

Università degli studi di Bergamo

Scuola di Ingegneria (Dolmine)

CCS Ingegneria Edile

LM-24 Ingegneria delle Costruzioni Edili

Complementi di Scienza delle Costruzioni

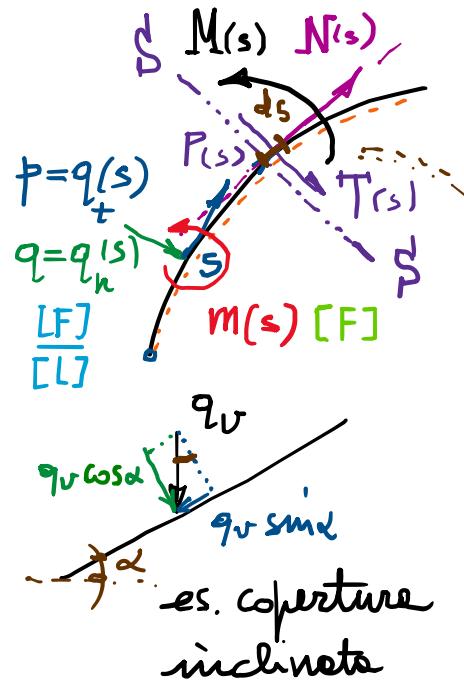
(ICAR/08 - SdC; 6 CFU)

prof. Egidio RIZZI

egidio.rizzi@unibg.it

LEZIONE 11

Azioni interne in alte curve - Equazioni indefinite di equilibrio delle alte curve (\dot{A}_S , d_S)



A.I. $\left\{ \begin{array}{l} N(s): \text{azione oriale o normale} \\ T(s): \rightarrow \text{tagliente o tangente} \\ M(s): " \text{ flettente o momento} \end{array} \right.$

$$ds = r(s) d\theta$$

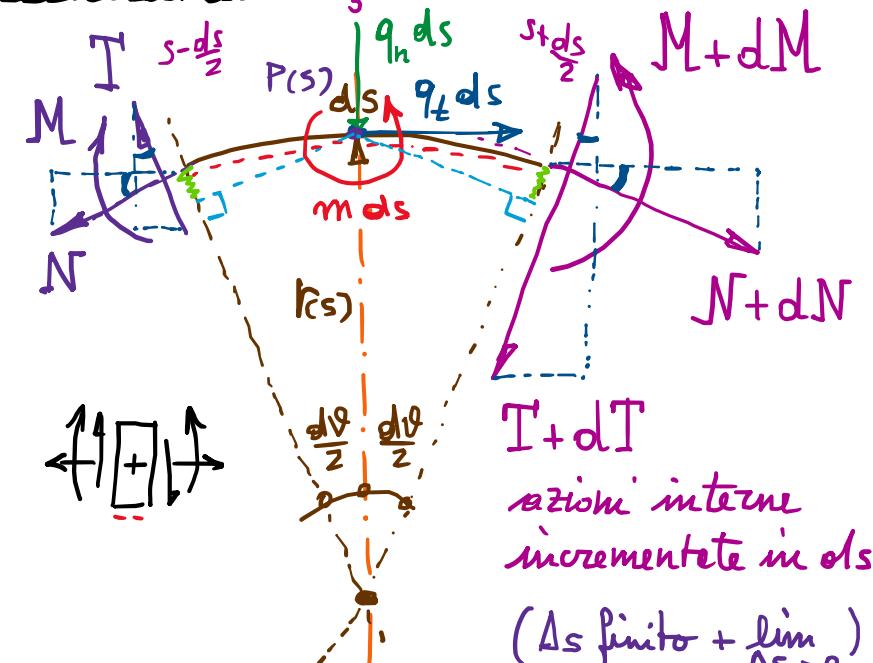
$$dV = \frac{1}{r(s)} ds$$

$$= \chi(s) ds$$

raggio di curvatura locale
 $r \approx l$ ($r \rightarrow \infty$, trave rettilinea)

r piccolo

elemento
"tozzo"
(non trova)



