DIAS

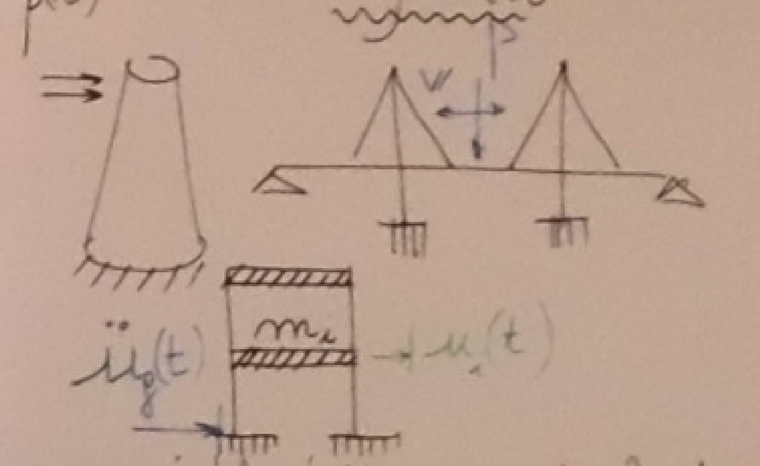
Dinamica, Instabilità e Anelasticità delle Strutture

Dynamics, Instability and Anelasticity of Structures

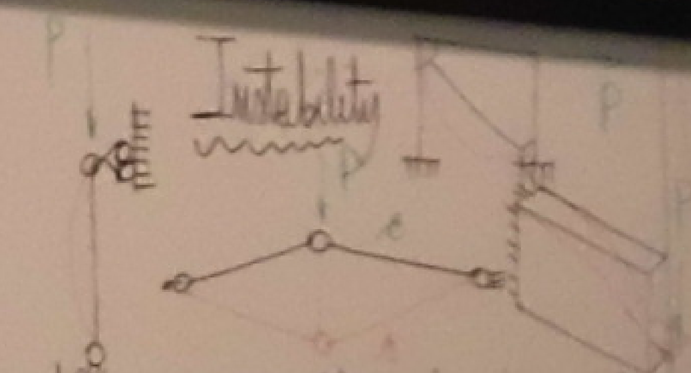
prof. Egidio RIZZI

www ⇒ (FDIS)

Dynamics



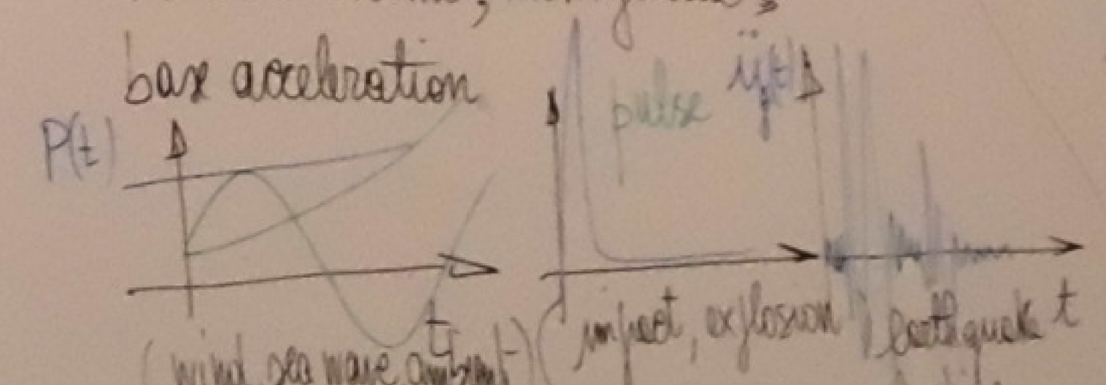
Instability



buckling; snap-through; torsional

Elasticity / Anelasticity

- Examples:
- Actions: – variable in time, moving load, base acceleration



- slowly variable in time, e.g. …; sudden; critical

- Characteristics: – time t is involved
  – inertia effects (masses) ⇒ equation(s) of motion

- equilibrium in the deformed configuration
- P-Δ effect, geometric non-linearities
- slender elements

