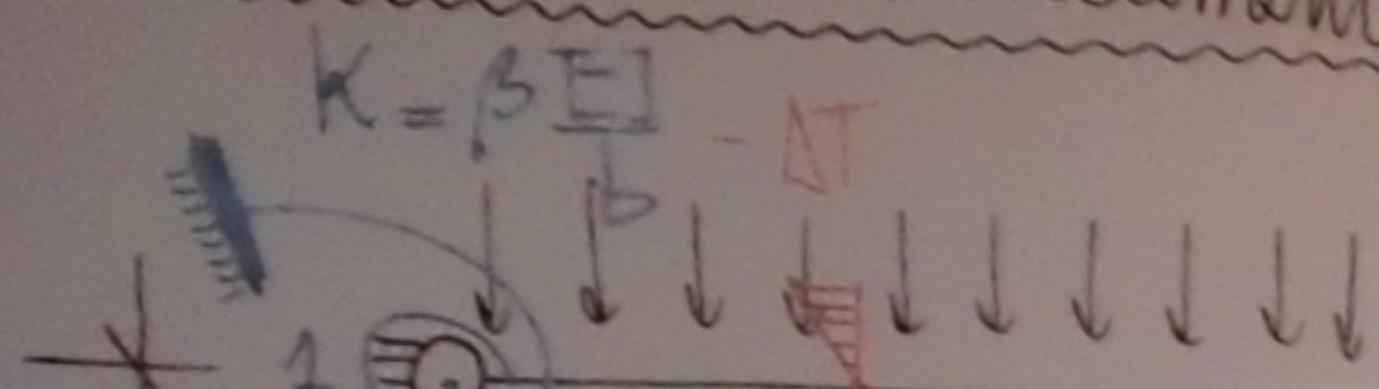


Strutture staticamente indeterminate



$$V = 4 \quad I = k + v - l = 1 \quad EA \rightarrow \infty \quad GA \rightarrow \infty$$

Vincolo rotazionale elastico

$$K \rightarrow 0 \quad \beta \rightarrow 0 \quad \text{no molla}$$

$$I = 0$$

$$K \text{ finita}$$

$$K \rightarrow \infty \quad I = 1$$

$$[EJ] = \frac{[F]}{[l]^2} \quad \text{- Rigidità flessionale} = [F][l] \quad X_i = \frac{M}{EJ} \rightarrow \frac{Fb}{EJ}$$