Equazioni di equilibrio : scacchiamento
↓ N, T

$$\sum F_t^{\text{ds}} = 0 \Rightarrow (\cancel{N} + dN - \cancel{N}) \cos \frac{d\theta}{2} - (T + dT + T) \sin \frac{d\theta}{2} + q$$

$$\sum F_n^{\text{ds}} = 0 \Rightarrow (\cancel{T} + dT - \cancel{T}) \cos \frac{\alpha \theta}{2} + (N + dN - N) \sin$$

$$\sum M_p^{\text{ds}} = 0 \Rightarrow (\cancel{N} + dN - \cancel{N}) - (T + dT + T) r \sin \frac{d\theta}{2} + \\ - (\cancel{N} + dN - \cancel{N}) r \left(1 - \cos \frac{d\theta}{2}\right) + m ds = 0$$

$$\Rightarrow \frac{dN}{ds} - \cancel{\frac{1}{2} T \frac{d\vartheta}{s}} - \cancel{dT \frac{\partial \vartheta}{2}} + q_t ds = 0 \Rightarrow N(s) = -q(s) + \frac{T(s)}{r(s)}$$

$$\Rightarrow \frac{dT}{ds} + \cancel{\frac{1}{2} N \frac{\partial \vartheta}{s}} + \cancel{dN \frac{\partial \vartheta}{2}} + q_h ds = 0 \Rightarrow T(s) = -q(s) - \frac{N(s)}{r(s)}$$

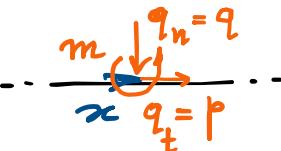
$$\Rightarrow \frac{dM}{ds} - \cancel{\frac{1}{2} Tr \frac{d\vartheta}{s}} - \cancel{dT r \frac{\partial \vartheta}{2}} + m ds = 0 \Rightarrow M(s) = -m(s) + T(s)$$

$$M(s) = -m(s) + T(s) = -\left(m(s) + q_h(s)\right) - \frac{N(s)}{r(s)} \quad \text{accoppiamento } T, M$$

Note:

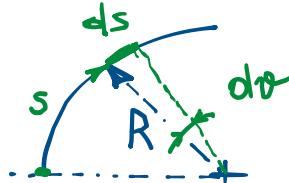
- nelle oste curve si registra accoppiamento N, T , in cui la variazione di ciascuna azione interna risulta accoppiata all'andamento dell'altra.
- si conferma l'accoppiamento T, M , già visto per oste rettilinee, ottenibili come segue -

- per oste rettilinee ($r \rightarrow \infty$):



$$\begin{cases} N'(x) = -p(x) \\ T'(x) = -q(x) \\ M''(x) = -m(x) + T(x) \end{cases} \Rightarrow M''(x) = -m'(x) + T'(x) = -(m'(x) + q(x))$$

- per oste circolari ($r(s) = R = \text{cost}$)



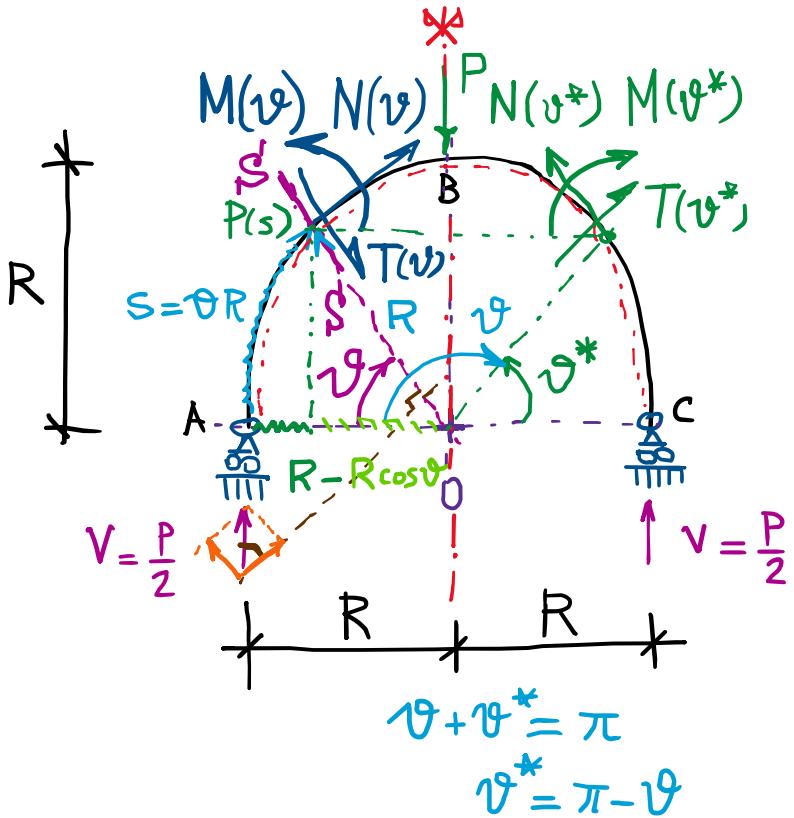
$$\frac{ds}{d\vartheta} = R \quad \Rightarrow \quad \frac{d}{ds} = \frac{1}{R} \frac{d}{d\vartheta} \quad \Rightarrow \quad \begin{cases} N'(v) = -q_n(v)R + T(v) \\ T'(v) = -q_h(v)R - N(v) \\ M''(v) = -m(v)R + T(v)R \end{cases} \quad (*)$$

