

Università degli studi di Bergamo

Scuola di Ingegneria (Dolmine)

CCS Ingegneria Edile

LM-24 Ingegneria delle Costruzioni Edili

Complementi di Scienza delle Costruzioni

( ICAR/08 - SdC ; 6 CFU )

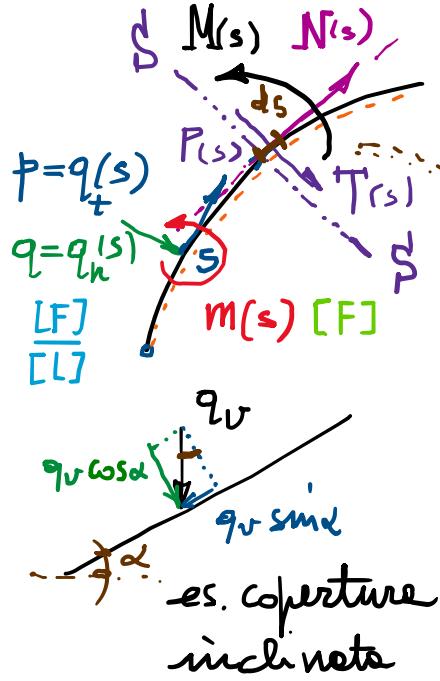
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LEZIONE 12

## Equazioni indefinite di equilibrio delleoste curve ( $N(s), ds$ )



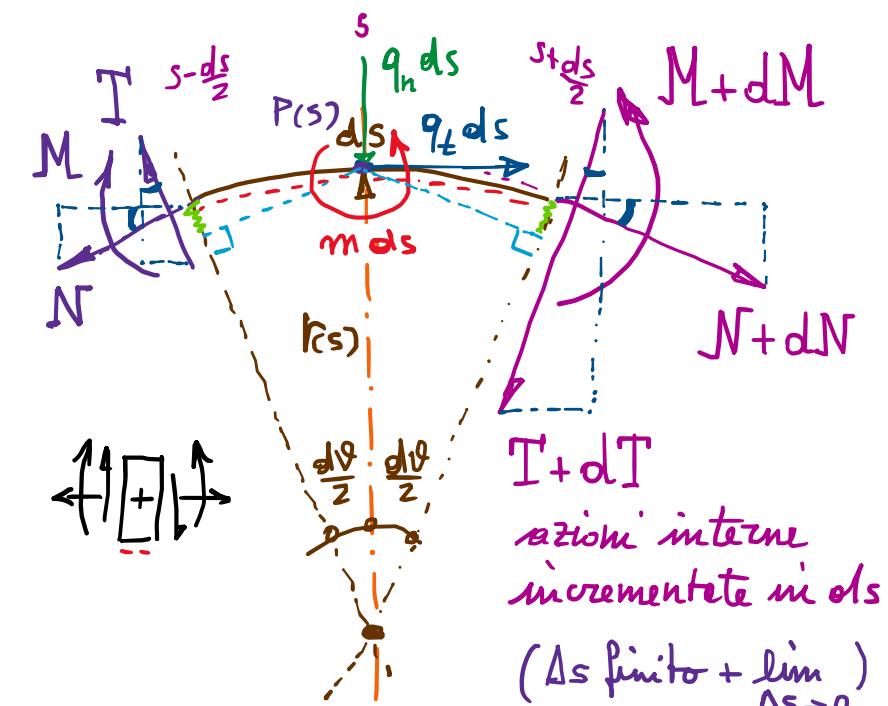
A.I.  $\left\{ \begin{array}{l} N(s): \text{azione axiale o normale} \\ T(s): \quad \text{,, tagliente o taglio} \\ M(s): \quad \text{,, flettente o momento} \end{array} \right.$

$ds = r(s) d\varphi$

$dV = \frac{1}{r(s)} ds$   
 $= \chi(s) ds$

raggio di curvatura locale  
 $r \approx l$  ( $r \rightarrow \infty$ , trave rettilinea)

elemento "tocco" (non tondo)



