

Università degli studi di Bergamo

Scuola di Ingegneria (Dolmine)

CCS Ingegneria Edile

LM-24 Ingegneria delle Costruzioni Edili

Complementi di Scienza delle Costruzioni

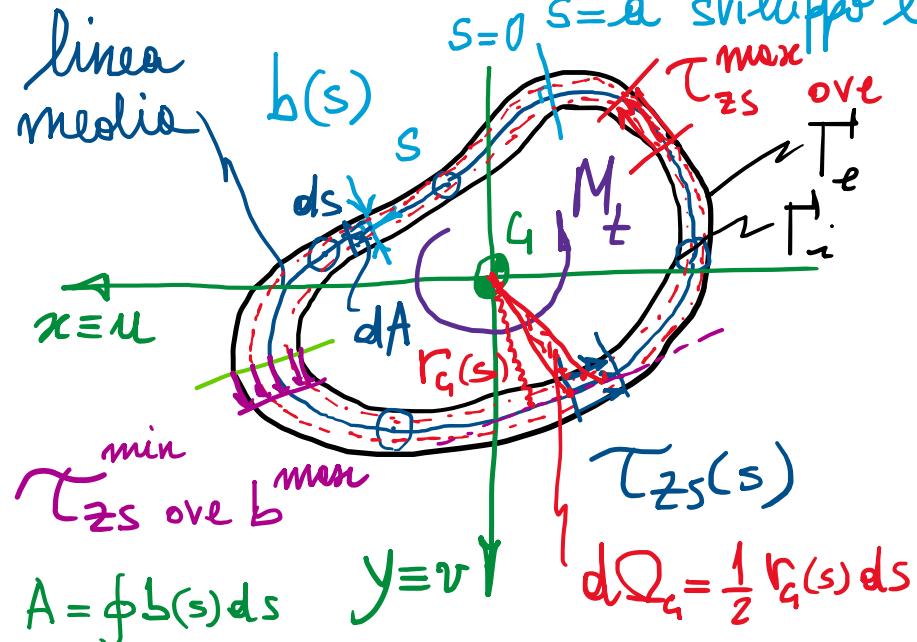
(ICAR/08 - SdC; 6 CFU)

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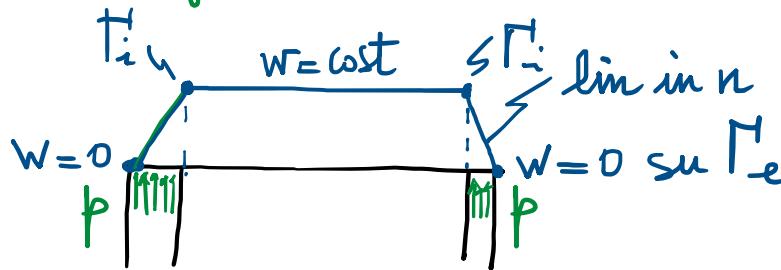
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LEZIONE 22

Torsione nei profili sottili chiusi (monocellulari)  biconnessi (2 tagli per separare) con continuità



- Analogie delle membrane:



$$\left\{ \begin{array}{l} T_{zs} = \frac{\partial \varphi}{\partial n} = \text{cost} \quad \text{tendenze cost sullo spessore} \\ T_{zn} = -\frac{\partial \varphi}{\partial s} \approx 0 \quad \text{tendenze nulle in s} \end{array} \right.$$

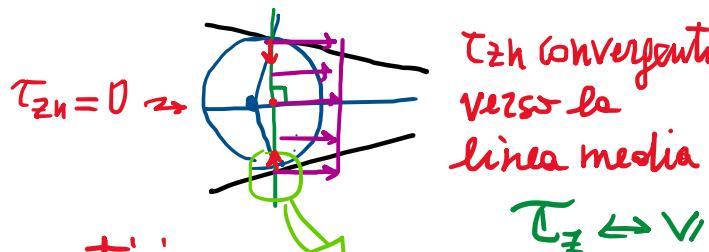
- Spessore $b(s)$ in generale variabile lungo lo sviluppo del profilo, rimanendo "piccolo": $b(s) \ll a \quad [|b'(s)| \ll 1] \quad (a > 10 b)$