- per tratto "scorico" ($q_t = q_n = m = 0$):

$$\begin{cases} N'(s) = \frac{T(s)}{r(s)} \\ T'(s) = -\frac{N(s)}{r(s)} \\ M''(s) = T(s) \end{cases} \Rightarrow M''(s) = T'(s) = -\frac{N(s)}{r(s)}$$

$$\begin{aligned} M''(v) &= -m'(v)R + T'(v)R \\ &= -m'(v)R - q_h(v)R^2 - N(v)R \\ &= -(m'(v) + q_h(v)R)R - N(v)R \end{aligned}$$

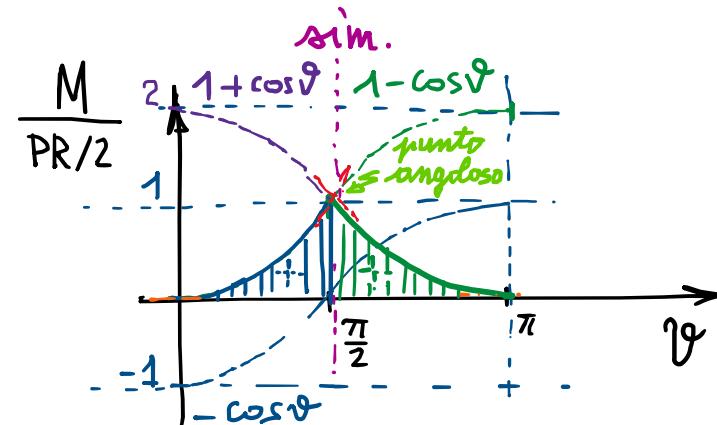
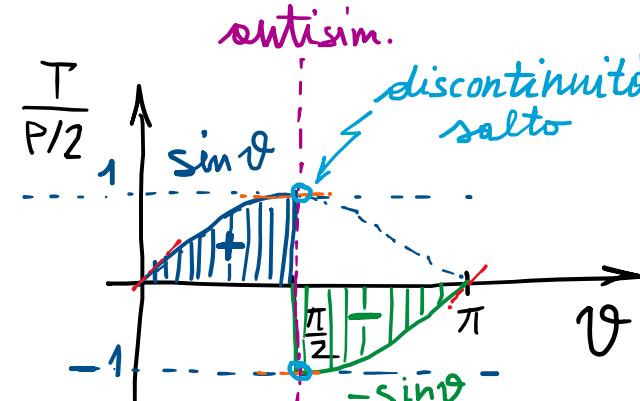
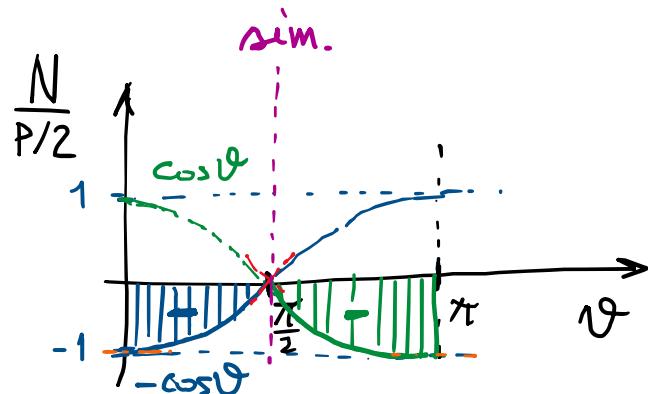
Arco semi circolare (con carico concentrato in chiuso): soluzione rappresentazione analitica (in coordinate polari)



$$\left\{ \begin{array}{l} 0 \leq \vartheta < \frac{\pi}{2} \\ \frac{\pi}{2} < \vartheta \leq \pi \end{array} \right. \quad \left\{ \begin{array}{l} N(\vartheta) = -\frac{P}{2} \cos \vartheta \\ T(\vartheta) = \frac{P}{2} \sin \vartheta \\ M(\vartheta) = \frac{P}{2} R (1 - \cos \vartheta) \end{array} \right.$$