- dissipation of energy

- Program: – SDOF systems → MDOF systems

- Discrete systems … continuous

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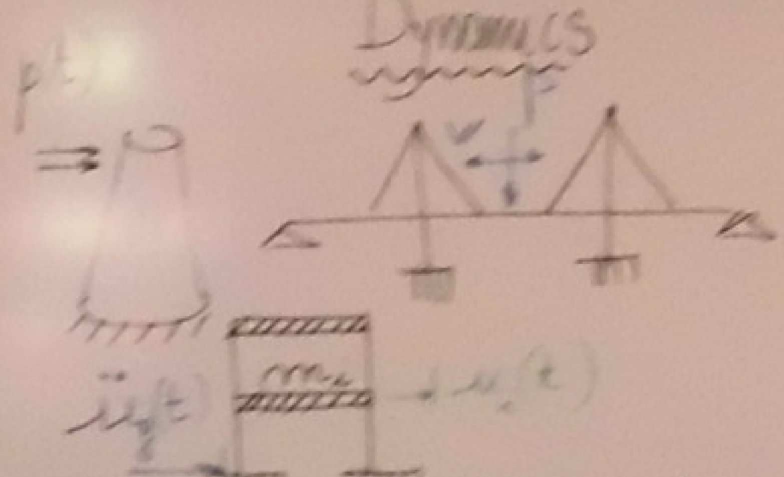
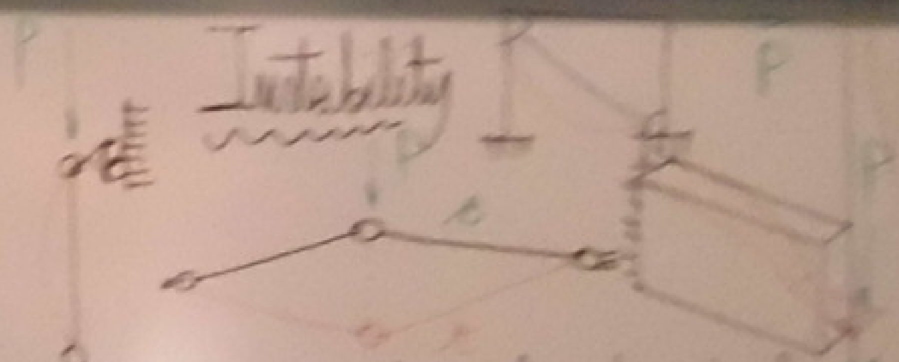
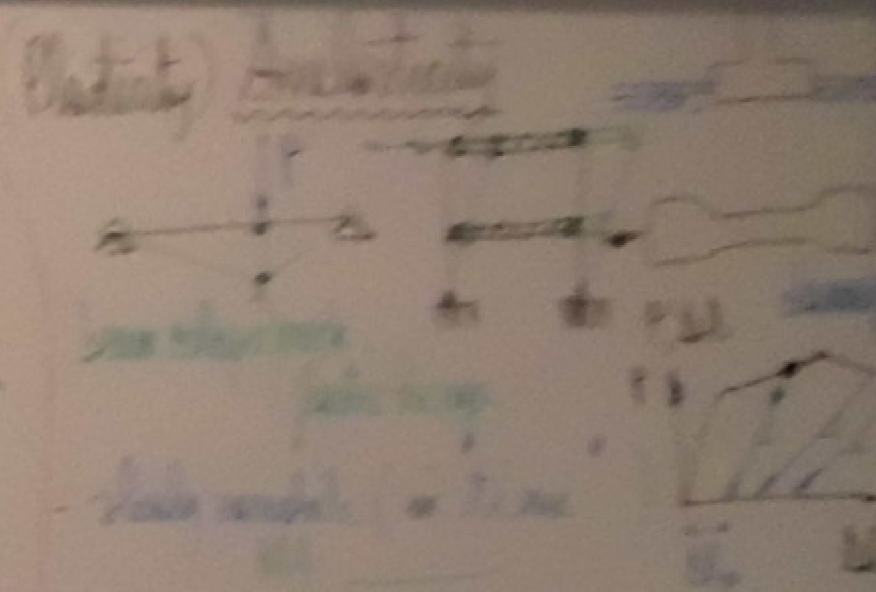