Equazioni di equilibrio: accoppiamento  $N, T$

$$\sum F_t^{\text{ds}} = 0 \Rightarrow (N + dN - N) \cos \frac{d\varphi}{2} - (T + dT - T) \sin \frac{d\varphi}{2} + q_t ds = 0 \Rightarrow \frac{dN}{ds} - T \frac{d\varphi}{2} - dT \frac{d\varphi}{2} + q_t ds = 0$$

$$\sum F_n^{\text{ds}} = 0 \Rightarrow (T + dT - T) \cos \frac{d\varphi}{2} + (N + dN - N) \sin \frac{d\varphi}{2} + q_h ds = 0 \Rightarrow \frac{dT}{ds} + N \frac{d\varphi}{2} + dN \frac{d\varphi}{2} + q_h ds = 0$$

$$\sum M_p^{\text{ds}} = 0 \Rightarrow (M + dM - M) - (T + dT - T) r \sin \frac{d\varphi}{2} - (N + dN - N) r (1 - \cos \frac{d\varphi}{2}) + m ds = 0$$

$$\frac{dN}{ds} - T \frac{d\varphi}{2} - dT \frac{d\varphi}{2} + q_t ds = 0 \Rightarrow N(s) = -q_t(s) + \frac{T(s)}{r(s)}$$

$$\frac{dT}{ds} + N \frac{d\varphi}{2} + dN \frac{d\varphi}{2} + q_h ds = 0 \Rightarrow T(s) = -q_h(s) - \frac{N(s)}{r(s)}$$

$$\frac{dM}{ds} - T r \frac{d\varphi}{2} - dT r \frac{d\varphi}{2} + m ds = 0 \Rightarrow M(s) = -m(s) + T(s)$$

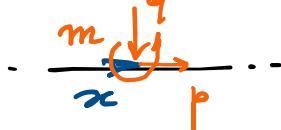
$$M''(s) = -m(s) + T'(s) = -(m(s) + q_h(s)) + \frac{N(s)}{r(s)}$$

accoppiamento  $T, M$

Note:

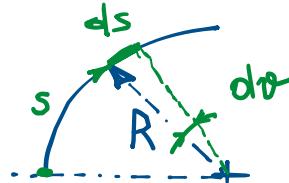
- nelle altre curve si registra accoppiamento  $N, T$ , in cui la variazione di ciascuna azione interna risulta accoppiata all'andamento dell'altra.
- si conferma l'accoppiamento  $T, M$ , già visto per altre rettilinee, ottenibili come segue -

- per altre rettilinee ( $r \rightarrow \infty$ ):



$$\begin{cases} N'(x) = -p(x) \\ T'(x) = -q(x) \\ M''(x) = -m(x) + T(x) \Rightarrow M''(x) = -\overset{\circ}{m}(x) + \overset{\circ}{T}(x) = -(m'(x) + q(x)) \end{cases}$$

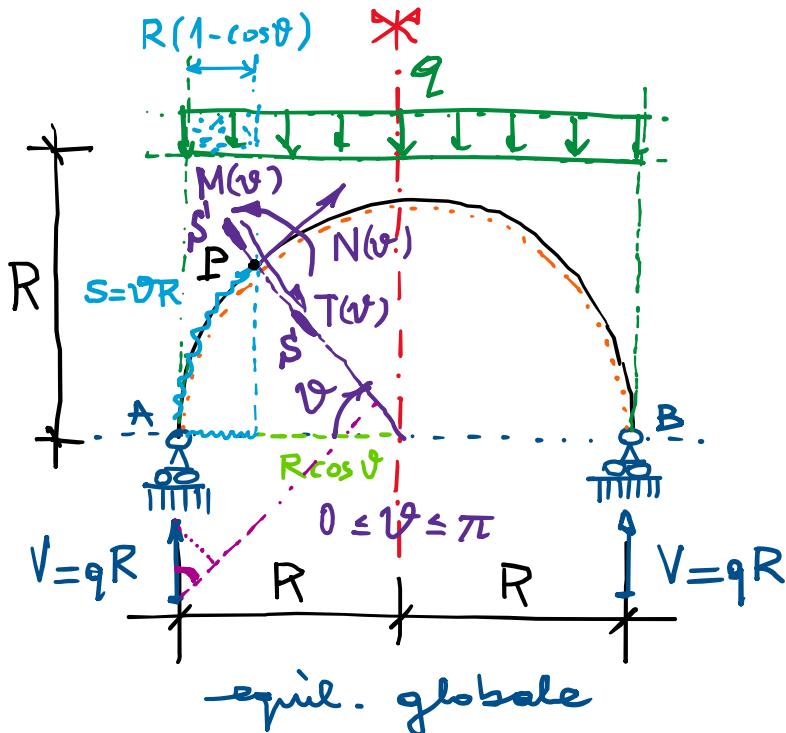
- per altre circolari ( $r(s) = R = \text{cost}$ )



