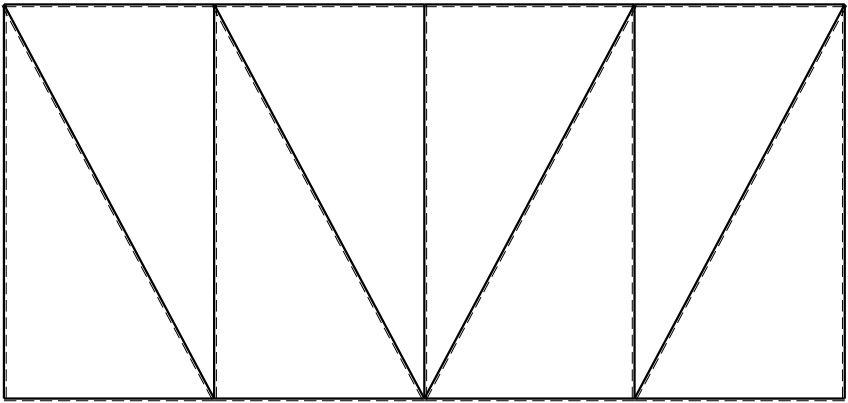
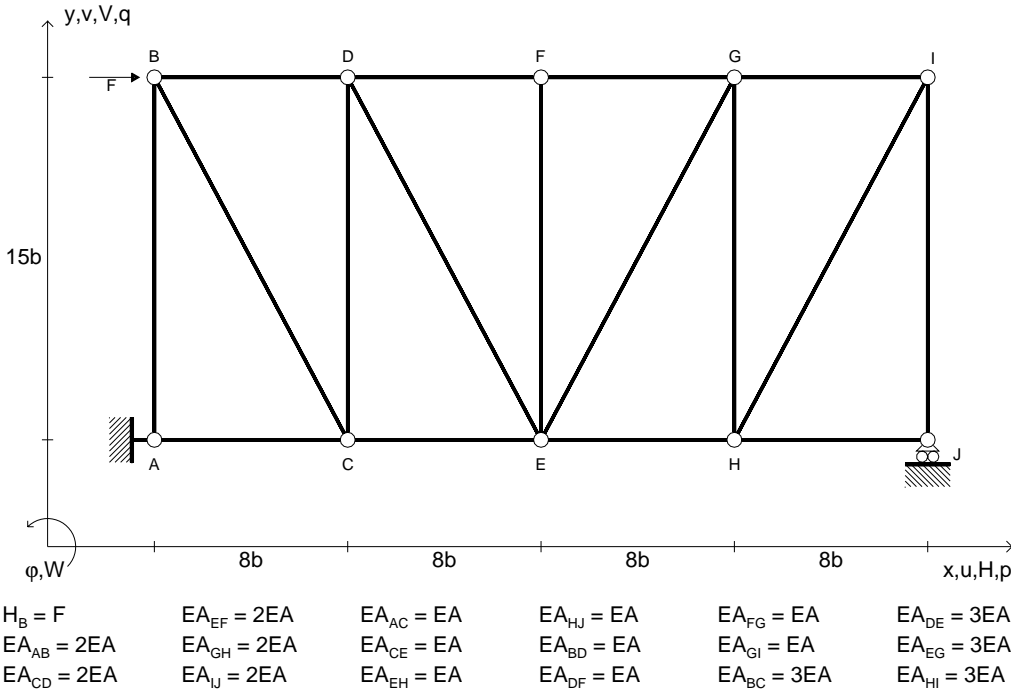
$u_A = 0$      $u_B = 64/3(Fb/EA)$      $u_C = -64/15(Fb/EA)$   
 $v_A = 0$      $v_B = -45/2(Fb/EA)$      $v_C = -68459/1350(Fb/EA)$

$u_D = 64/5(Fb/EA)$      $u_E = 0$      $u_F = 0$   
 $v_D = -88709/1350(Fb/EA)$      $v_E = -35917/450(Fb/EA)$      $v_F = -42667/450(Fb/EA)$

$u_G = -64/5(Fb/EA)$      $u_H = 64/15(Fb/EA)$      $u_I = -64/3(Fb/EA)$   
 $v_G = -88709/1350(Fb/EA)$      $v_H = -68459/1350(Fb/EA)$      $v_I = -45/2(Fb/EA)$

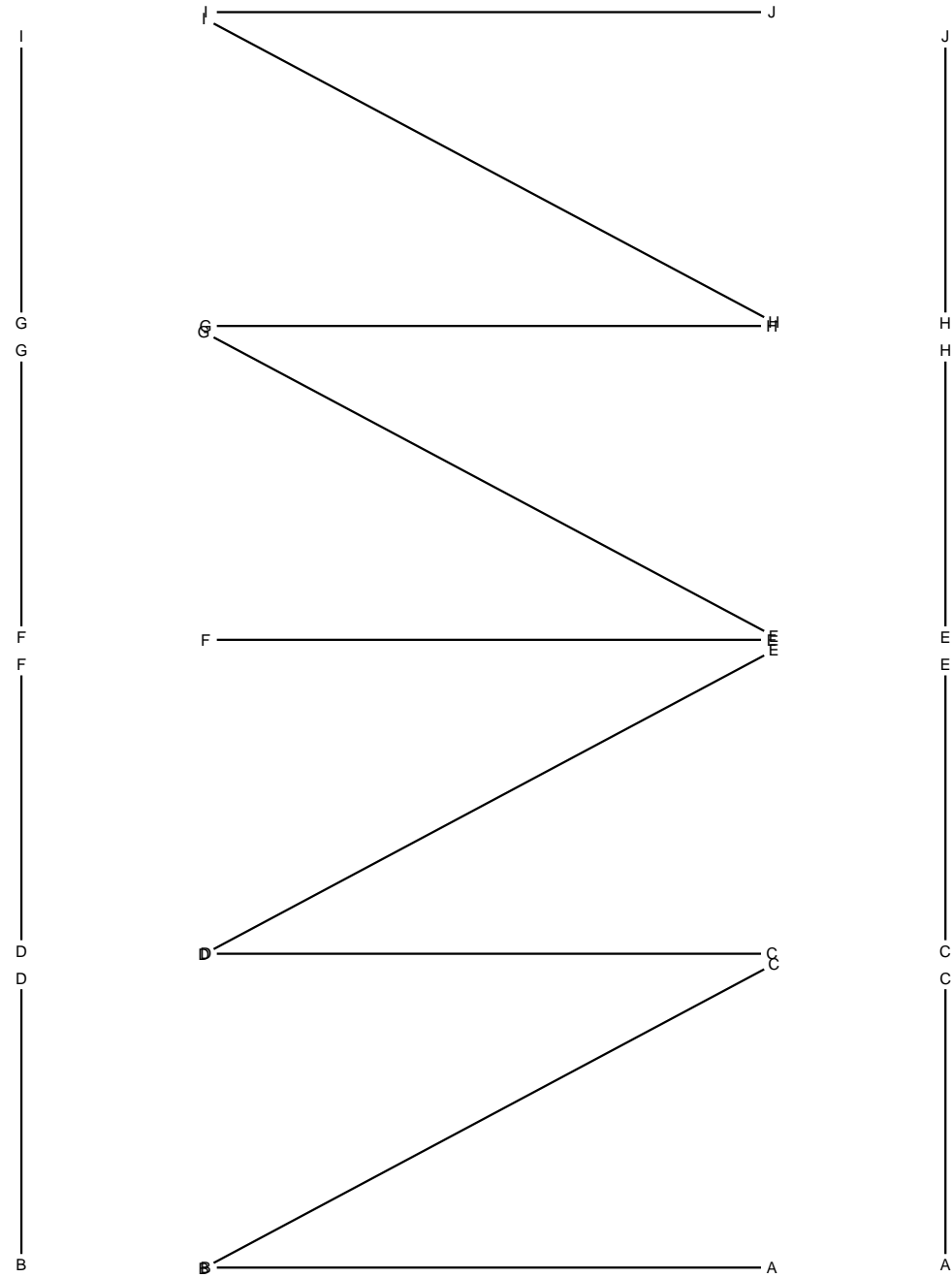
$u_J = 0$   
 $v_J = 0$



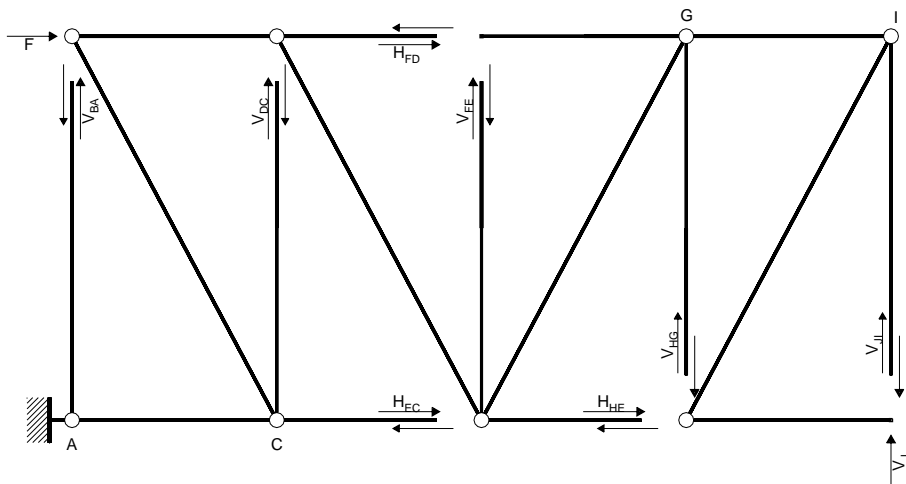


Svolgere l'analisi cinematica.  
Tracciare la deformata elastica.

Carichi e deformazioni date hanno verso efficace in disegno.  
Calcolare reazioni vincolari della struttura e delle aste.  
Tracciare i diagrammi delle azioni interne nelle aste.  
Calcolare spostamento e rotazione di tutti i nodi.  
 $A_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.24.05.07



REAZIONI					
$H_A =$	$V_A =$	$V_J =$			
$N_{AB} =$	$N_{CD} =$	$N_{EF} =$	$N_{GH} =$	$N_{IJ} =$	$N_{AC} =$
$N_{CE} =$	$N_{EH} =$	$N_{HJ} =$	$N_{BD} =$	$N_{DF} =$	$N_{FG} =$
$N_{GI} =$	$N_{BC} =$	$N_{DE} =$	$N_{EG} =$	$N_{HI} =$	
SPOSTAMENTI NODALI					
$u_A =$		$u_B =$		$u_C =$	
$v_A =$		$v_B =$		$v_C =$	
$u_D =$		$u_E =$		$u_F =$	
$v_D =$		$v_E =$		$v_F =$	
$u_G =$		$u_H =$		$u_I =$	
$v_G =$		$v_H =$		$v_I =$	
$u_{JJ} =$					
$v_J =$					



## EQUAZIONI DI EQUILIBRIO

Rotazione intorno a A: aste AC CD CE CB BD DF DE EF EH EG GH GF GI IJ IH HJ

$$32V_{Jb} = 15Fb$$

Rotazione intorno a C: aste CB BD DF DE EF EH EG GH GF GI IJ IH HJ

$$24V_{Jb} + 8V_{BA}b = 15Fb$$

Rotazione intorno a B: aste BD DF DE EF EH EG GH GF GI IJ IH HJ

$$32V_{Jb} - 8V_{DC}b - 15H_{EC}b = 0$$

Rotazione intorno a D: aste DE EF EH EG GH GF GI IJ IH HJ

$$24V_{Jb} - 15H_{EC}b = 0$$

Rotazione intorno a E: aste EG GH GF GI IJ IH HJ

$$16V_{Jb} + 15H_{FD}b = 0$$

Rotazione intorno a G: aste GF

$$8V_{FE}b = 0$$

Rotazione intorno a G: aste GI IJ IH HJ

$$8V_{Jb} - 15H_{HE}b = 0$$

Rotazione intorno a I: aste IH HJ

$$8V_{HG}b - 15H_{HE}b = 0$$

Rotazione intorno a H: aste HJ

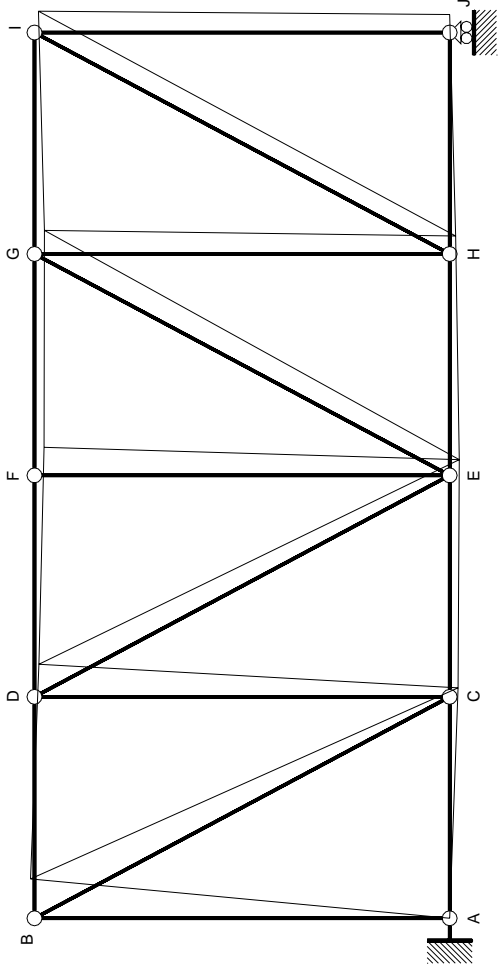
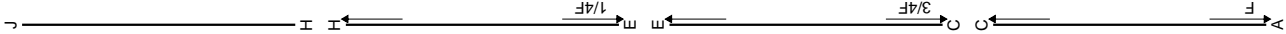
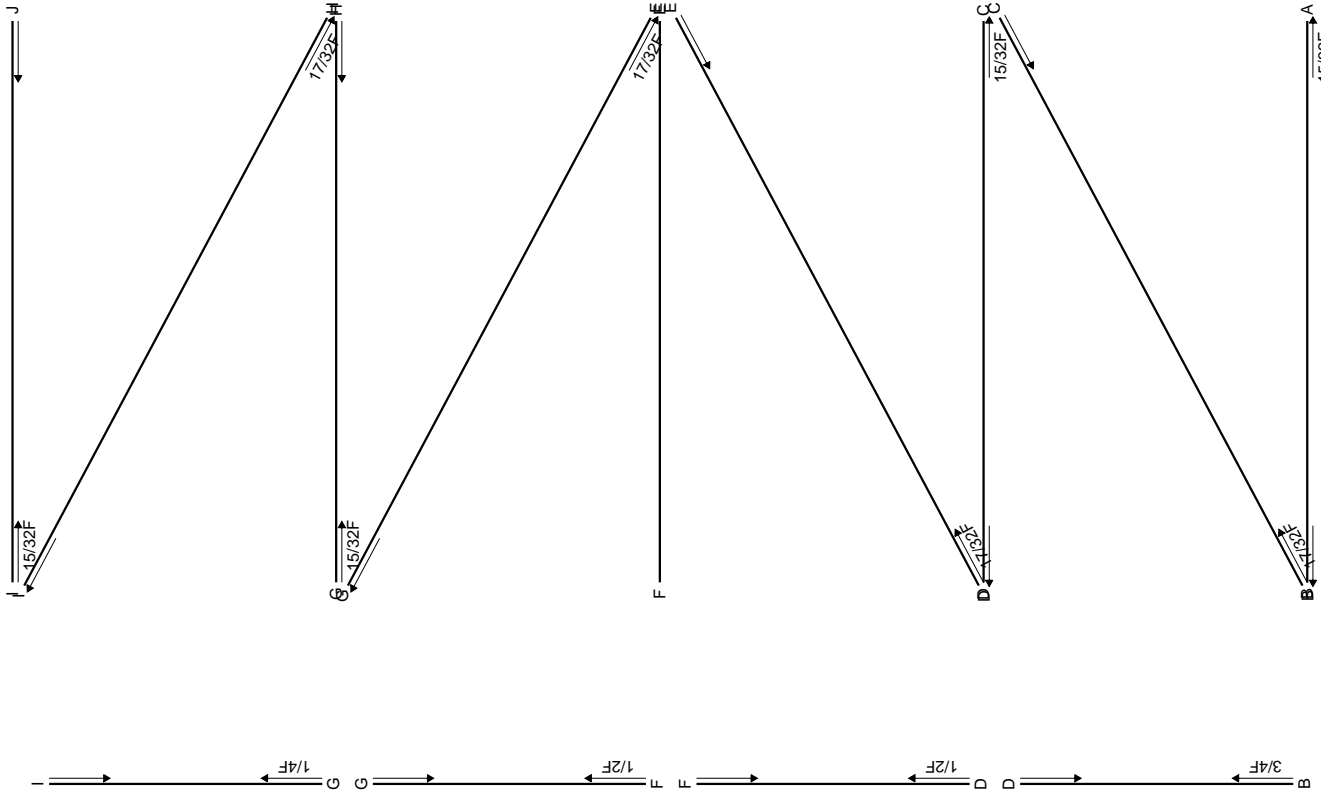
$$8V_{Jb} - 8V_{JI}b = 0$$

## Matrice di equilibrio

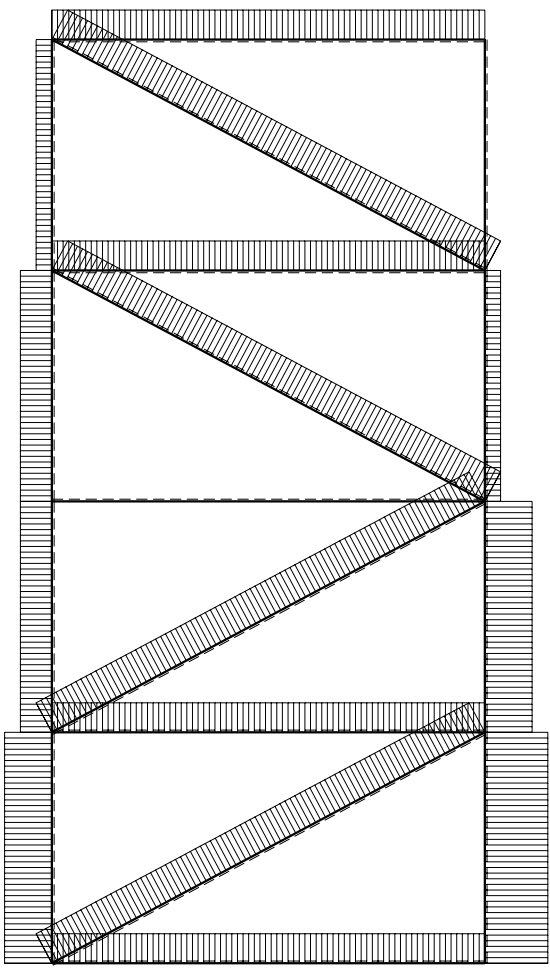
$$\begin{bmatrix}
 \varphi_{AC} \\
 \varphi_{CB} \\
 \varphi_{BD} \\
 \varphi_{DE} \\
 \varphi_{EG} \\
 \varphi_{GF} \\
 \varphi_{GI} \\
 \varphi_{IH} \\
 \varphi_{HJ}
 \end{bmatrix}
 \begin{bmatrix}
 V_{Jb} & V_{BA}b & V_{DC}b & V_{FE}b & V_{HG}b & V_{JI}b & H_{EC}b & H_{HE}b & H_{FD}b
 \end{bmatrix}
 =
 \begin{bmatrix}
 15 \\
 15 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0
 \end{bmatrix}$$

