

Università degli studi di Bergamo

Scuola di Ingegneria (Dolmine)

CCS Ingegneria Edile

LM-24 Ingegneria delle Costruzioni Edili

Complementi di Scienza delle Costruzioni

(ICAR/08 - SdC ; 6 CFU)

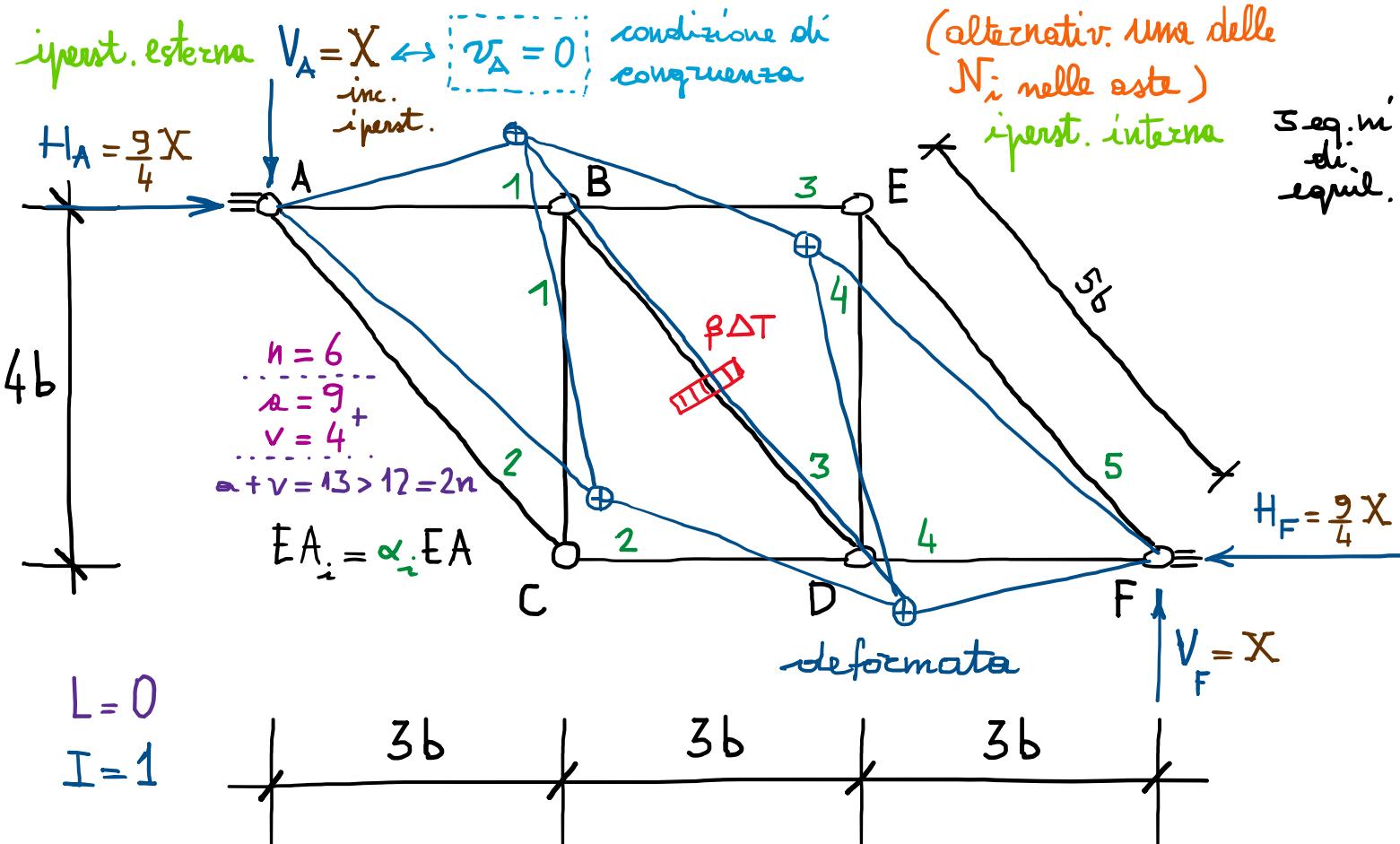
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prof. Egidio RIZZI