$$\begin{aligned} ds &= R d\vartheta \\ d\vartheta &= \frac{1}{R} ds \end{aligned} \rightarrow \frac{d(\ )}{ds} = \frac{1}{R} \frac{d(\ )}{d\vartheta} \Rightarrow \begin{cases} N'(\vartheta) = -q_r(\vartheta) R + T(\vartheta) \\ T'(\vartheta) = -q_h(\vartheta) R - N(\vartheta) \\ M''(\vartheta) = -m(\vartheta) R + T(\vartheta) R \end{cases} (*)$$

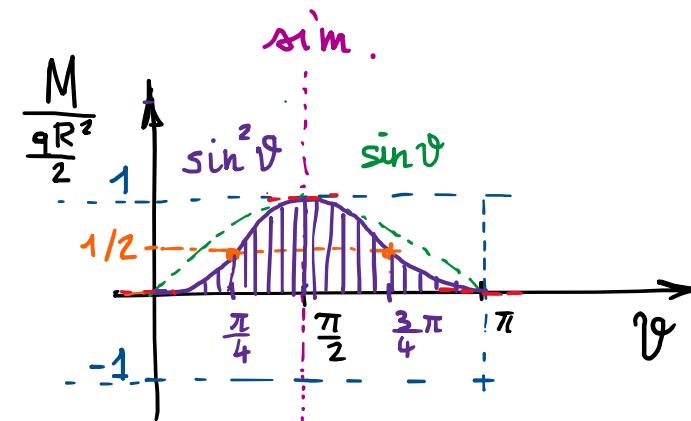
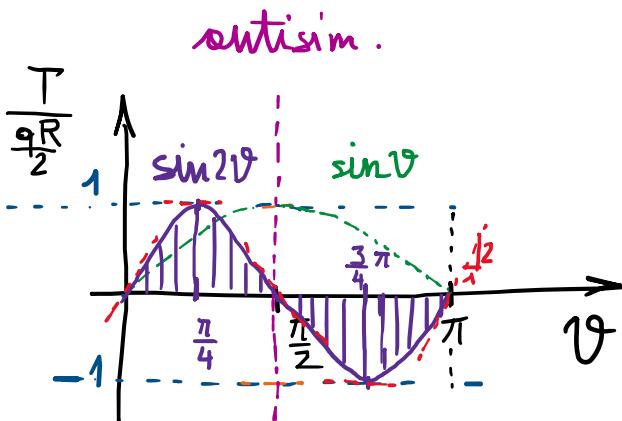
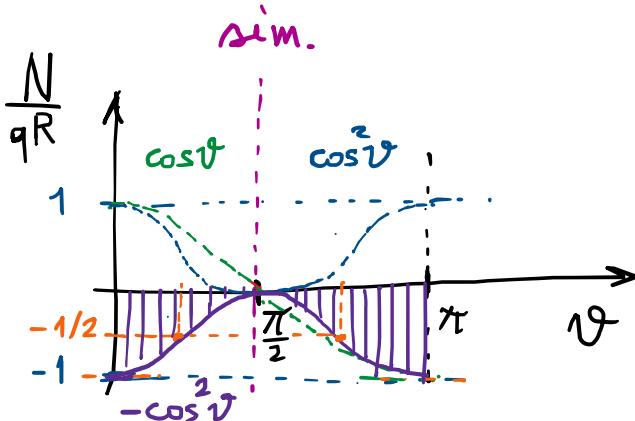
$$\begin{aligned} M''(\vartheta) &= -\overset{\circ}{m}(\vartheta) R + \overset{\circ}{T}(\vartheta) R \\ &= -m'(\vartheta) R - q_h(\vartheta) R^2 - N(\vartheta) R \\ &= -(m'(\vartheta) + q_h(\vartheta) R) R - N(\vartheta) R \end{aligned}$$