## Soluzione del sistema

$$\begin{bmatrix}
 V_{Jb} \\
 V_{BA}b \\
 V_{DC}b \\
 H_{EC}b \\
 H_{FD}b \\
 V_{FE}b \\
 H_{HE}b \\
 V_{HG}b \\
 V_{JI}b
 \end{bmatrix}
 =
 \begin{bmatrix}
 15/32 \\
 15/32 \\
 15/32 \\
 3/4 \\
 -1/2 \\
 0 \\
 1/4 \\
 15/32 \\
 15/32
 \end{bmatrix}$$



$67.24 \text{ Fb/EA}$



$\left[ \begin{array}{c} + \\ - \end{array} \right] \rightarrow \left[ \begin{array}{c} + \\ - \end{array} \right] 1.2 F$



REAZIONI

$H_A = -F$  $V_A = -15/32F$  $V_J = 15/32F$

$N_{AB} = 15/32F$  $N_{CD} = 15/32F$  $N_{EF} = 0$  $N_{GH} = -15/32F$  $N_{IJ} = -15/32F$  $N_{AC} = F$

$N_{CE} = 3/4F$  $N_{EH} = 1/4F$  $N_{HJ} = 0$  $N_{BD} = -3/4F$  $N_{DF} = -1/2F$  $N_{FG} = -1/2F$

$N_{GI} = -1/4F$  $N_{BC} = -17/32F$  $N_{DE} = -17/32F$  $N_{EG} = 17/32F$  $N_{HI} = 17/32F$

SPOSTAMENTI NODALI

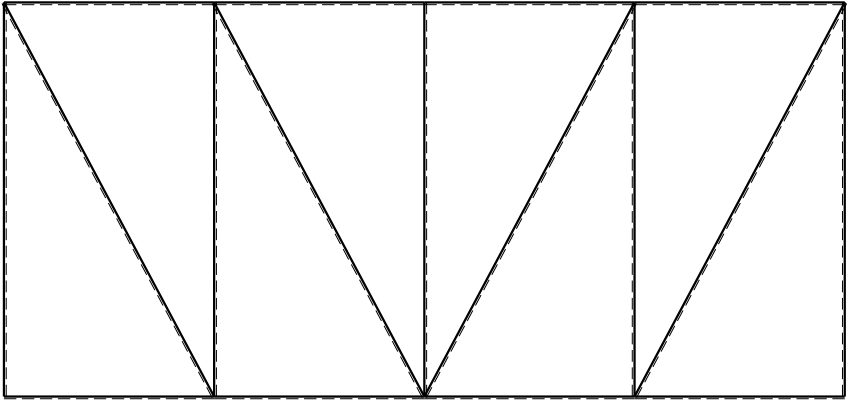
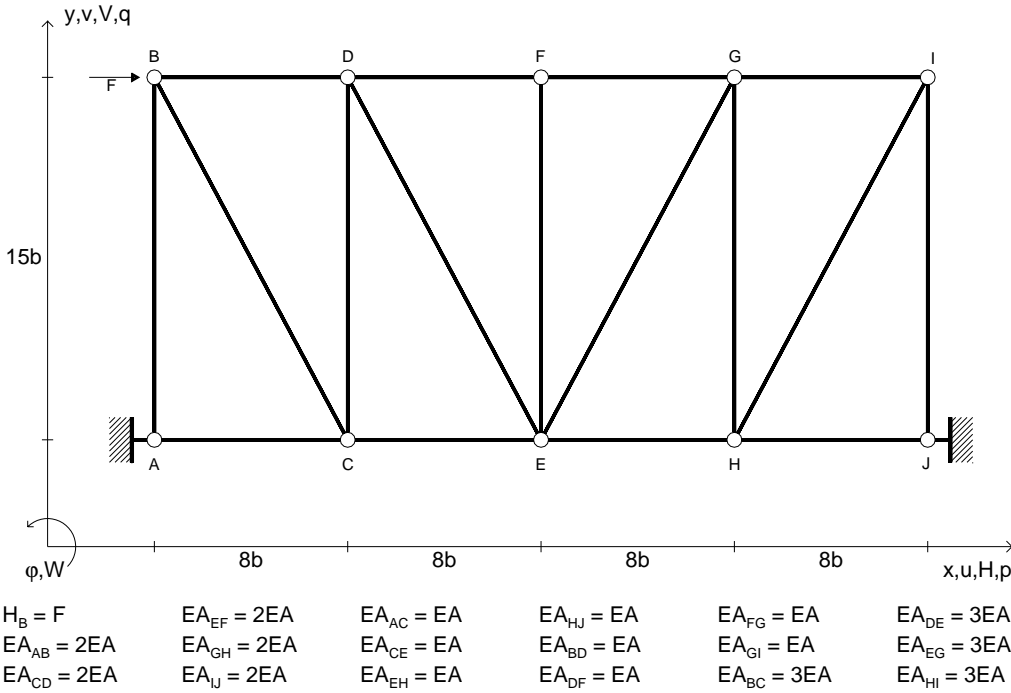
$u_A = 0$  $v_A = 0$  $u_B = 53743/1536(Fb/EA)$  $v_B = 225/64(Fb/EA)$  $u_C = 8(Fb/EA)$  $v_C = -112/15(Fb/EA)$

$u_D = 44527/1536(Fb/EA)$  $v_D = -3793/960(Fb/EA)$  $u_E = 14(Fb/EA)$  $v_E = -128/15(Fb/EA)$  $u_F = 38383/1536(Fb/EA)$  $v_F = -128/15(Fb/EA)$

$u_G = 32239/1536(Fb/EA)$  $v_G = -1699/192(Fb/EA)$  $u_H = 16(Fb/EA)$  $v_H = -16/3(Fb/EA)$  $u_I = 29167/1536(Fb/EA)$  $v_I = -225/64(Fb/EA)$

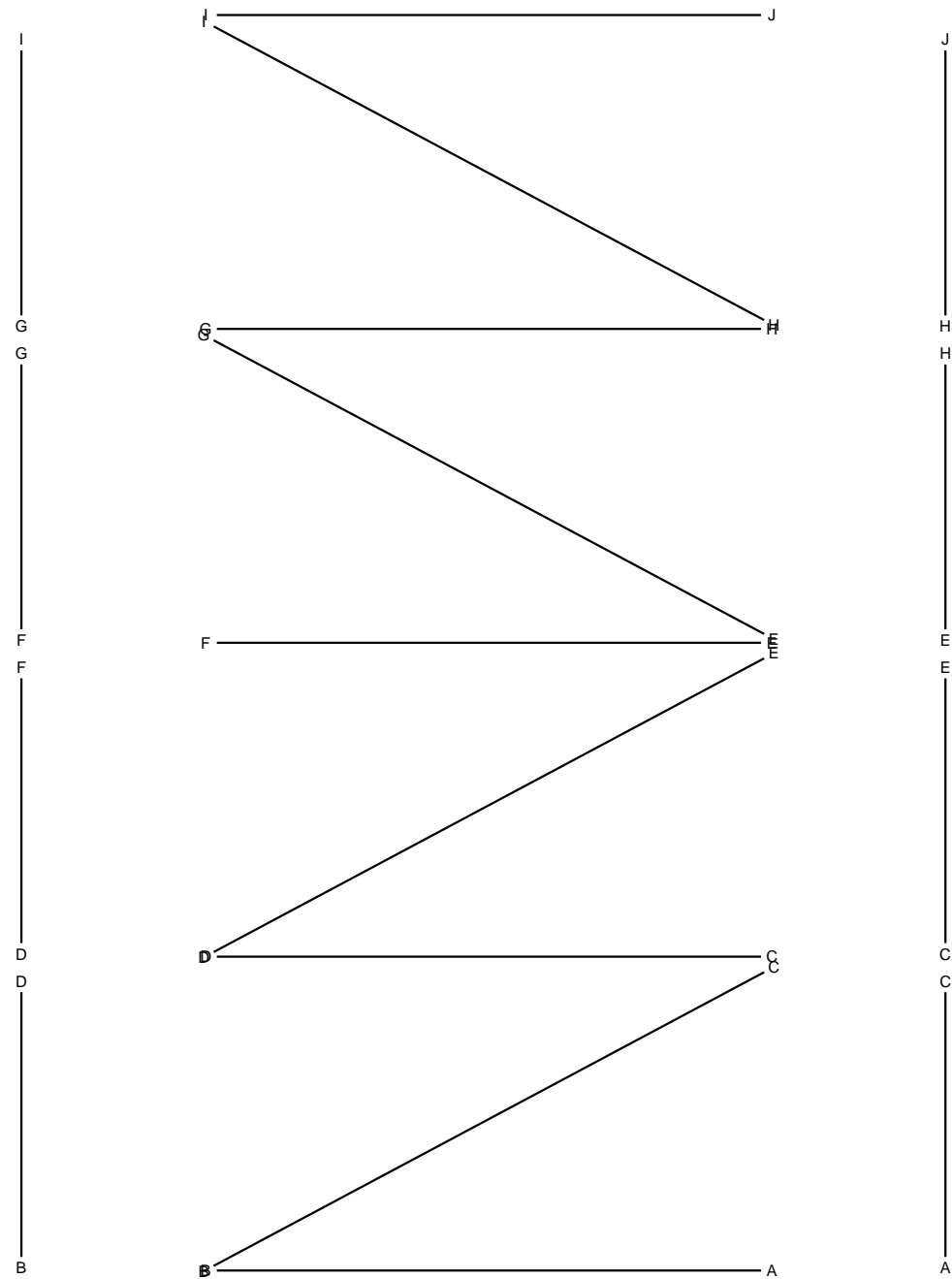
$u_{JJ} = 16(Fb/EA)$  $v_J = 0$



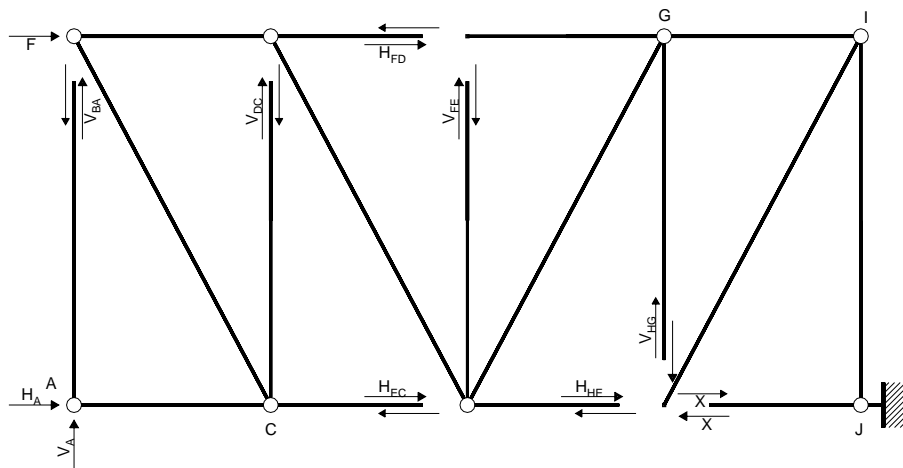


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@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.24.05.07



REAZIONI					
$H_A =$	$V_A =$	$H_J =$	$V_J =$		
$N_{AB} =$	$N_{CD} =$	$N_{EF} =$	$N_{GH} =$	$N_{IJ} =$	$N_{AC} =$
$N_{CE} =$	$N_{EH} =$	$N_{HJ} =$	$N_{BD} =$	$N_{DF} =$	$N_{FG} =$
$N_{GI} =$	$N_{BC} =$	$N_{DE} =$	$N_{EG} =$	$N_{HI} =$	
SPOSTAMENTI NODALI					
$u_A =$		$u_B =$		$u_C =$	
$v_A =$		$v_B =$		$v_C =$	
$u_D =$		$u_E =$		$u_F =$	
$v_D =$		$v_E =$		$v_F =$	
$u_G =$		$u_H =$		$u_I =$	
$v_G =$		$v_H =$		$v_I =$	
$u_J =$					
$v_J =$					



## EQUAZIONI DI EQUILIBRIO

Rotazione intorno a J: aste JI IG IH GH GF GE EF EH ED DB DF BC CD CA CE AB

$$-32V_A b = 15Fb$$

Rotazione intorno a I: aste IG GH GF GE EF EH ED DB DF BC CD CA CE AB

$$15H_A b - 32V_A b - 8V_{HG} b + 15H_{HE} b = 0$$

Rotazione intorno a I: aste IH

$$8V_{HG} b - 15H_{HE} b = 15Xb$$

Rotazione intorno a G: aste GF

$$8V_{FE} b = 0$$

Rotazione intorno a G: aste GE EF EH ED DB DF BC CD CA CE AB

$$15H_A b - 24V_A b - 8V_{FE} b + 15H_{HE} b = 0$$

Rotazione intorno a E: aste ED DB DF BC CD CA CE AB

$$-16V_A b - 15H_{FD} b = 15Fb$$

Rotazione intorno a D: aste DB BC CD CA CE AB

$$15H_A b - 8V_A b + 15H_{EC} b = 0$$

Rotazione intorno a B: aste BC CD CA CE AB

$$15H_A b + 8V_{DC} b + 15H_{EC} b = 0$$

Rotazione intorno a C: aste CA AB

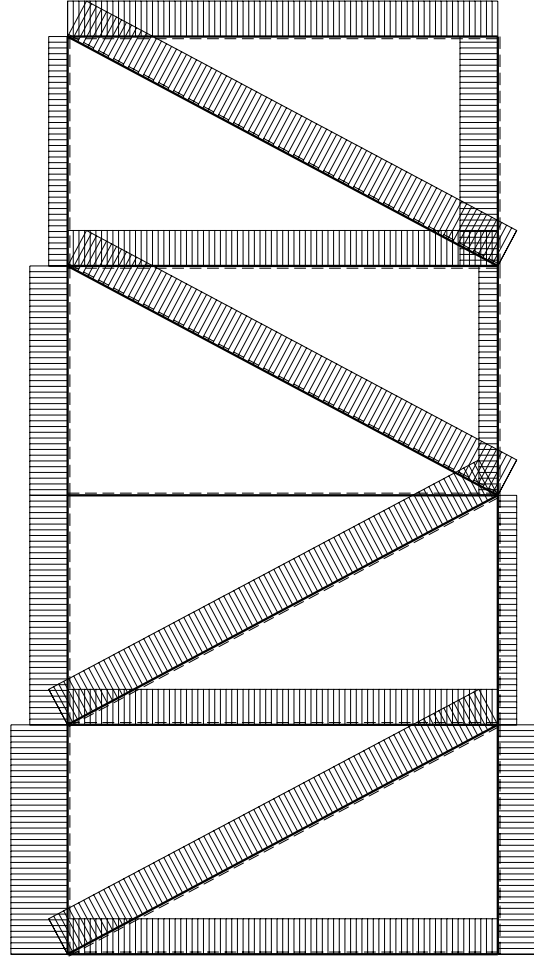
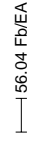
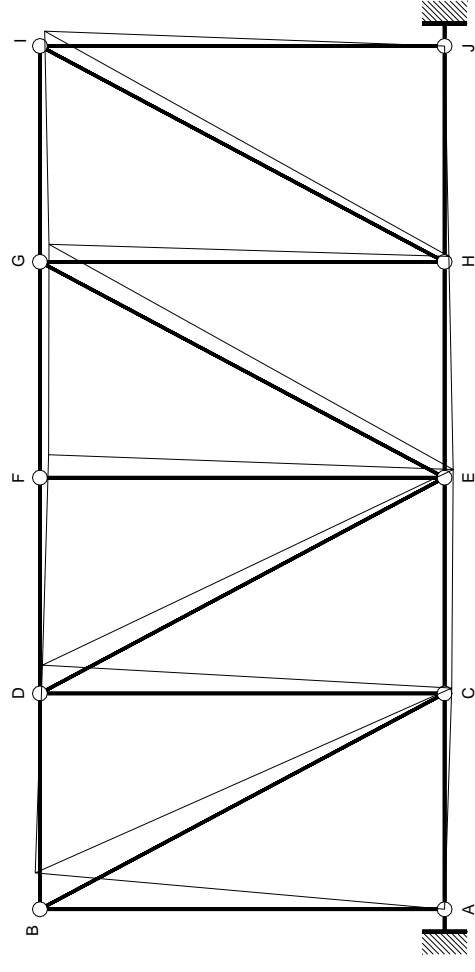
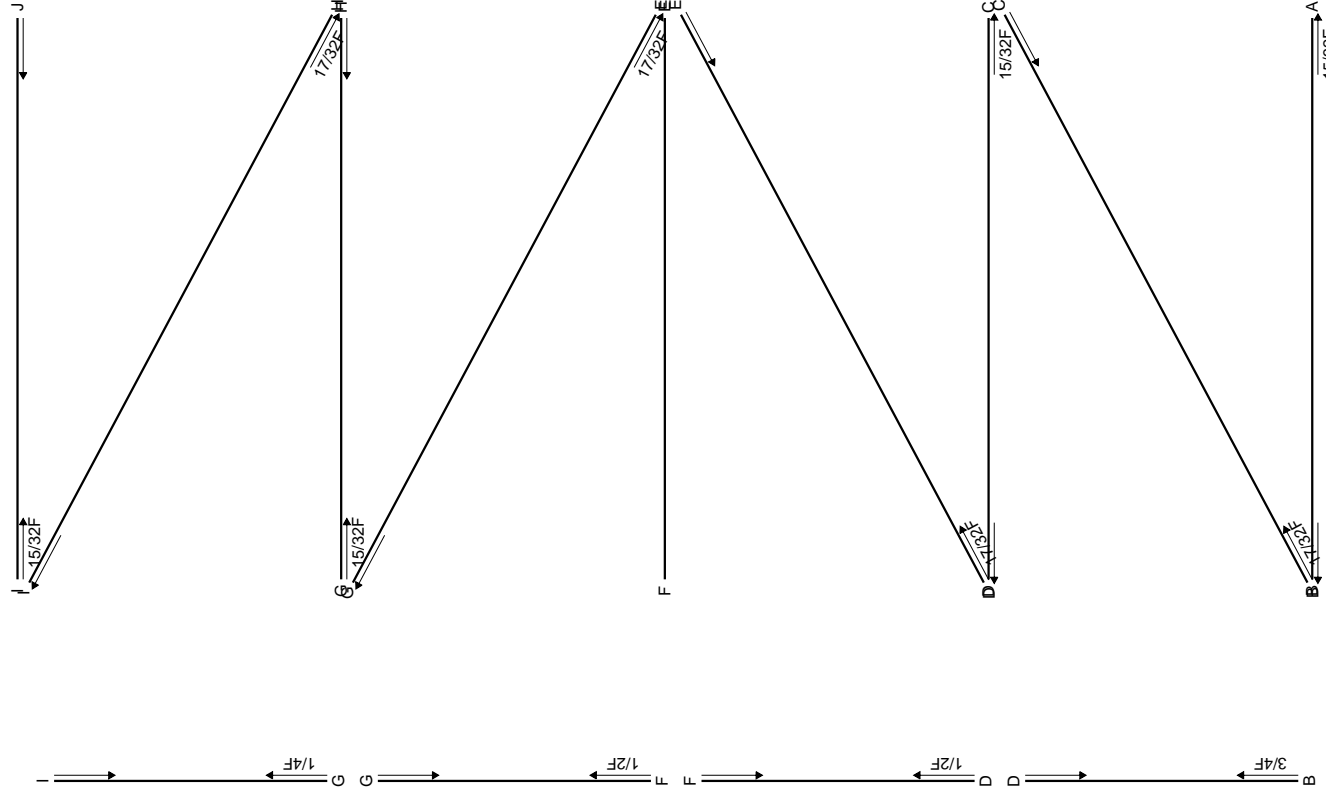
$$-8V_A b - 8V_{BA} b = 0$$