Instability

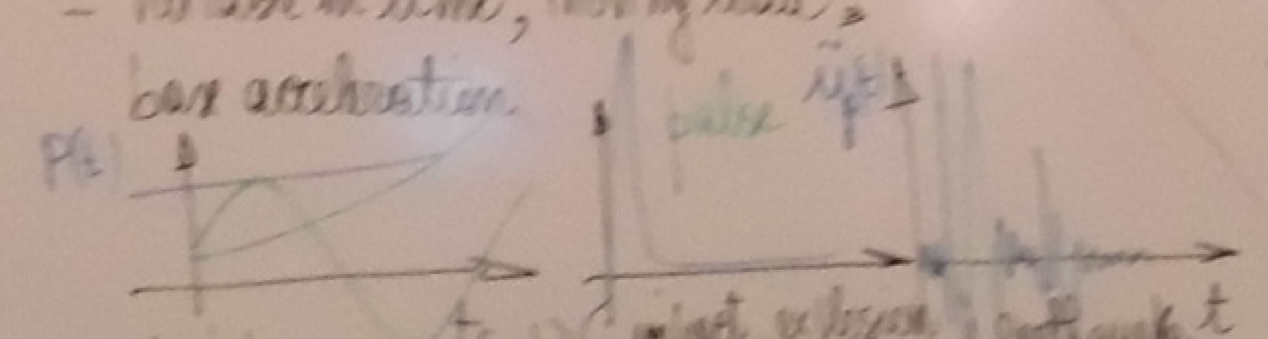


(Plasticity) Anelasticity



- Examples

- Actions
  - variable in time, moving load, base acceleration



  - buckling; snap-through; torsional ...
  - slowly variable in time, e.g. ... increasing; sudden; critical

- Characteristics
  - time t is involved
  - inertia effects (masses) => equation(s) of motion
  - equilibrium in the deformed configuration (...)
  - P-Δ effect, geometric non-linearity, ...
  - slender elements
  - non-linear material ...
  - dissipation of energy and ...

- Program
  - SDOF systems → MDOF systems
  - discrete systems ... continuous

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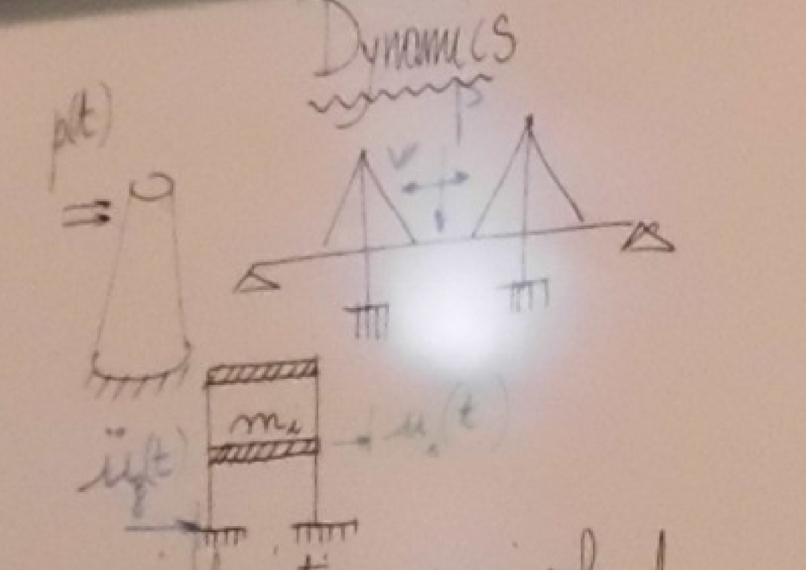
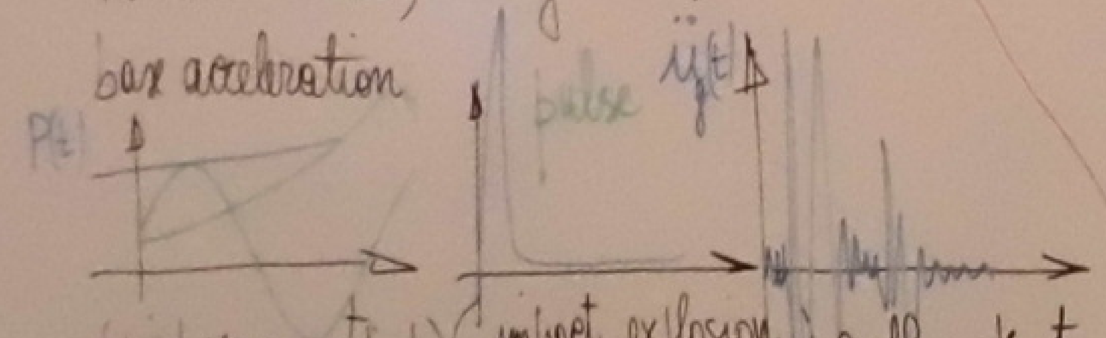
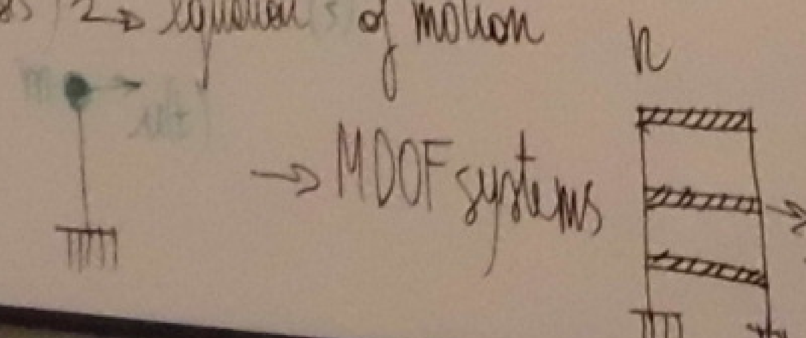


