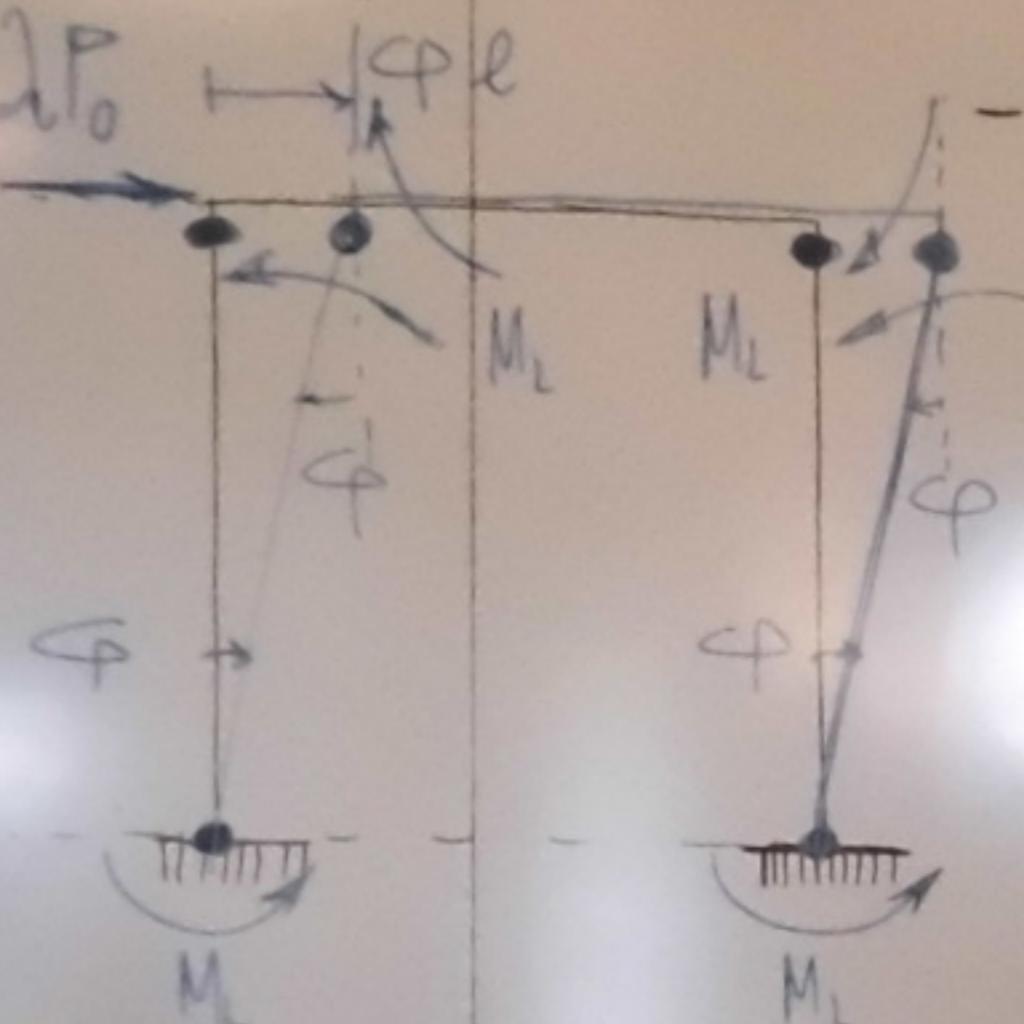


- Within the context of Limit Analysis of frames one would attempt:
 - a complete evolutive analysis, in the plastic hinge hypothesis, tracing the opening sequence of all plastic hinges and the consequent piece-wise linear force-displacement response (here elastic properties do matter);
 - an estimate, or exact evaluation, of the collapse load multiplier λ and associated collapse mechanism, by the methods of L.A. (here, only plastic properties do matter)

- "Manual" methods of collapse analysis

- Act by the "Kinematic" Theorem (Upper bound): $\lambda_L = \min\{\lambda^+\}$



- "Wall" mechanism ("complete" mechanism)

Energy balance considering the admissible mech.

$$\Delta_e = \frac{1}{2} \frac{M_L}{e} \varphi l - M_L (\varphi + \varphi + \varphi + \varphi) = 0$$

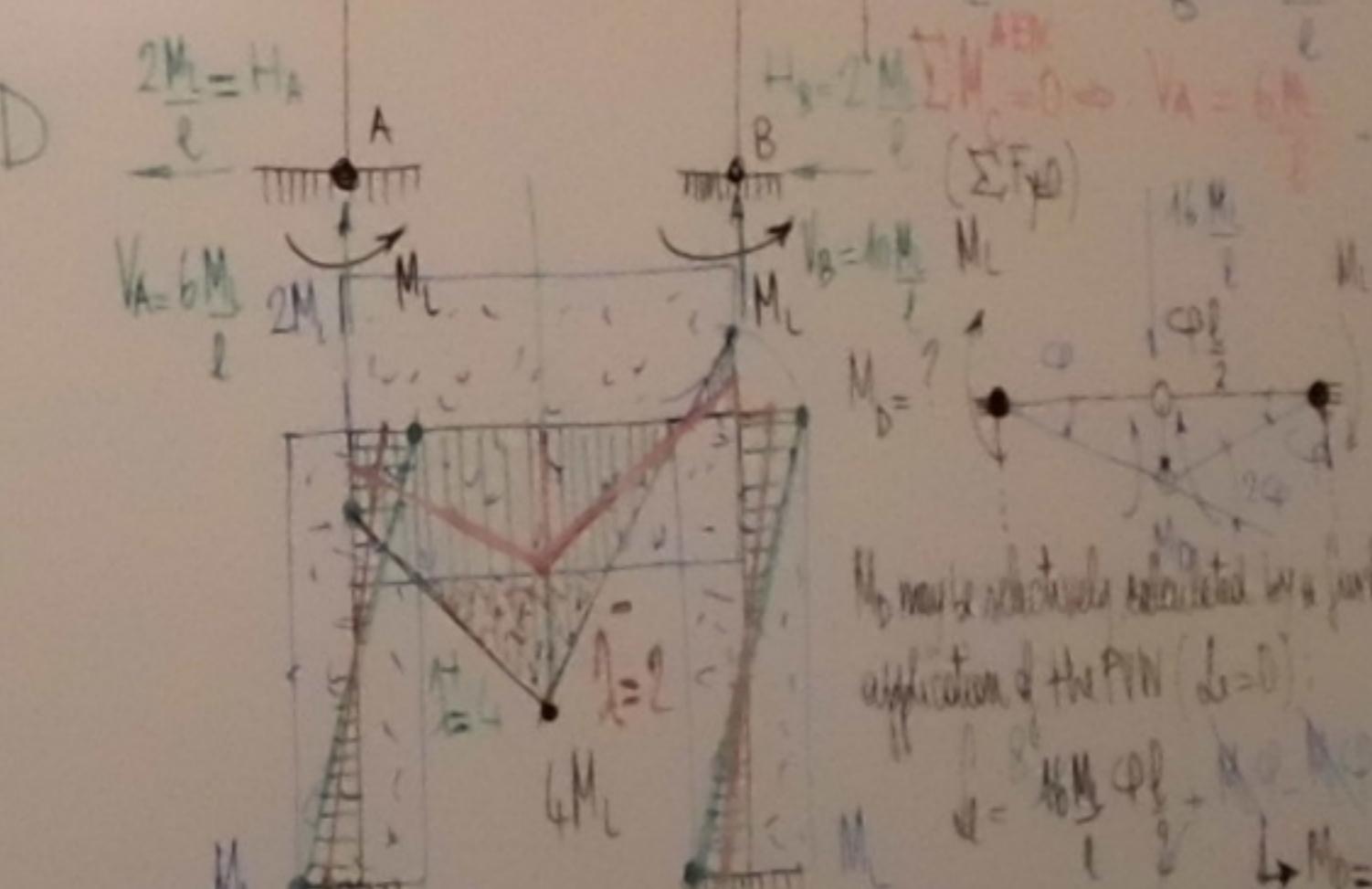
$$\Delta_e = \frac{1}{2} \frac{M_L}{e} \varphi l = 4M_L \varphi = \frac{P_0}{e} = D$$

$$\lambda_L \leq \lambda \Rightarrow \lambda = \lambda^+ = \frac{D > 0}{\Delta_e} = 4$$