## Matrice di equilibrio

$$\begin{bmatrix} H_A b & V_A b & V_{BA} b & V_{DC} b & V_{FE} b & V_{HG} b & H_{EC} b & H_{HE} b & H_{FD} b \\ \phi_{JI} & 0 & -32 & 0 & 0 & 0 & 0 & 0 & 0 \\ \phi_{IG} & 15 & -32 & 0 & 0 & 0 & -8 & 0 & 15 \\ \phi_{IH} & 0 & 0 & 0 & 0 & 0 & 8 & 0 & -15 \\ \phi_{GF} & 0 & 0 & 0 & 0 & 8 & 0 & 0 & 0 \\ \phi_{GE} & 15 & -24 & 0 & 0 & -8 & 0 & 0 & 15 \\ \phi_{ED} & 0 & -16 & 0 & 0 & 0 & 0 & 0 & -15 \\ \phi_{DB} & 15 & -8 & 0 & 0 & 0 & 15 & 0 & 0 \\ \phi_{BC} & 15 & 0 & 0 & 8 & 0 & 15 & 0 & 0 \\ \phi_{CA} & 0 & -8 & -8 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} Xb & Fb \end{bmatrix} = \begin{bmatrix} 0 & 15 \\ 0 & 0 \\ 15 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 15 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

## Soluzione del sistema

$$\begin{bmatrix} V_A b \\ H_A b \\ V_{HG} b \\ V_{FE} b \\ H_{HE} b \\ H_{FD} b \\ H_{EC} b \\ V_{DC} b \\ V_{BA} b \end{bmatrix} = \begin{bmatrix} Xb & Fb \\ 0 & -15/32 \\ 1 & -1 \\ 0 & 15/32 \\ 0 & 0 \\ -1 & 1/4 \\ 0 & -1/2 \\ -1 & 3/4 \\ 0 & 15/32 \end{bmatrix}$$



REAZIONI

$H_A = -1/2F$  $V_A = -15/32F$  $H_J = -1/2F$  $V_J = 15/32F$

$N_{AB} = 15/32F$  $N_{CD} = 15/32F$  $N_{EF} = 0$  $N_{GH} = -15/32F$  $N_{IJ} = -15/32F$  $N_{AC} = 1/2F$

$N_{CE} = 1/4F$  $N_{EH} = -1/4F$  $N_{HJ} = -1/2F$  $N_{BD} = -3/4F$  $N_{DF} = -1/2F$  $N_{FG} = -1/2F$

$N_{GI} = -1/4F$  $N_{BC} = -17/32F$  $N_{DE} = -17/32F$  $N_{EG} = 17/32F$  $N_{HI} = 17/32F$

SPOSTAMENTI NODALI

$u_A = 0$  $v_A = 0$  $u_B = 41455/1536(Fb/EA)$  $v_B = 225/64(Fb/EA)$  $u_C = 4(Fb/EA)$  $v_C = -16/3(Fb/EA)$

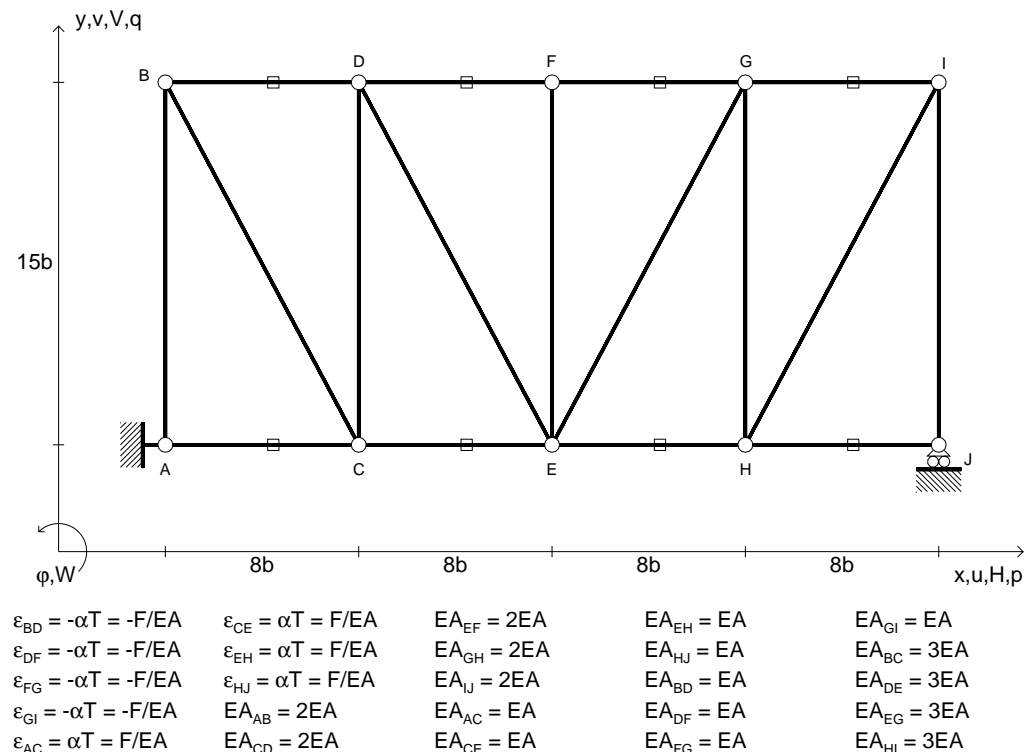
$u_D = 32239/1536(Fb/EA)$  $v_D = -349/192(Fb/EA)$  $u_E = 6(Fb/EA)$  $v_E = -32/5(Fb/EA)$  $u_F = 26095/1536(Fb/EA)$  $v_F = -32/5(Fb/EA)$

$u_G = 19951/1536(Fb/EA)$  $v_G = -2149/320(Fb/EA)$  $u_H = 4(Fb/EA)$  $v_H = -16/5(Fb/EA)$  $u_I = 16879/1536(Fb/EA)$  $v_I = -225/64(Fb/EA)$

$u_J = 0$  $v_J = 0$







Svolgere l'analisi cinematica.

Tracciare la deformata elastica.

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi delle azioni interne nelle aste.

Calcolare spostamento e rotazione di tutti i nodi.

$A_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.

Elongazione termica specifica  $\varepsilon$  assegnata su asta BD.

Elongazione termica specifica  $\varepsilon$  assegnata su asta DF.

Elongazione termica specifica  $\varepsilon$  assegnata su asta FG.

Elongazione termica specifica  $\varepsilon$  assegnata su asta GI.

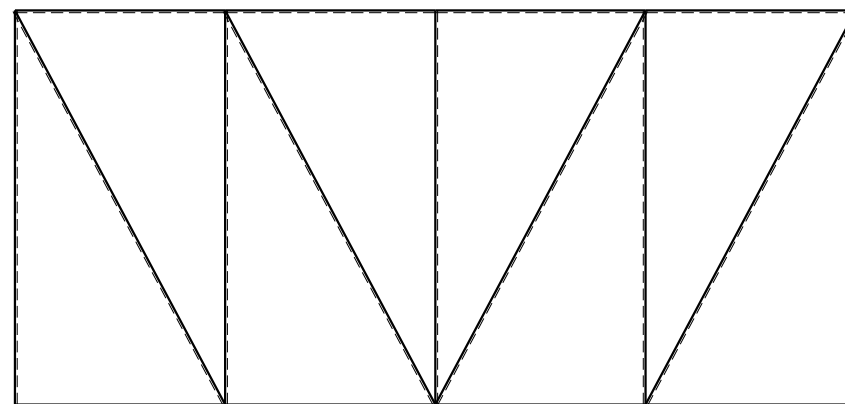
Elongazione termica specifica  $\varepsilon$  assegnata su asta AC.

Elongazione termica specifica  $\varepsilon$  assegnata su asta CE.

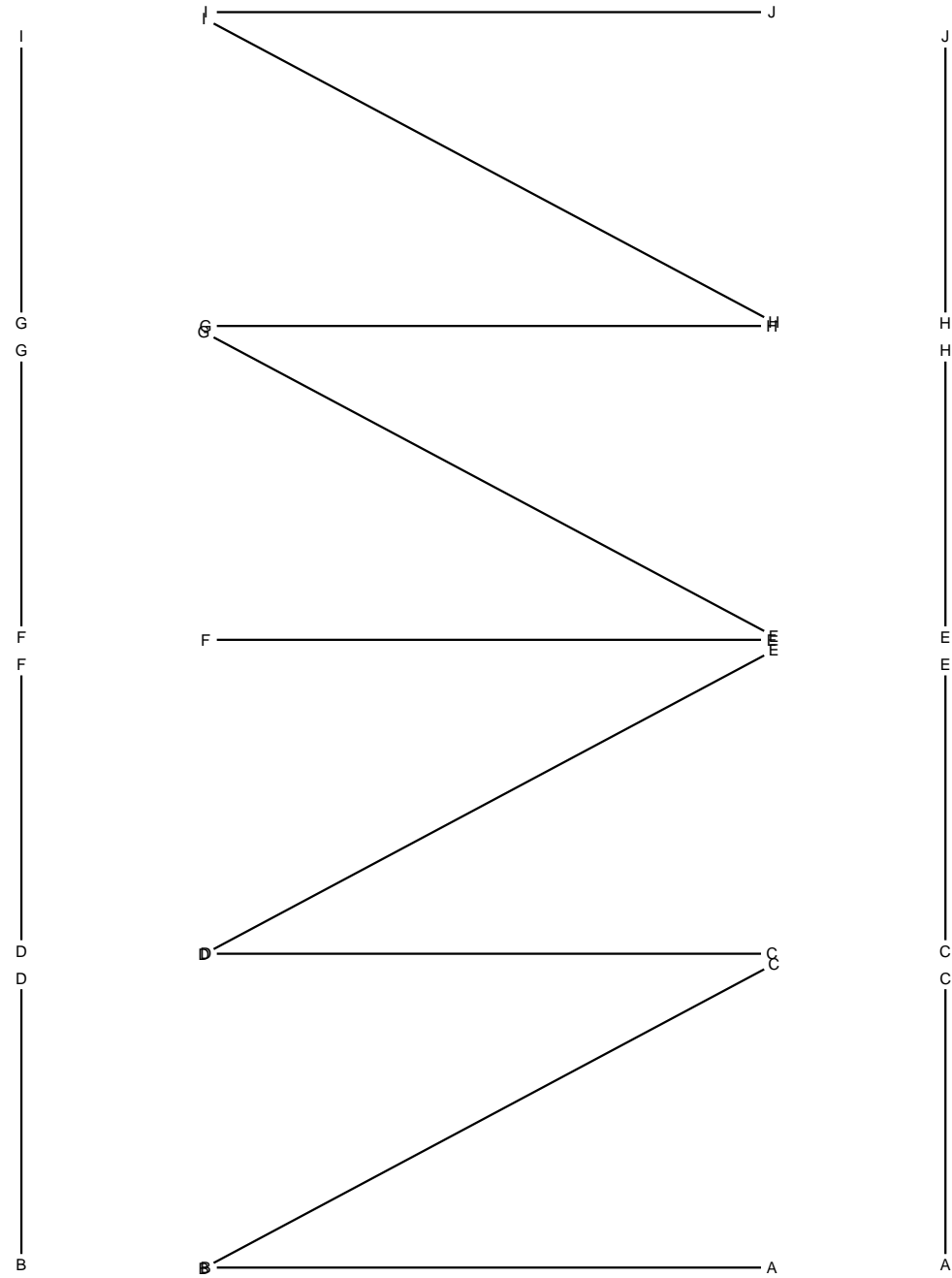
Elongazione termica specifica  $\varepsilon$  assegnata su asta EH.

Elongazione termica specifica  $\varepsilon$  assegnata su asta HJ.

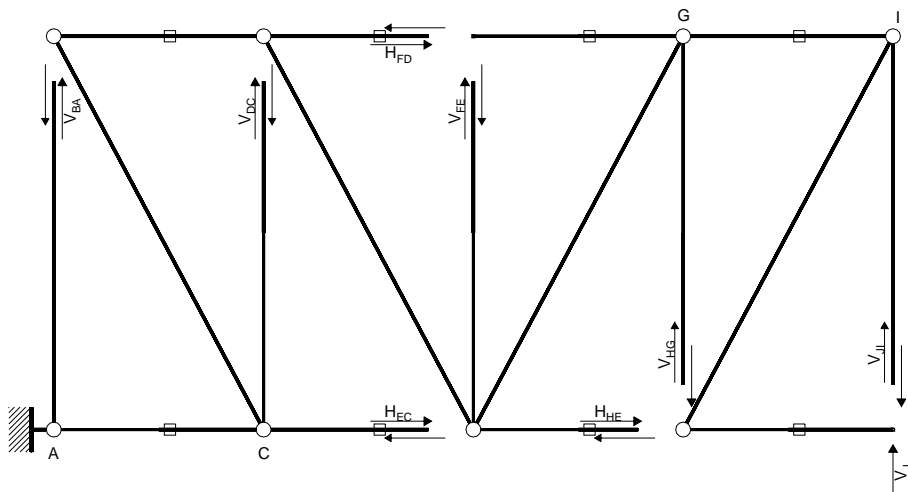
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.24.05.07



@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.24.05.07



REAZIONI								
$H_A =$	$V_A =$	$V_J =$						
$N_{AB} =$	$N_{CD} =$	$N_{EF} =$	$N_{GH} =$	$N_{IJ} =$	$N_{AC} =$	$N_{CE} =$	$N_{EH} =$	$N_{HJ} =$
$N_{BD} =$	$N_{DF} =$	$N_{FG} =$	$N_{GI} =$	$N_{BC} =$	$N_{DE} =$	$N_{EG} =$	$N_{HI} =$	
SPOSTAMENTI NODALI								
$u_A =$	$u_B =$	$u_C =$	$u_D =$	$u_E =$				
$v_A =$	$v_B =$	$v_C =$	$v_D =$	$v_E =$				
$u_F =$	$u_G =$	$u_H =$	$u_I =$	$u_{JJ} =$				
$v_F =$	$v_G =$	$v_H =$	$v_I =$	$v_J =$				



## EQUAZIONI DI EQUILIBRIO

Rotazione intorno a A: aste AC CD CE CB BD DF DE EF EH EG GH GF GI IJ IH HJ

$$32V_J b = 0$$

Rotazione intorno a C: aste CB BD DF DE EF EH EG GH GF GI IJ IH HJ

$$24V_J b + 8V_{BA} b = 0$$

Rotazione intorno a B: aste BD DF DE EF EH EG GH GF GI IJ IH HJ

$$32V_J b - 8V_{DC} b - 15H_{EC} b = 0$$

Rotazione intorno a D: aste DE EF EH EG GH GF GI IJ IH HJ

$$24V_J b - 15H_{EC} b = 0$$

Rotazione intorno a E: aste EG GH GF GI IJ IH HJ

$$16V_J b + 15H_{FD} b = 0$$

Rotazione intorno a G: aste GF

$$8V_{FE} b = 0$$

Rotazione intorno a G: aste GI IJ IH HJ

$$8V_J b - 15H_{HE} b = 0$$

Rotazione intorno a I: aste IH HJ

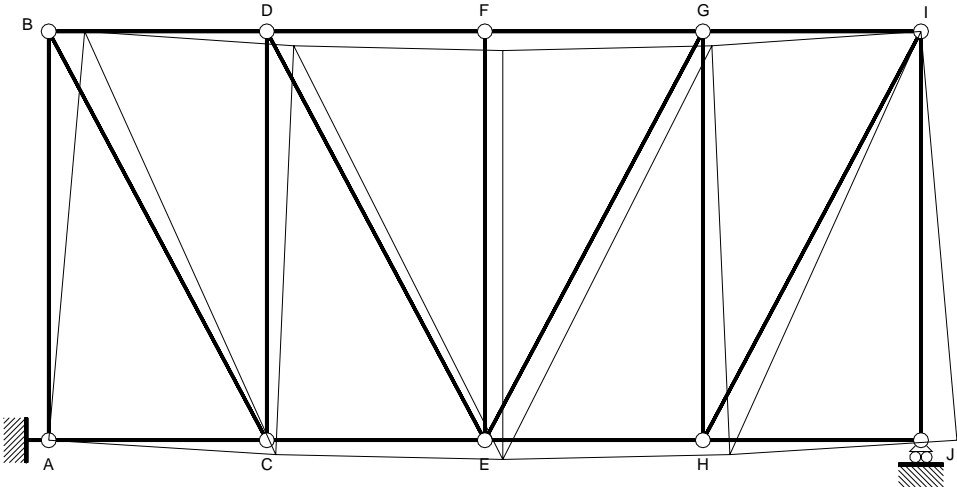
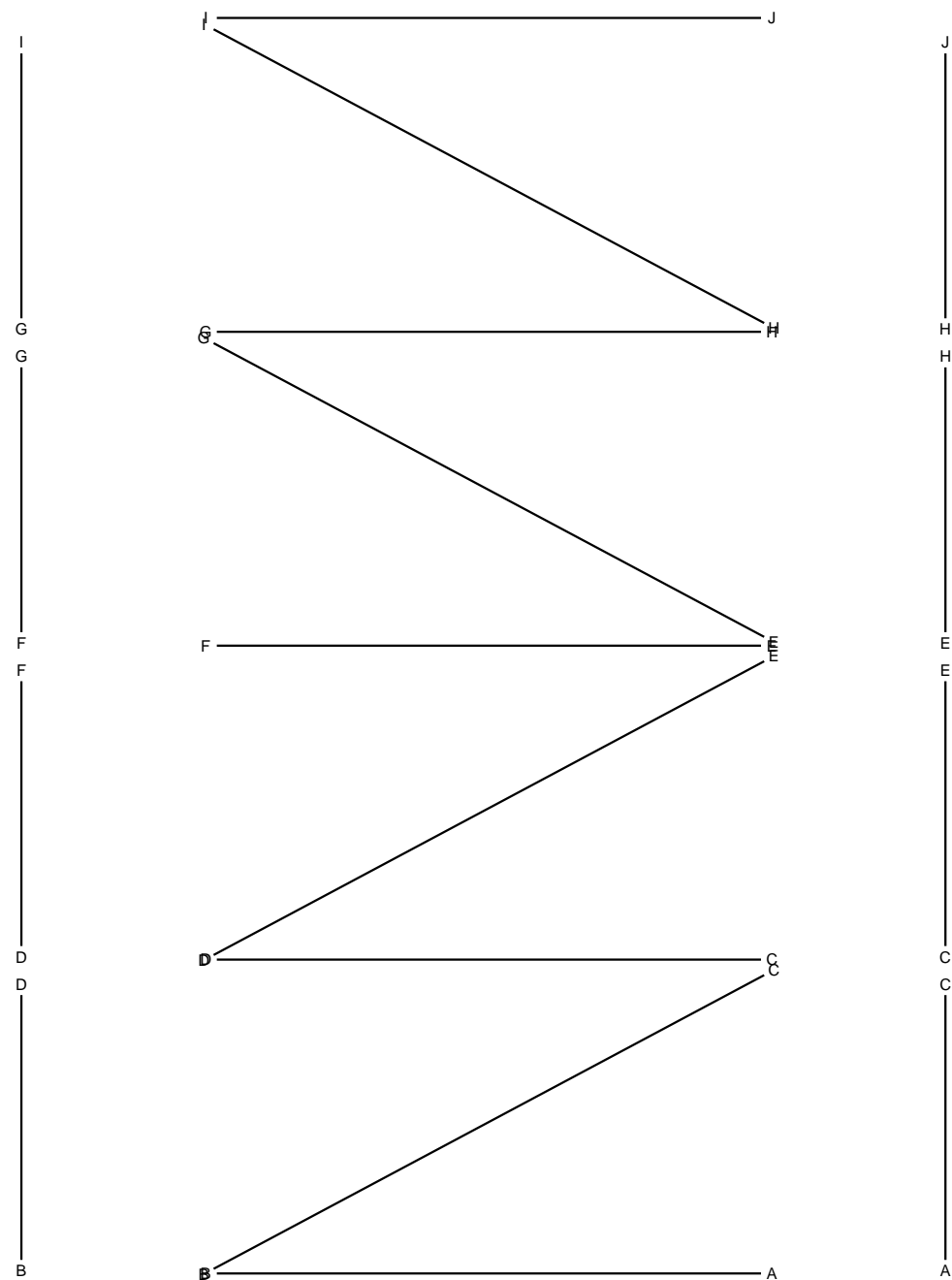
$$8V_{HG} b - 15H_{HE} b = 0$$

