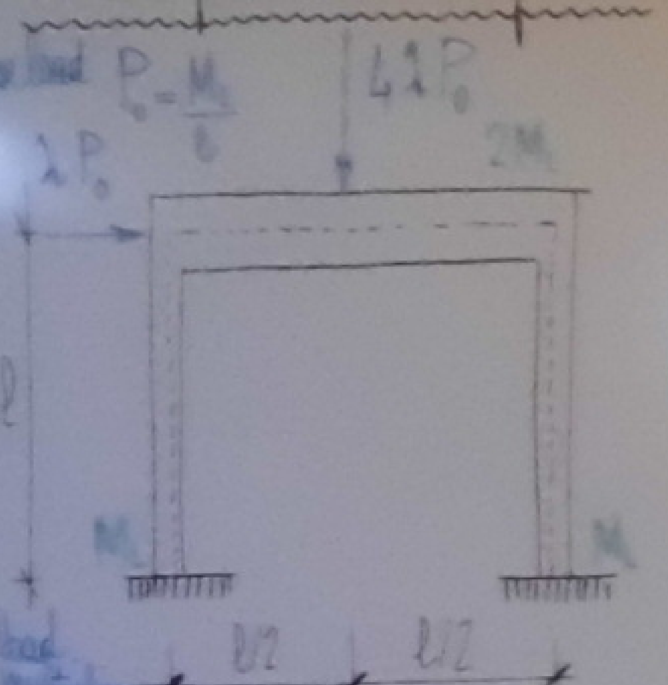
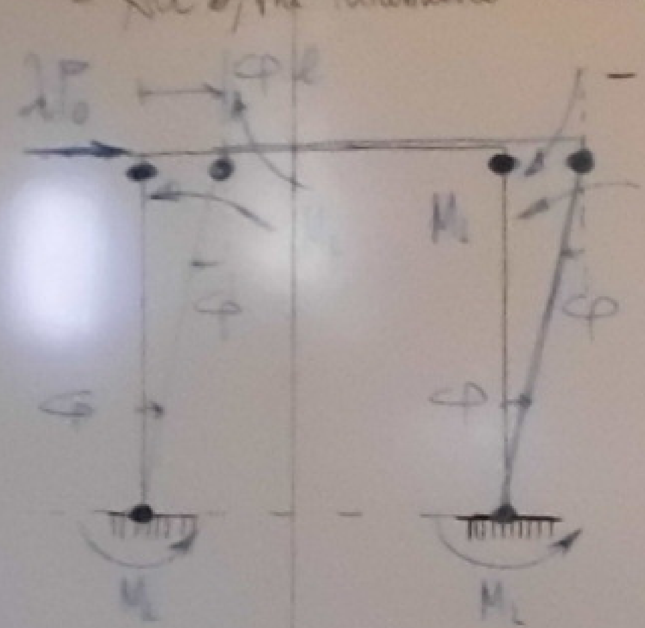
Example - Frame portal
base load
P0 = ML / l
4λP0
λP0
2ML
h=l
ML
ML
λ load multiplier
l/2
l/2
- Within the context of Limit Analysis of frames one would attempt:
- a complete evolutive analysis, in the plastic hinge hypothesis, tracing the opening sequence of all plastic hinges and the consequent piece-wise linear force-displacement response (here elastic properties do matter);
- an estimate, or exact evaluation, of the collapse load multiplier λL and associated collapse mechanism, by the theorems of L.A. (here, only plastic properties do matter)
- "Manual" methods of collapse analysis
- Act by the "Kinematic" theorem (upper bound): λL = min{λ+}
- "Wall" mechanism ("complete" mechanism)
Energy balance considering the admissible mech.
= 4
- Notice that, to attempt minimizing λ+, one seeks to minimize the ratio D/Le0 for different potential collapse mechanisms (minimize D and maximize Le0)
- One may check if this λ+ corresponds also to a static multiplier λ− (in such case one would have found λ− = λL = λ+ ("mixed" theorem)

Example - Frame portal



- Within the context of Limit Analysis of frames one would attempt:
  - a complete evolutive analysis, in the plastic hinge hypothesis, tracing the opening sequence of all plastic hinges, and the consequent piece-wise linear load-displacement response (here elastic properties do matter);
  - an estimate, or exact evaluation, of the collapse load multiplier λ_L and associated collapse mechanism, in the framework of LA (here, only plastic properties do matter)

- "Manual" methods of collapse analysis
- Act by the "Kinematic" Theorem (Upper bound) $\lambda_L = \min\{\lambda^+\}$



- "Wall" mechanism ("complete" mechanism)

Energy balance considering the admissible mech.