egidio.rizzi@unibg.it

LEZIONE 11

Soluzione di travatura reticolare iperstatico



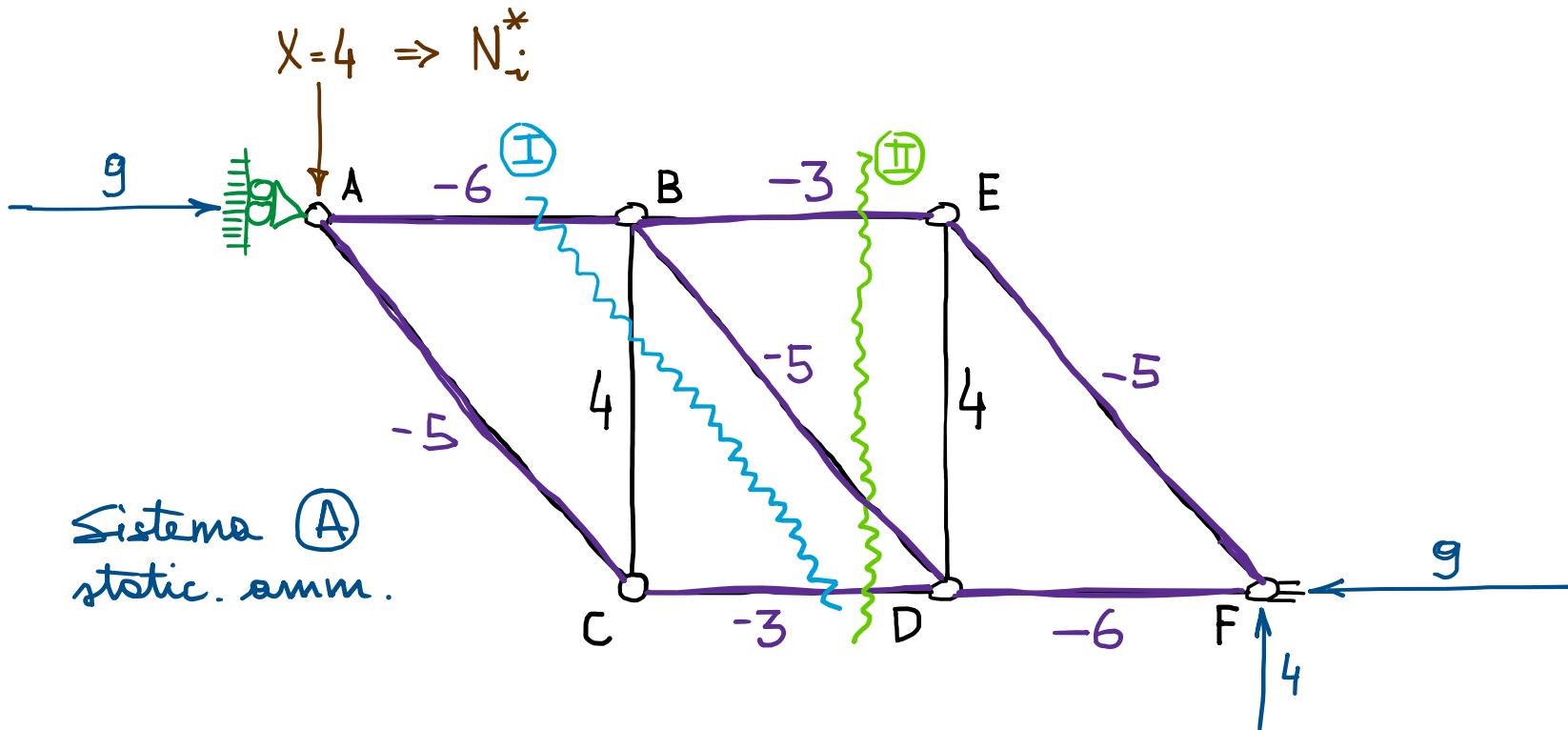
Sistema \textcircled{B} \rightarrow sistema reale
einem. amm.

