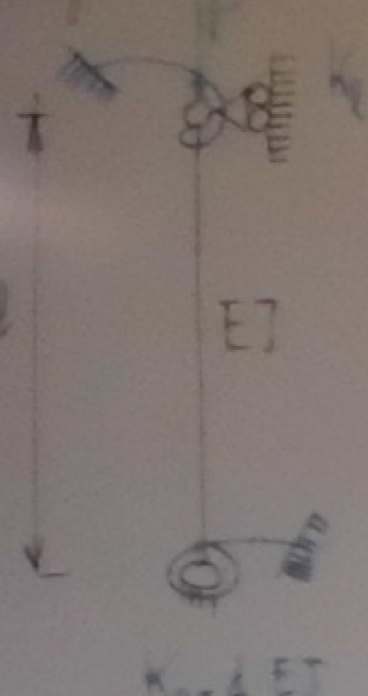
Analysis of different b.cs
Eq of the elastic line (in the deformed config.)
Boundary conditions
integr. constants to be
A, B, f determined
General integral
Imposing the b.cs
System of the b.cs
Singularity condition (leading to non-trivial solutions)
critical deformed shapes
corresponding to P=P cr,n
free inflection length
Different constraints
frames

Analysis of different b.c.s
Boundary conditions
A, B, f determined
Imposing the b.c.s
System of the b.c.s
Singularity condition (leading to non-trivial solutions)
critical deformed shapes
corresponding to P = P_cr,n
free inflection length
constraint coeff.
where l0 = l/√c
free inflection length
Different constraints (⇒ different b.c.s)
Euler beam
frames
from the beam to column

Different constraints (⇒ different b.c.s)
frames
free inflection length
critical deformed shapes corresponding to P = Pcr,n
EJb
EJc
tg αl = αl
αl = 4.493
= 1.431 π
(αl)² = 20.19
= 2.046 π²
≃ 2 π²
2l ≥ l0 ≥ l/2
from the beam to column stiffness ratio
compliance coeff.
μ
iso
hyper
for instance,
μ → ∞
μ → 0
c = 1/4
c = 1
c = 2
c = 4
l0 = 2l
l0 = l
l0 = l/2
l0 = 0.7 l
l/√2 ≥ l0 ≥ l/2

Generalized Euler beam
Example
$K_\ell = \frac{1}{\mu_\ell}\frac{EJ}{\ell}$
EJ
ℓ
$P_{cr}^{E} = c_N \pi^2 \frac{EJ}{\ell^2}$
$\mu_o = 0$
$\mu_\ell = \mu$
$c_N = 2\,\frac{0.4+1/3}{0.2+1/3} = 2\,\frac{1.2+1}{0.6+1} = 2\,\frac{2.2}{1.6} = 2\,\frac{1.1}{0.8} = \frac{11}{4} = 2.75$
Newmark formula
$c_N = \frac{(0.4+\mu_o)(0.4+\mu_\ell)}{(0.2+\mu_o)(0.2+\mu_\ell)}$
$K_o = \frac{1}{\mu_o}\frac{EJ}{\ell}$
μ_i compliance coefficients
$1/\mu_i$ stiffness coefficients
$\mu_i \to 0$ $c_N = 4$ ✓
$\mu_i \to \infty$ $c_N = 1$ ✓
- Exact solution in particular cases
- Approximate solution (good accuracy, err ~ 1.5%)
$K = 3\frac{EJ}{\ell} = \frac{1}{\frac{1}{3}}\frac{EJ}{\ell}$
Stability assessment (verifica di stabilità)
$\sigma = \frac{P}{A} \le \frac{P_{cr}^{E}}{A} = \sigma_{cr}^{E} = \frac{\pi^2}{A}\frac{EJ}{\ell_0^2} = \frac{\pi^2}{A}\frac{E}{\ell_0^2}Ar^2 = \pi^2\frac{E}{\left(\frac{\ell_0}{r}\right)^2} = \pi^2\frac{E}{\lambda^2}$
$P_{cr}^{E} = \pi^2\frac{EJ_{min}}{\ell_0^2}$; $J_{min} = A\,r_{min}^2$
slenderness
$r = \rho =$ inertia radius
Under no specific constraints, ruling the plane where flexural instability may take place, one shall rely only on the min. inertia moment of the cross section $J = J_{min}$ (rather than J_{max}!)
- Slenderness $\lambda = \frac{\ell_0}{r}$
- length ℓ
- cross section $r = \rho$
- boundary conditions
σ
Non-linear range
$\sigma_{cr}^{E} = \pi^2\frac{E}{\lambda^2}$
σ_0
σ_p
strength
$\lambda = \frac{\ell_0}{r}$
$\lambda_{lim} \approx 100$
slender elements
ε_p
ε
limit of proportionality
$\lambda = \lambda_{lim} \to \sigma_p = \pi^2\frac{E}{\lambda_{lim}^2}$; $\lambda_{lim} = \pi\sqrt{\frac{E}{\sigma_p}}$
Steel $E = 200000$ MPa
$\sigma_p = 200$ MPa
Strength assessment
$\sigma \le \sigma_p$
$\omega\sigma \le \sigma_p$
$\omega\frac{N}{A} \le \sigma_p$