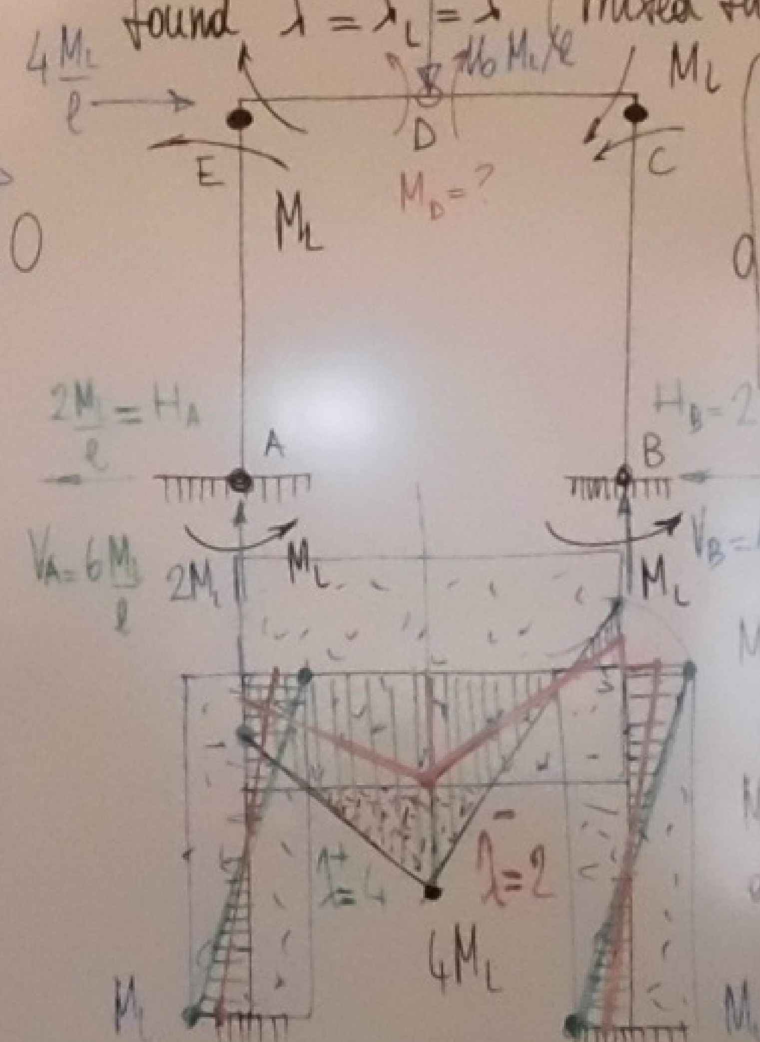
$$\mathcal{L}_e = \lambda \frac{M_L}{\ell}\varphi\ell - M_L(\varphi+\varphi+\varphi+\varphi) = 0 \quad \forall\varphi$$

$$\mathcal{L}_e = \lambda \underbrace{\frac{M_L}{\ell}\varphi\ell}_{\mathcal{L}_{e0} = P_0(\varphi\ell)} = 4M_L\varphi = \int \ldots = D$$

$$\lambda_L \leq \; \rightarrow \lambda = \lambda^+ = \frac{D_{>0}}{\mathcal{L}_{e0>0}} = 4$$

- Notice that, to attempt minimizing λ^+, one seeks to minimize the ratio $D/\mathcal{L}_{e0}$ for different potential collapse mechanisms (minimize D and maximize $\mathcal{L}_{e0}$)

- One may check if this λ^+ corresponds also to a static multiplier λ^- (in such case one would have found $\lambda^- = \lambda_L = \lambda^+$ ("mixed" theorem)