Equilibrio globale :

$$\left\{ \begin{array}{l} \sum F_x = 0 \Rightarrow H_A = H_F = \frac{9}{4} X \\ \sum F_y = 0 \Rightarrow V_A = V_F = X \\ \sum M_A = 0 \Rightarrow V_F 9b = H_F 4b \Rightarrow H_F = \frac{9}{4} V_F = \frac{9}{4} X \end{array} \right.$$

$$RV = RV(X)$$

Struttura fittizia



Metodo delle sezioni (Ritter)

$$\textcircled{I} \quad \sum M_B^{sn} = 0 \Rightarrow N_{CD}^* = -3 \checkmark$$

$$\sum M_C^{sn} = 0 \Rightarrow N_{AB}^* = -6 \checkmark$$

$$\sum F_y^{sn} = 0 \Rightarrow N_{BC}^* = 4 \checkmark$$

(v. astre tre-carrelli)

$$\textcircled{II} \quad \sum M_B^{sn} = 0 \Rightarrow N_{BC}^* = -3 \checkmark$$

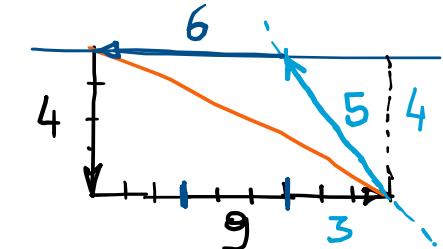
$$\sum M_D^{sn} = 0 \Rightarrow N_{BE}^* = -3 \checkmark$$

$$\sum F_y^{sn} = 0 \Rightarrow N_{BD}^* = -5 \quad (N_{BDy} = 4)$$

Metodo dei nodi

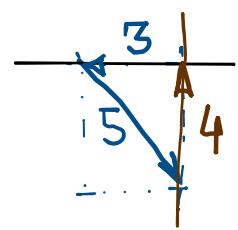
Nodo A:

(Nodo F)



poligono delle forze chiuso
(statica grafica)

Nodo C:
(Nodo E)



Scrittura del PLV (PFV, CS di congruenza):

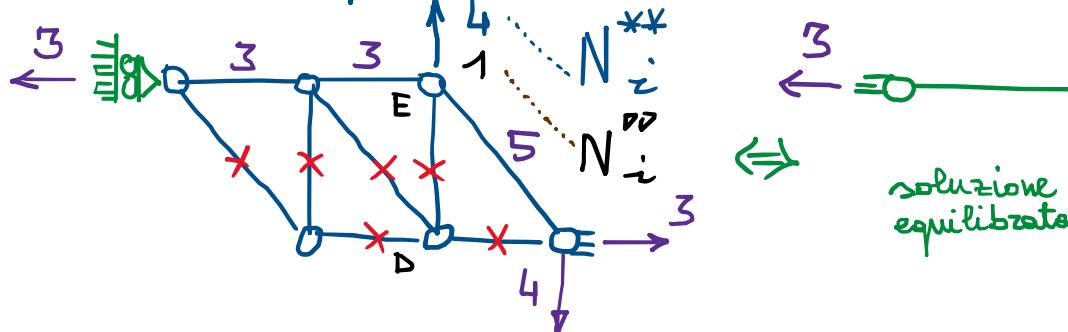
$$\int_e^{AB} = 1, \underset{v_A=0}{\underset{\text{str}}{\int}} = \int_{\text{str}} N_i^0 \frac{N_i^0 + X N_i^*}{EA_i} dx_i + \int_{BD} N_{BD} \alpha \frac{\beta \Delta T}{\sum_t} dx = \int_i^{AB}$$

$N_i = \text{cost nelle sezioni}$
 $(EA_i = \alpha_i EA \text{ cost})$

$$X = - \frac{\sum_i \frac{N_i^0 N_i^* l_i}{EA_i} + N_{BD} \alpha \beta \Delta T l_{BD}}{\sum_i \frac{N_i^0 l_i}{EA_i}} = - \frac{\sum_i \frac{N_i^0 N_i^* l_i}{\alpha_i} + N_{BD} \alpha \beta \Delta T l_{BD} EA}{\sum_i \frac{N_i^0 l_i}{\alpha_i}}$$

$$= - \frac{-5 \alpha \beta \Delta T 5b EA}{\frac{1100}{3} b} = + 100 \alpha \beta \Delta T EA \frac{3}{1100} = \boxed{\frac{3}{11} \alpha \beta \Delta T EA = X}$$