$$q = \frac{F}{b}$$

- carico uniforme di intensità $q = F/b$
- vincolo solido elasticamente in A

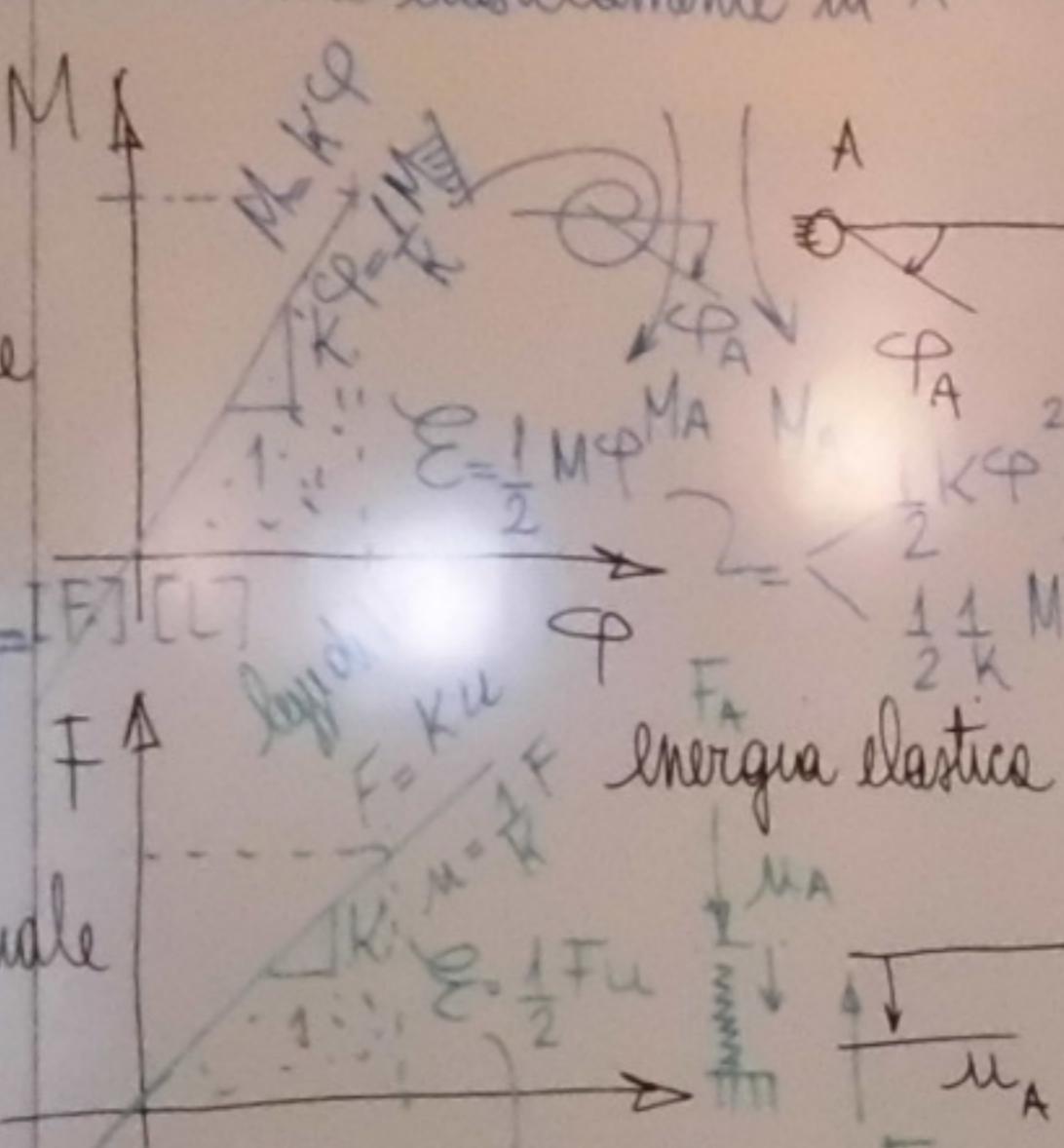
molla rotazionale

$$K_{rp} [k] = \frac{[M]}{[l]^2} = \frac{[F]}{[l]^2}$$

molla elongazionale

$$\delta_c = \frac{Fb^3}{EJ}$$

$$K_u$$



$$K = E, [k] = \frac{[F]}{[l]} \quad \Delta \theta_t = \frac{\theta_t}{l} \quad \delta_c = \frac{Fb^3}{EJ} \quad \varphi \approx \frac{Fb^2}{EJ}$$

$$[s] = \frac{Fb^3}{EJ} \quad [M] = \frac{Fb^2}{EJ}$$

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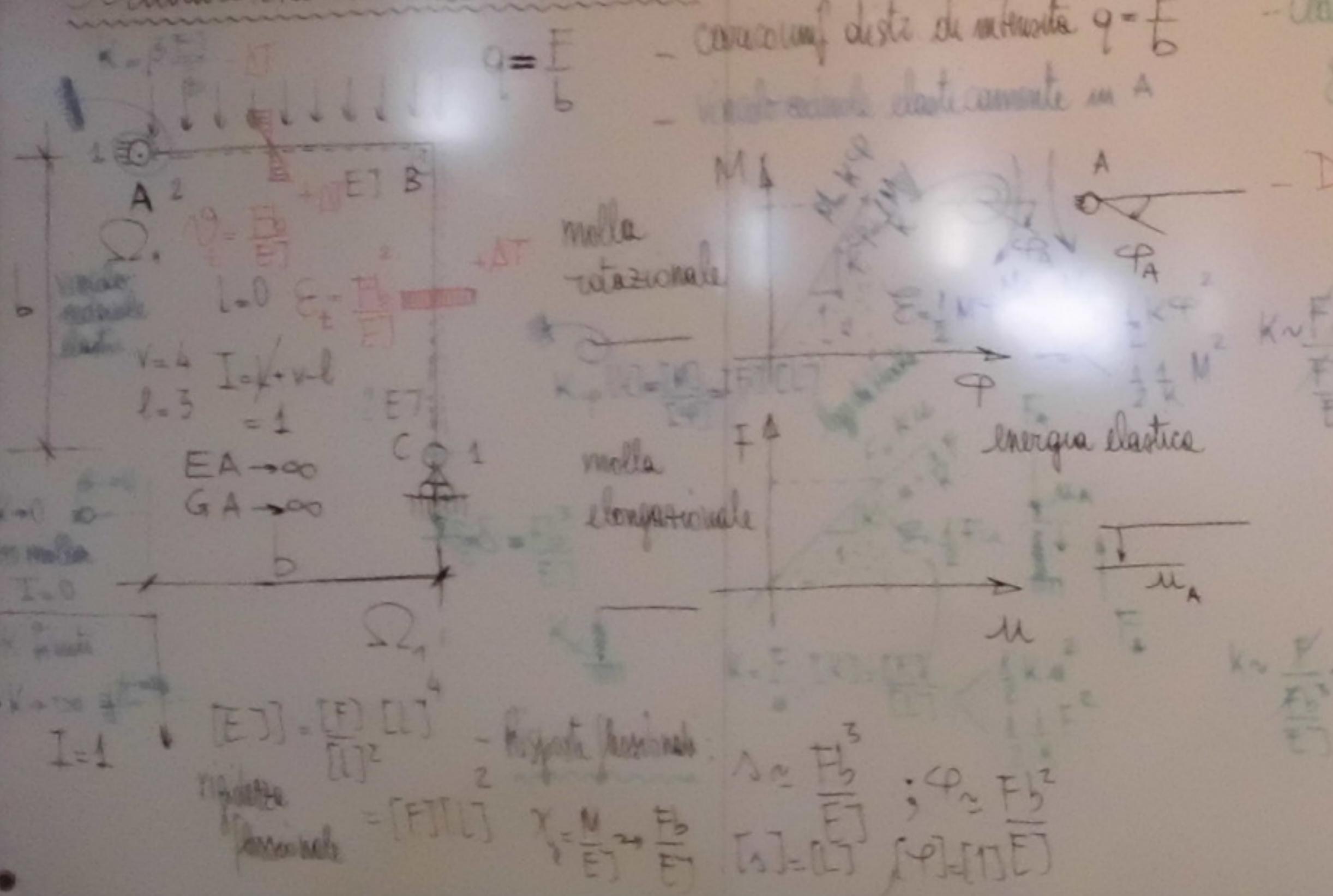
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Struttura staticamente indeterminata



- calore nello ambiente air

$$\delta_t = \frac{Fb^3}{EJ}$$

- Deformazione termica elongazionale

$$k = \frac{Fb^3}{EJ}$$

- Curvatura termica

$$d\varphi_t = \varepsilon_t dx \Rightarrow \varphi_t \sim \frac{Fb}{EJ}$$

$$X_t = \frac{\varepsilon_t}{EA}$$

no stress defom.

$$- \Delta T ISO$$

ΔT

+ ΔT

IPER

$X = \frac{B_t EJ}{l}$

$= 2 \Delta T EJ / l$

$X_t = \frac{B_t X}{EJ}$

$\frac{X_t}{EJ} = \varphi_t X$

Spost. no defom.