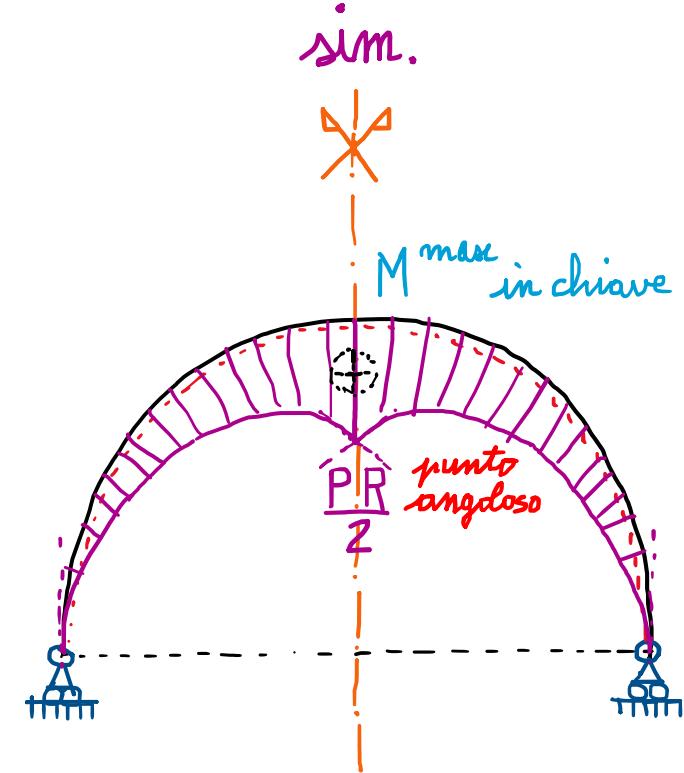
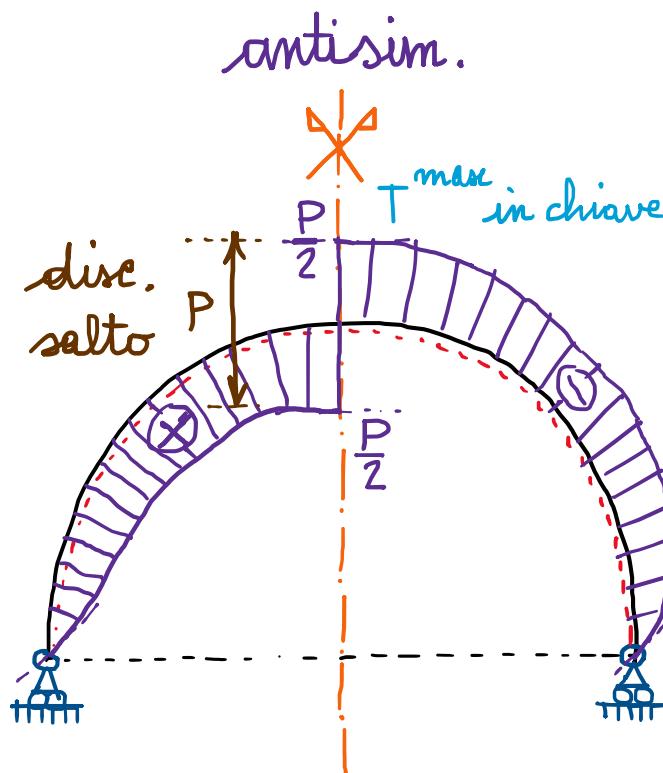
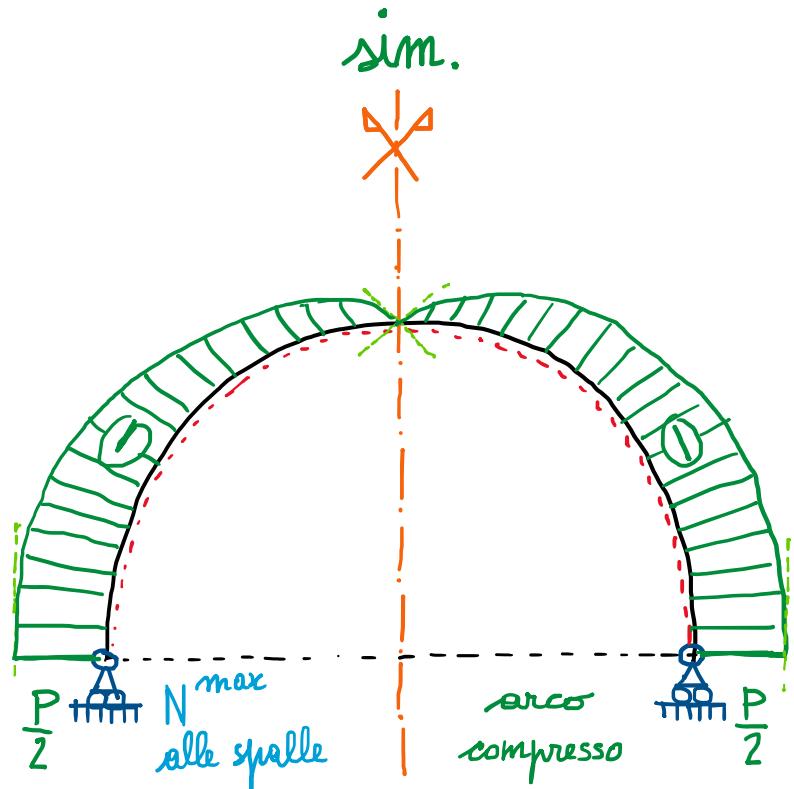
equilibrio locale
delle porzioni \widehat{AP}