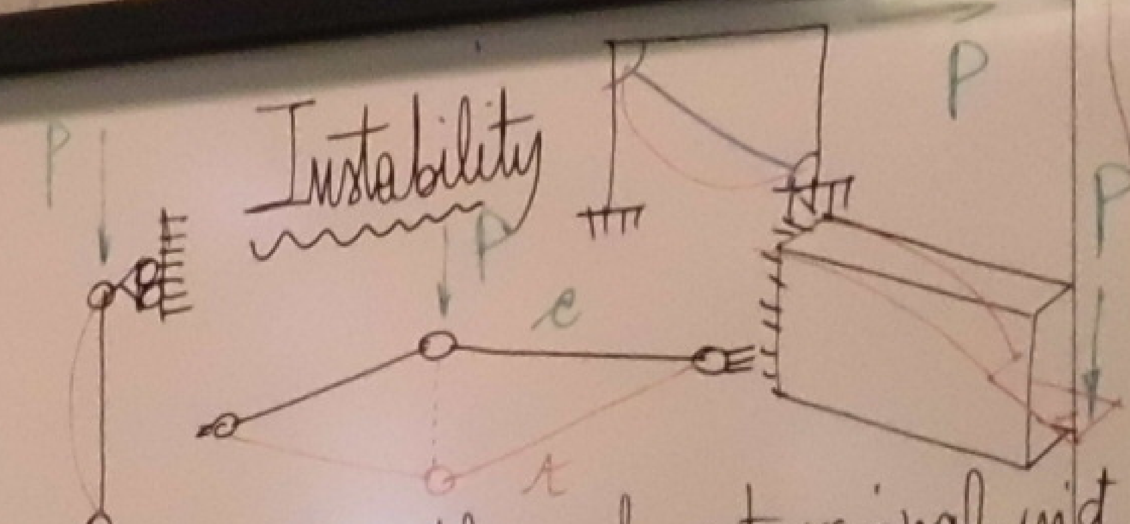
- Examples
- Actions – variable in time, moving load, base acceleration



- Characteristics – time t is involved
  - inertia effects (masses) => equation(s) of motion
- Program – SDOF systems → MDOF systems