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Spost. no defom.

Scelta della incognita x est.

$$M_A = X$$

$$V_C = \frac{F}{2} - \frac{M_A}{b} = \frac{F}{2} - \frac{X}{b}$$

$$V_A = \frac{F}{2} + \frac{M_A}{b} = \frac{F}{2} + \frac{X}{b}$$

$$RV = RV(q, X)$$

nel rispetto dello
equilibrio

RV "isostatica"

$$\sum F_x = 0 \Rightarrow H_A = 0 \quad \text{det. dal solo equil.}$$

$$\begin{cases} \sum M_A = 0 \rightarrow V_C b + M_A - \frac{qb^2}{2} = 0 \\ \sum M_C = 0 \rightarrow -V_A b + M_A + \frac{qb^2}{2} = 0 \end{cases} \rightarrow$$

il pb. risulta staticamente
indeterminato in V_A, V_C, M_A

$$\sum F_y = 0 \Rightarrow V_A + V_C - F = 0$$

combinaz.
lineare delle cond.

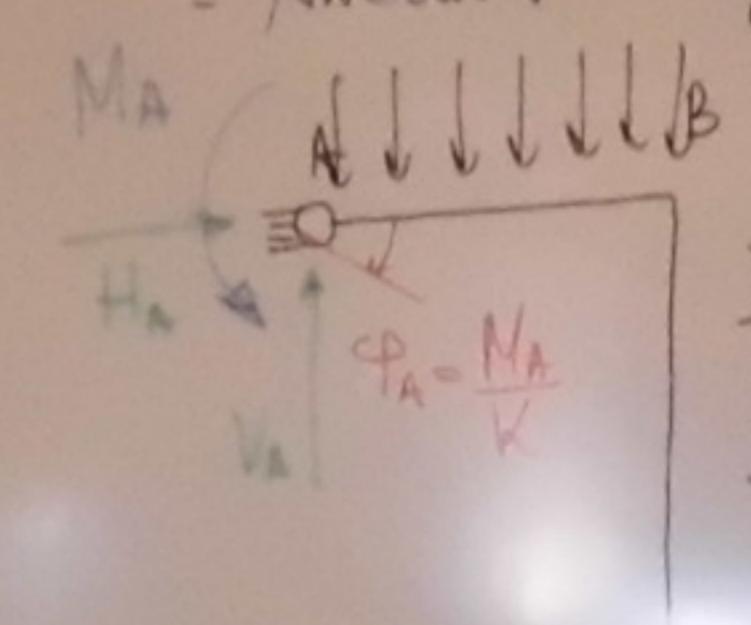
$$V_A + V_C = F$$

$$V_A + V_C = F \quad \checkmark$$

- Il pb. risulta staticamente indeterminato e, se pur risulta possibile det. $V_A, V_C \&$ valori di X , nel rispetto dell'equilibrio, non appare possibile, ed solo equil. det. $X \rightarrow$ Essa potrà essere det. solo nel rispetto della condizione di congruenza.
- Qualsiasi metodo (PV, E) utile a det. ϕ_n consente di risolvere X