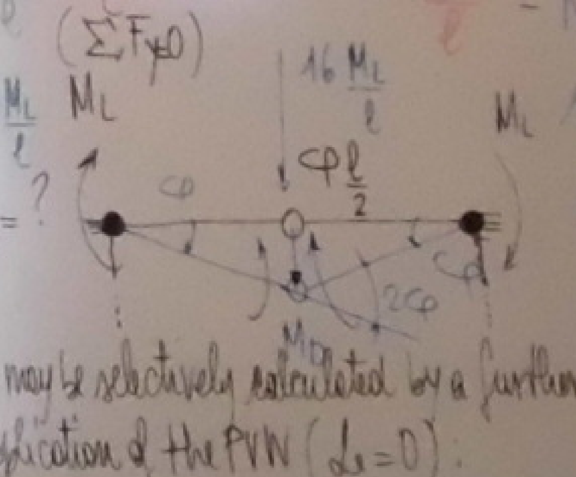


$$\sum M_C^{BC} = 0 \Rightarrow H_B = 2\frac{M_L}{\ell}$$

$$\sum F_x = 0 \Rightarrow H_A = 2\frac{M_L}{\ell}$$

$$\sum M_E^{BCDE} = 0 \Rightarrow V_B = 10\frac{M_L}{\ell}$$

$$\sum M_C^{AEDC} = 0 \Rightarrow V_A = 6\frac{M_L}{\ell} \quad (\sum F_y = 0)$$



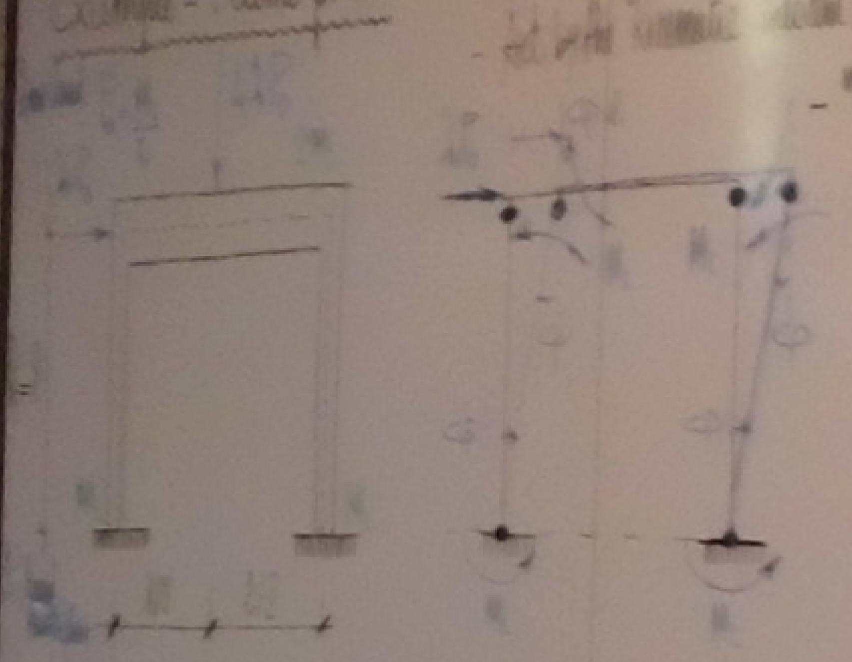
M_D may be selectively calculated by a further application of the PVW ($\mathcal{L}_i = 0$):

$$\mathcal{L}_i = 16\frac{M_L}{\ell}\varphi\frac{\ell}{2} + M_L\varphi - M_L\varphi - M_D 2\varphi = 0$$

$\rightarrow M_D = 4M_L$ "violation" of conformity: $\rho_m = 2$)

- Please notice that the distribution of internal actions at this stage is ruled just by equilibrium (for a complete potential collapse mechanism). Namely, the missing value M_D, is just fixed by equilibrium at $\lambda = \lambda^+ = 4$ for the assumed mechanism (fixing the values of $M = M_L$ in the plastic hinges at the extremes of the columns)
- Notice that, although conformity in the plastic hinges is automatically imposed, in general the resulting $M(x)$ is not conformed unless if we have truly found $\lambda^- = \lambda_L = \lambda^+$ the collapse multiplier [here $M_D = 2(2M_L) = \rho_m(2M_L)$ shows conformity violation, then λ^+ is not a λ^-; namely we have not found λ_L yet]
  - A way to reset conformity is to deflate $M(x)$ down to $M(x)/\rho_m$, to get estimate of $\lambda^- = \lambda^+/\rho_m = 4/2 = 2$ and provide bilateral delimitation $2 = \lambda^- \leq \lambda_L \leq \lambda^+ = 4$

Example - Frame portal



- "Wall" mechanism ("complete" mechanism)

Energy balance considering the admissible mech.