$$\left\{ \begin{array}{l} 0 \leq \vartheta^* < \frac{\pi}{2} \\ \frac{\pi}{2} < \vartheta^* \leq \pi \end{array} \right. \quad \left\{ \begin{array}{l} N(\vartheta^*) = -\frac{P}{2} \cos \vartheta^* \\ T(\vartheta^*) = -\frac{P}{2} \sin \vartheta^* \\ M(\vartheta^*) = \frac{PR}{2} (1 - \cos \vartheta^*) = \frac{PR}{2} (1 + \cos \vartheta) \end{array} \right. \quad \begin{array}{l} = \frac{P}{2} \cos \vartheta \\ = -\frac{P}{2} \sin \vartheta \end{array}$$



dipendenza
analitica
delle funzioni
di Azione Interna
e loro
rappresentazione

Diagrammi delle Azioni Interni N , T , M (andamenti funzionali rappresentati su fondamentali coincidenti con la struttura stessa)

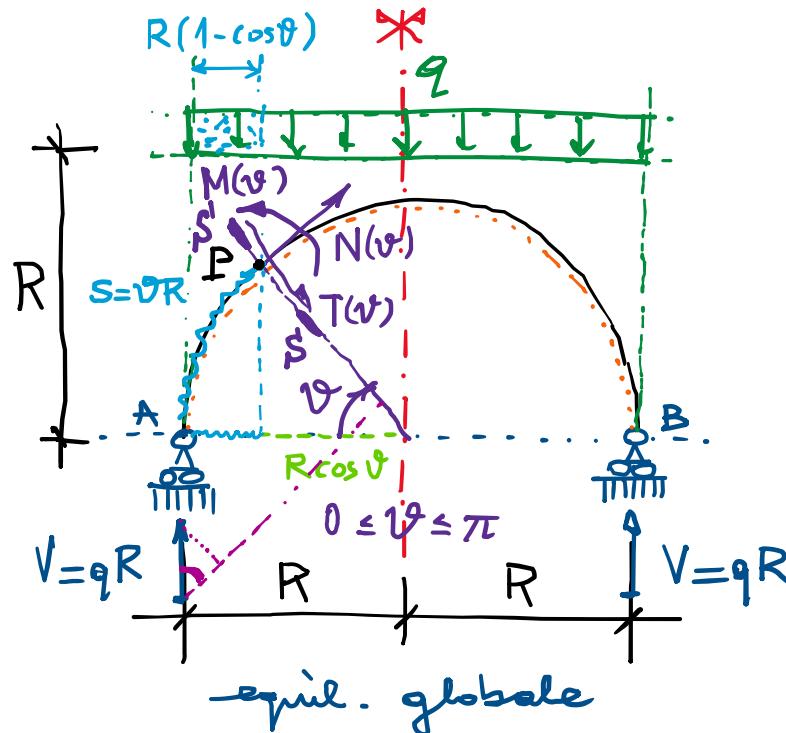


$$N(\vartheta)$$

$$T(\vartheta)$$

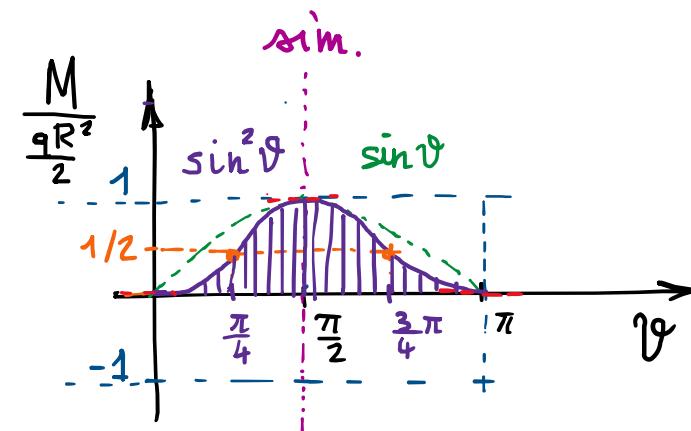
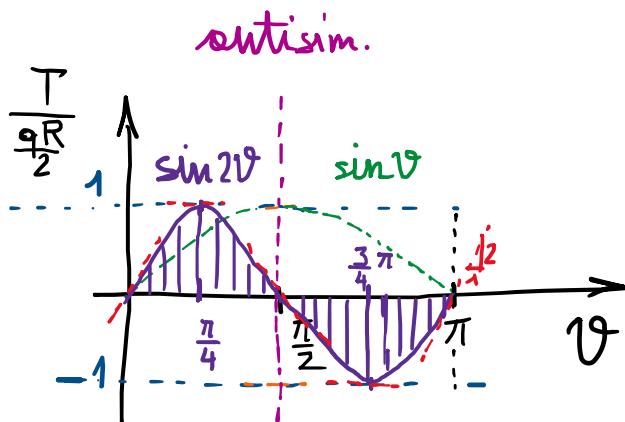
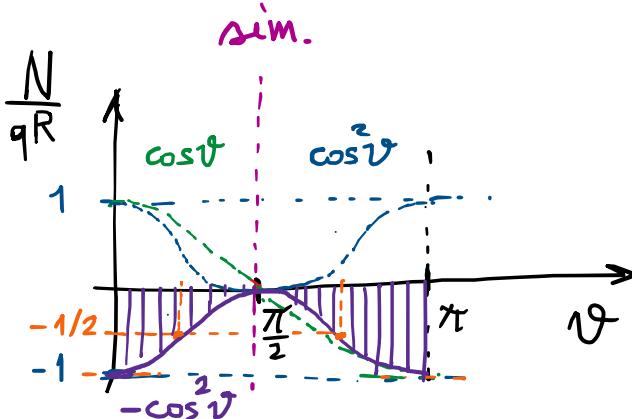
$$M(\vartheta)$$

Esempio: arco semicircolare con q_v distribuito (per unità di lunghezza in direz. orizzontale)