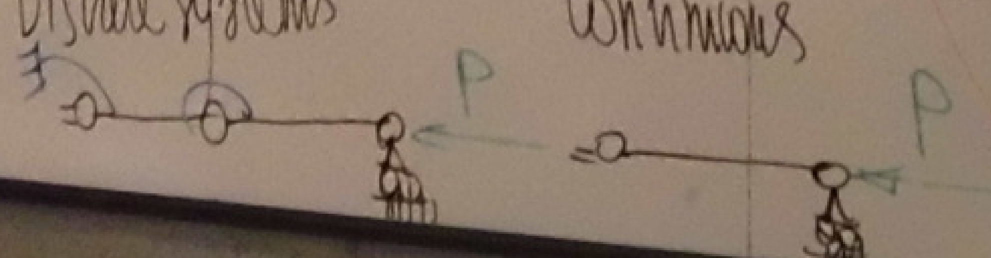


Instability

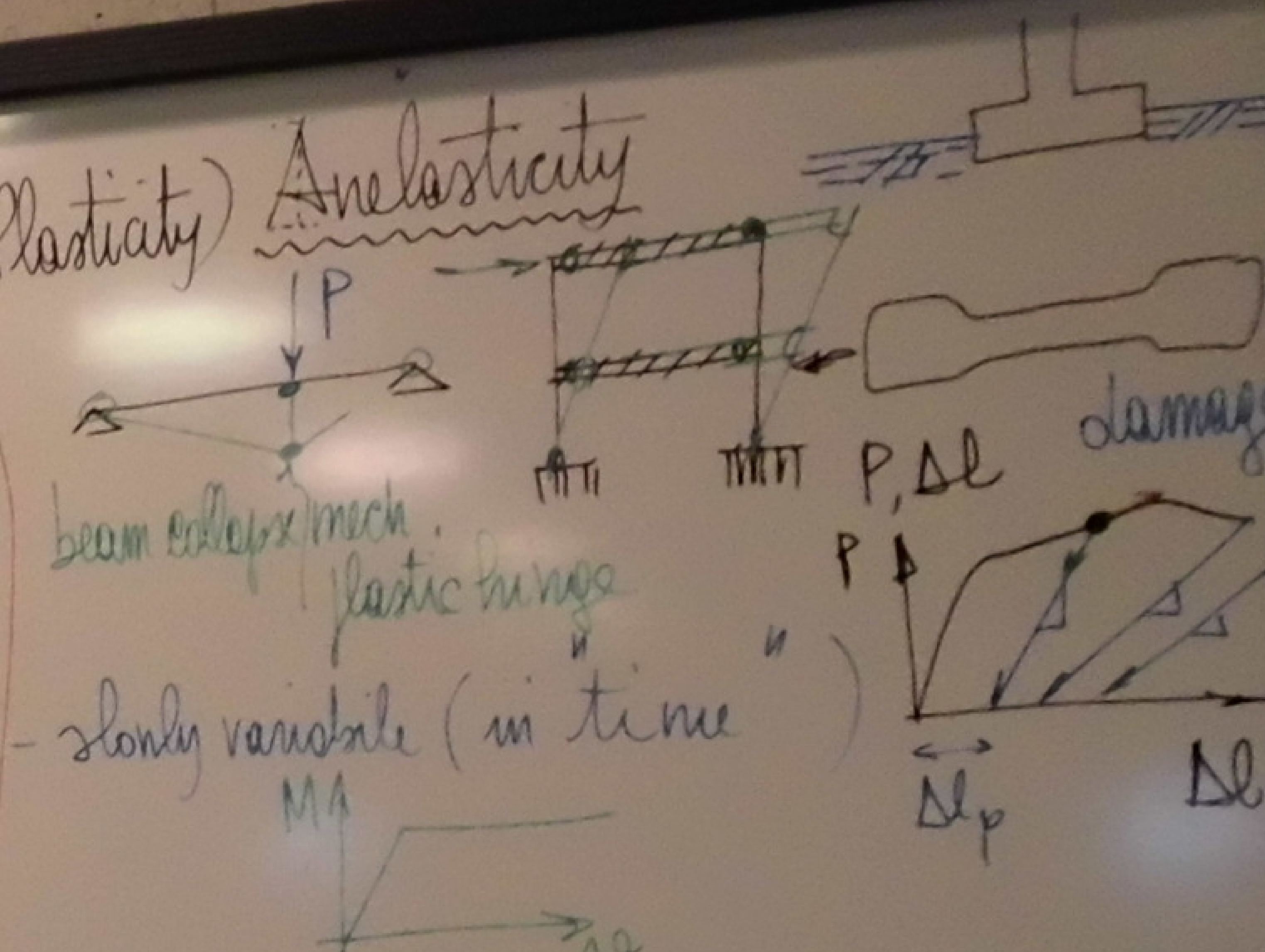


buckling; snap-through; torsional inst.

- slowly variable in time, e.g. linear increasing; sudden; critical
- equilibrium in the deformed configuration (interact. between forces & displ.)
  P-Δ effect, geometric non-linearity, 2nd-order
- slender elements
- Discrete systems — Continuous



(Plasticity) Anelasticity



- slowly variable ("in time")
- non-linear material (structural) behaviour
- dissipation of energy and irreversible processes
- Plasticity at the continuum scale