Calcolo degli spost. noduali (es. $v_E = ?$)



deformata

soluzione
equilibrata

$$N_i = \frac{X}{4} N_i^*$$

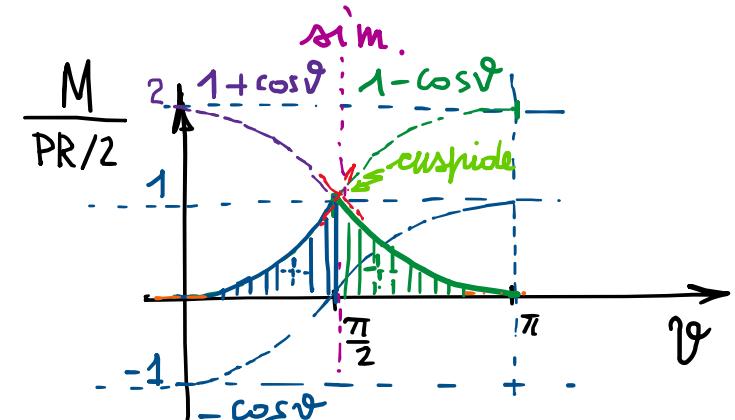
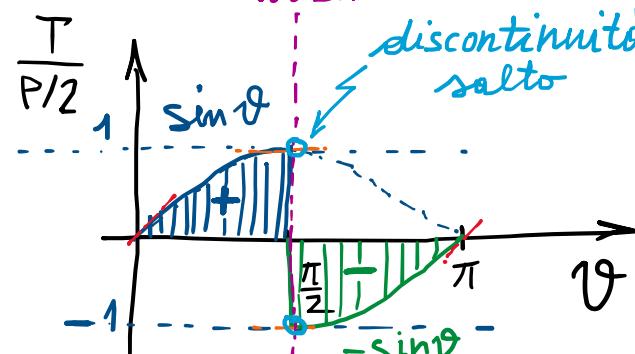
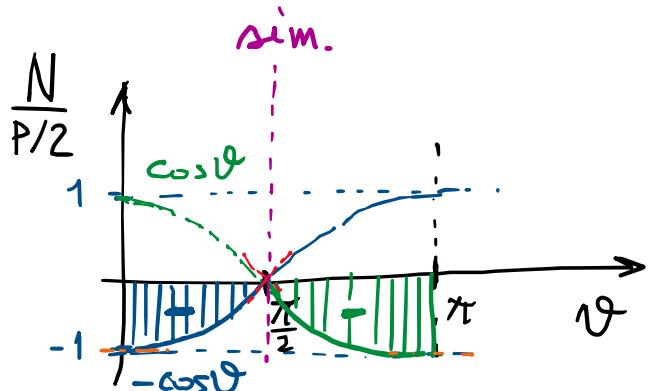
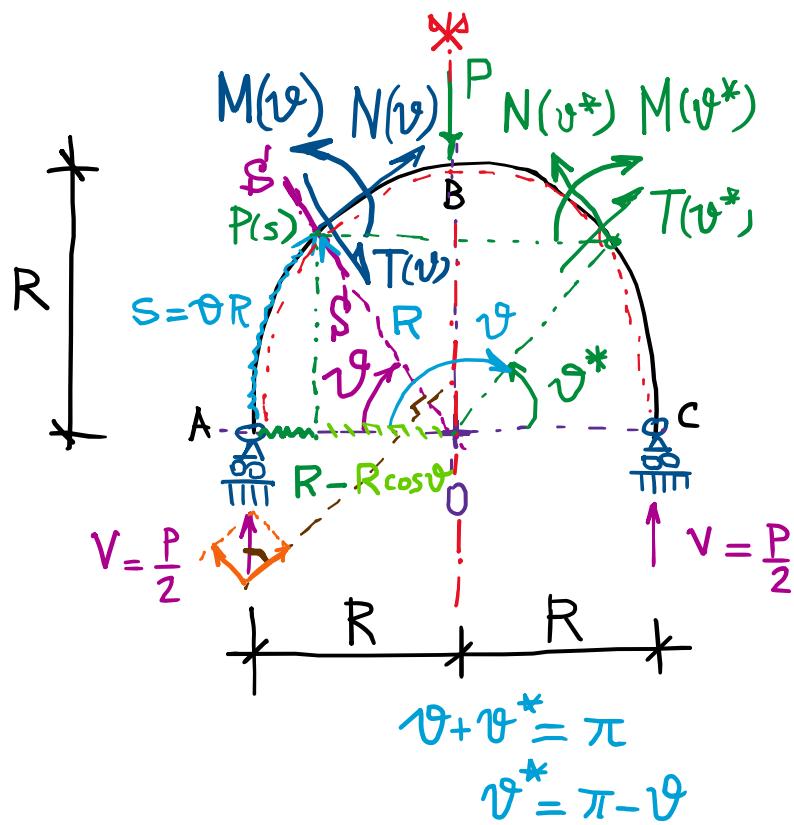
$$1. v_E = \sum_i \frac{N_i^0}{4} \left(N_i^0 + \frac{X N_i^*}{4} \right) \frac{l_i}{\alpha_i EA}$$