- Analogie idrodinamiche:

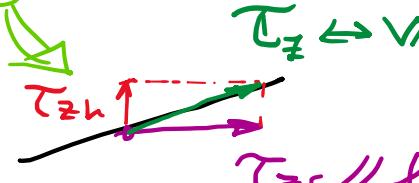
- traiettorie che abbracciano l'intero profilo $\sim \parallel$ alle linee medie.
- velocità \Rightarrow tens. tangenziali $\sim \parallel$ alle l.m. $\Rightarrow T_{zs}$ solo
- distribuz. di velocità (T_{zs}) cost. sullo spessore.
- "flusso delle tensioni tangenziali": portata costante

$$q(s) = T_{zs}(s) b(s) = q = \text{cost} \Leftrightarrow Q = VA = \text{cost}$$

$\rightarrow b \downarrow (b' < 0)$



antisimm. sullo spessore



$T_{zh} \ll T_{zs} \Rightarrow 0$



T_{zh} divergenti delle linee medie

$T_{zh} \ll T_{zs} \Rightarrow 0$

• Formula di Rudolf BREDT (~ 1896): [ragionamento "isostatico"] \rightarrow soluzione approssimata

- Equivalenza statica tra M_t e le $T_{zs}(s)$ come sopra discussi:

$$M_t = \oint \underbrace{T_{zs}(s) b(s)}_{\text{d}A} ds \cdot r_g(s)$$

$\Delta p: q(s) = q = \text{cost}$

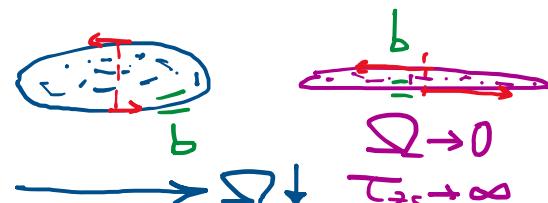
$$= 2q \oint \underbrace{\frac{1}{2} r_g(s) ds}_{d\Omega_g} \quad (\Omega_g(s): \text{arie settoriale riferita a } G)$$

$$\left\{ M_t = 2q \Sigma \right\} \Rightarrow q = T_{zs}(s) b(s) = \frac{M_t}{2\Sigma}$$

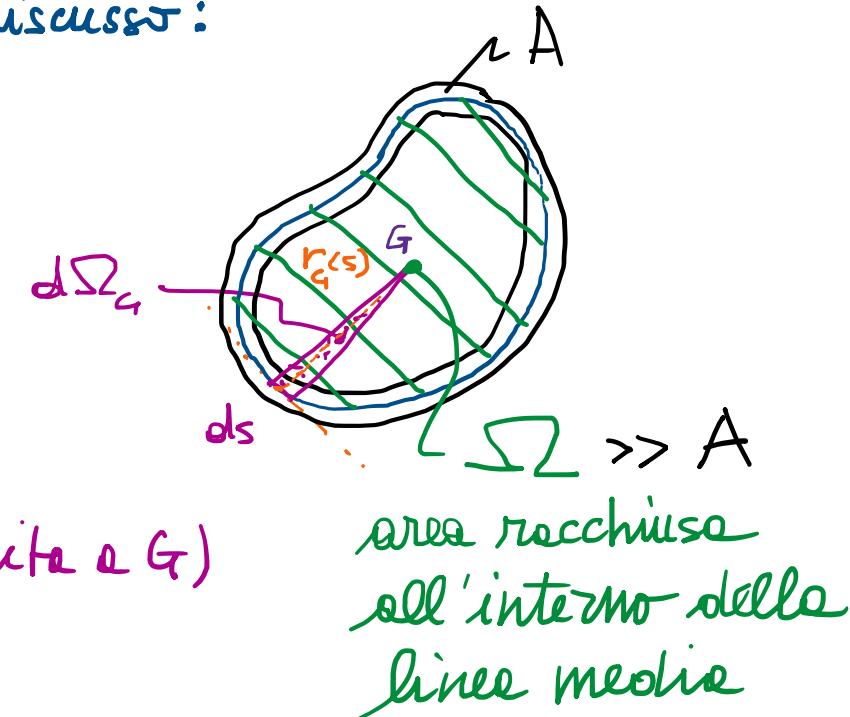
Riguardo l'entità delle T_{zs} , a parità di spessore, il param. geometrico cruciale è Σ

$$D = 2R$$

$$\begin{aligned} \Sigma &= \pi R^2 \\ \Sigma_{\text{spesi}}^{\text{max}} & \rightarrow \Sigma \downarrow \rightarrow b \downarrow \rightarrow \Sigma \rightarrow 0 \end{aligned}$$