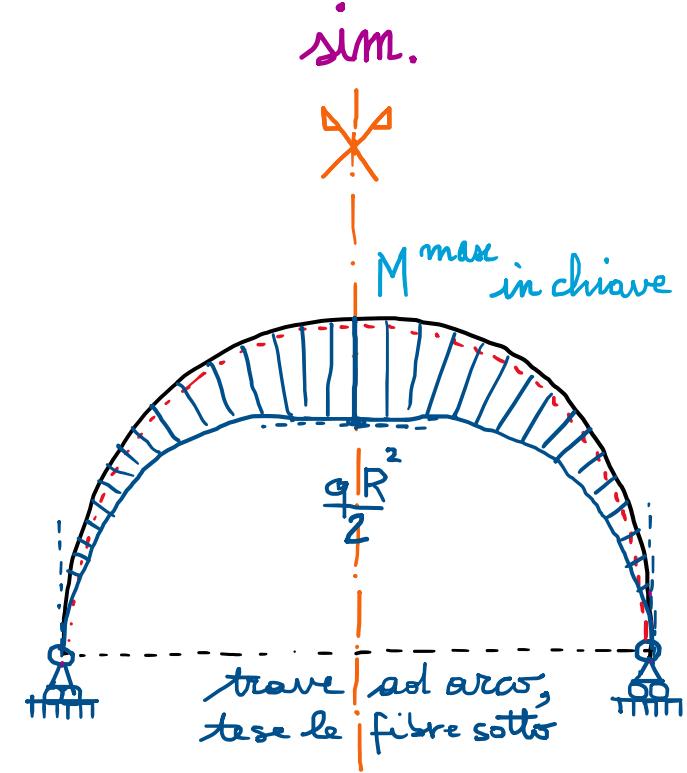
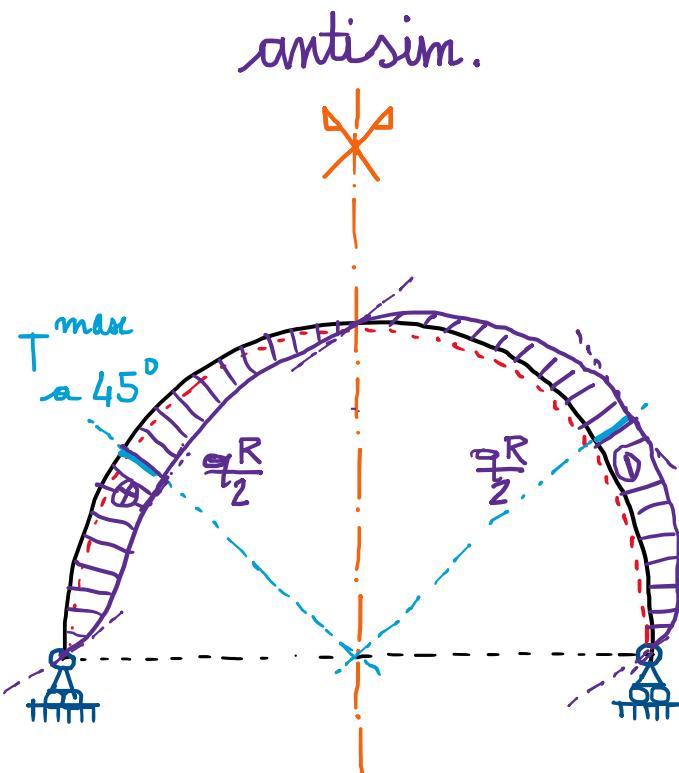
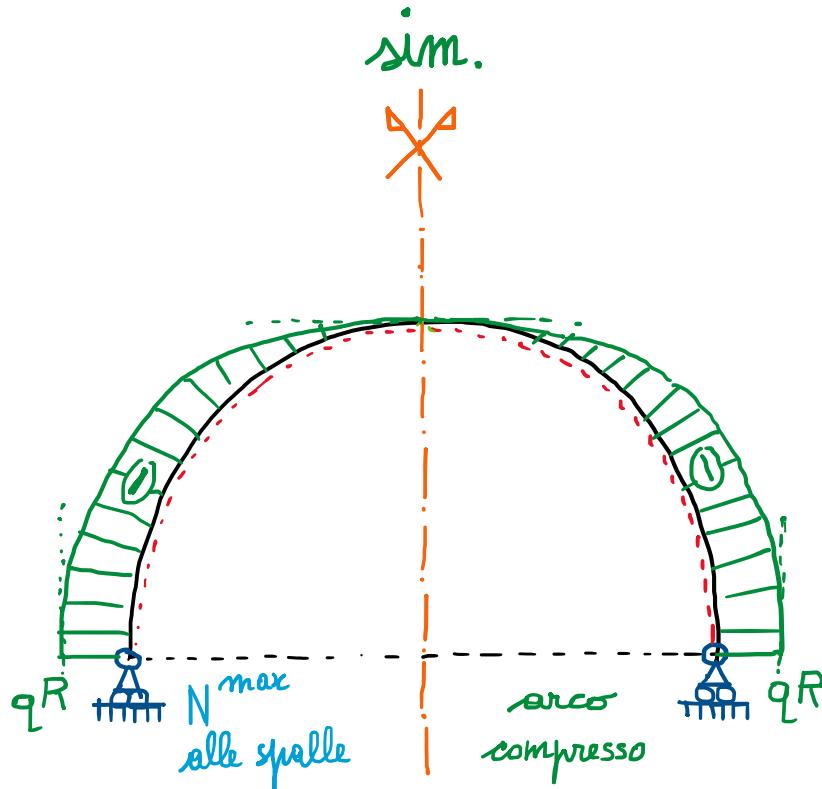
Equil. locale (AP):

$$\left\{ \begin{array}{l} N(v) = -qR \cos v + qR(1-\cos v) \cos v = -qR \cos^2 v = N \\ T(v) = qR \sin v - qR(1-\cos v) \sin v = \frac{qR}{2} \frac{2 \sin v \cos v}{\sin 2v} = \frac{qR \sin 2v}{2} = T \\ M(v) = qRR(1-\cos v) - qR(1-\cos v) \frac{R(1-\cos v)}{2} = \\ = qR^2(1-\cos v) \left(1 - \frac{1+\cos v}{2} \right) = \frac{qR^2}{2} (1-\cos v)(1+\cos v) \\ = \frac{qR^2}{2} (1-\cos^2 v) = \frac{qR^2}{2} \sin^2 v = M \end{array} \right.$$



dipendenza analitica delle funzioni di Azione Interna e loro rappresentazione

Diagrammi delle Azioni Interni N, T, M (andamenti funzionali rappresentati su fondamentali coincidenti con la struttura stessa)