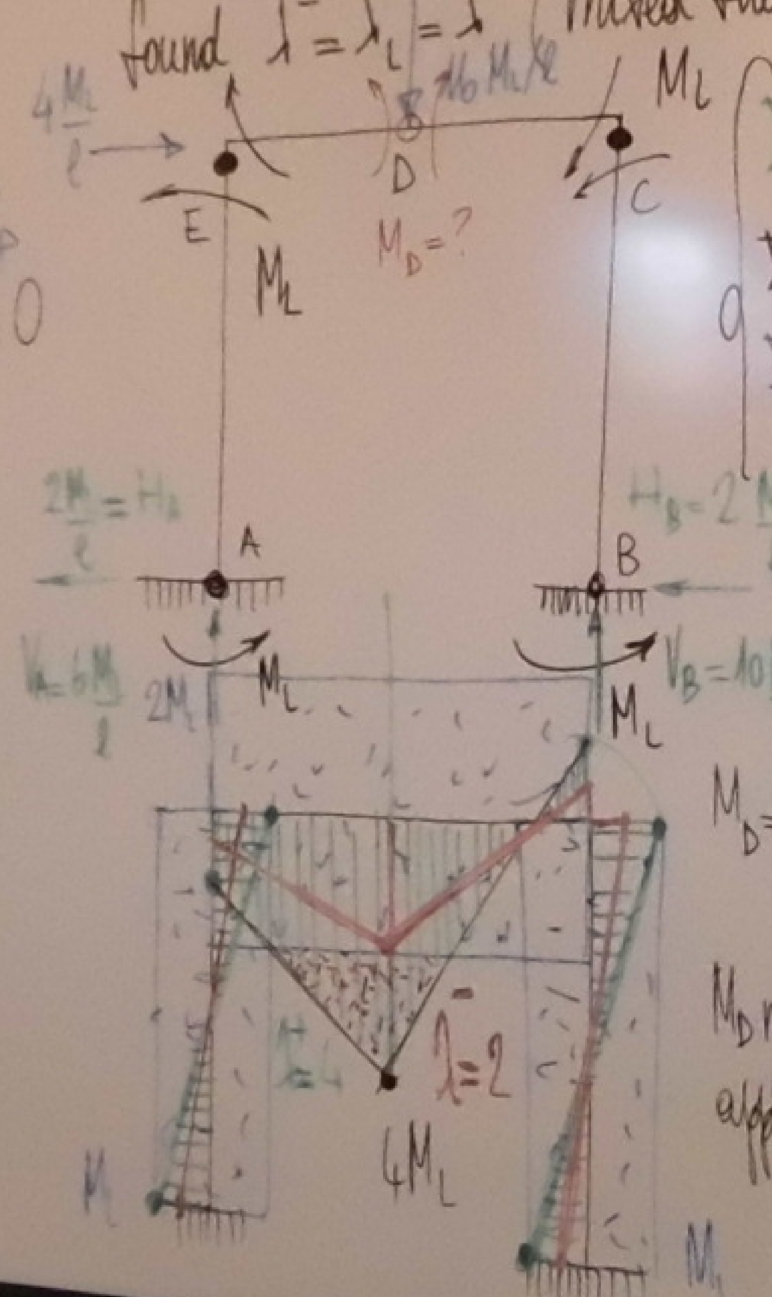
$$\mathcal{L}_e = \lambda \frac{M_L}{\ell}\varphi\ell - M_L(\varphi+\varphi+\varphi+\varphi) = 0$$

$$\mathcal{L}_e = \lambda \frac{M_L}{\ell}\varphi\ell = 4M_L\varphi = D$$

$$\lambda_L \leq \lambda = \lambda^{+} = \frac{D > 0}{\mathcal{L}_{e0} > 0} = 4$$

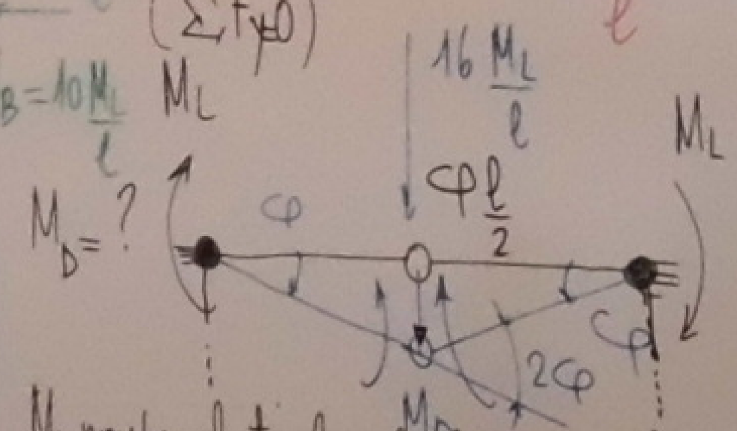
- Notice that, to attempt minimizing λ^+, one seeks to minimize the ratio $D/\mathcal{L}_{e0}$ for different potential collapse mechanisms (minimize D and maximize $\mathcal{L}_{e0}$)