$$= -\frac{3}{2} \alpha \beta \Delta T b = v_E$$

$$v_E = v_D + \Delta l_{ED}^e$$

$$v_D = v_E - \frac{N_{ED} l_{ED}}{EA_{ED}}$$

Azioni interne in este curve



← aguale rappresentazione analitica (in coordinate polari)

Arco circolare (con carico concentrato in chiave):

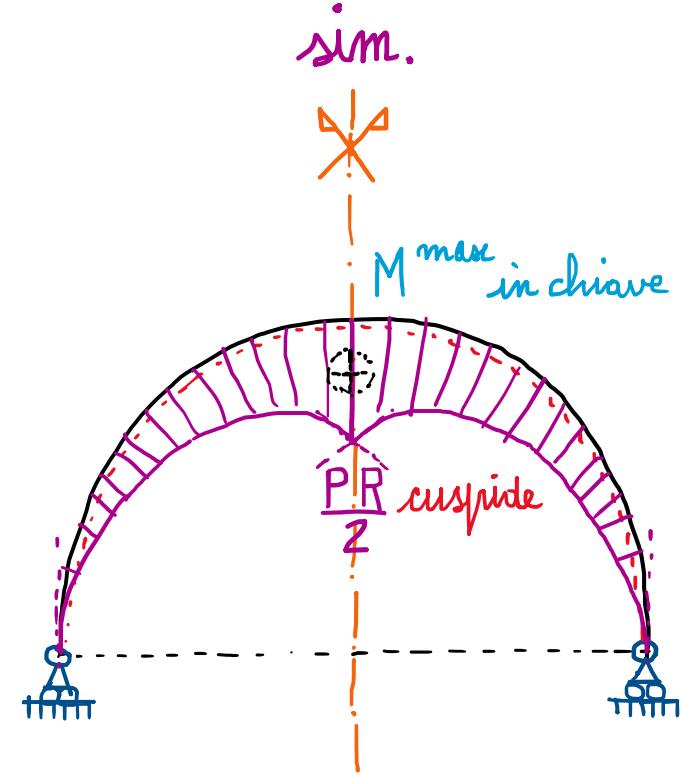
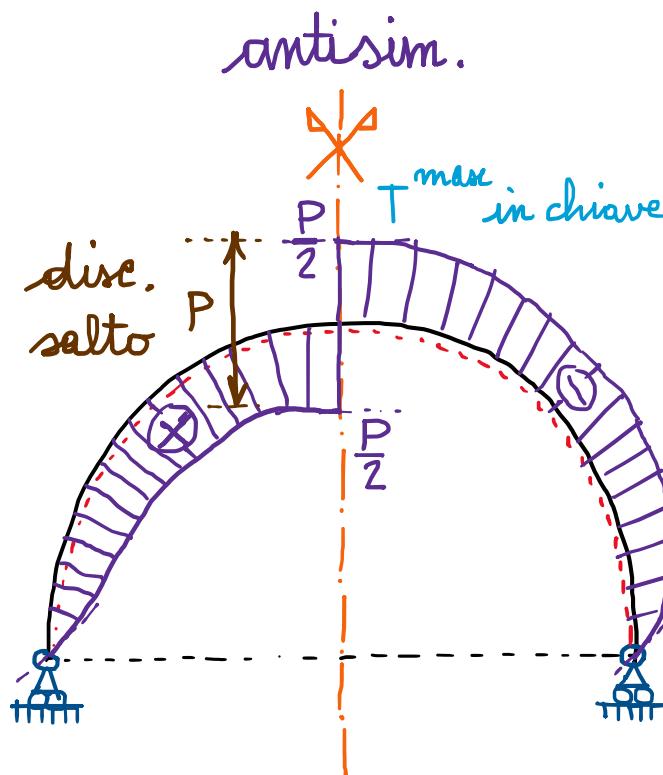
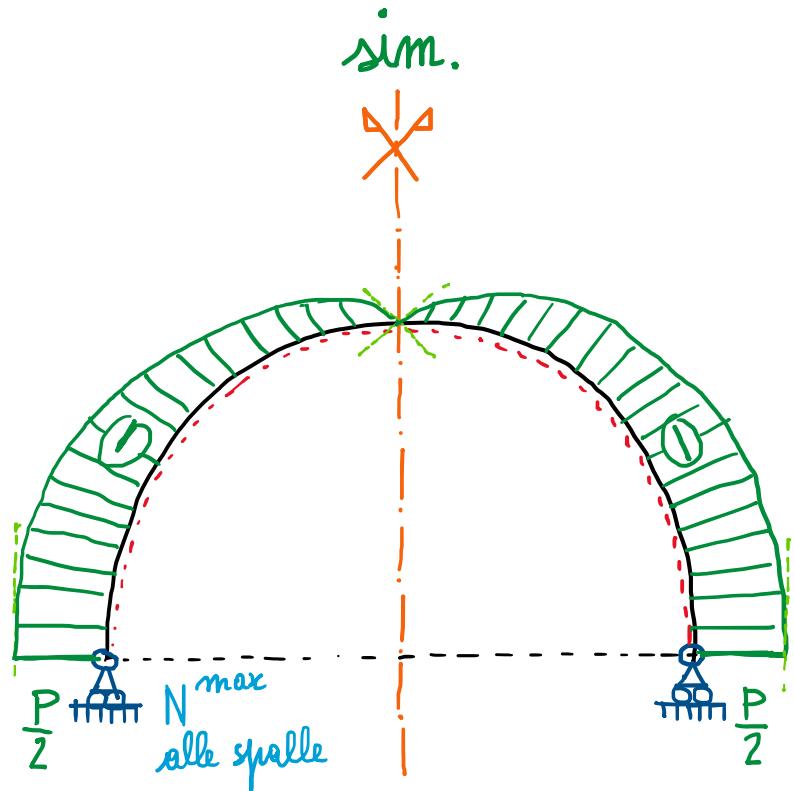
$$\left\{ \begin{array}{l} N(\vartheta) = -\frac{P}{2} \cos \vartheta \\ T(\vartheta) = \frac{P}{2} \sin \vartheta \\ M(\vartheta) = \frac{P}{2} R (1 - \cos \vartheta) \end{array} \right. \quad \begin{array}{l} 0 \leq \vartheta < \frac{\pi}{2} \\ \frac{\pi}{2} < \vartheta \leq \pi \end{array}$$

equilibrio locale
delle porzione \widehat{AP}

$$\left\{ \begin{array}{l} N(\vartheta^*) = -\frac{P}{2} \cos \vartheta^* \\ T(\vartheta^*) = -\frac{P}{2} \sin \vartheta^* \\ M(\vartheta^*) = \frac{PR}{2} (1 - \cos \vartheta^*) = \frac{PR}{2} (1 + \cos \vartheta) \end{array} \right. \quad \begin{array}{l} 0 \leq \vartheta^* < \frac{\pi}{2} \\ \frac{\pi}{2} < \vartheta^* \leq \pi \end{array}$$

dipendenza analitica
delle funzioni
di Azione Interna
e loro
rappresentazione

Diagrammi delle Azioni Interni N , T , M (andamenti funzionali rappresentati su fondamentali coincidenti con la struttura stessa)



$$N(\vartheta) \quad \longleftrightarrow \quad \boxed{+}$$

$$T(\vartheta) \quad \longleftrightarrow \quad \boxed{+}$$

$$M(\vartheta) \quad \longleftrightarrow \quad \boxed{+}$$