Esempio: arco semicircolare con  $q_v$  distribuito (per unità di lunghezza in direz. orizzontale)



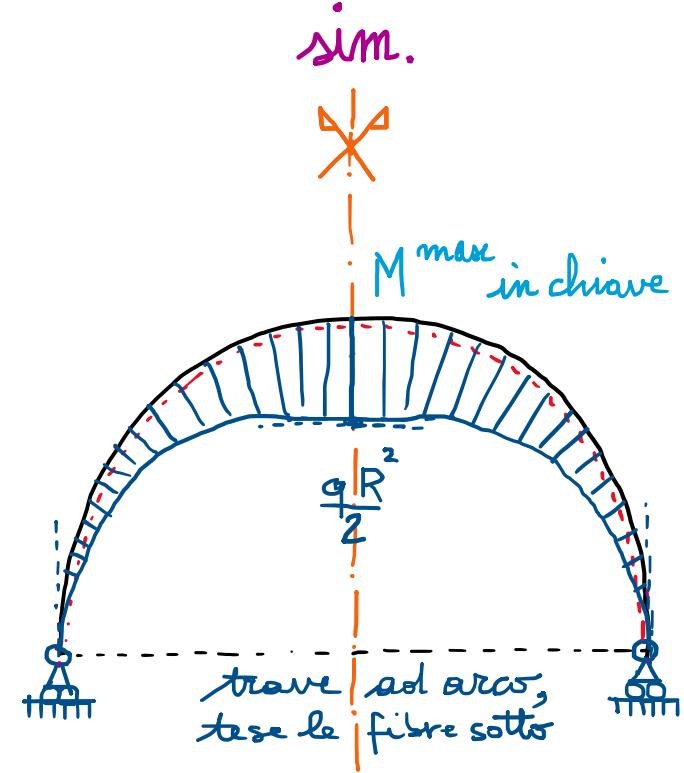