Rotazione intorno a H: aste HJ

$$8V_J b - 8V_{JI} b = 0$$

## Matrice di equilibrio

	$V_J b$	$V_{BA} b$	$V_{DC} b$	$V_{FE} b$	$V_{HG} b$	$V_{JI} b$	$H_{EC} b$	$H_{HE} b$	$H_{FD} b$
$\varphi_{AC}$	32	0	0	0	0	0	0	0	0
$\varphi_{CB}$	24	8	0	0	0	0	0	0	0
$\varphi_{BD}$	32	0	-8	0	0	0	-15	0	0
$\varphi_{DE}$	24	0	0	0	0	0	-15	0	0
$\varphi_{EG}$	16	0	0	0	0	0	0	0	15
$\varphi_{GF}$	0	0	0	8	0	0	0	0	0
$\varphi_{GI}$	8	0	0	0	0	0	0	-15	0
$\varphi_{IH}$	0	0	0	0	8	0	0	-15	0
$\varphi_{HJ}$	8	0	0	0	0	-8	0	0	0



1 67.24  $\alpha T_b$

REAZIONI

$H_A = 0$        $V_A = 0$        $V_J = 0$

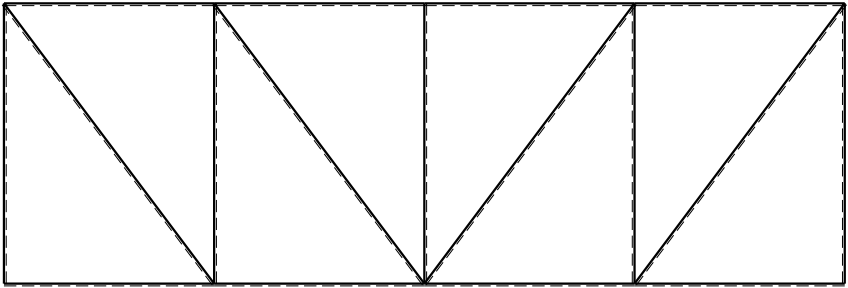
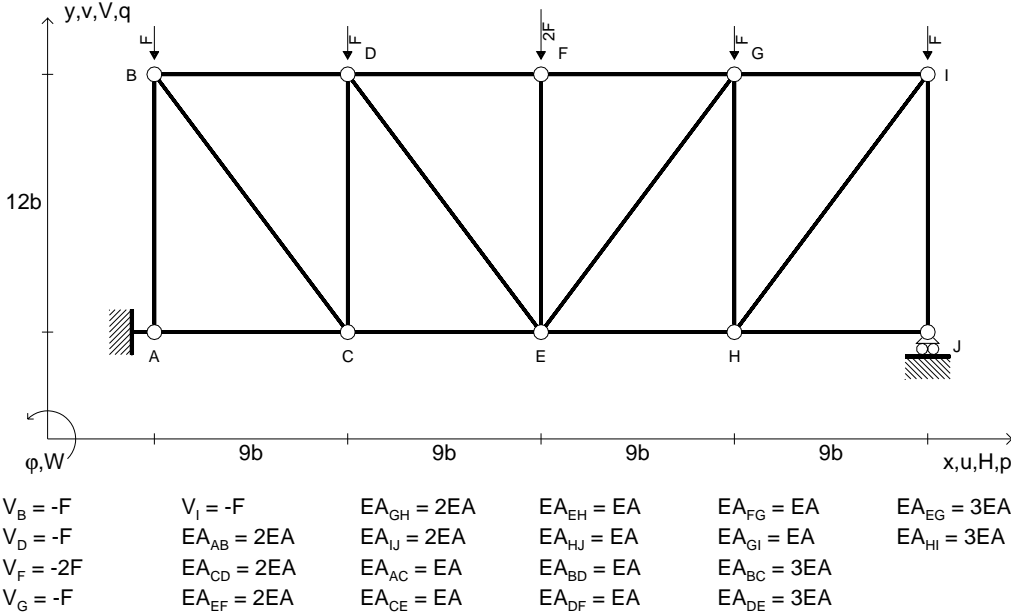
$N_{AB} = 0$      $N_{CD} = 0$      $N_{EF} = 0$      $N_{GH} = 0$      $N_{IJ} = 0$      $N_{AC} = 0$      $N_{CE} = 0$      $N_{EH} = 0$      $N_{HJ} = 0$

$N_{BD} = 0$      $N_{DF} = 0$      $N_{FG} = 0$      $N_{GI} = 0$      $N_{BC} = 0$      $N_{DE} = 0$      $N_{EG} = 0$      $N_{HI} = 0$

SPOSTAMENTI NODALI

$u_A = 0$	$u_B = 32\alpha Tb$	$u_C = 8\alpha Tb$	$u_D = 24\alpha Tb$	$u_E = 16\alpha Tb$
$v_A = 0$	$v_B = 0$	$v_C = -64/5\alpha Tb$	$v_D = -64/5\alpha Tb$	$v_E = -256/15\alpha Tb$
$u_F = 16\alpha Tb$	$u_G = 8\alpha Tb$	$u_H = 24\alpha Tb$	$u_I = 0$	$u_{JJ} = 32\alpha Tb$
$v_F = -256/15\alpha Tb$	$v_G = -64/5\alpha Tb$	$v_H = -64/5\alpha Tb$	$v_I = 0$	$v_J = 0$

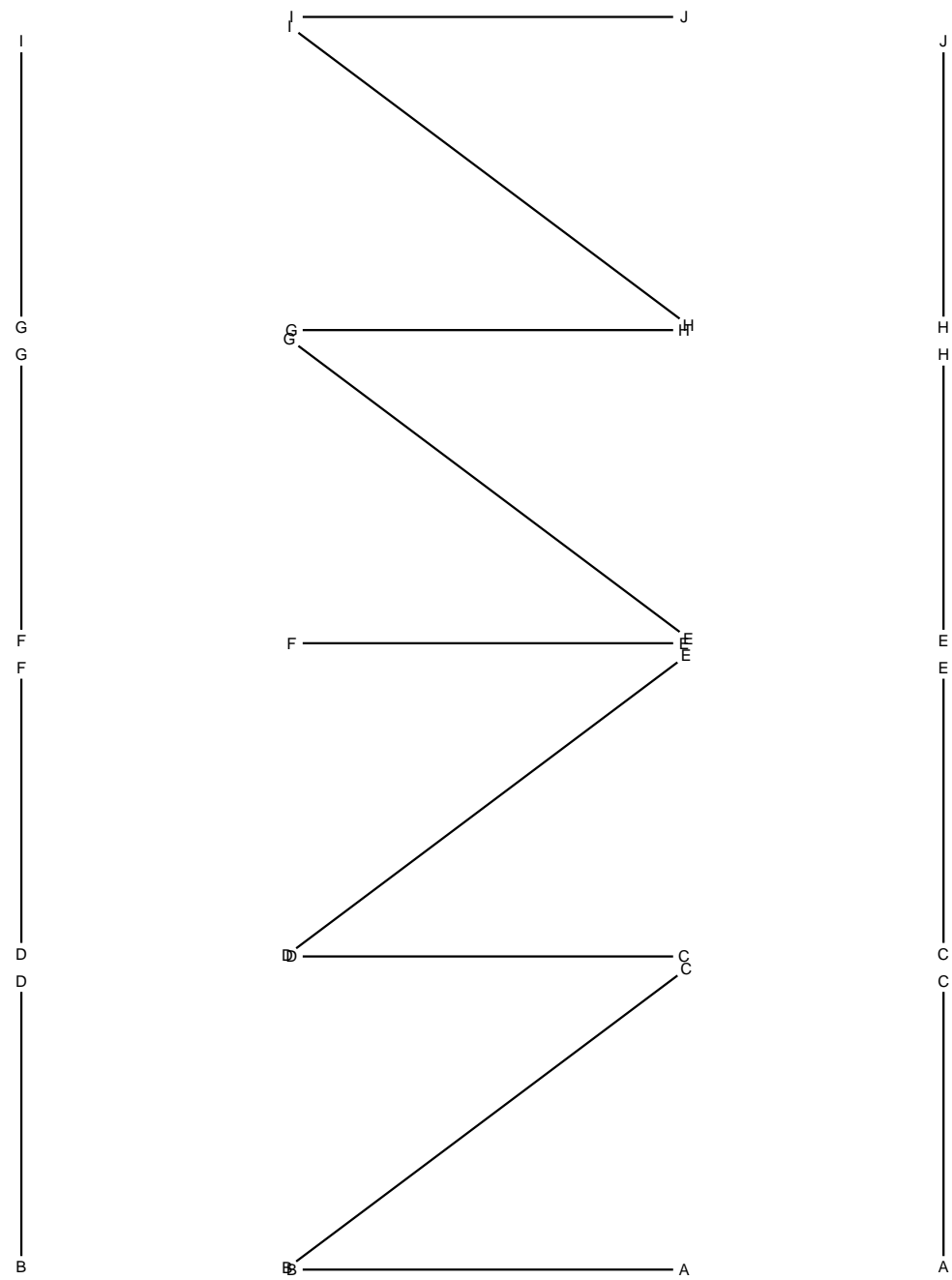




Svolgere l'analisi cinematica.  
Tracciare la deformata elastica.

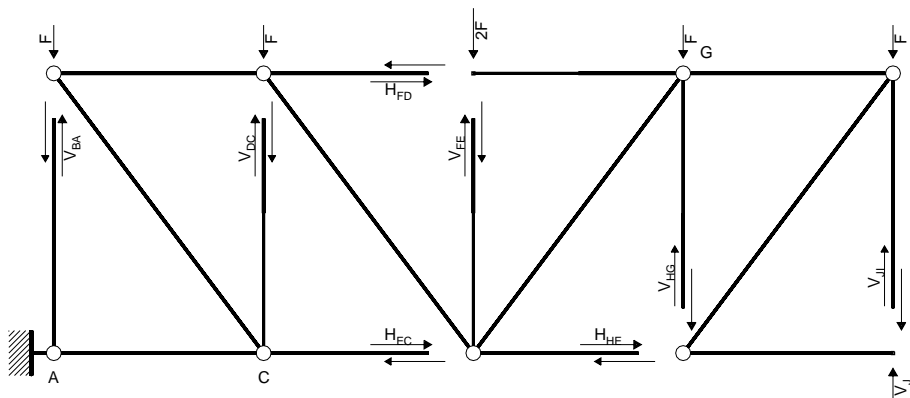
Carichi e deformazioni date hanno verso efficace in disegno.  
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 $A_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.24.05.07





REAZIONI						
$H_A =$	$V_A =$	$V_J =$				
$N_{AB} =$	$N_{CD} =$	$N_{EF} =$	$N_{GH} =$	$N_{IJ} =$	$N_{AC} =$	$N_{CE} =$
$N_{EH} =$	$N_{HJ} =$	$N_{BD} =$	$N_{DF} =$	$N_{FG} =$	$N_{GI} =$	$N_{BC} =$
$N_{DE} =$	$N_{EG} =$	$N_{HI} =$				
SPOSTAMENTI NODALI						
$u_A =$	$u_B =$	$u_C =$	$u_D =$			
$v_A =$	$v_B =$	$v_C =$	$v_D =$			
$u_E =$	$u_F =$	$u_G =$	$u_H =$			
$v_E =$	$v_F =$	$v_G =$	$v_H =$			
$u_I =$	$u_{JI} =$					
$v_I =$	$v_J =$					





## EQUAZIONI DI EQUILIBRIO

Rotazione intorno a A: aste AC CD CE CB BD DF DE EF EH EG GH GF GI IJ IH HJ

$$36V_{j,b} = 108Fb$$

Rotazione intorno a C: aste CB BD DF DE EF EH EG GH GF GI IJ IH HJ

$$27V_{j,b} + 9V_{BA,b} = 54Fb$$

Rotazione intorno a B: aste BD DF DE EF EH EG GH GF GI IJ IH HJ

$$36V_{j,b} - 9V_{DC,b} - 12H_{EC,b} = 108Fb$$

Rotazione intorno a D: aste DE EF EH EG GH GF GI IJ IH HJ

$$27V_{j,b} - 12H_{EC,b} = 63Fb$$

Rotazione intorno a E: aste EG GH GF GI IJ IH HJ

$$18V_{j,b} + 12H_{FD,b} = 27Fb$$

Rotazione intorno a G: aste GF

$$9V_{FE,b} = -18Fb$$

Rotazione intorno a G: aste GI IJ IH HJ

$$9V_{j,b} - 12H_{HE,b} = 9Fb$$

Rotazione intorno a I: aste IH HJ

$$9V_{HG,b} - 12H_{HE,b} = 0$$

Rotazione intorno a H: aste HJ

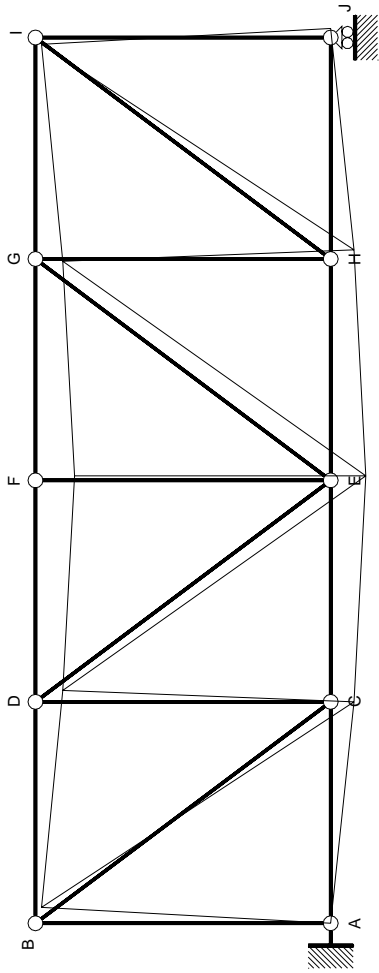
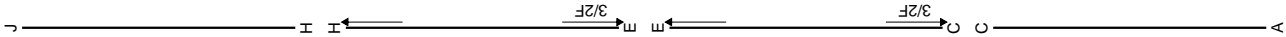
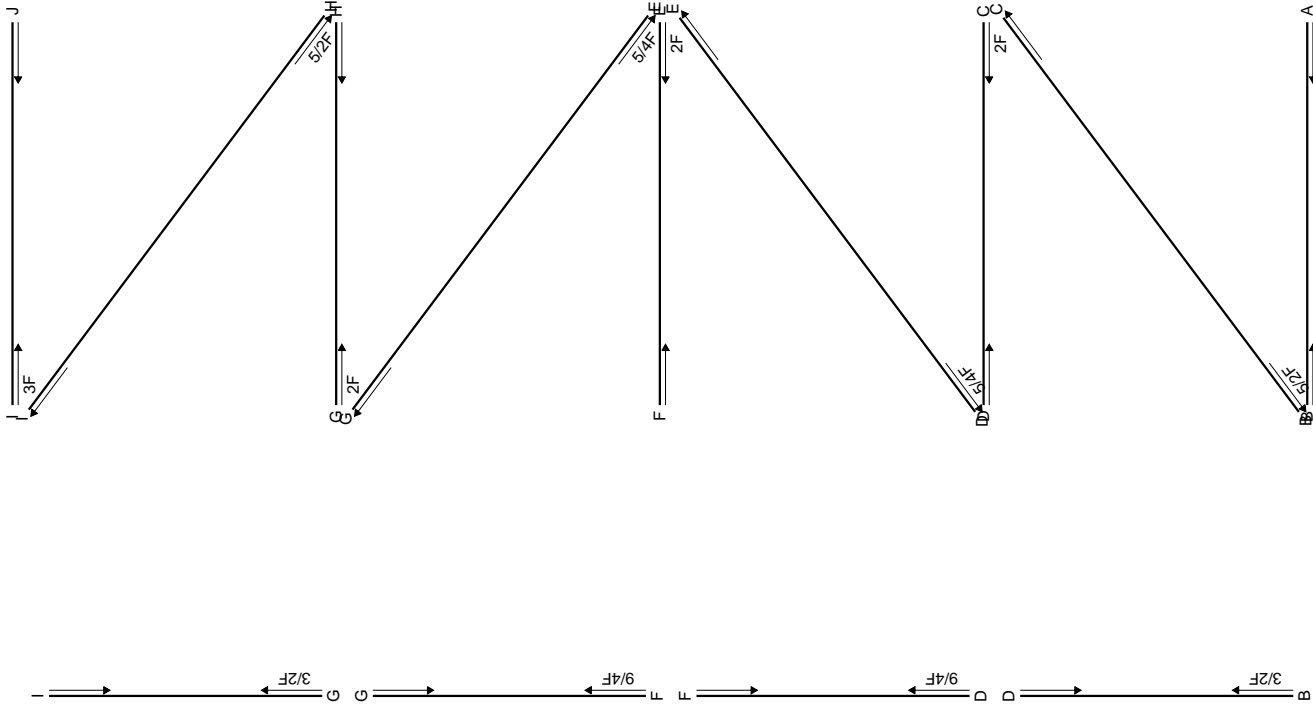
$$9V_{j,b} - 9V_{JI,b} = 0$$

## Matrice di equilibrio

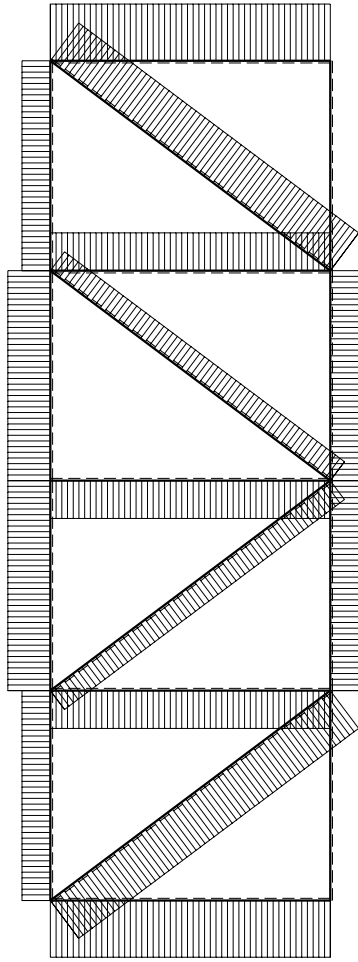
$$\begin{bmatrix} \varphi_{AC} \\ \varphi_{CB} \\ \varphi_{BD} \\ \varphi_{DE} \\ \varphi_{EG} \\ \varphi_{GF} \\ \varphi_{GI} \\ \varphi_{IH} \\ \varphi_{HJ} \end{bmatrix} \begin{bmatrix} V_{j,b} & V_{BA,b} & V_{DC,b} & V_{FE,b} & V_{HG,b} & V_{JI,b} & H_{EC,b} & H_{HE,b} & H_{FD,b} \end{bmatrix} = \begin{bmatrix} Fb \\ 108 \\ 54 \\ 108 \\ 63 \\ 27 \\ -18 \\ 9 \\ 0 \end{bmatrix}$$