- One may check if this λ^+ corresponds also to a static multiplier λ^- (in such case one would have found $\lambda^- = \lambda_L = \lambda^+$ ("mixed" theorem)



$$\left\{\begin{array}{l} \sum M_C^{BC} = 0 \Rightarrow H_B = 2\frac{M_L}{\ell} \\ \sum F_x = 0 \Rightarrow H_A = 2\frac{M_L}{\ell} \\ \sum M_E^{BCDE} = 0 \Rightarrow V_B = 10\frac{M_L}{\ell} \\ \sum M_C^{AEDC} = 0 \Rightarrow V_A = 6\frac{M_L}{\ell} \end{array}\right.$$

$(\sum F_y = 0)$



M_D may be selectively calculated by a further application of the PVW ($\mathcal{L}_e = 0$):

$$\mathcal{L}_e = \overset{8}{16}\frac{M_L}{\ell}\varphi\frac{\ell}{\cancel{2}} + \cancel{M_L\varphi} - \cancel{M_L\varphi} - M_D 2\varphi = 0$$

$\hookrightarrow M_D = 4M_L$ "violation" of conformity: $\rho_m = 2$)

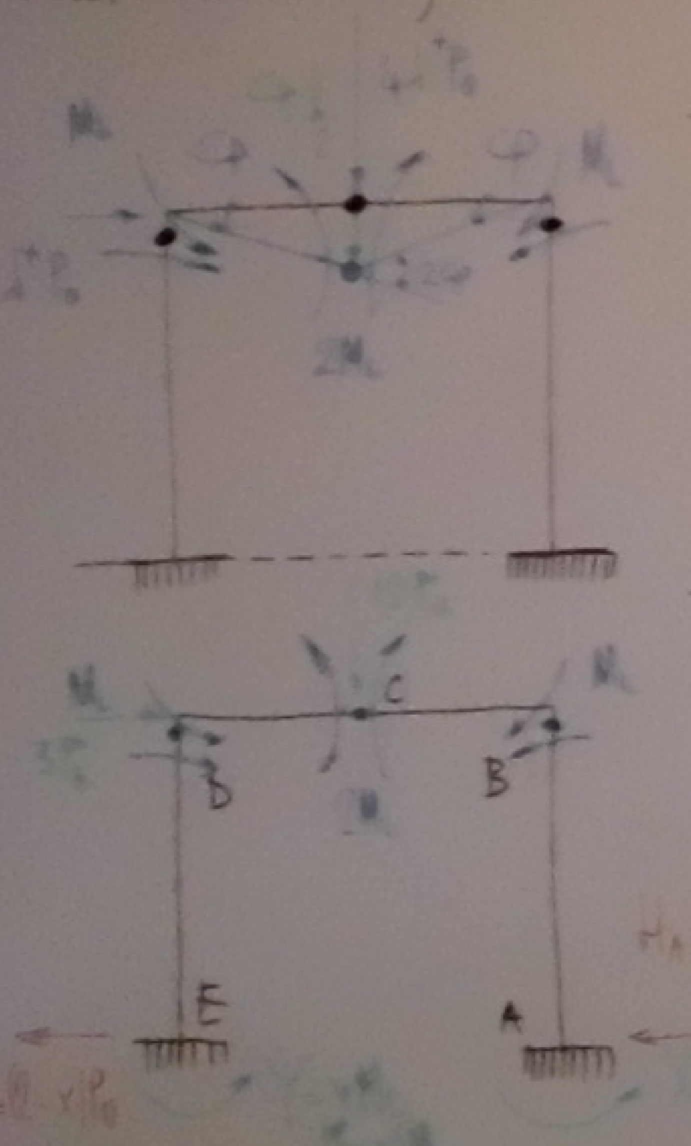
- Please notice that the distribution of internal actions at this stage is ruled just by equilibrium (for a complete potential collapse mechanism). Namely, the missing value M_D, is just fixed by equilibrium at $\lambda = \lambda^+ = 4$ for the assumed mechanism (fixing the values of $M = M_L$ in the plastic hinges at the extremes of the columns).