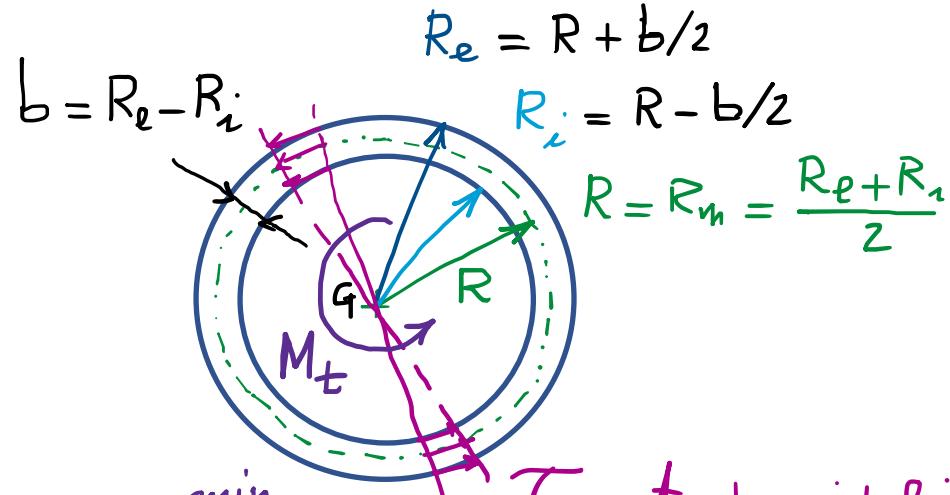
$$(idem per J) \quad T_{zs} \sim \Sigma^{-1}$$



Le $T_{zs}(s)$ risultano:

- dirett. prop. a M_t
 - invers. prop. a Σ
 - invers. prop. a $b(s)$
- $T_{zs}^{\text{max}} \text{ ove } b^{\text{min}} ; T_{zs}^{\text{min}} \text{ ove } b^{\text{max}}$

- Sezione circolare cava



$$\begin{aligned} r = R_i &: \tau^{\min} \\ r = R_m &: \tau^m \\ r = R_e &: \tau^{\max} \end{aligned}$$

τ_{zt} trapezoidali
nullo spessore
 $R_i \leq r \leq R_e$

flessione nei profili rettili chiusi

Bredt

$\tau^m = \frac{M_E}{2\pi R^2 b \left[1 + \frac{1}{4} \left(\frac{b}{R} \right)^2 \right]}$

$\tau \approx \frac{M_t}{2\pi R^2 b}$

$\tau^{\text{cost. nullo spessore}} \sim \Omega$ area racchiusa all'interno della linea media

$b \sim \frac{1}{10} R$

perete sottile

$\Rightarrow \text{se } \frac{b}{R} \rightarrow 0$

$$\tau_{zt} = \frac{M_t}{J_q} r$$

$$\begin{aligned} J_q &= \frac{\pi}{2} (R_e^4 - R_i^4) \\ &= \frac{\pi}{2} (R_e^2 + R_i^2)(R_e^2 - R_i^2) \\ &= \frac{\pi}{2} \left(\left(R + \frac{b}{2} \right)^2 + \left(R - \frac{b}{2} \right)^2 \right) \left(\left(R + \frac{b}{2} \right)^2 - \left(R - \frac{b}{2} \right)^2 \right) \\ &= \frac{\pi}{2} \left(R^2 + \frac{b^2}{4} + Rb + R^2 + \frac{b^2}{4} - Rb \right). \\ &\quad \left(R^2 + \frac{b^2}{4} + Rb - R^2 - \frac{b^2}{4} + Rb \right) \\ &= \frac{\pi}{2} 2 \left(R^2 + \frac{b^2}{4} \right) 2 R b = 2\pi R b \left(R^2 + \frac{b^2}{4} \right) \\ &= 2\pi R^3 b \left[1 + \frac{1}{4} \left(\frac{b}{R} \right)^2 \right] = J_q \end{aligned}$$