$$\phi_A = \frac{M_A}{R} - \frac{X}{K}$$

- Analisi statica

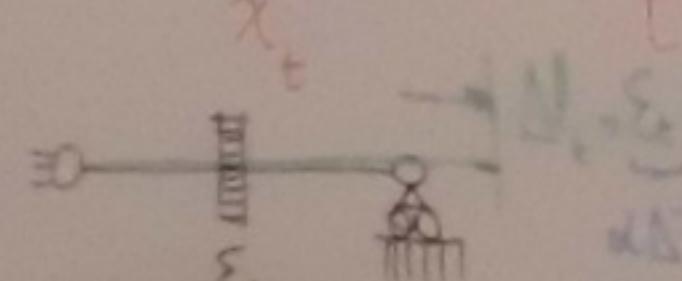


- Deformazione unica longitudinale

$$\delta u_t = \epsilon_t dx \approx \epsilon_t \sim \frac{Fb}{EJ}$$

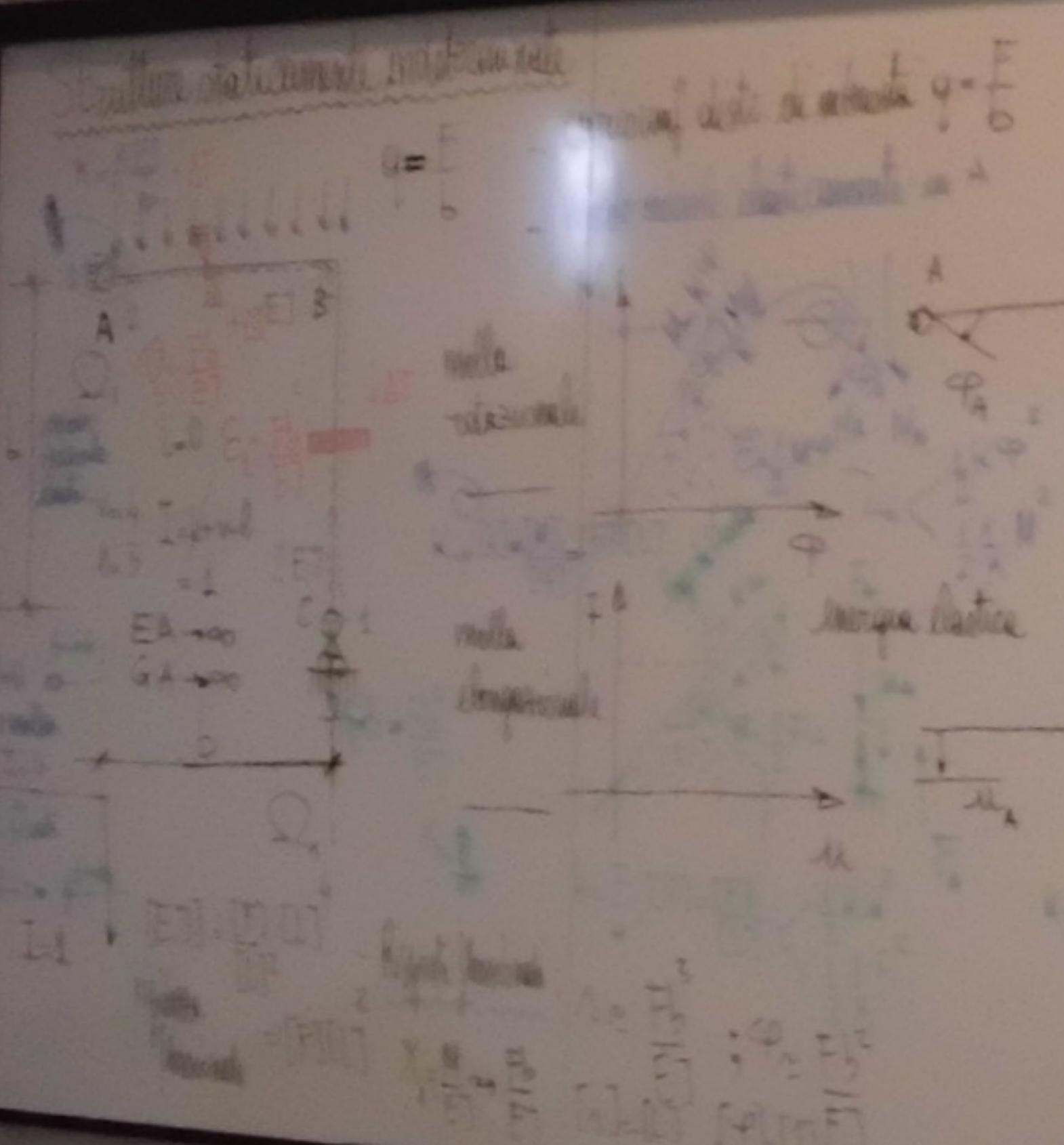
- Curvatura termica

$$\delta u_t = \theta_t dx \approx \theta_t \sim \frac{Fb}{EJ}$$