- Notice that, although conformity in the plastic hinges is automatically imposed, in general the resulting $M(x)$ is not conformal unless if we have truly found $\lambda^- = \lambda_L = \lambda^+$ the collapse multiplier [here $M_D = 2(2M_L) = \rho_m(2M_L)$ shows conformity violation, then λ^+ is not a λ^-; namely we have not found λ_L yet]

- A way to reset conformity is to deflate $M(x)$ down to $M(x)/\rho_m$, to get estimate of $\lambda^- = \lambda^+/\rho_m = 4/2 = 2$ and provide bilateral delimitation $\boxed{2 = \lambda^- \leq \lambda_L \leq \lambda^+ = 4}$


- Further estimates by the K.T.:
- Partial collapse mechanism
(beam mechanism)
= M_L φ + M_L φ + 2M_L 2φ = D
λ = λ+ = 3
- In this case, going to the static th., we have
one residual hyperstatic unknown (say X)
H_A = (1+x) P_0
H_B = (2-x) P_0
= 3/2 = 1.5
1.5 ≤ λ_L ≤ 3
- Complete collapse mechanism
(beam-wall mechanism)
λ = λ+ = 8/3 = 2.6 = 2.667
- Remarks

- Further estimates by the K.T.



- Partial collapse mechanism (beam mechanism)