- Momento d'inerzia torsionale: valutabile, a volte del calcolo delle tang. via PLV

$$\frac{dL_e}{dz} = M_t \beta = \oint \tau_{zs} \frac{\frac{dA}{G}}{b(s) ds} = \frac{dL_i}{dz}$$

$$\beta = \frac{M_t}{GJ}; \quad J = \frac{M_t}{G\beta}$$

$$G = \frac{E}{2(1+\nu)}$$

~~$$\frac{M_t}{GJ} = \oint \frac{\tau_{zs}^2(s)}{G} b(s) ds$$~~

$$\frac{1}{J} = \oint \frac{M_t^2}{(2\Omega b)^2} b(s) ds = \frac{1}{4\Omega^2} \oint \frac{ds}{b(s)} \Rightarrow$$

La C. di Bredt

$$J = \frac{4\Omega^2}{\oint \frac{ds}{b(s)}}$$

$$J \sim \Omega^2$$

$$= \frac{4\Omega^2 b}{a}$$

$b = \text{cost}$

$$a = \oint ds$$

$$\frac{b}{a} \sim \frac{1}{10}$$

- Ingobbamento (riferito alla linea media) \rightarrow Centro di torsione

$$d\Psi_g = \frac{\tau_{zs}(s)}{G\beta} ds - 2d\Omega_g$$

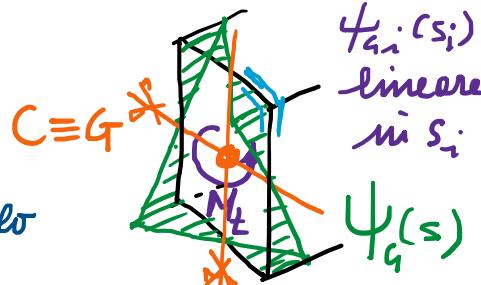
~~$$= \frac{J}{M_t} \frac{M_t}{2\Omega b(s)} ds - 2d\Omega_g$$~~

$$= \frac{2\Omega^2}{\oint \frac{ds}{b(s)}} \frac{1}{2\Omega} \frac{ds}{b(s)} - 2d\Omega_g$$

$$d\Psi_g = \frac{2\Omega}{\oint \frac{ds}{b(s)}} \frac{ds}{b(s)} - 2d\Omega_g \rightarrow$$

ingobb. medio nullo

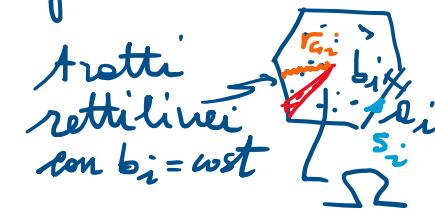
$$\bar{\Psi}_g = 0$$



$$\Psi_g(s) = \frac{2\Omega}{\oint \frac{ds}{b(s)}} \int_0^s \frac{ds}{b(s)} - 2\Omega_g(s) + \Psi_g^0$$

cost

Profili scatolari:



$$= \frac{4\Omega^2}{\sum_i b_i} = \frac{4\Omega^2}{\sum_i b_i} \cdot a = \sum_i s_i$$