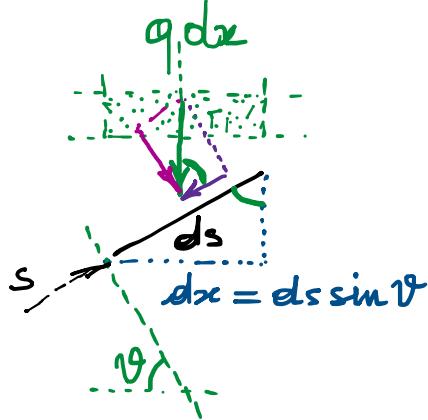


$$N(\theta)$$

$$T(\theta)$$

$$M(\theta)$$

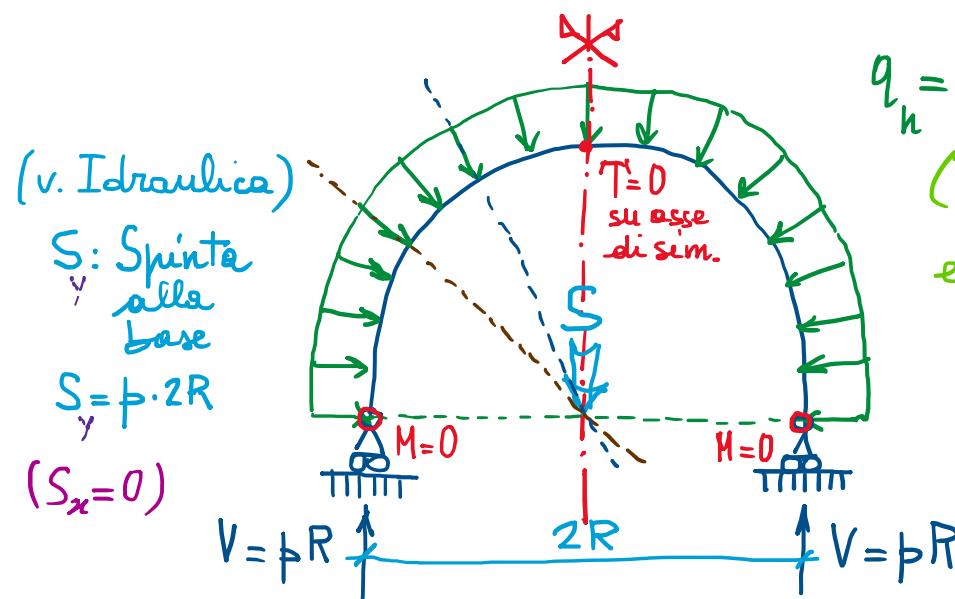
- Verificabili le relazioni differenziali viste (*), con q_t e q_n come segue:



$$q_t = -\frac{q dx \cos \theta}{ds} = -\frac{q \cancel{dx} \sin \theta \cos \theta}{\cancel{ds}} \frac{2}{2} = -\frac{q}{2} \sin 2\theta = q_t(\theta)$$

$$q_n = \frac{q dx \sin \theta}{ds} = \frac{q \cancel{dx} \sin^2 \theta}{\cancel{ds}} = q \sin^2 \theta = q_n(\theta)$$

- Arco semicircolare soggetto a pressione uniforme esterna:



$$q_n = p = \text{cost}$$

$$(q_t = 0 \text{ e } M = 0)$$

$$\left\{ \begin{array}{l} T = 0, \text{ per simmetria rispetto ad ogni} \\ \text{sezione radiale (prima ---,} \\ \text{poi ---, poi ---, ecc.)} \\ N = -pR = \text{cost} \quad (N' = 0) \\ \text{compressione} \\ M = 0 = \text{cost} \quad (M' = 0) \end{array} \right.$$

dalle (*):

$$\left\{ \begin{array}{l} N' = T \\ T' = -pR - N \\ M' = TR \end{array} \right.$$

come per tubo soggetto a pressione interna $\rightarrow N = pR$, traz.