$\ldots = M_L\varphi + M_L\varphi + 2M_L\, 2\varphi = D$

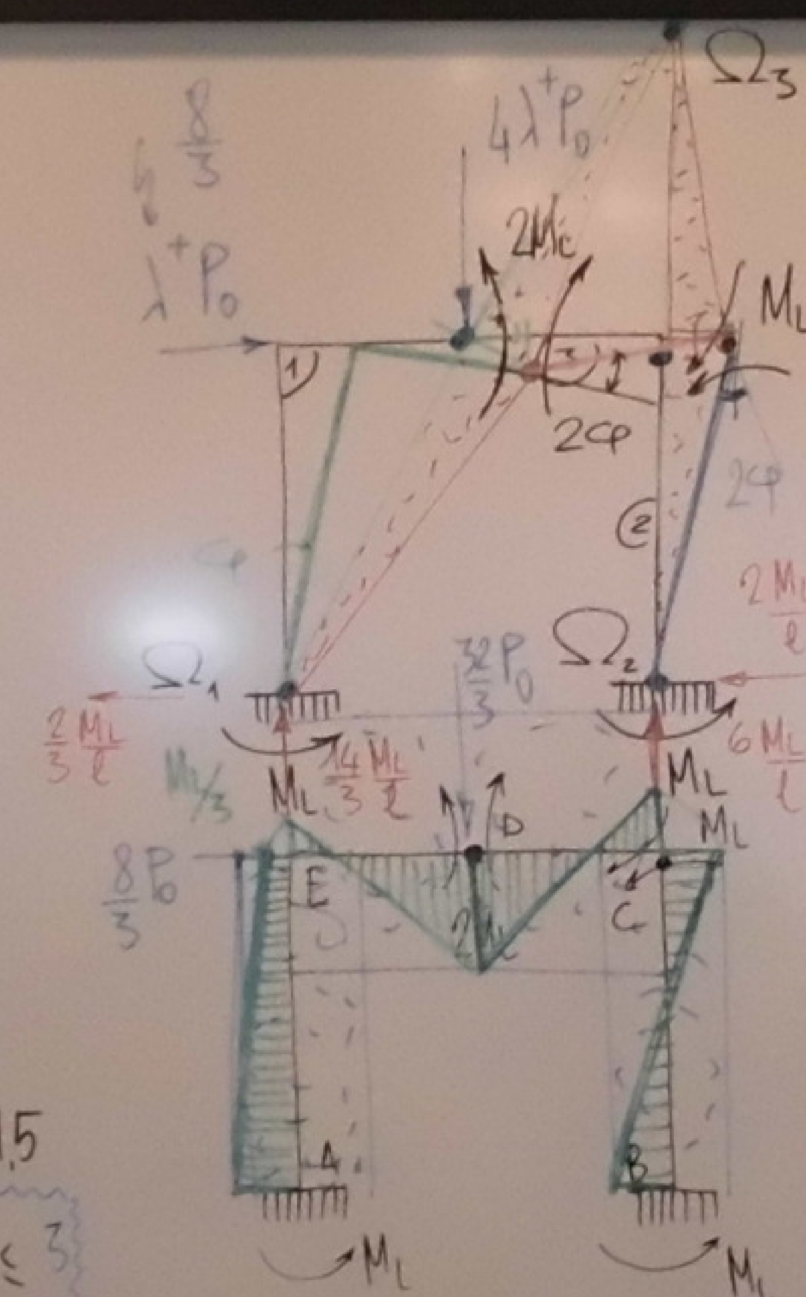
$\Rightarrow \lambda = \lambda^+ = 3$

- In this case, going to the static th., we have one residual hyperstatic unknown (say X)

$x = 2,\ y = 1$

$\Rightarrow \lambda^- = \lambda^+ / \rho_m = 3/2 = 1.5$

$\boxed{1.5 \leq \lambda_L \leq 3}$

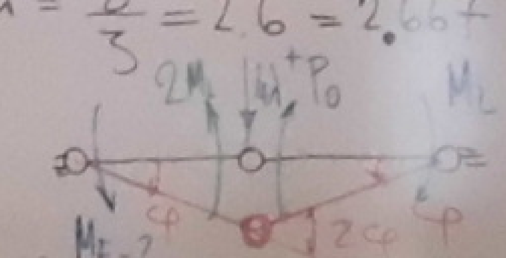


- Complete collapse mechanism ("beam-wall" mechanism)

$\mathcal{L}_e = \lambda P_0\, \varphi l + 4\lambda P_0\, \varphi \frac{l}{2} = M_L\varphi + M_L\varphi + M_L\, 2\varphi + 2M_L\, 2\varphi = D$

$\lambda = \lambda^+ = \frac{8}{3} = 2.\bar{6} = 2.667$

$M_E = ?$



$\lambda^+ = \frac{8}{3};\ 4\lambda^+ P_0 \varphi \frac{l}{2} - M_L\varphi - 2M_L\, 2\varphi - M_E\varphi = 0$

$\Rightarrow M_E = \frac{1}{3} M_L$

- Remarks:

- In general, by a few attempts of application of K.T. and/or S.T. one manages to find as strict as needed bilateral delimitations of the collapse load multiplier.
- If a $\lambda^- = \lambda^+$ is found, namely a ... load multiplier that is both statically and kinematically admissible, the collapse load multiplier is found (by the Mixed Theorem).
- These "manual" methods may be generalized, in the framework of Mathematical Programming, apt to solve either $\lambda_L = \max\{\lambda^-\}$ and/or $\lambda_L = \min\{\lambda^+\}$ as max/min problem in Operation Research (linear programming → Simplex Method)