## Soluzione del sistema

$$\begin{bmatrix} V_{j,b} \\ V_{BA,b} \\ V_{DC,b} \\ H_{EC,b} \\ H_{FD,b} \\ V_{FE,b} \\ H_{HE,b} \\ V_{HG,b} \\ V_{JI,b} \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ -2 \\ 3/2 \\ -9/4 \\ -2 \\ 3/2 \\ 2 \\ 3 \end{bmatrix}$$



$224.1 Fb/EA$



$\leftarrow \boxed{+} \rightarrow$   $\leftarrow \boxed{-} \rightarrow$   $4F$

REAZIONI

$H_A = 0 \quad V_A = 3F \quad V_J = 3F$

$N_{AB} = -3F \quad N_{CD} = -2F \quad N_{EF} = -2F \quad N_{GH} = -2F \quad N_{IJ} = -3F \quad N_{AC} = 0 \quad N_{CE} = 3/2F$

$N_{EH} = 3/2F \quad N_{HJ} = 0 \quad N_{BD} = -3/2F \quad N_{DF} = -9/4F \quad N_{FG} = -9/4F \quad N_{GI} = -3/2F \quad N_{BC} = 5/2F$

$N_{DE} = 5/4F \quad N_{EG} = 5/4F \quad N_{HI} = 5/2F$

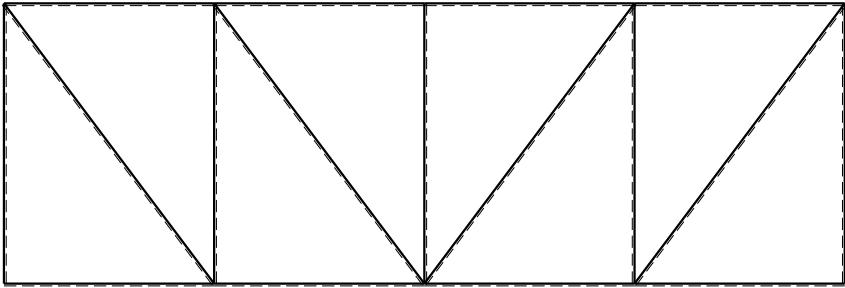
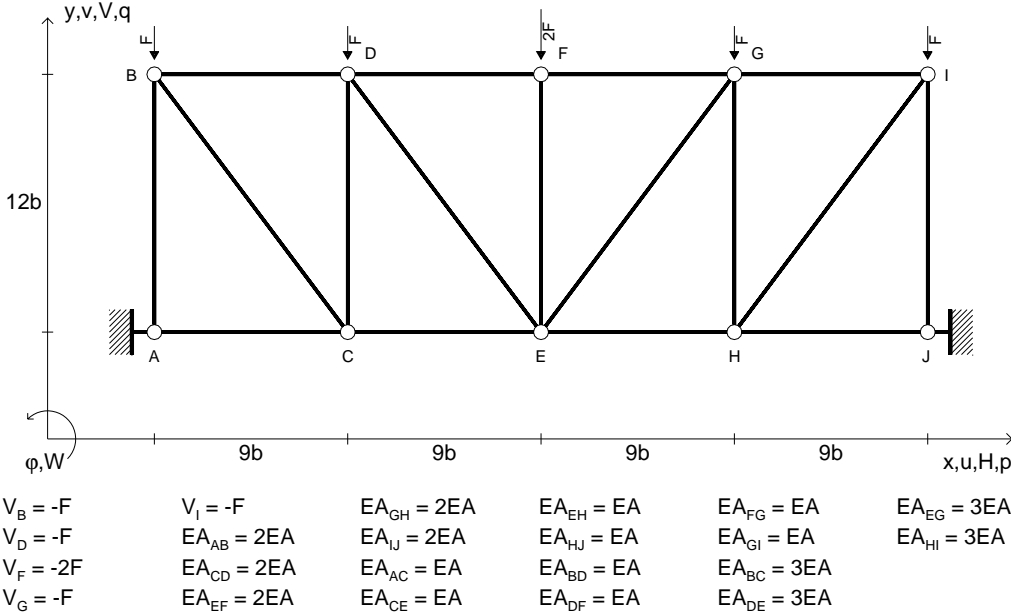
SPOSTAMENTI NODALI

$u_A = 0 \quad u_B = 189/4(Fb/EA) \quad u_C = 0 \quad u_D = 135/4(Fb/EA)$   
 $v_A = 0 \quad v_B = -18(Fb/EA) \quad v_C = -1105/16(Fb/EA) \quad v_D = -1297/16(Fb/EA)$

$u_E = 27/2(Fb/EA) \quad u_F = 27/2(Fb/EA) \quad u_G = -27/4(Fb/EA) \quad u_H = 27(Fb/EA)$   
 $v_E = -1665/16(Fb/EA) \quad v_F = -1857/16(Fb/EA) \quad v_G = -1297/16(Fb/EA) \quad v_H = -1105/16(Fb/EA)$

$u_I = -81/4(Fb/EA) \quad u_{JJ} = 27(Fb/EA)$   
 $v_I = -18(Fb/EA) \quad v_J = 0$

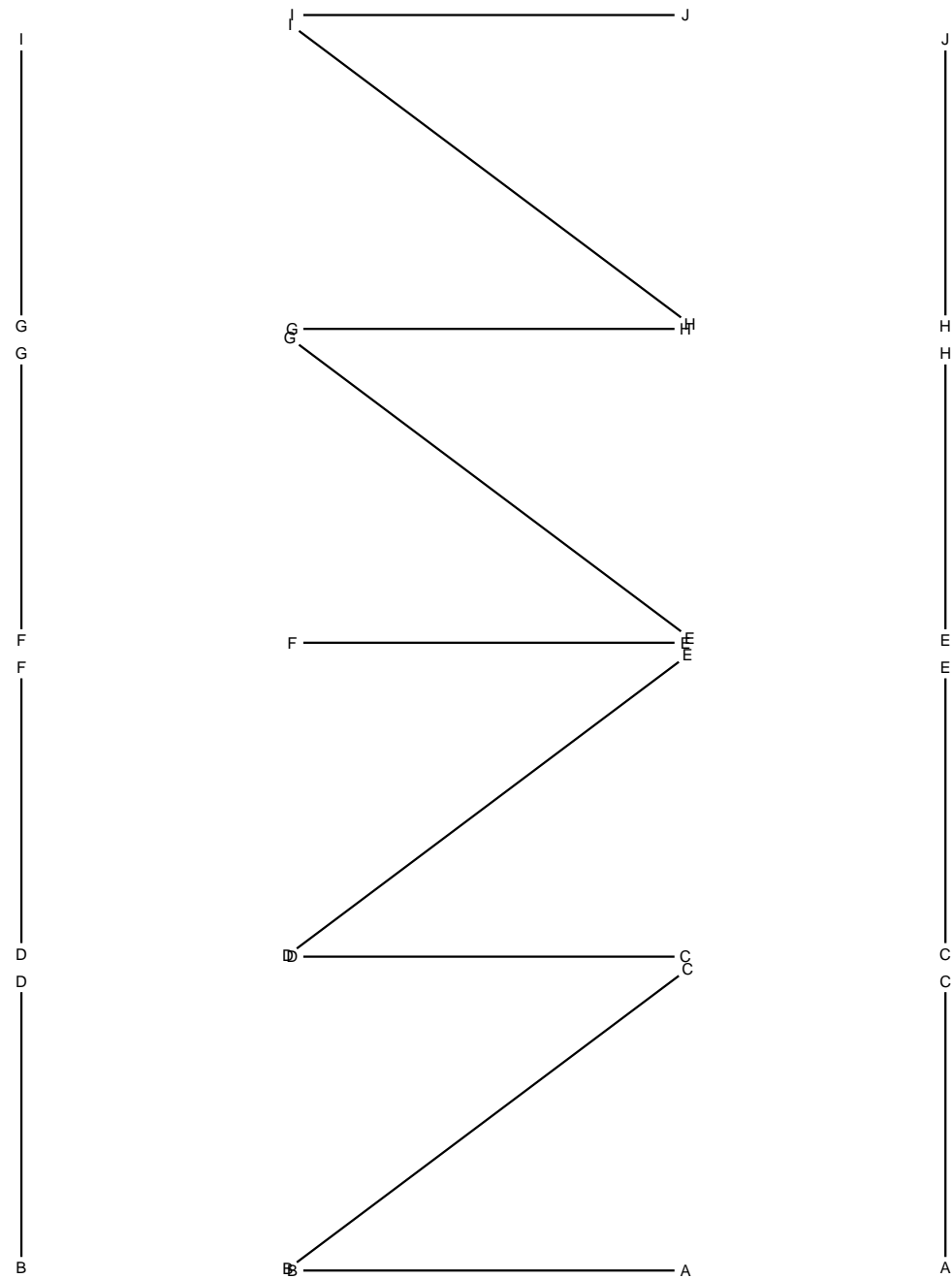




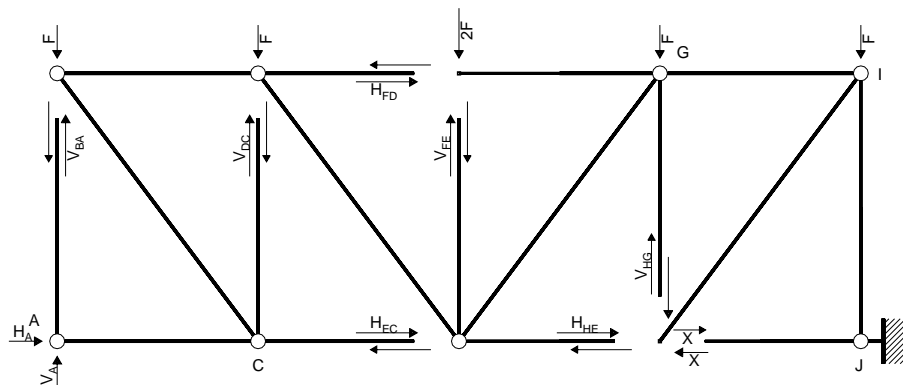
Svolgere l'analisi cinematica.  
Tracciare la deformata elastica.

Carichi e deformazioni date hanno verso efficace in disegno.  
Calcolare reazioni vincolari della struttura e delle aste.  
Tracciare i diagrammi delle azioni interne nelle aste.  
Calcolare spostamento e rotazione di tutti i nodi.  
 $A_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.24.05.07





REAZIONI						
$H_A =$	$V_A =$	$H_J =$	$V_J =$			
$N_{AB} =$	$N_{CD} =$	$N_{EF} =$	$N_{GH} =$	$N_{IJ} =$	$N_{AC} =$	$N_{CE} =$
$N_{EH} =$	$N_{HJ} =$	$N_{BD} =$	$N_{DF} =$	$N_{FG} =$	$N_{GI} =$	$N_{BC} =$
$N_{DE} =$	$N_{EG} =$	$N_{HI} =$				
SPOSTAMENTI NODALI						
$u_A =$	$u_B =$	$u_C =$	$u_D =$			
$v_A =$	$v_B =$	$v_C =$	$v_D =$			
$u_E =$	$u_F =$	$u_G =$	$u_H =$			
$v_E =$	$v_F =$	$v_G =$	$v_H =$			
$u_I =$	$u_J =$					
$v_I =$	$v_J =$					



## EQUAZIONI DI EQUILIBRIO

Rotazione intorno a J: aste JI IG IH GH GF GE EF EH ED DB DF BC CD CA CE AB

$$-36V_A b = -108Fb$$

Rotazione intorno a I: aste IG GH GF GE EF EH ED DB DF BC CD CA CE AB

$$12H_A b - 36V_A b - 9V_{HG} b + 12H_{HE} b = -108Fb$$

Rotazione intorno a I: aste IH

$$9V_{HG} b - 12H_{HE} b = 12Xb$$

Rotazione intorno a G: aste GF

$$9V_{FE} b = -18Fb$$

Rotazione intorno a G: aste GE EF EH ED DB DF BC CD CA CE AB

$$12H_A b - 27V_A b - 9V_{FE} b + 12H_{HE} b = -45Fb$$

Rotazione intorno a E: aste ED DB DF BC CD CA CE AB

$$-18V_A b - 12H_{FD} b = -27Fb$$

Rotazione intorno a D: aste DB BC CD CA CE AB

$$12H_A b - 9V_A b + 12H_{EC} b = -9Fb$$

Rotazione intorno a B: aste BC CD CA CE AB

$$12H_A b + 9V_{DC} b + 12H_{EC} b = 0$$

Rotazione intorno a C: aste CA AB

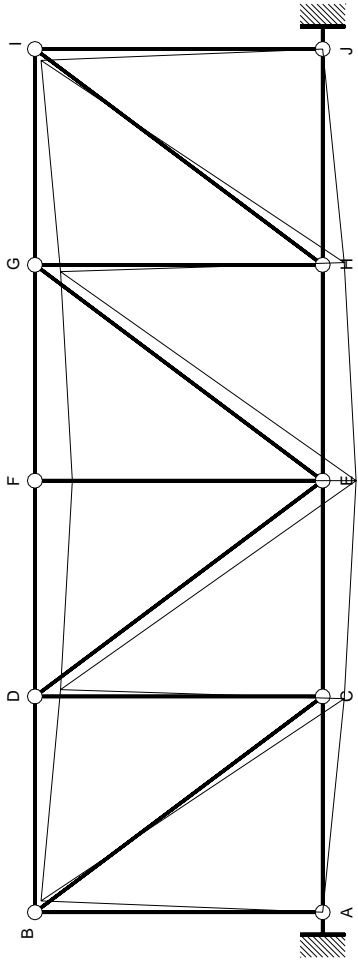
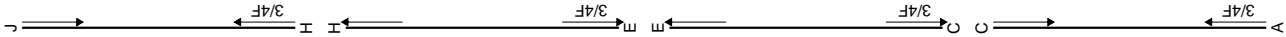
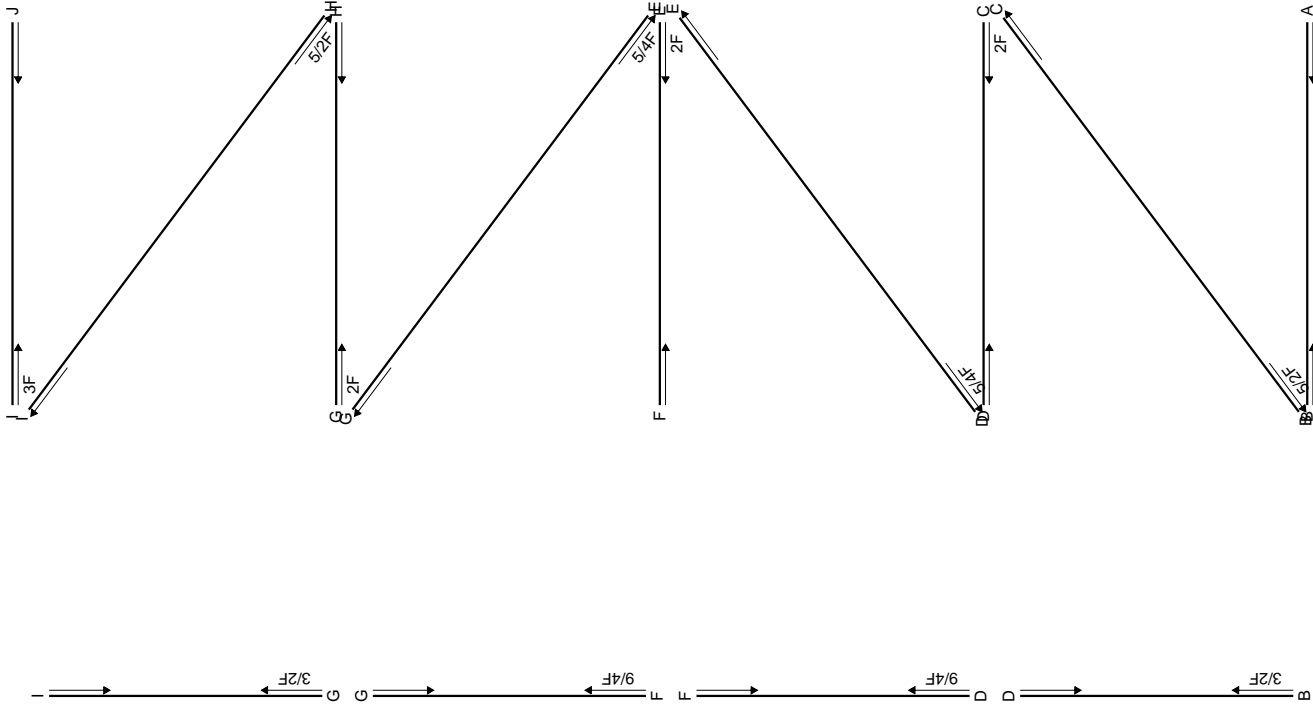
$$-9V_A b - 9V_{BA} b = 0$$

## Matrice di equilibrio

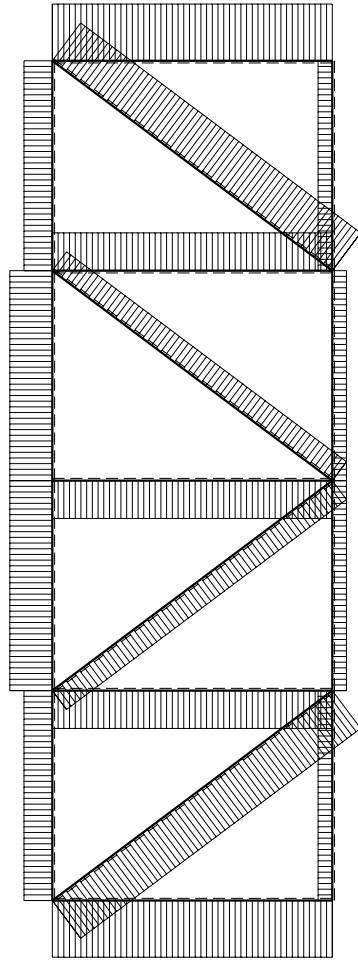
$$\begin{bmatrix} \varphi_{JI} \\ \varphi_{IG} \\ \varphi_{IH} \\ \varphi_{GF} \\ \varphi_{GE} \\ \varphi_{ED} \\ \varphi_{DB} \\ \varphi_{BC} \\ \varphi_{CA} \end{bmatrix} \begin{bmatrix} H_A b & V_A b & V_{BA} b & V_{DC} b & V_{FE} b & V_{HG} b & H_{EC} b & H_{HE} b & H_{FD} b \end{bmatrix} = \begin{bmatrix} Xb & Fb \end{bmatrix}$$

## Soluzione del sistema

$$\begin{bmatrix} V_A b \\ H_A b \\ V_{HG} b \\ V_{FE} b \\ H_{HE} b \\ H_{FD} b \\ H_{EC} b \\ V_{DC} b \\ V_{BA} b \end{bmatrix} = \begin{bmatrix} Xb & Fb \end{bmatrix}$$



$224.1 Fb/EA$



$\left[ \begin{array}{c} + \\ - \end{array} \right] \rightarrow \left[ \begin{array}{c} + \\ - \end{array} \right] 4F$



