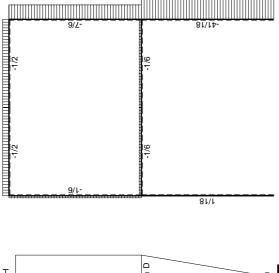
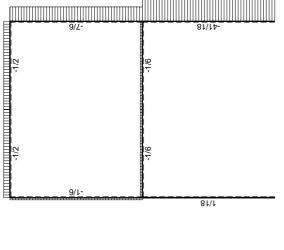
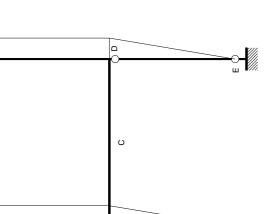
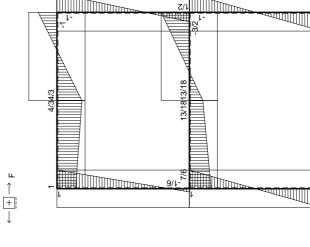
EQUILIBRIO Nome:

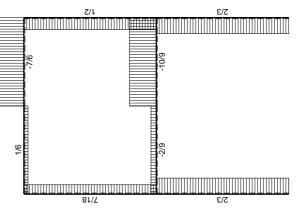
08.06.11

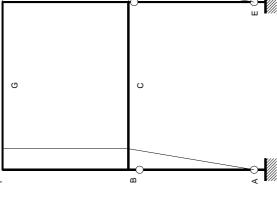


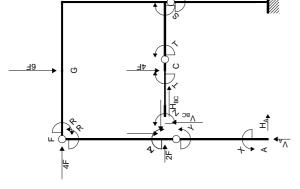












EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

 $3H_Ab - 4V_Ab = -Xb + Sb - 8Fb$

Rotazione intorno a C: aste CB

Rotazione intorno a F: aste FB BA $-2V_{BC}b = -Zb + Tb$

 $6H_Ab - 3H_{BC}b = -Xb + Zb - Rb - 6Fb$

Rotazione intorno a B: aste BA $3H_Ab = -Xb - Yb$

Matrice di equilibrio

Soluzione del sistema

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.11.05.11

PROGRAMMAZIONE LINEARE

Sia H_{ii} la matrice del simplesso, con m righe e n colonne.

Siano P_i le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.

Siano a riga m i coefficienti della funzione obiettivo primale $\max \Sigma_i H_{mi} P_{ii}$ $1 \le j < n$.

Siano a colonna n i coefficienti della funzione obiettivo duale $min \Sigma_i H_{in} D_i$, $1 \le i < m$.

Sequenza di operazioni pivotali:

- 1 Sia q (1 $\leq q < n$) la colonna pivot con massimo valore H_{mi} in riga m.
- 2 Sia p ($1 \le p < m$) la riga pivot di colonna q, a coefficiente negativo H_{in} , che minimizza il rapporto H_{in}/H_{in} .
- 3 Si ottiene il coefficiente pivotale H_{po} .
- 4 Si scambia la variabile primale P_q con la duale D_p .
- 5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
- 6 Si ridefiniscono i coefficienti della colonna pivot $q: H_{ig} = H_{ng} H_{ig}$, escluso il pivot H_{ng} .
- 7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ii} = H_{ii} H_{ii} + H_{pr}$
- 8 Si ridefiniscono i coefficienti della riga pivot $p: H_{oi} = -H_{oa} H_{oi}$, escluso il pivot H_{oc} .
- Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.

Giunti a questo punto, si individua la soluzione.

Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P, presenti sulla colonna di sinistra.

Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare *m*=6,*n*=4

SOLUZIONE DEL SIMPLESSO X=WAR Y=WRA Z=WRC T=WCD S=WDE R=WER

Tableau con variabili non vincolate in segno

	rabiea	ii vaiid	le III	segnic							
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		L X	Υ	Z	Т	S	R	αbF_{\perp}		└ Fb │	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{AB} -	¹	0	0	0	0	0	0	≥	-1	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{AB} +	1	0	0	0	0	0	0	≤	1	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W _{BA} -	0	1	0	0	0	0	0	≥	-1	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{BA} +	0	1	0	0	0	0	0	≤	1	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{BC} -	0	0	1	0	0	0	0	≥	-3/2	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W _{BC} +	0	0	1	0	0	0	0	≤	3/2	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{CD} -	0	0	0	1	0	0	0	≥	-3/2	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W _{CD} +	0	0	0	1	0	0	0	≤	3/2	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{DC} -	0	0	1	-2	0	0	-8	≥	-3/2	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{DC} +	0	0	1	-2	0	0	-8	≤	3/2	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{DE} -	0	0	0	0	1	0	0	≥	-1	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{DE} +	0	0	0	0	1	0	0	≤	1	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{ED} -	-1	-1	0	0	-1	0	18	≥	-1	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{ED} +	-1	-1	0	0	-1	0	18	≤	1	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{BF} -	0	-1	-1	0	0	0	0	≥	-1	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{BF} +	0	-1	-1	0	0	0	0	≤	1	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{FB} -	0	0	0	0	0	1	0	≥	-1	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{FB} +	0	0	0	0	0	1	0	≤	1	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{GH} -	0	1/2	1	-1	1/2	-1	-4	≥	-3/2	
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_GH +	0	1/2	1	-1	1/2	-1	-4	≤	3/2	
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{HD} -	0	1	2	-2	1	-1	4	≥	-1	
W_{DH}^{-} 0 0 -1 2 -1 0 8 \geq -1 W_{DH}^{+} 0 0 -1 2 -1 0 8 \leq 1	W_{HD} +	0	1	2	-2	1	-1	4	≤	1	
$W_{DH} + 0 0 -1 2 -1 0 8 \mid \leq 1 \mid$	W_{DH} -	0	0	-1	2	-1	0	8	≥	-1	
$Max \left[\begin{array}{ccccccccccccccccccccccccccccccccccc$	W_{DH} +	0	0	-1	2	-1	0	8	≤	1	
	Max	0	0	0	0	0	0	1	=	[0]	

08.06.11

Tableau con variabili vincolate in segno

Tableau con variabili non vincolate in segno

	[X	Υ	Z	Т	S	R	αbF]		[Fb]
W_{AB} -	1	0	0	0	0	0	0	≥	-1
W_{AB} +	-1	0	0	0	0	0	0	≥	-1
W_{BA} -	0	1	0	0	0	0	0	≥	-1
W_{BA} +	0	-1	0	0	0	0	0	≥	-1
W_{BC} -	0	0	1	0	0	0	0	≥	-3/2
W_{BC} +	0	0	-1	0	0	0	0	≥	-3/2
W _{CD} -	0	0	0	1	0	0	0	≥	-3/2
W_{CD} +	0	0	0	-1	0	0	0	≥	-3/2
W_{DC} -	0	0	1	-2	0	0	-8	≥	-3/2
W_{DC} +	0	0	-1	2	0	0	8	≥	-3/2
W_{DE} -	0	0	0	0	1	0	0	≥	-1
W_{DE} +	0	0	0	0	-1	0	0	≥	-1
W_{ED} -	-1	-1	0	0	-1	0	18	≥	-1
W_{ED} +	1	1	0	0	1	0	-18	≥	-1
W _{BF} -	0	-1	-1	0	0	0	0	≥	-1
W_{BF} +	0	1	1	0	0	0	0	≥	-1
W_{FB} -	0	0	0	0	0	1	0	≥	-1
W_{FB} +	0	0	0	0	0	-1	0	≥	-1
W_{GH} -	0	1/2	1	-1	1/2	-1	-4	≥	-3/2
W_{GH} +	0	-1/2	-1	1	-1/2	1	4	≥	-3/2
W_{HD} -	0	1	2	-2	1	-1	4	≥	-1
W_{HD} +	0	-1	-2	2	-1	1	-4	≥	-1
W_{DH} -	0	0	-1	2	-1	0	8	≥	-1
W_{DH} +	0	0	1	-2	1	0	-8	≥	-1
Max	0	0	0	0	0	0	1	=	0

	Ĺ X+	Y+	Z+	T+	S+	R+	Х-	Y-	Z-	T-	S-	R-	αbF_{\perp}		[Fb]	
W_{AB} -	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	[-1]	
W_{AB} +	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1	
W_{BA} -	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1	
W_{BA} +	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1	
W _{BC} -	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2	
W_{BC} +	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2	
W _{CD} -	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2	
W _{CD} +	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2	
W _{DC} -	0	0	1	-2	0	0	0	0	-1	2	0	0	-8	≥	-3/2	
W_{DC} +	0	0	-1	2	0	0	0	0	1	-2	0	0	8	≤	-3/2	
W_{DE} -	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1	
W_{DE} +	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1	
W_{ED} -	-1	-1	0	0	-1	0	1	1	0	0	1	0	18	≥	-1	
W_{ED} +	1	1	0	0	1	0	-1	-1	0	0	-1	0	-18	≤	-1	
W _{BF} -	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-1	
W_{BF} +	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-1	

0

0

≤ | -1≥ | -3/2

≤ -1

 \geq

-8 | ≤ | -1

-3/2 -1

0

Max

Tableau a variabili negative su X- e limitate

	[X	Υ	Z	Т	S	R	$\alpha b F$	X-		[Fb]
$\phi_{\text{AB}}\text{-}$	1	0	0	0	0	0	0	-1	≥	-1
$\phi_{AB}\textbf{+}$	-1	0	0	0	0	0	0	1	≥	-1
$\phi_{\text{BA}}\text{-}$	0	1	0	0	0	0	0	-1	≥	-1
ϕ_{BA} +	0	-1	0	0	0	0	0	1	≥	-1
ϕ_{BC} -	0	0	1	0	0	0	0	-1	≥	-3/2
ϕ_{BC} +	0	0	-1	0	0	0	0	1	≥	-3/2
ϕ_{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2
$\phi_{\text{CD}} \textbf{+}$	0	0	0	-1	0	0	0	1	≥	-3/2
$\phi_{\text{DC}}\text{-}$	0	0	1	-2	0	0	-8	1	≥	-3/2
$\phi_{DC} \textbf{+}$	0	0	-1	2	0	0	8	-1	≥	-3/2
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-1
ϕ_{DE} +	0	0	0	0	-1	0	0	1	≥	-1
$\phi_{\text{ED}}\text{-}$	-1	-1	0	0	-1	0	18	3	≥	-1
$\phi_{\text{ED}} \textbf{+}$	1	1	0	0	1	0	-18	-3	≥	-1
$\phi_{\text{BF}}\text{-}$	0	-1	-1	0	0	0	0	2	≥	-1
ϕ_{BF} +	0	1	1	0	0	0	0	-2	≥	-1
$\phi_{\text{FB}}\text{-}$	0	0	0	0	0	1	0	-1	≥	-1
$\phi_{\text{FB}}\text{+}$	0	0	0	0	0	-1	0	1	≥	-1
ϕ_{GH} -	0	1/2	1	-1	1/2	-1	-4	0	≥	-3/2
ϕ_{GH} +	0	-1/2	-1	1	-1/2	1	4	0	≥	-3/2
$\phi_{\text{HD}}\text{-}$	0	1	2	-2	1	-1	4	-1	≥	-1
$\phi_{\text{HD}} \textbf{+}$	0	-1	-2	2	-1	1	-4	1	≥	-1
$\phi_{\text{DH}}\text{-}$	0	0	-1	2	-1	0	8	0	≥	-1
ϕ_{DH} +	0	0	1	-2	1	0	-8	0	≥	-1
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	0	0	0	0	0	1	0 _	=	0

Scambio pivotale 14-7												
	[X	Υ	Z	Т	S	R	ϕ_{ED} +	X-]		[Fb]		
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	[-1]		
ϕ_{AB} +	-1	0	0	0	0	0	0	1	≥	-1		
ϕ_{BA} -	0	1	0	0	0	0	0	-1	≥	-1		
ϕ_{BA} +	0	-1	0	0	0	0	0	1	≥	-1		
ϕ_{BC} -	0	0	1	0	0	0	0	-1	≥	-3/2		
ϕ_{BC} +	0	0	-1	0	0	0	0	1	≥	-3/2		
ϕ_{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2		
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2		
ϕ_{DC} -	-4/9	-4/9	1	-2	-4/9	0	4/9	7/3	≥	-19/18		
ϕ_{DC} +	4/9	4/9	-1	2	4/9	0	-4/9	-7/3	≥	-35/18		
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-1		
ϕ_{DE} +	0	0	0	0	-1	0	0	1	≥	-1		
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2		
$\alpha b F$	1/18	1/18	0	0	1/18	0	-1/18	-1/6	≥	-1/18		
ϕ_{BF} -	0	-1	-1	0	0	0	0	2	≥	-1		
ϕ_{BF} +	0	1	1	0	0	0	0	-2	≥	-1		
ϕ_{FB} -	0	0	0	0	0	1	0	-1	≥	-1		
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1		
ϕ_{GH} -	-2/9	5/18	1	-1	5/18	-1	2/9	2/3	≥	-23/18		
$\phi_{\text{GH}} \textbf{+}$	2/9	-5/18	-1	1	-5/18	1	-2/9	-2/3	≥	-31/18		
$\phi_{\text{HD}}\text{-}$	2/9	11/9	2	-2	11/9	-1	-2/9	-5/3	≥	-11/9		
ϕ_{HD} +	-2/9	-11/9	-2	2	-11/9	1	2/9	5/3	≥	-7/9		
ϕ_{DH} -	4/9	4/9	-1	2	-5/9	0	-4/9	-4/3	≥	-13/9		
$\phi_{\text{DH}} \textbf{+}$	-4/9	-4/9	1	-2	5/9	0	4/9	4/3	≥	-5/9		
L_{x}	0	0	0	0	0	0	0	-1	≥	-3/2		
	1/18	1/18	0	0	1/18	0	-1/18	-1/6	=	-1/18		

Scambio pivotale 2-1

	$\left[\phi_{AB}\right]$	Υ	Z	Т	S	R	ϕ_{ED} +	X-]		[Fb]	
ϕ_{AB} -	-1	0	0	0	0	0	0	0	\geq	[- 2]	
Χ	-1	0	0	0	0	0	0	1	\geq	-1	
ϕ_{BA} -	0	1	0	0	0	0	0	-1	\geq	-1	
ϕ_{BA} +	0	-1	0	0	0	0	0	1	\geq	-1	
ϕ_{BC} -	0	0	1	0	0	0	0	-1	\geq	-3/2	
ϕ_{BC} +	0	0	-1	0	0	0	0	1	\geq	-3/2	
ϕ_{CD} -	0	0	0	1	0	0	0	-1	\geq	-3/2	
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2	
$\phi_{\text{DC}}\text{-}$	4/9	-4/9	1	-2	-4/9	0	4/9	17/9	\geq	-11/18	
ϕ_{DC} +	-4/9	4/9	-1	2	4/9	0	-4/9	-17/9	\geq	-43/18	
ϕ_{DE} -	0	0	0	0	1	0	0	-1	\geq	-1	
ϕ_{DE} +	0	0	0	0	-1	0	0	1	\geq	-1	
ϕ_{ED} -	0	0	0	0	0	0	-1	0	\geq	-2	
$\alpha b F$	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	\geq	-1/9	
ϕ_{BF} -	0	-1	-1	0	0	0	0	2	\geq	-1	
$\phi_{\text{BF}}\text{+}$	0	1	1	0	0	0	0	-2	\geq	-1	
$\phi_{\text{FB}}\text{-}$	0	0	0	0	0	1	0	-1	\geq	-1	
ϕ_{FB} +	0	0	0	0	0	-1	0	1	\geq	-1	
ϕ_{GH} -	2/9	5/18	1	-1	5/18	-1	2/9	4/9	\geq	-19/18	
ϕ_{GH} +	-2/9	-5/18	-1	1	-5/18	1	-2/9	-4/9	\geq	-35/18	
ϕ_{HD} -	-2/9	11/9	2	- 2	11/9	-1	-2/9	-13/9	\geq	-13/9	
ϕ_{HD} +	2/9	-11/9	-2	2	-11/9	1	2/9	13/9	\geq	-5/9	
ϕ_{DH} -	-4/9	4/9	-1	2	-5/9	0	-4/9	-8/9	\geq	-17/9	
ϕ_{DH} +	4/9	-4/9	1	-2	5/9	0	4/9	8/9	≥	-1/9	
L_{x}	0	0	0	0	0	0	0	-1	\geq	-3/2	
Max	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	=	-1/9	

	[φ _{AB} +	ϕ_{DH} +	Z	Т	S	R	ϕ_{ED} +	X-]	[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	[-2]
Χ	-1	0	0	0	0	0	0	1	≥	-1
ϕ_{BA} -	1	-9/4	9/4	-9/2	5/4	0	1	1	≥	-5/4
ϕ_{BA} +	-1	9/4	-9/4	9/2	-5/4	0	-1	-1	≥	-3/4
ϕ_{BC} -	0	0	1	0	0	0	0	-1	≥	-3/2
$\phi_{\text{BC}}\text{+}$	0	0	-1	0	0	0	0	1	≥	-3/2
ϕ_{CD}	0	0	0	1	0	0	0	-1	≥	-3/2
$\phi_{\text{CD}} \textbf{+}$	0	0	0	-1	0	0	0	1	≥	-3/2
$\phi_{\text{DC}}\text{-}$	0	1	0	0	-1	0	0	1	≥	-1/2
$\phi_{\text{DC}}\text{+}$	0	-1	0	0	1	0	0	-1	≥	-5/2
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-1
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	≥	-1
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	-1	0	≥	-2
$\alpha b F$	0	-1/8	1/8	-1/4	1/8	0	0	0	≥	-1/8
$\phi_{\text{BF}}\text{-}$	-1	9/4	-13/4	9/2	-5/4	0	-1	0	≥	-3/4
$\phi_{\text{BF}}\text{+}$	1	-9/4	13/4	-9/2	5/4	0	1	0	≥	-5/4
$\phi_{\text{FB}}\text{-}$	0	0	0	0	0	1	0	-1	≥	-1
$\phi_{\text{FB}}\text{+}$	0	0	0	0	0	-1	0	1	≥	-1
$\phi_{\text{GH}}\text{-}$	1/2	-5/8	13/8	-9/4	5/8	-1	1/2	1	≥	-9/8
ϕ_{GH} +	-1/2	5/8	-13/8	9/4	-5/8	1	-1/2	-1	≥	-15/8
$\phi_{\text{HD}}\text{-}$	1	-11/4	19/4	-15/2	11/4	-1	1	1	≥	-7/4
ϕ_{HD} +	-1	11/4	-19/4	15/2	-11/4	1	-1	-1	≥	-1/4
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-2
Υ	1	-9/4	9/4	-9/2	5/4	0	1	2	≥	-1/4
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	-1/8	1/8	-1/4	1/8	0	0	0	=	1/8]

Scambio pivotale 22-3

	- P. T	==	•					_		
	φ _{AB} +	ϕ_{DH} +	ϕ_{HD} +	Т	S	R	ϕ_{ED} +	X-]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	\geq	-2
X	-1	0	0	0	0	0	0	1	\geq	-1
$\phi_{\text{BA}}\text{-}$	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	10/19	≥	-26/19
$\phi_{\text{BA}} \textbf{+}$	-10/19	18/19	9/19	18/19	1/19	-9/19	-10/19	-10/19	≥	-12/19
ϕ_{BC} -	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-23/19	\geq	-59/38
$\phi_{\text{BC}} \textbf{+}$	4/19	-11/19	4/19	-30/19	11/19	-4/19	4/19	23/19	≥	-55/38
ϕ_{CD}	0	0	0	1	0	0	0	-1	≥	-3/2
ϕ_{CD} +	0	0	0	-1	0	0	0	1	\geq	-3/2
ϕ_{DC} -	0	1	0	0	-1	0	0	1	\geq	-1/2
$\phi_{\text{DC}}\text{+}$	0	-1	0	0	1	0	0	-1	\geq	-5/2
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	\geq	-1
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	\geq	-1
ϕ_{ED} -	0	0	0	0	0	0	-1	0	\geq	-2
$\alpha b F$	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	\geq	-5/38
ϕ_{BF} -	-6/19	7/19	13/19	-12/19	12/19	-13/19	-6/19	13/19	\geq	-11/19
ϕ_{BF} +	6/19	-7/19	-13/19	12/19	-12/19	13/19	6/19	-13/19	\geq	-27/19
$\phi_{\text{FB}}\text{-}$	0	0	0	0	0	1	0	-1	\geq	-1
ϕ_{FB} +	0	0	0	0	0	-1	0	1	\geq	-1
φ _{GH} -	3/19	6/19	-13/38	6/19	-6/19	-25/38	3/19	25/38	\geq	-23/19
ϕ_{GH} +	-3/19	-6/19	13/38	-6/19	6/19	25/38	-3/19	-25/38	\geq	-34/19
φ _{HD} -	0	0	-1	0	0	0	0	0	\geq	-2
Z	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-4/19	\geq	-1/19
ϕ_{DH} -	0	-1	0	0	0	0	0	0	\geq	-2
Υ	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	29/19	≥	-7/19
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	=	-5/38

Scambio pivotale 22-5

	[φ _{AB} +	ϕ_{DH} +	ϕ_{HD} +	Т	Z	R	ϕ_{ED} +	Χ-		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	[- 2]
Χ	-1	0	0	0	0	0	0	1	≥	-1
ϕ_{BA} -	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	6/11	≥	-15/11
ϕ_{BA} +	-6/11	1	5/11	12/11	-1/11	-5/11	-6/11	-6/11	≥	-7/11
ϕ_{BC} -	0	0	0	0	1	0	0	-1	≥	-3/2
ϕ_{BC} +	0	0	0	0	-1	0	0	1	≥	-3/2
φ _{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2
φ _{DC} -	4/11	0	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-9/22
ϕ_{DC} +	-4/11	0	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-57/22
ϕ_{DE} -	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-12/11
ϕ_{DE} +	4/11	-1	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-10/11
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	≥	-3/22
ϕ_{BF} -	-6/11	1	5/11	12/11	-12/11	-5/11	-6/11	5/11	≥	-7/11
ϕ_{BF} +	6/11	-1	-5/11	-12/11	12/11	5/11	6/11	-5/11	≥	-15/11
ϕ_{FB} -	0	0	0	0	0	1	0	-1	≥	-1
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1
ϕ_{GH} -	3/11	0	-5/22	-6/11	6/11	-17/22	3/11	17/22	≥	-13/11
ϕ_{GH} +	-3/11	0	5/22	6/11	-6/11	17/22	-3/11	-17/22	≥	-20/11
ϕ_{HD} -	0	0	-1	0	0	0	0	0	≥	-2
S	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-4/11	≥	-1/11
φ _{DH} -	0	-1	0	0	0	0	0	0	≥	-2
Υ	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	17/11	≥	-4/11
L_{x}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	=	-3/22

Scambio pivotale 9-4

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	φ_{AB} +	ϕ_{DH} +	ϕ_{HD} +	ϕ_{DC} -	Z	R	ϕ_{ED} +	X-		Fb]	
ϕ_{AB} -	-1	0	0	0	0	0	0	0] ≥	-2	
Χ	-1	0	0	0	0	0	0	1	≥	-1	
ϕ_{BA} -	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	0	≥	-6/5	
ϕ_{BA} +	-2/5	1	3/5	-2/5	3/5	-3/5	-2/5	0	≥	-4/5	
ϕ_{BC} -	0	0	0	0	1	0	0	-1	≥	-3/2	
ϕ_{BC} +	0	0	0	0	-1	0	0	1	≥	-3/2	
φ _{CD} -	2/15	0	2/15	-11/30	19/30	-2/15	2/15	-1/2	≥	-33/20	
ϕ_{CD} +	-2/15	0	-2/15	11/30	-19/30	2/15	-2/15	1/2	≥	-27/20	
Т	2/15	0	2/15	-11/30	19/30	-2/15	2/15	1/2	≥	-3/20	
ϕ_{DC} +	0	0	0	-1	0	0	0	0	≥	-3	
ϕ_{DE} -	0	1	0	-1	0	0	0	0	≥	-3/2	
ϕ_{DE} +	0	-1	0	1	0	0	0	0	≥	-1/2	
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2	
$\alpha b F$	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	≥	-3/20	
ϕ_{BF} -	-2/5	1	3/5	-2/5	-2/5	-3/5	-2/5	1	≥	-4/5	
ϕ_{BF} +	2/5	-1	-3/5	2/5	2/5	3/5	2/5	-1	≥	-6/5	
ϕ_{FB} -	0	0	0	0	0	1	0	-1	≥	-1	
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1	
φ _{GH} -	1/5	0	-3/10	1/5	1/5	-7/10	1/5	1/2	≥	-11/10	
ϕ_{GH} +	-1/5	0	3/10	-1/5	-1/5	7/10	-1/5	-1/2	≥	-19/10	
φ _{HD} -	0	0	-1	0	0	0	0	0	≥	-2	
S	0	1	0	-1	0	0	0	1	≥	-1/2	
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-2	
Υ	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	1	≥	-1/5	
L_{x}	0	0	0	0	0	0	0	-1	≥	-3/2	
Max	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	=	-3/20	

Scambio pivotale 18-6

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	φ _{AB} +	ϕ_{DH} +	ϕ_{HD} +	ϕ_{DC} -	Z	ϕ_{FB} +	ϕ_{ED} +	X-		[Fb]		
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	-2		
Χ	-1	0	0	0	0	0	0	1	≥	-1		
ϕ_{BA} -	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	3/5	≥	-9/5		
ϕ_{BA} +	-2/5	1	3/5	-2/5	3/5	3/5	-2/5	-3/5	≥	-1/5		
ϕ_{BC} -	0	0	0	0	1	0	0	-1	≥	-3/2		
ϕ_{BC} +	0	0	0	0	-1	0	0	1	≥	-3/2		
ϕ_{CD} -	2/15	0	2/15	-11/30	19/30	2/15	2/15	-19/30	≥	-91/60		
$\phi_{\text{CD}} \textbf{+}$	-2/15	0	-2/15	11/30	-19/30	-2/15	-2/15	19/30	≥	-89/60		
Т	2/15	0	2/15	-11/30	19/30	2/15	2/15	11/30	≥	-1/60		
$\phi_{\text{DC}}\text{+}$	0	0	0	-1	0	0	0	0	≥	-3		
ϕ_{DE} -	0	1	0	-1	0	0	0	0	≥	-3/2		
$\phi_{\text{DE}} \textbf{+}$	0	-1	0	1	0	0	0	0	≥	-1/2		
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2		
$\alpha b F$	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	≥	-11/60		
ϕ_{BF} -	-2/5	1	3/5	-2/5	-2/5	3/5	-2/5	2/5	≥	-1/5		
ϕ_{BF} +	2/5	-1	-3/5	2/5	2/5	-3/5	2/5	-2/5	≥	-9/5		
ϕ_{FB} -	0	0	0	0	0	-1	0	0	≥	-2		
R	0	0	0	0	0	-1	0	1	≥	-1		
ϕ_{GH} -	1/5	0	-3/10	1/5	1/5	7/10	1/5	-1/5	≥	-2/5		
$\phi_{\text{GH}} \textbf{+}$	-1/5	0	3/10	-1/5	-1/5	-7/10	-1/5	1/5	≥	-13/5		
ϕ_{HD} -	0	0	-1	0	0	0	0	0	≥	-2		
S	0	1	0	-1	0	0	0	1	≥	-1/2		
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-2		
Υ	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	8/5	≥	-4/5		
L_{x}	0	0	0	0	0	0	0	-1	≥	-3/2		
Max	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	=	-11/60		

Scambio pivotale 4-8

	[φ _{AB} +	φ _{DH} +	ϕ_{HD} +	φ _{DC} -	Z	φ _{FB} +	ϕ_{ED} +	φ_{BA} +]		[Fb]	
φ _{AB} -	-1	0	0	0	0	0	0	0	\geq	[-2]	1
X	-5/3	5/3	1	-2/3	1	1	-2/3	-5/3	≥	-4/3	
ϕ_{BA} -	0	0	0	0	0	0	0	-1	\geq	-2	
X-	-2/3	5/3	1	-2/3	1	1	-2/3	-5/3	≥	-1/3	
ϕ_{BC} -	2/3	-5/3	-1	2/3	0	-1	2/3	5/3	\geq	-7/6	
ϕ_{BC} +	-2/3	5/3	1	-2/3	0	1	-2/3	-5/3	\geq	-11/6	
φ _{CD} -	5/9	-19/18	-1/2	1/18	0	-1/2	5/9	19/18	\geq	-47/36	
ϕ_{CD} +	-5/9	19/18	1/2	-1/18	0	1/2	-5/9	-19/18	≥	-61/36	
Т	-1/9	11/18	1/2	-11/18	1	1/2	-1/9	-11/18	\geq	-5/36	
ϕ_{DC} +	0	0	0	-1	0	0	0	0	\geq	-3	
ϕ_{DE} -	0	1	0	-1	0	0	0	0	≥	-3/2	
$\phi_{\text{DE}}\text{+}$	0	-1	0	1	0	0	0	0	≥	-1/2	
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2	
$\alpha b F$	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	≥	-7/36	
ϕ_{BF} -	-2/3	5/3	1	-2/3	0	1	-2/3	-2/3	≥	-1/3	
ϕ_{BF} +	2/3	-5/3	-1	2/3	0	-1	2/3	2/3	≥	-5/3	
φ _{FB} -	0	0	0	0	0	-1	0	0	≥	-2	
R	-2/3	5/3	1	-2/3	1	0	-2/3	-5/3	≥	-4/3	
$\phi_{\text{GH}}\text{-}$	1/3	-1/3	-1/2	1/3	0	1/2	1/3	1/3	≥	-1/3	
ϕ_{GH} +	-1/3	1/3	1/2	-1/3	0	-1/2	-1/3	-1/3	≥	-8/3	
ϕ_{HD} -	0	0	-1	0	0	0	0	0	≥	-2	
S	-2/3	8/3	1	-5/3	1	1	-2/3	-5/3	≥	-5/6	
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-2	
Υ	-2/3	5/3	1	-2/3	1	1	-2/3	-8/3	≥	-4/3	
L_{x}	2/3	-5/3	-1	2/3	-1	-1	2/3	5/3	≥	-7/6	
Max	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	=	-7/36	

Scambio pivotale 12-2

	[φ _{AB} +	ϕ_{DE} +	ϕ_{HD} +	ϕ_{DC} -	Z	ϕ_{FB} +	ϕ_{ED} +	ϕ_{BA} +]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	-2
Χ	-5/3	-5/3	1	1	1	1	-2/3	-5/3	\geq	-13/6
ϕ_{BA} -	0	0	0	0	0	0	0	-1	\geq	-2
X-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	\geq	-7/6
ϕ_{BC} -	2/3	5/3	-1	-1	0	-1	2/3	5/3	\geq	-1/3
ϕ_{BC} +	-2/3	-5/3	1	1	0	1	-2/3	-5/3	\geq	-8/3
ϕ_{CD}^-	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	\geq	-7/9
$\phi_{\text{CD}}\text{+}$	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	\geq	-20/9
Т	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	\geq	-4/9
$\phi_{DC}\text{+}$	0	0	0	-1	0	0	0	0	\geq	-3
$\phi_{\text{DE}}\text{-}$	0	-1	0	0	0	0	0	0	\geq	-2
ϕ_{DH} +	0	-1	0	1	0	0	0	0	\geq	-1/2
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	-1/18	0	0	0	0	-1/18	-1/18	\geq	-2/9
ϕ_{BF} -	-2/3	-5/3	1	1	0	1	-2/3	-2/3	\geq	-7/6
ϕ_{BF} +	2/3	5/3	-1	-1	0	-1	2/3	2/3	\geq	-5/6
ϕ_{FB} -	0	0	0	0	0	-1	0	0	\geq	-2
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	\geq	-13/6
$\phi_{\text{GH}}\text{-}$	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	\geq	-1/6
ϕ_{GH} +	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	\geq	-17/6
ϕ_{HD} -	0	0	-1	0	0	0	0	0	\geq	-2
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	\geq	-13/6
ϕ_{DH} -	0	1	0	-1	0	0	0	0	\geq	-3/2
Υ	-2/3	-5/3	1	1	1	1	-2/3	-8/3	\geq	-13/6
L_{X}	2/3	5/3	-1	-1	-1	-1	2/3	5/3	\geq	-1/3
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	2/9 _

Tableau finale

	φ _{AB} +	ϕ_{DE} +	ϕ_{HD} +	ϕ_{DC} -	Z	ϕ_{FB} +	ϕ_{ED} +	φ_{BA} +		[Fb]		
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	[-2]		
Χ	-5/3	-5/3	1	1	1	1	-2/3	-5/3	\geq	-13/6		
ϕ_{BA} -	0	0	0	0	0	0	0	-1	\geq	-2		
X-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-7/6		
ϕ_{BC} -	2/3	5/3	-1	-1	0	-1	2/3	5/3	\geq	-1/3		
ϕ_{BC} +	-2/3	-5/3	1	1	0	1	-2/3	-5/3	≥	-8/3		
ϕ_{CD}	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	\geq	-7/9		
$\phi_{\text{CD}}\text{+}$	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	\geq	-20/9		
Т	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	≥	-4/9		
ϕ_{DC} +	0	0	0	-1	0	0	0	0	≥	-3		
$\phi_{\text{DE}}\text{-}$	0	-1	0	0	0	0	0	0	\geq	-2		
$\phi_{\text{DH}} \textbf{+}$	0	-1	0	1	0	0	0	0	\geq	-1/2		
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2		
$\alpha b F$	-1/18	-1/18	0	0	0	0	-1/18	-1/18	≥	-2/9		
$\phi_{\text{BF}}\text{-}$	-2/3	-5/3	1	1	0	1	-2/3	-2/3	≥	-7/6		
ϕ_{BF} +	2/3	5/3	-1	-1	0	-1	2/3	2/3	≥	-5/6		
ϕ_{FB} -	0	0	0	0	0	-1	0	0	\geq	-2		
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	≥	-13/6		
ϕ_{GH} -	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	≥	-1/6		
ϕ_{GH} +	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	≥	-17/6		
ϕ_{HD} -	0	0	-1	0	0	0	0	0	\geq	-2		
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	\geq	-13/6		
ϕ_{DH} -	0	1	0	-1	0	0	0	0	≥	-3/2		
Υ	-2/3	-5/3	1	1	1	1	-2/3	-8/3	≥	-13/6		
L_{X}	2/3	5/3	-1	-1	-1	-1	2/3	5/3	≥	-1/3		
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	2/9]		

Vettori soluzione della programmazione lineare

	[X	Υ	Z	T	s	R	αbF	Χ-		[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{AB}\textbf{+}$	0	0	0	0	0	0	0	0	≥	1/18
ϕ_{BA} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{BA}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	1/18
ϕ_{BC} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{BC}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{CD} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{CD}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DC}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DC}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DE}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	1/18
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{ED}}\text{+}$	0	0	0	0	0	0	0	0	≥	1/18
$\phi_{\text{BF}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{BF}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{FB} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{FB}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{GH}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{GH}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{HD}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{HD}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DH}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{DH} +	0	0	0	0	0	0	0	0	≥	0
L_{X}	0	0	0	0	0	0	0	0	≥	0
Max	13/6	13/6	0	4/9	13/6	13/6	2/9	7/6	=	2/9

Variabili soluzione dedotto il valore X-

 $u_G = 1/6\delta$

 $V_G = 0$

 $\varphi_G = 0$

Variabili soluzione differenza tra rotazioni

	_
ϕ_{AB}	1/18
ϕ_{BA}	1/18
ϕ_{BC}	0
ϕ_{CD}	0
ϕ_{DC}	0
ϕ_{DE}	1/18
$\begin{array}{c} \phi_{CD} \\ \phi_{DC} \\ \phi_{DE} \\ \phi_{ED} \end{array}$	1/18
ϕ_{BF}	0
ϕ_{FB}	0
ϕ_{GH}	0
ϕ_{HD}	0
$\begin{array}{c} \phi_{GH} \\ \phi_{HD} \\ \phi_{DH} \end{array}$	0 _

REAZIONI Fattore di collasso = 2/9

 $H_{\Delta} = -2/3F$

 $V_{\Delta}^{\prime} = -1/18F$

 $W_A = Fb$

 $H_{\rm F} = -2/3F$

 $V_{E} = 41/18F$

 $\overline{W_E} = Fb$

$H_{AB} = -2/3F$ $V_{AB} = -1/18F$ $W_{AB} = Fb$ $H_{BA} = 2/3F$ $V_{BA} = 1/18F$ $W_{BA} = Fb$	$H_{BC} = 1/6F$ $V_{BC} = -2/9F$ $W_{BC} = -7/6Fb$ $H_{CB} = -1/6F$ $V_{CB} = 2/9F$ $W_{CB} = 13/18Fb$	$H_{CD} = 1/6F$ $V_{CD} = -10/9F$ $W_{CD} = -13/18Fb$ $H_{DC} = -1/6F$ $V_{DC} = 10/9F$ $W_{DC} = -3/2Fb$	$H_{DE} = 2/3F$ $V_{DE} = -41/18F$ $W_{DE} = Fb$ $H_{ED} = -2/3F$ $V_{ED} = 41/18F$ $W_{ED} = Fb$	$H_{BF} = -7/18F$ $V_{BF} = 1/6F$ $W_{BF} = 1/6Fb$ $H_{FB} = 7/18F$ $V_{FB} = -1/6F$ $W_{FB} = Fb$
$H_{FG} = 1/2F$ $V_{FG} = 1/6F$ $W_{FG} = -Fb$ $H_{GF} = -1/2F$ $V_{GF} = -1/6F$ $W_{GF} = 4/3Fb$	$H_{GH} = 1/2F$ $V_{GH} = -7/6F$ $W_{GH} = -4/3Fb$ $H_{HG} = -1/2F$ $V_{HG} = 7/6F$ $W_{HG} = -Fb$	$H_{HD} = 1/2F$ $V_{HD} = -7/6F$ $W_{HD} = Fb$ $H_{DH} = -1/2F$ $V_{DH} = 7/6F$ $W_{DH} = 1/2Fb$		

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_B = 1/6\delta$	$u_C = 1/6\delta$	$u_D = 1/6\delta$	$u_{EED} = 0$
$V_{AAB} = 0$	$V_B = 0$	$v_C = 0$	$V_D = 0$	$V_{EED} = 0$
$\phi_{AAB} = -1/18\delta/b$	$\phi_{\rm p} = -1/18\delta/b$	$\varphi_{\rm C} = 0$	$\varphi_D = 0$	$\varphi_{\scriptscriptstyle \square} = -1/18\delta/b$

SPOSTAMENTI RIGIDI DELLE ASTE

 $u_F = 1/6\delta$

 $v_{F} = 0$

 $\phi_F = 0$

$U_{AAB} = 0$ $V_{AAB} = 0$ $V_{AAB} = -1/18\delta/b$	$u_{BBC} = 1/6\delta$ $v_{BBC} = 0$ $\phi_{BBC} = 0$	$V_{CCD} = 0$	$u_{DDE} = 1/6\delta$ $v_{DDE} = 0$ $\phi_{DDE} = -1/18\delta/b$	$u_{BBF} = 1/6\delta$ $v_{BBF} = 0$ $\phi_{BBF} = 0$
TAAB	LRRC -	TCCD -	TUDE	TBBF

 $u_H = 1/6\delta$

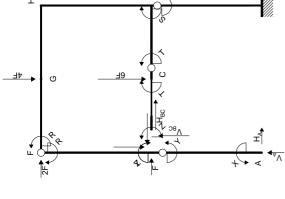
 $V_{H} = 0$

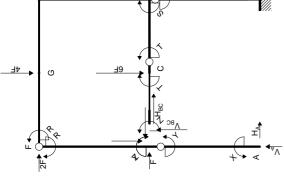
 $\phi_H = 0$

$u_{FFG} = 1/6\delta$	$u_{GGH} = 1/6\delta$	$u_{HHD} = 1/6\delta$
$V_{FFG} = 0$	$v_{GGH} = 0$	$V_{HHD} = 0$
$\phi_{FFG} = 0$	$\varphi_{GGH} = 0$	$\phi_{HHD} = 0$

EQUILIBRIO Nome:

(H)





EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

 $3H_Ab - 4V_Ab = -Xb + Sb - 14Fb$

Rotazione intorno a C: aste CB

 $-2V_{BC}b = -Zb + Tb$

Rotazione intorno a F: aste FB BA $6H_Ab - 3H_{BC}b = -Xb + Zb + Rb - 3Fb$

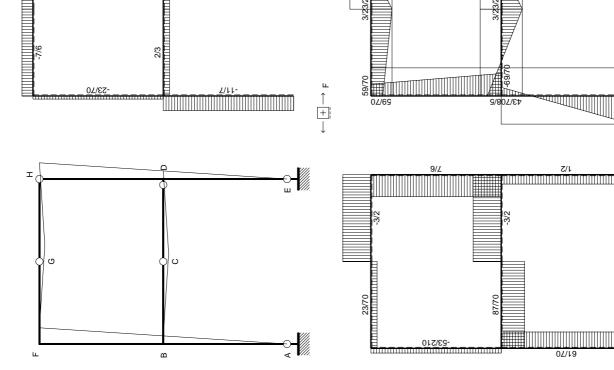
Rotazione intorno a B: aste BA $3H_Ab = -Xb - Yb$

Matrice di equilibrio

Soluzione del sistema

$$\begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1/3 & -1/3 & -1/2 & 0 & 0 & 0 & 0 \\ V_{Bc}b & 0 & 0 & 1/2 & -1/2 & 0 & 0 & 0 \\ H_{Bc}b & -1/3 & -2/3 & -1/3 & 0 & 0 & -1/3 & 1 \\ V_{A}b & 0 & -1/4 & 0 & 0 & -1/4 & 0 & 7/2 \end{bmatrix}$$

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.11.05.11



08.06.11

PROGRAMMAZIONE LINEARE

Sia H_{ii} la matrice del simplesso, con m righe e n colonne.

Siano P_i le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.

Siano a riga m i coefficienti della funzione obiettivo primale $\max \Sigma_i H_{mi} P_p$ $1 \le j < n$.

Siano a colonna n i coefficienti della funzione obiettivo duale $min \Sigma_i H_{in} D_{ir} 1 \le i < m$.

Sequenza di operazioni pivotali:

- 1 Sia q ($1 \le q < n$) la colonna pivot con massimo valore H_{mi} in riga m.
- 2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{io} , che minimizza il rapporto H_{in}/H_{io} .
- 3 Si ottiene il coefficiente pivotale H_{po}
- 4 Si scambia la variabile primale P_a con la duale D_a .
- 5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
- 6 Si ridefiniscono i coefficienti della colonna pivot $q: H_{ig} = H_{ng} H_{ig}$, escluso il pivot H_{ng} .
- 7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ii} = H_{ii} H_{ia} H_{pi}$
- 8 Si ridefiniscono i coefficienti della riga pivot $p: H_{pj} = -H_{pq} H_{pi}$, escluso il pivot H_{pq} .
- Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.

Giunti a questo punto, si individua la soluzione.

Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_i presenti sulla colonna di sinistra.

Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare *m*=6,*n*=4

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{BC}$ $T=W_{CD}$ $S=W_{DE}$ $R=W_{FG}$

Tableau con variabili non vincolate in segno

rableau con variabili non vincolate in Segno										
	[X	Υ	Z	Т	S	R	αbF		[Fb]	
W_{AB} -	1	0	0	0	0	0	0	\geq	-2	
W_{AB} +	1	0	0	0	0	0	0	\leq	2	
W _{BA} -	0	1	0	0	0	0	0	\geq	-2	
W_{BA} +	0	1	0	0	0	0	0	\leq	2	
W_{BC} -	0	0	1	0	0	0	0	\geq	-3/2	
W_{BC} +	0	0	1	0	0	0	0	\leq	3/2	
W _{CD} -	0	0	0	1	0	0	0	\geq	-3/2	
W _{CD} +	0	0	0	1	0	0	0	≤	3/2	
W _{DC} -	0	0	1	-2	0	0	-12	\geq	-3/2	
W_{DC} +	0	0	1	-2	0	0	-12	\leq	3/2	
W_{DE} -	0	0	0	0	1	0	0	\geq	-2	
W_{DE} +	0	0	0	0	1	0	0	≤	2	
W_{ED} -	-1	-1	0	0	-1	0	9	\geq	-2	
W_{ED} +	-1	-1	0	0	-1	0	9	\leq	2	
W_{BF} -	0	-1	-1	0	0	0	0	\geq	-2	
W_{BF} +	0	-1	-1	0	0	0	0	≤	2	
W _{FG} -	0	0	0	0	0	1	0	\geq	-3/2	
W_{FG} +	0	0	0	0	0	1	0	\leq	3/2	
W_{GH} -	0	1/2	1	-1	1/2	1	-7	\geq	-3/2	
W_GH +	0	1/2	1	-1	1/2	1	-7	≤	3/2	
W_{HG} -	0	-1	-2	2	-1	-1	6	\geq	-3/2	
W_{HG} +	0	-1	-2	2	-1	-1	6	≤	3/2	
W_{DH} -	0	0	-1	2	-1	0	12	\geq	-2	
W _{DH} +	0	0	-1	2	-1	0	12	≤	2	
Max	0	0	0	0	0	0	1	=	0	

Tableau con variabili non vincolate in segno

	[X	Υ	Z	Т	S	R	αbF]		[Fb]
W _{AB} -	1	0	0	0	0	0	0	≥	-2
W_{AB} +	-1	0	0	0	0	0	0	≥	-2
W_{BA} -	0	1	0	0	0	0	0	≥	-2
W_{BA} +	0	-1	0	0	0	0	0	≥	-2
W _{BC} -	0	0	1	0	0	0	0	≥	-3/2
W _{BC} +	0	0	-1	0	0	0	0	≥	-3/2
W _{CD} -	0	0	0	1	0	0	0	≥	-3/2
W_{CD} +	0	0	0	-1	0	0	0	≥	-3/2
W _{DC} -	0	0	1	-2	0	0	-12	≥	-3/2
W_{DC} +	0	0	-1	2	0	0	12	≥	-3/2
W_{DE} -	0	0	0	0	1	0	0	≥	-2
W_{DE} +	0	0	0	0	-1	0	0	≥	-2
W_{ED} -	-1	-1	0	0	-1	0	9	≥	-2
W_{ED} +	1	1	0	0	1	0	-9	≥	-2
W_{BF} -	0	-1	-1	0	0	0	0	≥	-2
W_{BF} +	0	1	1	0	0	0	0	≥	-2
W_{FG} -	0	0	0	0	0	1	0	≥	-3/2
W_{FG} +	0	0	0	0	0	-1	0	≥	-3/2
W_{GH} -	0	1/2	1	-1	1/2	1	-7	≥	-3/2
W_{GH} +	0	-1/2	-1	1	-1/2	-1	7	≥	-3/2
W_{HG} -	0	-1	-2	2	-1	-1	6	≥	-3/2
W_{HG} +	0	1	2	-2	1	1	-6	≥	-3/2
W_{DH} -	0	0	-1	2	-1	0	12	≥	-2
W_{DH} +	0	0	1	-2	1	0	-12	≥	-2
Max	0	0	0	0	0	0	1	=	0

Tableau	u con	varia	bili vi	ncola	te in	segno)
Г	V.	V.	7.	т.	е.	D.	V

	[X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		[Fb]
W_{AB} -	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-2
W_{AB} +	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-2
W_{BA} -	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-2
W_{BA} +	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-2
W_{BC} -	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2
W_{BC} +	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2
W_{CD} -	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2
W_{CD} +	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2
W_{DC} -	0	0	1	-2	0	0	0	0	-1	2	0	0	-12	≥	-3/2
W_{DC} +	0	0	-1	2	0	0	0	0	1	-2	0	0	12	≤	-3/2
W_{DF} -	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-2
W_{DE} +	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-2
W_{ED} -	-1	-1	0	0	-1	0	1	1	0	0	1	0	9	≥	-2
W_{FD} +	1	1	0	0	1	0	-1	-1	0	0	-1	0	-9	≤	-2
W_{BF} -	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-2
W_{BF} +	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-2
W_{FG} -	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-3/2
W_{FG} +	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-3/2
W_{GH} -	0	1/2	1	-1	1/2	1	0	-1/2	-1	1	-1/2	-1	-7	≥	-3/2
W_{GH} +	0	-1/2	-1	1	-1/2	-1	0	1/2	1	-1	1/2	1	7	≤	-3/2
W_{HG} -	0	-1	-2	2	-1	-1	0	1	2	-2	1	1	6	≥	-3/2
W_{HG} +	0	1	2	-2	1	1	0	-1	-2	2	-1	-1	-6	≤	-3/2
W_{DH} -	0	0	-1	2	-1	0	0	0	1	-2	1	0	12	≥	-2
W_{DH} +	0	0	1	-2	1	0	0	0	-1	2	-1	0	-12	≤	-2
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

Tableau a variabili negative su X- e limitate

	[X	Υ	Z	T	S	R	αbF	X- [[Fb]
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	-2
$\phi_{AB}\textbf{+}$	-1	0	0	0	0	0	0	1	≥	-2
ϕ_{BA} -	0	1	0	0	0	0	0	-1	≥	-2
ϕ_{BA} +	0	-1	0	0	0	0	0	1	≥	-2
$\phi_{\text{BC}}\text{-}$	0	0	1	0	0	0	0	-1	≥	-3/2
$\phi_{\text{BC}} \textbf{+}$	0	0	-1	0	0	0	0	1	≥	-3/2
ϕ_{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2
$\phi_{\text{CD}}\text{+}$	0	0	0	-1	0	0	0	1	≥	-3/2
$\phi_{\text{DC}}\text{-}$	0	0	1	-2	0	0	-12	1	≥	-3/2
$\phi_{\text{DC}}\text{+}$	0	0	-1	2	0	0	12	-1	≥	-3/2
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-2
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	≥	-2
$\phi_{\text{ED}}\text{-}$	-1	-1	0	0	-1	0	9	3	≥	-2
$\phi_{\text{ED}}\text{+}$	1	1	0	0	1	0	-9	-3	≥	-2
$\phi_{\text{BF}}\text{-}$	0	-1	-1	0	0	0	0	2	≥	-2
$\phi_{\text{BF}}\text{+}$	0	1	1	0	0	0	0	-2	≥	-2
$\phi_{\text{FG}}\text{-}$	0	0	0	0	0	1	0	-1	≥	-3/2
$\phi_{\text{FG}} \textbf{+}$	0	0	0	0	0	-1	0	1	≥	-3/2
$\phi_{\text{GH}}\text{-}$	0	1/2	1	-1	1/2	1	-7	-2	≥	-3/2
$\phi_{\text{GH}} \textbf{+}$	0	-1/2	-1	1	-1/2	-1	7	2	≥	-3/2
$\phi_{\text{HG}}\text{-}$	0	-1	-2	2	-1	-1	6	3	≥	-3/2
$\phi_{\text{HG}}\text{+}$	0	1	2	-2	1	1	-6	-3	≥	-3/2
$\phi_{\text{DH}}\text{-}$	0	0	-1	2	-1	0	12	0	≥	-2
$\phi_{\text{DH}} \textbf{+}$	0	0	1	-2	1	0	-12	0	≥	-2
L_{X}	0	0	0	0	0	0	0	-1	≥	-2
Max	0	0	0	0	0	0	1	0	=	0

Scam	bio pivo	otale 9-7	•								
	Χ	Υ	Z	Т	S	R	ϕ_{DC} -	X-]		[Fb]	
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	[-2]	
$\phi_{AB} \textbf{+}$	-1	0	0	0	0	0	0	1	≥	-2	
$\phi_{\text{BA}}\text{-}$	0	1	0	0	0	0	0	-1	≥	-2	
ϕ_{BA} +	0	-1	0	0	0	0	0	1	≥	-2	
ϕ_{BC} -	0	0	1	0	0	0	0	-1	≥	-3/2	
$\phi_{BC}\text{+}$	0	0	-1	0	0	0	0	1	≥	-3/2	
ϕ_{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2	
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2	
αbF	0	0	1/12	-1/6	0	0	-1/12	1/12	≥	-1/8	
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3	
ϕ_{DE} -	0	0	0	0	1	0	0	-1	≥	-2	
ϕ_{DE} +	0	0	0	0	-1	0	0	1	≥	-2	
$\phi_{\text{ED}}\text{-}$	-1	-1	3/4	-3/2	-1	0	-3/4	15/4	≥	-25/8	
ϕ_{ED} +	1	1	-3/4	3/2	1	0	3/4	-15/4	≥	-7/8	
ϕ_{BF} -	0	-1	-1	0	0	0	0	2	≥	-2	
ϕ_{BF} +	0	1	1	0	0	0	0	-2	≥	-2	
$\phi_{\text{FG}}\text{-}$	0	0	0	0	0	1	0	-1	≥	-3/2	
$\phi_{\text{FG}}\text{+}$	0	0	0	0	0	-1	0	1	≥	-3/2	
$\phi_{\text{GH}}\text{-}$	0	1/2	5/12	1/6	1/2	1	7/12	-31/12	≥	-5/8	
ϕ_{GH} +	0	-1/2	-5/12	-1/6	-1/2	-1	-7/12	31/12	≥	-19/8	
ϕ_{HG} -	0	-1	-3/2	1	-1	-1	-1/2	7/2	≥	-9/4	
ϕ_{HG} +	0	1	3/2	-1	1	1	1/2	-7/2	≥	-3/4	
ϕ_{DH} -	0	0	0	0	-1	0	-1	1	≥	-7/2	
ϕ_{DH} +	0	0	0	0	1	0	1	-1	≥	-1/2	
L_{x}	0	0	0	0	0	0	0	-1	≥	-2	
Max	0	0	1/12	-1/6	0	0	-1/12	1/12	=	1/8]	

Scambio pivotale 14-3

Scarri	DIO PIV	Jiaic 14-	,								
	[X	Υ	$\phi_{\text{ED}}\text{+}$	Т	S	R	ϕ_{DC} -	Χ-		[Fb]	
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	[-2]	
ϕ_{AB} +	-1	0	0	0	0	0	0	1	≥	-2	
$\phi_{\text{BA}}\text{-}$	0	1	0	0	0	0	0	-1	≥	-2	
$\phi_{BA}\textbf{+}$	0	-1	0	0	0	0	0	1	≥	-2	
ϕ_{BC} -	4/3	4/3	-4/3	2	4/3	0	1	-6	≥	-8/3	
ϕ_{BC} +	-4/3	-4/3	4/3	-2	-4/3	0	-1	6	≥	-1/3	
$\phi_{\text{CD}}\text{-}$	0	0	0	1	0	0	0	-1	≥	-3/2	
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2	
αbF	1/9	1/9	-1/9	0	1/9	0	0	-1/3	≥	-2/9	
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3	
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-2	
$\phi_{\text{DE}}\text{+}$	0	0	0	0	-1	0	0	1	≥	-2	
φ _{ED} -	0	0	-1	0	0	0	0	0	≥	-4	
Z	4/3	4/3	-4/3	2	4/3	0	1	-5	≥	-7/6	
ϕ_{BF} -	-4/3	-7/3	4/3	-2	-4/3	0	-1	7	≥	-5/6	
ϕ_{BF} +	4/3	7/3	-4/3	2	4/3	0	1	-7	≥	-19/6	
ϕ_{FG} -	0	0	0	0	0	1	0	-1	≥	-3/2	
ϕ_{FG} +	0	0	0	0	0	-1	0	1	≥	-3/2	
ϕ_{GH} -	5/9	19/18	-5/9	1	19/18	1	1	-14/3	≥	-10/9	
ϕ_{GH} +	-5/9	-19/18	5/9	-1	-19/18	-1	-1	14/3	≥	-17/9	
$\phi_{\text{HG}}\text{-}$	-2	-3	2	-2	-3	-1	-2	11	≥	-1/2	
ϕ_{HG} +	2	3	-2	2	3	1	2	-11	≥	-5/2	
φ _{DH} -	0	0	0	0	-1	0	-1	1	≥	-7/2	
ϕ_{DH} +	0	0	0	0	1	0	1	-1	≥	-1/2	
L_{x}	0	0	0	0	0	0	0	-1	≥	-2	
Max	1/9	1/9	-1/9	0	1/9	0	0	-1/3	=	2/9	

Scambio pivotale 6-1

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	φ_{BC} +	Υ	$\phi_{\text{ED}} \textbf{+}$	Т	S	R	$\phi_{\text{DC}}\text{-}$	X-]		[Fb]
ϕ_{AB} -	-3/4	-1	1	-3/2	-1	0	-3/4	7/2	\geq	-9/4
ϕ_{AB} +	3/4	1	-1	3/2	1	0	3/4	-7/2	\geq	-7/4
ϕ_{BA} -	0	1	0	0	0	0	0	-1	≥	-2
$\phi_{\text{BA}}\text{+}$	0	-1	0	0	0	0	0	1	\geq	-2
ϕ_{BC} -	-1	0	0	0	0	0	0	0	≥	-3
Χ	-3/4	-1	1	-3/2	-1	0	-3/4	9/2	≥	-1/4
$\phi_{\text{CD}}\text{-}$	0	0	0	1	0	0	0	-1	≥	-3/2
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2
α bF	-1/12	0	0	-1/6	0	0	-1/12	1/6	≥	-1/4
$\phi_{DC} \textbf{+}$	0	0	0	0	0	0	-1	0	≥	-3
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-2
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	\geq	-2
$\phi_{\text{ED}}\text{-}$	0	0	-1	0	0	0	0	0	≥	-4
Z	-1	0	0	0	0	0	0	1	≥	-3/2
$\phi_{\text{BF}}\text{-}$	1	-1	0	0	0	0	0	1	\geq	-1/2
ϕ_{BF} +	-1	1	0	0	0	0	0	-1	\geq	-7/2
$\phi_{\text{FG}}\text{-}$	0	0	0	0	0	1	0	-1	≥	-3/2
$\phi_{\text{FG}} \textbf{+}$	0	0	0	0	0	-1	0	1	\geq	-3/2
$\phi_{\text{GH}}\text{-}$	-5/12	1/2	0	1/6	1/2	1	7/12	-13/6	\geq	-5/4
ϕ_{GH} +	5/12	-1/2	0	-1/6	-1/2	-1	-7/12	13/6	\geq	-7/4
$\phi_{\text{HG}}\text{-}$	3/2	-1	0	1	-1	-1	-1/2	2	\geq	0
ϕ_{HG} +	-3/2	1	0	-1	1	1	1/2	-2	\geq	-3
ϕ_{DH} -	0	0	0	0	-1	0	-1	1	\geq	-7/2
ϕ_{DH} +	0	0	0	0	1	0	1	-1	\geq	-1/2
L_{X}	0	0	0	0	0	0	0	-1	≥	-2
Max	-1/12	0	0	-1/6	0	0	-1/12	1/6	=	-1/4

Scambio pivotale 2-8

								_		
	ϕ_{BC} +	Υ	ϕ_{ED} +	Т	S	R	ϕ_{DC} -	ϕ_{AB} +		[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	-4
Χ-	3/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1/2
$\phi_{\text{BA}}\text{-}$	-3/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2
$\phi_{\text{BA}} \textbf{+}$	3/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-5/2
ϕ_{BC} -	-1	0	0	0	0	0	0	0	≥	-3
Χ	3/14	2/7	-2/7	3/7	2/7	0	3/14	-9/7	≥	-5/2
ϕ_{CD} -	-3/14	-2/7	2/7	4/7	-2/7	0	-3/14	2/7	≥	-1
ϕ_{CD} +	3/14	2/7	-2/7	-4/7	2/7	0	3/14	-2/7	≥	-2
αbF	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	≥	-1/3
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3
ϕ_{DE} -	-3/14	-2/7	2/7	-3/7	5/7	0	-3/14	2/7	≥	-3/2
$\phi_{\text{DE}} \textbf{+}$	3/14	2/7	-2/7	3/7	-5/7	0	3/14	-2/7	≥	-5/2
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4
Z	-11/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-2
ϕ_{BF} -	17/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1
ϕ_{BF} +	-17/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3
ϕ_{FG} -	-3/14	-2/7	2/7	-3/7	-2/7	1	-3/14	2/7	≥	-1
ϕ_{FG} +	3/14	2/7	-2/7	3/7	2/7	-1	3/14	-2/7	≥	-2
ϕ_{GH} -	-37/42	-5/42	13/21	-16/21	-5/42	1	5/42	13/21	≥	-1/6
ϕ_{GH} +	37/42	5/42	-13/21	16/21	5/42	-1	-5/42	-13/21	≥	-17/6
ϕ_{HG} -	27/14	-3/7	-4/7	13/7	-3/7	-1	-1/14	-4/7	≥	-1
φ _{HG} +	-27/14	3/7	4/7	-13/7	3/7	1	1/14	4/7	≥	-2
φ _{DH} -	3/14	2/7	-2/7	3/7	-5/7	0	-11/14	-2/7	≥	-4
φ _{DH} +	-3/14	-2/7	2/7	-3/7	5/7	0	11/14	2/7	≥	0
L _X	-3/14	-2/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2
Max	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	=	-1/3

Scambio pivotale 24-2

	Г.,			-		ь			1	Г г ь -
	φ_{BC} +	φ_{DH} +	φ_{ED} +	Т	S	R	ϕ_{DC} -	φ _{AB} +]	Fb -
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	-4
Х-	0	-1	0	0	1	0	1	0	≥	-1/2
ϕ_{BA} -	-3/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3/2
ϕ_{BA} +	3/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-5/2
ϕ_{BC} -	-1	0	0	0	0	0	0	0	≥	-3
Χ	0	-1	0	0	1	0	1	-1	≥	-5/2
$\phi_{\text{CD}}\text{-}$	0	1	0	1	-1	0	-1	0	≥	-1
ϕ_{CD} +	0	-1	0	-1	1	0	1	0	≥	-2
αbF	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	≥	-1/3
$\phi_{\text{DC}}\text{+}$	0	0	0	0	0	0	-1	0	≥	-3
$\phi_{\text{DE}}\text{-}$	0	1	0	0	0	0	-1	0	≥	-3/2
ϕ_{DE} +	0	-1	0	0	0	0	1	0	≥	-5/2
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4
Z	-1	-1	0	0	1	0	1	0	≥	-2
ϕ_{BF} -	7/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-1
ϕ_{BF} +	-7/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3
ϕ_{FG} -	0	1	0	0	-1	1	-1	0	≥	-1
$\phi_{\text{FG}}\text{+}$	0	-1	0	0	1	-1	1	0	≥	-2
ϕ_{GH} -	-19/24	5/12	1/2	-7/12	-5/12	1	-5/24	1/2	≥	-1/6
ϕ_{GH} +	19/24	-5/12	-1/2	7/12	5/12	-1	5/24	-1/2	≥	-17/6
ϕ_{HG} -	9/4	3/2	-1	5/2	-3/2	-1	-5/4	-1	≥	-1
$\phi_{\text{HG}}\text{+}$	-9/4	-3/2	1	-5/2	3/2	1	5/4	1	≥	-2
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-4
Υ	-3/4	-7/2	1	-3/2	5/2	0	11/4	1	≥	0
L_{X}	0	1	0	0	-1	0	-1	0	≥	-3/2
Max	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	=	-1/3

Scambio pivotale 19-5

	φ _{BC} +	ϕ_{DH} +	φ _{ED} +	Т	φ _{GH} -	R	φ _{DC} -	φ _{AB} + _		[Fb]	
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	-4	
X-	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-9/10	
ϕ_{BA} -	-18/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-21/10	
ϕ_{BA} +	18/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-19/10	
ϕ_{BC} -	-1	0	0	0	0	0	0	0	≥	-3	
Χ	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	1/5	≥	-29/10	
$\phi_{\text{CD}}\text{-}$	19/10	0	-6/5	12/5	12/5	-12/5	-1/2	-6/5	≥	-3/5	
ϕ_{CD} +	-19/10	0	6/5	-12/5	-12/5	12/5	1/2	6/5	≥	-12/5	
αbF	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	≥	-2/5	
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3	
ϕ_{DE} -	0	1	0	0	0	0	-1	0	≥	-3/2	
$\phi_{\text{DE}} \textbf{+}$	0	-1	0	0	0	0	1	0	≥	-5/2	
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4	
Z	-29/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-12/5	
ϕ_{BF} -	23/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-2/5	
ϕ_{BF} +	-23/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-18/5	
$\phi_{\text{FG}}\text{-}$	19/10	0	-6/5	7/5	12/5	-7/5	-1/2	-6/5	≥	-3/5	
ϕ_{FG} +	-19/10	0	6/5	-7/5	-12/5	7/5	1/2	6/5	≥	-12/5	
S	-19/10	1	6/5	-7/5	-12/5	12/5	-1/2	6/5	≥	-2/5	
ϕ_{GH} +	0	0	0	0	-1	0	0	0	≥	-3	
$\phi_{\text{HG}}\text{-}$	51/10	0	-14/5	23/5	18/5	-23/5	-1/2	-14/5	≥	-2/5	
ϕ_{HG} +	-51/10	0	14/5	-23/5	-18/5	23/5	1/2	14/5	≥	-13/5	
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-4	
Υ	-11/2	-1	4	-5	-6	6	3/2	4	≥	-1	
L_{x}	19/10	0	-6/5	7/5	12/5	-12/5	-1/2	-6/5	≥	-11/10	
Max	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	=	2/5]	

Scambio pivotale 21-6

	[φ _{BC} +	ϕ_{DH} +	ϕ_{ED} +	Т	φ _{GH} -	φ _{HG} -	ϕ_{DC} -	ϕ_{AB} +]		[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	-1	\geq	「 -4]
X-	35/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	\geq	-51/46
ϕ_{BA} -	9/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	≥	-111/46
ϕ_{BA} +	-9/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	\geq	-73/46
ϕ_{BC} -	-1	0	0	0	0	0	0	0	\geq	-3
Χ	35/46	0	-6/23	1	-12/23	-12/23	11/46	-29/23	\geq	-143/46
ϕ_{CD} -	-35/46	0	6/23	0	12/23	12/23	-11/46	6/23	\geq	-9/23
ϕ_{CD} +	35/46	0	-6/23	0	-12/23	-12/23	11/46	-6/23	\geq	-60/23
αbF	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	≥	-10/23
ϕ_{DC} +	0	0	0	0	0	0	-1	0	\geq	-3
ϕ_{DE} -	0	1	0	0	0	0	-1	0	≥	-3/2
$\phi_{\text{DE}} \textbf{+}$	0	-1	0	0	0	0	1	0	\geq	-5/2
ϕ_{ED} -	0	0	-1	0	0	0	0	0	\geq	-4
Z	-11/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	\geq	-60/23
$\phi_{\text{BF}}\text{-}$	14/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	\geq	-2/23
ϕ_{BF} +	-14/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	\geq	-90/23
ϕ_{FG} -	8/23	0	-8/23	0	30/23	7/23	-8/23	-8/23	≥	-11/23
ϕ_{FG} +	-8/23	0	8/23	0	-30/23	-7/23	8/23	8/23	≥	-58/23
S	35/46	1	-6/23	1	-12/23	-12/23	-35/46	-6/23	\geq	-14/23
ϕ_{GH} +	0	0	0	0	-1	0	0	0	\geq	-3
R	51/46	0	-14/23	1	18/23	-5/23	-5/46	-14/23	\geq	-2/23
ϕ_{HG} +	0	0	0	0	0	-1	0	0	\geq	-3
ϕ_{DH} -	0	-1	0	0	0	0	0	0	\geq	-4
Υ	53/46	-1	8/23	1	-30/23	-30/23	39/46	8/23	\geq	-35/23
L_{x}	-35/46	0	6/23	-1	12/23	12/23	-11/46	6/23	≥	-41/46
Max	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	=	-10/23

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Scambio pivotale 7-1

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	φ _{CD} -	$\phi_{\text{DH}}\text{+}$	ϕ_{ED} +	Т	ϕ_{GH} -	ϕ_{HG} -	ϕ_{DC} -	φ_{AB} +		[Fb]	
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	「 -4]	
X-	-1	0	0	1	0	0	0	0	≥	-3/2	
ϕ_{BA} -	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	≥	-183/70	
ϕ_{BA} +	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	≥	-97/70	
ϕ_{BC} -	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	≥	-87/35	
Χ	-1	0	0	1	0	0	0	-1	≥	-7/2	
ϕ_{BC} +	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	≥	-18/35	
ϕ_{CD} +	-1	0	0	0	0	0	0	0	≥	-3	
αbF	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	≥	-16/35	
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3	
ϕ_{DE} -	0	1	0	0	0	0	-1	0	≥	-3/2	
ϕ_{DE} +	0	-1	0	0	0	0	1	0	≥	-5/2	
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4	
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	≥	-87/35	
ϕ_{BF} -	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5	
ϕ_{BF} +	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	≥	-18/5	
ϕ_{FG} -	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	≥	-23/35	
φ _{FG} +	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	≥	-82/35	
S	-1	1	0	1	0	0	-1	0	≥	-1	
ϕ_{GH} +	0	0	0	0	-1	0	0	0	≥	-3	
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	≥	-23/35	
ϕ_{HG} +	0	0	0	0	0	-1	0	0	≥	-3	
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-4	
Υ	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35	
L_{x}	1	0	0	-1	0	0	0	0	≥	-1/2	
Max	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	-16/35	

Table	eau fina	ᆈ

	[φ _{CD} -	ϕ_{DH} +	ϕ_{ED} +	Т	ϕ_{GH} -	φ _{HG} -	φ _{DC} -	ϕ_{AB} +		[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	-4
X-	-1	0	0	1	0	0	0	0	≥	-3/2
ϕ_{BA} -	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	≥	-183/70
ϕ_{BA} +	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	≥	-97/70
ϕ_{BC} -	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	≥	-87/35
Χ	-1	0	0	1	0	0	0	-1	≥	-7/2
ϕ_{BC} +	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	≥	-18/35
ϕ_{CD} +	-1	0	0	0	0	0	0	0	≥	-3
$\alpha b F$	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	≥	-16/35
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3
ϕ_{DE} -	0	1	0	0	0	0	-1	0	≥	-3/2
ϕ_{DE} +	0	-1	0	0	0	0	1	0	≥	-5/2
φ _{ED} -	0	0	-1	0	0	0	0	0	≥	-4
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	≥	-87/35
ϕ_{BF} -	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5
ϕ_{BF} +	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	≥	-18/5
ϕ_{FG} -	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	≥	-23/35
ϕ_{FG} +	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	≥	-82/35
S	-1	1	0	1	0	0	-1	0	≥	-1
ϕ_{GH} +	0	0	0	0	-1	0	0	0	≥	-3
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	≥	-23/35
ϕ_{HG} +	0	0	0	0	0	-1	0	0	≥	-3
φ _{DH} -	0	-1	0	0	0	0	0	0	≥	-4
Υ	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35
L_{x}	1	0	0	-1	0	0	0	0	≥	-1/2
	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	16/35 _

Vettori soluzione della programmazione lineare

	_ X	Υ	Z	Т	S	R	$\alpha b F$	Χ-]	[Fb]
$\phi_{\text{AB}}\text{-}$	0	0	0	0	0	0	0	0	≥	[0]
$\phi_{AB}\textbf{+}$	0	0	0	0	0	0	0	0	≥	1/35
$\phi_{\text{BA}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{BA} +	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{BC}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{BC} +	0	0	0	0	0	0	0	0	≥	0
ϕ_{CD} -	0	0	0	0	0	0	0	0	≥	2/35
ϕ_{CD} +	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DC}}\text{-}$	0	0	0	0	0	0	0	0	≥	2/35
$\phi_{\text{DC}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DE}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{ED}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	1/35
$\phi_{\text{BF}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{BF}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{FG} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{FG}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{GH} -	0	0	0	0	0	0	0	0	≥	2/35
$\phi_{\text{GH}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{HG}}\text{-}$	0	0	0	0	0	0	0	0	≥	2/35
$\phi_{\text{HG}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DH}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DH}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
L_{X}	0	0	0	0	0	0	0	0	≥	0
Max	7/2	74/35	87/35	0	1	23/35	16/35	3/2	=	-16/35

Variabili soluzione dedotto il valore X-

Variabili soluzione differenza tra rotazioni

ϕ_{AB}	1/35
ϕ_{BA}	0
ϕ_{BC}	0
ϕ_{CD}	-2/35
ϕ_{DC}	-2/35
ϕ_{DE}	0
ϕ_{ED}	1/35
ϕ_{BF}	0
ϕ_{FG}	0
ϕ_{GH}	-2/35
ϕ_{HG}	-2/35
ϕ_{DH}	[0]

REAZIONI Fattore di collasso = 16/35

 $H_{\Lambda} = -61/70F$

 $V_{\Delta} = 11/7F$

 $W_A = 2Fb$

 $H_{r} = -1/2F$

 $V_E = 3F$

 $W_F = 2Fb$

 $H_{BC} = -2/3F$ $H_{CD} = -2/3F$ $H_{AB} = -61/70F$ $H_{DF} = 1/2F$ $H_{RE} = 53/210F$ $V_{\Delta R} = 11/7F$ $V_{BC} = 87/70F$ $V_{CD} = -3/2F$ $V_{DF} = -3F$ $V_{RF} = 23/70F$ $W_{CD} = -3/2Fb$ $W_{\Delta R} = 2Fb$ $W_{BC} = 69/70 Fb$ $W_{DF} = -1/2Fb$ $W_{BF} = -8/5Fb$ $H_{DC} = 2/3F$ $H_{ED} = -1/2F$ $H_{BA} = 61/70F$ $H_{CR} = 2/3F$ $H_{FR} = -53/210F$ $V_{BA} = -11/7F$ $V_{CB} = -87/70F$ $V_{DC} = 3/2F$ $V_{ED} = 3F$ $V_{FR} = -23/70F$ $W_{BA} = 43/70 Fb$ $W_{CR} = 3/2Fb$ $W_{DC} = -3/2Fb$ $W_{ED} = 2Fb$ $W_{ER} = 59/70 Fb$

$$\begin{array}{llll} H_{FG} = 7/6F & H_{GH} = 7/6F & H_{HD} = 7/6F \\ V_{FG} = 23/70F & V_{GH} = -3/2F & V_{HD} = -3/2F \\ W_{FG} = -59/70Fb & W_{GH} = -3/2Fb & W_{HD} = 3/2Fb \\ H_{GF} = -7/6F & H_{HG} = -7/6F & H_{DH} = -7/6F \\ V_{GF} = -23/70F & V_{HG} = 3/2F & V_{DH} = 3/2F \\ W_{GF} = 3/2Fb & W_{HG} = -3/2Fb & W_{DH} = 2Fb \end{array}$$

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_B = 3/35\delta$	$u_{CCB} = 3/35\delta$	$u_D = 3/35\delta$	$u_{EED} = 0$
$V_{AAB} = 0$	$V_B = 0$	$V_{CCB} = -2/35\delta$	$V_D = 0$	$V_{EED} = 0$
$\phi_{AAB} = -1/35\delta/b$	$\phi_{\rm p} = -1/35\delta/b$	$\phi_{CCR} = -1/35\delta/b$	$\phi_{D} = 1/35\delta/b$	$\phi_{E} = -1/35\delta/b$

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$u_F = 6/35\delta$	$u_{GGF} = 6/35\delta$	$u_{HHG} = 6/35\delta$
$V_F = 0$	$V_{GGF} = -2/35\delta$	$V_{HHG} = 0$
$\varphi_F = -1/35\delta/b$	$\phi_{GGF} = -1/35\delta/b$	$\varphi_{HHD} = 1/35\delta/b$

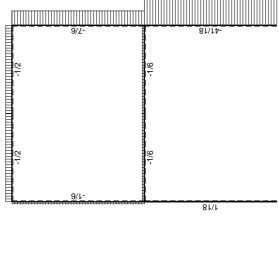
SPOSTAMENTI RIGIDI DELLE ASTE

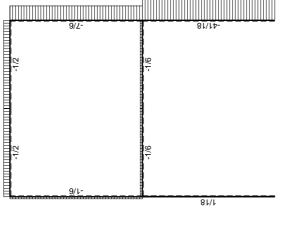
$u_{AAB} = 0$	$u_{BBC} = 3/35\delta$	$u_{CCD} = 3/35\delta$	$u_{DDE} = 3/35\delta$	$u_{BBF} = 3/35\delta$
$V_{AAB} = 0$	$V_{BBC} = 0$	$V_{CCD} = -2/35\delta$	$V_{DDE} = 0$	$V_{BBF} = 0$
$\phi_{AAB} = -1/35\delta/b$	$\phi_{BBC} = -1/35\delta/b$	$\varphi_{CCD} = 1/35\delta/b$	$\phi_{DDE} = -1/35\delta/b$	$\varphi_{BBF} = -1/35\delta/b$

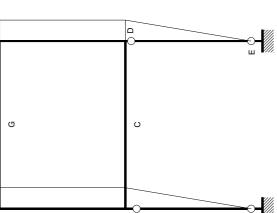
$$\begin{array}{lll} u_{FFG} = 6/35\delta & u_{GGH} = 6/35\delta & u_{HHD} = 6/35\delta \\ v_{FFG} = 0 & v_{GGH} = -2/35\delta & v_{HHD} = 0 \\ \phi_{FFG} = -1/35\delta/b & \phi_{GGH} = 1/35\delta/b & \phi_{HHD} = -1/35\delta/b \end{array}$$

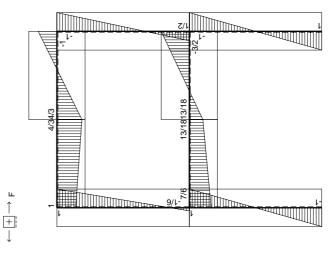
EQUILIBRIO Nome:

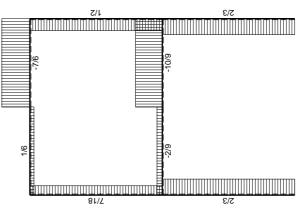


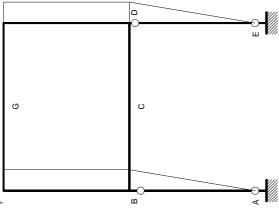


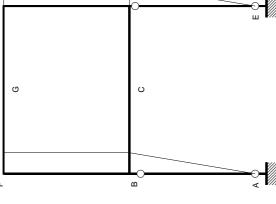


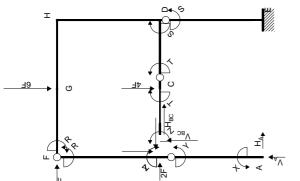












EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

 $3H_Ab - 4V_Ab = -Xb + Sb - 8Fb$

Rotazione intorno a C: aste CB

 $-2V_{BC}b = -Zb + Tb$

Rotazione intorno a F: aste FB BA $6H_Ab - 3H_{BC}b = -Xb + Zb - Rb - 6Fb$

Rotazione intorno a B: aste BA

 $3H_Ab = -Xb - Yb$

Matrice di equilibrio

Soluzione del sistema

$$\begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1/3 & -1/3 & 0 & 0 & 0 & 0 & 0 \\ V_{BC}b & 0 & 0 & 1/2 & -1/2 & 0 & 0 & 0 \\ H_{BC}b & -1/3 & -2/3 & -1/3 & 0 & 0 & 1/3 & 2 \\ V_{A}b & 0 & -1/4 & 0 & 0 & -1/4 & 0 & 2 \end{bmatrix}$$

PROGRAMMAZIONE LINEARE

Sia H_{ii} la matrice del simplesso, con m righe e n colonne.

Siano P_i le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.

Siano a riga m i coefficienti della funzione obiettivo primale $\max \Sigma_i H_{mi} P_{i}$, $1 \le j < n$.

Siano a colonna n i coefficienti della funzione obiettivo duale $min \Sigma_i H_{in} D_{it}$ $1 \le i < m$.

Sequenza di operazioni pivotali:

- 1 Sia q (1 $\leq q < n$) la colonna pivot con massimo valore H_{mi} in riga m.
- 2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{io} , che minimizza il rapporto H_{in}/H_{io} .
- 3 Si ottiene il coefficiente pivotale H_{po}
- 4 Si scambia la variabile primale P_q con la duale D_p .
- 5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
- 6 Si ridefiniscono i coefficienti della colonna pivot $q: H_{ig} = H_{ng} H_{ig}$, escluso il pivot H_{ng} .
- 7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ii} = H_{ii} H_{ia} H_{pi}$
- 8 Si ridefiniscono i coefficienti della riga pivot $p: H_{pj} = -H_{pq} H_{pi}$, escluso il pivot H_{pq} .
- Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.

Giunti a questo punto, si individua la soluzione.

Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_i presenti sulla colonna di sinistra.

Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare *m*=6,*n*=4

$$\begin{bmatrix} \mathsf{P}_1 & \mathsf{P}_2 & \mathsf{P}_3 \end{bmatrix} & \begin{bmatrix} \mathsf{MIN} \\ \mathsf{D}_1 & \begin{bmatrix} \mathsf{H}_{11} & \mathsf{H}_{12} & \mathsf{H}_{13} \end{bmatrix} \geq \begin{bmatrix} \mathsf{H}_{14} \\ \mathsf{H}_{21} & \mathsf{H}_{22} & \mathsf{H}_{23} \end{bmatrix} \geq \begin{bmatrix} \mathsf{H}_{14} \\ \mathsf{H}_{24} \\ \mathsf{D}_3 & \mathsf{H}_{31} & \mathsf{H}_{32} & \mathsf{H}_{33} \end{bmatrix} \geq \begin{bmatrix} \mathsf{H}_{34} \\ \mathsf{H}_{44} \\ \mathsf{D}_4 & \mathsf{H}_{41} & \mathsf{H}_{42} & \mathsf{H}_{43} \end{bmatrix} \geq \begin{bmatrix} \mathsf{H}_{44} \\ \mathsf{H}_{41} \\ \mathsf{D}_5 & \mathsf{H}_{51} & \mathsf{H}_{52} & \mathsf{H}_{53} \end{bmatrix} \geq \begin{bmatrix} \mathsf{H}_{54} \\ \mathsf{H}_{64} \end{bmatrix}$$

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{BC}$ $T=W_{CD}$ $S=W_{DE}$ $R=W_{FB}$

Tableau con variabili non vincolate in segno

rableau con variabili non vincolate in Segno									
	[X	Υ	Z	Т	S	R	αbF		[Fb]
W_{AB} -	1	0	0	0	0	0	0	≥	-1
W_{AB} +	1	0	0	0	0	0	0	≤	1
W _{BA} -	0	1	0	0	0	0	0	≥	-1
W_{RA} +	0	1	0	0	0	0	0	≤	1
W_{BC} -	0	0	1	0	0	0	0	≥	-3/2
W _{BC} +	0	0	1	0	0	0	0	≤	3/2
W _{CD} -	0	0	0	1	0	0	0	≥	-3/2
W _{CD} +	0	0	0	1	0	0	0	≤	3/2
W_{DC} -	0	0	1	-2	0	0	-8	≥	-3/2
W_{DC} +	0	0	1	-2	0	0	-8	≤	3/2
W_{DE} -	0	0	0	0	1	0	0	≥	-1
W_{DE} +	0	0	0	0	1	0	0	≤	1
W_{ED} -	-1	-1	0	0	-1	0	18	≥	-1
W_{ED} +	-1	-1	0	0	-1	0	18	≤	1
W_{BF} -	0	-1	-1	0	0	0	0	≥	-1
W_{RF} +	0	-1	-1	0	0	0	0	≤	1
W_{FB} -	0	0	0	0	0	1	0	≥	-1
W_{FB} +	0	0	0	0	0	1	0	≤	1
W_{GH} -	0	1/2	1	-1	1/2	-1	-4	≥	-3/2
W_{GH} +	0	1/2	1	-1	1/2	-1	-4	≤	3/2
W _{HD} -	0	1	2	-2	1	-1	4	≥	-1
W_{HD} +	0	1	2	-2	1	-1	4	≤	1
W_{DH} -	0	0	-1	2	-1	0	8	≥	-1
W_{DH} +	0	0	-1	2	-1	0	8	≤	1
Max	l o	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	[X	Υ	Z	Т	S	R	αbF]		[Fb]
W_{AB} -	1	0	0	0	0	0	0	≥	「-1
W_{AB} +	-1	0	0	0	0	0	0	≥	-1
W_{BA} -	0	1	0	0	0	0	0	≥	-1
W_{BA} +	0	-1	0	0	0	0	0	≥	-1
W_{BC} -	0	0	1	0	0	0	0	≥	-3/2
W_{BC} +	0	0	-1	0	0	0	0	≥	-3/2
W _{CD} -	0	0	0	1	0	0	0	≥	-3/2
W_{CD} +	0	0	0	-1	0	0	0	≥	-3/2
W_{DC} -	0	0	1	-2	0	0	-8	≥	-3/2
W_{DC} +	0	0	-1	2	0	0	8	≥	-3/2
W_{DE} -	0	0	0	0	1	0	0	≥	-1
W_{DE} +	0	0	0	0	-1	0	0	≥	-1
W_{ED} -	-1	-1	0	0	-1	0	18	≥	-1
W_{ED} +	1	1	0	0	1	0	-18	≥	-1
W _{BF} -	0	-1	-1	0	0	0	0	≥	-1
W_{BF} +	0	1	1	0	0	0	0	≥	-1
W_{FB} -	0	0	0	0	0	1	0	≥	-1
W_{FB} +	0	0	0	0	0	-1	0	≥	-1
W_{GH} -	0	1/2	1	-1	1/2	-1	-4	≥	-3/2
W_{GH} +	0	-1/2	-1	1	-1/2	1	4	≥	-3/2
W_{HD} -	0	1	2	-2	1	-1	4	≥	-1
W_{HD} +	0	-1	-2	2	-1	1	-4	≥	-1
W_{DH} -	0	0	-1	2	-1	0	8	≥	-1
W_{DH} +	0	0	1	-2	1	0	-8	≥	-1
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili vincolate in segno																
	[X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF]	[Fb]	
W_{AB} -	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	[-1]	
W_{AB} +	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1	
W_{BA} -	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1	
W_{BA} +	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1	
W_{BC} -	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2	
14/ .	_	^	4	^	^	^	^	^	4	^	^	^	^		2/2	

W W_{CD}-≥ -3/2 -3/2 W_{CD}+ 0 ≤ -3/2 ≥ W_{DC} + -3/2 ≤ W_{DE} -≥ -1 W_{DF} + ≤ -1 W_{ED}-≥ -1 W_{ED} + -1 \leq W_{BF} --1 \geq ≤ -1 W_{FB} -≥ -1 W_{FB} + 0 ≤ -1 W_{GH} -≥ -3/2 W_{GH} + -3/2 1/2 ≤ W_{HD} --2 \geq -1 W_{HD} + ≤ -1 W_{DH} -≥ W_{DH} + 0 0 -8 ≤ -1 0 Max

Tableau a variabili negative su X- e limitate

	X	Υ	Z	Ť	S	R	αbF	X		[Fb]
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	-1
ϕ_{AB} +	-1	0	0	0	0	0	0	1	≥	-1
$\phi_{\text{BA}}\text{-}$	0	1	0	0	0	0	0	-1	≥	-1
ϕ_{BA} +	0	-1	0	0	0	0	0	1	≥	-1
$\phi_{\text{BC}}\text{-}$	0	0	1	0	0	0	0	-1	≥	-3/2
ϕ_{BC} +	0	0	-1	0	0	0	0	1	≥	-3/2
ϕ_{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2
$\phi_{\text{CD}} \textbf{+}$	0	0	0	-1	0	0	0	1	≥	-3/2
$\phi_{\text{DC}}\text{-}$	0	0	1	-2	0	0	-8	1	≥	-3/2
$\phi_{\text{DC}} \textbf{+}$	0	0	-1	2	0	0	8	-1	≥	-3/2
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-1
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	≥	-1
$\phi_{\text{ED}}\text{-}$	-1	-1	0	0	-1	0	18	3	≥	-1
$\phi_{\text{ED}} \textbf{+}$	1	1	0	0	1	0	-18	-3	≥	-1
$\phi_{\text{BF}}\text{-}$	0	-1	-1	0	0	0	0	2	≥	-1
ϕ_{BF} +	0	1	1	0	0	0	0	-2	≥	-1
ϕ_{FB} -	0	0	0	0	0	1	0	-1	≥	-1
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1
ϕ_{GH} -	0	1/2	1	-1	1/2	-1	-4	0	≥	-3/2
$\phi_{\text{GH}} \textbf{+}$	0	-1/2	-1	1	-1/2	1	4	0	≥	-3/2
ϕ_{HD} -	0	1	2	-2	1	-1	4	-1	≥	-1
$\phi_{\text{HD}} \textbf{+}$	0	-1	-2	2	-1	1	-4	1	≥	-1
$\phi_{\text{DH}}\text{-}$	0	0	-1	2	-1	0	8	0	≥	-1
$\phi_{\text{DH}} \textbf{+}$	0	0	1	-2	1	0	-8	0	≥	-1
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	0	0	0	0	0	1	0 _	=	0