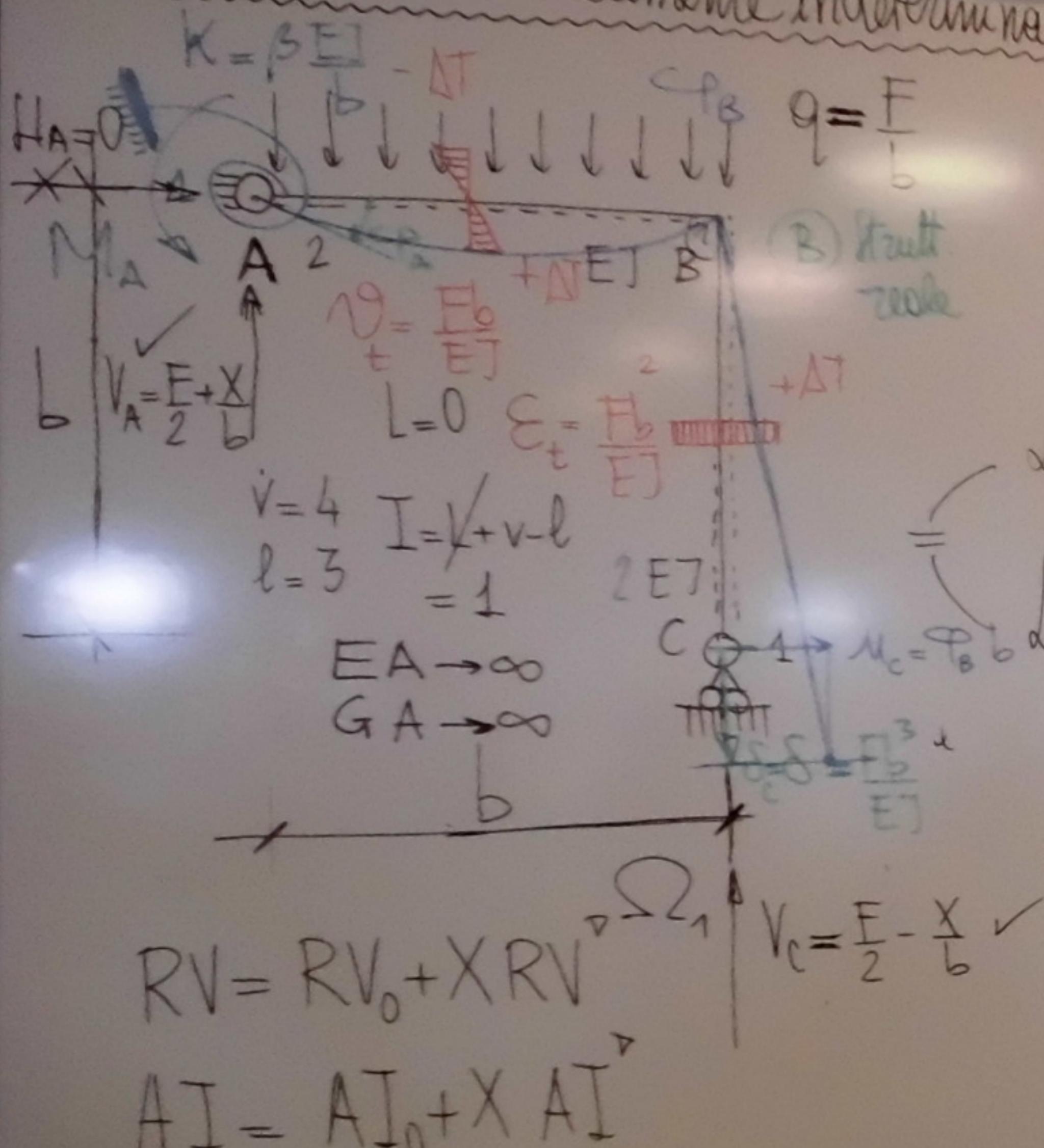


$$\begin{aligned} \frac{Xt}{EA} &= \alpha \Delta T \delta \\ X &= \alpha \Delta T EA \\ \text{Spazio di defl.} & \end{aligned}$$

$$\begin{aligned} X &= 0,5 \cdot \frac{Fb}{EJ} \cdot 1 \text{ PER} \\ &= 2,45 \cdot \frac{Fb}{EJ} \\ Xk &= V_t k \end{aligned}$$