## REAZIONI

$$H_A = 3/4F \quad V_A = 3F \quad H_J = -3/4F \quad V_J = 3F$$

$$N_{AB} = -3F \quad N_{CD} = -2F \quad N_{EF} = -2F \quad N_{GH} = -2F \quad N_{IJ} = -3F \quad N_{AC} = -3/4F \quad N_{CE} = 3/4F$$

$$N_{EH} = 3/4F \quad N_{HJ} = -3/4F \quad N_{BD} = -3/2F \quad N_{DF} = -9/4F \quad N_{FG} = -9/4F \quad N_{GI} = -3/2F \quad N_{BC} = 5/2F$$

$$N_{DE} = 5/4F \quad N_{EG} = 5/4F \quad N_{HI} = 5/2F$$

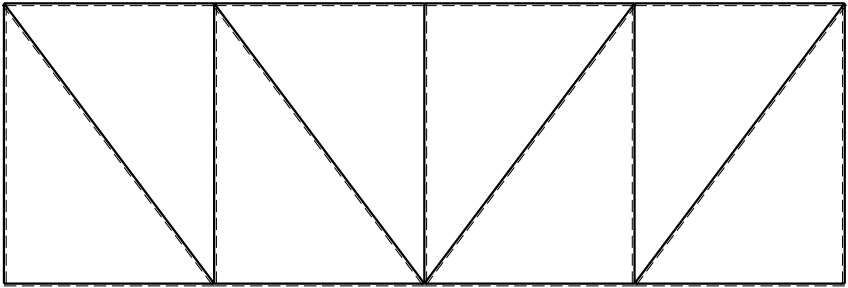
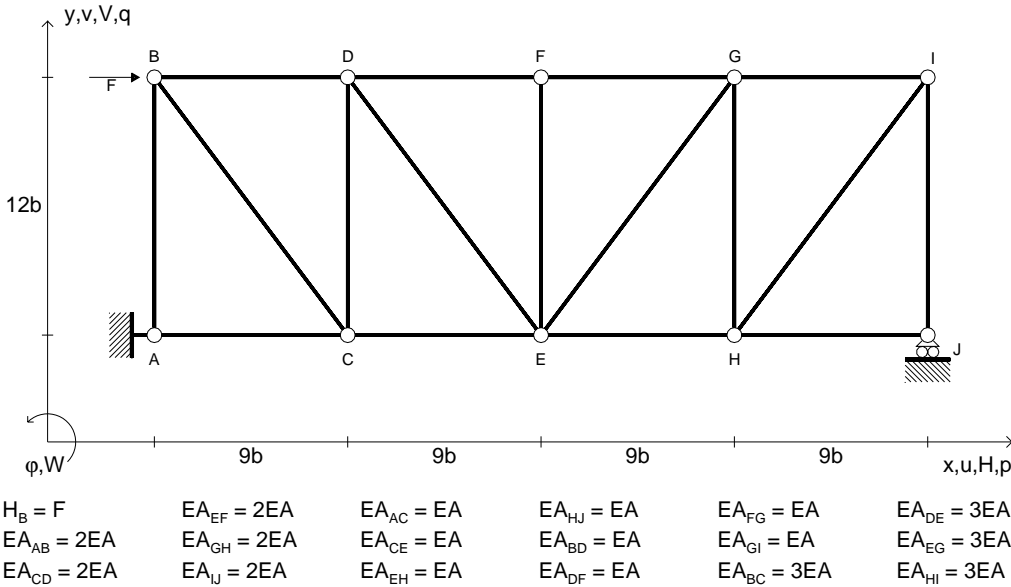
## SPOSTAMENTI NODALI

$$\begin{array}{llll} u_A = 0 & u_B = 135/4(Fb/EA) & u_C = -27/4(Fb/EA) & u_D = 81/4(Fb/EA) \\ v_A = 0 & v_B = -18(Fb/EA) & v_C = -64(Fb/EA) & v_D = -76(Fb/EA) \end{array}$$

$$\begin{array}{llll} u_E = 0 & u_F = 0 & u_G = -81/4(Fb/EA) & u_H = 27/4(Fb/EA) \\ v_E = -99(Fb/EA) & v_F = -111(Fb/EA) & v_G = -76(Fb/EA) & v_H = -64(Fb/EA) \end{array}$$

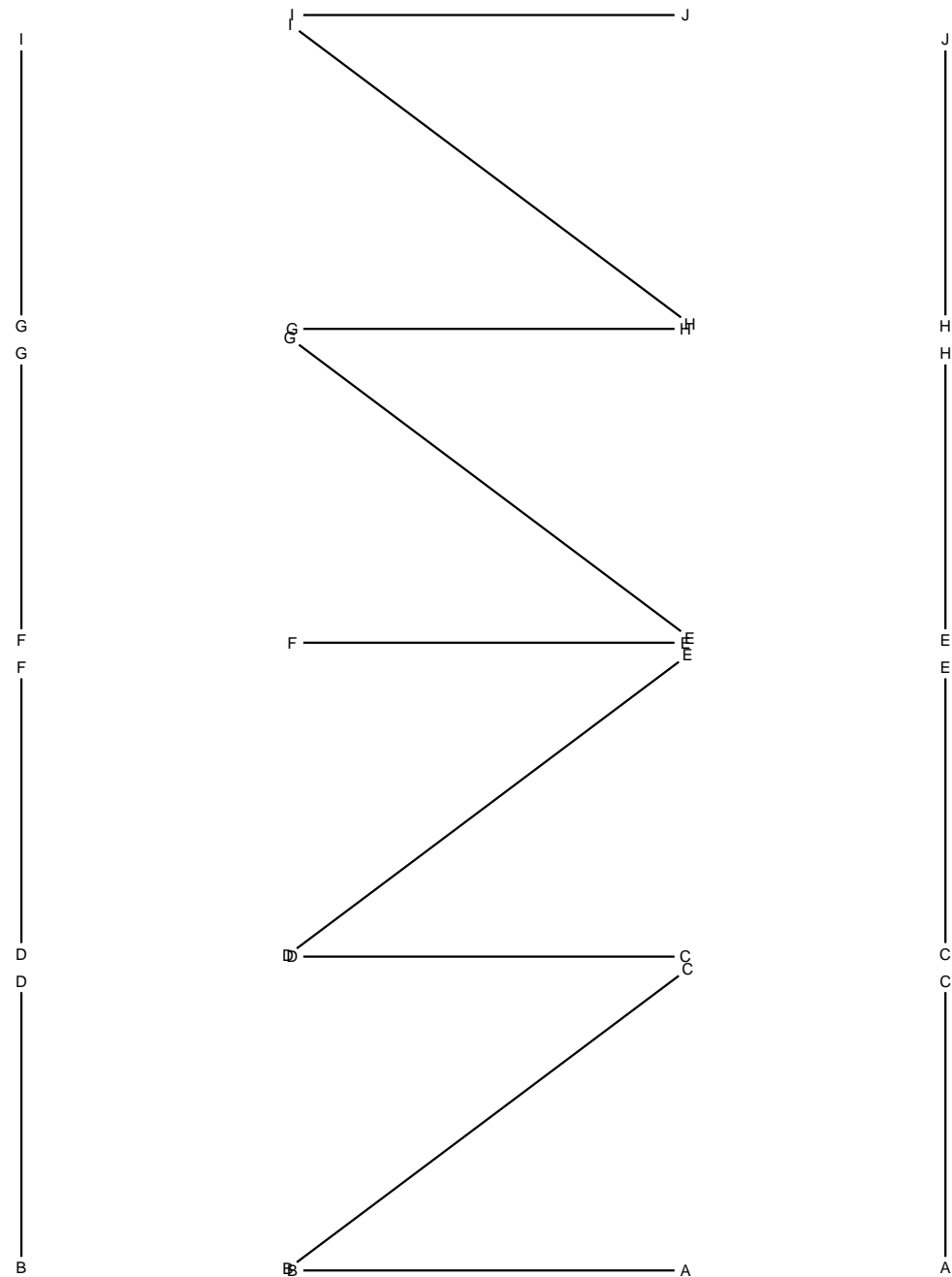
$$\begin{array}{ll} u_I = -135/4(Fb/EA) & u_J = 0 \\ v_I = -18(Fb/EA) & v_J = 0 \end{array}$$



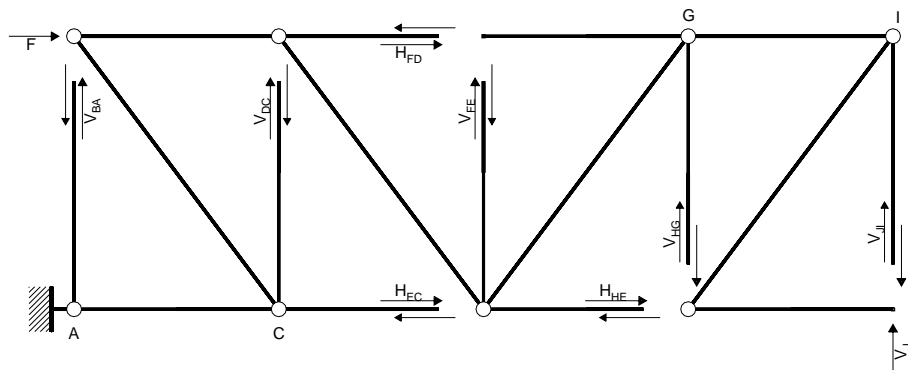


Svolgere l'analisi cinematica.  
Tracciare la deformata elastica.

Carichi e deformazioni date hanno verso efficace in disegno.  
Calcolare reazioni vincolari della struttura e delle aste.  
Tracciare i diagrammi delle azioni interne nelle aste.  
Calcolare spostamento e rotazione di tutti i nodi.  
 $A_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.24.05.07



REAZIONI					
$H_A =$	$V_A =$	$V_J =$			
$N_{AB} =$	$N_{CD} =$	$N_{EF} =$	$N_{GH} =$	$N_{IJ} =$	$N_{AC} =$
$N_{CE} =$	$N_{EH} =$	$N_{HJ} =$	$N_{BD} =$	$N_{DF} =$	$N_{FG} =$
$N_{GI} =$	$N_{BC} =$	$N_{DE} =$	$N_{EG} =$	$N_{HI} =$	
SPOSTAMENTI NODALI					
$u_A =$	$u_B =$	$u_C =$	$u_D =$		
$v_A =$	$v_B =$	$v_C =$	$v_D =$		
$u_E =$	$u_F =$	$u_G =$	$u_H =$		
$v_E =$	$v_F =$	$v_G =$	$v_H =$		
$u_I =$	$u_{JJ} =$				
$v_I =$	$v_J =$				



## EQUAZIONI DI EQUILIBRIO

Rotazione intorno a A: aste AC CD CE CB BD DF DE EF EH EG GH GF GI IJ IH HJ

$$36V_{Jb} = 12Fb$$

Rotazione intorno a C: aste CB BD DF DE EF EH EG GH GF GI IJ IH HJ

$$27V_{Jb} + 9V_{BA}b = 12Fb$$

Rotazione intorno a B: aste BD DF DE EF EH EG GH GF GI IJ IH HJ

$$36V_{Jb} - 9V_{DC}b - 12H_{EC}b = 0$$

Rotazione intorno a D: aste DE EF EH EG GH GF GI IJ IH HJ

$$27V_{Jb} - 12H_{EC}b = 0$$

Rotazione intorno a E: aste EG GH GF GI IJ IH HJ

$$18V_{Jb} + 12H_{FD}b = 0$$

Rotazione intorno a G: aste GF

$$9V_{FE}b = 0$$

Rotazione intorno a G: aste GI IJ IH HJ

$$9V_{Jb} - 12H_{HE}b = 0$$

Rotazione intorno a I: aste IH HJ

$$9V_{HG}b - 12H_{HE}b = 0$$

Rotazione intorno a H: aste HJ

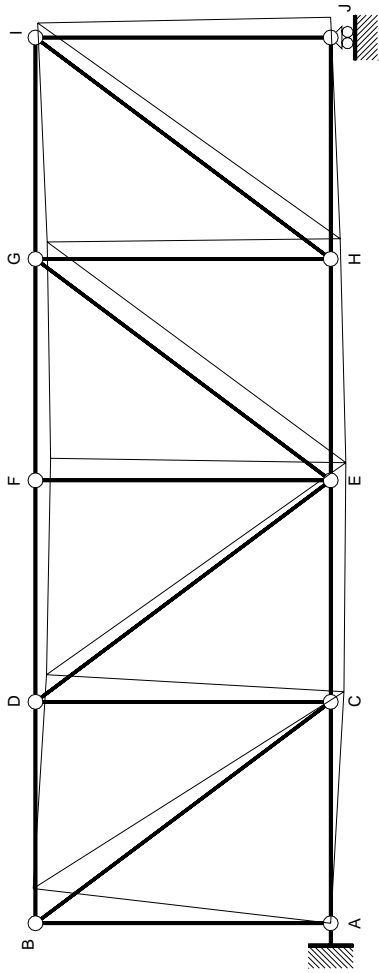
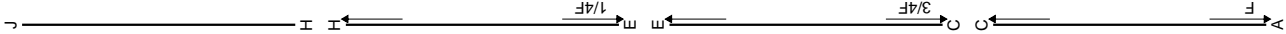
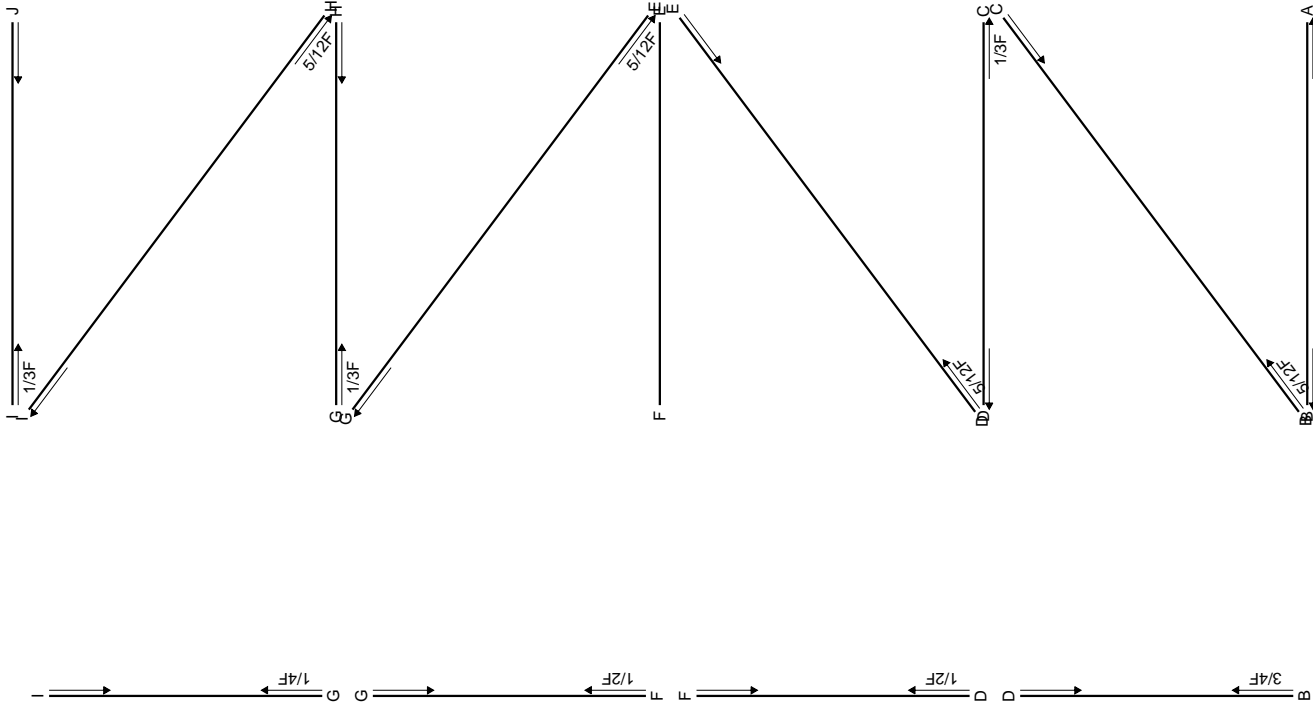
$$9V_{Jb} - 9V_{JI}b = 0$$

## Matrice di equilibrio

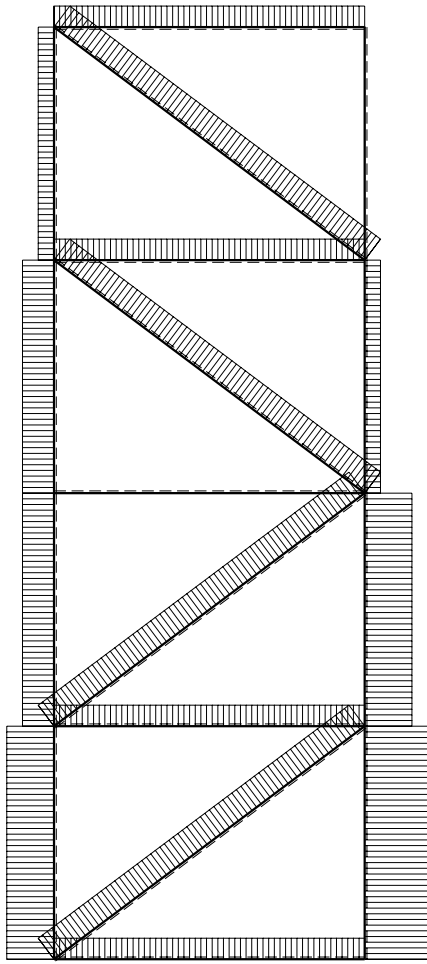
$$\begin{bmatrix} \varphi_{AC} & \varphi_{CB} & \varphi_{BD} & \varphi_{DE} & \varphi_{EG} & \varphi_{GF} & \varphi_{GI} & \varphi_{IH} & \varphi_{HJ} \end{bmatrix} \begin{bmatrix} V_{Jb} & V_{BA}b & V_{DC}b & V_{FE}b & V_{HG}b & V_{JI}b & H_{EC}b & H_{HE}b & H_{FD}b \end{bmatrix} = \begin{bmatrix} 12 \\ 12 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

## Soluzione del sistema

$$\begin{bmatrix} V_{Jb} \\ V_{BA}b \\ V_{DC}b \\ H_{EC}b \\ H_{FD}b \\ V_{FE}b \\ H_{HE}b \\ V_{HG}b \\ V_{JI}b \end{bmatrix} = \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \\ 3/4 \\ -1/2 \\ 0 \\ 1/4 \\ 1/3 \\ 1/3 \end{bmatrix}$$



$67.24 Fb/EA$



$\left[ \begin{array}{c} + \\ - \end{array} \right] \rightarrow \left[ \begin{array}{c} + \\ - \end{array} \right] 1.2 F$