Scam	Scambio pivotale 14-7											
	[X	Υ	Z	Т	S	R	ϕ_{ED} +	X-]		[Fb]		
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	[-1]		
ϕ_{AB} +	-1	0	0	0	0	0	0	1	≥	-1		
ϕ_{BA} -	0	1	0	0	0	0	0	-1	≥	-1		
ϕ_{BA} +	0	-1	0	0	0	0	0	1	≥	-1		
ϕ_{BC} -	0	0	1	0	0	0	0	-1	≥	-3/2		
ϕ_{BC} +	0	0	-1	0	0	0	0	1	≥	-3/2		
ϕ_{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2		
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2		
ϕ_{DC} -	-4/9	-4/9	1	-2	-4/9	0	4/9	7/3	≥	-19/18		
ϕ_{DC} +	4/9	4/9	-1	2	4/9	0	-4/9	-7/3	≥	-35/18		
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-1		
$\phi_{\text{DE}}\text{+}$	0	0	0	0	-1	0	0	1	≥	-1		
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2		
αbF	1/18	1/18	0	0	1/18	0	-1/18	-1/6	≥	-1/18		
ϕ_{BF} -	0	-1	-1	0	0	0	0	2	≥	-1		
ϕ_{BF} +	0	1	1	0	0	0	0	-2	≥	-1		
ϕ_{FB} -	0	0	0	0	0	1	0	-1	≥	-1		
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1		
ϕ_{GH} -	-2/9	5/18	1	-1	5/18	-1	2/9	2/3	≥	-23/18		
ϕ_{GH} +	2/9	-5/18	-1	1	-5/18	1	-2/9	-2/3	≥	-31/18		
ϕ_{HD} -	2/9	11/9	2	-2	11/9	-1	-2/9	-5/3	≥	-11/9		
ϕ_{HD} +	-2/9	-11/9	-2	2	-11/9	1	2/9	5/3	≥	-7/9		
ϕ_{DH} -	4/9	4/9	-1	2	-5/9	0	-4/9	-4/3	≥	-13/9		
ϕ_{DH} +	-4/9	-4/9	1	-2	5/9	0	4/9	4/3	≥	-5/9		
L_X	0	0	0	0	0	0	0	-1	≥	-3/2		
Max	1/18	1/18	0	0	1/18	0	-1/18	-1/6	=	-1/18		

Scambio pivotale 2-1

	[φ _{AB} +	Υ	Z	Т	S	R	ϕ_{ED} +	X-]		[Fb]	
ϕ_{AB} -	-1	0	0	0	0	0	0	0]	≥	[-2]	
Χ	-1	0	0	0	0	0	0	1	≥	-1	
ϕ_{BA} -	0	1	0	0	0	0	0	-1	\geq	-1	
ϕ_{BA} +	0	-1	0	0	0	0	0	1	≥	-1	
ϕ_{BC} -	0	0	1	0	0	0	0	-1	≥	-3/2	
ϕ_{BC} +	0	0	-1	0	0	0	0	1	\geq	-3/2	
ϕ_{CD} -	0	0	0	1	0	0	0	-1	\geq	-3/2	
ϕ_{CD} +	0	0	0	-1	0	0	0	1	\geq	-3/2	
φ _{DC} -	4/9	-4/9	1	-2	-4/9	0	4/9	17/9	≥	-11/18	
ϕ_{DC} +	-4/9	4/9	-1	2	4/9	0	-4/9	-17/9	\geq	-43/18	
ϕ_{DE} -	0	0	0	0	1	0	0	-1	\geq	-1	
ϕ_{DE} +	0	0	0	0	-1	0	0	1	\geq	-1	
ϕ_{ED} -	0	0	0	0	0	0	-1	0	\geq	-2	
αbF	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	\geq	-1/9	
ϕ_{BF} -	0	-1	-1	0	0	0	0	2	\geq	-1	
ϕ_{BF} +	0	1	1	0	0	0	0	-2	\geq	-1	
ϕ_{FB} -	0	0	0	0	0	1	0	-1	\geq	-1	
ϕ_{FB} +	0	0	0	0	0	-1	0	1	\geq	-1	
φ _{GH} -	2/9	5/18	1	-1	5/18	-1	2/9	4/9	\geq	-19/18	
ϕ_{GH} +	-2/9	-5/18	-1	1	-5/18	1	-2/9	-4/9	\geq	-35/18	
ϕ_{HD} -	-2/9	11/9	2	-2	11/9	-1	-2/9	-13/9	\geq	-13/9	
ϕ_{HD} +	2/9	-11/9	-2	2	-11/9	1	2/9	13/9	\geq	-5/9	
ϕ_{DH} -	-4/9	4/9	-1	2	-5/9	0	-4/9	-8/9	≥	-17/9	
ϕ_{DH} +	4/9	-4/9	1	-2	5/9	0	4/9	8/9	≥	-1/9	
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2	
Max	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	=	-1/9	

Scambio pivotale 24-2

	[φ _{AB} +	ϕ_{DH} +	Z	Т	S	R	ϕ_{ED} +	X-]	[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	-2
Χ	-1	0	0	0	0	0	0	1	≥	-1
$\phi_{\text{BA}}\text{-}$	1	-9/4	9/4	-9/2	5/4	0	1	1	≥	-5/4
ϕ_{BA} +	-1	9/4	-9/4	9/2	-5/4	0	-1	-1	≥	-3/4
$\phi_{\text{BC}}\text{-}$	0	0	1	0	0	0	0	-1	≥	-3/2
ϕ_{BC} +	0	0	-1	0	0	0	0	1	≥	-3/2
$\phi_{\text{CD}}\text{-}$	0	0	0	1	0	0	0	-1	≥	-3/2
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2
$\phi_{\text{DC}}\text{-}$	0	1	0	0	-1	0	0	1	≥	-1/2
$\phi_{\text{DC}}\text{+}$	0	-1	0	0	1	0	0	-1	≥	-5/2
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-1
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	≥	-1
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
$\alpha b F$	0	-1/8	1/8	-1/4	1/8	0	0	0	≥	-1/8
$\phi_{\text{BF}}\text{-}$	-1	9/4	-13/4	9/2	-5/4	0	-1	0	≥	-3/4
ϕ_{BF} +	1	-9/4	13/4	-9/2	5/4	0	1	0	≥	-5/4
$\phi_{\text{FB}}\text{-}$	0	0	0	0	0	1	0	-1	≥	-1
$\phi_{\text{FB}}\text{+}$	0	0	0	0	0	-1	0	1	≥	-1
$\phi_{\text{GH}}\text{-}$	1/2	-5/8	13/8	-9/4	5/8	-1	1/2	1	≥	-9/8
$\phi_{\text{GH}} \textbf{+}$	-1/2	5/8	-13/8	9/4	-5/8	1	-1/2	-1	≥	-15/8
ϕ_{HD} -	1	-11/4	19/4	-15/2	11/4	-1	1	1	≥	-7/4
ϕ_{HD} +	-1	11/4	-19/4	15/2	-11/4	1	-1	-1	≥	-1/4
$\phi_{\text{DH}}\text{-}$	0	-1	0	0	0	0	0	0	≥	-2
Υ	1	-9/4	9/4	-9/2	5/4	0	1	2	≥	-1/4
L_{x}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	-1/8	1/8	-1/4	1/8	0	0	0	=	-1/8

Scambio pivotale 22-3

	[φ _{AB} +	ϕ_{DH} +	ϕ_{HD} +	Т	S	R	ϕ_{ED} +	X-]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0]	\geq	-2
Χ	-1	0	0	0	0	0	0	1	\geq	-1
ϕ_{BA} -	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	10/19	≥	-26/19
$\phi_{\text{BA}} \textbf{+}$	-10/19	18/19	9/19	18/19	1/19	-9/19	-10/19	-10/19	\geq	-12/19
ϕ_{BC} -	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-23/19	\geq	-59/38
$\phi_{\text{BC}} \textbf{+}$	4/19	-11/19	4/19	-30/19	11/19	-4/19	4/19	23/19	\geq	-55/38
ϕ_{CD}	0	0	0	1	0	0	0	-1	≥	-3/2
$\phi_{\text{CD}}\text{+}$	0	0	0	-1	0	0	0	1	\geq	-3/2
$\phi_{\text{DC}}\text{-}$	0	1	0	0	-1	0	0	1	\geq	-1/2
$\phi_{DC} \textbf{+}$	0	-1	0	0	1	0	0	-1	\geq	-5/2
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	\geq	-1
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	\geq	-1
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	-1	0	\geq	-2
$\alpha b F$	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	\geq	-5/38
$\phi_{\text{BF}}\text{-}$	-6/19	7/19	13/19	-12/19	12/19	-13/19	-6/19	13/19	\geq	-11/19
$\phi_{\text{BF}}\text{+}$	6/19	-7/19	-13/19	12/19	-12/19	13/19	6/19	-13/19	\geq	-27/19
$\phi_{\text{FB}}\text{-}$	0	0	0	0	0	1	0	-1	\geq	-1
$\phi_{\text{FB}}\text{+}$	0	0	0	0	0	-1	0	1	\geq	-1
ϕ_{GH} -	3/19	6/19	-13/38	6/19	-6/19	-25/38	3/19	25/38	\geq	-23/19
$\phi_{\text{GH}} \textbf{+}$	-3/19	-6/19	13/38	-6/19	6/19	25/38	-3/19	-25/38	\geq	-34/19
ϕ_{HD} -	0	0	-1	0	0	0	0	0	\geq	-2
Z	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-4/19	\geq	-1/19
$\phi_{\text{DH}}\text{-}$	0	-1	0	0	0	0	0	0	\geq	-2
Υ	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	29/19	\geq	-7/19
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	=	5/38]

Scambio pivotale 22-5

	[φ _{AB} +	ϕ_{DH} +	ϕ_{HD} +	Т	Z	R	ϕ_{ED} +	X-]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	2
Χ	-1	0	0	0	0	0	0	1	≥	-1
ϕ_{BA} -	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	6/11	≥	-15/11
ϕ_{BA} +	-6/11	1	5/11	12/11	-1/11	-5/11	-6/11	-6/11	≥	-7/11
ϕ_{BC} -	0	0	0	0	1	0	0	-1	≥	-3/2
ϕ_{BC} +	0	0	0	0	-1	0	0	1	≥	-3/2
φ _{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2
φ _{DC} -	4/11	0	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-9/22
ϕ_{DC} +	-4/11	0	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-57/22
ϕ_{DE} -	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-12/11
ϕ_{DE} +	4/11	-1	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-10/11
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
$\alpha b F$	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	≥	-3/22
ϕ_{BF} -	-6/11	1	5/11	12/11	-12/11	-5/11	-6/11	5/11	≥	-7/11
ϕ_{BF} +	6/11	-1	-5/11	-12/11	12/11	5/11	6/11	-5/11	≥	-15/11
ϕ_{FB} -	0	0	0	0	0	1	0	-1	≥	-1
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1
ϕ_{GH} -	3/11	0	-5/22	-6/11	6/11	-17/22	3/11	17/22	≥	-13/11
ϕ_{GH} +	-3/11	0	5/22	6/11	-6/11	17/22	-3/11	-17/22	≥	-20/11
φ _{HD} -	0	0	-1	0	0	0	0	0	≥	-2
S	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-4/11	≥	-1/11
φ _{DH} -	0	-1	0	0	0	0	0	0	≥	-2
Υ	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	17/11	≥	-4/11
L_{x}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	=	-3/22

Scambio pivotale 9-4

Scarri	ocambio pivotale 9-4										
	φ _{AB} +	ϕ_{DH} +	ϕ_{HD} +	ϕ_{DC} -	Z	R	ϕ_{ED} +	Χ-		[Fb]	
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	[- 2]	
Χ	-1	0	0	0	0	0	0	1	≥	-1	
ϕ_{BA} -	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	0	≥	-6/5	
ϕ_{BA} +	-2/5	1	3/5	-2/5	3/5	-3/5	-2/5	0	≥	-4/5	
ϕ_{BC} -	0	0	0	0	1	0	0	-1	≥	-3/2	
ϕ_{BC} +	0	0	0	0	-1	0	0	1	≥	-3/2	
ϕ_{CD} -	2/15	0	2/15	-11/30	19/30	-2/15	2/15	-1/2	≥	-33/20	
ϕ_{CD} +	-2/15	0	-2/15	11/30	-19/30	2/15	-2/15	1/2	≥	-27/20	
Т	2/15	0	2/15	-11/30	19/30	-2/15	2/15	1/2	≥	-3/20	
ϕ_{DC} +	0	0	0	-1	0	0	0	0	≥	-3	
$\phi_{\text{DE}}\text{-}$	0	1	0	-1	0	0	0	0	≥	-3/2	
ϕ_{DE} +	0	-1	0	1	0	0	0	0	≥	-1/2	
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2	
αbF	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	≥	-3/20	
ϕ_{BF} -	-2/5	1	3/5	-2/5	-2/5	-3/5	-2/5	1	≥	-4/5	
ϕ_{BF} +	2/5	-1	-3/5	2/5	2/5	3/5	2/5	-1	≥	-6/5	
ϕ_{FB} -	0	0	0	0	0	1	0	-1	≥	-1	
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1	
ϕ_{GH} -	1/5	0	-3/10	1/5	1/5	-7/10	1/5	1/2	≥	-11/10	
ϕ_{GH} +	-1/5	0	3/10	-1/5	-1/5	7/10	-1/5	-1/2	≥	-19/10	
φ _{HD} -	0	0	-1	0	0	0	0	0	≥	-2	
S	0	1	0	-1	0	0	0	1	≥	-1/2	
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-2	
Υ	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	1	≥	-1/5	
L_{x}	0	0	0	0	0	0	0	-1	≥	-3/2	
Max	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	=	-3/20	

Scambio pivotale 18-6

	[φ _{AB} +	φ _{DH} +	φ _{HD} +	ϕ_{DC} -	Z	ϕ_{FB} +	ϕ_{ED} +	X-]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	-2
Χ	-1	0	0	0	0	0	0	1	≥	-1
ϕ_{BA} -	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	3/5	≥	-9/5
ϕ_{BA} +	-2/5	1	3/5	-2/5	3/5	3/5	-2/5	-3/5	≥	-1/5
ϕ_{BC} -	0	0	0	0	1	0	0	-1	≥	-3/2
$\phi_{\text{BC}}\text{+}$	0	0	0	0	-1	0	0	1	≥	-3/2
ϕ_{CD} -	2/15	0	2/15	-11/30	19/30	2/15	2/15	-19/30	≥	-91/60
$\phi_{\text{CD}}\text{+}$	-2/15	0	-2/15	11/30	-19/30	-2/15	-2/15	19/30	≥	-89/60
Т	2/15	0	2/15	-11/30	19/30	2/15	2/15	11/30	≥	-1/60
ϕ_{DC} +	0	0	0	-1	0	0	0	0	≥	-3
ϕ_{DE} -	0	1	0	-1	0	0	0	0	≥	-3/2
ϕ_{DE} +	0	-1	0	1	0	0	0	0	≥	-1/2
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
$\alpha b F$	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	≥	-11/60
ϕ_{BF} -	-2/5	1	3/5	-2/5	-2/5	3/5	-2/5	2/5	≥	-1/5
ϕ_{BF} +	2/5	-1	-3/5	2/5	2/5	-3/5	2/5	-2/5	≥	-9/5
ϕ_{FB} -	0	0	0	0	0	-1	0	0	≥	-2
R	0	0	0	0	0	-1	0	1	≥	-1
$\phi_{\text{GH}}\text{-}$	1/5	0	-3/10	1/5	1/5	7/10	1/5	-1/5	≥	-2/5
ϕ_{GH} +	-1/5	0	3/10	-1/5	-1/5	-7/10	-1/5	1/5	≥	-13/5
ϕ_{HD} -	0	0	-1	0	0	0	0	0	≥	-2
S	0	1	0	-1	0	0	0	1	≥	-1/2
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-2
Υ	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	8/5	≥	-4/5
L_{x}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	=	-11/60

Scambio pivotale 4-8

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	φ _{AB} +	ϕ_{DH} +	$\phi_{\text{HD}} \textbf{+}$	ϕ_{DC} -	Z	ϕ_{FB} +	ϕ_{ED} +	φ_{BA} +		[Fb]	
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	-2	
X	-5/3	5/3	1	-2/3	1	1	-2/3	-5/3	≥	-4/3	
ϕ_{BA} -	0	0	0	0	0	0	0	-1	≥	-2	
X-	-2/3	5/3	1	-2/3	1	1	-2/3	-5/3	≥	-1/3	
ϕ_{BC} -	2/3	-5/3	-1	2/3	0	-1	2/3	5/3	≥	-7/6	
ϕ_{BC} +	-2/3	5/3	1	-2/3	0	1	-2/3	-5/3	≥	-11/6	
ϕ_{CD} -	5/9	-19/18	-1/2	1/18	0	-1/2	5/9	19/18	≥	-47/36	
ϕ_{CD} +	-5/9	19/18	1/2	-1/18	0	1/2	-5/9	-19/18	≥	-61/36	
Т	-1/9	11/18	1/2	-11/18	1	1/2	-1/9	-11/18	≥	-5/36	
ϕ_{DC} +	0	0	0	-1	0	0	0	0	≥	-3	
$\phi_{\text{DE}}\text{-}$	0	1	0	-1	0	0	0	0	≥	-3/2	
$\phi_{\text{DE}}\text{+}$	0	-1	0	1	0	0	0	0	≥	-1/2	
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2	
$\alpha b F$	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	≥	-7/36	
ϕ_{BF} -	-2/3	5/3	1	-2/3	0	1	-2/3	-2/3	≥	-1/3	
ϕ_{BF} +	2/3	-5/3	-1	2/3	0	-1	2/3	2/3	≥	-5/3	
ϕ_{FB} -	0	0	0	0	0	-1	0	0	≥	-2	
R	-2/3	5/3	1	-2/3	1	0	-2/3	-5/3	≥	-4/3	
ϕ_{GH} -	1/3	-1/3	-1/2	1/3	0	1/2	1/3	1/3	≥	-1/3	
ϕ_{GH} +	-1/3	1/3	1/2	-1/3	0	-1/2	-1/3	-1/3	≥	-8/3	
$\phi_{\text{HD}}\text{-}$	0	0	-1	0	0	0	0	0	≥	-2	
S	-2/3	8/3	1	-5/3	1	1	-2/3	-5/3	≥	-5/6	
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-2	
Υ	-2/3	5/3	1	-2/3	1	1	-2/3	-8/3	≥	-4/3	
L_{x}	2/3	-5/3	-1	2/3	-1	-1	2/3	5/3	≥	-7/6	
Max	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	=	7/36]	

Scambio pivotale 12-2

	[φ _{AB} +	ϕ_{DE} +	ϕ_{HD} +	ϕ_{DC} -	Z	ϕ_{FB} +	ϕ_{ED} +	ϕ_{BA} +]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	\geq	-2
Χ	-5/3	-5/3	1	1	1	1	-2/3	-5/3	\geq	-13/6
ϕ_{BA} -	0	0	0	0	0	0	0	-1	\geq	-2
X-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-7/6
ϕ_{BC} -	2/3	5/3	-1	-1	0	-1	2/3	5/3	≥	-1/3
ϕ_{BC} +	-2/3	-5/3	1	1	0	1	-2/3	-5/3	≥	-8/3
ϕ_{CD} -	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	≥	-7/9
ϕ_{CD} +	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	≥	-20/9
Т	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	≥	-4/9
ϕ_{DC} +	0	0	0	-1	0	0	0	0	≥	-3
$\phi_{\text{DE}}\text{-}$	0	-1	0	0	0	0	0	0	≥	-2
ϕ_{DH} +	0	-1	0	1	0	0	0	0	≥	-1/2
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	-1/18	0	0	0	0	-1/18	-1/18	≥	-2/9
ϕ_{BF} -	-2/3	-5/3	1	1	0	1	-2/3	-2/3	≥	-7/6
ϕ_{BF} +	2/3	5/3	-1	-1	0	-1	2/3	2/3	≥	-5/6
φ _{FB} -	0	0	0	0	0	-1	0	0	≥	-2
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	≥	-13/6
$\phi_{\text{GH}}\text{-}$	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	≥	-1/6
ϕ_{GH} +	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	≥	-17/6
ϕ_{HD} -	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	≥	-13/6
ϕ_{DH} -	0	1	0	-1	0	0	0	0	≥	-3/2
Υ	-2/3	-5/3	1	1	1	1	-2/3	-8/3	≥	-13/6
L_{X}	2/3	5/3	-1	-1	-1	-1	2/3	5/3	≥	-1/3
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	2/9

Tableau finale

i abio	Tablead III ale									
	[φ _{AB} +	$\phi_{\text{DE}} \textbf{+}$	ϕ_{HD} +	ϕ_{DC}	Z	ϕ_{FB} +	ϕ_{ED} +	φ_{BA} +]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	[-2]
Χ	-5/3	-5/3	1	1	1	1	-2/3	-5/3	\geq	-13/6
ϕ_{BA} -	0	0	0	0	0	0	0	-1	\geq	-2
X-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	\geq	-7/6
ϕ_{BC} -	2/3	5/3	-1	-1	0	-1	2/3	5/3	\geq	-1/3
$\phi_{BC}\text{+}$	-2/3	-5/3	1	1	0	1	-2/3	-5/3	\geq	-8/3
ϕ_{CD} -	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	≥	-7/9
ϕ_{CD} +	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	\geq	-20/9
Т	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	≥	-4/9
ϕ_{DC} +	0	0	0	-1	0	0	0	0	≥	-3
ϕ_{DE} -	0	-1	0	0	0	0	0	0	≥	-2
$\phi_{\text{DH}} \textbf{+}$	0	-1	0	1	0	0	0	0	≥	-1/2
ϕ_{ED} -	0	0	0	0	0	0	-1	0	\geq	-2
αbF	-1/18	-1/18	0	0	0	0	-1/18	-1/18	\geq	-2/9
ϕ_{BF} -	-2/3	-5/3	1	1	0	1	-2/3	-2/3	\geq	-7/6
ϕ_{BF} +	2/3	5/3	-1	-1	0	-1	2/3	2/3	≥	-5/6
ϕ_{FB} -	0	0	0	0	0	-1	0	0	\geq	-2
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	\geq	-13/6
ϕ_{GH} -	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	≥	-1/6
ϕ_{GH} +	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	\geq	-17/6
ϕ_{HD} -	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	≥	-13/6
ϕ_{DH} -	0	1	0	-1	0	0	0	0	≥	-3/2
Υ	-2/3	-5/3	1	1	1	1	-2/3	-8/3	≥	-13/6
L_{x}	2/3	5/3	-1	-1	-1	-1	2/3	5/3	≥	-1/3
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	2/9]

Vettori soluzione della programmazione lineare

	[X	Υ	Z	T	s	R	αbF	X-]		[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	0	≥	[0]
$\phi_{AB}\textbf{+}$	0	0	0	0	0	0	0	0	≥	1/18
ϕ_{BA} -	0	0	0	0	0	0	0	0	≥	0
ϕ_{BA} +	0	0	0	0	0	0	0	0	≥	1/18
ϕ_{BC} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{BC}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{CD} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{CD}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DC}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DC}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DE}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	1/18
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{ED}}\text{+}$	0	0	0	0	0	0	0	0	≥	1/18
ϕ_{BF} -	0	0	0	0	0	0	0	0	≥	0
ϕ_{BF} +	0	0	0	0	0	0	0	0	≥	0
ϕ_{FB} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{FB}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{GH}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{GH} +	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{HD}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{HD}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DH}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DH}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
L_{X}	0	0	0	0	0	0	0	0	≥	0
Max	13/6	13/6	0	4/9	13/6	13/6	2/9	7/6	=	2/9

Variabili soluzione dedotto il valore X-

 $u_G = 1/6\delta$

 $V_G = 0$

 $\varphi_G = 0$

Variabili soluzione differenza tra rotazioni

ϕ_{AB}	1/18
ϕ_{BA}	1/18
ϕ_{BC}	0
ϕ_{CD}	0
ϕ_{DC}	0
ϕ_{DE}	1/18
ϕ_{ED}	1/18
ϕ_{BF}	0
ϕ_{FB}	0
ϕ_{GH}	0
ϕ_{HD}	0
ϕ_{DH}	0

REAZIONI Fattore di collasso = 2/9

 $H_{\Delta} = -2/3F$

 $V_{\Delta}^{\prime} = -1/18F$

 $W_A = Fb$

 $H_{\rm F} = -2/3F$

 $V_{\rm F} = 41/18F$

 $W_F = Fb$

$H_{AB} = -2/3F$ $V_{AB} = -1/18F$ $W_{AB} = Fb$ $H_{BA} = 2/3F$ $V_{BA} = 1/18F$ $W_{BA} = Fb$	$H_{BC} = 1/6F$ $V_{BC} = -2/9F$ $W_{BC} = -7/6Fb$ $H_{CB} = -1/6F$ $V_{CB} = 2/9F$ $W_{CB} = 13/18Fb$	$H_{CD} = 1/6F$ $V_{CD} = -10/9F$ $W_{CD} = -13/18Fb$ $H_{DC} = -1/6F$ $V_{DC} = 10/9F$ $W_{DC} = -3/2Fb$	$H_{DE} = 2/3F$ $V_{DE} = -41/18F$ $W_{DE} = Fb$ $H_{ED} = -2/3F$ $V_{ED} = 41/18F$ $W_{ED} = Fb$	$H_{BF} = -7/18F$ $V_{BF} = 1/6F$ $W_{BF} = 1/6Fb$ $H_{FB} = 7/18F$ $V_{FB} = -1/6F$ $W_{FB} = Fb$
$H_{FG} = 1/2F$ $V_{FG} = 1/6F$ $W_{FG} = -Fb$ $H_{GF} = -1/2F$ $V_{GF} = -1/6F$ $W_{GF} = 4/3Fb$	$H_{GH} = 1/2F$ $V_{GH} = -7/6F$ $W_{GH} = -4/3Fb$ $H_{HG} = -1/2F$ $V_{HG} = 7/6F$ $W_{HG} = -Fb$	$H_{HD} = 1/2F$ $V_{HD} = -7/6F$ $W_{HD} = Fb$ $H_{DH} = -1/2F$ $V_{DH} = 7/6F$ $W_{DH} = 1/2Fb$		

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_B = 1/6\delta$	$u_C = 1/6\delta$	$u_D = 1/6\delta$	$u_{EED} = 0$
$V_{AAB} = 0$	$V_B = 0$	$v_C = 0$	$v_D = 0$	$v_{EED} = 0$
$\phi_{AAB} = -1/18\delta/b$	$\phi_{\rm p} = -1/18\delta/b$	$\varphi_{\rm C} = 0$	$\varphi_D = 0$	$\phi_{=} = -1/18\delta/b$

SPOSTAMENTI RIGIDI DELLE ASTE

 $u_F = 1/6\delta$

 $V_F = 0$

 $\phi_F = 0$

$u_{AAB} = 0$	$u_{BBC} = 1/6\delta$	$u_{CCD} = 1/6\delta$	$u_{DDE} = 1/6\delta$	$u_{BBF} = 1/6\delta$
$V_{AAB} = 0$	$V_{BBC} = 0$	$V_{CCD} = 0$	$V_{DDE} = 0$	$V_{BBF} = 0$
$\phi_{AAB} = -1/18\delta/b$	$\varphi_{BBC} = 0$	$\varphi_{CCD} = 0$	$\phi_{DDE} = -1/18\delta/b$	$\varphi_{BBF} = 0$

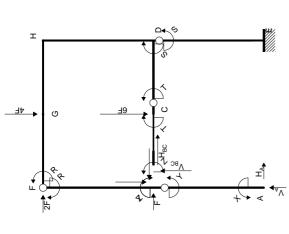
 $u_H = 1/6\delta$

 $V_{H} = 0$

 $\phi_H = 0$

$u_{FFG} = 1/6\delta$	$u_{GGH} = 1/6\delta$	$u_{HHD} = 1/6\delta$
$V_{FFG} = 0$	$v_{GGH} = 0$	$V_{HHD} = 0$
$\phi_{FFG} = 0$	$\varphi_{GGH} = 0$	$\phi_{HHD} = 0$

EQUILIBRIO Nome:



EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

 $3H_Ab - 4V_Ab = -Xb + Sb - 14Fb$

Rotazione intorno a C: aste CB

Rotazione intorno a F: aste FB BA $-2V_{BC}b = -Zb + Tb$

 $6H_Ab - 3H_{BC}b = -Xb + Zb + Rb - 3Fb$ Rotazione intorno a B: aste BA

 $3H_Ab = -Xb - Yb$

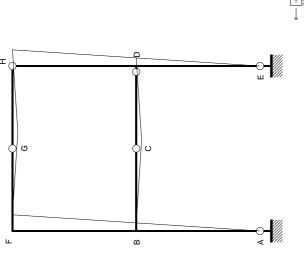
Matrice di equilibrio

년 - 0 양 0

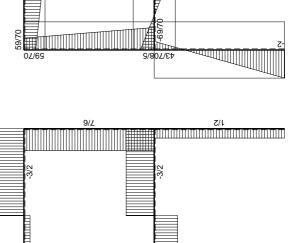
Soluzione del sistema

$$\begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1/3 & -1/3 & 0 & 0 & 0 & 0 & 0 \\ V_{BC}b & 0 & 0 & 1/2 & -1/2 & 0 & 0 & 0 \\ H_{BC}b & 0 & -1/3 & -2/3 & -1/3 & 0 & 0 & -1/3 & 1 \\ V_Ab & 0 & -1/4 & 0 & 0 & -1/4 & 0 & 7/2 \end{bmatrix}$$









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PROGRAMMAZIONE LINEARE

Sia H_{ii} la matrice del simplesso, con m righe e n colonne.

Siano P_i le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.

Siano a riga m i coefficienti della funzione obiettivo primale $\max \Sigma_i H_{mi} P_{i}$, $1 \le j < n$.

Siano a colonna n i coefficienti della funzione obiettivo duale $min \Sigma_i H_{in} D_{it}$ $1 \le i < m$.

Sequenza di operazioni pivotali:

- 1 Sia q (1 $\leq q < n$) la colonna pivot con massimo valore H_{mi} in riga m.
- 2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{io} , che minimizza il rapporto H_{in}/H_{io} .
- 3 Si ottiene il coefficiente pivotale H_{po}
- 4 Si scambia la variabile primale P_a con la duale D_p .
- 5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
- 6 Si ridefiniscono i coefficienti della colonna pivot $q: H_{ig} = H_{ng} H_{ig}$, escluso il pivot H_{ng} .
- 7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ii} = H_{ii} H_{ii} + H_{pr}$
- 8 Si ridefiniscono i coefficienti della riga pivot $p: H_{pj} = -H_{pq} H_{pj}$, escluso il pivot H_{pq} .
- Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.

Giunti a questo punto, si individua la soluzione.

Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_i presenti sulla colonna di sinistra.

Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare *m*=6,*n*=4

$$\begin{bmatrix} \mathsf{P}_1 & \mathsf{P}_2 & \mathsf{P}_3 \end{bmatrix} & \begin{bmatrix} \mathsf{MIN} \\ \mathsf{D}_1 & \begin{bmatrix} \mathsf{H}_{11} & \mathsf{H}_{12} & \mathsf{H}_{13} \end{bmatrix} \geq \begin{bmatrix} \mathsf{H}_{14} \\ \mathsf{H}_{21} & \mathsf{H}_{22} & \mathsf{H}_{23} \end{bmatrix} \geq \begin{bmatrix} \mathsf{H}_{14} \\ \mathsf{H}_{24} \\ \mathsf{D}_3 & \mathsf{H}_{31} & \mathsf{H}_{32} & \mathsf{H}_{33} \end{bmatrix} \geq \begin{bmatrix} \mathsf{H}_{34} \\ \mathsf{H}_{44} \\ \mathsf{D}_4 & \mathsf{H}_{41} & \mathsf{H}_{42} & \mathsf{H}_{43} \end{bmatrix} \geq \begin{bmatrix} \mathsf{H}_{44} \\ \mathsf{H}_{41} \\ \mathsf{D}_5 & \mathsf{H}_{51} & \mathsf{H}_{52} & \mathsf{H}_{53} \end{bmatrix} \geq \begin{bmatrix} \mathsf{H}_{54} \\ \mathsf{H}_{64} \end{bmatrix}$$

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{BC}$ $T=W_{CD}$ $S=W_{DE}$ $R=W_{FG}$

Tableau con variabili non vincolate in segno

Tableau con variabili non vincolate in segno										
_	Υ	Z	T	_	R			[Fb]		
1	0	0	0	0	0	0	≥	-2		
1	0	0	0	0	0	0	≤	2		
0	1	0	0	0	0	0	≥	-2		
0	1	0	0	0	0	0	≤	2		
0	0	1	0	0	0	0	≥	-3/2		
0	0	1	0	0	0	0	≤	3/2		
0	0	0	1	0	0	0	≥	-3/2		
0	0	0	1	0	0	0	≤	3/2		
0	0	1	-2	0	0	-12	≥	-3/2		
0	0	1	-2	0	0	-12	≤	3/2		
0	0	0	0	1	0	0	≥	-2		
0	0	0	0	1	0	0	≤	2		
-1	-1	0	0	-1	0	9	≥	-2		
-1	-1	0	0	-1	0	9	≤	2		
0	-1	-1	0	0	0	0	≥	-2		
0	-1	-1	0	0	0	0	≤	2		
0	0	0	0	0	1	0	≥	-3/2		
0	0	0	0	0	1	0	≤	3/2		
0	1/2	1	-1	1/2	1	-7	≥	-3/2		
0	1/2	1	-1	1/2	1	-7	≤	3/2		
0	-1	-2	2	-1	-1	6	≥	-3/2		
0	-1	-2	2	-1	-1	6	≤	3/2		
0	0	-1	2	-1	0	12	≥	-2		
0	0	-1	2	-1	0	12	≤	2		
0	0	0	0	0	0	1	=	0		
	X 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0	X Y 1 0 1 0 0 1 0 0 0 1 0 0 0 0 0 0 0 0 0 0	X Y Z 1 0 0 1 0 0 0 1 0 0 1 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0	X Y Z T 1 0 0 0 1 0 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 0 1 0 0 0 0 0 0 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 0 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 1	X Y Z T S 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 0 0 0 0 1 -2 0 0 0 0 1 -2 0 0 0 0 1 -2 0 0 0 0 1 -2 0 0 0 0 1 -1 -1 0 0 -1 -1 -1 0 0 -1 -1 -1 0 0 0 0 0 1/2 1 -1 1/2 0 1/2 1 -1 1/2 0 1/2 1 -1 1/2 0 -1 -2 2 -1 0 0 -1 2 -1	X Y Z T S R 1 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 0 1 -2 0	[X Y Z T S R αbF] 1 0 0 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 1 0 0 0 0	$ \begin{bmatrix} X & Y & Z & T & S & R & \alpha bF \end{bmatrix} \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 0 & 0 & -12 \\ 0 & 0 & 1 & -2 & 0 & 0 & -12 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 9 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0$		

Tableau con variabili non vincolate in segno

	[X	Υ	Z	Т	S	R	αbF		[Fb]	
W_{AB} -	1	0	0	0	0	0	0	≥	-2	
W_{AB} +	-1	0	0	0	0	0	0	≥	-2	
W_{BA} -	0	1	0	0	0	0	0	≥	-2	
W_{BA} +	0	-1	0	0	0	0	0	≥	-2	
W_{BC} -	0	0	1	0	0	0	0	≥	-3/2	
W _{BC} +	0	0	-1	0	0	0	0	≥	-3/2	
W _{CD} -	0	0	0	1	0	0	0	≥	-3/2	
W_{CD} +	0	0	0	-1	0	0	0	≥	-3/2	
W_{DC} -	0	0	1	-2	0	0	-12	≥	-3/2	
W_{DC} +	0	0	-1	2	0	0	12	≥	-3/2	
W_{DE} -	0	0	0	0	1	0	0	≥	-2	
W_{DE} +	0	0	0	0	-1	0	0	≥	-2	
W_{ED} -	-1	-1	0	0	-1	0	9	≥	-2	
W_{ED} +	1	1	0	0	1	0	-9	≥	-2	
W_{BF} -	0	-1	-1	0	0	0	0	≥	-2	
W_{BF} +	0	1	1	0	0	0	0	≥	-2	
W_{FG} -	0	0	0	0	0	1	0	≥	-3/2	
W_{FG} +	0	0	0	0	0	-1	0	≥	-3/2	
W_{GH} -	0	1/2	1	-1	1/2	1	-7	≥	-3/2	
W_{GH} +	0	-1/2	-1	1	-1/2	-1	7	≥	-3/2	
W_{HG} -	0	-1	-2	2	-1	-1	6	≥	-3/2	
W_{HG} +	0	1	2	-2	1	1	-6	≥	-3/2	
W_{DH} -	0	0	-1	2	-1	0	12	≥	-2	
W_{DH} +	0	0	1	-2	1	0	-12	≥	-2	
Max	0	0	0	0	0	0	1	=	0	

Tableau con variabili vincolate in segno																
	[X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		[Fb]	
W_{AB} -	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	[-2]	
W_{AB} +	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-2	
W_{BA} -	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-2	
W_{BA} +	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-2	
W_{BC} -	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2	
W_{BC} +	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2	
W_{CD} -	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2	
W _{CD} +	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2	
W_{DC} -	0	0	1	-2	0	0	0	0	-1	2	0	0	-12	≥	-3/2	
W _{DC} +	0	0	-1	2	0	0	0	0	1	-2	0	0	12	≤	-3/2	
W _{DE} -	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-2	
W_{DE} +	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-2	
W _{ED} -	-1	-1	0	0	-1	0	1	1	0	0	1	0	9	≥	-2	
W_{FD} +	1	1	0	0	1	0	-1	-1	0	0	-1	0	-9	≤	-2	
W _{BF} -	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-2	
W_{BF} +	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-2	
W_{FG} -	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-3/2	
W_{FG} +	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-3/2	
W _{GH} -	0	1/2	1	-1	1/2	1	0	-1/2	-1	1	-1/2	-1	-7	≥	-3/2	
W_{GH} +	0	-1/2	-1	1	-1/2	-1	0	1/2	1	-1	1/2	1	7	≤	-3/2	
W_{HG} -	0	-1	-2	2	-1	-1	0	1	2	-2	1	1	6	≥	-3/2	
W_{HG} +	0	1	2	-2	1	1	0	-1	-2	2	-1	-1	-6	≤	-3/2	
W _{DH} -	0	0	-1	2	-1	0	0	0	1	-2	1	0	12	≥	-2	
W_{DH} +	0	0	1	-2	1	0	0	0	-1	2	-1	0	-12	≤	-2	

Tableau a variabili negative su X- e limitate

	[X	Υ	Z	Ť	S	R	αbF	Χ-]	[Fb]
ϕ_{AB} -	1	0	0	0	0	0	0	-1 ⁼	_ ≥	-2
ϕ_{AB} +	-1	0	0	0	0	0	0	1	≥	-2
φ _{BA} -	0	1	0	0	0	0	0	-1	≥	-2
ϕ_{BA} +	0	-1	0	0	0	0	0	1	≥	-2
φ _{BC} -	0	0	1	0	0	0	0	-1	≥	-3/2
ϕ_{BC} +	0	0	-1	0	0	0	0	1	≥	-3/2
ϕ_{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2
ϕ_{DC} -	0	0	1	-2	0	0	-12	1	≥	-3/2
$\phi_{\text{DC}} \textbf{+}$	0	0	-1	2	0	0	12	-1	≥	-3/2
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-2
ϕ_{DE} +	0	0	0	0	-1	0	0	1	≥	-2
ϕ_{ED} -	-1	-1	0	0	-1	0	9	3	≥	-2
ϕ_{ED} +	1	1	0	0	1	0	-9	-3	≥	-2
$\phi_{\text{BF}}\text{-}$	0	-1	-1	0	0	0	0	2	≥	-2
ϕ_{BF} +	0	1	1	0	0	0	0	-2	≥	-2
ϕ_{FG} -	0	0	0	0	0	1	0	-1	≥	-3/2
ϕ_{FG} +	0	0	0	0	0	-1	0	1	≥	-3/2
ϕ_{GH} -	0	1/2	1	-1	1/2	1	-7	-2	≥	-3/2
ϕ_{GH} +	0	-1/2	-1	1	-1/2	-1	7	2	≥	-3/2
$\phi_{\text{HG}}\text{-}$	0	-1	-2	2	-1	-1	6	3	≥	-3/2
$\phi_{\text{HG}}\text{+}$	0	1	2	-2	1	1	-6	-3	≥	-3/2
$\phi_{\text{DH}}\text{-}$	0	0	-1	2	-1	0	12	0	≥	-2
ϕ_{DH} +	0	0	1	-2	1	0	-12	0	≥	-2
L_{X}	0	0	0	0	0	0	0	-1	≥	-2
Max	0	0	0	0	0	0	1	0	=	0

Scaml	Scambio pivotale 9-7									
[Χ	Υ	Z	Т	S	R	ϕ_{DC} -	X-]		[Fb]
φ _{AB} -	1	0	0	0	0	0	0	-1	≥	[- 2]
ϕ_{AB} +	-1	0	0	0	0	0	0	1	\geq	-2
ϕ_{BA} -	0	1	0	0	0	0	0	-1	\geq	-2
ϕ_{BA} +	0	-1	0	0	0	0	0	1	\geq	-2
ϕ_{BC} -	0	0	1	0	0	0	0	-1	\geq	-3/2
ϕ_{BC} +	0	0	-1	0	0	0	0	1	\geq	-3/2
φ _{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2
αbF	0	0	1/12	-1/6	0	0	-1/12	1/12	\geq	-1/8
ϕ_{DC} +	0	0	0	0	0	0	-1	0	\geq	-3
ϕ_{DE} -	0	0	0	0	1	0	0	-1	≥	-2
ϕ_{DE} +	0	0	0	0	-1	0	0	1	≥	-2
ϕ_{ED} -	-1	-1	3/4	-3/2	-1	0	-3/4	15/4	\geq	-25/8
ϕ_{ED} +	1	1	-3/4	3/2	1	0	3/4	-15/4	≥	-7/8
ϕ_{BF} -	0	-1	-1	0	0	0	0	2	≥	-2
ϕ_{BF} +	0	1	1	0	0	0	0	-2	≥	-2
ϕ_{FG} -	0	0	0	0	0	1	0	-1	\geq	-3/2
ϕ_{FG} +	0	0	0	0	0	-1	0	1	≥	-3/2
ϕ_{GH} -	0	1/2	5/12	1/6	1/2	1	7/12	-31/12	\geq	-5/8
ϕ_{GH} +	0	-1/2	-5/12	-1/6	-1/2	-1	-7/12	31/12	≥	-19/8
ϕ_{HG} -	0	-1	-3/2	1	-1	-1	-1/2	7/2	\geq	-9/4
ϕ_{HG} +	0	1	3/2	-1	1	1	1/2	-7/2	\geq	-3/4
ϕ_{DH} -	0	0	0	0	-1	0	-1	1	≥	-7/2
ϕ_{DH} +	0	0	0	0	1	0	1	-1	≥	-1/2
L_{x}	0	0	0	0	0	0	0	-1	\geq	-2
Max	_ 0	0	1/12	-1/6	0	0	-1/12	1/12	=	-1/8

Scambio pivotale 14-3

Coairii	zeamble protate 110												
	_ X	Υ	$\phi_{\text{ED}} \textbf{+}$	Т	S	R	ϕ_{DC} -	X-		[Fb]			
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	[-2]			
$\phi_{AB}\textbf{+}$	-1	0	0	0	0	0	0	1	≥	-2			
$\phi_{\text{BA}}\text{-}$	0	1	0	0	0	0	0	-1	≥	-2			
$\phi_{\text{BA}} \textbf{+}$	0	-1	0	0	0	0	0	1	≥	-2			
ϕ_{BC} -	4/3	4/3	-4/3	2	4/3	0	1	-6	≥	-8/3			
$\phi_{BC} \textbf{+}$	-4/3	-4/3	4/3	-2	-4/3	0	-1	6	≥	-1/3			
ϕ_{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2			
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2			
$\alpha b F$	1/9	1/9	-1/9	0	1/9	0	0	-1/3	≥	-2/9			
$\phi_{\text{DC}}\text{+}$	0	0	0	0	0	0	-1	0	≥	-3			
ϕ_{DE} -	0	0	0	0	1	0	0	-1	≥	-2			
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	≥	-2			
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4			
Z	4/3	4/3	-4/3	2	4/3	0	1	-5	≥	-7/6			
$\phi_{\text{BF}}\text{-}$	-4/3	-7/3	4/3	-2	-4/3	0	-1	7	≥	-5/6			
$\phi_{\text{BF}}\text{+}$	4/3	7/3	-4/3	2	4/3	0	1	-7	≥	-19/6			
$\phi_{\text{FG}}\text{-}$	0	0	0	0	0	1	0	-1	≥	-3/2			
$\phi_{\text{FG}} \textbf{+}$	0	0	0	0	0	-1	0	1	≥	-3/2			
$\phi_{\text{GH}}\text{-}$	5/9	19/18	-5/9	1	19/18	1	1	-14/3	≥	-10/9			
ϕ_{GH} +	-5/9	-19/18	5/9	-1	-19/18	-1	-1	14/3	≥	-17/9			
$\phi_{\text{HG}}\text{-}$	-2	-3	2	-2	-3	-1	-2	11	≥	-1/2			
$\phi_{\text{HG}}\text{+}$	2	3	-2	2	3	1	2	-11	≥	-5/2			
ϕ_{DH} -	0	0	0	0	-1	0	-1	1	≥	-7/2			
ϕ_{DH} +	0	0	0	0	1	0	1	-1	≥	-1/2			
L_{x}	0	0	0	0	0	0	0	-1	≥	-2			
Max	1/9	1/9	-1/9	0	1/9	0	0	-1/3	=	2/9			