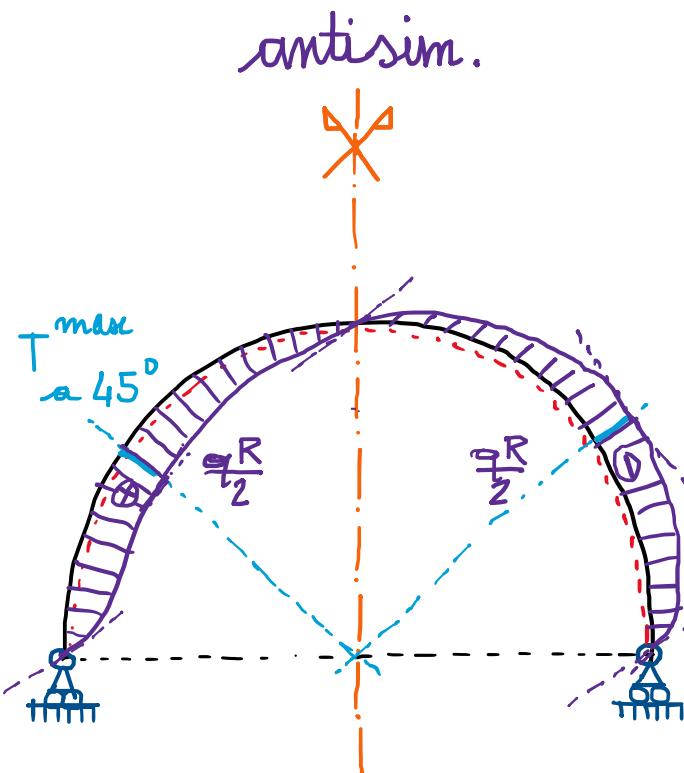
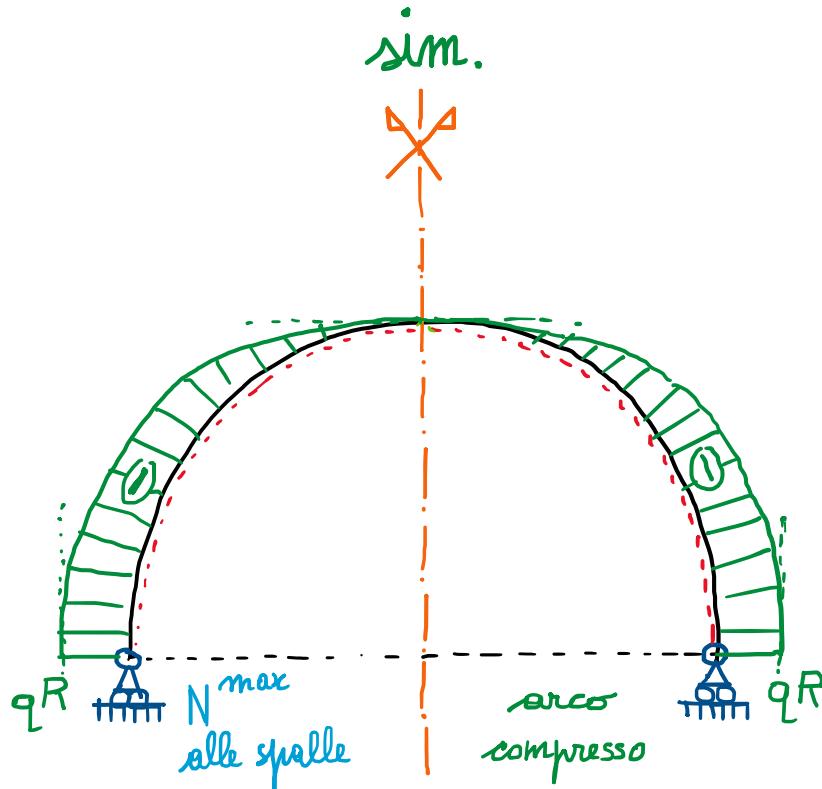
Equil. locale (AP):

