Università degli studi di Bergamo

Scuola di Ingegneria (Dolmine)

CCS Ingegneria Edile

LM-24 Ingegneria delle Costruzioni Edili

Dinamica (, Instabilità) e Anelasticità delle Strutture

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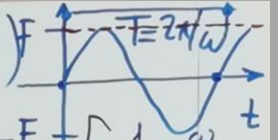
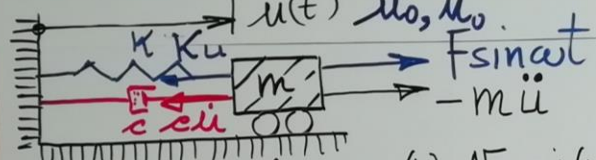
LEZIONE 05

5. Risposta smorzata a forzante armonica ($F\sin\omega t$)



$f = \frac{1}{T} = \frac{\omega}{2\pi}$



Eq. del moto (d'Alembert)

$$\cancel{m}\ddot u + \underbrace{\frac{c}{m}}\dot u + \underbrace{\frac{K}{m}}u = \underbrace{\frac{K}{m}}\underbrace{\frac{F}{K}}\sin\omega t \qquad \zeta \ll 1$$

$$\ddot u + 2\zeta\omega_1\dot u + \omega_1^2 u = \omega_1^2 u_{st}\sin\omega t$$

Integrale particolare: $u_p(t) = N u_{st}\sin(\omega t - \xi)$

fattore di amplif. dinam. (N) — fase (ξ)

$$\dot u_p(t) = \omega N u_{st}\cos(\omega t - \xi);$$

$$\ddot u_p(t) = -\omega^2 \underbrace{N u_{st}\sin(\omega t - \xi)}_{u_p(t)} = -\omega^2 u_p(t)$$

$$\frac{(\omega_1^2 - \omega^2)}{\omega_1^2}\sin(\omega t - \xi) + 2\zeta\frac{\omega_1\omega}{\omega_1^2}\cos(\omega t - \xi) = \frac{\omega_1^2}{N}\frac{u_{st}}{u_{st}\,\omega_1^2}\sin\omega t$$

$$\beta = \frac{\omega}{\omega_1} = \frac{2\pi f}{2\pi f_1} = \frac{f}{f_1}$$

$$(1-\beta^2)\sin(\omega t - \xi) + 2\zeta\beta\cos(\omega t - \xi) = \frac{1}{N}\sin\omega t \qquad \forall t$$

$$(1-\beta^2)(\cos\xi\sin\omega t - \sin\xi\cos\omega t) + 2\zeta\beta(\cos\xi\cos\omega t + \sin\xi\sin\omega t) = \frac{1}{N}\sin\omega t$$

$$\begin{cases} (1-\beta^2)\cos\xi + 2\zeta\beta\sin\xi = \frac{1}{N} & (1) \\ -(1-\beta^2)\sin\xi + 2\zeta\beta\cos\xi = 0 & (2) \end{cases}$$

(2) $\Rightarrow \tan\xi = \frac{2\zeta\beta}{1-\beta^2}$; $\boxed{\xi = \operatorname{atan}\frac{2\zeta\beta}{1-\beta^2} = \xi(\beta;\zeta)}$ fase

Sost.:

$$\cos\xi = \frac{1}{\sqrt{1+\tan^2\xi}} = \frac{1-\beta^2}{\sqrt{\underbrace{(1-\beta^2)^2 + (2\zeta\beta)^2}_{D(\beta;\zeta)}}} = \frac{1-\beta^2}{\sqrt{D}}; \quad \sin\xi = \tan\xi\cos\xi = \frac{2\zeta\beta}{\sqrt{D}}$$

$$(1-\beta^2)\frac{(1-\beta^2)}{\sqrt{D}} + 2\zeta\beta\frac{2\zeta\beta}{\sqrt{D}} = \frac{1}{N} = \frac{D}{\sqrt{D}} = \sqrt{D}$$

$$\boxed{N = \frac{1}{\sqrt{D}} = \frac{1}{\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}} = N(\beta;\zeta)}$$ fattore di amplificaz. dinamica

In condiz. di risonanza ($\zeta = 0$), $\beta = 1$:

$$N(\beta = 1) = \frac{1}{2\zeta} = \frac{1/\zeta}{2} \quad ; \quad \xi(\beta = 1) = \frac{\pi}{2}$$

ζ	0	1%	2%	5%
N	∞	50	25	10

risposta $u_p(t)$ in quadratura in ritardo

Alternativamente:

$+\begin{matrix}(1)\sin\xi \\ (2)\cos\xi\end{matrix}$ $\quad 2\zeta\beta = \frac{\sin\xi}{N}$; $\sin\xi = 2\zeta\beta N$

$-\begin{matrix}(1)\cos\xi \\ (2)\sin\xi\end{matrix}$ $\quad (1-\beta^2) = \frac{\cos\xi}{N}$; $\cos\xi = (1-\beta^2)N$

sost. nella (1) $\Rightarrow N = \frac{1}{\sqrt{D}}$

$$\frac{1}{\sqrt{D}} = \frac{1}{\sqrt{\underbrace{(1-\beta^2)^2+(2\zeta\beta)^2}_{D}}} = N(\beta;\zeta) = N$$