Scambio pivotale 6-1

Ooaiii	Courible product of 1											
	φ_{BC} +	Υ	$\phi_{\text{ED}}\text{+}$	Т	S	R	ϕ_{DC}	X-]		[Fb]		
ϕ_{AB} -	-3/4	-1	1	-3/2	-1	0	-3/4	7/2	≥	-9/4		
ϕ_{AB} +	3/4	1	-1	3/2	1	0	3/4	-7/2	≥	-7/4		
$\phi_{\text{BA}}\text{-}$	0	1	0	0	0	0	0	-1	≥	-2		
$\phi_{\text{BA}} \textbf{+}$	0	-1	0	0	0	0	0	1	≥	-2		
ϕ_{BC} -	-1	0	0	0	0	0	0	0	≥	-3		
Χ	-3/4	-1	1	-3/2	-1	0	-3/4	9/2	≥	-1/4		
ϕ_{CD}	0	0	0	1	0	0	0	-1	≥	-3/2		
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2		
$\alpha b F$	-1/12	0	0	-1/6	0	0	-1/12	1/6	≥	-1/4		
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3		
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-2		
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	≥	-2		
$\phi_{\text{ED}}\text{-}$	0	0	-1	0	0	0	0	0	≥	-4		
Z	-1	0	0	0	0	0	0	1	≥	-3/2		
$\phi_{\text{BF}}\text{-}$	1	-1	0	0	0	0	0	1	≥	-1/2		
$\phi_{\text{BF}}\text{+}$	-1	1	0	0	0	0	0	-1	≥	-7/2		
$\phi_{\text{FG}}\text{-}$	0	0	0	0	0	1	0	-1	≥	-3/2		
$\phi_{\text{FG}}\text{+}$	0	0	0	0	0	-1	0	1	≥	-3/2		
ϕ_{GH} -	-5/12	1/2	0	1/6	1/2	1	7/12	-13/6	≥	-5/4		
$\phi_{\text{GH}} \textbf{+}$	5/12	-1/2	0	-1/6	-1/2	-1	-7/12	13/6	≥	-7/4		
$\phi_{\text{HG}}\text{-}$	3/2	-1	0	1	-1	-1	-1/2	2	≥	0		
$\phi_{\text{HG}} \textbf{+}$	-3/2	1	0	-1	1	1	1/2	-2	≥	-3		
$\phi_{\text{DH}}\text{-}$	0	0	0	0	-1	0	-1	1	≥	-7/2		
$\phi_{\text{DH}} \textbf{+}$	0	0	0	0	1	0	1	-1	≥	-1/2		
L_{X}	0	0	0	0	0	0	0	-1	≥	-2		
Max	-1/12	0	0	-1/6	0	0	-1/12	1/6	=	-1/4		

Scambio pivotale 2-8

Coambio protato 2 c											
	ϕ_{BC} +	Υ	ϕ_{ED} +	Т	S	R	ϕ_{DC} -	ϕ_{AB} +		[Fb]	
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	[-4]	
X-	3/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1/2	
$\phi_{\text{BA}}\text{-}$	-3/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2	
ϕ_{BA} +	3/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-5/2	
ϕ_{BC} -	-1	0	0	0	0	0	0	0	≥	-3	
Χ	3/14	2/7	-2/7	3/7	2/7	0	3/14	-9/7	≥	-5/2	
ϕ_{CD} -	-3/14	-2/7	2/7	4/7	-2/7	0	-3/14	2/7	≥	-1	
ϕ_{CD} +	3/14	2/7	-2/7	-4/7	2/7	0	3/14	-2/7	≥	-2	
αbF	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	≥	-1/3	
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3	
ϕ_{DE} -	-3/14	-2/7	2/7	-3/7	5/7	0	-3/14	2/7	≥	-3/2	
ϕ_{DE} +	3/14	2/7	-2/7	3/7	-5/7	0	3/14	-2/7	≥	-5/2	
φ _{ED} -	0	0	-1	0	0	0	0	0	≥	-4	
Z	-11/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-2	
ϕ_{BF} -	17/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1	
ϕ_{BF} +	-17/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3	
ϕ_{FG} -	-3/14	-2/7	2/7	-3/7	-2/7	1	-3/14	2/7	≥	-1	
φ _{FG} +	3/14	2/7	-2/7	3/7	2/7	-1	3/14	-2/7	≥	-2	
ϕ_{GH} -	-37/42	-5/42	13/21	-16/21	-5/42	1	5/42	13/21	≥	-1/6	
ϕ_{GH} +	37/42	5/42	-13/21	16/21	5/42	-1	-5/42	-13/21	≥	-17/6	
ϕ_{HG} -	27/14	-3/7	-4/7	13/7	-3/7	-1	-1/14	-4/7	≥	-1	
φ _{HG} +	-27/14	3/7	4/7	-13/7	3/7	1	1/14	4/7	≥	-2	
φ _{DH} -	3/14	2/7	-2/7	3/7	-5/7	0	-11/14	-2/7	≥	-4	
φ _{DH} +	-3/14	-2/7	2/7	-3/7	5/7	0	11/14	2/7	≥	0	
L _X	-3/14	-2/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2	
Max	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	=	-1/3	

Scambio pivotale 24-2

	Г.,			-		Ъ			1	Г г ь -
	φ_{BC} +	φ_{DH} +	φ_{ED} +	T	S	R	ϕ_{DC} -	φ _{AB} +]	Fb -
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	-4
Х-	0	-1	0	0	1	0	1	0	≥	-1/2
ϕ_{BA} -	-3/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3/2
ϕ_{BA} +	3/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-5/2
ϕ_{BC} -	-1	0	0	0	0	0	0	0	≥	-3
Χ	0	-1	0	0	1	0	1	-1	≥	-5/2
$\phi_{\text{CD}}\text{-}$	0	1	0	1	-1	0	-1	0	≥	-1
ϕ_{CD} +	0	-1	0	-1	1	0	1	0	≥	-2
αbF	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	≥	-1/3
$\phi_{\text{DC}}\text{+}$	0	0	0	0	0	0	-1	0	≥	-3
$\phi_{\text{DE}}\text{-}$	0	1	0	0	0	0	-1	0	≥	-3/2
ϕ_{DE} +	0	-1	0	0	0	0	1	0	≥	-5/2
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4
Z	-1	-1	0	0	1	0	1	0	≥	-2
ϕ_{BF} -	7/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-1
ϕ_{BF} +	-7/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3
ϕ_{FG} -	0	1	0	0	-1	1	-1	0	≥	-1
$\phi_{\text{FG}}\text{+}$	0	-1	0	0	1	-1	1	0	≥	-2
ϕ_{GH} -	-19/24	5/12	1/2	-7/12	-5/12	1	-5/24	1/2	≥	-1/6
ϕ_{GH} +	19/24	-5/12	-1/2	7/12	5/12	-1	5/24	-1/2	≥	-17/6
ϕ_{HG} -	9/4	3/2	-1	5/2	-3/2	-1	-5/4	-1	≥	-1
$\phi_{\text{HG}}\text{+}$	-9/4	-3/2	1	-5/2	3/2	1	5/4	1	≥	-2
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-4
Υ	-3/4	-7/2	1	-3/2	5/2	0	11/4	1	≥	0
L_X	0	1	0	0	-1	0	-1	0	≥	-3/2
Max	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	=	-1/3

Scambio pivotale 19-5

-	[φ _{BC} +	φ _{DH} +	φ _{ED} +	Т	φ _{GH} -	R	φ _{DC} -	φ _{AB} + _		[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	[-4]
X-	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-9/10
ϕ_{BA} -	-18/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-21/10
ϕ_{BA} +	18/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-19/10
ϕ_{BC} -	-1	0	0	0	0	0	0	0	≥	-3
X	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	1/5	≥	-29/10
ϕ_{CD} -	19/10	0	-6/5	12/5	12/5	-12/5	-1/2	-6/5	≥	-3/5
ϕ_{CD} +	-19/10	0	6/5	-12/5	-12/5	12/5	1/2	6/5	≥	-12/5
αbF	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	≥	-2/5
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3
ϕ_{DE} -	0	1	0	0	0	0	-1	0	≥	-3/2
ϕ_{DE} +	0	-1	0	0	0	0	1	0	≥	-5/2
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4
Z	-29/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-12/5
ϕ_{BF} -	23/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-2/5
ϕ_{BF} +	-23/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-18/5
ϕ_{FG} -	19/10	0	-6/5	7/5	12/5	-7/5	-1/2	-6/5	≥	-3/5
ϕ_{FG} +	-19/10	0	6/5	-7/5	-12/5	7/5	1/2	6/5	≥	-12/5
S	-19/10	1	6/5	-7/5	-12/5	12/5	-1/2	6/5	≥	-2/5
$\phi_{\text{GH}} \textbf{+}$	0	0	0	0	-1	0	0	0	≥	-3
ϕ_{HG} -	51/10	0	-14/5	23/5	18/5	-23/5	-1/2	-14/5	≥	-2/5
ϕ_{HG} +	-51/10	0	14/5	-23/5	-18/5	23/5	1/2	14/5	≥	-13/5
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-4
Υ	-11/2	-1	4	-5	-6	6	3/2	4	≥	-1
L_{x}	19/10	0	-6/5	7/5	12/5	-12/5	-1/2	-6/5	≥	-11/10
Max	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	=	-2/5

Scambio pivotale 21-6

	φ_{BC} +	ϕ_{DH} +	ϕ_{ED} +	Т	φ _{GH} -	φ _{HG} -	ϕ_{DC} -	ϕ_{AB} +		[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	-4
X-	35/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	\geq	-51/46
ϕ_{BA} -	9/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	≥	-111/46
ϕ_{BA} +	-9/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	\geq	-73/46
ϕ_{BC} -	-1	0	0	0	0	0	0	0	\geq	-3
Χ	35/46	0	-6/23	1	-12/23	-12/23	11/46	-29/23	\geq	-143/46
φ _{CD} -	-35/46	0	6/23	0	12/23	12/23	-11/46	6/23	\geq	-9/23
ϕ_{CD} +	35/46	0	-6/23	0	-12/23	-12/23	11/46	-6/23	≥	-60/23
αbF	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	≥	-10/23
ϕ_{DC} +	0	0	0	0	0	0	-1	0	\geq	-3
ϕ_{DE} -	0	1	0	0	0	0	-1	0	\geq	-3/2
$\phi_{\text{DE}}\text{+}$	0	-1	0	0	0	0	1	0	≥	-5/2
ϕ_{ED} -	0	0	-1	0	0	0	0	0	\geq	-4
Z	-11/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	\geq	-60/23
ϕ_{BF} -	14/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	\geq	-2/23
ϕ_{BF} +	-14/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	\geq	-90/23
ϕ_{FG} -	8/23	0	-8/23	0	30/23	7/23	-8/23	-8/23	\geq	-11/23
ϕ_{FG} +	-8/23	0	8/23	0	-30/23	-7/23	8/23	8/23	≥	-58/23
S	35/46	1	-6/23	1	-12/23	-12/23	-35/46	-6/23	\geq	-14/23
ϕ_{GH} +	0	0	0	0	-1	0	0	0	\geq	-3
R	51/46	0	-14/23	1	18/23	-5/23	-5/46	-14/23	\geq	-2/23
ϕ_{HG} +	0	0	0	0	0	-1	0	0	\geq	-3
ϕ_{DH} -	0	-1	0	0	0	0	0	0	\geq	-4
Υ	53/46	-1	8/23	1	-30/23	-30/23	39/46	8/23	\geq	-35/23
L_{x}	-35/46	0	6/23	-1	12/23	12/23	-11/46	6/23	≥	-41/46
Max	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	=	10/23]

Scambio pivotale 7-1

Ocam	DIO PIVO										
	φ _{CD} -	$\phi_{\text{DH}} \textbf{+}$	ϕ_{ED} +	Т	ϕ_{GH} -	ϕ_{HG} -	ϕ_{DC} -	φ_{AB} +		[Fb]	
ϕ_{AB} -	0	0	0	0	0	0	0	-1	\geq	[-4]	
X-	-1	0	0	1	0	0	0	0	\geq	-3/2	
ϕ_{BA} -	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	\geq	-183/70	
ϕ_{BA} +	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	\geq	-97/70	
ϕ_{BC} -	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	\geq	-87/35	
Χ	-1	0	0	1	0	0	0	-1	\geq	-7/2	
$\phi_{\text{BC}}\text{+}$	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	\geq	-18/35	
ϕ_{CD} +	-1	0	0	0	0	0	0	0	\geq	-3	
$\alpha b F$	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	\geq	-16/35	
ϕ_{DC} +	0	0	0	0	0	0	-1	0	\geq	-3	
ϕ_{DE} -	0	1	0	0	0	0	-1	0	\geq	-3/2	
$\phi_{\text{DE}} \textbf{+}$	0	-1	0	0	0	0	1	0	≥	-5/2	
$\phi_{\text{ED}}\text{-}$	0	0	-1	0	0	0	0	0	\geq	-4	
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	\geq	-87/35	
ϕ_{BF} -	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5	
ϕ_{BF} +	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	\geq	-18/5	
ϕ_{FG} -	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	\geq	-23/35	
ϕ_{FG} +	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	\geq	-82/35	
S	-1	1	0	1	0	0	-1	0	≥	-1	
ϕ_{GH} +	0	0	0	0	-1	0	0	0	≥	-3	
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	\geq	-23/35	
ϕ_{HG} +	0	0	0	0	0	-1	0	0	\geq	-3	
ϕ_{DH} -	0	-1	0	0	0	0	0	0	\geq	-4	
Υ	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35	
L_{x}	1	0	0	-1	0	0	0	0	≥	-1/2	
Max	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	-16/35	

Tableau finale

	φ _{CD} -	ϕ_{DH} +	ϕ_{ED} +	Т	φ _{GH} -	φ _{HG} -	φ _{DC} -	ϕ_{AB} +		[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	「 -4]
X-	-1	0	0	1	0	0	0	0	≥	-3/2
ϕ_{BA} -	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	≥	-183/70
ϕ_{BA} +	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	≥	-97/70
ϕ_{BC} -	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	≥	-87/35
Χ	-1	0	0	1	0	0	0	-1	≥	-7/2
ϕ_{BC} +	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	≥	-18/35
ϕ_{CD} +	-1	0	0	0	0	0	0	0	≥	-3
$\alpha b F$	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	≥	-16/35
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3
ϕ_{DE} -	0	1	0	0	0	0	-1	0	≥	-3/2
$\phi_{\text{DE}} \textbf{+}$	0	-1	0	0	0	0	1	0	≥	-5/2
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	≥	-87/35
$\phi_{\text{BF}}\text{-}$	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5
ϕ_{BF} +	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	≥	-18/5
ϕ_{FG} -	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	≥	-23/35
ϕ_{FG} +	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	≥	-82/35
S	-1	1	0	1	0	0	-1	0	≥	-1
ϕ_{GH} +	0	0	0	0	-1	0	0	0	≥	-3
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	≥	-23/35
ϕ_{HG} +	0	0	0	0	0	-1	0	0	≥	-3
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-4
Υ	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35
L_{x}	1	0	0	-1	0	0	0	0	≥	-1/2
Max	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	-16/35

Vettori soluzione della programmazione lineare

	_ X	Υ	Z	Т	S	R	$\alpha b F$	Χ-]	[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	0	≥	[0]
$\phi_{AB}\textbf{+}$	0	0	0	0	0	0	0	0	≥	1/35
$\phi_{\text{BA}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{BA} +	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{BC}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{BC} +	0	0	0	0	0	0	0	0	≥	0
ϕ_{CD} -	0	0	0	0	0	0	0	0	≥	2/35
ϕ_{CD} +	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DC}}\text{-}$	0	0	0	0	0	0	0	0	≥	2/35
$\phi_{\text{DC}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DE}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{ED}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	1/35
$\phi_{\text{BF}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{BF}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{FG} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{FG}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{GH} -	0	0	0	0	0	0	0	0	≥	2/35
$\phi_{\text{GH}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{HG}}\text{-}$	0	0	0	0	0	0	0	0	≥	2/35
$\phi_{\text{HG}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DH}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DH}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
L_{X}	0	0	0	0	0	0	0	0	≥	0
Max	7/2	74/35	87/35	0	1	23/35	16/35	3/2	=	-16/35

Variabili soluzione dedotto il valore X-

Variabili soluzione differenza tra rotazioni

ϕ_{AB}	1/35
ϕ_{BA}	0
ϕ_{BC}	0
ϕ_{CD}	-2/35
ϕ_{DC}	-2/35
ϕ_{DE}	0
ϕ_{ED}	1/35
ϕ_{BF}	0
ϕ_{FG}	0
ϕ_{GH}	-2/35
ϕ_{HG}	-2/35
ϕ_{DH}	0

REAZIONI Fattore di collasso = 16/35

 $H_{\Lambda} = -61/70F$

 $V_{\Delta} = 11/7F$

 $W_A = 2Fb$

 $H_F = -1/2F$

 $V_E = 3F$

 $W_F = 2Fb$

$$\begin{array}{llll} H_{FG} = 7/6F & H_{GH} = 7/6F & H_{HD} = 7/6F \\ V_{FG} = 23/70F & V_{GH} = -3/2F & V_{HD} = -3/2F \\ W_{FG} = -59/70Fb & W_{GH} = -3/2Fb & W_{HD} = 3/2Fb \\ H_{GF} = -7/6F & H_{HG} = -7/6F & H_{DH} = -7/6F \\ V_{GF} = -23/70F & V_{HG} = 3/2F & V_{DH} = 3/2F \\ W_{GF} = 3/2Fb & W_{HG} = -3/2Fb & W_{DH} = 2Fb \end{array}$$

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_B = 3/35\delta$	$u_{CCB} = 3/35\delta$	$u_D = 3/35\delta$	$u_{EED} = 0$
$V_{AAB} = 0$	$V_B = 0$	$V_{CCB} = -2/35\delta$	$V_D = 0$	$V_{EED} = 0$
$\varphi_{\Delta\Delta B} = -1/35\delta/b$	$\varphi_{\rm B} = -1/35\delta/b$	$\varphi_{CCB} = -1/35\delta/b$	$\phi_{D} = 1/35\delta/b$	$\phi_{E} = -1/35\delta/b$

$u_F = 6/35\delta$	$u_{GGF} = 6/35\delta$	$u_{HHG} = 6/35\delta$
$V_F = 0$	$V_{GGF} = -2/35\delta$	$V_{HHG} = 0$
$\varphi_F = -1/35\delta/b$	$\phi_{GGF} = -1/35\delta/b$	$\varphi_{HHD} = 1/35\delta/b$

SPOSTAMENTI RIGIDI DELLE ASTE

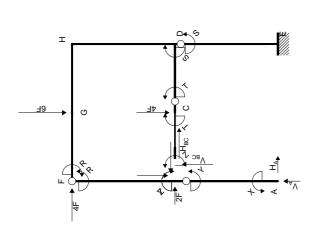
$u_{AAB} = 0$ $v_{AAB} = 0$	$u_{BBC} = 3/35\delta$ $v_{BBC} = 0$	$u_{CCD} = 3/35\delta$ $v_{CCD} = -2/35\delta$	$u_{DDE} = 3/35\delta$ $v_{DDE} = 0$	$u_{BBF} = 3/35\delta$ $v_{BBF} = 0$
$\phi_{AAB} = -1/35\delta/b$	$\phi_{BBC} = -1/35\delta/b$	$\varphi_{CCD} = 1/35\delta/b$	$\phi_{DDE} = -1/35\delta/b$	$\phi_{BBF} = -1/35\delta/b$

$u_{FFG} = 6/35\delta$	$u_{GGH} = 6/35\delta$	$u_{HHD} = 6/35\delta$
$V_{FFG} = 0$	$V_{GGH} = -2/35\delta$	$V_{HHD} = 0$
$\phi_{FFG} = -1/35\delta/b$	$\phi_{GGH} = 1/35\delta/b$	$\phi_{HHD} = -1/35\delta/b$

08.06.11

EQUILIBRIO Nome:

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.11.05.11



EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

 $3H_Ab - 4V_Ab = -Xb + Sb - 8Fb$

Rotazione intorno a C: aste CB

Rotazione intorno a F: aste FB BA $-2V_{BC}b = -Zb + Tb$

 $6H_Ab - 3H_{BC}b = -Xb + Zb - Rb - 6Fb$

Rotazione intorno a B: aste BA

 $3H_Ab = -Xb - Yb$

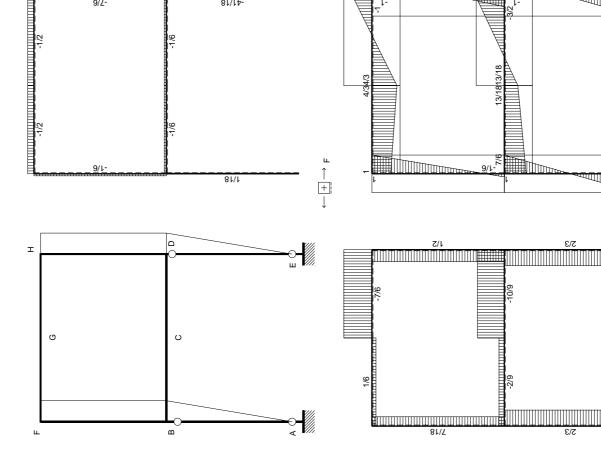
Matrice di equilibrio

Soluzione del sistema

$$\begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1/3 & -1/3 & 0 & 0 & 0 & 0 & 0 \\ V_{BC}b & 0 & 0 & 1/2 & -1/2 & 0 & 0 & 0 \\ +1/3 & -2/3 & -1/3 & 0 & 0 & 1/3 & 2 \\ 0 & -1/4 & 0 & 0 & -1/4 & 0 & 2 \end{bmatrix}$$

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.11.05.11 08.06.11

08.06.11



08.06.11

PROGRAMMAZIONE LINEARE

Sia H_{ii} la matrice del simplesso, con m righe e n colonne.

Siano P_i le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.

Siano a riga m i coefficienti della funzione obiettivo primale $\max \Sigma_i H_{mi} P_p$ $1 \le j < n$.

Siano a colonna n i coefficienti della funzione obiettivo duale $min \Sigma_i H_{in} D_{ii}$, $1 \le i < m$.

Sequenza di operazioni pivotali:

- 1 Sia q (1 $\leq q < n$) la colonna pivot con massimo valore H_{mi} in riga m.
- 2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{io} , che minimizza il rapporto H_{in}/H_{io} .
- 3 Si ottiene il coefficiente pivotale H_{po}
- 4 Si scambia la variabile primale P_a con la duale D_a .
- 5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
- 6 Si ridefiniscono i coefficienti della colonna pivot $q: H_{ig} = H_{ng} H_{ig}$, escluso il pivot H_{ng} .
- 7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ii} = H_{ii} H_{ii} + H_{pr}$
- 8 Si ridefiniscono i coefficienti della riga pivot $p: H_{pj} = -H_{pq} H_{pj}$, escluso il pivot H_{pq} .
- Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.

Giunti a questo punto, si individua la soluzione.

Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_i presenti sulla colonna di sinistra.

Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare *m*=6,*n*=4

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{BC}$ $T=W_{CD}$ $S=W_{DE}$ $R=W_{FB}$

Tableau con variabili non vincolate in segno

i abiea	u co	n varia	adili n	ion vi	ncola	te in	segno)		
	[X	Υ	Z	Т	S	R	αbF		[Fb]	
W_{AB} -	1	0	0	0	0	0	0	≥	[-1]	
W_{AB} +	1	0	0	0	0	0	0	≤	1	
W_{BA} -	0	1	0	0	0	0	0	≥	-1	
W_{BA} +	0	1	0	0	0	0	0	≤	1	
W_{BC} -	0	0	1	0	0	0	0	≥	-3/2	
W_{BC} +	0	0	1	0	0	0	0	≤	3/2	
W _{CD} -	0	0	0	1	0	0	0	≥	-3/2	
W _{CD} +	0	0	0	1	0	0	0	≤	3/2	
W_{DC} -	0	0	1	-2	0	0	-8	≥	-3/2	
W_{DC} +	0	0	1	-2	0	0	-8	≤	3/2	
W_{DE} -	0	0	0	0	1	0	0	≥	-1	
W_{DE} +	0	0	0	0	1	0	0	≤	1	
W_{ED} -	-1	-1	0	0	-1	0	18	≥	-1	
W_{ED} +	-1	-1	0	0	-1	0	18	≤	1	
W_{BF} -	0	-1	-1	0	0	0	0	≥	-1	
W_{BF} +	0	-1	-1	0	0	0	0	≤	1	
W _{FB} -	0	0	0	0	0	1	0	≥	-1	
W_{FB} +	0	0	0	0	0	1	0	≤	1	
W_{GH} -	0	1/2	1	-1	1/2	-1	-4	≥	-3/2	
W_{GH} +	0	1/2	1	-1	1/2	-1	-4	≤	3/2	
W_{HD} -	0	1	2	-2	1	-1	4	≥	-1	
W_{HD} +	0	1	2	-2	1	-1	4	≤	1	
W_{DH} -	0	0	-1	2	-1	0	8	≥	-1	
W _{DH} +	0	0	-1	2	-1	0	8	≤	1	
Max	0	0	0	0	0	0	1 _	=	[0]	

Tableau con variabili non vincolate in segno

	[X	Υ	Z	Т	S	R	αbF]		[Fb]
W_{AB} -	1	0	0	0	0	0	0	≥	「-1
W_{AB} +	-1	0	0	0	0	0	0	≥	-1
W_{BA} -	0	1	0	0	0	0	0	≥	-1
W_{BA} +	0	-1	0	0	0	0	0	≥	-1
W_{BC} -	0	0	1	0	0	0	0	≥	-3/2
W_{BC} +	0	0	-1	0	0	0	0	≥	-3/2
W _{CD} -	0	0	0	1	0	0	0	≥	-3/2
W_{CD} +	0	0	0	-1	0	0	0	≥	-3/2
W_{DC} -	0	0	1	-2	0	0	-8	≥	-3/2
W_{DC} +	0	0	-1	2	0	0	8	≥	-3/2
W_{DE} -	0	0	0	0	1	0	0	≥	-1
W_{DE} +	0	0	0	0	-1	0	0	≥	-1
W_{ED} -	-1	-1	0	0	-1	0	18	≥	-1
W_{ED} +	1	1	0	0	1	0	-18	≥	-1
W _{BF} -	0	-1	-1	0	0	0	0	≥	-1
W_{BF} +	0	1	1	0	0	0	0	≥	-1
W_{FB} -	0	0	0	0	0	1	0	≥	-1
W_{FB} +	0	0	0	0	0	-1	0	≥	-1
W_{GH} -	0	1/2	1	-1	1/2	-1	-4	≥	-3/2
W_{GH} +	0	-1/2	-1	1	-1/2	1	4	≥	-3/2
W_{HD} -	0	1	2	-2	1	-1	4	≥	-1
W_{HD} +	0	-1	-2	2	-1	1	-4	≥	-1
W_{DH} -	0	0	-1	2	-1	0	8	≥	-1
W_{DH} +	0	0	1	-2	1	0	-8	≥	-1
Max	0	0	0	0	0	0	1	=	0

Tableau	con '	varial	oili vir	ncolat	e in s	segno	
г		v.	7 .	т.	ο.	ъ.	\

	[X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		[Fb]
W_{AB} -	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	[-1]
W_{AB} +	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1
W_{BA} -	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1
W_{BA} +	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1
W_{BC} -	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2
W_{BC} +	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2
W_{CD} -	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2
W _{CD} +	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2
W_{DC} -	0	0	1	-2	0	0	0	0	-1	2	0	0	-8	≥	-3/2
W_{DC} +	0	0	-1	2	0	0	0	0	1	-2	0	0	8	≤	-3/2
W_{DF} -	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1
W_{DE} +	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1
W_{ED} -	-1	-1	0	0	-1	0	1	1	0	0	1	0	18	≥	-1
W_{FD} +	1	1	0	0	1	0	-1	-1	0	0	-1	0	-18	≤	-1
W_{BF} -	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-1
W_{BF} +	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-1
W_{FB} -	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-1
W_{FB} +	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-1
W_{GH} -	0	1/2	1	-1	1/2	-1	0	-1/2	-1	1	-1/2	1	-4	≥	-3/2
W_{GH} +	0	-1/2	-1	1	-1/2	1	0	1/2	1	-1	1/2	-1	4	≤	-3/2
W_{HD} -	0	1	2	-2	1	-1	0	-1	-2	2	-1	1	4	≥	-1
W_{HD} +	0	-1	-2	2	-1	1	0	1	2	-2	1	-1	-4	≤	-1
W_{DH} -	0	0	-1	2	-1	0	0	0	1	-2	1	0	8	≥	-1
W_{DH} +	0	0	1	-2	1	0	0	0	-1	2	-1	0	-8	≤	-1
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	[0]

Tableau a variabili negative su X- e limitate

	X	Υ	Z	Ť	S	R	αbF	X		[Fb]
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	-1
ϕ_{AB} +	-1	0	0	0	0	0	0	1	≥	-1
$\phi_{\text{BA}}\text{-}$	0	1	0	0	0	0	0	-1	≥	-1
ϕ_{BA} +	0	-1	0	0	0	0	0	1	≥	-1
$\phi_{\text{BC}}\text{-}$	0	0	1	0	0	0	0	-1	≥	-3/2
ϕ_{BC} +	0	0	-1	0	0	0	0	1	≥	-3/2
ϕ_{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2
$\phi_{\text{CD}} \textbf{+}$	0	0	0	-1	0	0	0	1	≥	-3/2
$\phi_{\text{DC}}\text{-}$	0	0	1	-2	0	0	-8	1	≥	-3/2
$\phi_{\text{DC}} \textbf{+}$	0	0	-1	2	0	0	8	-1	≥	-3/2
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-1
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	≥	-1
$\phi_{\text{ED}}\text{-}$	-1	-1	0	0	-1	0	18	3	≥	-1
$\phi_{\text{ED}} \textbf{+}$	1	1	0	0	1	0	-18	-3	≥	-1
$\phi_{\text{BF}}\text{-}$	0	-1	-1	0	0	0	0	2	≥	-1
ϕ_{BF} +	0	1	1	0	0	0	0	-2	≥	-1
$\phi_{\text{FB}}\text{-}$	0	0	0	0	0	1	0	-1	≥	-1
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1
ϕ_{GH} -	0	1/2	1	-1	1/2	-1	-4	0	≥	-3/2
$\phi_{\text{GH}} \textbf{+}$	0	-1/2	-1	1	-1/2	1	4	0	≥	-3/2
ϕ_{HD} -	0	1	2	-2	1	-1	4	-1	≥	-1
$\phi_{\text{HD}} \textbf{+}$	0	-1	-2	2	-1	1	-4	1	≥	-1
$\phi_{\text{DH}}\text{-}$	0	0	-1	2	-1	0	8	0	≥	-1
$\phi_{\text{DH}} \textbf{+}$	0	0	1	-2	1	0	-8	0	≥	-1
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	0	0	0	0	0	1	0 _	=	0

Scam	bio piv	otale 1	4-7							
	[X	Υ	Z	Т	S	R	ϕ_{ED} +	Χ-		[Fb]
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	[-1]
ϕ_{AB} +	-1	0	0	0	0	0	0	1	≥	-1
ϕ_{BA} -	0	1	0	0	0	0	0	-1	≥	-1
$\phi_{\text{BA}} \textbf{+}$	0	-1	0	0	0	0	0	1	≥	-1
ϕ_{BC} -	0	0	1	0	0	0	0	-1	≥	-3/2
ϕ_{BC} +	0	0	-1	0	0	0	0	1	≥	-3/2
$\phi_{\text{CD}}\text{-}$	0	0	0	1	0	0	0	-1	≥	-3/2
$\phi_{\text{CD}}\text{+}$	0	0	0	-1	0	0	0	1	≥	-3/2
$\phi_{\text{DC}}\text{-}$	-4/9	-4/9	1	-2	-4/9	0	4/9	7/3	≥	-19/18
ϕ_{DC} +	4/9	4/9	-1	2	4/9	0	-4/9	-7/3	≥	-35/18
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-1
ϕ_{DE} +	0	0	0	0	-1	0	0	1	≥	-1
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
αbF	1/18	1/18	0	0	1/18	0	-1/18	-1/6	≥	-1/18
ϕ_{BF} -	0	-1	-1	0	0	0	0	2	≥	-1
ϕ_{BF} +	0	1	1	0	0	0	0	-2	≥	-1
ϕ_{FB} -	0	0	0	0	0	1	0	-1	≥	-1
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1
ϕ_{GH} -	-2/9	5/18	1	-1	5/18	-1	2/9	2/3	≥	-23/18
ϕ_{GH} +	2/9	-5/18	-1	1	-5/18	1	-2/9	-2/3	≥	-31/18
$\phi_{\text{HD}}\text{-}$	2/9	11/9	2	-2	11/9	-1	-2/9	-5/3	≥	-11/9
ϕ_{HD} +	-2/9	-11/9	-2	2	-11/9	1	2/9	5/3	≥	-7/9
ϕ_{DH} -	4/9	4/9	-1	2	-5/9	0	-4/9	-4/3	≥	-13/9
ϕ_{DH} +	-4/9	-4/9	1	-2	5/9	0	4/9	4/3	≥	-5/9
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	1/18	1/18	0	0	1/18	0	-1/18	-1/6	=	1/18]

Scambio pivotale 2-1

	[φ _{AB} +	Υ	Z	Т	S	R	ϕ_{ED} +	X-]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	[-2]
Χ	-1	0	0	0	0	0	0	1	≥	-1
ϕ_{BA} -	0	1	0	0	0	0	0	-1	\geq	-1
ϕ_{BA} +	0	-1	0	0	0	0	0	1	\geq	-1
ϕ_{BC} -	0	0	1	0	0	0	0	-1	\geq	-3/2
ϕ_{BC} +	0	0	-1	0	0	0	0	1	\geq	-3/2
ϕ_{CD}	0	0	0	1	0	0	0	-1	\geq	-3/2
ϕ_{CD} +	0	0	0	-1	0	0	0	1	\geq	-3/2
ϕ_{DC} -	4/9	-4/9	1	-2	-4/9	0	4/9	17/9	≥	-11/18
ϕ_{DC} +	-4/9	4/9	-1	2	4/9	0	-4/9	-17/9	≥	-43/18
ϕ_{DE} -	0	0	0	0	1	0	0	-1	\geq	-1
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	\geq	-1
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
$\alpha b F$	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	≥	-1/9
$\phi_{\text{BF}}\text{-}$	0	-1	-1	0	0	0	0	2	\geq	-1
ϕ_{BF} +	0	1	1	0	0	0	0	-2	\geq	-1
ϕ_{FB} -	0	0	0	0	0	1	0	-1	\geq	-1
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1
ϕ_{GH} -	2/9	5/18	1	-1	5/18	-1	2/9	4/9	\geq	-19/18
ϕ_{GH} +	-2/9	-5/18	-1	1	-5/18	1	-2/9	-4/9	\geq	-35/18
ϕ_{HD} -	-2/9	11/9	2	-2	11/9	-1	-2/9	-13/9	≥	-13/9
ϕ_{HD} +	2/9	-11/9	-2	2	-11/9	1	2/9	13/9	≥	-5/9
ϕ_{DH} -	-4/9	4/9	-1	2	-5/9	0	-4/9	-8/9	≥	-17/9
ϕ_{DH} +	4/9	-4/9	1	-2	5/9	0	4/9	8/9	≥	-1/9
L_{x}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	=	-1/9

Scambio pivotale 24-2

Scarri	DIO PIV	Otale 2	4-2							
	φ_{AB} +	$\phi_{\text{DH}} \textbf{+}$	Z	Т	S	R	$\phi_{\text{ED}}\text{+}$	Χ-		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	-2
Χ	-1	0	0	0	0	0	0	1	≥	-1
$\phi_{\text{BA}}\text{-}$	1	-9/4	9/4	-9/2	5/4	0	1	1	≥	-5/4
$\phi_{\text{BA}} \textbf{+}$	-1	9/4	-9/4	9/2	-5/4	0	-1	-1	≥	-3/4
ϕ_{BC} -	0	0	1	0	0	0	0	-1	≥	-3/2
ϕ_{BC} +	0	0	-1	0	0	0	0	1	≥	-3/2
ϕ_{CD}	0	0	0	1	0	0	0	-1	≥	-3/2
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2
$\phi_{\text{DC}}\text{-}$	0	1	0	0	-1	0	0	1	≥	-1/2
ϕ_{DC} +	0	-1	0	0	1	0	0	-1	≥	-5/2
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-1
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	≥	-1
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
$\alpha b F$	0	-1/8	1/8	-1/4	1/8	0	0	0	≥	-1/8
$\phi_{\text{BF}}\text{-}$	-1	9/4	-13/4	9/2	-5/4	0	-1	0	≥	-3/4
ϕ_{BF} +	1	-9/4	13/4	-9/2	5/4	0	1	0	≥	-5/4
$\phi_{\text{FB}}\text{-}$	0	0	0	0	0	1	0	-1	≥	-1
$\phi_{\text{FB}}\text{+}$	0	0	0	0	0	-1	0	1	≥	-1
$\phi_{\text{GH}}\text{-}$	1/2	-5/8	13/8	-9/4	5/8	-1	1/2	1	≥	-9/8
$\phi_{\text{GH}} \textbf{+}$	-1/2	5/8	-13/8	9/4	-5/8	1	-1/2	-1	≥	-15/8
$\phi_{\text{HD}}\text{-}$	1	-11/4	19/4	-15/2	11/4	-1	1	1	≥	-7/4
$\phi_{\text{HD}}\text{+}$	-1	11/4	-19/4	15/2	-11/4	1	-1	-1	≥	-1/4
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-2
Υ	1	-9/4	9/4	-9/2	5/4	0	1	2	≥	-1/4
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	-1/8	1/8	-1/4	1/8	0	0	0	=	-1/8

Scambio pivotale 22-3

	[φ _{AB} +	ϕ_{DH} +	ϕ_{HD} +	Т	S	R	ϕ_{ED} +	X-]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0]	\geq	-2
Χ	-1	0	0	0	0	0	0	1	\geq	-1
ϕ_{BA} -	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	10/19	≥	-26/19
$\phi_{\text{BA}} \textbf{+}$	-10/19	18/19	9/19	18/19	1/19	-9/19	-10/19	-10/19	\geq	-12/19
ϕ_{BC} -	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-23/19	\geq	-59/38
$\phi_{BC} \textbf{+}$	4/19	-11/19	4/19	-30/19	11/19	-4/19	4/19	23/19	\geq	-55/38
ϕ_{CD}	0	0	0	1	0	0	0	-1	≥	-3/2
$\phi_{\text{CD}}\text{+}$	0	0	0	-1	0	0	0	1	\geq	-3/2
$\phi_{\text{DC}}\text{-}$	0	1	0	0	-1	0	0	1	\geq	-1/2
$\phi_{DC} \textbf{+}$	0	-1	0	0	1	0	0	-1	\geq	-5/2
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	\geq	-1
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	\geq	-1
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	-1	0	\geq	-2
$\alpha b F$	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	\geq	-5/38
$\phi_{\text{BF}}\text{-}$	-6/19	7/19	13/19	-12/19	12/19	-13/19	-6/19	13/19	\geq	-11/19
$\phi_{\text{BF}}\text{+}$	6/19	-7/19	-13/19	12/19	-12/19	13/19	6/19	-13/19	\geq	-27/19
$\phi_{\text{FB}}\text{-}$	0	0	0	0	0	1	0	-1	\geq	-1
$\phi_{\text{FB}}\text{+}$	0	0	0	0	0	-1	0	1	\geq	-1
ϕ_{GH} -	3/19	6/19	-13/38	6/19	-6/19	-25/38	3/19	25/38	\geq	-23/19
$\phi_{\text{GH}} \textbf{+}$	-3/19	-6/19	13/38	-6/19	6/19	25/38	-3/19	-25/38	\geq	-34/19
ϕ_{HD} -	0	0	-1	0	0	0	0	0	\geq	-2
Z	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-4/19	\geq	-1/19
$\phi_{\text{DH}}\text{-}$	0	-1	0	0	0	0	0	0	\geq	-2
Υ	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	29/19	\geq	-7/19
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	=	5/38]

Scambio pivotale 22-5

	[φ _{AB} +	φ _{DH} +	ϕ_{HD} +	Т	Z	R	ϕ_{ED} +	Χ-		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	-2
Χ	-1	0	0	0	0	0	0	1	≥	-1
ϕ_{BA} -	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	6/11	≥	-15/11
ϕ_{BA} +	-6/11	1	5/11	12/11	-1/11	-5/11	-6/11	-6/11	≥	-7/11
ϕ_{BC} -	0	0	0	0	1	0	0	-1	≥	-3/2
$\phi_{BC}\text{+}$	0	0	0	0	-1	0	0	1	≥	-3/2
φ _{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2
ϕ_{DC} -	4/11	0	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-9/22
ϕ_{DC} +	-4/11	0	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-57/22
ϕ_{DE} -	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-12/11
$\phi_{\text{DE}} +$	4/11	-1	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-10/11
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
$\alpha b F$	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	≥	-3/22
ϕ_{BF} -	-6/11	1	5/11	12/11	-12/11	-5/11	-6/11	5/11	≥	-7/11
ϕ_{BF} +	6/11	-1	-5/11	-12/11	12/11	5/11	6/11	-5/11	≥	-15/11
ϕ_{FB} -	0	0	0	0	0	1	0	-1	≥	-1
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1
ϕ_{GH} -	3/11	0	-5/22	-6/11	6/11	-17/22	3/11	17/22	≥	-13/11
ϕ_{GH} +	-3/11	0	5/22	6/11	-6/11	17/22	-3/11	-17/22	≥	-20/11
ϕ_{HD} -	0	0	-1	0	0	0	0	0	≥	-2
S	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-4/11	≥	-1/11
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-2
Υ	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	17/11	≥	-4/11
L_{x}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	=	-3/22

Scambio pivotale 9-4

Scarri	DIO PIVO	tale 3-4									
	φ _{AB} +	ϕ_{DH} +	ϕ_{HD} +	ϕ_{DC} -	Z	R	ϕ_{ED} +	Χ-		[Fb]	
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	[- 2]	
Χ	-1	0	0	0	0	0	0	1	≥	-1	
$\phi_{\text{BA}}\text{-}$	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	0	≥	-6/5	
ϕ_{BA} +	-2/5	1	3/5	-2/5	3/5	-3/5	-2/5	0	≥	-4/5	
ϕ_{BC} -	0	0	0	0	1	0	0	-1	≥	-3/2	
$\phi_{\text{BC}} \textbf{+}$	0	0	0	0	-1	0	0	1	≥	-3/2	
ϕ_{CD} -	2/15	0	2/15	-11/30	19/30	-2/15	2/15	-1/2	≥	-33/20	
$\phi_{\text{CD}} \textbf{+}$	-2/15	0	-2/15	11/30	-19/30	2/15	-2/15	1/2	≥	-27/20	
Τ	2/15	0	2/15	-11/30	19/30	-2/15	2/15	1/2	≥	-3/20	
ϕ_{DC} +	0	0	0	-1	0	0	0	0	≥	-3	
ϕ_{DE} -	0	1	0	-1	0	0	0	0	≥	-3/2	
φ _{DE} +	0	-1	0	1	0	0	0	0	≥	-1/2	
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	-1	0	≥	-2	
αbF	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	≥	-3/20	
$\phi_{\text{BF}}\text{-}$	-2/5	1	3/5	-2/5	-2/5	-3/5	-2/5	1	≥	-4/5	
ϕ_{BF} +	2/5	-1	-3/5	2/5	2/5	3/5	2/5	-1	≥	-6/5	
$\phi_{\text{FB}}\text{-}$	0	0	0	0	0	1	0	-1	≥	-1	
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1	
φ _{GH} -	1/5	0	-3/10	1/5	1/5	-7/10	1/5	1/2	≥	-11/10	
φ _{GH} +	-1/5	0	3/10	-1/5	-1/5	7/10	-1/5	-1/2	≥	-19/10	
φ _{HD} -	0	0	-1	0	0	0	0	0	≥	-2	
S	0	1	0	-1	0	0	0	1	≥	-1/2	
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-2	
Y	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	1	≥	-1/5	
L_{x}	0	0	0	0	0	0	0	-1	≥	-3/2	
Max	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	=	-3/20	