- Generalized Euler beam



$K_l = \frac{1}{\mu_l}\frac{EJ}{l}$

$K_0 = \frac{1}{\mu_0}\frac{EJ}{l}$

Newmark formula

$c_N = \frac{(0.4+\mu_0)(0.4+\mu_l)}{(0.2+\mu_0)(0.2+\mu_l)}$

μ_i compliance coefficients

$1/\mu_i$ stiffness coefficients

$\mu_i \to 0 \quad c_N = 4$ ✓

$\mu_i \to \infty \quad c_N = 1$ ✓

- Exact solution in particular cases
- Approximate solution (error ~ 1.5%)

Example

$\mu_0 = 0$

$\mu_l = \mu$

$$c_N = 2\,\frac{0.4 + 1/3}{0.2 + 1/3} = 2\,\frac{1.2+1}{0.6+1} = 2\,\frac{2.2}{1.6} = \cancel{2}\,\frac{1.1}{\cancel{0.8}\,0.4} = \frac{11}{4} = 2.75$$

$K = 3\frac{EJ}{l} = \frac{1}{\frac{1}{3}}\frac{EJ}{l}$

- **Stability assessment** (verifica di stabilità)

$$\sigma = \frac{P}{A} \le \frac{P^E_{cr}}{A} = \sigma^E_{cr} = \frac{\pi^2}{A}\frac{EJ}{l_0^2} = \frac{\pi^2}{A}\frac{E}{l_0^2}Ar^2 = \pi^2\frac{E}{\underbrace{\left(\frac{l_0}{r}\right)^2}_{\lambda}} = \pi^2\frac{E}{\lambda^2}$$

$$P^E_{cr} = \pi^2\frac{EJ_{min}}{l^2} \; ; \quad J_{min} = A\,r^2_{min}$$

slenderness λ

$r = \rho =$ inertia radius

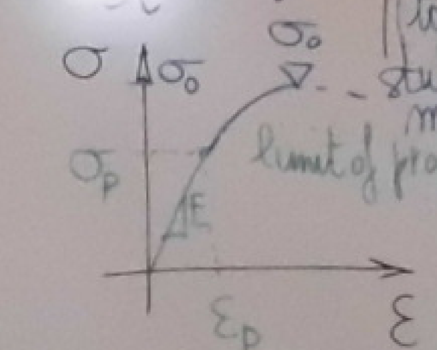
- Under no specific constraints, ruling the plane where flexural instability may take place, one shall rely only on the min. inertia moment of the cross section $J = J_{min}$ (rather than J_{max}!)