- Notice that, to attempt minimizing λ^+ , one seeks to minimize the ratio D/Δ_e for different potential collapse mechanisms (minimize D and maximize Δ_e)

- One may check if this λ corresponds also to a static multiplier λ^- (in such case one would have found $\lambda = \lambda_L = \lambda^+$ (Mixed theorem))

$$\begin{aligned} \sum M_C = 0 &\Rightarrow H_B = 2M_L \\ \sum F_x = 0 &\Rightarrow H_A = 2M_L \\ \sum M_E = 0 &\Rightarrow V_B = 10M_L \end{aligned}$$



- Please notice that the distribution of actions of the frame is valid just for equilibrium for a simple initial configuration. Namely, the missing value H_D is not fixed, so again we act $\lambda = \lambda^+ + \lambda^-$ for the second mechanism, using the values of $H - H_D$ as the plastic moment increments of the solutions.

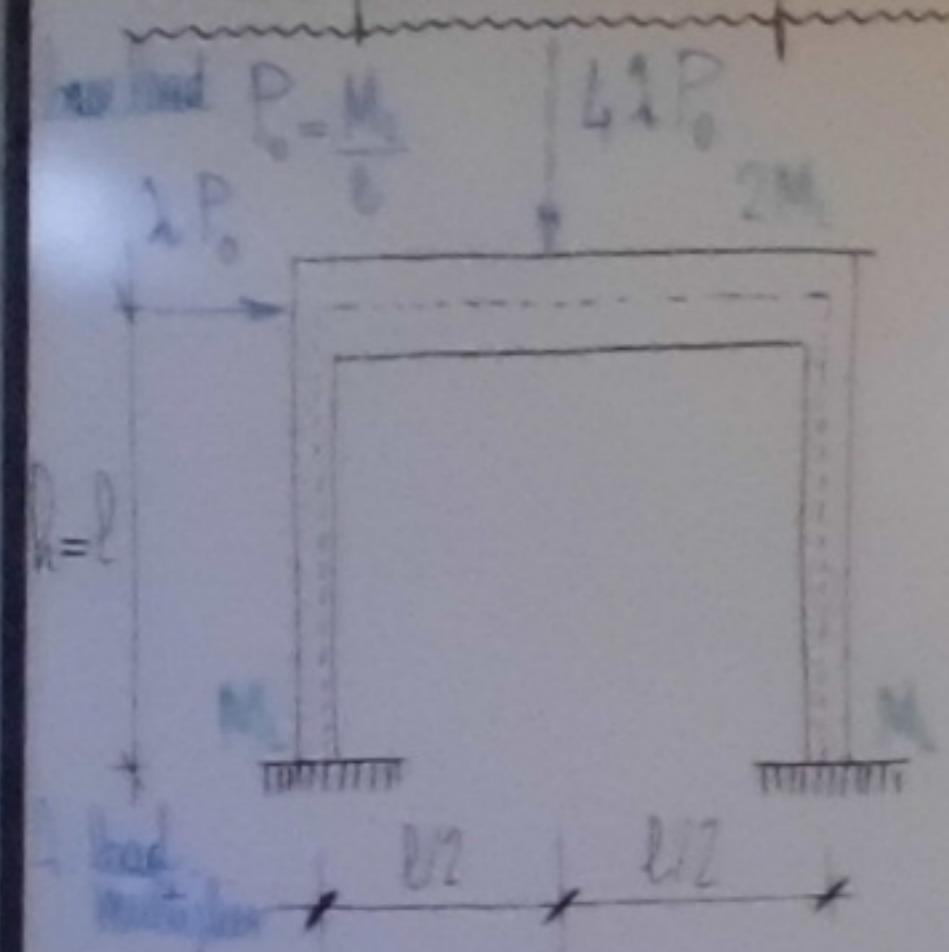
- Notice that, although capacity in the plastic hinge is automatically increased, around the reaction H_D no additional safety margin is left (but from $\lambda = \lambda^+ + \lambda^-$ the collapse multiplier is $\lambda = \lambda^+ + \lambda^-$).

The multiplier is increased by further application of the P.W. ($\lambda = \lambda^+$)

A next target capacity is to add the shear force, at point of $\lambda = \lambda^+ + \lambda^-$. This requires some calculations.

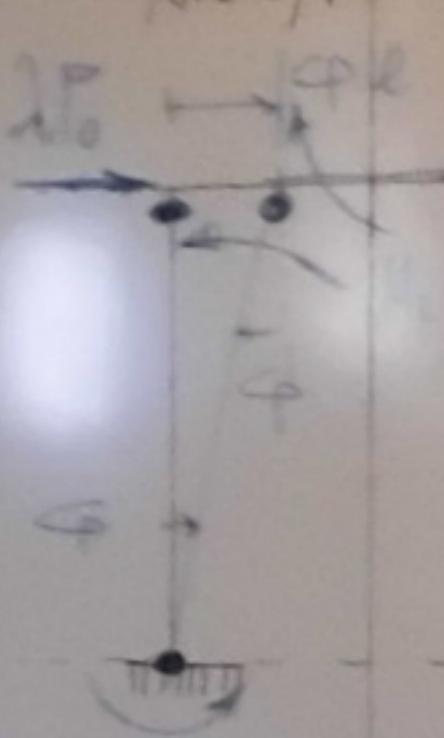


Example - Frame portal



"Manual" methods of collapse analysis

- Act by the "Kinematic" theorem (upper bound). $\lambda_L = \min\{\lambda^+\}$



"Wall" mechanism ("complete" mechanism)

Energy balance considering the admissible mech.

$$\mathcal{L}_e = \frac{1}{2} \frac{M_L}{l} CP_L - M_L (\varphi + \varphi + \varphi + \varphi) = 0$$

$$\mathcal{L}_e = \frac{1}{2} \frac{M_L}{l} CP_L = 4M_L CP = \int_{-P}^{P_h} D$$

$$D = P_0 (CP)$$

$$\lambda_L \leq L, \lambda = \lambda^+ = \frac{D > 0}{l} = 4$$

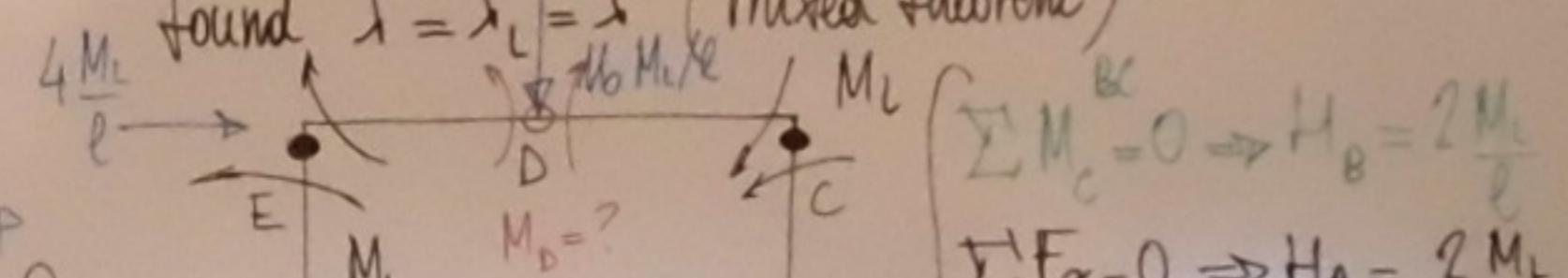
Notice that, to attempt minimizing λ^+ , one seeks to minimize the ratio D/l_e for different potential collapse mechanisms (minimize D and maximize P_0)