Scambio pivotale 18-6

	[φ _{AB} +	φ _{DH} +	φ _{HD} +	ϕ_{DC}	Z	ϕ_{FB} +	ϕ_{ED} +	Χ-		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	[- 2]
X	-1	0	0	0	0	0	0	1	≥	-1
ϕ_{BA} -	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	3/5	\geq	-9/5
ϕ_{BA} +	-2/5	1	3/5	-2/5	3/5	3/5	-2/5	-3/5	\geq	-1/5
ϕ_{BC} -	0	0	0	0	1	0	0	-1	\geq	-3/2
$\phi_{BC}\text{+}$	0	0	0	0	-1	0	0	1	\geq	-3/2
ϕ_{CD} -	2/15	0	2/15	-11/30	19/30	2/15	2/15	-19/30	\geq	-91/60
ϕ_{CD} +	-2/15	0	-2/15	11/30	-19/30	-2/15	-2/15	19/30	\geq	-89/60
Т	2/15	0	2/15	-11/30	19/30	2/15	2/15	11/30	≥	-1/60
ϕ_{DC} +	0	0	0	-1	0	0	0	0	≥	-3
$\phi_{\text{DE}}\text{-}$	0	1	0	-1	0	0	0	0	≥	-3/2
ϕ_{DE} +	0	-1	0	1	0	0	0	0	≥	-1/2
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	≥	-11/60
ϕ_{BF} -	-2/5	1	3/5	-2/5	-2/5	3/5	-2/5	2/5	≥	-1/5
ϕ_{BF} +	2/5	-1	-3/5	2/5	2/5	-3/5	2/5	-2/5	≥	-9/5
ϕ_{FB} -	0	0	0	0	0	-1	0	0	≥	-2
R	0	0	0	0	0	-1	0	1	≥	-1
ϕ_{GH} -	1/5	0	-3/10	1/5	1/5	7/10	1/5	-1/5	≥	-2/5
ϕ_{GH} +	-1/5	0	3/10	-1/5	-1/5	-7/10	-1/5	1/5	≥	-13/5
ϕ_{HD} -	0	0	-1	0	0	0	0	0	≥	-2
S	0	1	0	-1	0	0	0	1	≥	-1/2
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-2
Υ	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	8/5	≥	-4/5
L_{x}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	=	-11/60

Scambio pivotale 4-8

	[φ _{AB} +	φ _{DH} +	ϕ_{HD} +	φ _{DC} -	Z	ϕ_{FB} +	ϕ_{ED} +	ϕ_{BA} +]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	[-2]
Χ	-5/3	5/3	1	-2/3	1	1	-2/3	-5/3	≥	-4/3
ϕ_{BA} -	0	0	0	0	0	0	0	-1	≥	-2
X-	-2/3	5/3	1	-2/3	1	1	-2/3	-5/3	≥	-1/3
ϕ_{BC} -	2/3	-5/3	-1	2/3	0	-1	2/3	5/3	\geq	-7/6
ϕ_{BC} +	-2/3	5/3	1	-2/3	0	1	-2/3	-5/3	\geq	-11/6
ϕ_{CD}	5/9	-19/18	-1/2	1/18	0	-1/2	5/9	19/18	\geq	-47/36
$\phi_{\text{CD}}\text{+}$	-5/9	19/18	1/2	-1/18	0	1/2	-5/9	-19/18	\geq	-61/36
Т	-1/9	11/18	1/2	-11/18	1	1/2	-1/9	-11/18	≥	-5/36
ϕ_{DC} +	0	0	0	-1	0	0	0	0	\geq	-3
$\phi_{\text{DE}}\text{-}$	0	1	0	-1	0	0	0	0	\geq	-3/2
$\phi_{\text{DE}} \textbf{+}$	0	-1	0	1	0	0	0	0	\geq	-1/2
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	-1	0	\geq	-2
$\alpha b F$	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	\geq	-7/36
ϕ_{BF} -	-2/3	5/3	1	-2/3	0	1	-2/3	-2/3	\geq	-1/3
$\phi_{\text{BF}}\text{+}$	2/3	-5/3	-1	2/3	0	-1	2/3	2/3	\geq	-5/3
ϕ_{FB} -	0	0	0	0	0	-1	0	0	\geq	-2
R	-2/3	5/3	1	-2/3	1	0	-2/3	-5/3	≥	-4/3
ϕ_{GH} -	1/3	-1/3	-1/2	1/3	0	1/2	1/3	1/3	\geq	-1/3
ϕ_{GH} +	-1/3	1/3	1/2	-1/3	0	-1/2	-1/3	-1/3	\geq	-8/3
ϕ_{HD} -	0	0	-1	0	0	0	0	0	\geq	-2
S	-2/3	8/3	1	-5/3	1	1	-2/3	-5/3	\geq	-5/6
$\phi_{\text{DH}}\text{-}$	0	-1	0	0	0	0	0	0	\geq	-2
Υ	-2/3	5/3	1	-2/3	1	1	-2/3	-8/3	≥	-4/3
L_{X}	2/3	-5/3	-1	2/3	-1	-1	2/3	5/3	≥	-7/6
Max	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	=	-7/36

Scambio pivotale 12-2

	[φ _{AB} +	ϕ_{DE} +	ϕ_{HD} +	ϕ_{DC} -	Z	ϕ_{FB} +	ϕ_{ED} +	ϕ_{BA} +]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	-2
Χ	-5/3	-5/3	1	1	1	1	-2/3	-5/3	\geq	-13/6
ϕ_{BA} -	0	0	0	0	0	0	0	-1	\geq	-2
X-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	\geq	-7/6
ϕ_{BC} -	2/3	5/3	-1	-1	0	-1	2/3	5/3	\geq	-1/3
$\phi_{BC}\text{+}$	-2/3	-5/3	1	1	0	1	-2/3	-5/3	\geq	-8/3
ϕ_{CD}	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	\geq	-7/9
$\phi_{\text{CD}}\text{+}$	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	\geq	-20/9
Т	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	\geq	-4/9
$\phi_{DC}\text{+}$	0	0	0	-1	0	0	0	0	\geq	-3
$\phi_{\text{DE}}\text{-}$	0	-1	0	0	0	0	0	0	\geq	-2
ϕ_{DH} +	0	-1	0	1	0	0	0	0	\geq	-1/2
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	-1/18	0	0	0	0	-1/18	-1/18	\geq	-2/9
ϕ_{BF} -	-2/3	-5/3	1	1	0	1	-2/3	-2/3	\geq	-7/6
ϕ_{BF} +	2/3	5/3	-1	-1	0	-1	2/3	2/3	\geq	-5/6
ϕ_{FB} -	0	0	0	0	0	-1	0	0	\geq	-2
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	\geq	-13/6
$\phi_{\text{GH}}\text{-}$	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	\geq	-1/6
ϕ_{GH} +	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	\geq	-17/6
ϕ_{HD} -	0	0	-1	0	0	0	0	0	\geq	-2
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	\geq	-13/6
ϕ_{DH} -	0	1	0	-1	0	0	0	0	\geq	-3/2
Υ	-2/3	-5/3	1	1	1	1	-2/3	-8/3	\geq	-13/6
L_{X}	2/3	5/3	-1	-1	-1	-1	2/3	5/3	\geq	-1/3
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	2/9 _

Tableau finale

IFB IED IDA]
0 0 0 ≥ -2	
1 -2/3 -5/3 ≥ -13/6	
0 0 -1 ≥ -2	
1 -2/3 -5/3 ≥ -7/6	
-1 2/3 5/3 ≥ -1/3	
1 -2/3 -5/3 ≥ -8/3	
-1/2 5/9 19/18 ≥ -7/9	
1/2 -5/9 -19/18 ≥ -20/9	
1/2 -1/9 -11/18 ≥ -4/9	
0 0 0 ≥ -3	
0 0 0 ≥ -2	
0 0 0 ≥ -1/2	
0 -1 0 ≥ -2	
0 -1/18 -1/18 ≥ -2/9	
1 -2/3 -2/3 ≥ -7/6	
-1 2/3 2/3 ≥ -5/6	
-1 0 0 ≥ -2	
0 -2/3 -5/3 ≥ -13/6	
1/2 1/3 1/3 ≥ -1/6	
-1/2 -1/3 -1/3 ≥ -17/6	
0 0 0 ≥ -2	
1 -2/3 -5/3 ≥ -13/6	
0 0 0 ≥ -3/2	
1 -2/3 -8/3 ≥ -13/6	
-1 2/3 5/3 ≥ -1/3	
0 -1/18 -1/18 = -2/9	
	$\begin{array}{cccccccccccccccccccccccccccccccccccc$

Vettori soluzione della programmazione lineare

	[X	Υ	Z	Т	S	R	$\alpha b F$	Χ-		[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{AB}\textbf{+}$	0	0	0	0	0	0	0	0	≥	1/18
ϕ_{BA} -	0	0	0	0	0	0	0	0	≥	0
ϕ_{BA} +	0	0	0	0	0	0	0	0	≥	1/18
ϕ_{BC} -	0	0	0	0	0	0	0	0	≥	0
ϕ_{BC} +	0	0	0	0	0	0	0	0	≥	0
ϕ_{CD} -	0	0	0	0	0	0	0	0	≥	0
ϕ_{CD} +	0	0	0	0	0	0	0	0	≥	0
$\phi_{DC}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DC}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DE}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{DE} +	0	0	0	0	0	0	0	0	≥	1/18
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{ED}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	1/18
$\phi_{\text{BF}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{BF}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{FB} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{FB}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{GH}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{GH} +	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{HD}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{HD} +	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DH}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{DH} +	0	0	0	0	0	0	0	0	≥	0
L_{x}	0	0	0	0	0	0	0	0	≥	0
Max	13/6	13/6	0	4/9	13/6	13/6	2/9	7/6	=	2/9_

Variabili soluzione dedotto il valore X-

 $u_F = 1/6\delta$

 $v_F = 0$ $\phi_F = 0$

 $u_{AAB} = 0$

 $V_{AAB} = 0$

 $u_{FFG} = 1/6\delta$

 $V_{FFG} = 0$

 $\varphi_{FFG} = 0$

 $\phi_{AAB} = -1/18\delta/b$

 $u_G = 1/6\delta$

 $u_{BBC} = 1/6\delta$

 $u_{GGH} = 1/6\delta$

 $V_{BBC} = 0$

 $\varphi_{BBC} = 0$

 $v_{GGH} = 0$

 $\varphi_{GGH} = 0$

 $v_G = 0$

 $\varphi_G = 0$

SPOSTAMENTI RIGIDI DELLE ASTE

 $u_{H} = 1/6\delta$ $v_{H} = 0$

 $u_{CCD} = 1/6\delta$

 $u_{HHD} = 1/6\delta$

 $V_{HHD} = 0$

 $\varphi_{HHD} = 0$

 $V_{CCD} = 0$

 $\phi_{CCD} = 0$

 $\phi_H = 0$

 $u_{BBF} = 1/6\delta$

 $V_{BBF} = 0$

 $\phi_{BBF} = 0$

 $u_{DDE} = 1/6\delta$

 $\phi_{DDE} = -1/18\delta/b$

 $V_{DDE} = 0$

Variabili soluzione differenza tra rotazioni

ϕ_{AB}	1/18	
ϕ_{BA}	1/18	
ϕ_{BC}	0	
ϕ_{CD}	0	
ϕ_{DC}	0	
ϕ_{DE}	1/18	
ϕ_{ED}	1/18	
ϕ_{BF}	0	
ϕ_{FB}	0	
ϕ_{GH}	0	
ϕ_{HD}	0	
ϕ_{DH}	0 _	

REAZIONI Fattore di collasso = 2/9

 $H_A = -2/3F$

 $V_{\Delta} = -1/18F$

 $W_{\Delta} = Fb$

 $H_{\rm F} = -2/3F$

 $V_{E} = 41/18F$

 $W_F = Fb$

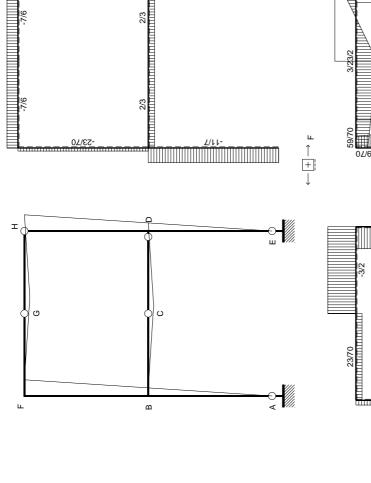
$H_{AB} = -2/3F$	$H_{BC} = 1/6F$	$H_{CD} = 1/6F$	$H_{DE} = 2/3F$	$H_{BF} = -7/18F$
$V_{AB} = -1/18F$	$V_{BC} = -2/9F$	$V_{CD} = -10/9F$	$V_{DE} = -41/18F$	$V_{BF} = 1/6F$
$W_{AB} = Fb$	$W_{BC} = -7/6Fb$	$W_{CD} = -13/18Fb$	$W_{DE} = Fb$	$W_{BF} = 1/6Fb$
$H_{BA} = 2/3F$	$H_{CB} = -1/6F$	$H_{DC} = -1/6F$	$H_{ED} = -2/3F$	$H_{FB} = 7/18F$
$V_{BA} = 1/18F$	$V_{CB} = 2/9F$	$V_{DC} = 10/9F$	$V_{ED} = 41/18F$	$V_{FB} = -1/6F$
$W_{BA} = Fb$	$W_{CB} = 13/18Fb$	$W_{DC} = -3/2Fb$	$W_{ED} = Fb$	$W_{FB} = Fb$
H _{FG} = 1/2F	H _{GH} = 1/2F	H _{HD} = 1/2F		
	$V_{GH} = -7/6F$	$V_{HD} = -7/6F$		
$V_{FG} = 1/6F$				
$W_{FG} = -Fb$	$W_{GH} = -4/3Fb$	$W_{HD} = Fb$		
$H_{GF} = -1/2F$	$H_{HG} = -1/2F$	$H_{DH} = -1/2F$		
$V_{GF} = -1/6F$	$V_{HG} = 7/6F$	$V_{DH} = 7/6F$		
$W_{CF} = 4/3 Fb$	$W_{HC} = -Fb$	$W_{DH} = 1/2Fb$		

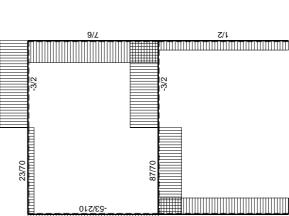
SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_B = 1/6\delta$	$u_C = 1/6\delta$	$u_D = 1/6\delta$	$u_{EED} = 0$
$V_{AAB} = 0$	$V_B = 0$	$v_C = 0$	$V_D = 0$	$V_{EED} = 0$
$\phi_{AAB} = -1/18\delta/b$	$\phi_{\rm B} = -1/18\delta/b$	$\varphi_{c} = 0$	$\varphi_D = 0$	$\varphi_{E} = -1/18\delta/b$

EQUILIBRIO Nome:

(H)





년 - 0 양 0

Rotazione intorno a D: aste DC DH CB HG GF FB BA

EQUAZIONI DI EQUILIBRIO

Rotazione intorno a F: aste FB BA $6H_Ab - 3H_{BC}b = -Xb + Zb + Rb - 3Fb$

Rotazione intorno a B: aste BA

 $3H_Ab = -Xb - Yb$

Matrice di equilibrio

Rotazione intorno a C: aste CB

 $-2V_{BC}b = -Zb + Tb$

 $3H_Ab - 4V_Ab = -Xb + Sb - 14Fb$



08.06.11

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.11.05.11

[Xb Yb Zb Tb Sb Rb Fb]
-1/3 -1/3 0 0 0 0 0
0 0 1/2 -1/2 0 0 0
-1/3 -2/3 -1/3 0 0 -1/3 1
0 -1/4 0 0 -1/4 0 7/2

Soluzione del sistema

PROGRAMMAZIONE LINEARE

Sia H_{ii} la matrice del simplesso, con m righe e n colonne.

Siano P_i le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.

Siano a riga m i coefficienti della funzione obiettivo primale $\max \Sigma_i H_{mi} P_{ir}$ $1 \le j < n$.

Siano a colonna n i coefficienti della funzione obiettivo duale $min \Sigma_i H_{in} D_{in} 1 \le i < m$.

Sequenza di operazioni pivotali:

- 1 Sia q ($1 \le q < n$) la colonna pivot con massimo valore H_{mi} in riga m.
- 2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{io} , che minimizza il rapporto H_{in}/H_{io} .
- 3 Si ottiene il coefficiente pivotale H_{po}
- 4 Si scambia la variabile primale P_a con la duale D_a .
- 5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
- 6 Si ridefiniscono i coefficienti della colonna pivot $q: H_{ig} = H_{ng} H_{ig}$, escluso il pivot H_{ng} .
- 7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ii} = H_{ii} H_{ii} + H_{pr}$
- 8 Si ridefiniscono i coefficienti della riga pivot $p: H_{pj} = -H_{pq} H_{pj}$, escluso il pivot H_{pq} .
- Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.

Giunti a questo punto, si individua la soluzione.

Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_i presenti sulla colonna di sinistra.

Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare *m*=6,*n*=4

$$\begin{bmatrix} \mathsf{P}_1 & \mathsf{P}_2 & \mathsf{P}_3 \end{bmatrix} & \begin{bmatrix} \mathsf{MIN} \\ \mathsf{D}_1 & \begin{bmatrix} \mathsf{H}_{11} & \mathsf{H}_{12} & \mathsf{H}_{13} \\ \mathsf{H}_{21} & \mathsf{H}_{22} & \mathsf{H}_{23} \end{bmatrix} \geq \begin{bmatrix} \mathsf{H}_{14} \\ \mathsf{H}_{24} \\ \mathsf{D}_3 & \mathsf{H}_{31} & \mathsf{H}_{32} & \mathsf{H}_{33} \\ \mathsf{D}_4 & \mathsf{H}_{41} & \mathsf{H}_{42} & \mathsf{H}_{43} \\ \mathsf{D}_5 & \mathsf{H}_{51} & \mathsf{H}_{52} & \mathsf{H}_{53} \\ \mathsf{MAX} & \begin{bmatrix} \mathsf{H}_{61} & \mathsf{H}_{62} & \mathsf{H}_{63} \end{bmatrix} = \begin{bmatrix} \mathsf{H}_{64} \end{bmatrix}$$

SOLUZIONE DEL SIMPLESSO X=WAR Y=WRA Z=WRC T=WCD S=WDE R=WEG

Tableau con variabili non vincolate in segno

	rableau con variabili non vincolate in segno									
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		[X	Υ	Z	Т	S	R	αbF		[Fb]
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{AB} -	1	0	0	0	0	0	0	≥	-2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{AB} +	1	0	0	0	0	0	0	≤	2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W _{BA} -	0	1	0	0	0	0	0	≥	-2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{BA} +	0	1	0	0	0	0	0	≤	2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{BC} -	0	0	1	0	0	0	0	≥	-3/2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{BC} +	0	0	1	0	0	0	0	≤	3/2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{CD} -	0	0	0	1	0	0	0	≥	-3/2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W _{CD} +	0	0	0	1	0	0	0	≤	3/2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{DC} -	0	0	1	-2	0	0	-12	≥	-3/2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{DC} +	0	0	1	-2	0	0	-12	≤	3/2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{DE} -	0	0	0	0	1	0	0	≥	-2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{DE} +	0	0	0	0	1	0	0	≤	2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{ED} -	-1	-1	0	0	-1	0	9	≥	-2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{ED} +	-1	-1	0	0	-1	0	9	≤	2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{BF} -	0	-1	-1	0	0	0	0	≥	-2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{BF} +	0	-1	-1	0	0	0	0	≤	2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{FG} -	0	0	0	0	0	1	0	≥	-3/2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{FG} +	0	0	0	0	0	1	0	≤	3/2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{GH} -	0	1/2	1	-1	1/2	1	-7	≥	-3/2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{GH} +	0	1/2	1	-1	1/2	1	-7	≤	3/2
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	W_{HG} -	0	-1	-2	2	-1	-1	6	≥	-3/2
W_{DH}^{-} 0 0 -1 2 -1 0 12 \geq -2 W_{DH}^{+} 0 0 -1 2 -1 0 12 \leq 2	W_{HG} +	0	-1	-2	2	-1	-1	6	≤	3/2
$W_{DH} + 0 0 -1 2 -1 0 12 \leq 2 $	W_{DH} -	0	0	-1	2	-1	0	12	≥	-2
$Max \left[\begin{array}{ccccccccccccccccccccccccccccccccccc$	W_{DH} +	0	0	-1	2	-1	0	12	≤	2
	Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	[X	Υ	Z	Т	S	R	αbF]		[Fb]	
W_{AB} -	1	0	0	0	0	0	0	≥	-2	
W_{AB} +	-1	0	0	0	0	0	0	≥	-2	
W_{BA} -	0	1	0	0	0	0	0	≥	-2	
W_{BA} +	0	-1	0	0	0	0	0	≥	-2	
W_{BC} -	0	0	1	0	0	0	0	≥	-3/2	
W _{BC} +	0	0	-1	0	0	0	0	≥	-3/2	
W _{CD} -	0	0	0	1	0	0	0	≥	-3/2	
W_{CD} +	0	0	0	-1	0	0	0	≥	-3/2	
W_{DC} -	0	0	1	-2	0	0	-12	≥	-3/2	
W_{DC} +	0	0	-1	2	0	0	12	≥	-3/2	
W_{DE} -	0	0	0	0	1	0	0	≥	-2	
W_{DE} +	0	0	0	0	-1	0	0	≥	-2	
W_{ED} -	-1	-1	0	0	-1	0	9	≥	-2	
W_{ED} +	1	1	0	0	1	0	-9	≥	-2	
W_{BF} -	0	-1	-1	0	0	0	0	≥	-2	
W_{BF} +	0	1	1	0	0	0	0	≥	-2	
W_{FG} -	0	0	0	0	0	1	0	≥	-3/2	
W_{FG} +	0	0	0	0	0	-1	0	≥	-3/2	
W_{GH} -	0	1/2	1	-1	1/2	1	-7	≥	-3/2	
W_{GH} +	0	-1/2	-1	1	-1/2	-1	7	≥	-3/2	
W_{HG} -	0	-1	-2	2	-1	-1	6	≥	-3/2	
W_{HG} +	0	1	2	-2	1	1	-6	≥	-3/2	
W_{DH} -	0	0	-1	2	-1	0	12	≥	-2	
W_{DH} +	0	0	1	-2	1	0	-12	≥	-2	
Max	0	0	0	0	0	0	1	=	0	

able	au cor	n vari	abili v	incol	ate in	segn	0		
	X+	Y+	Z+	T+	S+	R+	X-	Y-	

	_ X+	Y+	Z+	T+	S+	R+	Х-	Y-	Z-	T-	S-	R-	αbF_{\perp}		[Fb]
W_{AB} -	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	[-2]
W_{AB} +	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-2
W_{BA} -	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-2
W_{BA} +	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-2
W_{BC} -	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2
W_{BC} +	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2
W_{CD} -	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2
W _{CD} +	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2
W_{DC} -	0	0	1	-2	0	0	0	0	-1	2	0	0	-12	≥	-3/2
W_{DC} +	0	0	-1	2	0	0	0	0	1	-2	0	0	12	≤	-3/2
W_{DE} -	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-2
W_{DE} +	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-2
W_{ED} -	-1	-1	0	0	-1	0	1	1	0	0	1	0	9	≥	-2
W_{ED} +	1	1	0	0	1	0	-1	-1	0	0	-1	0	-9	≤	-2
W_{BF} -	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-2
W_{BF} +	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-2
W_{FG} -	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-3/2
W_{FG} +	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-3/2
W_{GH} -	0	1/2	1	-1	1/2	1	0	-1/2	-1	1	-1/2	-1	-7	≥	-3/2
W_{GH} +	0	-1/2	-1	1	-1/2	-1	0	1/2	1	-1	1/2	1	7	≤	-3/2
W_{HG} -	0	-1	-2	2	-1	-1	0	1	2	-2	1	1	6	≥	-3/2
W_{HG} +	0	1	2	-2	1	1	0	-1	-2	2	-1	-1	-6	≤	-3/2
W_{DH} -	0	0	-1	2	-1	0	0	0	1	-2	1	0	12	≥	-2
W_{DH} +	0	0	1	-2	1	0	0	0	-1	2	-1	0	-12	≤	-2
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	[0]

Tableau a variabili negative su X- e limitate

	[X	Υ	Z	T	S	R	αbF	X- [[Fb]
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	-2
$\phi_{AB}\textbf{+}$	-1	0	0	0	0	0	0	1	≥	-2
ϕ_{BA} -	0	1	0	0	0	0	0	-1	≥	-2
ϕ_{BA} +	0	-1	0	0	0	0	0	1	≥	-2
$\phi_{\text{BC}}\text{-}$	0	0	1	0	0	0	0	-1	≥	-3/2
$\phi_{\text{BC}} \textbf{+}$	0	0	-1	0	0	0	0	1	≥	-3/2
ϕ_{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2
$\phi_{\text{CD}}\text{+}$	0	0	0	-1	0	0	0	1	≥	-3/2
$\phi_{\text{DC}}\text{-}$	0	0	1	-2	0	0	-12	1	≥	-3/2
$\phi_{\text{DC}}\text{+}$	0	0	-1	2	0	0	12	-1	≥	-3/2
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-2
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	≥	-2
$\phi_{\text{ED}}\text{-}$	-1	-1	0	0	-1	0	9	3	≥	-2
$\phi_{\text{ED}}\text{+}$	1	1	0	0	1	0	-9	-3	≥	-2
$\phi_{\text{BF}}\text{-}$	0	-1	-1	0	0	0	0	2	≥	-2
$\phi_{\text{BF}}\text{+}$	0	1	1	0	0	0	0	-2	≥	-2
$\phi_{\text{FG}}\text{-}$	0	0	0	0	0	1	0	-1	≥	-3/2
$\phi_{\text{FG}} \textbf{+}$	0	0	0	0	0	-1	0	1	≥	-3/2
$\phi_{\text{GH}}\text{-}$	0	1/2	1	-1	1/2	1	-7	-2	≥	-3/2
$\phi_{\text{GH}} \textbf{+}$	0	-1/2	-1	1	-1/2	-1	7	2	≥	-3/2
$\phi_{\text{HG}}\text{-}$	0	-1	-2	2	-1	-1	6	3	≥	-3/2
$\phi_{\text{HG}}\text{+}$	0	1	2	-2	1	1	-6	-3	≥	-3/2
$\phi_{\text{DH}}\text{-}$	0	0	-1	2	-1	0	12	0	≥	-2
$\phi_{\text{DH}} \textbf{+}$	0	0	1	-2	1	0	-12	0	≥	-2
L_{X}	0	0	0	0	0	0	0	-1	≥	-2
Max	0	0	0	0	0	0	1	0	=	0

Scam	bio pivo	otale 9-7	,								
	[X	Υ	Z	Т	S	R	ϕ_{DC} -	X-]		[Fb]	
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	[-2]	
$\phi_{AB} \textbf{+}$	-1	0	0	0	0	0	0	1	≥	-2	
$\phi_{\text{BA}}\text{-}$	0	1	0	0	0	0	0	-1	≥	-2	
ϕ_{BA} +	0	-1	0	0	0	0	0	1	≥	-2	
ϕ_{BC} -	0	0	1	0	0	0	0	-1	≥	-3/2	
$\phi_{BC}\text{+}$	0	0	-1	0	0	0	0	1	≥	-3/2	
$\phi_{\text{CD}}\text{-}$	0	0	0	1	0	0	0	-1	≥	-3/2	
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2	
αbF	0	0	1/12	-1/6	0	0	-1/12	1/12	≥	-1/8	
$\phi_{\text{DC}}\text{+}$	0	0	0	0	0	0	-1	0	≥	-3	
ϕ_{DE} -	0	0	0	0	1	0	0	-1	≥	-2	
$\phi_{\text{DE}}\text{+}$	0	0	0	0	-1	0	0	1	≥	-2	
$\phi_{\text{ED}}\text{-}$	-1	-1	3/4	-3/2	-1	0	-3/4	15/4	≥	-25/8	
φ _{ED} +	1	1	-3/4	3/2	1	0	3/4	-15/4	≥	-7/8	
φ _{BF} -	0	-1	-1	0	0	0	0	2	≥	-2	
ϕ_{BF} +	0	1	1	0	0	0	0	-2	≥	-2	
ϕ_{FG} -	0	0	0	0	0	1	0	-1	≥	-3/2	
ϕ_{FG} +	0	0	0	0	0	-1	0	1	≥	-3/2	
ϕ_{GH} -	0	1/2	5/12	1/6	1/2	1	7/12	-31/12	≥	-5/8	
ϕ_{GH} +	0	-1/2	-5/12	-1/6	-1/2	-1	-7/12	31/12	≥	-19/8	
ϕ_{HG} -	0	-1	-3/2	1	-1	-1	-1/2	7/2	≥	-9/4	
ϕ_{HG} +	0	1	3/2	-1	1	1	1/2	-7/2	≥	-3/4	
φ _{DH} -	0	0	0	0	-1	0	-1	1	≥	-7/2	
ϕ_{DH} +	0	0	0	0	1	0	1	-1	≥	-1/2	
L _X	0	0	0	0	0	0	0	-1	≥	-2	
Max	0	0	1/12	-1/6	0	0	-1/12	1/12	=	-1/8	

Scambio pivotale 14-3

ocambio pivotale 14-3											
	_ X	Υ	ϕ_{ED} +	Т	S	R	ϕ_{DC} -	X-]		[Fb]	
$\phi_{\text{AB}}\text{-}$	1	0	0	0	0	0	0	-1	≥	[-2]	
$\phi_{AB}\textbf{+}$	-1	0	0	0	0	0	0	1	≥	-2	
ϕ_{BA} -	0	1	0	0	0	0	0	-1	≥	-2	
$\phi_{\text{BA}} \textbf{+}$	0	-1	0	0	0	0	0	1	≥	-2	
$\phi_{\text{BC}}\text{-}$	4/3	4/3	-4/3	2	4/3	0	1	-6	≥	-8/3	
$\phi_{BC} \textbf{+}$	-4/3	-4/3	4/3	-2	-4/3	0	-1	6	≥	-1/3	
ϕ_{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2	
$\phi_{\text{CD}}\text{+}$	0	0	0	-1	0	0	0	1	≥	-3/2	
$\alpha b F$	1/9	1/9	-1/9	0	1/9	0	0	-1/3	≥	-2/9	
$\phi_{\text{DC}}\text{+}$	0	0	0	0	0	0	-1	0	≥	-3	
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-2	
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	≥	-2	
$\phi_{\text{ED}}\text{-}$	0	0	-1	0	0	0	0	0	≥	-4	
Z	4/3	4/3	-4/3	2	4/3	0	1	-5	≥	-7/6	
$\phi_{\text{BF}}\text{-}$	-4/3	-7/3	4/3	-2	-4/3	0	-1	7	≥	-5/6	
$\phi_{\text{BF}}\text{+}$	4/3	7/3	-4/3	2	4/3	0	1	-7	≥	-19/6	
$\phi_{\text{FG}}\text{-}$	0	0	0	0	0	1	0	-1	≥	-3/2	
ϕ_{FG} +	0	0	0	0	0	-1	0	1	≥	-3/2	
$\phi_{\text{GH}}\text{-}$	5/9	19/18	-5/9	1	19/18	1	1	-14/3	≥	-10/9	
$\phi_{\text{GH}} \textbf{+}$	-5/9	-19/18	5/9	-1	-19/18	-1	-1	14/3	≥	-17/9	
$\phi_{\text{HG}}\text{-}$	-2	-3	2	-2	-3	-1	-2	11	≥	-1/2	
$\phi_{\text{HG}}\text{+}$	2	3	-2	2	3	1	2	-11	≥	-5/2	
ϕ_{DH} -	0	0	0	0	-1	0	-1	1	≥	-7/2	
$\phi_{\text{DH}} \textbf{+}$	0	0	0	0	1	0	1	-1	≥	-1/2	
L_{X}	0	0	0	0	0	0	0	-1	≥	-2	
Max	1/9	1/9	-1/9	0	1/9	0	0	-1/3	=	-2/9	

Scambio pivotale 6-1

Ooaiii	DIO PIT	olalo (
	φ_{BC} +	Υ	$\phi_{\text{ED}} \textbf{+}$	Т	S	R	ϕ_{DC}	X-]		[Fb]
$\phi_{AB}\text{-}$	-3/4	-1	1	-3/2	-1	0	-3/4	7/2	\geq	[-9/4]
ϕ_{AB} +	3/4	1	-1	3/2	1	0	3/4	-7/2	\geq	-7/4
ϕ_{BA} -	0	1	0	0	0	0	0	-1	\geq	-2
$\phi_{\text{BA}} \textbf{+}$	0	-1	0	0	0	0	0	1	\geq	-2
ϕ_{BC} -	-1	0	0	0	0	0	0	0	\geq	-3
Χ	-3/4	-1	1	-3/2	-1	0	-3/4	9/2	\geq	-1/4
$\phi_{\text{CD}}\text{-}$	0	0	0	1	0	0	0	-1	\geq	-3/2
$\phi_{\text{CD}}\text{+}$	0	0	0	-1	0	0	0	1	\geq	-3/2
αbF	-1/12	0	0	-1/6	0	0	-1/12	1/6	\geq	-1/4
$\phi_{\text{DC}}\text{+}$	0	0	0	0	0	0	-1	0	≥	-3
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	\geq	-2
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	\geq	-2
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4
Z	-1	0	0	0	0	0	0	1	\geq	-3/2
$\phi_{\text{BF}}\text{-}$	1	-1	0	0	0	0	0	1	≥	-1/2
ϕ_{BF} +	-1	1	0	0	0	0	0	-1	\geq	-7/2
$\phi_{\text{FG}}\text{-}$	0	0	0	0	0	1	0	-1	≥	-3/2
ϕ_{FG} +	0	0	0	0	0	-1	0	1	≥	-3/2
$\phi_{\text{GH}}\text{-}$	-5/12	1/2	0	1/6	1/2	1	7/12	-13/6	≥	-5/4
$\phi_{\text{GH}} \textbf{+}$	5/12	-1/2	0	-1/6	-1/2	-1	-7/12	13/6	≥	-7/4
$\phi_{\text{HG}}\text{-}$	3/2	-1	0	1	-1	-1	-1/2	2	\geq	0
$\phi_{\text{HG}}\text{+}$	-3/2	1	0	-1	1	1	1/2	-2	\geq	-3
$\phi_{\text{DH}}\text{-}$	0	0	0	0	-1	0	-1	1	\geq	-7/2
$\phi_{\text{DH}} \textbf{+}$	0	0	0	0	1	0	1	-1	≥	-1/2
L_{X}	0	0	0	0	0	0	0	-1	≥	-2
Max	-1/12	0	0	-1/6	0	0	-1/12	1/6	=	-1/4

Scambio pivotale 2-8

	г .			-	_	_		. 7		Г 🗝 П	
	φ_{BC} +	Υ	ϕ_{ED} +	Т	S	R	ϕ_{DC} -	φ_{AB} +		[Fb]	
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	-4	
Х-	3/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1/2	
ϕ_{BA} -	-3/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2	
$\phi_{\text{BA}} \textbf{+}$	3/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-5/2	
ϕ_{BC} -	-1	0	0	0	0	0	0	0	≥	-3	
Χ	3/14	2/7	-2/7	3/7	2/7	0	3/14	-9/7	≥	-5/2	
ϕ_{CD} -	-3/14	-2/7	2/7	4/7	-2/7	0	-3/14	2/7	≥	-1	
ϕ_{CD} +	3/14	2/7	-2/7	-4/7	2/7	0	3/14	-2/7	≥	-2	
αbF	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	≥	-1/3	
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3	
$\phi_{\text{DE}}\text{-}$	-3/14	-2/7	2/7	-3/7	5/7	0	-3/14	2/7	≥	-3/2	
$\phi_{\text{DE}} \textbf{+}$	3/14	2/7	-2/7	3/7	-5/7	0	3/14	-2/7	≥	-5/2	
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4	
Z	-11/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-2	
ϕ_{BF} -	17/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1	
ϕ_{BF} +	-17/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3	
ϕ_{FG} -	-3/14	-2/7	2/7	-3/7	-2/7	1	-3/14	2/7	≥	-1	
ϕ_{FG} +	3/14	2/7	-2/7	3/7	2/7	-1	3/14	-2/7	≥	-2	
ϕ_{GH} -	-37/42	-5/42	13/21	-16/21	-5/42	1	5/42	13/21	≥	-1/6	
ϕ_{GH} +	37/42	5/42	-13/21	16/21	5/42	-1	-5/42	-13/21	≥	-17/6	
ϕ_{HG} -	27/14	-3/7	-4/7	13/7	-3/7	-1	-1/14	-4/7	≥	-1	
ϕ_{HG} +	-27/14	3/7	4/7	-13/7	3/7	1	1/14	4/7	≥	-2	
ϕ_{DH} -	3/14	2/7	-2/7	3/7	-5/7	0	-11/14	-2/7	≥	-4	
$\phi_{\text{DH}}\text{+}$	-3/14	-2/7	2/7	-3/7	5/7	0	11/14	2/7	≥	0	
L_{X}	-3/14	-2/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2	
Max	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	=	-1/3	

Scambio pivotale 24-2

	φ _{BC} +	ϕ_{DH} +	ϕ_{ED} +	Т	S	R	ϕ_{DC}	φ _{AB} +]	[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	-1] ≥	-4
X-	0	-1	0	0	1	0	1	0	≥	-1/2
ϕ_{BA} -	-3/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3/2
ϕ_{BA} +	3/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-5/2
ϕ_{BC} -	-1	0	0	0	0	0	0	0	≥	-3
Χ	0	-1	0	0	1	0	1	-1	≥	-5/2
$\phi_{\text{CD}}\text{-}$	0	1	0	1	-1	0	-1	0	≥	-1
$\phi_{\text{CD}}\text{+}$	0	-1	0	-1	1	0	1	0	≥	-2
αbF	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	≥	-1/3
$\phi_{\text{DC}}\text{+}$	0	0	0	0	0	0	-1	0	≥	-3
ϕ_{DE} -	0	1	0	0	0	0	-1	0	≥	-3/2
ϕ_{DE} +	0	-1	0	0	0	0	1	0	≥	-5/2
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4
Z	-1	-1	0	0	1	0	1	0	≥	-2
$\phi_{\text{BF}}\text{-}$	7/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-1
ϕ_{BF} +	-7/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3
ϕ_{FG} -	0	1	0	0	-1	1	-1	0	≥	-1
$\phi_{\text{FG}}\text{+}$	0	-1	0	0	1	-1	1	0	≥	-2
$\phi_{\text{GH}}\text{-}$	-19/24	5/12	1/2	-7/12	-5/12	1	-5/24	1/2	≥	-1/6
$\phi_{\text{GH}} \textbf{+}$	19/24	-5/12	-1/2	7/12	5/12	-1	5/24	-1/2	≥	-17/6
$\phi_{\text{HG}}\text{-}$	9/4	3/2	-1	5/2	-3/2	-1	-5/4	-1	≥	-1
ϕ_{HG} +	-9/4	-3/2	1	-5/2	3/2	1	5/4	1	≥	-2
$\phi_{\text{DH}}\text{-}$	0	-1	0	0	0	0	0	0	≥	-4
Υ	-3/4	-7/2	1	-3/2	5/2	0	11/4	1	≥	0
L_{X}	0	1	0	0	-1	0	-1	0	≥	-3/2
Max	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	=	1/3 _

Scambio pivotale 19-5

	φ _{BC} +	ϕ_{DH} +	ϕ_{ED} +	Т	ϕ_{GH} -	R	ϕ_{DC} -	ϕ_{AB} +		[Fb]	
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	[-4]	
X-	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-9/10	
ϕ_{BA} -	-18/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-21/10	
ϕ_{BA} +	18/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-19/10	
ϕ_{BC} -	-1	0	0	0	0	0	0	0	≥	-3	
Χ	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	1/5	≥	-29/10	
ϕ_{CD} -	19/10	0	-6/5	12/5	12/5	-12/5	-1/2	-6/5	≥	-3/5	
ϕ_{CD} +	-19/10	0	6/5	-12/5	-12/5	12/5	1/2	6/5	≥	-12/5	
αbF	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	≥	-2/5	
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3	
ϕ_{DE} -	0	1	0	0	0	0	-1	0	≥	-3/2	
ϕ_{DE} +	0	-1	0	0	0	0	1	0	≥	-5/2	
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4	
Z	-29/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-12/5	
ϕ_{BF} -	23/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-2/5	
$\phi_{\text{BF}}\text{+}$	-23/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-18/5	
ϕ_{FG} -	19/10	0	-6/5	7/5	12/5	-7/5	-1/2	-6/5	≥	-3/5	
ϕ_{FG} +	-19/10	0	6/5	-7/5	-12/5	7/5	1/2	6/5	≥	-12/5	
S	-19/10	1	6/5	-7/5	-12/5	12/5	-1/2	6/5	≥	-2/5	
ϕ_{GH} +	0	0	0	0	-1	0	0	0	≥	-3	
ϕ_{HG} -	51/10	0	-14/5	23/5	18/5	-23/5	-1/2	-14/5	≥	-2/5	
ϕ_{HG} +	-51/10	0	14/5	-23/5	-18/5	23/5	1/2	14/5	≥	-13/5	
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-4	
Υ	-11/2	-1	4	-5	-6	6	3/2	4	≥	-1	
L_{x}	19/10	0	-6/5	7/5	12/5	-12/5	-1/2	-6/5	≥	-11/10	
Max	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	=	2/5]	

Scambio pivotale 21-6

	φ _{BC} +	ϕ_{DH} +	ϕ_{ED} +	Т	ϕ_{GH} -	ϕ_{HG} -	ϕ_{DC} -	ϕ_{AB} +		[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	-1	\geq	-4
X-	35/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	\geq	-51/46
ϕ_{BA} -	9/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	\geq	-111/46
ϕ_{BA} +	-9/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	\geq	-73/46
ϕ_{BC} -	-1	0	0	0	0	0	0	0	\geq	-3
Χ	35/46	0	-6/23	1	-12/23	-12/23	11/46	-29/23	\geq	-143/46
ϕ_{CD} -	-35/46	0	6/23	0	12/23	12/23	-11/46	6/23	\geq	-9/23
ϕ_{CD} +	35/46	0	-6/23	0	-12/23	-12/23	11/46	-6/23	\geq	-60/23
αbF	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	≥	-10/23
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3
ϕ_{DE} -	0	1	0	0	0	0	-1	0	≥	-3/2
ϕ_{DE} +	0	-1	0	0	0	0	1	0	\geq	-5/2
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4
Z	-11/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	≥	-60/23
ϕ_{BF} -	14/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	≥	-2/23
ϕ_{BF} +	-14/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	≥	-90/23
ϕ_{FG} -	8/23	0	-8/23	0	30/23	7/23	-8/23	-8/23	\geq	-11/23
ϕ_{FG} +	-8/23	0	8/23	0	-30/23	-7/23	8/23	8/23	≥	-58/23
S	35/46	1	-6/23	1	-12/23	-12/23	-35/46	-6/23	≥	-14/23
ϕ_{GH} +	0	0	0	0	-1	0	0	0	≥	-3
R	51/46	0	-14/23	1	18/23	-5/23	-5/46	-14/23	\geq	-2/23
ϕ_{HG} +	0	0	0	0	0	-1	0	0	\geq	-3
ϕ_{DH} -	0	-1	0	0	0	0	0	0	\geq	-4
Υ	53/46	-1	8/23	1	-30/23	-30/23	39/46	8/23	≥	-35/23
L_{X}	-35/46	0	6/23	-1	12/23	12/23	-11/46	6/23	≥	-41/46
Max	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	=	-10/23