Strutture staticamente indeterminate



- Soluzione framme PLV (PFV)

Sistema A: strutt. fittiza

$\Delta_e = \int_{AB} \Delta_i$

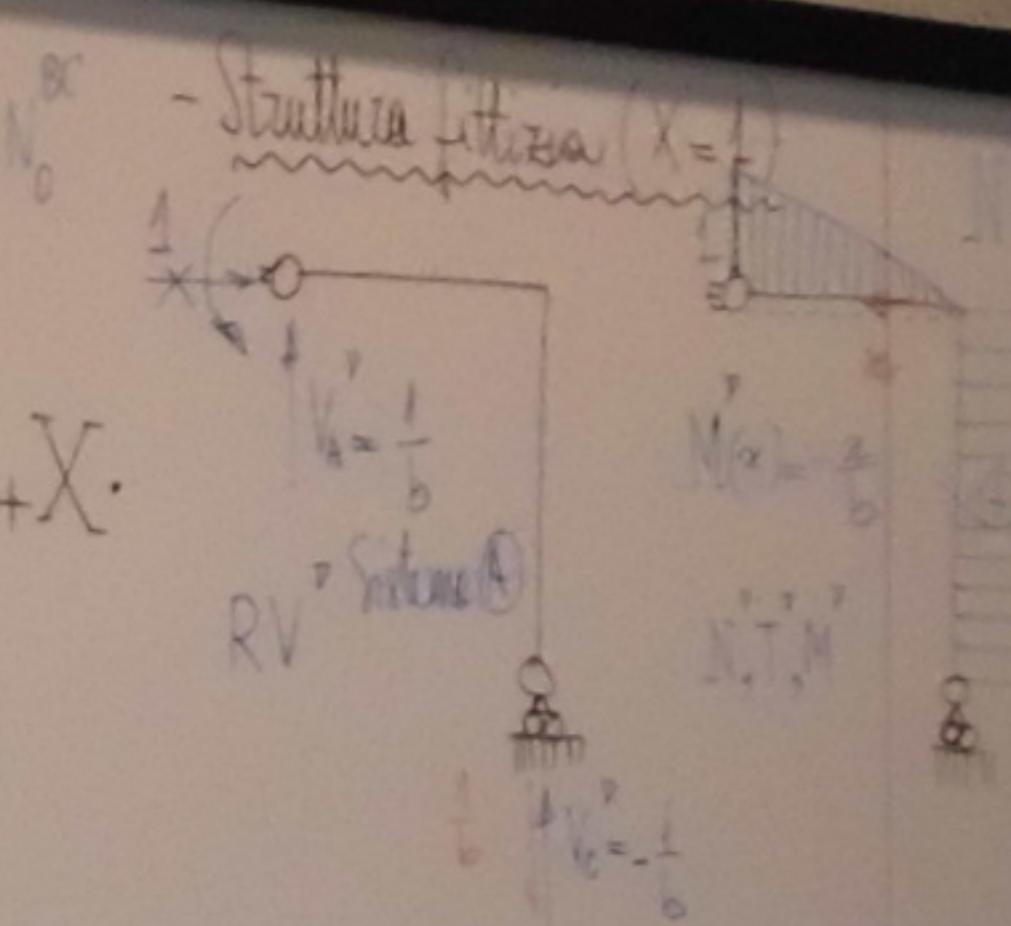
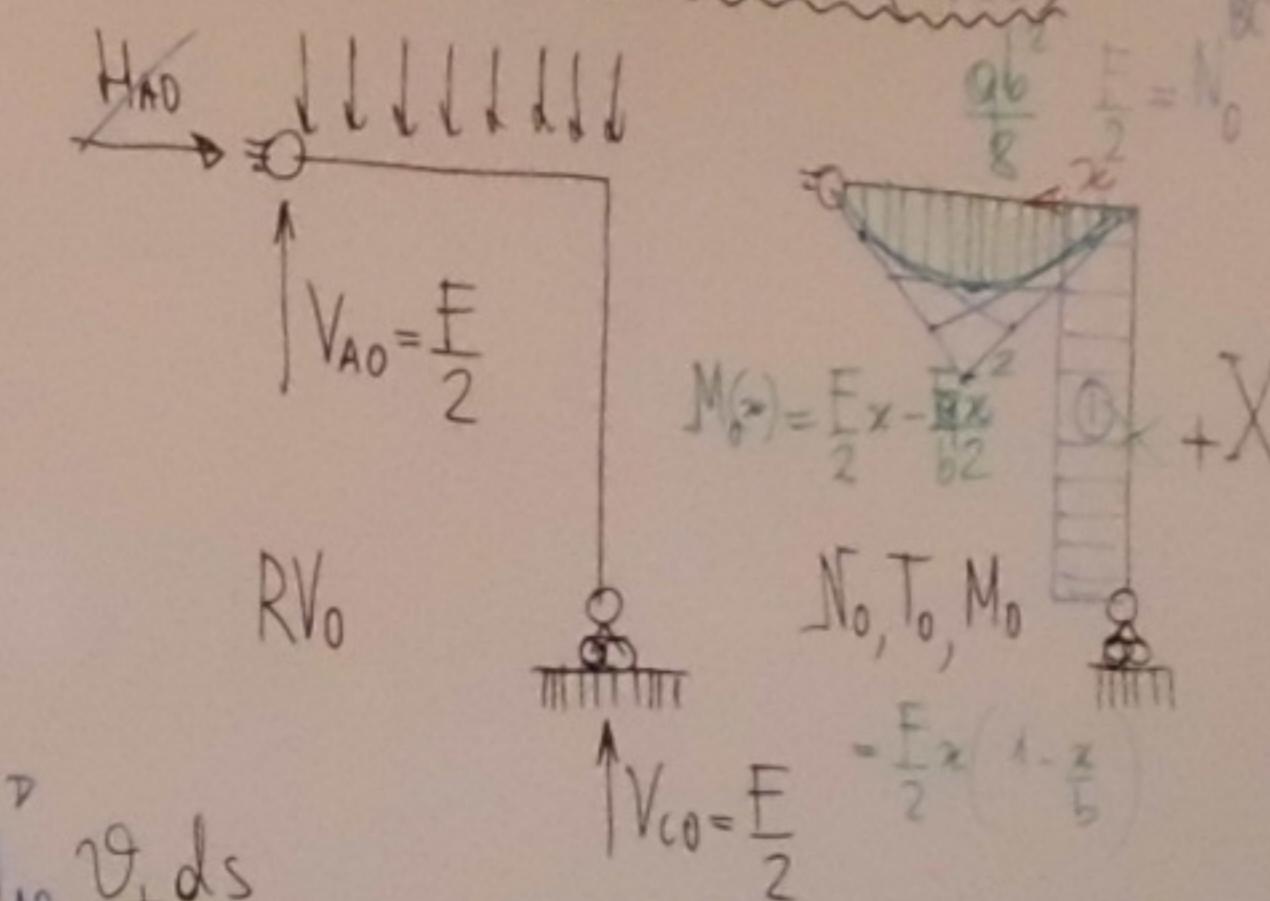
$$x_e = \frac{1}{b} x + \left(-\frac{1}{b}\right)(-\delta) + 1$$