Amplif. massima: cond. di staz. $N'(\beta)=0$

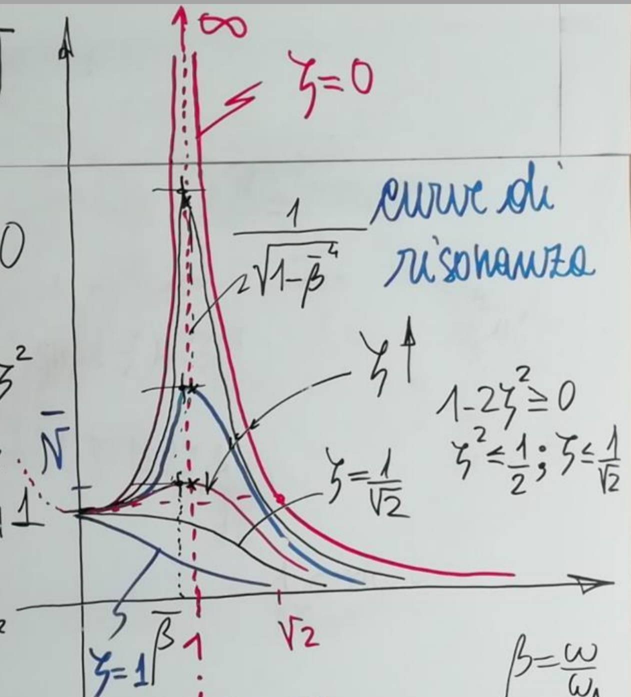
$$N' = -\frac{1}{2\sqrt{D}}\frac{D'}{D} = 0 \quad \{D'=0\} \quad \cancel{2}(1-\beta^2)(-\cancel{2}\beta)+\cancel{2}(2\zeta\beta)\cancel{2}\zeta = 0$$

$$\beta(-1+\beta^2+2\zeta^2)=0 \quad \begin{cases}\beta=0\\ \beta^2=1-2\zeta^2\end{cases}$$

$$\bar\beta = \sqrt{1-2\zeta^2} \simeq 1 \quad \zeta \ll 1$$

$$\bar D(\bar\beta) = (1-\bar\beta^2)^2+(2\zeta\bar\beta)^2 =$$
$$= 4\zeta^4 + 4\zeta^2(1-2\zeta^2)$$
$$= 4\zeta^2(\zeta^2+1-2\zeta^2) = 4\zeta^2(1-\zeta^2)$$

$$\bar N = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \simeq \frac{1}{2\zeta}$$

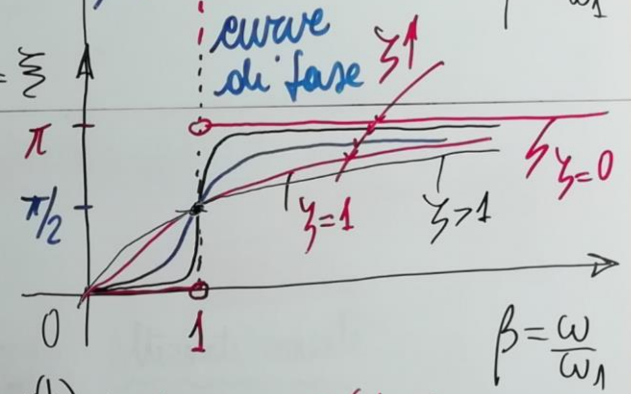


Rappresentaz. dell'integrale generale:

$$u_p(t) = N u_{st}\sin(\omega t-\xi) \qquad \arctan\frac{2\zeta\beta}{1-\beta^2} = \xi$$
$$= N u_{st}(\cos\xi\sin\omega t - \sin\xi\cos\omega t)$$
$$= \underbrace{N u_{st}\cos\xi}\sin\omega t - \underbrace{N u_{st}\sin\xi}\cos\omega t$$

$$Z_1 = N u_{st}\frac{1-\beta^2}{\sqrt{D}} = u_{st}\frac{1-\beta^2}{D} \qquad Z_2 = N u_{st}\frac{2\zeta\beta}{\sqrt{D}} = u_{st}\frac{2\zeta\beta}{D}$$

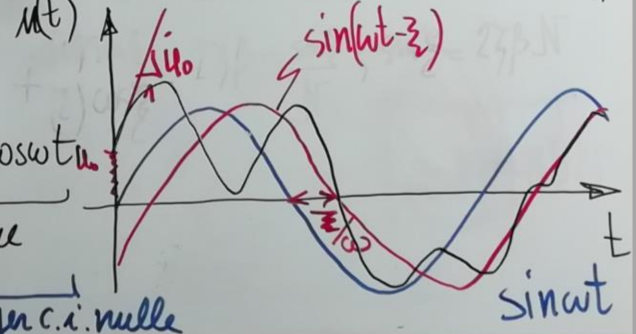
$$= Z_1\sin\omega t - Z_2\cos\omega t$$



$$u(t) = u_{po}(t) + u_p(t) \qquad \omega_d = \omega_1\sqrt{1-\zeta^2}$$

$$= \underbrace{e^{-\zeta\omega_1 t}(A\sin\omega_d t + B\cos\omega_d t)}_{\text{risp. transiente}} + \underbrace{Z_1\sin\omega t - Z_2\cos\omega t}_{\text{risp. a regime}}$$

c.i. $\dot u_0, u_0$ = risposta alle sole c.i. + risposta a F per c.i. nulle



SOMMARIO (Lez. 05)

- Risposta smorzata a forzante armonica.
- Effetto dello smorzamento su curve di risonanza e di fase.
- Picco finito di ampiezza in condizioni di risonanza; risposta in quadratura della risposta rispetto alla forzante.
- Risposta a regime in componenti $\sin\omega t$ e $\cos\omega t$.
- Integrale generale con risposta transiente e a regime.
- Next step: trattazione unificata in variabili complesse per risposta a $F\sin\omega t$ e/o $F\cos\omega t$.