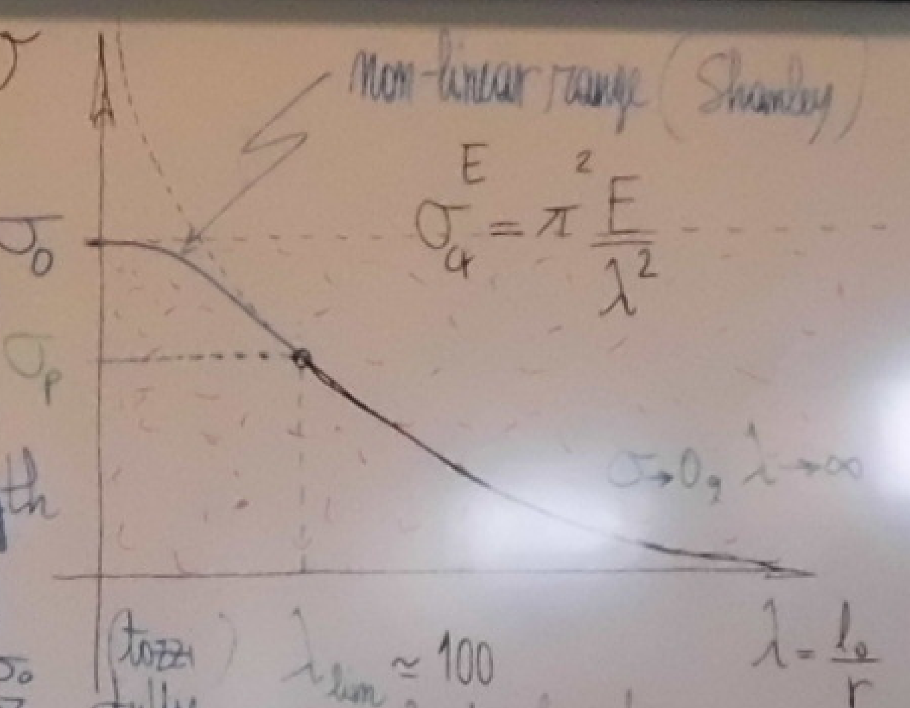
- Slenderness $\lambda = \frac{l_0}{r}$
  - length l
  - cross section $r = \rho$
  - boundary conditions

σ_0

σ_p

strength





$$\sigma^E_{cr} = \pi^2\frac{E}{\lambda^2}$$

(tozzi) stubby members ⟵ $\lambda_{lim} \simeq 100$ ⟶ slender elements (snelli)

⟶ linear behaviour

$\lambda \ge \lambda_{lim}$

$$\lambda = \lambda_{lim} \to \sigma_p = \pi^2\frac{E}{\lambda^2_{lim}} \; ; \quad \lambda_{lim} = \pi\sqrt{\frac{E}{\sigma_p}} \simeq \pi\sqrt{1000} = \pi\,10\sqrt{10} = \sqrt{\pi^2 10}\;10 \simeq 10 \cdot 10 = 100$$

steel $E \simeq 200\,000$ MPa

$\sigma_p \simeq 200$ MPa

- **Strength assessment** (verifica di resistenza)

$\sigma \le \sigma_0$

$\le \frac{\sigma_{cr}}{\sigma_0}$ $\qquad = \frac{1}{\omega}\sigma_0$

⇓

$\omega\,\sigma \le \sigma_0$

$\omega\,\frac{N}{A} \le \sigma_0$

Omega method: amplification coeff for the axial force N in view of performing the stability assessment instead of the strength assessment

$$(\lambda)\;\omega = \frac{\sigma_0}{\sigma_{cr}} = \frac{\sigma_0}{\pi^2\frac{E}{\lambda^2}} = \frac{1}{\pi^2}\frac{\sigma_0}{E}\lambda^2$$

$\lambda \ge \lambda_{lim}$

$\simeq 100$

steel $\simeq \frac{1}{\pi^2}\frac{1}{1000}\lambda^2 \simeq \frac{1}{10\,000}\lambda^2$

See standards, where $\omega(\lambda)$ is given based on structural properties and boundary conditions

$\lambda = \lambda_{lim} \simeq 100 \quad (\sigma_p \to \sigma_0)$

$\omega = 1$

- **Stability assessment** (verifica di stabilità)

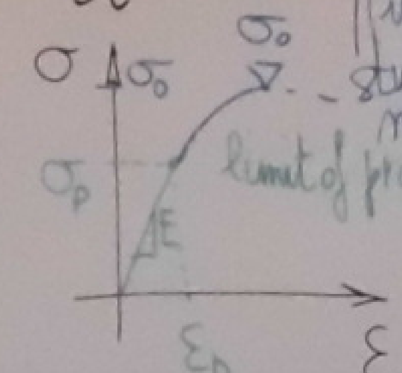
$$P \le \frac{P_{cr}^E}{A} = \sigma_{cr} = \frac{\pi^2}{A}\frac{EJ}{l_0^2} = \frac{\pi^2}{A}\frac{E}{l_0^2}Ar^2 = \pi^2\frac{E}{\underbrace{\left(\frac{l_0}{r}\right)^2}_{\lambda}} = \pi^2\frac{E}{\lambda^2}$$

$$P_{cr}^E = \pi^2\frac{EJ_{min}}{l_0^2} \; ; \; J_{min} = A\,r_{min}^2$$ slenderness