$$\int_{St_2} M^0 \frac{M_0 + XM^0}{EJ} ds + \int_B^C N_{BC} \varepsilon_t ds + \int_A^B M_{AB} \vartheta_t ds$$

$$X = \frac{F_b^2}{EJ} - \int_{St_2} \frac{M^0 M_0}{EJ} ds - \int_B^C N_{BC} \varepsilon_t ds - \int_A^B M_{AB} \vartheta_t ds$$

$$\frac{1}{K} + \int_{St_2} \frac{M^0 ds}{EJ}$$

- Struttura principale isotatica ($X=0$)

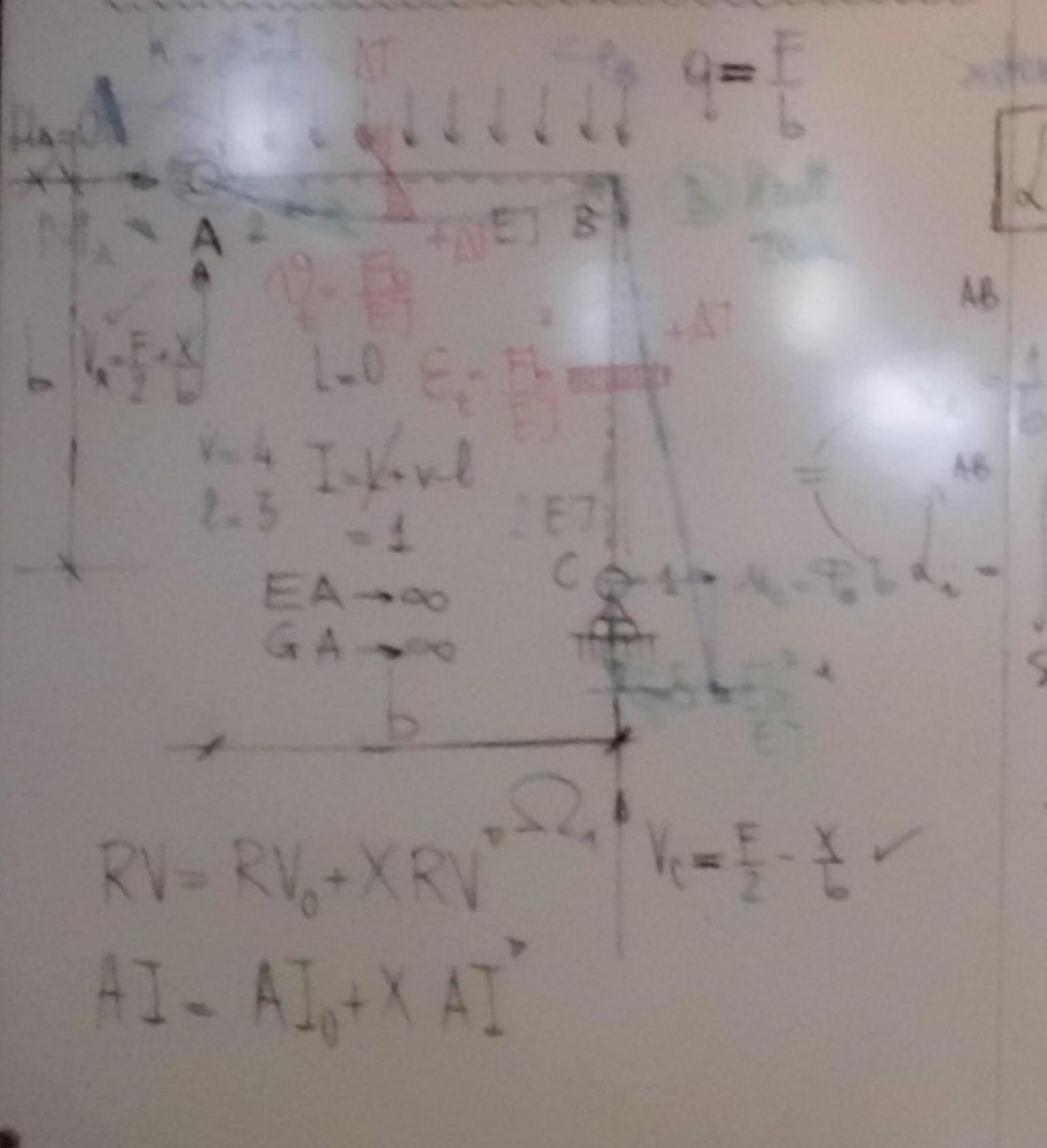


$$\int_{St_2} M^0 \frac{M_0}{EJ} ds = -\frac{1}{24} \frac{E b^2}{EJ}$$

$$\int_{St_2} M^0 \frac{ds}{EJ} = \int_0^b \frac{x^2}{b^2 EJ} ds = \frac{b}{3 EJ} = \frac{1+1}{1+3} = \frac{1}{4}$$

$$F_b = \frac{b}{4} \frac{3b}{3+b} F_b = \frac{3}{8} \frac{b}{3+b} F_b = X$$

Strutture staticamente indeterminate



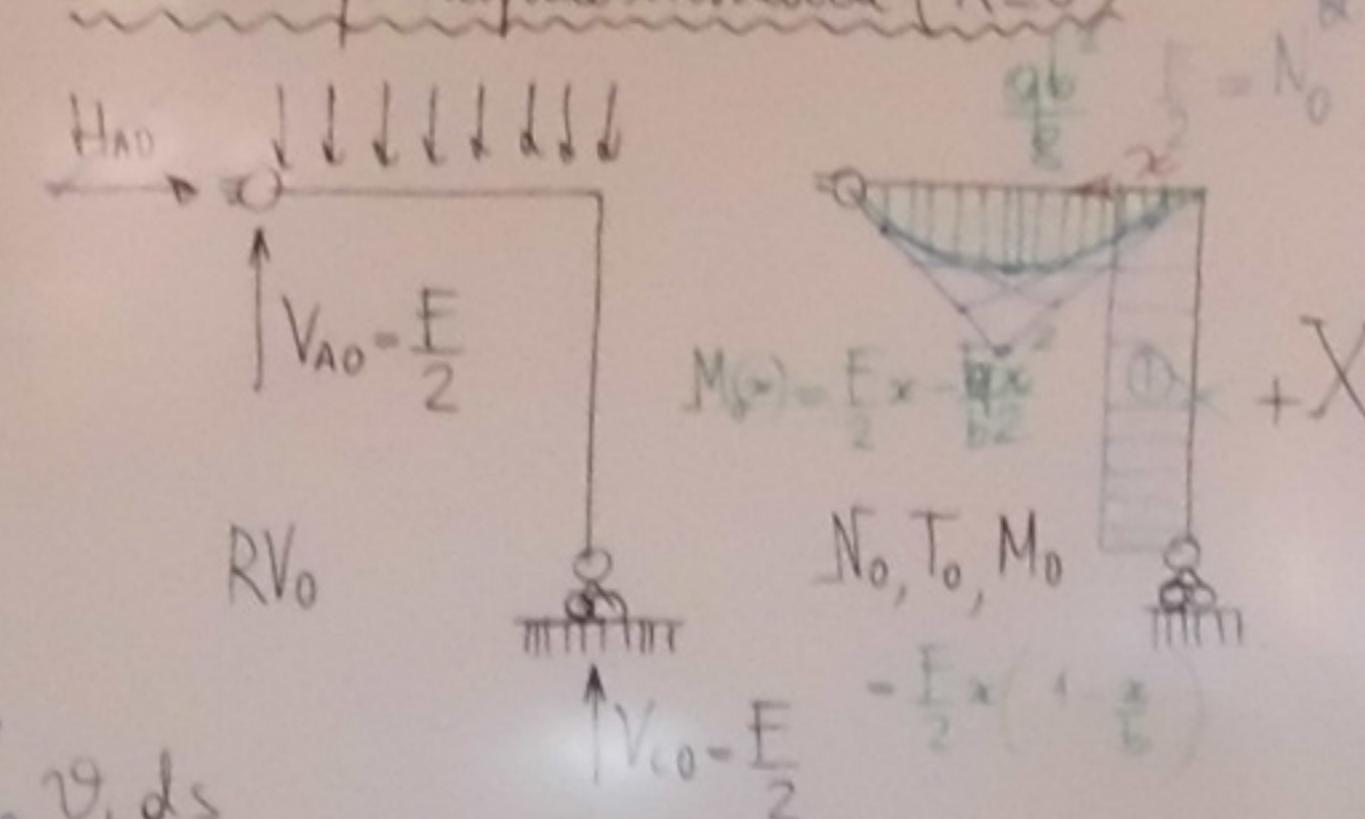
- Soluzioni finite PLV (PFV)