- Within the context of limit analysis of frames one would attempt:

- a specific mechanism, as the plastic hinge hypothesis, forcing the opening ratios of all plastic hinges, and the consequent zero-increase of displacement (zero plastic properties demand).

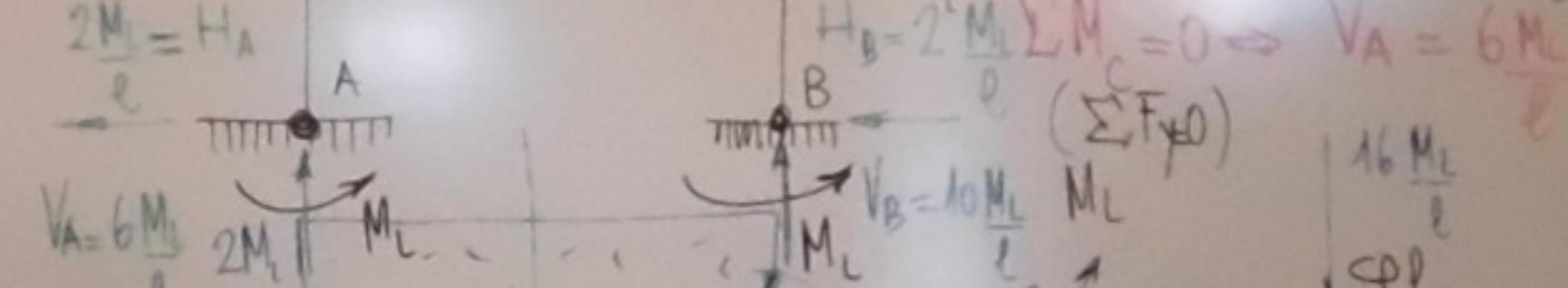
- an ultimate stress calculation of the collapse load multiplier, and a created collapse mechanism (the problem of A: zero plastic properties demand).

- One may check if this λ^+ corresponds also to a static multiplier λ^- (in such case one would have found $\lambda^- = \lambda_L = \lambda^+$ (Mixed theorem))



$$\begin{aligned} \sum M_C = 0 &\Rightarrow H_B = 2 \frac{M_L}{l} \\ \sum F_x = 0 &\Rightarrow H_A = 2 \frac{M_L}{l} \\ \sum M_{BCDE} &= 0 \Rightarrow V_B = 10 \frac{M_L}{l} \end{aligned}$$

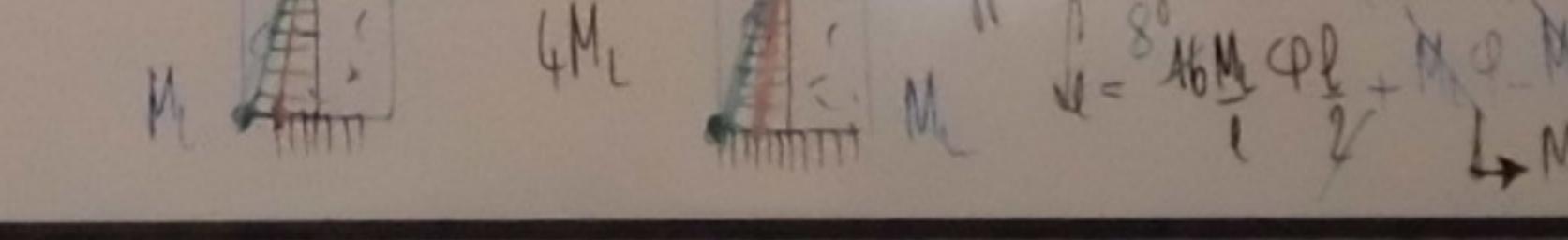
$$H_B = 2 \frac{M_L}{l}, \sum M_C = 0 \Rightarrow V_A = 6 \frac{M_L}{l}$$



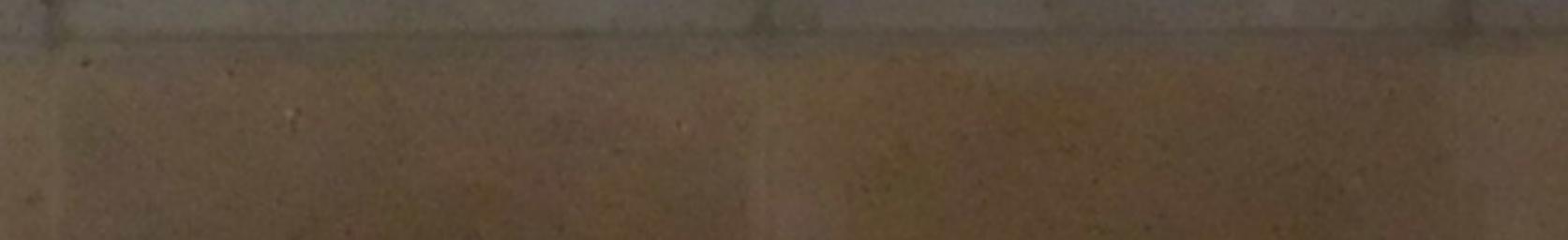
$$V_A = 6 \frac{M_L}{l}, H_A = 2 \frac{M_L}{l}, V_B = 10 \frac{M_L}{l}, M_L$$



$$H_B = 2 \frac{M_L}{l}, \sum M_C = 0 \Rightarrow M_D = ?$$



$$M_D = ?$$



$$M_D = ?$$

- Please notice that the distribution of internal actions at the stage is ruled just by equilibrium (for a complete potential collapse mechanism).