$$\left\{ \begin{array}{l} N(v) = -qR \cos^2 \theta + qR(1-\cos \theta) \cos \theta = -qR \cos^2 \theta = N \\ T(v) = qR \sin \theta - qR(1-\cos \theta) \sin \theta = \frac{qR}{2} \frac{2 \sin \theta \cos \theta}{\sin 2\theta} = \frac{qR \sin 2\theta}{2} = T \\ M(v) = qR R(1-\cos \theta) - qR(1-\cos \theta) \frac{R(1-\cos \theta)}{2} = \\ = qR^2 (1-\cos \theta) \left( 1 - \frac{1+\cos \theta}{2} \right) = \frac{qR^2}{2} (1-\cos \theta)(1+\cos \theta) \\ = \frac{qR^2}{2} (1-\cos^2 \theta) = \frac{qR^2}{2} \sin^2 \theta = M \end{array} \right.$$



dipendenza analitica delle funzioni di Azione Interna e loro rappresentazione

Diagrammi delle Azioni Interni N, T, M  
 (andamenti funzionali rappresentati su fondamentali coincidenti con la struttura stessa)

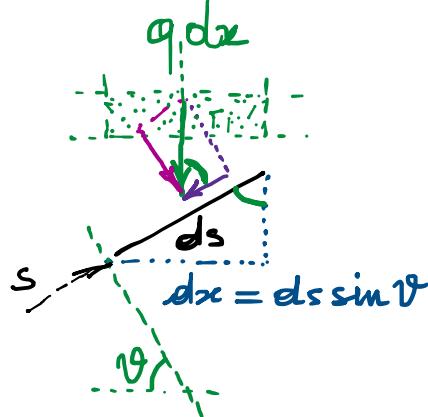


$$N(\theta)$$

$$T(\theta)$$

$$M(\theta)$$

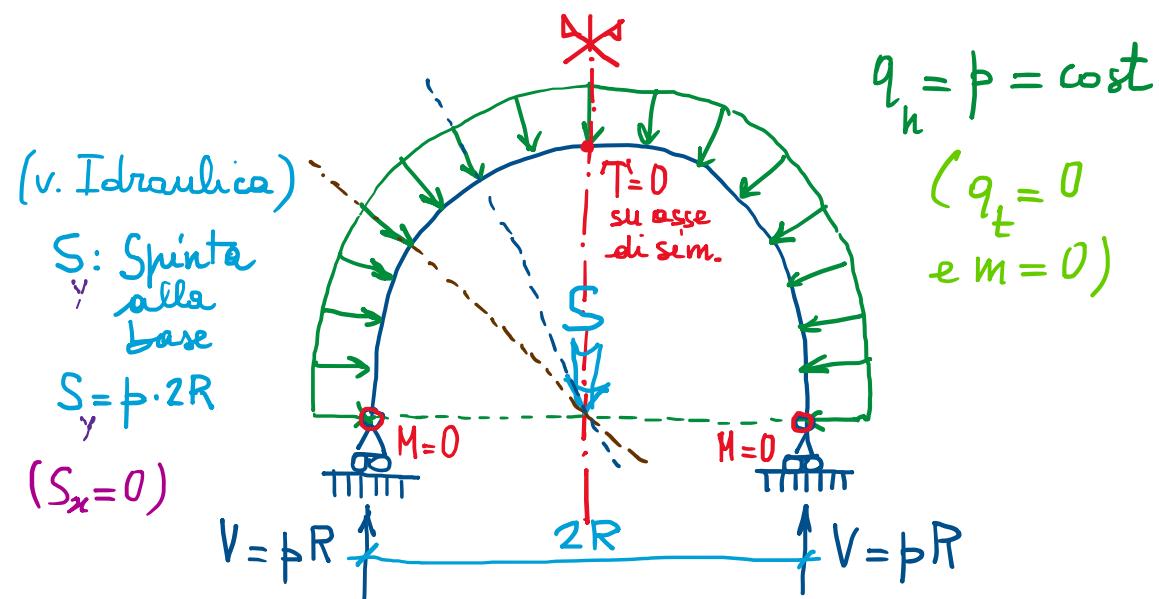
- Verificabili le relazioni differenziali viste (\*), con  $q_t$  e  $q_n$  come segue:



$$q_t = -\frac{q dx \cos \theta}{ds} = -\frac{q ds \sin \theta \cos \theta}{ds} \frac{2}{2} = -\frac{q}{2} \sin 2\theta = q_t(\theta)$$

$$q_n = \frac{q dx \sin \theta}{ds} = \frac{q ds \sin \theta \sin \theta}{ds} = q \sin^2 \theta = q_n(\theta)$$

- Arco circolare soggetto a pressione uniforme esterna:



$T \equiv 0$ , per simmetria rispetto ad ogni sezione radiale (prima --, poi --, poi --, ecc.)

$N = -pR = \text{cost}$  ( $N' = 0$ ) compressione

$M \equiv 0 = \text{cost}$  ( $M' = 0$ )

dalle (\*):

$$\begin{cases} N' = T \\ T' = -pR - N \\ M' = TR \end{cases}$$

come per tubo soggetto a pressione interna  $\rightarrow N = pR$ , traz.