REAZIONI

$H_A = -F$  $V_A = -1/3F$  $V_J = 1/3F$

$N_{AB} = 1/3F$  $N_{CD} = 1/3F$  $N_{EF} = 0$  $N_{GH} = -1/3F$  $N_{IJ} = -1/3F$  $N_{AC} = F$

$N_{CE} = 3/4F$  $N_{EH} = 1/4F$  $N_{HJ} = 0$  $N_{BD} = -3/4F$  $N_{DF} = -1/2F$  $N_{FG} = -1/2F$

$N_{GI} = -1/4F$  $N_{BC} = -5/12F$  $N_{DE} = -5/12F$  $N_{EG} = 5/12F$  $N_{HI} = 5/12F$

SPOSTAMENTI NODALI

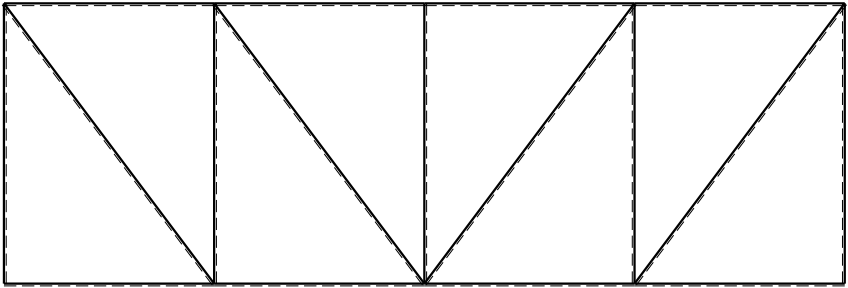
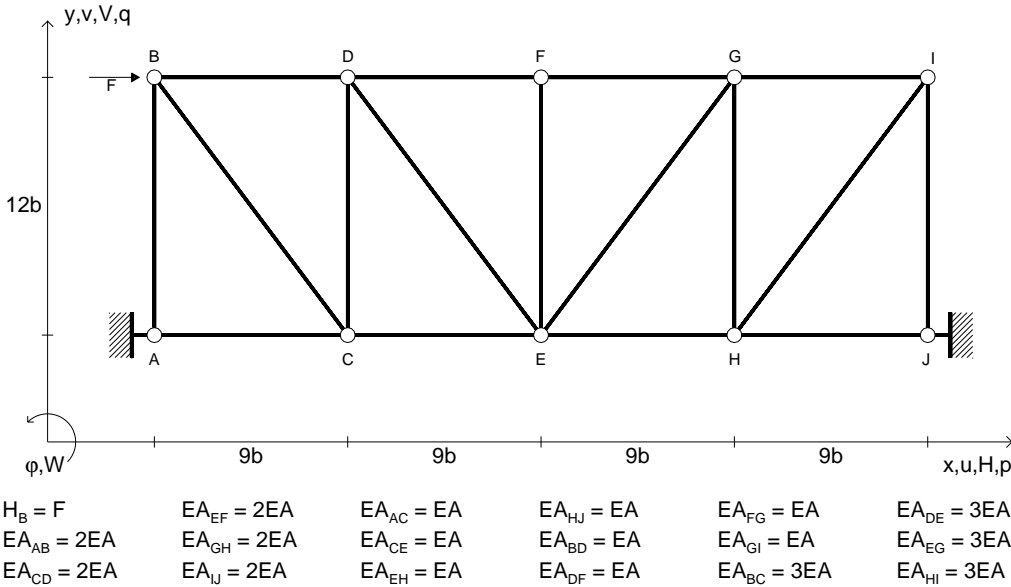
$u_A = 0$  $v_A = 0$  $u_B = 278/9(Fb/EA)$  $v_B = 2(Fb/EA)$  $u_C = 9(Fb/EA)$  $v_C = -189/16(Fb/EA)$  $u_D = 869/36(Fb/EA)$  $v_D = -157/16(Fb/EA)$

$u_E = 63/4(Fb/EA)$  $v_E = -27/2(Fb/EA)$  $u_F = 707/36(Fb/EA)$  $v_F = -27/2(Fb/EA)$  $u_G = 545/36(Fb/EA)$  $v_G = -167/16(Fb/EA)$  $u_H = 18(Fb/EA)$  $v_H = -135/16(Fb/EA)$

$u_I = 116/9(Fb/EA)$  $v_I = -2(Fb/EA)$  $u_{JJ} = 18(Fb/EA)$  $v_J = 0$



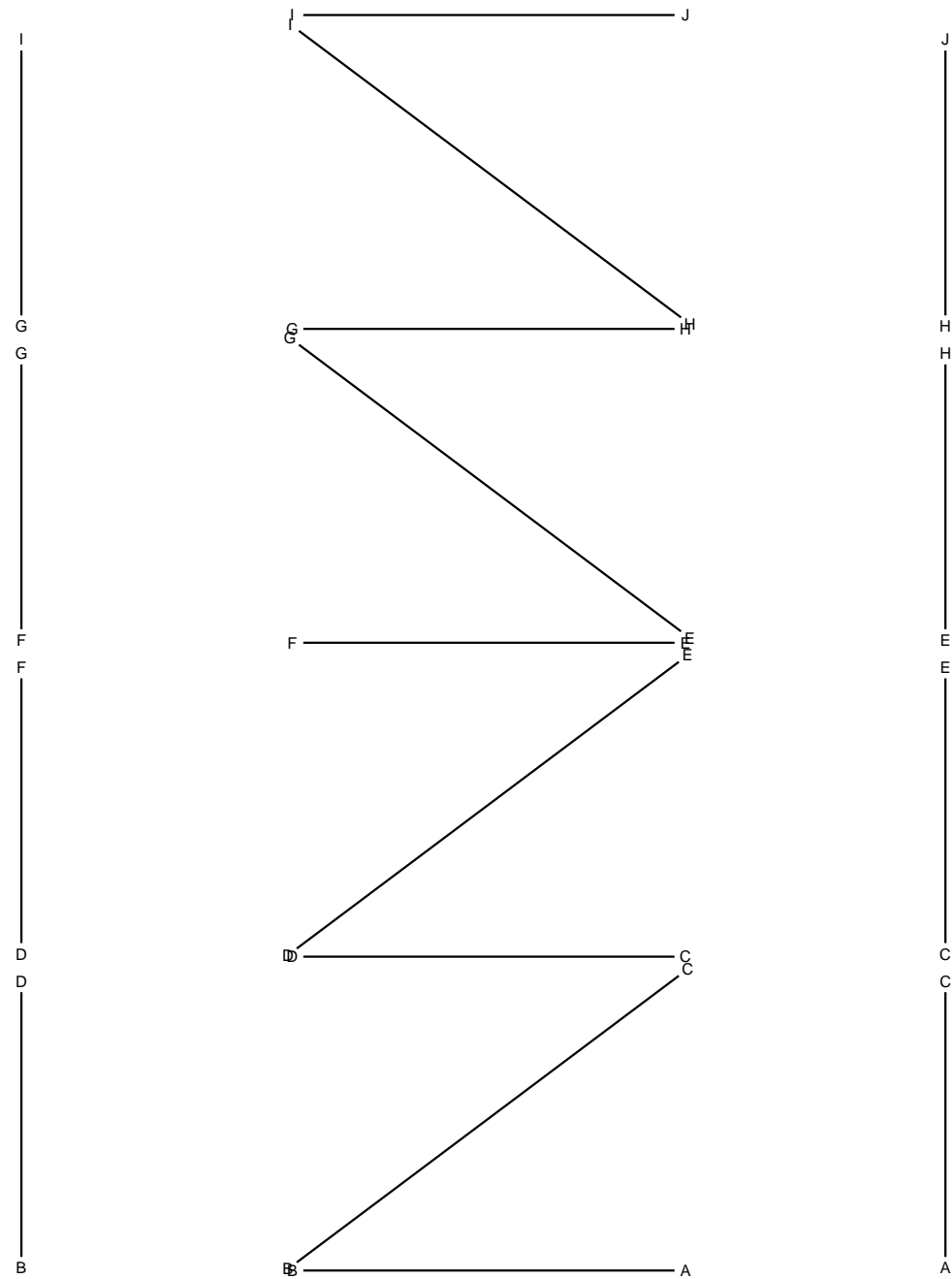




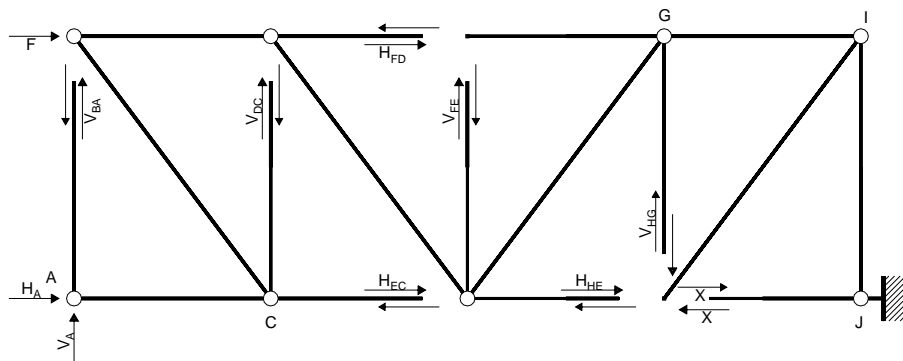
Svolgere l'analisi cinematica.  
Tracciare la deformata elastica.

Carichi e deformazioni date hanno verso efficace in disegno.  
Calcolare reazioni vincolari della struttura e delle aste.  
Tracciare i diagrammi delle azioni interne nelle aste.  
Calcolare spostamento e rotazione di tutti i nodi.  
 $A_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.  
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.24.05.07





REAZIONI					
$H_A =$	$V_A =$	$H_J =$	$V_J =$		
$N_{AB} =$	$N_{CD} =$	$N_{EF} =$	$N_{GH} =$	$N_{IJ} =$	$N_{AC} =$
$N_{CE} =$	$N_{EH} =$	$N_{HJ} =$	$N_{BD} =$	$N_{DF} =$	$N_{FG} =$
$N_{GI} =$	$N_{BC} =$	$N_{DE} =$	$N_{EG} =$	$N_{HI} =$	
SPOSTAMENTI NODALI					
$u_A =$	$u_B =$	$u_C =$	$u_D =$		
$v_A =$	$v_B =$	$v_C =$	$v_D =$		
$u_E =$	$u_F =$	$u_G =$	$u_H =$		
$v_E =$	$v_F =$	$v_G =$	$v_H =$		
$u_I =$	$u_J =$				
$v_I =$	$v_J =$				



## EQUAZIONI DI EQUILIBRIO

Rotazione intorno a J: aste JI IG IH GH GF GE EF EH ED DB DF BC CD CA CE AB

$$-36V_A b = 12Fb$$

Rotazione intorno a I: aste IG GH GF GE EF EH ED DB DF BC CD CA CE AB

$$12H_A b - 36V_A b - 9V_{HG} b + 12H_{HE} b = 0$$

Rotazione intorno a I: aste IH

$$9V_{HG} b - 12H_{HE} b = 12Xb$$

Rotazione intorno a G: aste GF

$$9V_{FE} b = 0$$

Rotazione intorno a G: aste GE EF EH ED DB DF BC CD CA CE AB

$$12H_A b - 27V_A b - 9V_{FE} b + 12H_{HE} b = 0$$

Rotazione intorno a E: aste ED DB DF BC CD CA CE AB

$$-18V_A b - 12H_{FD} b = 12Fb$$

Rotazione intorno a D: aste DB BC CD CA CE AB

$$12H_A b - 9V_A b + 12H_{EC} b = 0$$

Rotazione intorno a B: aste BC CD CA CE AB

$$12H_A b + 9V_{DC} b + 12H_{EC} b = 0$$

Rotazione intorno a C: aste CA AB

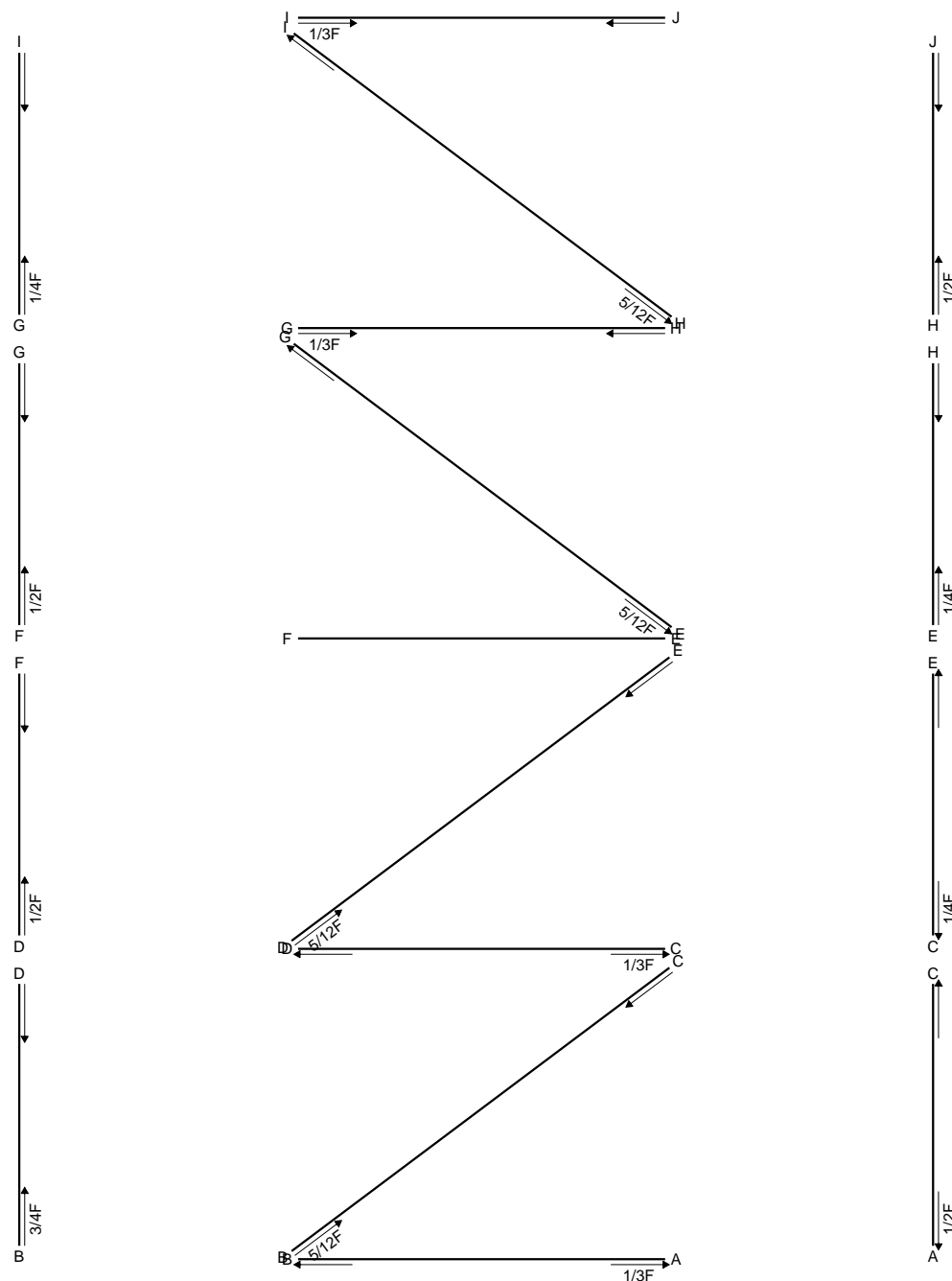
$$-9V_A b - 9V_{BA} b = 0$$

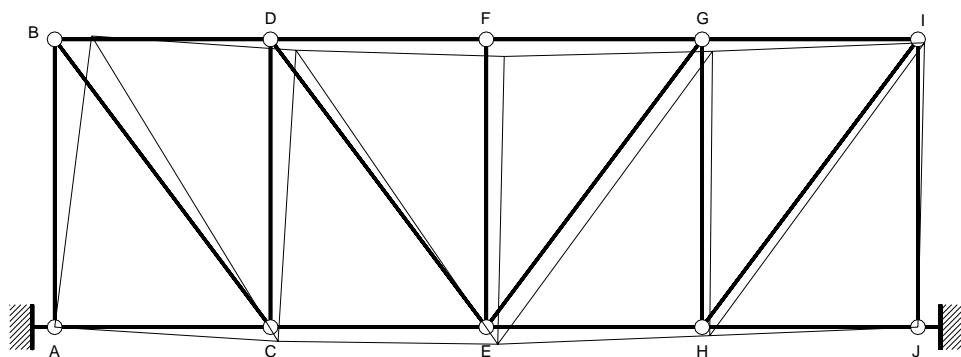
## Matrice di equilibrio

$$\begin{bmatrix} \phi_{JI} \\ \phi_{IG} \\ \phi_{IH} \\ \phi_{GF} \\ \phi_{GE} \\ \phi_{ED} \\ \phi_{DB} \\ \phi_{BC} \\ \phi_{CA} \end{bmatrix} \begin{bmatrix} H_A b & V_A b & V_{BA} b & V_{DC} b & V_{FE} b & V_{HG} b & H_{EC} b & H_{HE} b & H_{FD} b \end{bmatrix} = \begin{bmatrix} Xb & Fb \end{bmatrix}$$

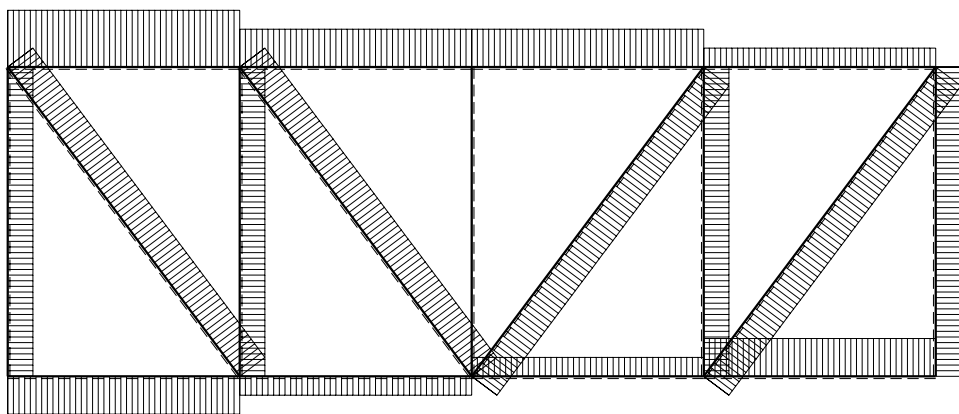
## Soluzione del sistema

$$\begin{bmatrix} V_A b \\ H_A b \\ V_{HG} b \\ V_{FE} b \\ H_{HE} b \\ H_{FD} b \\ H_{EC} b \\ V_{DC} b \\ V_{BA} b \end{bmatrix} = \begin{bmatrix} Xb & Fb \end{bmatrix}$$





— 44.83 Fb/EA



← ⊕ → — 1 F

## REAZIONI

$$H_A = -1/2F \quad V_A = -1/3F \quad H_J = -1/2F \quad V_J = 1/3F$$

$$N_{AB} = 1/3F \quad N_{CD} = 1/3F \quad N_{EF} = 0 \quad N_{GH} = -1/3F \quad N_{IJ} = -1/3F \quad N_{AC} = 1/2F$$

$$N_{CE} = 1/4F \quad N_{EH} = -1/4F \quad N_{HJ} = -1/2F \quad N_{BD} = -3/4F \quad N_{DF} = -1/2F \quad N_{FG} = -1/2F$$

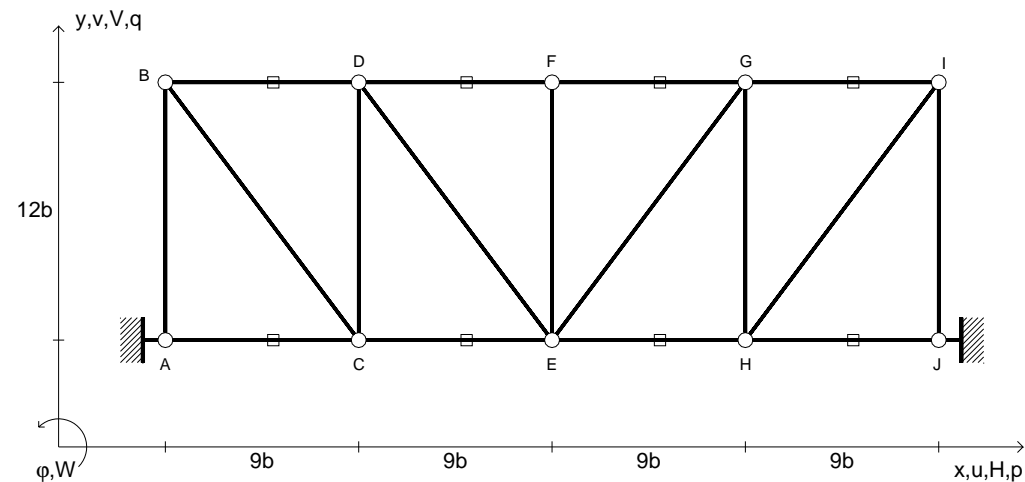
$$N_{GI} = -1/4F \quad N_{BC} = -5/12F \quad N_{DE} = -5/12F \quad N_{EG} = 5/12F \quad N_{HI} = 5/12F$$