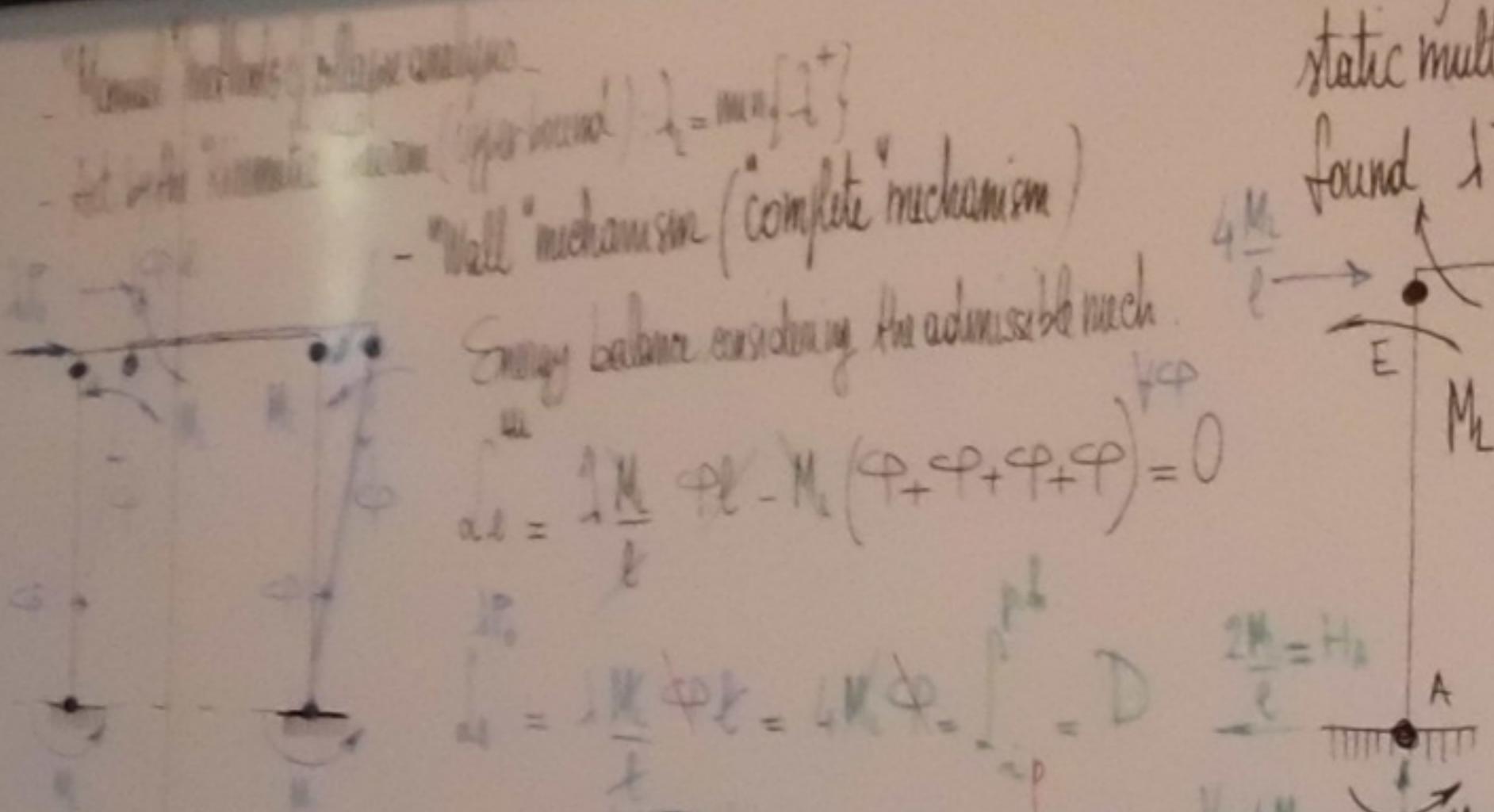
Namely, the missing value M_D is just fixed by equilibrium at $\lambda = \lambda^+ = 4$ for the assumed mechanism (fixing the values of $M = M_L$ in the plastic hinges at the extrema of the columns).

- Notice that, although conformity in the plastic hinge is automatically imposed, in general the resulting M_{ex} is not conformal unless if we have truly found $\lambda^- = \lambda_L = \lambda^+$ (true collapse multiplier [here $M_D = 2(2M_L) = P_0(2M_L)$]), otherwise

M_D may be selectively calculated by a further variation, then λ^- is not a λ ; usually we have not found it yet application of the PVW ($\lambda = 0$):

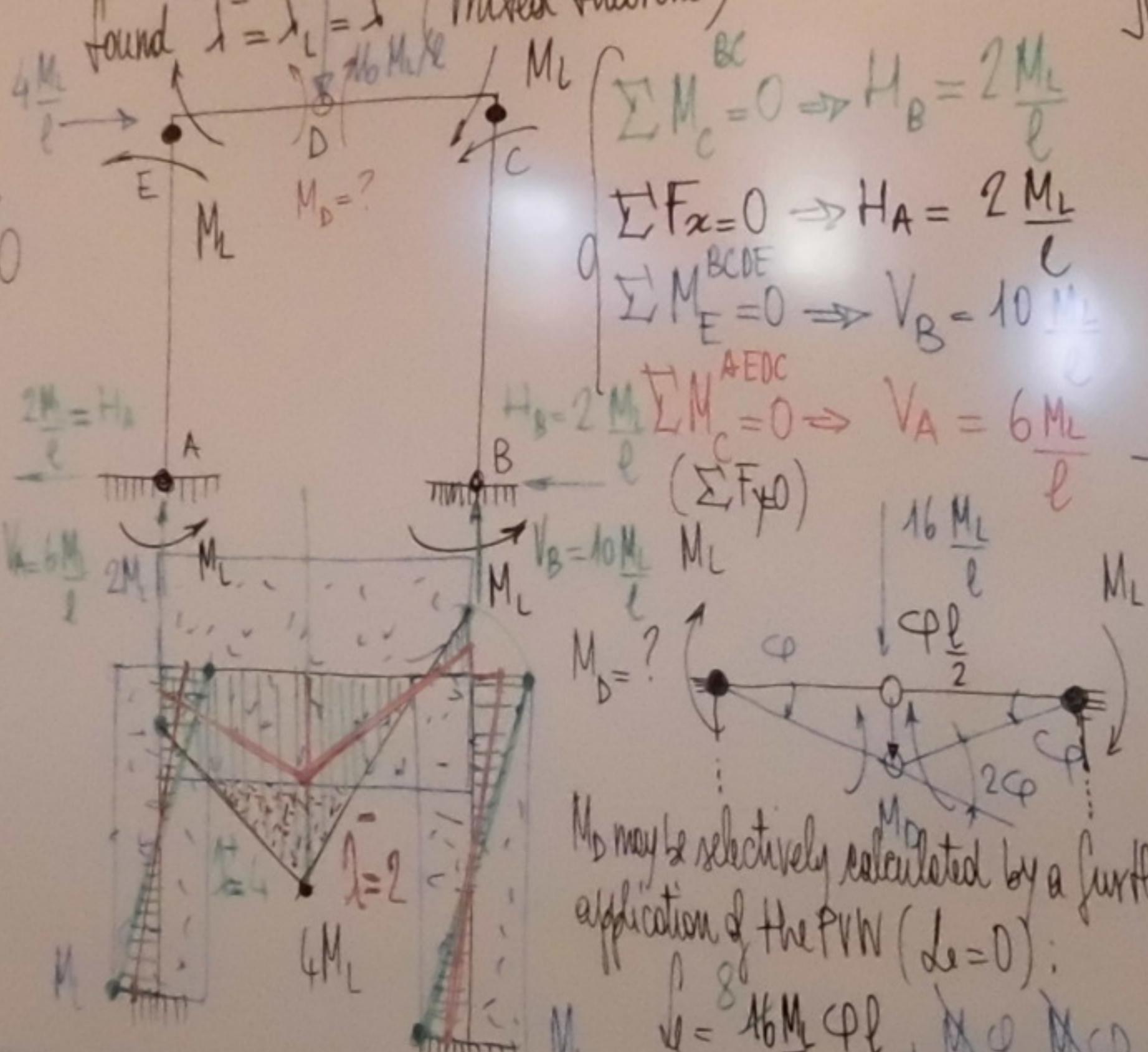
$$\begin{aligned} \int \frac{8}{l} 16 M CP + M_C 0 - M_D 20 = 0 &\Rightarrow \text{get estimate of } \lambda = \lambda^+ / e = 4/2 = 2 \text{ and provide bilateral} \\ \rightarrow M_D = 4 M_L &\text{ violation of compatibility: } P_0 = 2 \text{ elimination } 2 = 1 \text{ contradiction} \end{aligned}$$