Scambio pivotale 7-1

Ocam	DIO PIVO										
	φ _{CD} -	$\phi_{\text{DH}} \textbf{+}$	ϕ_{ED} +	Т	ϕ_{GH} -	ϕ_{HG} -	ϕ_{DC} -	φ_{AB} +		[Fb]	
ϕ_{AB} -	0	0	0	0	0	0	0	-1	\geq	[-4]	
X-	-1	0	0	1	0	0	0	0	\geq	-3/2	
ϕ_{BA} -	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	\geq	-183/70	
ϕ_{BA} +	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	\geq	-97/70	
ϕ_{BC} -	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	\geq	-87/35	
Χ	-1	0	0	1	0	0	0	-1	\geq	-7/2	
$\phi_{\text{BC}}\text{+}$	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	\geq	-18/35	
ϕ_{CD} +	-1	0	0	0	0	0	0	0	\geq	-3	
$\alpha b F$	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	\geq	-16/35	
ϕ_{DC} +	0	0	0	0	0	0	-1	0	\geq	-3	
ϕ_{DE} -	0	1	0	0	0	0	-1	0	\geq	-3/2	
$\phi_{\text{DE}} \textbf{+}$	0	-1	0	0	0	0	1	0	≥	-5/2	
$\phi_{\text{ED}}\text{-}$	0	0	-1	0	0	0	0	0	\geq	-4	
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	\geq	-87/35	
ϕ_{BF} -	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5	
ϕ_{BF} +	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	\geq	-18/5	
ϕ_{FG} -	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	\geq	-23/35	
ϕ_{FG} +	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	\geq	-82/35	
S	-1	1	0	1	0	0	-1	0	≥	-1	
ϕ_{GH} +	0	0	0	0	-1	0	0	0	≥	-3	
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	\geq	-23/35	
ϕ_{HG} +	0	0	0	0	0	-1	0	0	\geq	-3	
ϕ_{DH} -	0	-1	0	0	0	0	0	0	\geq	-4	
Υ	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35	
L_{x}	1	0	0	-1	0	0	0	0	≥	-1/2	
Max	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	-16/35	

Tab	 fina	_

	[φ _{CD} -	ϕ_{DH} +	ϕ_{ED} +	Т	ϕ_{GH} -	φ _{HG} -	φ _{DC} -	ϕ_{AB} +		[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	-4
X-	-1	0	0	1	0	0	0	0	≥	-3/2
ϕ_{BA} -	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	≥	-183/70
ϕ_{BA} +	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	≥	-97/70
ϕ_{BC} -	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	≥	-87/35
Χ	-1	0	0	1	0	0	0	-1	≥	-7/2
ϕ_{BC} +	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	≥	-18/35
ϕ_{CD} +	-1	0	0	0	0	0	0	0	≥	-3
$\alpha b F$	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	≥	-16/35
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3
ϕ_{DE} -	0	1	0	0	0	0	-1	0	≥	-3/2
ϕ_{DE} +	0	-1	0	0	0	0	1	0	≥	-5/2
φ _{ED} -	0	0	-1	0	0	0	0	0	≥	-4
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	≥	-87/35
ϕ_{BF} -	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5
ϕ_{BF} +	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	≥	-18/5
ϕ_{FG} -	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	≥	-23/35
ϕ_{FG} +	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	≥	-82/35
S	-1	1	0	1	0	0	-1	0	≥	-1
ϕ_{GH} +	0	0	0	0	-1	0	0	0	≥	-3
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	≥	-23/35
ϕ_{HG} +	0	0	0	0	0	-1	0	0	≥	-3
φ _{DH} -	0	-1	0	0	0	0	0	0	≥	-4
Υ	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35
L_{x}	1	0	0	-1	0	0	0	0	≥	-1/2
	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	16/35 _

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Vettori soluzione della programmazione lineare

v Otto	1 00102		ma prog		a_10110	milouro					
	X	Υ	Z	Τ	S	R	$\alpha b F$	X-		[Fb]	
ϕ_{AB} -	0	0	0	0	0	0	0	0	≥	[0]	
ϕ_{AB} +	0	0	0	0	0	0	0	0	≥	1/35	
$\phi_{\text{BA}}\text{-}$	0	0	0	0	0	0	0	0	≥	0	
ϕ_{BA} +	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{BC}}\text{-}$	0	0	0	0	0	0	0	0	≥	0	
ϕ_{BC} +	0	0	0	0	0	0	0	0	≥	0	
ϕ_{CD}	0	0	0	0	0	0	0	0	≥	2/35	
ϕ_{CD} +	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{DC}}\text{-}$	0	0	0	0	0	0	0	0	≥	2/35	
ϕ_{DC} +	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{DE}}\text{-}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0	
ϕ_{ED} -	0	0	0	0	0	0	0	0	≥	0	
ϕ_{ED} +	0	0	0	0	0	0	0	0	≥	1/35	
$\phi_{\text{BF}}\text{-}$	0	0	0	0	0	0	0	0	≥	0	
ϕ_{BF} +	0	0	0	0	0	0	0	0	≥	0	
ϕ_{FG} -	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{FG}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0	
ϕ_{GH} -	0	0	0	0	0	0	0	0	≥	2/35	
ϕ_{GH} +	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{HG}}\text{-}$	0	0	0	0	0	0	0	0	≥	2/35	
ϕ_{HG} +	0	0	0	0	0	0	0	0	≥	0	
ϕ_{DH} -	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{DH}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0	
L_X	0	0	0	0	0	0	0	0	≥	0	
Max	7/2	74/35	87/35	0	1	23/35	16/35	3/2	=	-16/35	

Variabili soluzione dedotto il valore X-

Variabili soluzione differenza tra rotazioni

ϕ_{AB}	1/35
ϕ_{BA}	0
ϕ_{BC}	0
ϕ_{CD}	-2/35
ϕ_{DC}	-2/35
ϕ_{DE}	0
ϕ_{ED}	1/35
ϕ_{BF}	0
ϕ_{FG}	0
ϕ_{GH}	-2/35
ϕ_{HG}	-2/35
ϕ_{DH}	0

REAZIONI Fattore di collasso = 16/35

 $H_A = -61/70F$

 $V_A = 11/7F$

 $W_A = 2Fb$

 $H_{r} = -1/2F$

 $V_E = 3F$

 $W_F = 2Fb$

$$\begin{array}{llll} H_{FG} = 7/6F & H_{GH} = 7/6F & H_{HD} = 7/6F \\ V_{FG} = 23/70F & V_{GH} = -3/2F & V_{HD} = -3/2F \\ W_{FG} = -59/70Fb & W_{GH} = -3/2Fb & W_{HD} = 3/2Fb \\ H_{GF} = -7/6F & H_{HG} = -7/6F & H_{DH} = -7/6F \\ V_{GF} = -23/70F & V_{HG} = 3/2F & V_{DH} = 3/2F \\ W_{GF} = 3/2Fb & W_{HG} = -3/2Fb & W_{DH} = 2Fb \end{array}$$

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_B = 3/35\delta$	$u_{CCB} = 3/35\delta$	$u_D = 3/35\delta$	$u_{EED} = 0$
$V_{AAB} = 0$	$V_B = 0$	$V_{CCB} = -2/35\delta$	$V_D = 0$	$V_{EED} = 0$
$\phi_{AAB} = -1/35\delta/b$	$\phi_{\rm p} = -1/35 \delta/b$	$\phi_{CCR} = -1/35\delta/b$	$\phi_{D} = 1/35\delta/b$	$\phi_{E} = -1/35\delta/b$

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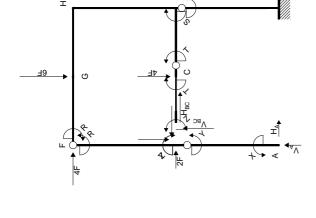
$u_F = 6/35\delta$	$u_{GGF} = 6/35\delta$	$u_{HHG} = 6/35\delta$
$V_F = 0$	$V_{GGF} = -2/35\delta$	$V_{HHG} = 0$
$\varphi_F = -1/35\delta/b$	$\varphi_{GGF} = -1/35\delta/b$	$\phi_{HHD} = 1/35\delta/b$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$	$u_{BBC} = 3/35\delta$	$u_{CCD} = 3/35\delta$	$u_{DDE} = 3/35\delta$	$u_{BBF} = 3/35\delta$
$V_{AAB} = 0$	$V_{BBC} = 0$	$V_{CCD} = -2/35\delta$	$V_{DDE} = 0$	$V_{BBF} = 0$
$\phi_{AAB} = -1/35\delta/b$	$\phi_{BBC} = -1/35\delta/b$	$\varphi_{CCD} = 1/35\delta/b$	ϕ_{DDE} = -1/35 δ /b	$\phi_{BBF} = -1/35\delta/b$

$$\begin{array}{lll} u_{FFG} = 6/35\delta & u_{GGH} = 6/35\delta & u_{HHD} = 6/35\delta \\ v_{FFG} = 0 & v_{GGH} = -2/35\delta & v_{HHD} = 0 \\ \phi_{FFG} = -1/35\delta/b & \phi_{GGH} = 1/35\delta/b & \phi_{HHD} = -1/35\delta/b \end{array}$$

EQUILIBRIO Nome:



EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

 $3H_Ab - 4V_Ab = -Xb + Sb - 8Fb$

Rotazione intorno a C: aste CB

 $-2V_{BC}b = -Zb + Tb$

Rotazione intorno a F: aste FB BA $6H_Ab - 3H_{BC}b = -Xb + Zb - Rb - 6Fb$

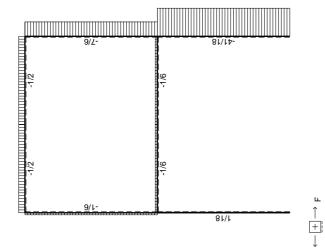
Rotazione intorno a B: aste BA

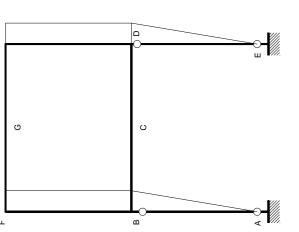
 $3H_Ab = -Xb - Yb$

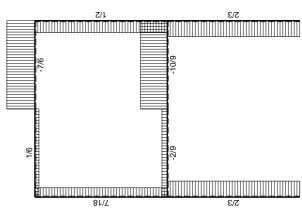
Matrice di equilibrio

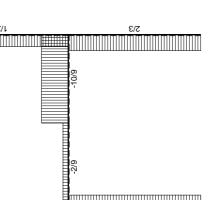
Soluzione del sistema

$$\begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1/3 & -1/3 & 0 & 0 & 0 & 0 & 0 \\ V_{BC}b & 0 & 0 & 1/2 & -1/2 & 0 & 0 & 0 \\ H_{BC}b & 0 & 0 & -1/3 & -2/3 & -1/3 & 0 & 0 & 1/3 & 2 \\ V_{A}b & 0 & -1/4 & 0 & 0 & -1/4 & 0 & 2 \end{bmatrix}$$













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PROGRAMMAZIONE LINEARE

Sia H_{ii} la matrice del simplesso, con m righe e n colonne.

Siano P_i le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.

Siano a riga m i coefficienti della funzione obiettivo primale $\max \Sigma_i H_{mi} P_{ii}$ $1 \le j < n$.

Siano a colonna n i coefficienti della funzione obiettivo duale $min \Sigma_i H_{in} D_i$, $1 \le i < m$.

Sequenza di operazioni pivotali:

- 1 Sia q (1 $\leq q < n$) la colonna pivot con massimo valore H_{mi} in riga m.
- 2 Sia p ($1 \le p < m$) la riga pivot di colonna q, a coefficiente negativo H_{in} , che minimizza il rapporto H_{in}/H_{in} .
- 3 Si ottiene il coefficiente pivotale H_{po} .
- 4 Si scambia la variabile primale P_q con la duale D_p .
- 5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
- 6 Si ridefiniscono i coefficienti della colonna pivot $q: H_{ig} = H_{ng} H_{ig}$, escluso il pivot H_{ng} .
- 7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ii} = H_{ii} H_{ii} + H_{pr}$
- 8 Si ridefiniscono i coefficienti della riga pivot $p: H_{oi} = -H_{oa} H_{oi}$, escluso il pivot H_{oc} .
- Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.

Giunti a questo punto, si individua la soluzione.

Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P, presenti sulla colonna di sinistra.

Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare *m*=6,*n*=4

$$\begin{bmatrix} \mathsf{P}_1 & \mathsf{P}_2 & \mathsf{P}_3 \end{bmatrix} & \begin{bmatrix} \mathsf{MIN} \\ \mathsf{D}_1 & \begin{bmatrix} \mathsf{H}_{11} & \mathsf{H}_{12} & \mathsf{H}_{13} \\ \mathsf{H}_{21} & \mathsf{H}_{22} & \mathsf{H}_{23} \end{bmatrix} \geq \begin{bmatrix} \mathsf{H}_{14} \\ \mathsf{H}_{24} \\ \mathsf{D}_3 & \mathsf{H}_{31} & \mathsf{H}_{32} & \mathsf{H}_{33} \end{bmatrix} \geq \mathsf{H}_{34} \\ \mathsf{D}_4 & \mathsf{H}_{41} & \mathsf{H}_{42} & \mathsf{H}_{43} \end{bmatrix} \geq \mathsf{H}_{44} \\ \mathsf{D}_5 & \mathsf{H}_{51} & \mathsf{H}_{52} & \mathsf{H}_{53} \end{bmatrix} \geq \mathsf{H}_{54} \\ \mathsf{MAX} \begin{bmatrix} \mathsf{H}_{61} & \mathsf{H}_{62} & \mathsf{H}_{63} \end{bmatrix} = \begin{bmatrix} \mathsf{H}_{64} \end{bmatrix}$$

SOLUZIONE DEL SIMPLESSO X=WAR Y=WRA Z=WRC T=WCD S=WDE R=WER

Tableau con variabili non vincolate in segno

Tablea	u co	ii valid	ו וווטג	IOII VI	licola	le III	segnic		
	[X	Υ	Z	Τ	S	R	αbF		[Fb]
W_{AB} -	1	0	0	0	0	0	0	≥	-1
W_{AB} +	1	0	0	0	0	0	0	≤	1
W _{BA} -	0	1	0	0	0	0	0	≥	-1
W _{BA} +	0	1	0	0	0	0	0	≤	1
W_{BC} -	0	0	1	0	0	0	0	≥	-3/2
W_{BC} +	0	0	1	0	0	0	0	≤	3/2
W _{CD} -	0	0	0	1	0	0	0	≥	-3/2
W _{CD} +	0	0	0	1	0	0	0	≤	3/2
W_{DC} -	0	0	1	-2	0	0	-8	≥	-3/2
W_{DC} +	0	0	1	-2	0	0	-8	≤	3/2
W_{DE} -	0	0	0	0	1	0	0	≥	-1
W_{DE} +	0	0	0	0	1	0	0	≤	1
W_{ED} -	-1	-1	0	0	-1	0	18	≥	-1
W_{ED} +	-1	-1	0	0	-1	0	18	≤	1
W_{BF} -	0	-1	-1	0	0	0	0	≥	-1
W_{BF} +	0	-1	-1	0	0	0	0	≤	1
W_{FB} -	0	0	0	0	0	1	0	≥	-1
W_{FB} +	0	0	0	0	0	1	0	≤	1
W_{GH} -	0	1/2	1	-1	1/2	-1	-4	≥	-3/2
W_{GH} +	0	1/2	1	-1	1/2	-1	-4	≤	3/2
W _{HD} -	0	1	2	-2	1	-1	4	≥	-1
W_{HD} +	0	1	2	-2	1	-1	4	≤	1
W_{DH} -	0	0	-1	2	-1	0	8	≥	-1
W_{DH} +	0	0	-1	2	-1	0	8	≤	1
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili vincolate in segno

Tableau con variabili non vincolate in segno

	[X	Υ	Z	Т	S	R	αbF]		[Fb]
W_{AB} -	1	0	0	0	0	0	0	≥	-1
W_{AB} +	-1	0	0	0	0	0	0	≥	-1
W_{BA} -	0	1	0	0	0	0	0	≥	-1
W_{BA} +	0	-1	0	0	0	0	0	≥	-1
W_{BC} -	0	0	1	0	0	0	0	≥	-3/2
W_{BC} +	0	0	-1	0	0	0	0	≥	-3/2
W _{CD} -	0	0	0	1	0	0	0	≥	-3/2
W_{CD} +	0	0	0	-1	0	0	0	≥	-3/2
W_{DC} -	0	0	1	-2	0	0	-8	≥	-3/2
W_{DC} +	0	0	-1	2	0	0	8	≥	-3/2
W_{DE} -	0	0	0	0	1	0	0	≥	-1
W_{DE} +	0	0	0	0	-1	0	0	≥	-1
W_{ED} -	-1	-1	0	0	-1	0	18	≥	-1
W_{ED} +	1	1	0	0	1	0	-18	≥	-1
W _{BF} -	0	-1	-1	0	0	0	0	≥	-1
W_{BF} +	0	1	1	0	0	0	0	≥	-1
W_{FB} -	0	0	0	0	0	1	0	≥	-1
W_{FB} +	0	0	0	0	0	-1	0	≥	-1
W_{GH} -	0	1/2	1	-1	1/2	-1	-4	≥	-3/2
W_{GH} +	0	-1/2	-1	1	-1/2	1	4	≥	-3/2
W_{HD} -	0	1	2	-2	1	-1	4	≥	-1
W_{HD} +	0	-1	-2	2	-1	1	-4	≥	-1
W_{DH} -	0	0	-1	2	-1	0	8	≥	-1
W_{DH} +	0	0	1	-2	1	0	-8	≥	-1
Max	0	0	0	0	0	0	1	=	0

	[X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		[Fb]	
W_{AB} -	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	[-1]	
W_{AB} +	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1	
W_{BA} -	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1	
W_{BA} +	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1	
W_{BC} -	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2	
W_{BC} +	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2	
W _{CD} -	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2	
W_{CD} +	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2	
W _{DC} -	0	0	1	-2	0	0	0	0	-1	2	0	0	-8	≥	-3/2	
W_{DC} +	0	0	-1	2	0	0	0	0	1	-2	0	0	8	≤	-3/2	
W _{DE} -	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1	
W_{DE} +	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1	
W _{ED} -	-1	-1	0	0	-1	0	1	1	0	0	1	0	18	≥	-1	
W_{ED} +	1	1	0	0	1	0	-1	-1	0	0	-1	0	-18	≤	-1	
W _{BF} -	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-1	
W_{BF} +	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-1	
W _{FB} -	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-1	
W_{FB} +	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-1	
W_{GH} -	0	1/2	1	-1	1/2	-1	0	-1/2	-1	1	-1/2	1	-4	≥	-3/2	
W_{GH} +	0	-1/2	-1	1	-1/2	1	0	1/2	1	-1	1/2	-1	4	≤	-3/2	
١٨/ -	0	1	2	-2	1	-1	Ω	-1	-2	2	-1	1	4	>	-1	

| ≤ | -1

| ≤ | -1

Tableau a variabili negative su X- e limitate

	X	Υ	Z	Ť	S	R	αbF	X		[Fb]
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	-1
ϕ_{AB} +	-1	0	0	0	0	0	0	1	≥	-1
$\phi_{\text{BA}}\text{-}$	0	1	0	0	0	0	0	-1	≥	-1
ϕ_{BA} +	0	-1	0	0	0	0	0	1	≥	-1
$\phi_{\text{BC}}\text{-}$	0	0	1	0	0	0	0	-1	≥	-3/2
ϕ_{BC} +	0	0	-1	0	0	0	0	1	≥	-3/2
ϕ_{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2
$\phi_{\text{CD}} \textbf{+}$	0	0	0	-1	0	0	0	1	≥	-3/2
$\phi_{\text{DC}}\text{-}$	0	0	1	-2	0	0	-8	1	≥	-3/2
$\phi_{\text{DC}} \textbf{+}$	0	0	-1	2	0	0	8	-1	≥	-3/2
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-1
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	≥	-1
$\phi_{\text{ED}}\text{-}$	-1	-1	0	0	-1	0	18	3	≥	-1
$\phi_{\text{ED}} \textbf{+}$	1	1	0	0	1	0	-18	-3	≥	-1
$\phi_{\text{BF}}\text{-}$	0	-1	-1	0	0	0	0	2	≥	-1
ϕ_{BF} +	0	1	1	0	0	0	0	-2	≥	-1
ϕ_{FB} -	0	0	0	0	0	1	0	-1	≥	-1
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1
ϕ_{GH} -	0	1/2	1	-1	1/2	-1	-4	0	≥	-3/2
$\phi_{\text{GH}} \textbf{+}$	0	-1/2	-1	1	-1/2	1	4	0	≥	-3/2
ϕ_{HD} -	0	1	2	-2	1	-1	4	-1	≥	-1
$\phi_{\text{HD}} \textbf{+}$	0	-1	-2	2	-1	1	-4	1	≥	-1
$\phi_{\text{DH}}\text{-}$	0	0	-1	2	-1	0	8	0	≥	-1
$\phi_{\text{DH}} \textbf{+}$	0	0	1	-2	1	0	-8	0	≥	-1
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	0	0	0	0	0	1	0 _	=	0

Scam	Scambio pivotale 14-7 X Y Z T S R ϕ_{En} + X-] Fb]												
	[X	Υ	ϕ_{ED} +	Χ-		[Fb]							
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	[-1]			
ϕ_{AB} +	-1	0	0	0	0	0	0	1	≥	-1			
ϕ_{BA} -	0	1	0	0	0	0	0	-1	≥	-1			
$\phi_{\text{BA}} \textbf{+}$	0	-1	0	0	0	0	0	1	≥	-1			
ϕ_{BC} -	0	0	1	0	0	0	0	-1	≥	-3/2			
ϕ_{BC} +	0	0	-1	0	0	0	0	1	≥	-3/2			
$\phi_{\text{CD}}\text{-}$	0	0	0	1	0	0	0	-1	≥	-3/2			
$\phi_{\text{CD}}\text{+}$	0	0	0	-1	0	0	0	1	≥	-3/2			
$\phi_{\text{DC}}\text{-}$	-4/9	-4/9	1	-2	-4/9	0	4/9	7/3	≥	-19/18			
ϕ_{DC} +	4/9	4/9	-1	2	4/9	0	-4/9	-7/3	≥	-35/18			
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-1			
ϕ_{DE} +	0	0	0	0	-1	0	0	1	≥	-1			
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2			
αbF	1/18	1/18	0	0	1/18	0	-1/18	-1/6	≥	-1/18			
ϕ_{BF} -	0	-1	-1	0	0	0	0	2	≥	-1			
ϕ_{BF} +	0	1	1	0	0	0	0	-2	≥	-1			
ϕ_{FB} -	0	0	0	0	0	1	0	-1	≥	-1			
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1			
ϕ_{GH} -	-2/9	5/18	1	-1	5/18	-1	2/9	2/3	≥	-23/18			
ϕ_{GH} +	2/9	-5/18	-1	1	-5/18	1	-2/9	-2/3	≥	-31/18			
$\phi_{\text{HD}}\text{-}$	2/9	11/9	2	-2	11/9	-1	-2/9	-5/3	≥	-11/9			
ϕ_{HD} +	-2/9	-11/9	-2	2	-11/9	1	2/9	5/3	≥	-7/9			
ϕ_{DH} -	4/9	4/9	-1	2	-5/9	0	-4/9	-4/3	≥	-13/9			
ϕ_{DH} +	-4/9	-4/9	1	-2	5/9	0	4/9	4/3	≥	-5/9			
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2			
Max	1/18	1/18	0	0	1/18	0	-1/18	-1/6	=	1/18]			

Scambio pivotale 2-1

	$\left[\phi_{AB}\right]$	Υ	Z	Т	S	R	$\phi_{\text{ED}}\text{+}$	X-]		[Fb]	
φ _{AB} -	-1	0	0	0	0	0	0	0	\geq	[-2]	
Χ	-1	0	0	0	0	0	0	1	\geq	-1	
ϕ_{BA} -	0	1	0	0	0	0	0	-1	≥	-1	
ϕ_{BA} +	0	-1	0	0	0	0	0	1	≥	-1	
ϕ_{BC} -	0	0	1	0	0	0	0	-1	≥	-3/2	
φ _{BC} +	0	0	-1	0	0	0	0	1	≥	-3/2	
φ _{CD} -	0	0	0	1	0	0	0	-1	\geq	-3/2	
ϕ_{CD} +	0	0	0	-1	0	0	0	1	\geq	-3/2	
ϕ_{DC} -	4/9	-4/9	1	-2	-4/9	0	4/9	17/9	≥	-11/18	
ϕ_{DC} +	-4/9	4/9	-1	2	4/9	0	-4/9	-17/9	≥	-43/18	
ϕ_{DE} -	0	0	0	0	1	0	0	-1	≥	-1	
ϕ_{DE} +	0	0	0	0	-1	0	0	1	≥	-1	
ϕ_{ED} -	0	0	0	0	0	0	-1	0	\geq	-2	
α bF	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	\geq	-1/9	
ϕ_{BF} -	0	-1	-1	0	0	0	0	2	≥	-1	
ϕ_{BF} +	0	1	1	0	0	0	0	-2	≥	-1	
ϕ_{FB} -	0	0	0	0	0	1	0	-1	\geq	-1	
ϕ_{FB} +	0	0	0	0	0	-1	0	1	\geq	-1	
ϕ_{GH} -	2/9	5/18	1	-1	5/18	-1	2/9	4/9	≥	-19/18	
ϕ_{GH} +	-2/9	-5/18	-1	1	-5/18	1	-2/9	-4/9	\geq	-35/18	
ϕ_{HD} -	-2/9	11/9	2	-2	11/9	-1	-2/9	-13/9	\geq	-13/9	
ϕ_{HD} +	2/9	-11/9	-2	2	-11/9	1	2/9	13/9	\geq	-5/9	
ϕ_{DH} -	-4/9	4/9	-1	2	-5/9	0	-4/9	-8/9	\geq	-17/9	
ϕ_{DH} +	4/9	-4/9	1	-2	5/9	0	4/9	8/9	≥	-1/9	
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2	
Max	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	=	1/9]	

an	ibio biv	otale 2	4-2						
	$\left[\phi_{AB} + \right]$	ϕ_{DH} +	Z	Т	S	R	ϕ_{ED} +	X-]	
_B -	-1	0	0	0	0	0	0	0]	

[Fb]

	L YAB'	ΨDH.	_	•	_		ΨED.	· · -	J	∟ • ~ .
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	-2
Χ	-1	0	0	0	0	0	0	1	≥	-1
ϕ_{BA} -	1	-9/4	9/4	-9/2	5/4	0	1	1	≥	-5/4
ϕ_{BA} +	-1	9/4	-9/4	9/2	-5/4	0	-1	-1	≥	-3/4
ϕ_{BC} -	0	0	1	0	0	0	0	-1	≥	-3/2
ϕ_{BC} +	0	0	-1	0	0	0	0	1	≥	-3/2
ϕ_{CD}	0	0	0	1	0	0	0	-1	≥	-3/2
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2
ϕ_{DC} -	0	1	0	0	-1	0	0	1	≥	-1/2
ϕ_{DC} +	0	-1	0	0	1	0	0	-1	≥	-5/2
ϕ_{DE} -	0	0	0	0	1	0	0	-1	≥	-1
ϕ_{DE} +	0	0	0	0	-1	0	0	1	≥	-1
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
αbF	0	-1/8	1/8	-1/4	1/8	0	0	0	≥	-1/8
ϕ_{BF} -	-1	9/4	-13/4	9/2	-5/4	0	-1	0	≥	-3/4
ϕ_{BF} +	1	-9/4	13/4	-9/2	5/4	0	1	0	≥	-5/4
ϕ_{FB} -	0	0	0	0	0	1	0	-1	≥	-1
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1
ϕ_{GH} -	1/2	-5/8	13/8	-9/4	5/8	-1	1/2	1	≥	-9/8
ϕ_{GH} +	-1/2	5/8	-13/8	9/4	-5/8	1	-1/2	-1	≥	-15/8
ϕ_{HD} -	1	-11/4	19/4	-15/2	11/4	-1	1	1	≥	-7/4
ϕ_{HD} +	-1	11/4	-19/4	15/2	-11/4	1	-1	-1	≥	-1/4
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-2
Υ	1	-9/4	9/4	-9/2	5/4	0	1	2	≥	-1/4
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	-1/8	1/8	-1/4	1/8	0	0	0	=	-1/8

Scambio pivotale 22-3

	[φ _{AB} +	ϕ_{DH} +	ϕ_{HD} +	Т	S	R	ϕ_{ED} +	X-]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0]	\geq	-2
Χ	-1	0	0	0	0	0	0	1	\geq	-1
ϕ_{BA} -	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	10/19	≥	-26/19
$\phi_{\text{BA}} \textbf{+}$	-10/19	18/19	9/19	18/19	1/19	-9/19	-10/19	-10/19	\geq	-12/19
ϕ_{BC} -	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-23/19	\geq	-59/38
$\phi_{BC} \textbf{+}$	4/19	-11/19	4/19	-30/19	11/19	-4/19	4/19	23/19	\geq	-55/38
ϕ_{CD}	0	0	0	1	0	0	0	-1	≥	-3/2
$\phi_{\text{CD}}\text{+}$	0	0	0	-1	0	0	0	1	\geq	-3/2
$\phi_{\text{DC}}\text{-}$	0	1	0	0	-1	0	0	1	\geq	-1/2
$\phi_{DC} \textbf{+}$	0	-1	0	0	1	0	0	-1	\geq	-5/2
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	\geq	-1
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	\geq	-1
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	-1	0	\geq	-2
$\alpha b F$	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	\geq	-5/38
$\phi_{\text{BF}}\text{-}$	-6/19	7/19	13/19	-12/19	12/19	-13/19	-6/19	13/19	\geq	-11/19
$\phi_{\text{BF}}\text{+}$	6/19	-7/19	-13/19	12/19	-12/19	13/19	6/19	-13/19	\geq	-27/19
$\phi_{\text{FB}}\text{-}$	0	0	0	0	0	1	0	-1	\geq	-1
$\phi_{\text{FB}}\text{+}$	0	0	0	0	0	-1	0	1	\geq	-1
$\phi_{\text{GH}}\text{-}$	3/19	6/19	-13/38	6/19	-6/19	-25/38	3/19	25/38	\geq	-23/19
$\phi_{\text{GH}} \textbf{+}$	-3/19	-6/19	13/38	-6/19	6/19	25/38	-3/19	-25/38	\geq	-34/19
ϕ_{HD} -	0	0	-1	0	0	0	0	0	\geq	-2
Z	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-4/19	\geq	-1/19
$\phi_{\text{DH}}\text{-}$	0	-1	0	0	0	0	0	0	\geq	-2
Υ	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	29/19	\geq	-7/19
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	=	5/38]

Scambio pivotale 22-5

Courn	Бю рио Г.			_	7	_		v		r - . 1
	φ _{AB} +	ϕ_{DH} +	ϕ_{HD} +	Т	Ζ	R	ϕ_{ED} +	X-		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	-2
Χ	-1	0	0	0	0	0	0	1	≥	-1
ϕ_{BA} -	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	6/11	≥	-15/11
ϕ_{BA} +	-6/11	1	5/11	12/11	-1/11	-5/11	-6/11	-6/11	≥	-7/11
ϕ_{BC} -	0	0	0	0	1	0	0	-1	≥	-3/2
ϕ_{BC} +	0	0	0	0	-1	0	0	1	≥	-3/2
ϕ_{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2
ϕ_{DC} -	4/11	0	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-9/22
ϕ_{DC} +	-4/11	0	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-57/22
ϕ_{DE} -	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-12/11
ϕ_{DE} +	4/11	-1	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-10/11
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
α bF	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	≥	-3/22
ϕ_{BF} -	-6/11	1	5/11	12/11	-12/11	-5/11	-6/11	5/11	≥	-7/11
ϕ_{BF} +	6/11	-1	-5/11	-12/11	12/11	5/11	6/11	-5/11	≥	-15/11
ϕ_{FB} -	0	0	0	0	0	1	0	-1	≥	-1
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1
ϕ_{GH} -	3/11	0	-5/22	-6/11	6/11	-17/22	3/11	17/22	≥	-13/11
ϕ_{GH} +	-3/11	0	5/22	6/11	-6/11	17/22	-3/11	-17/22	≥	-20/11
ϕ_{HD} -	0	0	-1	0	0	0	0	0	≥	-2
S	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-4/11	≥	-1/11
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-2
Υ	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	17/11	≥	-4/11
L_{x}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	=	-3/22

Scambio pivotale 9-4

Scarri	DIO PIVO	tale 3-4									
	φ _{AB} +	ϕ_{DH} +	$\phi_{\text{HD}} \textbf{+}$	ϕ_{DC} -	Z	R	ϕ_{ED} +	Χ-		[Fb]	
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	[-2]	
Χ	-1	0	0	0	0	0	0	1	≥	-1	
ϕ_{BA} -	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	0	≥	-6/5	
$\phi_{\text{BA}} \textbf{+}$	-2/5	1	3/5	-2/5	3/5	-3/5	-2/5	0	≥	-4/5	
ϕ_{BC} -	0	0	0	0	1	0	0	-1	≥	-3/2	
$\phi_{\text{BC}} \textbf{+}$	0	0	0	0	-1	0	0	1	≥	-3/2	
ϕ_{CD} -	2/15	0	2/15	-11/30	19/30	-2/15	2/15	-1/2	≥	-33/20	
$\phi_{\text{CD}} \textbf{+}$	-2/15	0	-2/15	11/30	-19/30	2/15	-2/15	1/2	≥	-27/20	
Τ	2/15	0	2/15	-11/30	19/30	-2/15	2/15	1/2	≥	-3/20	
ϕ_{DC} +	0	0	0	-1	0	0	0	0	≥	-3	
ϕ_{DE} -	0	1	0	-1	0	0	0	0	≥	-3/2	
$\phi_{\text{DE}} \textbf{+}$	0	-1	0	1	0	0	0	0	≥	-1/2	
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	-1	0	≥	-2	
$\alpha b F$	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	≥	-3/20	
ϕ_{BF} -	-2/5	1	3/5	-2/5	-2/5	-3/5	-2/5	1	≥	-4/5	
$\phi_{\text{BF}}\text{+}$	2/5	-1	-3/5	2/5	2/5	3/5	2/5	-1	≥	-6/5	
ϕ_{FB} -	0	0	0	0	0	1	0	-1	≥	-1	
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1	
$\phi_{\text{GH}}\text{-}$	1/5	0	-3/10	1/5	1/5	-7/10	1/5	1/2	≥	-11/10	
ϕ_{GH} +	-1/5	0	3/10	-1/5	-1/5	7/10	-1/5	-1/2	≥	-19/10	
ϕ_{HD} -	0	0	-1	0	0	0	0	0	≥	-2	
S	0	1	0	-1	0	0	0	1	≥	-1/2	
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-2	
Υ	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	1	≥	-1/5	
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2	
Max	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	=	3/20]	

Scambio pivotale 18-6

	[φ _{AB} +	φ _{DH} +	φ _{HD} +	ϕ_{DC} -	Z	ϕ_{FB} +	ϕ_{ED} +	X-]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	-2
Χ	-1	0	0	0	0	0	0	1	≥	-1
ϕ_{BA} -	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	3/5	≥	-9/5
ϕ_{BA} +	-2/5	1	3/5	-2/5	3/5	3/5	-2/5	-3/5	≥	-1/5
ϕ_{BC} -	0	0	0	0	1	0	0	-1	≥	-3/2
$\phi_{\text{BC}}\text{+}$	0	0	0	0	-1	0	0	1	≥	-3/2
φ _{CD} -	2/15	0	2/15	-11/30	19/30	2/15	2/15	-19/30	≥	-91/60
$\phi_{\text{CD}}\text{+}$	-2/15	0	-2/15	11/30	-19/30	-2/15	-2/15	19/30	≥	-89/60
Т	2/15	0	2/15	-11/30	19/30	2/15	2/15	11/30	≥	-1/60
ϕ_{DC} +	0	0	0	-1	0	0	0	0	≥	-3
ϕ_{DE} -	0	1	0	-1	0	0	0	0	≥	-3/2
ϕ_{DE} +	0	-1	0	1	0	0	0	0	≥	-1/2
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
$\alpha b F$	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	≥	-11/60
ϕ_{BF} -	-2/5	1	3/5	-2/5	-2/5	3/5	-2/5	2/5	≥	-1/5
ϕ_{BF} +	2/5	-1	-3/5	2/5	2/5	-3/5	2/5	-2/5	≥	-9/5
ϕ_{FB} -	0	0	0	0	0	-1	0	0	≥	-2
R	0	0	0	0	0	-1	0	1	≥	-1
$\phi_{\text{GH}}\text{-}$	1/5	0	-3/10	1/5	1/5	7/10	1/5	-1/5	≥	-2/5
ϕ_{GH} +	-1/5	0	3/10	-1/5	-1/5	-7/10	-1/5	1/5	≥	-13/5
ϕ_{HD} -	0	0	-1	0	0	0	0	0	≥	-2
S	0	1	0	-1	0	0	0	1	≥	-1/2
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-2
Υ	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	8/5	≥	-4/5
L_{x}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	=	-11/60

Scambio pivotale 4-8

Ooaiii											
	φ _{AB} +	ϕ_{DH} +	$\phi_{\text{HD}} \textbf{+}$	ϕ_{DC} -	Z	ϕ_{FB} +	ϕ_{ED} +	φ_{BA} +		[Fb]	
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	[-2]	
Χ	-5/3	5/3	1	-2/3	1	1	-2/3	-5/3	\geq	-4/3	
$\phi_{\text{BA}}\text{-}$	0	0	0	0	0	0	0	-1	\geq	-2	
X-	-2/3	5/3	1	-2/3	1	1	-2/3	-5/3	\geq	-1/3	
ϕ_{BC} -	2/3	-5/3	-1	2/3	0	-1	2/3	5/3	\geq	-7/6	
ϕ_{BC} +	-2/3	5/3	1	-2/3	0	1	-2/3	-5/3	≥	-11/6	
ϕ_{CD} -	5/9	-19/18	-1/2	1/18	0	-1/2	5/9	19/18	≥	-47/36	
ϕ_{CD} +	-5/9	19/18	1/2	-1/18	0	1/2	-5/9	-19/18	\geq	-61/36	
Т	-1/9	11/18	1/2	-11/18	1	1/2	-1/9	-11/18	≥	-5/36	
ϕ_{DC} +	0	0	0	-1	0	0	0	0	≥	-3	
ϕ_{DE} -	0	1	0	-1	0	0	0	0	≥	-3/2	
$\phi_{\text{DE}}\text{+}$	0	-1	0	1	0	0	0	0	\geq	-1/2	
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2	
$\alpha b F$	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	≥	-7/36	
ϕ_{BF} -	-2/3	5/3	1	-2/3	0	1	-2/3	-2/3	≥	-1/3	
ϕ_{BF} +	2/3	-5/3	-1	2/3	0	-1	2/3	2/3	≥	-5/3	
ϕ_{FB} -	0	0	0	0	0	-1	0	0	≥	-2	
R	-2/3	5/3	1	-2/3	1	0	-2/3	-5/3	≥	-4/3	
$\phi_{\text{GH}}\text{-}$	1/3	-1/3	-1/2	1/3	0	1/2	1/3	1/3	≥	-1/3	
$\phi_{\text{GH}} \textbf{+}$	-1/3	1/3	1/2	-1/3	0	-1/2	-1/3	-1/3	\geq	-8/3	
ϕ_{HD} -	0	0	-1	0	0	0	0	0	\geq	-2	
S	-2/3	8/3	1	-5/3	1	1	-2/3	-5/3	\geq	-5/6	
ϕ_{DH} -	0	-1	0	0	0	0	0	0	\geq	-2	
Υ	-2/3	5/3	1	-2/3	1	1	-2/3	-8/3	\geq	-4/3	
L_{x}	2/3	-5/3	-1	2/3	-1	-1	2/3	5/3	≥	-7/6	
Max	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	=	7/36]	

Scambio pivotale 12-2

	[φ _{AB} +	ϕ_{DE} +	ϕ_{HD} +	ϕ_{DC}	Z	ϕ_{FB} +	ϕ_{ED} +	ϕ_{BA} +]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	\geq	-2
Χ	-5/3	-5/3	1	1	1	1	-2/3	-5/3	\geq	-13/6
ϕ_{BA} -	0	0	0	0	0	0	0	-1	\geq	-2
X-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	\geq	-7/6
ϕ_{BC} -	2/3	5/3	-1	-1	0	-1	2/3	5/3	\geq	-1/3
$\phi_{\text{BC}}\text{+}$	-2/3	-5/3	1	1	0	1	-2/3	-5/3	\geq	-8/3
ϕ_{CD}	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	\geq	-7/9
$\phi_{\text{CD}}\text{+}$	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	\geq	-20/9
Т	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	\geq	-4/9
$\phi_{\text{DC}} \textbf{+}$	0	0	0	-1	0	0	0	0	\geq	-3
$\phi_{\text{DE}}\text{-}$	0	-1	0	0	0	0	0	0	\geq	-2
ϕ_{DH} +	0	-1	0	1	0	0	0	0	\geq	-1/2
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	-1	0	\geq	-2
$\alpha b F$	-1/18	-1/18	0	0	0	0	-1/18	-1/18	\geq	-2/9
ϕ_{BF} -	-2/3	-5/3	1	1	0	1	-2/3	-2/3	\geq	-7/6
ϕ_{BF} +	2/3	5/3	-1	-1	0	-1	2/3	2/3	\geq	-5/6
ϕ_{FB} -	0	0	0	0	0	-1	0	0	\geq	-2
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	\geq	-13/6
$\phi_{\text{GH}}\text{-}$	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	\geq	-1/6
ϕ_{GH} +	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	\geq	-17/6
ϕ_{HD} -	0	0	-1	0	0	0	0	0	\geq	-2
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	\geq	-13/6
ϕ_{DH}	0	1	0	-1	0	0	0	0	\geq	-3/2
Υ	-2/3	-5/3	1	1	1	1	-2/3	-8/3	\geq	-13/6
L_{X}	2/3	5/3	-1	-1	-1	-1	2/3	5/3	\geq	-1/3
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	2/9 _

Tableau finale

	[φ _{AB} +	ϕ_{DE} +	ϕ_{HD} +	ϕ_{DC} -	Z	ϕ_{FB} +	ϕ_{ED} +	ϕ_{BA} +]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0]	\geq	[-2]
Χ	-5/3	-5/3	1	1	1	1	-2/3	-5/3	\geq	-13/6
ϕ_{BA} -	0	0	0	0	0	0	0	-1	\geq	-2
Χ-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	\geq	-7/6
ϕ_{BC} -	2/3	5/3	-1	-1	0	-1	2/3	5/3	\geq	-1/3
ϕ_{BC} +	-2/3	-5/3	1	1	0	1	-2/3	-5/3	\geq	-8/3
$\phi_{\text{CD}}\text{-}$	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	\geq	-7/9
ϕ_{CD} +	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	\geq	-20/9
Т	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	≥	-4/9
ϕ_{DC} +	0	0	0	-1	0	0	0	0	≥	-3
ϕ_{DE} -	0	-1	0	0	0	0	0	0	≥	-2
ϕ_{DH} +	0	-1	0	1	0	0	0	0	≥	-1/2
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
$\alpha b F$	-1/18	-1/18	0	0	0	0	-1/18	-1/18	≥	-2/9
ϕ_{BF} -	-2/3	-5/3	1	1	0	1	-2/3	-2/3	≥	-7/6
ϕ_{BF} +	2/3	5/3	-1	-1	0	-1	2/3	2/3	\geq	-5/6
ϕ_{FB} -	0	0	0	0	0	-1	0	0	≥	-2
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	≥	-13/6
ϕ_{GH} -	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	≥	-1/6
ϕ_{GH} +	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	\geq	-17/6
ϕ_{HD} -	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	≥	-13/6
ϕ_{DH} -	0	1	0	-1	0	0	0	0	≥	-3/2
Υ	-2/3	-5/3	1	1	1	1	-2/3	-8/3	≥	-13/6
L_{x}	2/3	5/3	-1	-1	-1	-1	2/3	5/3	≥	-1/3
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	2/9