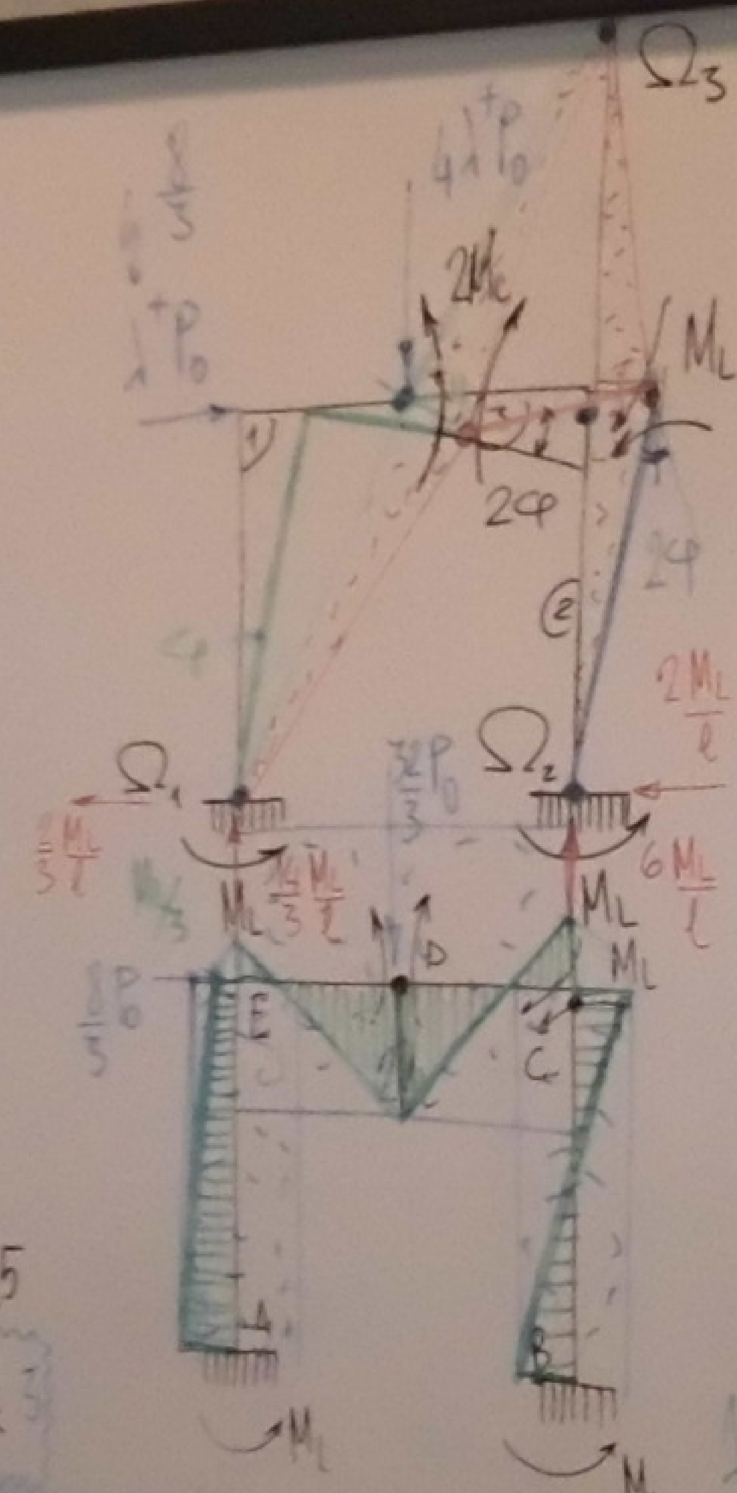
- Partial collapse mechanism
(beam mechanism)

$\lambda = \lambda^+ = 3$

- In this case, going to the static Th., we have one residual hyperstatic unknown (say X)

$\Rightarrow \lambda^- = \lambda^+ / P_0$

$= 3M_L = 1.5$

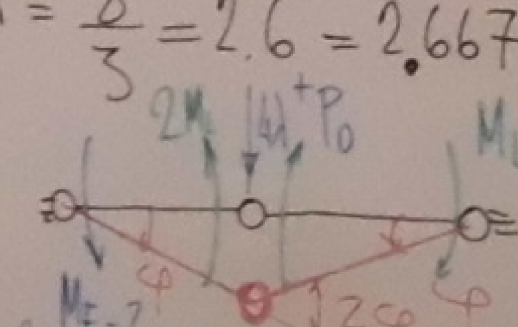


- Complete collapse mechanism
("beam-wall" mechanism)

$$= \underbrace{M_L\varphi + M_L\varphi + M_L 2\varphi + 2M_L 2\varphi}_{8M_L\varphi} = D$$

$$\lambda = \lambda^+ = \frac{8}{3} = 2.\bar{6} = 2.667$$

$M_E = ?$



$$\lambda^+ = \frac{8}{3};\quad 4\lambda^+ P_0 \varphi \frac{l}{2} - M_L\varphi - 2M_L 2\varphi - M_E\varphi = 0$$

$$\Rightarrow M_E = \frac{1}{3} M_L \quad \text{conformal!}$$

$$\Rightarrow \lambda = \lambda^- = \lambda^+ = \frac{8}{3}$$

collapse load multiplier

- Remarks:

- In general, by a few attempts of application of K.T. and/or S.T. one manages to find as strict as needed bilateral delimitations of the collapse load multiplier.
- If a $\lambda^- = \lambda^+$ is found, namely a plastically-admissible load multiplier that is both statically- and kinematically-admissible, the collapse load multiplier is found (by the Mixed Theorem).
- These "manual" methods may be generalized, in the framework of Mathematical Programming, apt to solve either $\lambda_L = \max\{\lambda^-\}$ and/or $\lambda_L = \min\{\lambda^+\}$ as a max/min problem in Operation Research (Linear programming $\Rightarrow$ Simplex Method)