Example - Frame vertical



$\lambda \leq \lambda^+ \Rightarrow \lambda = \lambda^+ = \frac{D > 0}{P} = 4$
Notice that, to attempt minimizing λ^+ , one seeks to minimize the ratio D/P_m for different potential collapse mechanisms (minimize D and maximize P_m)

- One may check if this λ^+ corresponds also to a static multiplier λ^- (in such case one would have found $\lambda^- = \lambda_L = \lambda^+$ (mixed "theorem")



M_D may be selectively calculated by a further application of the PVW ($\Delta = 0$):

$$\Delta = \frac{8}{l} M_L \frac{\phi l}{2} + M_l \phi - M_D 2\phi = 0 \Rightarrow M_D = 4M_L$$

"Violation" of conformity: $P_m = 2$ delimitation

- Please notice that the distribution of internal actions at this stage is ruled just by equilibrium (for a complete potential collapse mechanism).

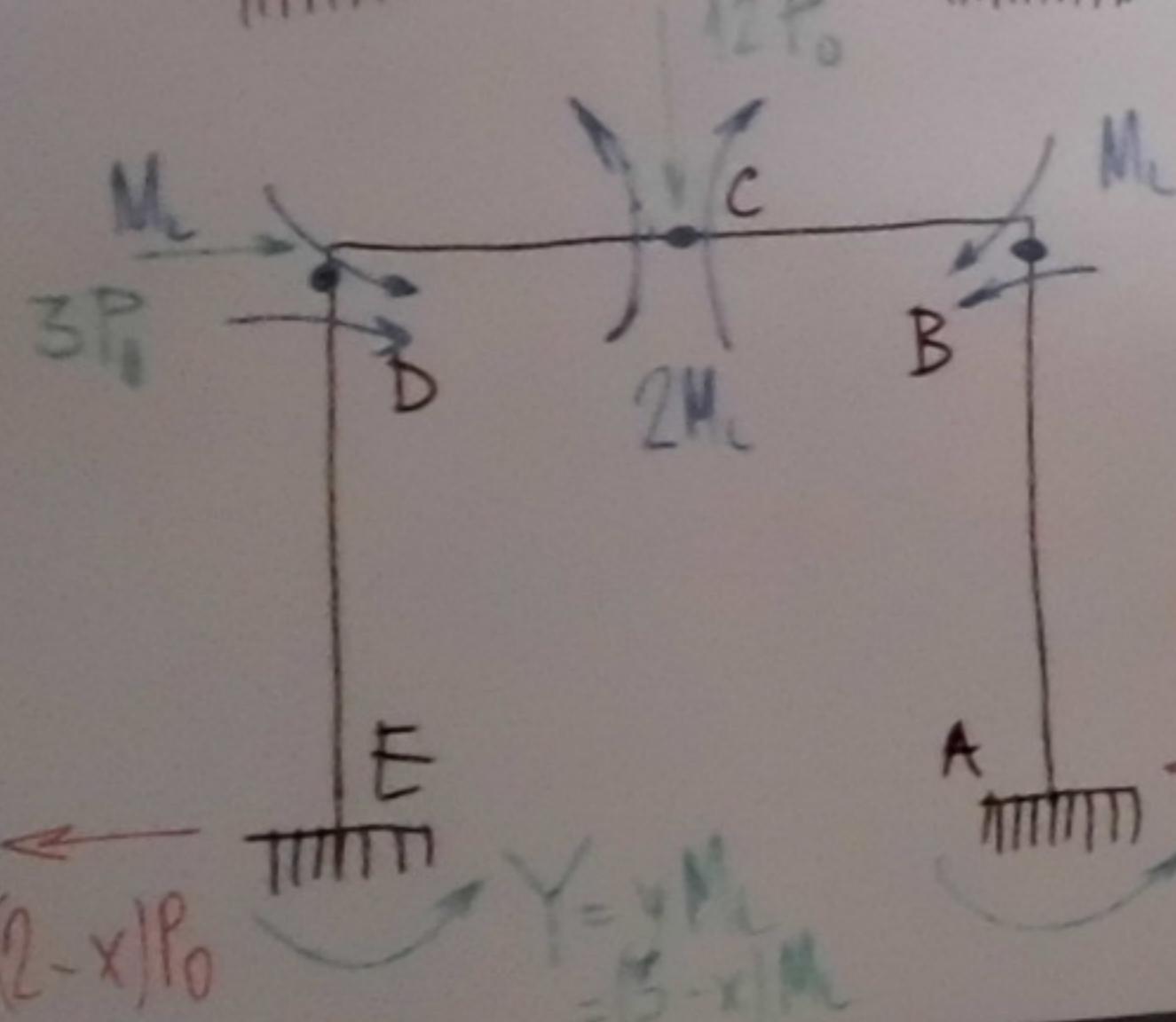
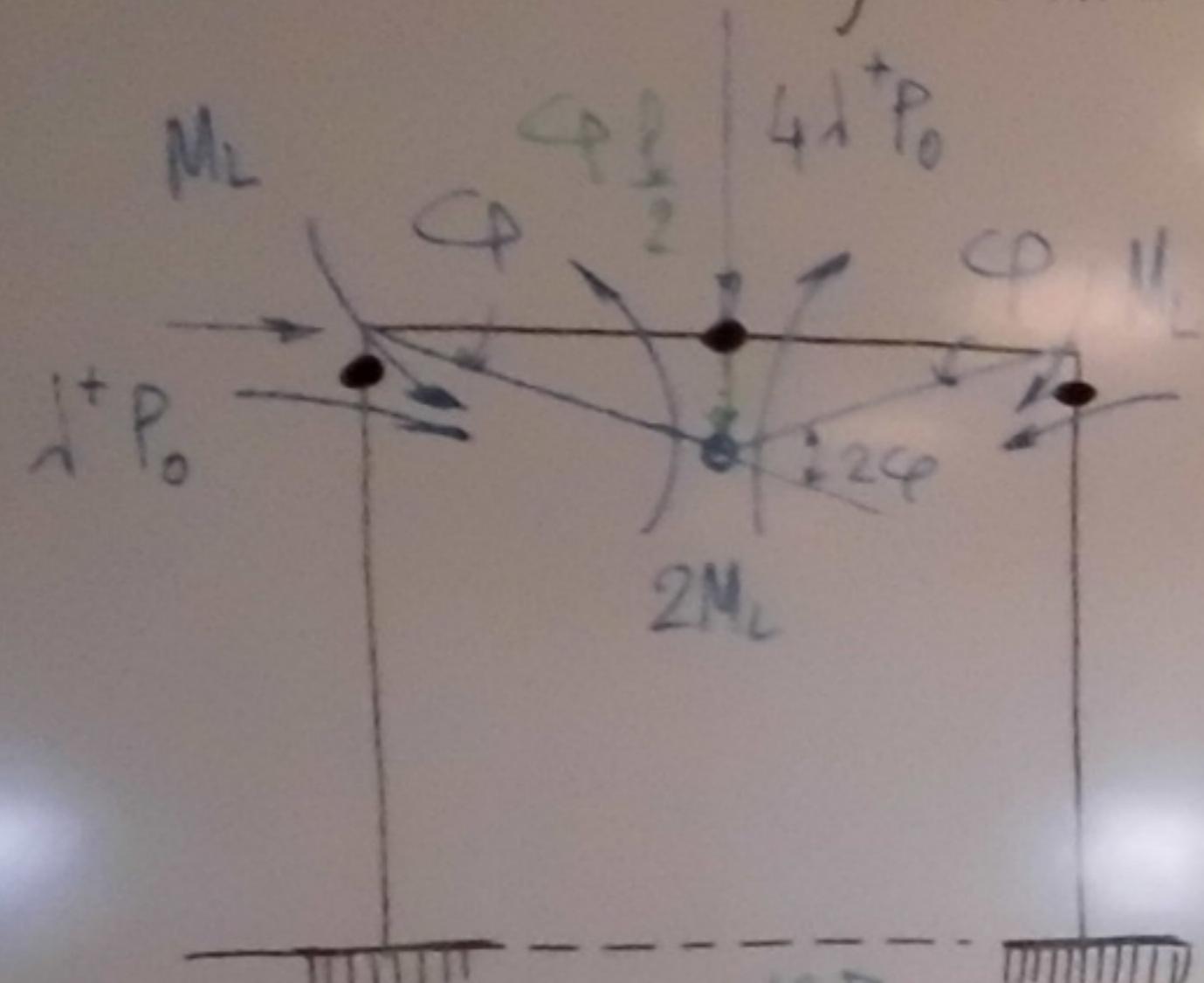
Namely, the missing value M_D is just fixed by equilibrium at $\lambda = \lambda^+ = 4$ for the assumed mechanism (fixing the values of $M = M_L$ in the plastic hinges at the extremes of the columns).