Vettori soluzione della programmazione lineare

	[X	Υ	Z	T	s	R	$\alpha b F$	Χ-		[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{AB}\textbf{+}$	0	0	0	0	0	0	0	0	≥	1/18
ϕ_{BA} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{BA}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	1/18
ϕ_{BC} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{BC}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{CD} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{CD}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DC}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DC}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DE}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	1/18
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{ED}}\text{+}$	0	0	0	0	0	0	0	0	≥	1/18
$\phi_{\text{BF}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{BF}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{FB} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{FB}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{GH}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{GH}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{HD}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{HD}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DH}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{DH} +	0	0	0	0	0	0	0	0	≥	0
L_{X}	0	0	0	0	0	0	0	0	≥	0
Max	13/6	13/6	0	4/9	13/6	13/6	2/9	7/6	=	2/9

Variabili soluzione dedotto il valore X-

 $u_G = 1/6\delta$

 $V_G = 0$

 $\varphi_G = 0$

Variabili soluzione differenza tra rotazioni

ϕ_{AB}	[1/18]	
ϕ_{BA}	1/18	
ϕ_{BC}	0	
ϕ_{CD}	0	
ϕ_{DC}	0	
ϕ_{DE}	1/18	
ϕ_{ED}	1/18	
ϕ_{BF}	0	
ϕ_{FB}	0	
ϕ_{GH}	0	
ϕ_{HD}	0	
ϕ_{DH}	[0]	

REAZIONI Fattore di collasso = 2/9

 $H_{\Delta} = -2/3F$

 $V_{\Delta}^{\prime} = -1/18F$

 $W_A = Fb$

 $H_{\rm F} = -2/3F$

 $V_{E} = 41/18F$

 $W_F = Fb$

$H_{AB} = -2/3F$	$H_{BC} = 1/6F$	$H_{CD} = 1/6F$	$H_{DE} = 2/3F$	$H_{BF} = -7/18F$
$V_{AB} = -1/18F$	$V_{BC} = -2/9F$	$V_{CD} = -10/9F$	$V_{DE} = -41/18F$	$V_{BF} = 1/6F$
$W_{AB} = Fb$	$W_{BC} = -7/6Fb$	$W_{CD} = -13/18Fb$	$W_{DE} = Fb$	$W_{BF} = 1/6Fb$
$H_{BA} = 2/3F$	$H_{CB} = -1/6F$	$H_{DC} = -1/6F$	$H_{ED} = -2/3F$	$H_{FB} = 7/18F$
$V_{BA} = 1/18F$	$V_{CB} = 2/9F$	$V_{DC} = 10/9F$	$V_{ED} = 41/18F$	$V_{FB} = -1/6F$
$W_{BA} = Fb$	$W_{CB} = 13/18Fb$	$W_{DC} = -3/2Fb$	$W_{ED} = Fb$	$W_{FB} = Fb$
$H_{FG} = 1/2F$	$H_{GH} = 1/2F$	$H_{HD} = 1/2F$		
$V_{FG} = 1/6F$	$V_{GH} = -7/6F$	$V_{HD} = -7/6F$		
$W_{FG} = -Fb$	$W_{GH} = -4/3Fb$	$W_{HD} = Fb$		
$H_{GF} = -1/2F$	$H_{HG} = -1/2F$	$H_{DH} = -1/2F$		
$V_{GF} = -1/6F$	$V_{HG} = 7/6F$	$V_{DH} = 7/6F$		
$W_{GF} = 4/3Fb$	$W_{HG} = -Fb$	$W_{DH} = 1/2Fb$		

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_B = 1/6\delta$	$u_C = 1/6\delta$	$u_D = 1/6\delta$	$u_{EED} = 0$
$V_{AAB} = 0$	$V_B = 0$	$v_C = 0$	$V_D = 0$	$V_{EED} = 0$
$\phi_{AAB} = -1/18\delta/b$	$\phi_{\rm p} = -1/18\delta/b$	$\varphi_{c} = 0$	$\varphi_D = 0$	$\phi_{r} = -1/18\delta/b$

SPOSTAMENTI RIGIDI DELLE ASTE

 $u_F = 1/6\delta$

 $V_F = 0$

 $\phi_F = 0$

$u_{AAB} = 0$	$u_{BBC} = 1/6\delta$	$u_{CCD} = 1/6\delta$	$u_{DDE} = 1/6\delta$	$u_{BBF} = 1/6\delta$
$V_{AAB} = 0$	$V_{BBC} = 0$		$V_{DDE} = 0$	$V_{BBF} = 0$
$\phi_{AAB} = -1/18\delta/b$	$\varphi_{BBC} = 0$	$\varphi_{CCD} = 0$	$\phi_{\text{DDE}} = -1/18\delta/b$	$\varphi_{BBF} = 0$

 $u_H = 1/6\delta$

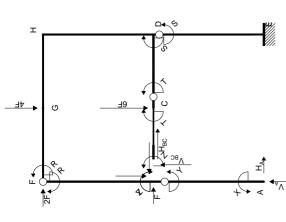
 $V_{H} = 0$

 $\varphi_H = 0$

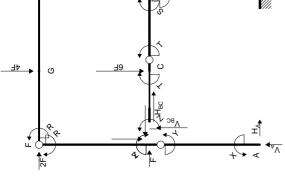
$u_{FFG} = 1/6\delta$	$u_{GGH} = 1/6\delta$	$u_{HHD} = 1/6\delta$
$V_{FFG} = 0$	$V_{GGH} = 0$	$V_{HHD} = 0$
$\phi_{FFG} = 0$	$\phi_{GGH} = 0$	$\phi_{HHD} = 0$

EQUILIBRIO Nome:

08.06.11



М



EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

 $3H_Ab - 4V_Ab = -Xb + Sb - 14Fb$

Rotazione intorno a C: aste CB

Rotazione intorno a F: aste FB BA $-2V_{BC}b = -Zb + Tb$

 $6H_Ab - 3H_{BC}b = -Xb + Zb + Rb - 3Fb$

Rotazione intorno a B: aste BA

 $3H_Ab = -Xb - Yb$

Matrice di equilibrio

$$\begin{cases}
H_A b & V_A b & H_{BC} b & V_{BC} b
\end{cases} & \begin{bmatrix} X b & Y b & Z b & T b & S b & R t \\ 3 & -4 & 0 & 0 \\ \phi_{CD} & 0 & 0 & -2 \\ 6 & 0 & -3 & 0 \\ 3 & 0 & 0 & 0 & 1
\end{cases}$$

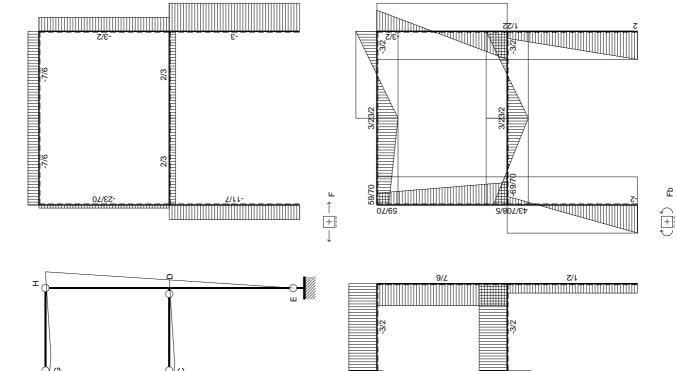
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87/70

Soluzione del sistema

$$\begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1/3 & -1/3 & 0 & 0 & 0 & 0 & 0 \\ V_{BC}b & 0 & 0 & 1/2 & -1/2 & 0 & 0 & 0 \\ H_{BC}b & -1/3 & -2/3 & -1/3 & 0 & -1/3 & 1 \\ V_Ab & 0 & -1/4 & 0 & 0 & -1/4 & 0 & 7/2 \end{bmatrix}$$

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.11.05.11



@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.11.05.11

08.06.11

PROGRAMMAZIONE LINEARE

Sia H_{ii} la matrice del simplesso, con m righe e n colonne.

Siano P_i le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.

Siano a riga m i coefficienti della funzione obiettivo primale $\max \Sigma_i H_{mi} P_{ii}$ $1 \le j < n$.

Siano a colonna n i coefficienti della funzione obiettivo duale $min \Sigma_i H_{in} D_i$, $1 \le i < m$.

Sequenza di operazioni pivotali:

- 1 Sia q (1 $\leq q < n$) la colonna pivot con massimo valore H_{mi} in riga m.
- 2 Sia p ($1 \le p < m$) la riga pivot di colonna q, a coefficiente negativo H_{in} , che minimizza il rapporto H_{in}/H_{in} .
- 3 Si ottiene il coefficiente pivotale H_{po} .
- 4 Si scambia la variabile primale P_q con la duale D_p .
- 5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
- 6 Si ridefiniscono i coefficienti della colonna pivot $q: H_{ig} = H_{ng} H_{ig}$, escluso il pivot H_{ng} .
- 7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ii} = H_{ii} H_{ii} + H_{pr}$
- 8 Si ridefiniscono i coefficienti della riga pivot $p: H_{pj} = -H_{pq} H_{pj}$, escluso il pivot H_{pq} .
- Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.

Giunti a questo punto, si individua la soluzione.

Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P, presenti sulla colonna di sinistra.

Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare *m*=6,*n*=4

$$\begin{bmatrix} \mathsf{P}_1 & \mathsf{P}_2 & \mathsf{P}_3 \end{bmatrix} & \begin{bmatrix} \mathsf{MIN} \\ \mathsf{D}_1 & \begin{bmatrix} \mathsf{H}_{11} & \mathsf{H}_{12} & \mathsf{H}_{13} \\ \mathsf{D}_2 & \mathsf{H}_{21} & \mathsf{H}_{22} & \mathsf{H}_{23} \\ \mathsf{D}_3 & \mathsf{H}_{31} & \mathsf{H}_{32} & \mathsf{H}_{33} \\ \mathsf{D}_4 & \mathsf{H}_{41} & \mathsf{H}_{42} & \mathsf{H}_{43} \\ \mathsf{D}_5 & \mathsf{H}_{51} & \mathsf{H}_{52} & \mathsf{H}_{53} \\ \mathsf{MAX} & \begin{bmatrix} \mathsf{H}_{61} & \mathsf{H}_{62} & \mathsf{H}_{63} \end{bmatrix} = \begin{bmatrix} \mathsf{MIN} \\ \mathsf{H}_{41} \\ \mathsf{H}_{42} & \mathsf{H}_{43} \\ \mathsf{H}_{64} \end{bmatrix}$$

SOLUZIONE DEL SIMPLESSO X=WAR Y=WRA Z=WRC T=WCD S=WDE R=WEG

Tableau con variabili non vincolate in segno

	rableau con variabili non vincolate in segno								
	[X	Υ	Z	Т	S	R	αbF		[Fb]
W_{AB} -	1	0	0	0	0	0	0	≥	-2
W_{AB} +	1	0	0	0	0	0	0	≤	2
W _{BA} -	0	1	0	0	0	0	0	≥	-2
W_{BA} +	0	1	0	0	0	0	0	≤	2
W_{BC} -	0	0	1	0	0	0	0	≥	-3/2
W _{BC} +	0	0	1	0	0	0	0	≤	3/2
W _{CD} -	0	0	0	1	0	0	0	≥	-3/2
W _{CD} +	0	0	0	1	0	0	0	≤	3/2
W_{DC} -	0	0	1	-2	0	0	-12	≥	-3/2
W_{DC} +	0	0	1	-2	0	0	-12	≤	3/2
W_{DE} -	0	0	0	0	1	0	0	≥	-2
W_{DE} +	0	0	0	0	1	0	0	≤	2
W_{ED} -	-1	-1	0	0	-1	0	9	≥	-2
W_{ED} +	-1	-1	0	0	-1	0	9	≤	2
W_{BF} -	0	-1	-1	0	0	0	0	≥	-2
W_{BF} +	0	-1	-1	0	0	0	0	≤	2
W_{FG} -	0	0	0	0	0	1	0	≥	-3/2
W_{FG} +	0	0	0	0	0	1	0	≤	3/2
W_{GH} -	0	1/2	1	-1	1/2	1	-7	≥	-3/2
W_{GH} +	0	1/2	1	-1	1/2	1	-7	≤	3/2
W_{HG} -	0	-1	-2	2	-1	-1	6	≥	-3/2
W_{HG} +	0	-1	-2	2	-1	-1	6	≤	3/2
W_{DH} -	0	0	-1	2	-1	0	12	≥	-2
W _{DH} +	0	0	-1	2	-1	0	12	≤	2
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	[X	Υ	Z	Т	S	R	αbF]		[Fb]
W_{AB} -	1	0	0	0	0	0	0	≥	-2
W_{AB} +	-1	0	0	0	0	0	0	≥	-2
W_{BA} -	0	1	0	0	0	0	0	≥	-2
W_{BA} +	0	-1	0	0	0	0	0	≥	-2
W_{BC} -	0	0	1	0	0	0	0	≥	-3/2
W_{BC} +	0	0	-1	0	0	0	0	≥	-3/2
W _{CD} -	0	0	0	1	0	0	0	≥	-3/2
W _{CD} +	0	0	0	-1	0	0	0	≥	-3/2
W _{DC} -	0	0	1	-2	0	0	-12	≥	-3/2
W_{DC} +	0	0	-1	2	0	0	12	≥	-3/2
W_{DE} -	0	0	0	0	1	0	0	≥	-2
W_{DE} +	0	0	0	0	-1	0	0	≥	-2
W_{ED} -	-1	-1	0	0	-1	0	9	≥	-2
W_{ED} +	1	1	0	0	1	0	-9	≥	-2
W_{BF} -	0	-1	-1	0	0	0	0	≥	-2
W_{BF} +	0	1	1	0	0	0	0	≥	-2
W_{FG} -	0	0	0	0	0	1	0	≥	-3/2
W_{FG} +	0	0	0	0	0	-1	0	≥	-3/2
W_{GH} -	0	1/2	1	-1	1/2	1	-7	≥	-3/2
W_{GH} +	0	-1/2	-1	1	-1/2	-1	7	≥	-3/2
W_{HG} -	0	-1	-2	2	-1	-1	6	≥	-3/2
W_{HG} +	0	1	2	-2	1	1	-6	≥	-3/2
W_{DH} -	0	0	-1	2	-1	0	12	≥	-2
W_{DH} +	0	0	1	-2	1	0	-12	≥	-2
Max	0	0	0	0	0	0	1	=	0

ableau con	varia	adili v	incola	ate in	segn	0				
[X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	

	Ĺ X+	Y+	Z+	T+	S+	R+	Х-	Y-	Z-	T-	S-	R-	αbF_{\perp}		[Fb]
W_{AB} -	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	[-2]
W_{AB} +	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-2
W_{BA} -	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-2
W_{BA} +	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-2
W_{BC} -	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2
W_{BC} +	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2
W_{CD} -	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2
W_{CD} +	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2
W_{DC} -	0	0	1	-2	0	0	0	0	-1	2	0	0	-12	≥	-3/2
W_{DC} +	0	0	-1	2	0	0	0	0	1	-2	0	0	12	≤	-3/2
W_{DE} -	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-2
W_{DE} +	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-2
W_{ED} -	-1	-1	0	0	-1	0	1	1	0	0	1	0	9	≥	-2
W _{ED} +	1	1	0	0	1	0	-1	-1	0	0	-1	0	-9	≤	-2
W_{BF} -	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-2
$W_{\rm BF}$ +	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-2
W_{FG} -	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-3/2
W_{FG} +	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-3/2
W_{GH} -	0	1/2	1	-1	1/2	1	0	-1/2	-1	1	-1/2	-1	-7	≥	-3/2
W_{GH} +	0	-1/2	-1	1	-1/2	-1	0	1/2	1	-1	1/2	1	7	≤	-3/2
W_{HG} -	0	-1	-2	2	-1	-1	0	1	2	-2	1	1	6	≥	-3/2
W_{HG} +	0	1	2	-2	1	1	0	-1	-2	2	-1	-1	-6	≤	-3/2
W_{DH} -	0	0	-1	2	-1	0	0	0	1	-2	1	0	12	≥	-2
W_{DH} +	0	0	1	-2	1	0	0	0	-1	2	-1	0	-12	≤	-2
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	[0]

Tableau a variabili negative su X- e limitate

	[X	Υ	Z	Т	S	R	$\alpha b F$	X-]	[Fb]
$\phi_{AB}\text{-}$	1	0	0	0	0	0	0	-1	≥	-2
ϕ_{AB} +	-1	0	0	0	0	0	0	1	≥	-2
$\phi_{\text{BA}}\text{-}$	0	1	0	0	0	0	0	-1	≥	-2
$\phi_{\text{BA}} \textbf{+}$	0	-1	0	0	0	0	0	1	≥	-2
$\phi_{\text{BC}}\text{-}$	0	0	1	0	0	0	0	-1	≥	-3/2
$\phi_{\text{BC}}\text{+}$	0	0	-1	0	0	0	0	1	≥	-3/2
ϕ_{CD}	0	0	0	1	0	0	0	-1	≥	-3/2
$\phi_{\text{CD}} \textbf{+}$	0	0	0	-1	0	0	0	1	≥	-3/2
$\phi_{\text{DC}}\text{-}$	0	0	1	-2	0	0	-12	1	≥	-3/2
$\phi_{DC} \textbf{+}$	0	0	-1	2	0	0	12	-1	≥	-3/2
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-2
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	≥	-2
$\phi_{\text{ED}}\text{-}$	-1	-1	0	0	-1	0	9	3	≥	-2
$\phi_{\text{ED}} \textbf{+}$	1	1	0	0	1	0	-9	-3	≥	-2
$\phi_{\text{BF}}\text{-}$	0	-1	-1	0	0	0	0	2	≥	-2
$\phi_{\text{BF}}\text{+}$	0	1	1	0	0	0	0	-2	≥	-2
$\phi_{\text{FG}}\text{-}$	0	0	0	0	0	1	0	-1	≥	-3/2
$\phi_{\text{FG}} \textbf{+}$	0	0	0	0	0	-1	0	1	≥	-3/2
$\phi_{\text{GH}}\text{-}$	0	1/2	1	-1	1/2	1	-7	-2	≥	-3/2
$\phi_{\text{GH}} \textbf{+}$	0	-1/2	-1	1	-1/2	-1	7	2	≥	-3/2
$\phi_{\text{HG}}\text{-}$	0	-1	-2	2	-1	-1	6	3	≥	-3/2
$\phi_{\text{HG}} \textbf{+}$	0	1	2	-2	1	1	-6	-3	≥	-3/2
$\phi_{\text{DH}}\text{-}$	0	0	-1	2	-1	0	12	0	≥	-2
$\phi_{\text{DH}} \textbf{+}$	0	0	1	-2	1	0	-12	0	≥	-2
L_{X}	0	0	0	0	0	0	0	-1	≥	-2
Max	0	0	0	0	0	0	1	0	=	0

Scam	bio pivo	otale 9-7	,								
	[X	Υ	Z	Т	S	R	ϕ_{DC} -	X-		[Fb]	
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	[-2]	
ϕ_{AB} +	-1	0	0	0	0	0	0	1	≥	-2	
$\phi_{\text{BA}}\text{-}$	0	1	0	0	0	0	0	-1	≥	-2	
ϕ_{BA} +	0	-1	0	0	0	0	0	1	≥	-2	
ϕ_{BC} -	0	0	1	0	0	0	0	-1	≥	-3/2	
ϕ_{BC} +	0	0	-1	0	0	0	0	1	≥	-3/2	
ϕ_{CD}	0	0	0	1	0	0	0	-1	≥	-3/2	
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2	
αbF	0	0	1/12	-1/6	0	0	-1/12	1/12	≥	-1/8	
$\phi_{\text{DC}}\text{+}$	0	0	0	0	0	0	-1	0	≥	-3	
ϕ_{DE} -	0	0	0	0	1	0	0	-1	≥	-2	
$\phi_{\text{DE}}\text{+}$	0	0	0	0	-1	0	0	1	≥	-2	
$\phi_{\text{ED}}\text{-}$	-1	-1	3/4	-3/2	-1	0	-3/4	15/4	≥	-25/8	
ϕ_{ED} +	1	1	-3/4	3/2	1	0	3/4	-15/4	≥	-7/8	
$\phi_{\text{BF}}\text{-}$	0	-1	-1	0	0	0	0	2	≥	-2	
$\phi_{\text{BF}}\text{+}$	0	1	1	0	0	0	0	-2	≥	-2	
ϕ_{FG} -	0	0	0	0	0	1	0	-1	≥	-3/2	
ϕ_{FG} +	0	0	0	0	0	-1	0	1	≥	-3/2	
$\phi_{\text{GH}}\text{-}$	0	1/2	5/12	1/6	1/2	1	7/12	-31/12	≥	-5/8	
ϕ_{GH} +	0	-1/2	-5/12	-1/6	-1/2	-1	-7/12	31/12	≥	-19/8	
ϕ_{HG} -	0	-1	-3/2	1	-1	-1	-1/2	7/2	≥	-9/4	
φ _{HG} +	0	1	3/2	-1	1	1	1/2	-7/2	≥	-3/4	
ϕ_{DH} -	0	0	0	0	-1	0	-1	1	≥	-7/2	
ϕ_{DH} +	0	0	0	0	1	0	1	-1	≥	-1/2	
L _X	0	0	0	0	0	0	0	-1	≥	-2	
Max	0	0	1/12	-1/6	0	0	-1/12	1/12	=	-1/8	

Scambio pivotale 14-3

	_ X	Υ	ϕ_{ED} +	Т	S	R	ϕ_{DC} -	X-		[Fb]
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	[-2]
$\phi_{AB}\text{+}$	-1	0	0	0	0	0	0	1	≥	-2
ϕ_{BA} -	0	1	0	0	0	0	0	-1	≥	-2
$\phi_{\text{BA}} \textbf{+}$	0	-1	0	0	0	0	0	1	≥	-2
ϕ_{BC} -	4/3	4/3	-4/3	2	4/3	0	1	-6	≥	-8/3
ϕ_{BC} +	-4/3	-4/3	4/3	-2	-4/3	0	-1	6	≥	-1/3
$\phi_{\text{CD}}\text{-}$	0	0	0	1	0	0	0	-1	≥	-3/2
$\phi_{\text{CD}}\text{+}$	0	0	0	-1	0	0	0	1	≥	-3/2
αbF	1/9	1/9	-1/9	0	1/9	0	0	-1/3	≥	-2/9
$\phi_{\text{DC}}\text{+}$	0	0	0	0	0	0	-1	0	≥	-3
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-2
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	≥	-2
$\phi_{\text{ED}}\text{-}$	0	0	-1	0	0	0	0	0	≥	-4
Z	4/3	4/3	-4/3	2	4/3	0	1	-5	≥	-7/6
$\phi_{\text{BF}}\text{-}$	-4/3	-7/3	4/3	-2	-4/3	0	-1	7	≥	-5/6
ϕ_{BF} +	4/3	7/3	-4/3	2	4/3	0	1	-7	≥	-19/6
ϕ_{FG} -	0	0	0	0	0	1	0	-1	≥	-3/2
ϕ_{FG} +	0	0	0	0	0	-1	0	1	≥	-3/2
$\phi_{\text{GH}}\text{-}$	5/9	19/18	-5/9	1	19/18	1	1	-14/3	≥	-10/9
$\phi_{\text{GH}} \textbf{+}$	-5/9	-19/18	5/9	-1	-19/18	-1	-1	14/3	≥	-17/9
$\phi_{\text{HG}}\text{-}$	-2	-3	2	-2	-3	-1	-2	11	≥	-1/2
ϕ_{HG} +	2	3	-2	2	3	1	2	-11	≥	-5/2
$\phi_{\text{DH}}\text{-}$	0	0	0	0	-1	0	-1	1	≥	-7/2
$\phi_{\text{DH}}\text{+}$	0	0	0	0	1	0	1	-1	≥	-1/2
L_{x}	0	0	0	0	0	0	0	-1	≥	-2
Max	_ 1/9	1/9	-1/9	0	1/9	0	0	-1/3	=	2/9]

Scambio pivotale 6-1

Ooaiii										
	φ_{BC} +	Υ	$\phi_{\text{ED}}\text{+}$	Т	S	R	ϕ_{DC}	X-]		[Fb]
ϕ_{AB} -	-3/4	-1	1	-3/2	-1	0	-3/4	7/2	\geq	-9/4
ϕ_{AB} +	3/4	1	-1	3/2	1	0	3/4	-7/2	\geq	-7/4
$\phi_{\text{BA}}\text{-}$	0	1	0	0	0	0	0	-1	\geq	-2
$\phi_{\text{BA}}\text{+}$	0	-1	0	0	0	0	0	1	\geq	-2
ϕ_{BC} -	-1	0	0	0	0	0	0	0	\geq	-3
Χ	-3/4	-1	1	-3/2	-1	0	-3/4	9/2	\geq	-1/4
ϕ_{CD}	0	0	0	1	0	0	0	-1	\geq	-3/2
ϕ_{CD} +	0	0	0	-1	0	0	0	1	\geq	-3/2
$\alpha b F$	-1/12	0	0	-1/6	0	0	-1/12	1/6	\geq	-1/4
ϕ_{DC} +	0	0	0	0	0	0	-1	0	\geq	-3
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	\geq	-2
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	\geq	-2
$\phi_{\text{ED}}\text{-}$	0	0	-1	0	0	0	0	0	\geq	-4
Z	-1	0	0	0	0	0	0	1	\geq	-3/2
$\phi_{\text{BF}}\text{-}$	1	-1	0	0	0	0	0	1	\geq	-1/2
$\phi_{\text{BF}}\text{+}$	-1	1	0	0	0	0	0	-1	\geq	-7/2
$\phi_{\text{FG}}\text{-}$	0	0	0	0	0	1	0	-1	\geq	-3/2
$\phi_{\text{FG}}\text{+}$	0	0	0	0	0	-1	0	1	\geq	-3/2
ϕ_{GH} -	-5/12	1/2	0	1/6	1/2	1	7/12	-13/6	\geq	-5/4
$\phi_{\text{GH}} \textbf{+}$	5/12	-1/2	0	-1/6	-1/2	-1	-7/12	13/6	\geq	-7/4
$\phi_{\text{HG}}\text{-}$	3/2	-1	0	1	-1	-1	-1/2	2	\geq	0
$\phi_{\text{HG}} \textbf{+}$	-3/2	1	0	-1	1	1	1/2	-2	\geq	-3
$\phi_{\text{DH}}\text{-}$	0	0	0	0	-1	0	-1	1	\geq	-7/2
$\phi_{\text{DH}} \textbf{+}$	0	0	0	0	1	0	1	-1	\geq	-1/2
L_{X}	0	0	0	0	0	0	0	-1	\geq	-2
Max	-1/12	0	0	-1/6	0	0	-1/12	1/6	=	-1/4

Scambio pivotale 2-8

	φ _{BC} +	Υ	φ _{ED} +	Т	S	R	φ _{DC} -	ϕ_{AB} +		[Fb]	
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	-4	
X-	3/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1/2	
ϕ_{BA} -	-3/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2	
ϕ_{BA} +	3/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-5/2	
ϕ_{BC} -	-1	0	0	0	0	0	0	0	≥	-3	
Χ	3/14	2/7	-2/7	3/7	2/7	0	3/14	-9/7	≥	-5/2	
φ _{CD} -	-3/14	-2/7	2/7	4/7	-2/7	0	-3/14	2/7	≥	-1	
ϕ_{CD} +	3/14	2/7	-2/7	-4/7	2/7	0	3/14	-2/7	≥	-2	
αbF	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	≥	-1/3	
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3	
ϕ_{DE} -	-3/14	-2/7	2/7	-3/7	5/7	0	-3/14	2/7	≥	-3/2	
ϕ_{DE} +	3/14	2/7	-2/7	3/7	-5/7	0	3/14	-2/7	≥	-5/2	
φ _{ED} -	0	0	-1	0	0	0	0	0	≥	-4	
Z	-11/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-2	
ϕ_{BF} -	17/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1	
ϕ_{BF} +	-17/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3	
ϕ_{FG} -	-3/14	-2/7	2/7	-3/7	-2/7	1	-3/14	2/7	≥	-1	
φ _{FG} +	3/14	2/7	-2/7	3/7	2/7	-1	3/14	-2/7	≥	-2	
φ _{GH} -	-37/42	-5/42	13/21	-16/21	-5/42	1	5/42	13/21	≥	-1/6	
φ _{GH} +	37/42	5/42	-13/21	16/21	5/42	-1	-5/42	-13/21	≥	-17/6	
ϕ_{HG} -	27/14	-3/7	-4/7	13/7	-3/7	-1	-1/14	-4/7	≥	-1	
φ _{HG} +	-27/14	3/7	4/7	-13/7	3/7	1	1/14	4/7	≥	-2	
φ _{DH} -	3/14	2/7	-2/7	3/7	-5/7	0	-11/14	-2/7	≥	-4	
ϕ_{DH} +	-3/14	-2/7	2/7	-3/7	5/7	0	11/14	2/7	≥	0	
L _X	-3/14	-2/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2	
Max	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	=	-1/3	

Scambio pivotale 24-2

	Г.,			-		ь			1	Г г ь -
	φ_{BC} +	φ_{DH} +	φ_{ED} +	Т	S	R	ϕ_{DC} -	φ _{AB} +]	Fb -
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	-4
Х-	0	-1	0	0	1	0	1	0	≥	-1/2
ϕ_{BA} -	-3/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3/2
ϕ_{BA} +	3/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-5/2
ϕ_{BC} -	-1	0	0	0	0	0	0	0	≥	-3
Χ	0	-1	0	0	1	0	1	-1	≥	-5/2
$\phi_{\text{CD}}\text{-}$	0	1	0	1	-1	0	-1	0	≥	-1
ϕ_{CD} +	0	-1	0	-1	1	0	1	0	≥	-2
αbF	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	≥	-1/3
$\phi_{\text{DC}}\text{+}$	0	0	0	0	0	0	-1	0	≥	-3
$\phi_{\text{DE}}\text{-}$	0	1	0	0	0	0	-1	0	≥	-3/2
ϕ_{DE} +	0	-1	0	0	0	0	1	0	≥	-5/2
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4
Z	-1	-1	0	0	1	0	1	0	≥	-2
ϕ_{BF} -	7/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-1
ϕ_{BF} +	-7/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3
ϕ_{FG} -	0	1	0	0	-1	1	-1	0	≥	-1
$\phi_{\text{FG}}\text{+}$	0	-1	0	0	1	-1	1	0	≥	-2
ϕ_{GH} -	-19/24	5/12	1/2	-7/12	-5/12	1	-5/24	1/2	≥	-1/6
ϕ_{GH} +	19/24	-5/12	-1/2	7/12	5/12	-1	5/24	-1/2	≥	-17/6
ϕ_{HG} -	9/4	3/2	-1	5/2	-3/2	-1	-5/4	-1	≥	-1
$\phi_{\text{HG}}\text{+}$	-9/4	-3/2	1	-5/2	3/2	1	5/4	1	≥	-2
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-4
Υ	-3/4	-7/2	1	-3/2	5/2	0	11/4	1	≥	0
L_{X}	0	1	0	0	-1	0	-1	0	≥	-3/2
Max	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	=	-1/3

Scambio pivotale 19-5

	[φ _{BC} +	φ _{DH} +	φ _{ED} +	Т	φ _{GH} -	R	φ _{DC} -	φ _{AB} + _		[Fb]	
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	「 -4]	
X-	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-9/10	
ϕ_{BA} -	-18/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-21/10	
ϕ_{BA} +	18/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-19/10	
ϕ_{BC} -	-1	0	0	0	0	0	0	0	≥	-3	
Χ	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	1/5	≥	-29/10	
φ _{CD} -	19/10	0	-6/5	12/5	12/5	-12/5	-1/2	-6/5	≥	-3/5	
ϕ_{CD} +	-19/10	0	6/5	-12/5	-12/5	12/5	1/2	6/5	≥	-12/5	
αbF	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	≥	-2/5	
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3	
ϕ_{DE} -	0	1	0	0	0	0	-1	0	≥	-3/2	
ϕ_{DE} +	0	-1	0	0	0	0	1	0	≥	-5/2	
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4	
Z	-29/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-12/5	
ϕ_{BF} -	23/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-2/5	
ϕ_{BF} +	-23/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-18/5	
ϕ_{FG} -	19/10	0	-6/5	7/5	12/5	-7/5	-1/2	-6/5	≥	-3/5	
φ _{FG} +	-19/10	0	6/5	-7/5	-12/5	7/5	1/2	6/5	≥	-12/5	
S	-19/10	1	6/5	-7/5	-12/5	12/5	-1/2	6/5	≥	-2/5	
ϕ_{GH} +	0	0	0	0	-1	0	0	0	≥	-3	
ϕ_{HG} -	51/10	0	-14/5	23/5	18/5	-23/5	-1/2	-14/5	≥	-2/5	
φ _{HG} +	-51/10	0	14/5	-23/5	-18/5	23/5	1/2	14/5	≥	-13/5	
φ _{DH} -	0	-1	0	0	0	0	0	0	≥	-4	
Υ	-11/2	-1	4	-5	-6	6	3/2	4	≥	-1	
L_{x}	19/10	0	-6/5	7/5	12/5	-12/5	-1/2	-6/5	≥	-11/10	
Max	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	=	2/5]	

Scambio pivotale 21-6

	φ _{BC} +	ϕ_{DH} +	ϕ_{ED} +	Т	ϕ_{GH} -	ϕ_{HG} -	ϕ_{DC} -	φ_{AB} +		[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	-1	\geq	-4
X-	35/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	\geq	-51/46
ϕ_{BA} -	9/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	\geq	-111/46
ϕ_{BA} +	-9/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	\geq	-73/46
ϕ_{BC} -	-1	0	0	0	0	0	0	0	\geq	-3
Χ	35/46	0	-6/23	1	-12/23	-12/23	11/46	-29/23	\geq	-143/46
ϕ_{CD} -	-35/46	0	6/23	0	12/23	12/23	-11/46	6/23	\geq	-9/23
ϕ_{CD} +	35/46	0	-6/23	0	-12/23	-12/23	11/46	-6/23	\geq	-60/23
αbF	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	≥	-10/23
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3
ϕ_{DE} -	0	1	0	0	0	0	-1	0	≥	-3/2
ϕ_{DE} +	0	-1	0	0	0	0	1	0	\geq	-5/2
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4
Z	-11/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	≥	-60/23
ϕ_{BF} -	14/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	≥	-2/23
ϕ_{BF} +	-14/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	≥	-90/23
φ _{FG} -	8/23	0	-8/23	0	30/23	7/23	-8/23	-8/23	\geq	-11/23
ϕ_{FG} +	-8/23	0	8/23	0	-30/23	-7/23	8/23	8/23	≥	-58/23
S	35/46	1	-6/23	1	-12/23	-12/23	-35/46	-6/23	≥	-14/23
ϕ_{GH} +	0	0	0	0	-1	0	0	0	≥	-3
R	51/46	0	-14/23	1	18/23	-5/23	-5/46	-14/23	\geq	-2/23
ϕ_{HG} +	0	0	0	0	0	-1	0	0	\geq	-3
ϕ_{DH} -	0	-1	0	0	0	0	0	0	\geq	-4
Υ	53/46	-1	8/23	1	-30/23	-30/23	39/46	8/23	≥	-35/23
L_{X}	-35/46	0	6/23	-1	12/23	12/23	-11/46	6/23	≥	-41/46
Max	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	=	-10/23

Scambio pivotale 7-1

Ocam											
	φ _{CD} -	$\phi_{\text{DH}} \textbf{+}$	ϕ_{ED} +	Т	ϕ_{GH} -	ϕ_{HG} -	ϕ_{DC} -	φ_{AB} +		[Fb]	
ϕ_{AB} -	0	0	0	0	0	0	0	-1	\geq	[-4]	
X-	-1	0	0	1	0	0	0	0	\geq	-3/2	
ϕ_{BA} -	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	\geq	-183/70	
ϕ_{BA} +	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	\geq	-97/70	
ϕ_{BC} -	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	\geq	-87/35	
Χ	-1	0	0	1	0	0	0	-1	\geq	-7/2	
$\phi_{\text{BC}}\text{+}$	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	\geq	-18/35	
ϕ_{CD} +	-1	0	0	0	0	0	0	0	\geq	-3	
$\alpha b F$	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	\geq	-16/35	
ϕ_{DC} +	0	0	0	0	0	0	-1	0	\geq	-3	
ϕ_{DE} -	0	1	0	0	0	0	-1	0	\geq	-3/2	
$\phi_{\text{DE}} \textbf{+}$	0	-1	0	0	0	0	1	0	≥	-5/2	
$\phi_{\text{ED}}\text{-}$	0	0	-1	0	0	0	0	0	\geq	-4	
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	\geq	-87/35	
ϕ_{BF} -	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5	
ϕ_{BF} +	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	\geq	-18/5	
ϕ_{FG} -	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	\geq	-23/35	
ϕ_{FG} +	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	\geq	-82/35	
S	-1	1	0	1	0	0	-1	0	≥	-1	
ϕ_{GH} +	0	0	0	0	-1	0	0	0	≥	-3	
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	\geq	-23/35	
ϕ_{HG} +	0	0	0	0	0	-1	0	0	\geq	-3	
ϕ_{DH} -	0	-1	0	0	0	0	0	0	\geq	-4	
Υ	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35	
L_{x}	1	0	0	-1	0	0	0	0	≥	-1/2	
Max	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	-16/35	

Tableau finale

	φ _{CD} -	ϕ_{DH} +	ϕ_{ED} +	Т	ϕ_{GH} -	φ _{HG} -	φ _{DC} -	ϕ_{AB} +		[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	-4
Χ-	-1	0	0	1	0	0	0	0	≥	-3/2
ϕ_{BA} -	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	≥	-183/70
ϕ_{BA} +	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	≥	-97/70
φ _{BC} -	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	≥	-87/35
Χ	-1	0	0	1	0	0	0	-1	≥	-7/2
ϕ_{BC} +	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	≥	-18/35
ϕ_{CD} +	-1	0	0	0	0	0	0	0	≥	-3
αbF	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	≥	-16/35
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3
ϕ_{DE} -	0	1	0	0	0	0	-1	0	≥	-3/2
$\phi_{\text{DE}} \textbf{+}$	0	-1	0	0	0	0	1	0	≥	-5/2
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	≥	-87/35
ϕ_{BF} -	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5
ϕ_{BF} +	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	≥	-18/5
ϕ_{FG} -	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	≥	-23/35
ϕ_{FG} +	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	≥	-82/35
S	-1	1	0	1	0	0	-1	0	≥	-1
ϕ_{GH} +	0	0	0	0	-1	0	0	0	≥	-3
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	≥	-23/35
ϕ_{HG} +	0	0	0	0	0	-1	0	0	≥	-3
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-4
Υ	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35
L_{x}	1	0	0	-1	0	0	0	0	≥	-1/2
Max	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	16/35]

Vettori soluzione della programmazione lineare

	[X	Υ	Z	Т	S	R	αbF	Χ-]	[Fb]	
ϕ_{AB} -	0	0	0	0	0	0	0	0] ≥	[0]	
$\phi_{AB}\textbf{+}$	0	0	0	0	0	0	0	0	≥	1/35	
$\phi_{\text{BA}}\text{-}$	0	0	0	0	0	0	0	0	≥	0	
ϕ_{BA} +	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{BC}}\text{-}$	0	0	0	0	0	0	0	0	≥	0	
ϕ_{BC} +	0	0	0	0	0	0	0	0	≥	0	
ϕ_{CD} -	0	0	0	0	0	0	0	0	≥	2/35	
ϕ_{CD} +	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{DC}}\text{-}$	0	0	0	0	0	0	0	0	≥	2/35	
$\phi_{\text{DC}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{DE}}\text{-}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{ED}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	1/35	
$\phi_{\text{BF}}\text{-}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{BF}}\text{+}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{FG}}\text{-}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{FG}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0	
ϕ_{GH} -	0	0	0	0	0	0	0	0	≥	2/35	
$\phi_{\text{GH}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{HG}}\text{-}$	0	0	0	0	0	0	0	0	≥	2/35	
$\phi_{\text{HG}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{DH}}\text{-}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{DH}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0	
L_{X}	0	0	0	0	0	0	0	0	≥	0	
Max	7/2	74/35	87/35	0	1	23/35	16/35	3/2	=	16/35	

Variabili soluzione dedotto il valore X-

Variabili soluzione differenza tra rotazioni

ϕ_{AB}	1/35
ϕ_{BA}	0
ϕ_{BC}	0
ϕ_{CD}	-2/35
ϕ_{DC}	-2/35
ϕ_{DE}	0
ϕ_{ED}	1/35
ϕ_{BF}	0
ϕ_{FG}	0
ϕ_{GH}	-2/35
ϕ_{HG}	-2/35
ϕ_{DH}	0

REAZIONI Fattore di collasso = 16/35

 $H_{\Lambda} = -61/70F$

 $V_{\Delta} = 11/7F$

 $W_A = 2Fb$

 $H_{r} = -1/2F$

 $V_E = 3F$

 $W_F = 2Fb$

$$\begin{array}{llll} H_{FG} = 7/6F & H_{GH} = 7/6F & H_{HD} = 7/6F \\ V_{FG} = 23/70F & V_{GH} = -3/2F & V_{HD} = -3/2F \\ W_{FG} = -59/70Fb & W_{GH} = -3/2Fb & W_{HD} = 3/2Fb \\ H_{GF} = -7/6F & H_{HG} = -7/6F & H_{DH} = -7/6F \\ V_{GF} = -23/70F & V_{HG} = 3/2F & V_{DH} = 3/2F \\ W_{GF} = 3/2Fb & W_{HG} = -3/2Fb & W_{DH} = 2Fb \end{array}$$

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_B = 3/35\delta$	$u_{CCB} = 3/35\delta$	$u_D = 3/35\delta$	$u_{EED} = 0$
$V_{AAB} = 0$	$V_B = 0$	$V_{CCB} = -2/35\delta$	$V_D = 0$	$V_{EED} = 0$
$\varphi_{\Delta\Delta B} = -1/35\delta/b$	$\varphi_{\rm B} = -1/35\delta/b$	$\varphi_{CCB} = -1/35\delta/b$	$\phi_{D} = 1/35\delta/b$	$\phi_{E} = -1/35\delta/b$

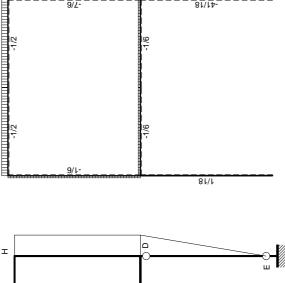
$u_F = 6/35\delta$	$u_{GGF} = 6/35\delta$	$u_{HHG} = 6/35\delta$
$V_F = 0$	$V_{GGF} = -2/35\delta$	$V_{HHG} = 0$
$\varphi_F = -1/35\delta/b$	$\phi_{GGF} = -1/35\delta/b$	$\varphi_{HHD} = 1/35\delta/b$

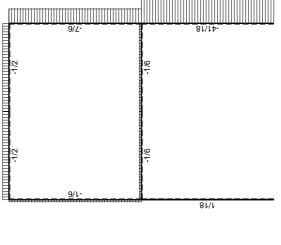
SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$	$u_{BBC} = 3/35\delta$	$u_{CCD} = 3/35\delta$	$u_{DDE} = 3/35\delta$	$u_{BBF} = 3/35\delta$
$V_{AAB} = 0$ $\phi_{AAB} = -1/35\delta/b$		$v_{CCD} = -2/35\delta$ $\phi_{CCD} = 1/35\delta/b$	$v_{DDE} = 0$ $\phi_{DDE} = -1/35\delta/b$	$V_{BBF} = 0$ $\phi_{BBF} = -1/35\delta/b$
Ψ _{AAB} = -1/330/b	Ψ _{BBC} = -1/330/b	Ψ _{CCD} = 1/330/b	Ψ _{DDE} = -1/330/b	Ψ _{BBF} = -1/330/b

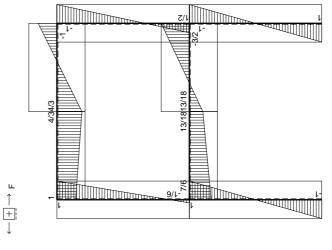
$$\begin{array}{lll} u_{FFG} = 6/35\delta & u_{GGH} = 6/35\delta & u_{HHD} = 6/35\delta \\ v_{FFG} = 0 & v_{GGH} = -2/35\delta & v_{HHD} = 0 \\ \phi_{FFG} = -1/35\delta/b & \phi_{GGH} = 1/35\delta/b & \phi_{HHD} = -1/35\delta/b \end{array}$$

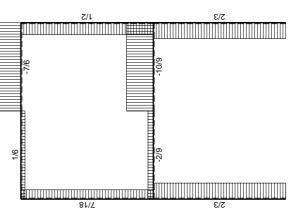
EQUILIBRIO Nome:

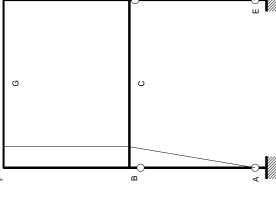


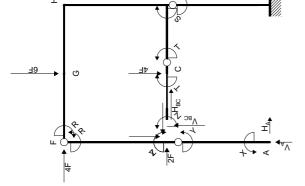












EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

 $3H_Ab - 4V_Ab = -Xb + Sb - 8Fb$

Rotazione intorno a C: aste CB

Rotazione intorno a F: aste FB BA $-2V_{BC}b = -Zb + Tb$

 $6H_Ab - 3H_{BC}b = -Xb + Zb - Rb - 6Fb$ Rotazione intorno a B: aste BA

 $3H_Ab = -Xb - Yb$

Matrice di equilibrio

$$\begin{cases} H_Ab & V_Ab & H_{BC}b & V_{BC}b \end{bmatrix} & [Xb Yb Zb Tb Sb Rb F] \\ \phi_{DE} & 3 & -4 & 0 & 0 \\ \phi_{CD} & 0 & 0 & -2 \\ \phi_{FB} & 6 & 0 & -3 & 0 \\ \phi_{BA} & 3 & 0 & 0 & 0 \end{bmatrix} = \begin{vmatrix} -1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 & 0 \end{vmatrix}$$

Soluzione del sistema

$$\begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1/3 & -1/3 & -1/3 & 0 & 0 & 0 & 0 & 0 \\ V_{BC}b & 0 & 0 & 1/2 & -1/2 & 0 & 0 & 0 \\ H_{BC}b & -1/3 & -2/3 & -1/3 & 0 & 0 & 1/3 & 2 \\ V_Ab & 0 & -1/4 & 0 & 0 & -1/4 & 0 & 2 \end{bmatrix}$$

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.11.05.11

PROGRAMMAZIONE LINEARE

Sia H_{ii} la matrice del simplesso, con m righe e n colonne.