## SPOSTAMENTI NODALI

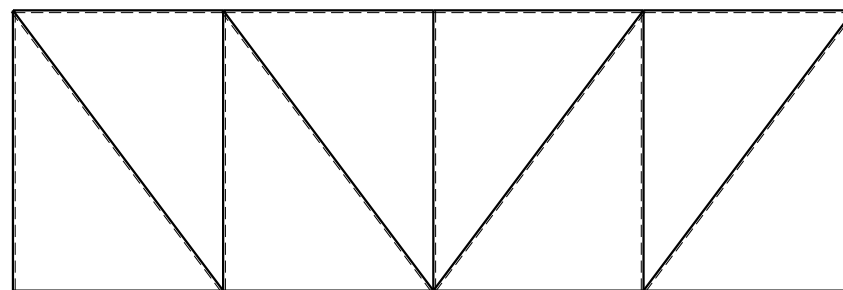
$$\begin{aligned} u_A &= 0 & u_B &= 197/9(Fb/EA) & u_C &= 9/2(Fb/EA) & u_D &= 545/36(Fb/EA) \\ v_A &= 0 & v_B &= 2(Fb/EA) & v_C &= -135/16(Fb/EA) & v_D &= -103/16(Fb/EA) \end{aligned}$$

$$\begin{aligned} u_E &= 27/4(Fb/EA) & u_F &= 383/36(Fb/EA) & u_G &= 221/36(Fb/EA) & u_H &= 9/2(Fb/EA) \\ v_E &= -81/8(Fb/EA) & v_F &= -81/8(Fb/EA) & v_G &= -113/16(Fb/EA) & v_H &= -81/16(Fb/EA) \end{aligned}$$

$$\begin{aligned} u_I &= 35/9(Fb/EA) & u_J &= 0 \\ v_I &= -2(Fb/EA) & v_J &= 0 \end{aligned}$$



$\varepsilon_{BD} = -\alpha T = -F/EA$	$\varepsilon_{CE} = \alpha T = F/EA$	$EA_{EF} = 2EA$	$EA_{EH} = EA$	$EA_{GI} = EA$
$\varepsilon_{DF} = -\alpha T = -F/EA$	$\varepsilon_{EH} = \alpha T = F/EA$	$EA_{GH} = 2EA$	$EA_{HJ} = EA$	$EA_{BC} = 3EA$
$\varepsilon_{FG} = -\alpha T = -F/EA$	$\varepsilon_{HJ} = \alpha T = F/EA$	$EA_{IJ} = 2EA$	$EA_{BD} = EA$	$EA_{DE} = 3EA$
$\varepsilon_{GI} = -\alpha T = -F/EA$	$EA_{AB} = 2EA$	$EA_{AC} = EA$	$EA_{DF} = EA$	$EA_{EG} = 3EA$
$\varepsilon_{AC} = \alpha T = F/EA$	$EA_{CD} = 2EA$	$EA_{CE} = EA$	$EA_{FG} = EA$	$EA_{HI} = 3EA$



Svolgere l'analisi cinematica.  
Tracciare la deformata elastica.

Carichi e deformazioni date hanno verso efficace in disegno.

Calcolare reazioni vincolari della struttura e delle aste.

Tracciare i diagrammi delle azioni interne nelle aste.

Calcolare spostamento e rotazione di tutti i nodi.

$A_{YZ} - x_{YZ} - \theta_{YZ}$  riferimento locale asta YZ con origine in Y.

Elongazione termica specifica  $\varepsilon$  assegnata su asta BD.

Elongazione termica specifica  $\varepsilon$  assegnata su asta DF.

Elongazione termica specifica  $\varepsilon$  assegnata su asta FG.

Elongazione termica specifica  $\varepsilon$  assegnata su asta GI.

Elongazione termica specifica  $\varepsilon$  assegnata su asta AC.

Elongazione termica specifica  $\varepsilon$  assegnata su asta CE.

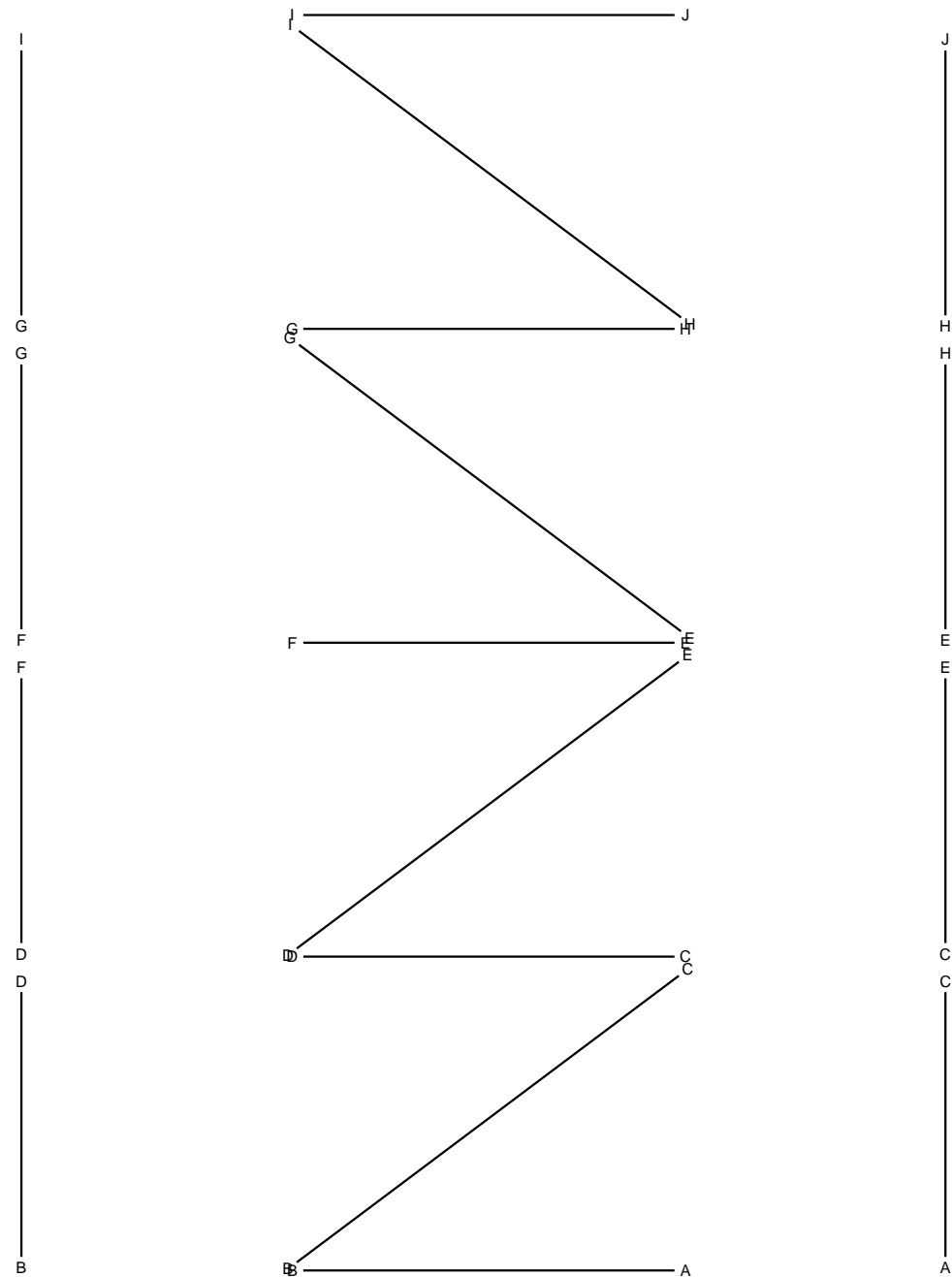
Elongazione termica specifica  $\varepsilon$  assegnata su asta EH.

Elongazione termica specifica  $\varepsilon$  assegnata su asta HJ.

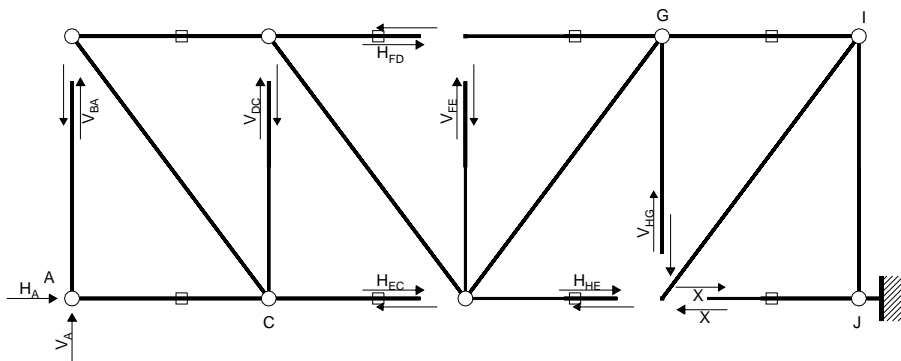
@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.24.05.07



@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.24.05.07



REAZIONI					
$H_A =$	$V_A =$	$H_J =$	$V_J =$		
$N_{AB} =$	$N_{CD} =$	$N_{EF} =$	$N_{GH} =$	$N_{IJ} =$	$N_{AC} =$
$N_{CE} =$	$N_{EH} =$	$N_{HJ} =$	$N_{BD} =$	$N_{DF} =$	$N_{FG} =$
$N_{GI} =$	$N_{BC} =$	$N_{DE} =$	$N_{EG} =$	$N_{HI} =$	
SPOSTAMENTI NODALI					
$u_A =$	$u_B =$	$u_C =$	$u_D =$	$u_E =$	$u_F =$
$v_A =$	$v_B =$	$v_C =$	$v_D =$	$v_E =$	$v_F =$
$u_G =$	$u_H =$	$u_I =$	$u_J =$		
$v_G =$	$v_H =$	$v_I =$	$v_J =$		



## EQUAZIONI DI EQUILIBRIO

Rotazione intorno a J: aste JI IG IH GH GF GE EF EH ED DB DF BC CD CA CE AB

$$-36V_A b = 0$$

Rotazione intorno a I: aste IG GH GF GE EF EH ED DB DF BC CD CA CE AB

$$12H_A b - 36V_A b - 9V_{HG} b + 12H_{HE} b = 0$$

Rotazione intorno a I: aste IH

$$9V_{HG} b - 12H_{HE} b = 12Xb$$

Rotazione intorno a G: aste GF

$$9V_{FE} b = 0$$

Rotazione intorno a G: aste GE EF EH ED DB DF BC CD CA CE AB

$$12H_A b - 27V_A b - 9V_{FE} b + 12H_{HE} b = 0$$

Rotazione intorno a E: aste ED DB DF BC CD CA CE AB

$$-18V_A b - 12H_{FD} b = 0$$

Rotazione intorno a D: aste DB BC CD CA CE AB

$$12H_A b - 9V_A b + 12H_{EC} b = 0$$

Rotazione intorno a B: aste BC CD CA CE AB

$$12H_A b + 9V_{DC} b + 12H_{EC} b = 0$$

Rotazione intorno a C: aste CA AB

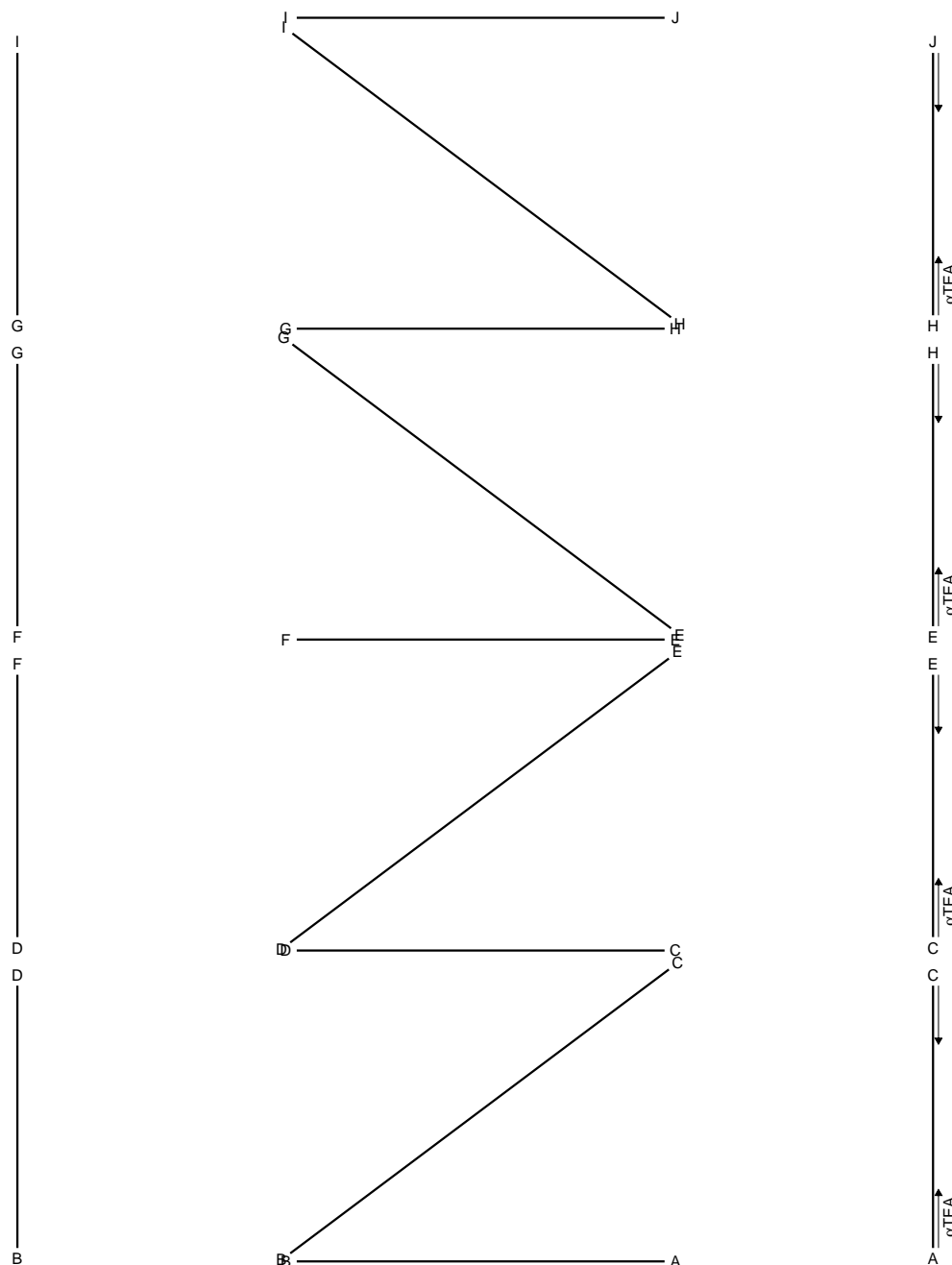
$$-9V_A b - 9V_{BA} b = 0$$

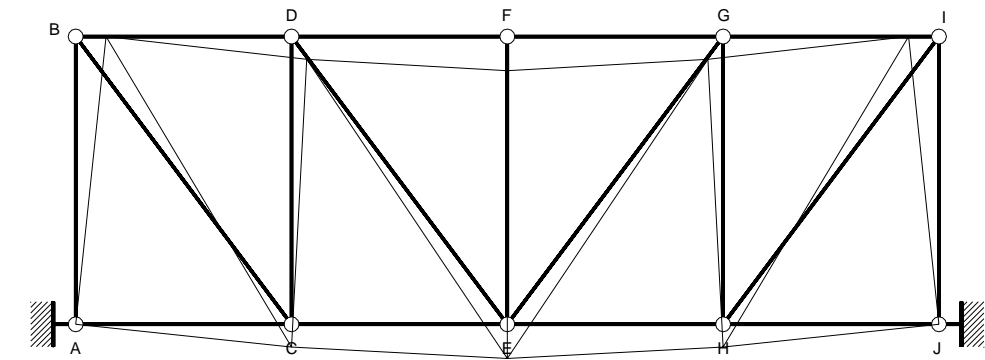
## Matrice di equilibrio

$$\begin{bmatrix} \phi_{JI} \\ \phi_{IG} \\ \phi_{IH} \\ \phi_{GF} \\ \phi_{GE} \\ \phi_{ED} \\ \phi_{DB} \\ \phi_{BC} \\ \phi_{CA} \end{bmatrix} \begin{bmatrix} H_A b & V_A b & V_{BA} b & V_{DC} b & V_{FE} b & V_{HG} b & H_{EC} b & H_{HE} b & H_{FD} b \end{bmatrix} = \begin{bmatrix} Xb \\ 0 \\ 12 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

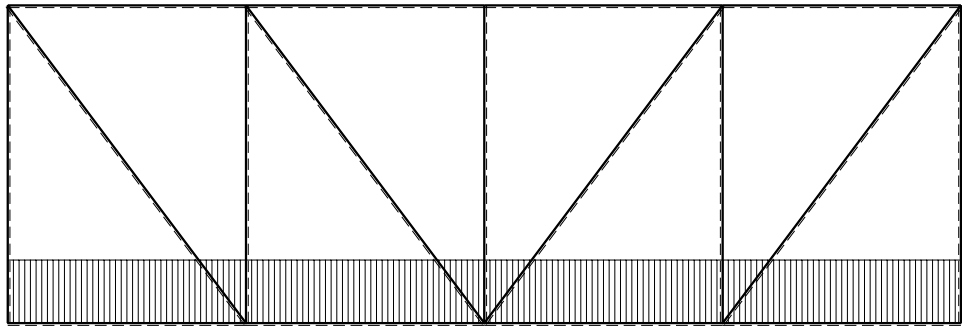
## Soluzione del sistema

$$\begin{bmatrix} V_A b \\ H_A b \\ V_{HG} b \\ V_{FE} b \\ H_{HE} b \\ H_{FD} b \\ H_{EC} b \\ V_{DC} b \\ V_{BA} b \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ -1 \\ -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$





← 44.83  $\alpha T b$



←  $\boxed{+}$  →    ← 1.2  $\alpha T E A$

REAZIONI

$H_A = \alpha T E A$      $V_A = 0$      $H_J = -\alpha T E A$      $V_J = 0$

$N_{AB} = 0$	$N_{CD} = 0$	$N_{EF} = 0$	$N_{GH} = 0$	$N_{IJ} = 0$	$N_{AC} = -\alpha T E A$
$N_{CE} = -\alpha T E A$	$N_{EH} = -\alpha T E A$	$N_{HJ} = -\alpha T E A$	$N_{BD} = 0$	$N_{DF} = 0$	$N_{FG} = 0$
$N_{GI} = 0$	$N_{BC} = 0$	$N_{DE} = 0$	$N_{EG} = 0$	$N_{HI} = 0$	

SPOSTAMENTI NODALI

$u_A = 0$	$u_B = 18 \alpha T b$	$u_C = 0$	$u_D = 9 \alpha T b$	$u_E = 0$	$u_F = 0$
$v_A = 0$	$v_B = 0$	$v_C = -27/2 \alpha T b$	$v_D = -27/2 \alpha T b$	$v_E = -81/4 \alpha T b$	$v_F = -81/4 \alpha T b$
$u_G = -9 \alpha T b$	$u_H = 0$	$u_I = -18 \alpha T b$	$u_J = 0$		
$v_G = -27/2 \alpha T b$	$v_H = -27/2 \alpha T b$	$v_I = 0$	$v_J = 0$		