- Notice that, although conformity in the plastic hinges is automatically imposed, in general the resulting $M(\phi)$ is not conformal unless if we have truly found $\lambda^- = \lambda_L = \lambda^+$ the collapse multiplier [here $M_D = 2(2M_L) = P_m(2M_L)$ shows conformity violation, then λ is not a λ ; namely we have not found λ_L yet]

- A way to respect conformity is to deflate $M(\phi)$ down to $M(\phi)/P_m$, to get estimate of $\lambda = \lambda^+ / P_m = 4/2 = 2$ and provide bilateral

$$2 = \lambda^- \leq \lambda_L \leq \lambda^+ = 4$$

- Further estimates by the K.T.:



- Partial collapse mechanism
(beam mechanism)

$$\lambda^2 = \frac{1}{4} \lambda P_0 \quad \frac{9L}{2} = M_L \varphi + M_L \varphi + 2M_L 2\varphi = D$$

$$\Rightarrow \lambda = \lambda^+ = 3$$

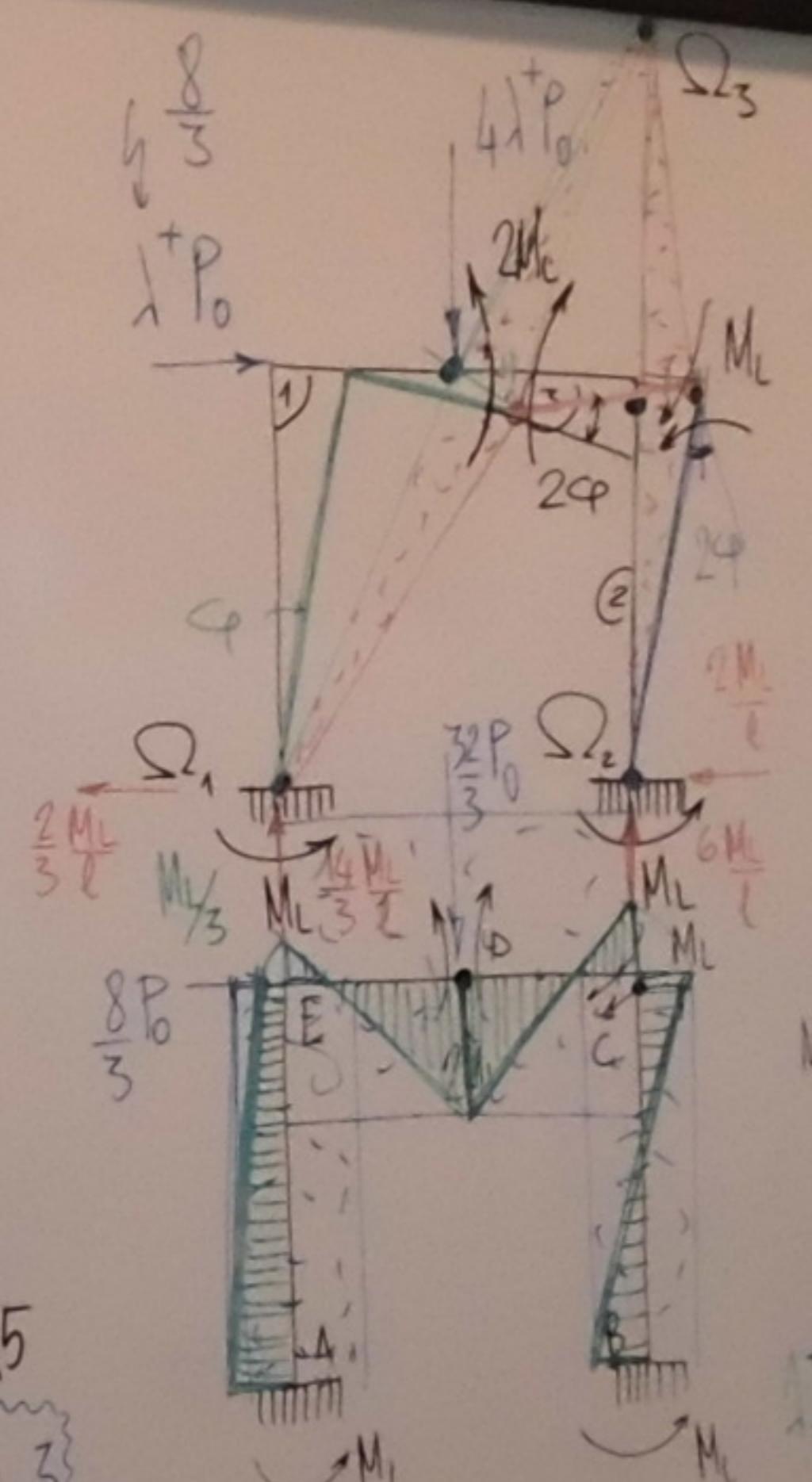
- In this case, going to the static th., we have one residual hyperstatic unknown (say X)

$$x=1, y=1 \quad ; \quad x=2, y=1$$

$$\rho_m = 2 \quad \rho_m = 2 \quad \Rightarrow \lambda = \lambda^+ / \rho_m$$

conf. in A unconf. in B conf. in E $= 3/2 = 1.5$

$\lambda \in [1.5 \leq \lambda \leq 3]$



- Complete collapse mechanism
(beam-wall mechanism)

$$\lambda^2 = 1P_0 \cdot \Omega_1 + 6P_0 \cdot \frac{\Omega_2}{2} =$$

$$= M_L \varphi + M_L \varphi + M_L 2\varphi + 2M_L 24 = D$$

$$\lambda = \lambda^+ = \frac{8}{3} = 2.6 = 2.667$$

$$M_E = ?$$

$$\lambda = \frac{8}{3} \cdot 6 \lambda^+ \varphi / 2 - M_L \varphi - M_L \varphi - M_L \varphi = 0$$

$$\lambda = \frac{8}{3} \cdot 6 \lambda^+ \varphi / 2 - 2M_L \varphi = 1M_L \varphi \quad \Rightarrow \quad \lambda = 1.5$$