$\dot{\varepsilon}_p = \dot{\lambda}\, m$ – limit analysis of frames

Example: time-invariant
P > 0
P
varied configuration
B
reference configuration
ϑ = 0
μ = m/l
l
plastic branch
M = K ϑ
linear branch
M
K
A
M(ϑ)
concentrated deformation
element (spring)
Spring behaviour
angular position
velocity
acceleration
F = m a ⇒ F − m a = 0
Equation of motion (dynamic equilibrium, through D'Alembert principle)
Eq of motion
statics → dynamics
equil
Inertia forces
eq of motion
equilibrium
P l sin ϑ = K ϑ +
destabilizing effect
(overturning moment)
stabilizing effects
inertia effects → eq of motion
moments
Initial conditions
small perturbation
linearization
small rotations
moment of inertia of the bar wrt A
wrt ↔ with respect to

Example
F = m a ⇒ F − m a = 0
Equation of motion
Eq. of motion
angular
position
velocity
acceleration
equilibrium
Inertia
forces
Initial conditions
small
perturbation

$F = ma \Rightarrow F \underbrace{- ma}_{F_i} = 0$

Equation of motion (dynamic equilibrium, through D'Alembert principle)

angular position ϑ

velocity $\dot\vartheta$

acceleration $\ddot\vartheta$

static equil — equilibrium $Pl\sin\vartheta = K\vartheta$

dynamics — Inertia forces $(-m a_i)$ — eq. of motion

$(-dm)\,\ddot\vartheta_x \cdot x$

→ inertia effects → eq. of motion

stabilizing effects

$I_A = \int_0^l \mu x^2\, dx > 0$

moment of inertia of the bar w.r.t. A $= \mu \int_0^l x^2\, dx = \frac{1}{3}\mu l^3$

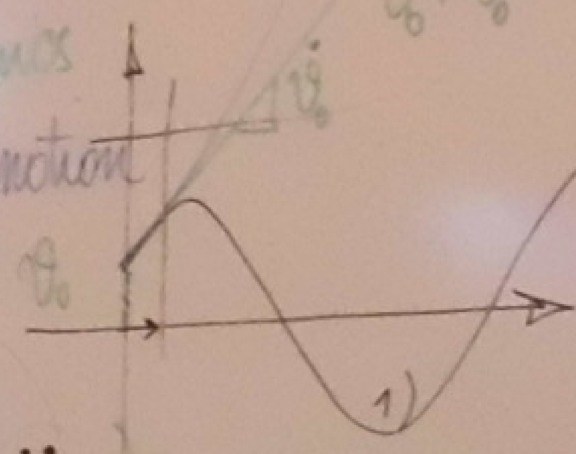
w.r.t. → with respect to

Eq of motion:

$Pl\sin\vartheta = K\vartheta + I\ddot\vartheta$

Initial conditions $(t = t_0 = 0)$

$\vartheta(t) = \vartheta_0 + \dot\vartheta_0 t$



$$\begin{cases} \vartheta(0) = \vartheta_0 \\ \dot\vartheta(0) = \dot\vartheta_0 \end{cases}$$ small perturbation

linearization

$\sin\vartheta \simeq \vartheta + \dots$

small rotations

$|\vartheta| \ll 1$; $I\ddot\vartheta + \underbrace{(K - Pl)}_{\text{total stiffness}}\vartheta = 0$

elastic stiffness — geometrical stiffness

1) $P < \frac{K}{l}$, $K - Pl > 0$, $\omega_1^2 = \frac{K - Pl}{I}$

subcritical condition $\ddot\vartheta + \omega_1^2 \vartheta = 0$ harmonic motion

$\rightarrow \vartheta(t) = \vartheta_0 \cos\omega_1 t + \frac{\dot\vartheta_0}{\omega_1}\sin\omega_1 t$

Liapunov sense → STABLE

Critical load $P = \frac{K}{l} = P_{cr}$

2) $P = \frac{K}{l}$, $K - Pl = 0$

critical condition $\ddot\vartheta = 0 \Rightarrow \vartheta = \vartheta_0 + \dot\vartheta_0 t$

3) $P > \frac{K}{l}$, $K - Pl < 0$ UNSTABLE

supercritical condition $\ddot\vartheta - \alpha^2\vartheta = 0$, $\alpha^2 = \frac{Pl - K}{I}$

$\rightarrow \vartheta(t) = \frac{1}{2}\left(\vartheta_0 + \frac{\dot\vartheta_0}{\alpha}\right)e^{\alpha t} + \frac{1}{2}\left(\vartheta_0 - \frac{\dot\vartheta_0}{\alpha}\right)e^{-\alpha t}$