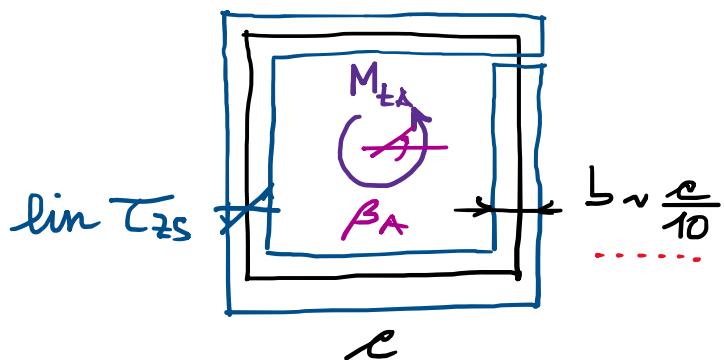
$$\left\{ \begin{array}{l} x_c = -\frac{1}{J_x} \oint \Psi_g(s) y(s) b(s) ds \\ y_c = \frac{1}{J_y} \oint \Psi_g(s) x(s) b(s) ds \end{array} \right.$$

coordinate del C. di To.
(se Esse di sim, C è stesse esse)

Confronto a torsione tra profilo sottile aperto e profilo sottile chiuso

$$\beta = \frac{M_t}{GJ}$$

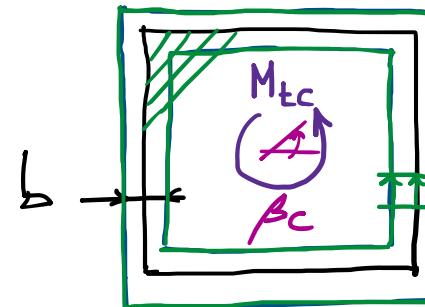
APERTO



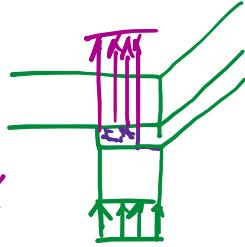
$\text{lin } \tau_{zs}$

the winner is...
profilo chiuso!

CHIUSO



saldate
difettose:
innescio di
frazioni



$\tau_{zs} \text{ cost}$

$$T \cdot b = \text{cost}$$

$$J_A = \sum_i \frac{1}{3} \pi_i b_i^3 = \frac{4}{3} \pi b^3$$

$$\frac{r}{b} \sim 10$$

$$\frac{r}{b} > 1$$

... inerzia torsionale

$$J_c = \frac{4 \Omega^2 b}{\sum_i \pi_i} = \frac{4 (\pi^2)^2 L}{4 \pi} = \pi^3 b$$

$$\tau_A = \frac{M_{tA}}{J_A} b = \frac{3}{4} \frac{M_{tA}}{\pi b^2}$$

... tensione tangenziale
(angolo unitario di)

"torsione"

$$\tau_c = \frac{M_{tc}}{2 \Omega b} = \frac{M_{tc}}{2 \pi^2 b}$$

Ese:

- a parità di M_t : $M_{tA} = M_{tc}$

$$- \frac{\tau_A}{\tau_c} = \frac{3}{2} \frac{\pi}{5} \sim 1.5 \cdot 10$$

$$- \frac{\beta_A}{\beta_c} = \frac{3}{4} \left(\frac{\pi}{5} \right) \sim 0.75 \cdot 100$$

- a parità di β : $\beta_A = \beta_c$

$$- \frac{M_{tc}}{M_{tA}} = \frac{3}{4} \left(\frac{\pi}{5} \right)^2 \sim 0.75 \cdot 100$$

Rapporti:

$$\frac{J_c}{J_A} = \frac{\pi^3 b}{4/3 \pi b^3} = \frac{3}{4} \left(\frac{\pi}{5} \right)^2 \ggg 1 \sim 0.75 \cdot 100 = 75$$

$$\sim \left(\frac{\pi}{5} \right)^2$$

$$\frac{\tau_A}{\tau_c} = \frac{3}{2} \frac{M_{tA}}{M_{tc}} \cancel{\frac{2 \pi^2 b}{M_{tc}}} = \frac{M_{tA}}{M_{tc}} \frac{3}{2} \left(\frac{\pi}{5} \right)^2 \sim 1.5 \cdot 10 = 15$$

$$\sim \left(\frac{\pi}{5} \right)^2$$

$$\frac{\beta_A}{\beta_c} = \frac{M_{tA}}{M_{tc}} \frac{J_c}{J_A} = \frac{M_{tA}}{M_{tc}} \frac{3}{4} \left(\frac{\pi}{5} \right)^2$$

$$\text{un ordine di grandezza}$$