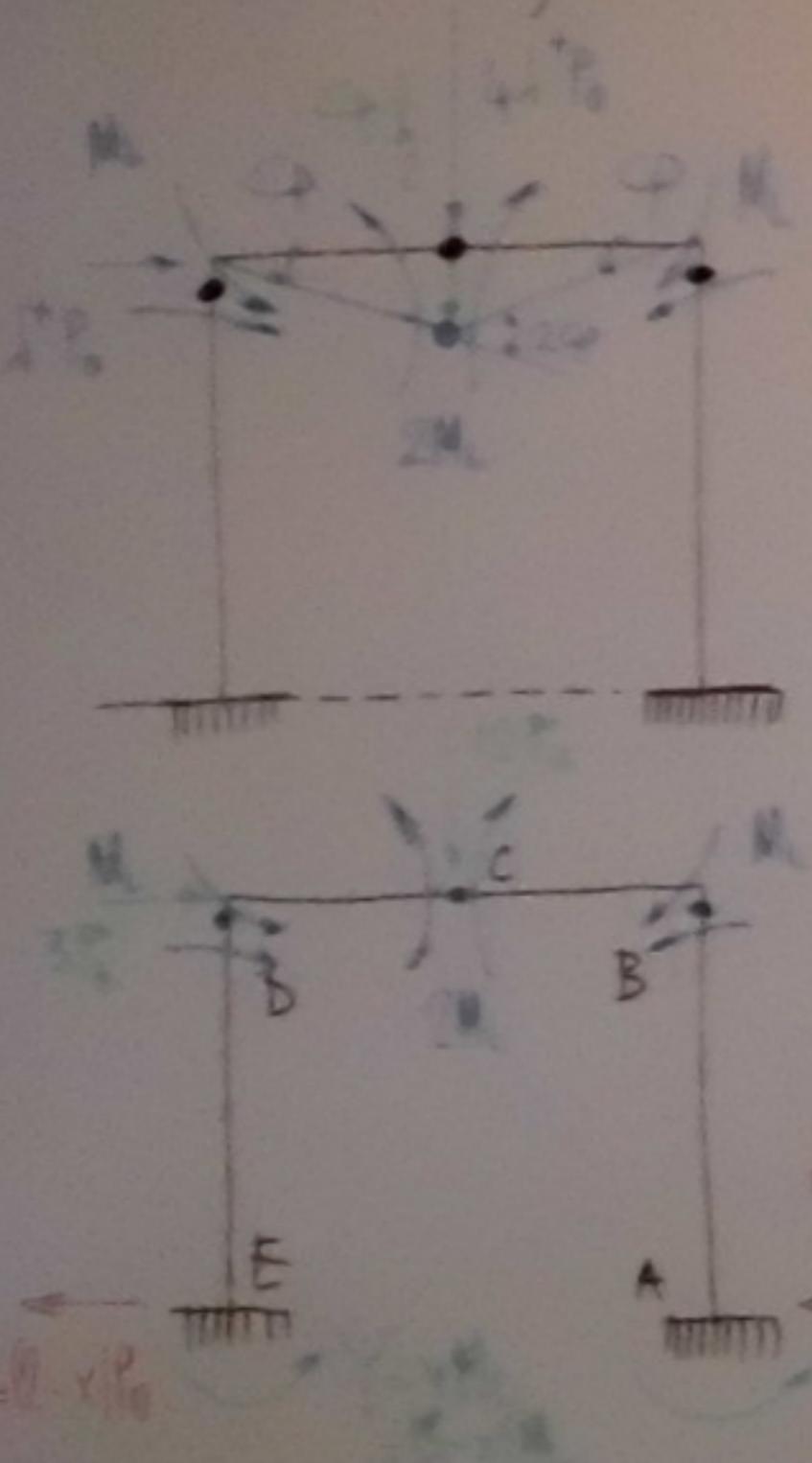
- Remarks

- In general, we are interested in collapse of K.T. under the same loading as initial, as well as determining the collapse load multiple.

- A 1+ mechanism is more likely to occur than a 1- mechanism, because it is more difficult to reach equilibrium with a 1- mechanism.

- This 'beam' method can be applied, in the framework of Mathematical Programming, to other structures, such as shells, as well as problems of optimal design.

- Further estimates by the K.T.



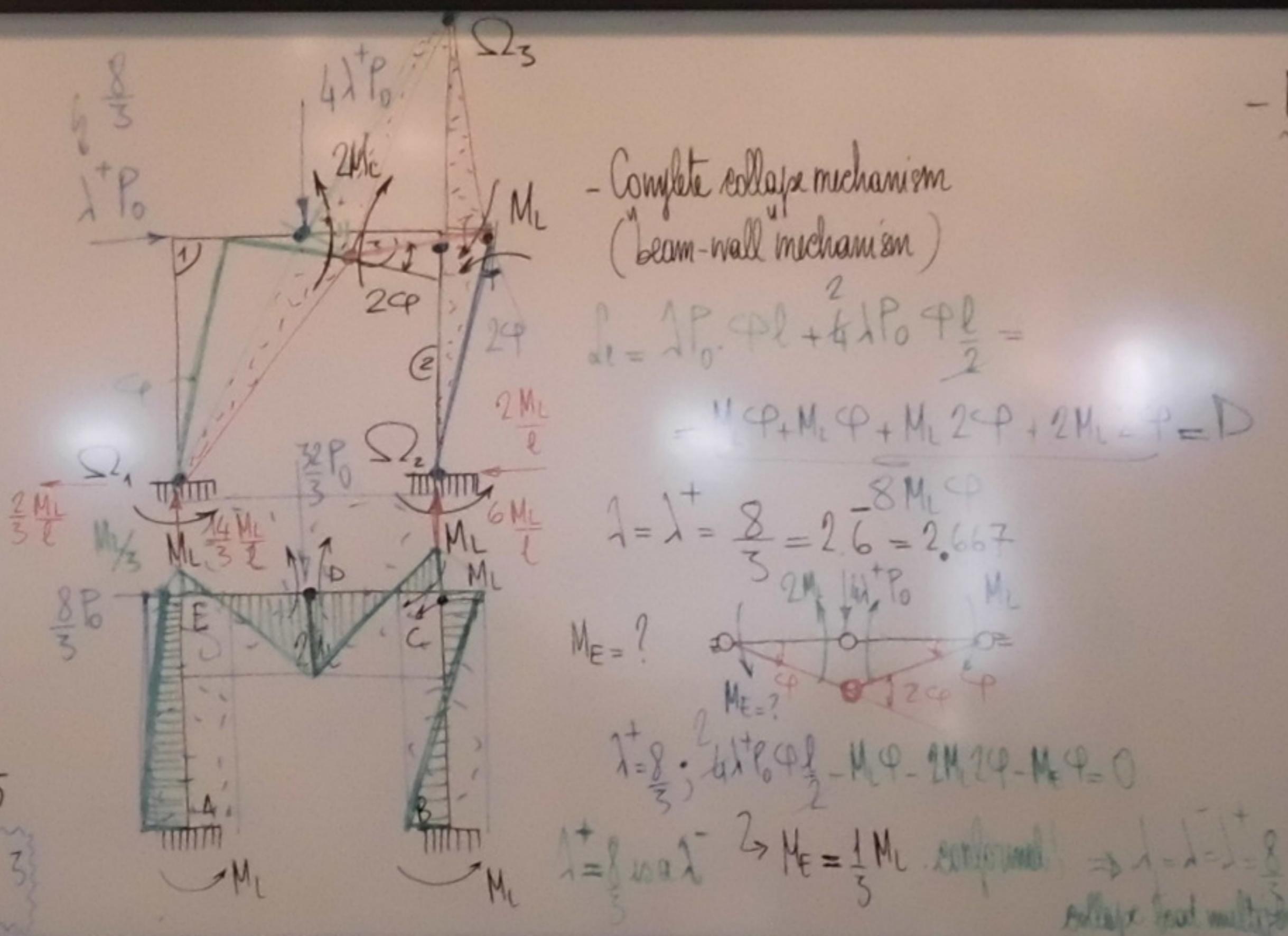
- Partial collapse mechanism
(beam mechanism)