$$\left\{ \begin{array}{l} \text{a) struttura} \\ \text{b) congruente} \end{array} \right. \Rightarrow \left\{ \begin{array}{l} P_A = N_A \\ K \end{array} \right.$$

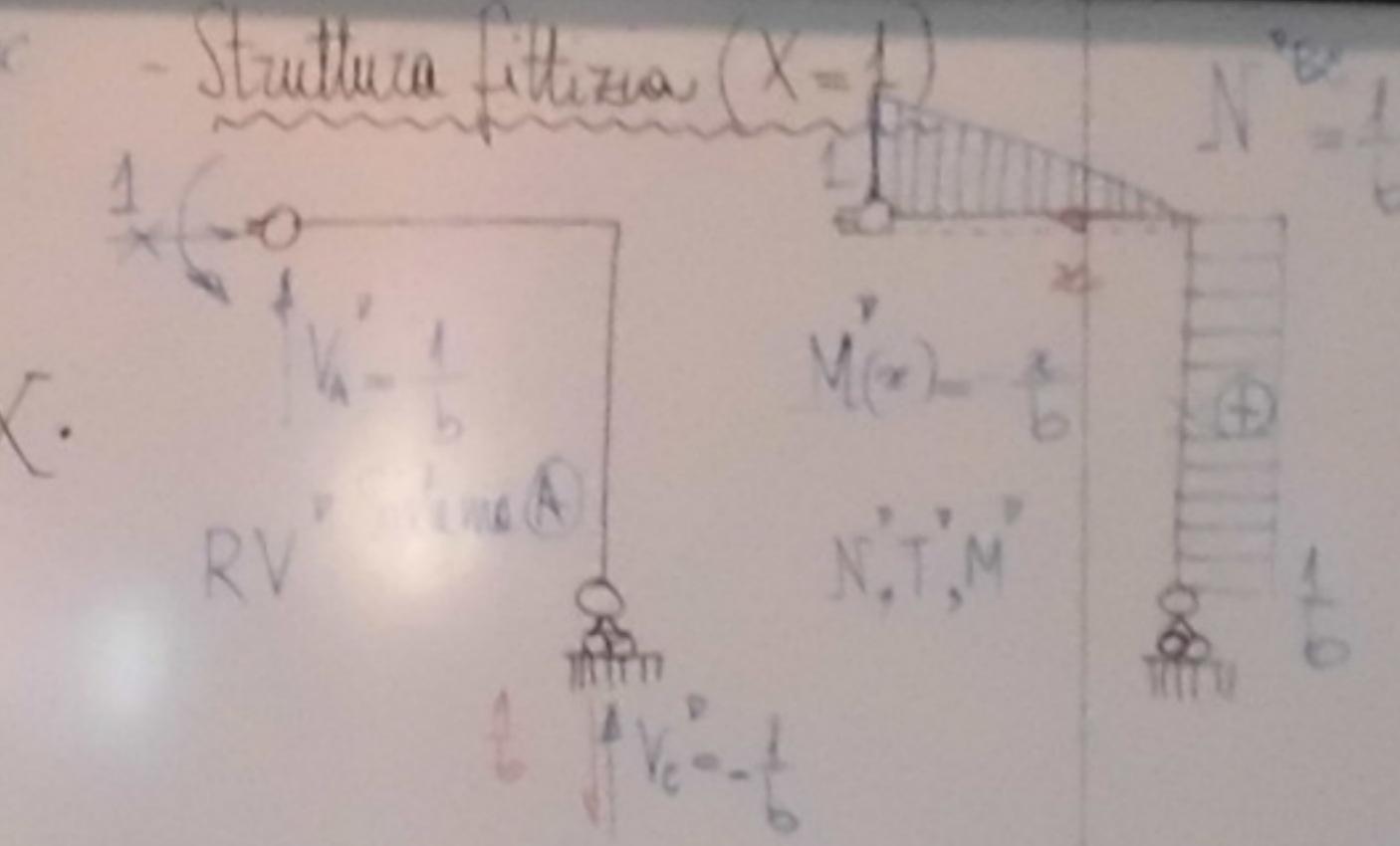
$$X = \frac{\int M^2 ds}{EI} - \frac{\int M^2 ds}{EI} - \frac{\int N_{BC} \varepsilon_t ds}{EI} - \frac{\int M_{AB} \varepsilon_t ds}{EI}$$

$$X = \frac{1}{K} + \frac{\int M^2 ds}{EI}$$

- Struttura principale isostatica ($X=0$)



- Struttura fittizia ($X \neq 0$)



N, T, M''

Calcolo di Φ_B
2 nuova applicazione
del PLV (PFV)

$$RV(X) < V_B - V_C(\beta)$$

$$V_A - V_B(\beta)$$

N, T, M finali