$r = \rho =$ inertia radius

σ_0, σ_p, strength

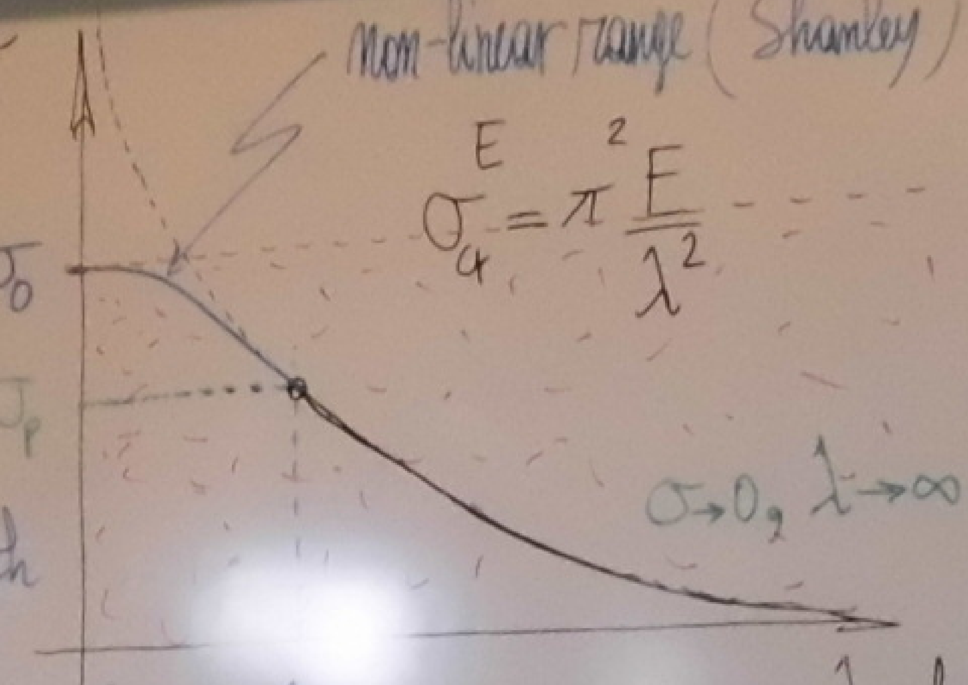
- Under no specific constraints, ruling the plane where flexural instability may take place, one shall rely only on the min. inertia moment of the cross section $J = J_{min}$ (rather than J_{max}!)
- Slenderness $\lambda = \frac{l_0}{r}$
  - length l
  - cross section $r = \rho$
  - boundary conditions



$\lambda = \lambda_{lim} \rightarrow \sigma_p = \pi^2\frac{E}{\lambda_{lim}^2} \; ; \; \lambda_{lim} = \pi\sqrt{\frac{E}{\sigma_p}} \simeq \pi\sqrt{1000} = \pi\,10\sqrt{10} = \sqrt{\pi^2 10}\;10 \simeq 10 \cdot 10 = 100$

steel $E \simeq 200\,000$ MPa

$\sigma_p \simeq 200$ MPa



$\sigma_{cr}^E = \pi^2\frac{E}{\lambda^2}$

$\lambda \ge \lambda_{lim}$

$\lambda \ge \lambda_{lim} \simeq 100$

- **Strength assessment** (verifica di resistenze)

$\sigma \le \sigma_0$

stability $\sigma \le \frac{\sigma_{cr}}{\sigma_0}$ $\quad \sigma_0 = \frac{1}{\omega}\sigma_0$

$\Downarrow$

$\omega\sigma \le \sigma_0$

$\omega\frac{N}{A} \le \sigma_0$

Omega method

amplification coeff. for the axial force N in view of performing the stability assessment instead of the strength assessm.

$(\lambda)\;\omega = \frac{\sigma_0}{\sigma_{cr}} = \frac{\sigma_0}{\pi^2\frac{E}{\lambda^2}} = \frac{1}{\pi^2}\frac{\sigma_0}{E}\lambda^2$

steel $\simeq \frac{1}{\pi^2}\frac{1}{1000}\lambda^2 \simeq \frac{1}{10\,000}\lambda^2$

See standards, where $\omega(\lambda)$ is given based on structural properties and boundary conditions

$\lambda = \lambda_{lim} \simeq 100 \quad (\sigma_p \rightarrow \sigma_0)$

$\omega = 1$