$$\lambda^+ = M_L \varphi + M_L \varphi + 2M_L 2\varphi = D$$

$$2 - 1 - \lambda^+ = 3$$

- In this case, going to the static th, we have one residual kinematic unknown (say X)

$$\begin{aligned} x=1 & ; \quad x=2, y=1 \\ \lambda^+ = \lambda & = \lambda / \rho_m \\ \lambda & = 3/2 = 1.5 \\ 1.5 \leq \lambda & \leq 3 \end{aligned}$$



- Remarks:

In general, by a few attempts of application of K.T. and/or S.T. one manages to find an structure needed initial eliminations of the collapse load multipliers.

If a $\lambda = \lambda^+$ is found, namely a technically-admissible load multiplier that is both statically and kinematically admissible, the collapse load multiplier is found (via the Mixed Theorem).

- These "manual" methods may be generalized, in the framework of Mathematical Programming, up to solve either $\lambda = \max(\lambda)$, and/or $\lambda = \min(\lambda)$, as max/min problem in Operation Research (via programming \rightarrow Simplex method).

Lateral collapse mechanism
beam mechanism

$$\lambda = 10 \cdot M_c \cdot 24 \cdot \varphi = D$$

$$\lambda = \lambda^+$$

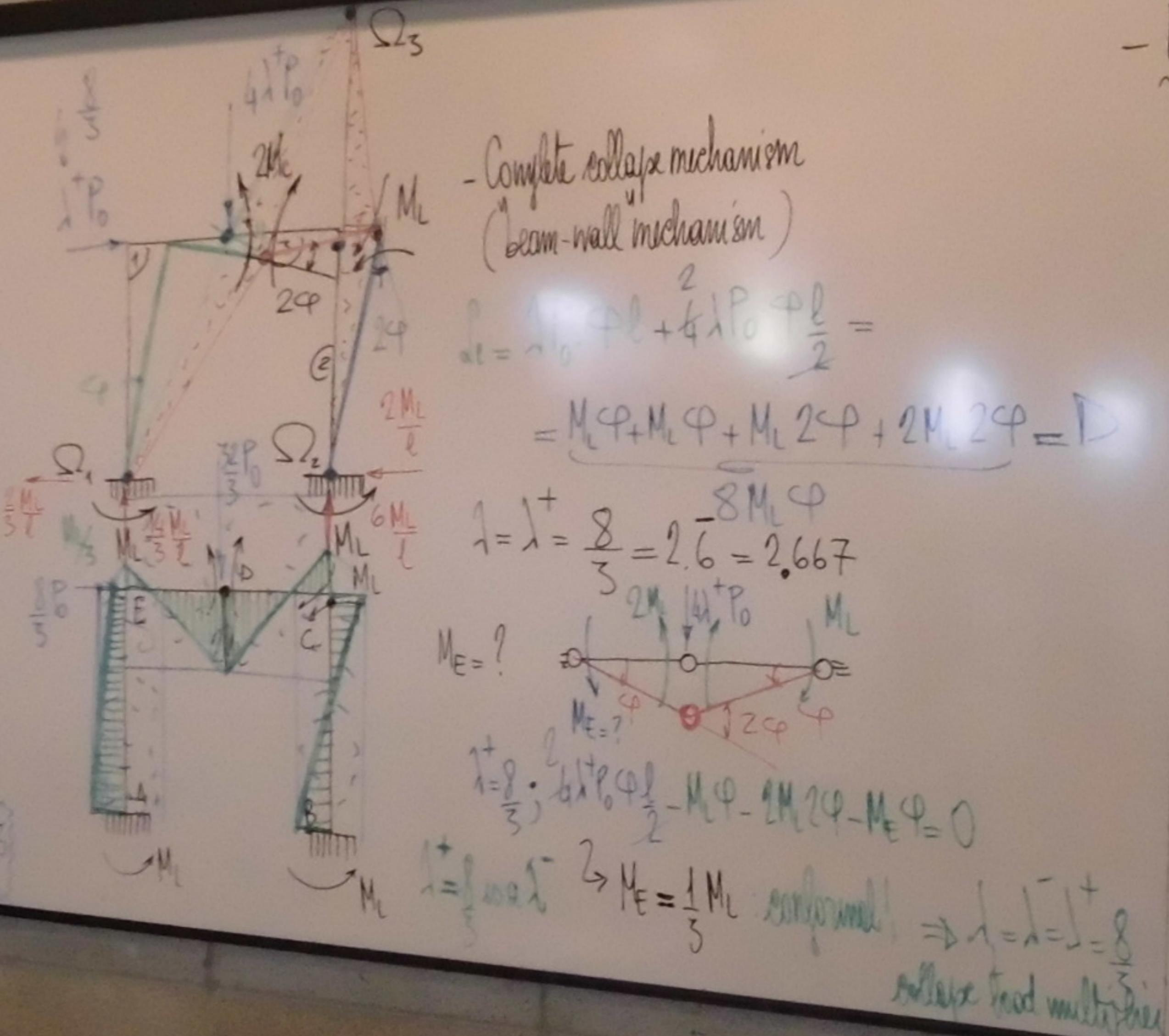
In this case, going to the state Ω_0 , we have
one residual hinge mechanism (say X)

$$x_1 = 2, x_2 = 1$$

$$\lambda = \lambda^+$$

$$= 3/2 = 1.5$$

$$\lambda \leq \lambda^+$$



- Complete collapse mechanism
(beam-wall mechanism)

$$\lambda = \lambda^+ = l + \frac{2}{3}l P_0 + \frac{l}{2} =$$

$$= M_c \varphi + M_c \varphi + M_c 2 \varphi + 2 M_c 2 \varphi = D$$

$$\lambda = \lambda^+ = \frac{8}{3} M_c \varphi$$

$$M_E = ?$$

$$\lambda^+ = \frac{8}{3} ; \quad \lambda^+ P_0 \frac{l}{2} - M_c \varphi - 2 M_c 2 \varphi - M_E \varphi = 0$$

$$\lambda^+ = \frac{8}{3} \Rightarrow M_E = \frac{1}{3} M_c \text{ (conformal)} \Rightarrow \lambda = \lambda^+ = \frac{8}{3}$$

collapse load multiplier

- Remarks:

In general, by a few attempts of application of K.T. and/or S.T. one manages to find as strict as needed bilateral delimitations of the collapse load multiplier.

If a $\lambda = \lambda^+$ is found, namely a plastically-admissible load multiplier that is both statically and kinematically admissible, the collapse load multiplier is found (by the Mixed Theorem).

These "manual" methods may be generalized, in the framework of Mathematical Programming, opt to solve either $\lambda^+ = \max\{\lambda\}$ and/or $\lambda^- = \min\{\lambda\}$ use max/min problem in Operation Research (Linear programming \rightarrow Simplex Method)