Siano P_i le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.

Siano a riga m i coefficienti della funzione obiettivo primale $\max \Sigma_i H_{mi} P_{ii}$ $1 \le j < n$.

Siano a colonna n i coefficienti della funzione obiettivo duale $min \Sigma_i H_{in} D_{it}$ $1 \le i < m$.

Sequenza di operazioni pivotali:

- 1 Sia q ($1 \le q < n$) la colonna pivot con massimo valore H_{mi} in riga m.
- 2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{io} , che minimizza il rapporto H_{in}/H_{io} .
- 3 Si ottiene il coefficiente pivotale H_{po}
- 4 Si scambia la variabile primale P_q con la duale D_p .
- 5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
- 6 Si ridefiniscono i coefficienti della colonna pivot $q: H_{ig} = H_{ng} H_{ig}$, escluso il pivot H_{ng} .
- 7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ii} = H_{ii} H_{ii} + H_{pr}$
- 8 Si ridefiniscono i coefficienti della riga pivot $p: H_{oi} = -H_{oa} H_{oi}$, escluso il pivot H_{oc} .
- Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.

Giunti a questo punto, si individua la soluzione.

Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_i presenti sulla colonna di sinistra.

Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare *m*=6,*n*=4

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{BC}$ $T=W_{CD}$ $S=W_{DE}$ $R=W_{FB}$

Tableau con variabili non vincolate in segno

Tablea	_	ii vaiid	ו וווטג	ion vi	licola		segnic			
	[X	Υ	Z	Т	S	R	$\alpha bF_{}$		[Fb]	
W_{AB} -	¹	0	0	0	0	0	0	≥	-1	
W_{AB} +	1	0	0	0	0	0	0	≤	1	
W _{BA} -	0	1	0	0	0	0	0	≥	-1	
W_{BA} +	0	1	0	0	0	0	0	≤	1	
W_{BC} -	0	0	1	0	0	0	0	≥	-3/2	
W _{BC} +	0	0	1	0	0	0	0	≤	3/2	
W_{CD} -	0	0	0	1	0	0	0	≥	-3/2	
W _{CD} +	0	0	0	1	0	0	0	≤	3/2	
W_{DC} -	0	0	1	-2	0	0	-8	≥	-3/2	
W_{DC} +	0	0	1	-2	0	0	-8	≤	3/2	
W_{DE} -	0	0	0	0	1	0	0	≥	-1	
W_{DE} +	0	0	0	0	1	0	0	≤	1	
W_{ED} -	-1	-1	0	0	-1	0	18	≥	-1	
W_{ED} +	-1	-1	0	0	-1	0	18	≤	1	
W_{BF} -	0	-1	-1	0	0	0	0	≥	-1	
W_{BF} +	0	-1	-1	0	0	0	0	≤	1	
W_{FB} -	0	0	0	0	0	1	0	≥	-1	
W_{FB} +	0	0	0	0	0	1	0	≤	1	
W_{GH} -	0	1/2	1	-1	1/2	-1	-4	≥	-3/2	
W_{GH} +	0	1/2	1	-1	1/2	-1	-4	≤	3/2	
W _{HD} -	0	1	2	-2	1	-1	4	≥	-1	
W_{HD} +	0	1	2	-2	1	-1	4	≤	1	
W_{DH} -	0	0	-1	2	-1	0	8	≥	-1	
W_{DH} +	0	0	-1	2	-1	0	8	≤	1	
Max	0	0	0	0	0	0	1 _	=	0	

Tableau con variabili non vincolate in segno

	[X	Υ	Z	Т	S	R	αbF]		[Fb]
W_{AB} -	1	0	0	0	0	0	0	≥	「-1 - 1
W_{AB} +	-1	0	0	0	0	0	0	≥	-1
W_{BA} -	0	1	0	0	0	0	0	≥	-1
W_{BA} +	0	-1	0	0	0	0	0	≥	-1
W_{BC} -	0	0	1	0	0	0	0	≥	-3/2
W_{BC} +	0	0	-1	0	0	0	0	≥	-3/2
W _{CD} -	0	0	0	1	0	0	0	≥	-3/2
W_{CD} +	0	0	0	-1	0	0	0	≥	-3/2
W_{DC} -	0	0	1	-2	0	0	-8	≥	-3/2
W_{DC} +	0	0	-1	2	0	0	8	≥	-3/2
W_{DE} -	0	0	0	0	1	0	0	≥	-1
W_{DE} +	0	0	0	0	-1	0	0	≥	-1
W_{ED} -	-1	-1	0	0	-1	0	18	≥	-1
W_{ED} +	1	1	0	0	1	0	-18	≥	-1
W _{BF} -	0	-1	-1	0	0	0	0	≥	-1
W_{BF} +	0	1	1	0	0	0	0	≥	-1
W_{FB} -	0	0	0	0	0	1	0	≥	-1
W_{FB} +	0	0	0	0	0	-1	0	≥	-1
W_{GH} -	0	1/2	1	-1	1/2	-1	-4	≥	-3/2
W_{GH} +	0	-1/2	-1	1	-1/2	1	4	≥	-3/2
W_{HD} -	0	1	2	-2	1	-1	4	≥	-1
W_{HD} +	0	-1	-2	2	-1	1	-4	≥	-1
W_{DH} -	0	0	-1	2	-1	0	8	≥	-1
W_{DH} +	0	0	1	-2	1	0	-8	≥	-1
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili vincolate in segno													
	[X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF
W_{AB} -	1	0	0	0	0	0	-1	0	0	0	0	0	0

	[X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		[Fb]	
W_{AB} -	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	[-1]	
W _{AB} +	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1	
W _{BA} -	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1	
W _{BA} +	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1	
W_{BC} -	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2	
W_{BC} +	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2	
W_{CD} -	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2	
W _{CD} +	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2	
W_{DC} -	0	0	1	-2	0	0	0	0	-1	2	0	0	-8	≥	-3/2	
W_{DC} +	0	0	-1	2	0	0	0	0	1	-2	0	0	8	≤	-3/2	
W_{DE} -	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1	
W_{DE} +	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1	
W_{ED} -	-1	-1	0	0	-1	0	1	1	0	0	1	0	18	≥	-1	
W_{ED} +	1	1	0	0	1	0	-1	-1	0	0	-1	0	-18	≤	-1	
W_{BF} -	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-1	
W_{BF} +	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-1	
W_{FB} -	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-1	
W_{FR} +	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-1	
W_{GH} -	0	1/2	1	-1	1/2	-1	0	-1/2	-1	1	-1/2	1	-4	≥	-3/2	
W_{GH} +	0	-1/2	-1	1	-1/2	1	0	1/2	1	-1	1/2	-1	4	≤	-3/2	
W_{HD} -	0	1	2	-2	1	-1	0	-1	-2	2	-1	1	4	≥	-1	
W_{HD} +	0	-1	-2	2	-1	1	0	1	2	-2	1	-1	-4	≤	-1	
W_{DH} -	0	0	-1	2	-1	0	0	0	1	-2	1	0	8	≥	-1	
W_{DH} +	0	0	1	-2	1	0	0	0	-1	2	-1	0	-8	≤	-1	
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0	

08.06.11

Tableau a variabili negative su X- e limitate

	[X	Υ	Z	Т	S	R	$\alpha b F$	X		[Fb]
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	-1
ϕ_{AB} +	-1	0	0	0	0	0	0	1	≥	-1
ϕ_{BA} -	0	1	0	0	0	0	0	-1	≥	-1
ϕ_{BA} +	0	-1	0	0	0	0	0	1	≥	-1
ϕ_{BC} -	0	0	1	0	0	0	0	-1	≥	-3/2
ϕ_{BC} +	0	0	-1	0	0	0	0	1	≥	-3/2
ϕ_{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2
ϕ_{DC} -	0	0	1	-2	0	0	-8	1	≥	-3/2
ϕ_{DC} +	0	0	-1	2	0	0	8	-1	≥	-3/2
ϕ_{DE} -	0	0	0	0	1	0	0	-1	≥	-1
ϕ_{DE} +	0	0	0	0	-1	0	0	1	≥	-1
ϕ_{ED} -	-1	-1	0	0	-1	0	18	3	≥	-1
ϕ_{ED} +	1	1	0	0	1	0	-18	-3	≥	-1
ϕ_{BF} -	0	-1	-1	0	0	0	0	2	≥	-1
ϕ_{BF} +	0	1	1	0	0	0	0	-2	≥	-1
ϕ_{FB} -	0	0	0	0	0	1	0	-1	≥	-1
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1
ϕ_{GH} -	0	1/2	1	-1	1/2	-1	-4	0	≥	-3/2
ϕ_{GH} +	0	-1/2	-1	1	-1/2	1	4	0	≥	-3/2
ϕ_{HD} -	0	1	2	-2	1	-1	4	-1	≥	-1
ϕ_{HD} +	0	-1	-2	2	-1	1	-4	1	≥	-1
ϕ_{DH} -	0	0	-1	2	-1	0	8	0	≥	-1
ϕ_{DH} +	0	0	1	-2	1	0	-8	0	≥	-1
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	0	0	0	0	0	1	0 _	=	0

Scam	bio piv	otale 1	4-7							
	X	Υ	Z	Т	S	R	ϕ_{ED} +	X-		[Fb]
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	[-1]
φ _{AB} +	-1	0	0	0	0	0	0	1	≥	-1
ϕ_{BA} -	0	1	0	0	0	0	0	-1	≥	-1
ϕ_{BA} +	0	-1	0	0	0	0	0	1	≥	-1
ϕ_{BC} -	0	0	1	0	0	0	0	-1	≥	-3/2
ϕ_{BC} +	0	0	-1	0	0	0	0	1	≥	-3/2
$\phi_{\text{CD}}\text{-}$	0	0	0	1	0	0	0	-1	≥	-3/2
$\phi_{\text{CD}}\text{+}$	0	0	0	-1	0	0	0	1	≥	-3/2
ϕ_{DC}	-4/9	-4/9	1	-2	-4/9	0	4/9	7/3	≥	-19/18
ϕ_{DC} +	4/9	4/9	-1	2	4/9	0	-4/9	-7/3	≥	-35/18
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-1
$\phi_{\text{DE}}\text{+}$	0	0	0	0	-1	0	0	1	≥	-1
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
αbF	1/18	1/18	0	0	1/18	0	-1/18	-1/6	≥	-1/18
ϕ_{BF} -	0	-1	-1	0	0	0	0	2	≥	-1
ϕ_{BF} +	0	1	1	0	0	0	0	-2	≥	-1
$\phi_{\text{FB}}\text{-}$	0	0	0	0	0	1	0	-1	≥	-1
$\phi_{\text{FB}}\text{+}$	0	0	0	0	0	-1	0	1	≥	-1
$\phi_{\text{GH}}\text{-}$	-2/9	5/18	1	-1	5/18	-1	2/9	2/3	≥	-23/18
$\phi_{\text{GH}} \textbf{+}$	2/9	-5/18	-1	1	-5/18	1	-2/9	-2/3	≥	-31/18
ϕ_{HD} -	2/9	11/9	2	-2	11/9	-1	-2/9	-5/3	≥	-11/9
$\phi_{\text{HD}}\text{+}$	-2/9	-11/9	-2	2	-11/9	1	2/9	5/3	≥	-7/9
ϕ_{DH} -	4/9	4/9	-1	2	-5/9	0	-4/9	-4/3	≥	-13/9
$\phi_{\text{DH}}\text{+}$	-4/9	-4/9	1	-2	5/9	0	4/9	4/3	≥	-5/9
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	1/18	1/18	0	0	1/18	0	-1/18	-1/6	=	-1/18

Scambio pivotale 2-1

	[φ _{AB} +	Υ	Z	Т	S	R	ϕ_{ED} +	X-]		[Fb]	
ϕ_{AB} -	-1	0	0	0	0	0	0	0	\geq	「 -2]	
Χ	-1	0	0	0	0	0	0	1	\geq	-1	
ϕ_{BA} -	0	1	0	0	0	0	0	-1	≥	-1	
ϕ_{BA} +	0	-1	0	0	0	0	0	1	\geq	-1	
ϕ_{BC} -	0	0	1	0	0	0	0	-1	\geq	-3/2	
ϕ_{BC} +	0	0	-1	0	0	0	0	1	≥	-3/2	
ϕ_{CD}	0	0	0	1	0	0	0	-1	≥	-3/2	
ϕ_{CD} +	0	0	0	-1	0	0	0	1	\geq	-3/2	
φ _{DC} -	4/9	-4/9	1	-2	-4/9	0	4/9	17/9	\geq	-11/18	
ϕ_{DC} +	-4/9	4/9	-1	2	4/9	0	-4/9	-17/9	\geq	-43/18	
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-1	
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	\geq	-1	
ϕ_{ED} -	0	0	0	0	0	0	-1	0	\geq	-2	
$\alpha b F$	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	\geq	-1/9	
$\phi_{\text{BF}}\text{-}$	0	-1	-1	0	0	0	0	2	\geq	-1	
ϕ_{BF} +	0	1	1	0	0	0	0	-2	\geq	-1	
ϕ_{FB} -	0	0	0	0	0	1	0	-1	\geq	-1	
ϕ_{FB} +	0	0	0	0	0	-1	0	1	\geq	-1	
ϕ_{GH} -	2/9	5/18	1	-1	5/18	-1	2/9	4/9	\geq	-19/18	
ϕ_{GH} +	-2/9	-5/18	-1	1	-5/18	1	-2/9	-4/9	≥	-35/18	
φ _{HD} -	-2/9	11/9	2	-2	11/9	-1	-2/9	-13/9	\geq	-13/9	
ϕ_{HD} +	2/9	-11/9	-2	2	-11/9	1	2/9	13/9	\geq	-5/9	
ϕ_{DH} -	-4/9	4/9	-1	2	-5/9	0	-4/9	-8/9	≥	-17/9	
ϕ_{DH} +	4/9	-4/9	1	-2	5/9	0	4/9	8/9	\geq	-1/9	
L_{x}	0	0	0	0	0	0	0	-1	\geq	-3/2	
Max	-1/18	1/18	0	0	1/18	0	-1/18	-1/9	=	-1/9	

Scambio pivotale 24-2	Scambio	pivotale	24-2
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	[φ _{AB} +	ϕ_{DH} +	Z	Т	S	R	ϕ_{ED} +	Χ-		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	-2]
Χ	-1	0	0	0	0	0	0	1	≥	-1
ϕ_{BA} -	1	-9/4	9/4	-9/2	5/4	0	1	1	≥	-5/4
ϕ_{BA} +	-1	9/4	-9/4	9/2	-5/4	0	-1	-1	≥	-3/4
$\phi_{\text{BC}}\text{-}$	0	0	1	0	0	0	0	-1	≥	-3/2
$\phi_{\text{BC}}\text{+}$	0	0	-1	0	0	0	0	1	≥	-3/2
$\phi_{\text{CD}}\text{-}$	0	0	0	1	0	0	0	-1	≥	-3/2
$\phi_{\text{CD}}\text{+}$	0	0	0	-1	0	0	0	1	≥	-3/2
ϕ_{DC} -	0	1	0	0	-1	0	0	1	≥	-1/2
$\phi_{\text{DC}}\text{+}$	0	-1	0	0	1	0	0	-1	≥	-5/2
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-1
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	≥	-1
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
$\alpha b F$	0	-1/8	1/8	-1/4	1/8	0	0	0	≥	-1/8
ϕ_{BF} -	-1	9/4	-13/4	9/2	-5/4	0	-1	0	≥	-3/4
ϕ_{BF} +	1	-9/4	13/4	-9/2	5/4	0	1	0	≥	-5/4
ϕ_{FB} -	0	0	0	0	0	1	0	-1	≥	-1
$\phi_{\text{FB}} \textbf{+}$	0	0	0	0	0	-1	0	1	≥	-1
$\phi_{\text{GH}}\text{-}$	1/2	-5/8	13/8	-9/4	5/8	-1	1/2	1	≥	-9/8
$\phi_{\text{GH}} \textbf{+}$	-1/2	5/8	-13/8	9/4	-5/8	1	-1/2	-1	≥	-15/8
ϕ_{HD} -	1	-11/4	19/4	-15/2	11/4	-1	1	1	≥	-7/4
ϕ_{HD} +	-1	11/4	-19/4	15/2	-11/4	1	-1	-1	≥	-1/4
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-2
Υ	1	-9/4	9/4	-9/2	5/4	0	1	2	≥	-1/4
L_{x}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	0	-1/8	1/8	-1/4	1/8	0	0	0	=	-1/8

Scambio pivotale 22-3

	- P. T	==	•					_		
	φ _{AB} +	ϕ_{DH} +	ϕ_{HD} +	Т	S	R	ϕ_{ED} +	X-]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	\geq	-2
X	-1	0	0	0	0	0	0	1	\geq	-1
$\phi_{\text{BA}}\text{-}$	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	10/19	≥	-26/19
$\phi_{\text{BA}} \textbf{+}$	-10/19	18/19	9/19	18/19	1/19	-9/19	-10/19	-10/19	≥	-12/19
ϕ_{BC} -	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-23/19	\geq	-59/38
$\phi_{\text{BC}} \textbf{+}$	4/19	-11/19	4/19	-30/19	11/19	-4/19	4/19	23/19	≥	-55/38
ϕ_{CD}	0	0	0	1	0	0	0	-1	≥	-3/2
ϕ_{CD} +	0	0	0	-1	0	0	0	1	\geq	-3/2
ϕ_{DC} -	0	1	0	0	-1	0	0	1	\geq	-1/2
$\phi_{DC}\text{+}$	0	-1	0	0	1	0	0	-1	\geq	-5/2
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	\geq	-1
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	\geq	-1
ϕ_{ED} -	0	0	0	0	0	0	-1	0	\geq	-2
$\alpha b F$	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	\geq	-5/38
ϕ_{BF} -	-6/19	7/19	13/19	-12/19	12/19	-13/19	-6/19	13/19	\geq	-11/19
ϕ_{BF} +	6/19	-7/19	-13/19	12/19	-12/19	13/19	6/19	-13/19	\geq	-27/19
$\phi_{\text{FB}}\text{-}$	0	0	0	0	0	1	0	-1	\geq	-1
ϕ_{FB} +	0	0	0	0	0	-1	0	1	\geq	-1
φ _{GH} -	3/19	6/19	-13/38	6/19	-6/19	-25/38	3/19	25/38	\geq	-23/19
ϕ_{GH} +	-3/19	-6/19	13/38	-6/19	6/19	25/38	-3/19	-25/38	\geq	-34/19
φ _{HD} -	0	0	-1	0	0	0	0	0	\geq	-2
Z	-4/19	11/19	-4/19	30/19	-11/19	4/19	-4/19	-4/19	\geq	-1/19
ϕ_{DH} -	0	-1	0	0	0	0	0	0	\geq	-2
Υ	10/19	-18/19	-9/19	-18/19	-1/19	9/19	10/19	29/19	≥	-7/19
L_{X}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/38	-1/19	-1/38	-1/19	1/19	1/38	-1/38	-1/38	=	-5/38

Scambio pivotale 22-5

	[φ _{AB} +	φ _{DH} +	ϕ_{HD} +	Т	Z	R	ϕ_{ED} +	Χ-		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	-2
Χ	-1	0	0	0	0	0	0	1	≥	-1
ϕ_{BA} -	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	6/11	≥	-15/11
ϕ_{BA} +	-6/11	1	5/11	12/11	-1/11	-5/11	-6/11	-6/11	≥	-7/11
ϕ_{BC} -	0	0	0	0	1	0	0	-1	≥	-3/2
$\phi_{BC}\text{+}$	0	0	0	0	-1	0	0	1	≥	-3/2
φ _{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2
ϕ_{DC} -	4/11	0	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-9/22
ϕ_{DC} +	-4/11	0	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-57/22
ϕ_{DE} -	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-15/11	≥	-12/11
$\phi_{\text{DE}} +$	4/11	-1	4/11	-30/11	19/11	-4/11	4/11	15/11	≥	-10/11
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
$\alpha b F$	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	≥	-3/22
ϕ_{BF} -	-6/11	1	5/11	12/11	-12/11	-5/11	-6/11	5/11	≥	-7/11
ϕ_{BF} +	6/11	-1	-5/11	-12/11	12/11	5/11	6/11	-5/11	≥	-15/11
ϕ_{FB} -	0	0	0	0	0	1	0	-1	≥	-1
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1
ϕ_{GH} -	3/11	0	-5/22	-6/11	6/11	-17/22	3/11	17/22	≥	-13/11
ϕ_{GH} +	-3/11	0	5/22	6/11	-6/11	17/22	-3/11	-17/22	≥	-20/11
ϕ_{HD} -	0	0	-1	0	0	0	0	0	≥	-2
S	-4/11	1	-4/11	30/11	-19/11	4/11	-4/11	-4/11	≥	-1/11
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-2
Υ	6/11	-1	-5/11	-12/11	1/11	5/11	6/11	17/11	≥	-4/11
L_{x}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/22	0	-1/22	1/11	-1/11	1/22	-1/22	-1/22	=	-3/22

Scambio pivotale 9-4

Scam	pio bivo	tale 9-4									
	$\left[\phi_{AB} + \right]$	ϕ_{DH} +	ϕ_{HD} +	ϕ_{DC} -	Z	R	ϕ_{ED} +	X-]	[Fb]	
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	[-2]	
Χ	-1	0	0	0	0	0	0	1	≥	-1	
ϕ_{BA} -	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	0	≥	-6/5	
$\phi_{\text{BA}}\text{+}$	-2/5	1	3/5	-2/5	3/5	-3/5	-2/5	0	≥	-4/5	
ϕ_{BC} -	0	0	0	0	1	0	0	-1	≥	-3/2	
$\phi_{\text{BC}} \textbf{+}$	0	0	0	0	-1	0	0	1	≥	-3/2	
φ _{CD} -	2/15	0	2/15	-11/30	19/30	-2/15	2/15	-1/2	≥	-33/20	
ϕ_{CD} +	-2/15	0	-2/15	11/30	-19/30	2/15	-2/15	1/2	≥	-27/20	
T	2/15	0	2/15	-11/30	19/30	-2/15	2/15	1/2	≥	-3/20	
ϕ_{DC} +	0	0	0	-1	0	0	0	0	≥	-3	
φ _{DE} -	0	1	0	-1	0	0	0	0	≥	-3/2	
φ _{DE} +	0	-1	0	1	0	0	0	0	≥	-1/2	
φ _{ED} -	0	0	0	0	0	0	-1	0	≥	-2	
αbF	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	≥	-3/20	
ϕ_{BF} -	-2/5	1	3/5	-2/5	-2/5	-3/5	-2/5	1	≥	-4/5	
φ _{BF} +	2/5	-1	-3/5	2/5	2/5	3/5	2/5	-1	≥	-6/5	
ϕ_{FB} -	0	0	0	0	0	1	0	-1	≥	-1	
ϕ_{FB} +	0	0	0	0	0	-1	0	1	≥	-1	
φ _{GH} -	1/5	0	-3/10	1/5	1/5	-7/10	1/5	1/2	≥	-11/10	
φ _{GH} +	-1/5	0	3/10	-1/5	-1/5	7/10	-1/5	-1/2	≥	-19/10	
φ _{HD} -	0	0	-1	0	0	0	0	0	≥	-2	
S	0	1	0	-1	0	0	0	1	≥	-1/2	
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-2	
Y	2/5	-1	-3/5	2/5	-3/5	3/5	2/5	1	≥	-1/5	
L_{x}	0	0	0	0	0	0	0	-1	≥	-3/2	
Max	-1/30	0	-1/30	-1/30	-1/30	1/30	-1/30	0	=	-3/20	

Scambio pivotale 18-6

	[φ _{AB} +	ϕ_{DH} +	ϕ_{HD} +	ϕ_{DC} -	Z	ϕ_{FB} +	ϕ_{ED} +	Χ-		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	-2
Χ	-1	0	0	0	0	0	0	1	≥	-1
ϕ_{BA} -	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	3/5	≥	-9/5
ϕ_{BA} +	-2/5	1	3/5	-2/5	3/5	3/5	-2/5	-3/5	≥	-1/5
ϕ_{BC} -	0	0	0	0	1	0	0	-1	≥	-3/2
ϕ_{BC} +	0	0	0	0	-1	0	0	1	≥	-3/2
ϕ_{CD} -	2/15	0	2/15	-11/30	19/30	2/15	2/15	-19/30	≥	-91/60
φ _{CD} +	-2/15	0	-2/15	11/30	-19/30	-2/15	-2/15	19/30	≥	-89/60
T	2/15	0	2/15	-11/30	19/30	2/15	2/15	11/30	≥	-1/60
ϕ_{DC} +	0	0	0	-1	0	0	0	0	≥	-3
φ _{DE} -	0	1	0	-1	0	0	0	0	≥	-3/2
φ _{DE} +	0	-1	0	1	0	0	0	0	≥	-1/2
φ _{ED} -	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	≥	-11/60
ϕ_{BF} -	-2/5	1	3/5	-2/5	-2/5	3/5	-2/5	2/5	≥	-1/5
φ _{BF} +	2/5	-1	-3/5	2/5	2/5	-3/5	2/5	-2/5	≥	-9/5
ϕ_{FB} -	0	0	0	0	0	-1	0	0	≥	-2
R	0	0	0	0	0	-1	0	1	≥	-1
ϕ_{GH} -	1/5	0	-3/10	1/5	1/5	7/10	1/5	-1/5	≥	-2/5
ϕ_{GH} +	-1/5	0	3/10	-1/5	-1/5	-7/10	-1/5	1/5	≥	-13/5
φ _{HD} -	0	0	-1	0	0	0	0	0	≥	-2
S	0	1	0	-1	0	0	0	1	≥	-1/2
φ _{DH} -	0	-1	0	0	0	0	0	0	≥	-2
Υ	2/5	-1	-3/5	2/5	-3/5	-3/5	2/5	8/5	≥	-4/5
L_{x}	0	0	0	0	0	0	0	-1	≥	-3/2
Max	-1/30	0	-1/30	-1/30	-1/30	-1/30	-1/30	1/30	=	_11/60_

Scambio pivotale 4-8

	[φ _{AB} +	φ _{DH} +	ϕ_{HD} +	φ _{DC} -	Z	ϕ_{FB} +	ϕ_{ED} +	ϕ_{BA} +]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	≥	[-2]
Χ	-5/3	5/3	1	-2/3	1	1	-2/3	-5/3	≥	-4/3
ϕ_{BA} -	0	0	0	0	0	0	0	-1	≥	-2
X-	-2/3	5/3	1	-2/3	1	1	-2/3	-5/3	≥	-1/3
ϕ_{BC} -	2/3	-5/3	-1	2/3	0	-1	2/3	5/3	\geq	-7/6
ϕ_{BC} +	-2/3	5/3	1	-2/3	0	1	-2/3	-5/3	\geq	-11/6
$\phi_{\text{CD}}\text{-}$	5/9	-19/18	-1/2	1/18	0	-1/2	5/9	19/18	\geq	-47/36
$\phi_{\text{CD}}\text{+}$	-5/9	19/18	1/2	-1/18	0	1/2	-5/9	-19/18	\geq	-61/36
Т	-1/9	11/18	1/2	-11/18	1	1/2	-1/9	-11/18	≥	-5/36
$\phi_{\text{DC}}\text{+}$	0	0	0	-1	0	0	0	0	\geq	-3
$\phi_{\text{DE}}\text{-}$	0	1	0	-1	0	0	0	0	\geq	-3/2
$\phi_{\text{DE}} \textbf{+}$	0	-1	0	1	0	0	0	0	\geq	-1/2
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	-1	0	\geq	-2
$\alpha b F$	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	\geq	-7/36
ϕ_{BF} -	-2/3	5/3	1	-2/3	0	1	-2/3	-2/3	\geq	-1/3
$\phi_{\text{BF}}\text{+}$	2/3	-5/3	-1	2/3	0	-1	2/3	2/3	\geq	-5/3
ϕ_{FB} -	0	0	0	0	0	-1	0	0	\geq	-2
R	-2/3	5/3	1	-2/3	1	0	-2/3	-5/3	≥	-4/3
ϕ_{GH} -	1/3	-1/3	-1/2	1/3	0	1/2	1/3	1/3	\geq	-1/3
ϕ_{GH} +	-1/3	1/3	1/2	-1/3	0	-1/2	-1/3	-1/3	\geq	-8/3
ϕ_{HD} -	0	0	-1	0	0	0	0	0	\geq	-2
S	-2/3	8/3	1	-5/3	1	1	-2/3	-5/3	\geq	-5/6
$\phi_{\text{DH}}\text{-}$	0	-1	0	0	0	0	0	0	\geq	-2
Υ	-2/3	5/3	1	-2/3	1	1	-2/3	-8/3	≥	-4/3
L_{X}	2/3	-5/3	-1	2/3	-1	-1	2/3	5/3	≥	-7/6
Max	-1/18	1/18	0	-1/18	0	0	-1/18	-1/18	=	-7/36

Scambio pivotale 12-2

	[φ _{AB} +	ϕ_{DE} +	ϕ_{HD} +	ϕ_{DC} -	Z	ϕ_{FB} +	ϕ_{ED} +	φ_{BA} +]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	\geq	[-2]
Χ	-5/3	-5/3	1	1	1	1	-2/3	-5/3	\geq	-13/6
ϕ_{BA} -	0	0	0	0	0	0	0	-1	\geq	-2
X-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	≥	-7/6
ϕ_{BC} -	2/3	5/3	-1	-1	0	-1	2/3	5/3	≥	-1/3
ϕ_{BC} +	-2/3	-5/3	1	1	0	1	-2/3	-5/3	≥	-8/3
ϕ_{CD} -	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	≥	-7/9
$\phi_{\text{CD}}\text{+}$	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	≥	-20/9
T	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	≥	-4/9
ϕ_{DC} +	0	0	0	-1	0	0	0	0	≥	-3
$\phi_{\text{DE}}\text{-}$	0	-1	0	0	0	0	0	0	≥	-2
ϕ_{DH} +	0	-1	0	1	0	0	0	0	≥	-1/2
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/18	-1/18	0	0	0	0	-1/18	-1/18	≥	-2/9
ϕ_{BF} -	-2/3	-5/3	1	1	0	1	-2/3	-2/3	≥	-7/6
ϕ_{BF} +	2/3	5/3	-1	-1	0	-1	2/3	2/3	≥	-5/6
φ _{FB} -	0	0	0	0	0	-1	0	0	\geq	-2
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	≥	-13/6
ϕ_{GH} -	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	≥	-1/6
ϕ_{GH} +	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	≥	-17/6
ϕ_{HD} -	0	0	-1	0	0	0	0	0	\geq	-2
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	\geq	-13/6
$\phi_{\text{DH}}\text{-}$	0	1	0	-1	0	0	0	0	\geq	-3/2
Υ	-2/3	-5/3	1	1	1	1	-2/3	-8/3	≥	-13/6
L_X	2/3	5/3	-1	-1	-1	-1	2/3	5/3	≥	-1/3
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	-2/9

Tableau finale

lable	au iii iai	C								
	[φ _{AB} +	ϕ_{DE} +	ϕ_{HD} +	ϕ_{DC} -	Z	ϕ_{FB} +	ϕ_{ED} +	φ_{BA} +]		[Fb]
ϕ_{AB} -	-1	0	0	0	0	0	0	0	\geq	[-2]
X	-5/3	-5/3	1	1	1	1	-2/3	-5/3	\geq	-13/6
ϕ_{BA} -	0	0	0	0	0	0	0	-1	\geq	-2
Χ-	-2/3	-5/3	1	1	1	1	-2/3	-5/3	\geq	-7/6
ϕ_{BC} -	2/3	5/3	-1	-1	0	-1	2/3	5/3	\geq	-1/3
ϕ_{BC} +	-2/3	-5/3	1	1	0	1	-2/3	-5/3	\geq	-8/3
ϕ_{CD} -	5/9	19/18	-1/2	-1	0	-1/2	5/9	19/18	\geq	-7/9
ϕ_{CD} +	-5/9	-19/18	1/2	1	0	1/2	-5/9	-19/18	\geq	-20/9
Т	-1/9	-11/18	1/2	0	1	1/2	-1/9	-11/18	≥	-4/9
ϕ_{DC} +	0	0	0	-1	0	0	0	0	≥	-3
ϕ_{DE} -	0	-1	0	0	0	0	0	0	\geq	-2
ϕ_{DH} +	0	-1	0	1	0	0	0	0	\geq	-1/2
ϕ_{ED} -	0	0	0	0	0	0	-1	0	≥	-2
$\alpha b F$	-1/18	-1/18	0	0	0	0	-1/18	-1/18	≥	-2/9
ϕ_{BF} -	-2/3	-5/3	1	1	0	1	-2/3	-2/3	≥	-7/6
ϕ_{BF} +	2/3	5/3	-1	-1	0	-1	2/3	2/3	\geq	-5/6
ϕ_{FB} -	0	0	0	0	0	-1	0	0	≥	-2
R	-2/3	-5/3	1	1	1	0	-2/3	-5/3	≥	-13/6
ϕ_{GH} -	1/3	1/3	-1/2	0	0	1/2	1/3	1/3	≥	-1/6
ϕ_{GH} +	-1/3	-1/3	1/2	0	0	-1/2	-1/3	-1/3	\geq	-17/6
ϕ_{HD} -	0	0	-1	0	0	0	0	0	≥	-2
S	-2/3	-8/3	1	1	1	1	-2/3	-5/3	≥	-13/6
ϕ_{DH} -	0	1	0	-1	0	0	0	0	\geq	-3/2
Υ	-2/3	-5/3	1	1	1	1	-2/3	-8/3	\geq	-13/6
L_{x}	2/3	5/3	-1	-1	-1	-1	2/3	5/3	≥	-1/3
Max	-1/18	-1/18	0	0	0	0	-1/18	-1/18	=	-2/9

Vettori soluzione della programmazione lineare

	X	Υ	Z	Т	S	R	$\alpha b F$	X-		[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{AB}\textbf{+}$	0	0	0	0	0	0	0	0	≥	1/18
ϕ_{BA} -	0	0	0	0	0	0	0	0	≥	0
ϕ_{BA} +	0	0	0	0	0	0	0	0	≥	1/18
ϕ_{BC} -	0	0	0	0	0	0	0	0	≥	0
ϕ_{BC} +	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{CD}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{CD}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{DC}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DC}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DE}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	1/18
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{ED}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	1/18
$\phi_{\text{BF}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{BF} +	0	0	0	0	0	0	0	0	≥	0
ϕ_{FB} -	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{FB}}\text{+}$	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{GH}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{GH} +	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{HD}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{HD} +	0	0	0	0	0	0	0	0	≥	0
$\phi_{\text{DH}}\text{-}$	0	0	0	0	0	0	0	0	≥	0
ϕ_{DH} +	0	0	0	0	0	0	0	0	≥	0
L_{X}	0	0	0	0	0	0	0	0	≥	0
Max	13/6	13/6	0	4/9	13/6	13/6	2/9	7/6	=	2/9

Variabili soluzione dedotto il valore X-

 $u_G = 1/6\delta$

 $V_G = 0$

 $\phi_G = 0$

Variabili soluzione differenza tra rotazioni

ϕ_{AB}	[1/18]
ϕ_{BA}	1/18
ϕ_{BC}	0
ϕ_{CD}	0
ϕ_{DC}	0
ϕ_{DE}	1/18
ϕ_{ED}	1/18
ϕ_{BF}	0
ϕ_{FB}	0
ϕ_{GH}	0
ϕ_{HD}	0
ϕ_{DH}	0 _

REAZIONI Fattore di collasso = 2/9

 $H_{\Delta} = -2/3F$

 $V_{\Delta} = -1/18F$

 $W_A = Fb$

 $H_{\rm F} = -2/3F$

 $V_{E} = 41/18F$

 $W_F = Fb$

$H_{AB} = -2/3F$	$H_{BC} = 1/6F$	$H_{CD} = 1/6F$	$H_{DE} = 2/3F$	$H_{BF} = -7/18F$
$V_{AB} = -1/18F$	$V_{BC} = -2/9F$	$V_{CD} = -10/9F$	$V_{DE} = -41/18F$	$V_{BF} = 1/6F$
$W_{AB} = Fb$	$W_{BC} = -7/6Fb$	$W_{CD} = -13/18Fb$	$W_{DE} = Fb$	$W_{BF} = 1/6Fb$
$H_{BA} = 2/3F$	$H_{CB} = -1/6F$	$H_{DC} = -1/6F$	$H_{ED} = -2/3F$	$H_{FB} = 7/18F$
$V_{BA} = 1/18F$	$V_{CB} = 2/9F$	$V_{DC} = 10/9F$	$V_{ED} = 41/18F$	$V_{FB} = -1/6F$
$W_{BA} = Fb$	$W_{CB} = 13/18Fb$	$W_{DC} = -3/2Fb$	$W_{ED} = Fb$	$W_{FB} = Fb$
LL 4/2F	Ц 4/2Г	Ц 1/2Г		
$H_{FG} = 1/2F$	$H_{GH} = 1/2F$	$H_{HD} = 1/2F$		
$V_{FG} = 1/6F$	$V_{GH} = -7/6F$	$V_{HD} = -7/6F$		
$W_{FG} = -Fb$	$W_{GH} = -4/3Fb$	$W_{HD} = Fb$		
$H_{GF} = -1/2F$	$H_{HG} = -1/2F$	$H_{DH} = -1/2F$		
$V_{GF} = -1/6F$	$V_{HG} = 7/6F$	$V_{DH} = 7/6F$		
$W_{cr} = 4/3$ Fb	$W_{uc} = -Fb$	$W_{DU} = 1/2Fb$		

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_B = 1/6\delta$	$u_C = 1/6\delta$	$u_D = 1/6\delta$	$u_{EED} = 0$
$V_{AAB} = 0$	$V_B = 0$	$v_C = 0$	$V_D = 0$	$V_{EED} = 0$
$\phi_{AAB} = -1/18\delta/b$	$\phi_{\rm p} = -1/18\delta/b$	$\varphi_{\rm C} = 0$	$\varphi_D = 0$	$\varphi_{\scriptscriptstyle \square} = -1/18\delta/b$

SPOSTAMENTI RIGIDI DELLE ASTE

 $u_F = 1/6\delta$

 $V_F = 0$

 $\phi_F = 0$

$u_{AAB} = 0$	$u_{BBC} = 1/6\delta$	$u_{CCD} = 1/6\delta$	$u_{DDE} = 1/6\delta$	$u_{BBF} = 1/6\delta$
$V_{AAB} = 0$	$V_{BBC} = 0$	$V_{CCD} = 0$	$V_{DDE} = 0$	$V_{BBF} = 0$
$\phi_{AAB} = -1/18\delta/b$	$\phi_{BBC} = 0$	$\phi_{CCD} = 0$	$\phi_{DDE} = -1/18\delta/b$	$\phi_{BBF} = 0$

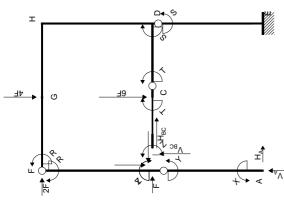
 $u_H = 1/6\delta$

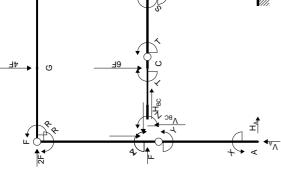
 $V_{H} = 0$

 $\phi_H = 0$

$u_{FFG} = 1/6\delta$	$u_{GGH} = 1/6\delta$	$u_{HHD} = 1/6\delta$
$V_{FFG} = 0$	$v_{GGH} = 0$	$V_{HHD} = 0$
$\varphi_{FFG} = 0$	$\varphi_{GGH} = 0$	$\varphi_{HHD} = 0$

EQUILIBRIO Nome:





М

EQUAZIONI DI EQUILIBRIO

Rotazione intorno a D: aste DC DH CB HG GF FB BA

 $3H_Ab - 4V_Ab = -Xb + Sb - 14Fb$

Rotazione intorno a C: aste CB

 $-2V_{BC}b = -Zb + Tb$

Rotazione intorno a F: aste FB BA $6H_Ab - 3H_{BC}b = -Xb + Zb + Rb - 3Fb$

Rotazione intorno a B: aste BA

 $3H_Ab = -Xb - Yb$

Matrice di equilibrio

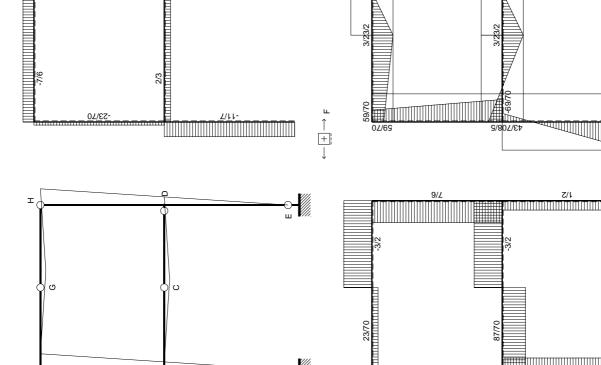
$$\begin{cases} H_A b & V_A b & H_B c b & V_B c b \end{bmatrix} \quad \begin{bmatrix} X b & Y b & Z b & T b & S b & R t \\ 3 & -4 & 0 & 0 \\ \phi_{CD} & 0 & 0 & -2 \\ 6 & 0 & -3 & 0 \\ 3 & 0 & 0 & 0 & 0 \end{cases}$$

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Soluzione del sistema

$$\begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1/3 & -1/3 & 0 & 0 & 0 & 0 & 0 \\ V_{BC}b & 0 & 0 & 1/2 & -1/2 & 0 & 0 & 0 \\ -1/3 & -2/3 & -1/3 & 0 & 0 & -1/3 & 1 \\ V_{A}b & 0 & -1/4 & 0 & 0 & -1/4 & 0 & 7/2 \end{bmatrix}$$

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.11.05.11





08.06.11

PROGRAMMAZIONE LINEARE

Sia H_{ii} la matrice del simplesso, con m righe e n colonne.

Siano P_i le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.

Siano a riga m i coefficienti della funzione obiettivo primale $\max \Sigma_i H_{mi} P_{i}$, $1 \le j < n$.

Siano a colonna n i coefficienti della funzione obiettivo duale $min \Sigma_i H_{in} D_{it}$ $1 \le i < m$.

Sequenza di operazioni pivotali:

- 1 Sia q (1 $\leq q < n$) la colonna pivot con massimo valore H_{mi} in riga m.
- 2 Sia p (1 $\leq p < m$) la riga pivot di colonna q, a coefficiente negativo H_{in} , che minimizza il rapporto H_{in}/H_{in} .
- 3 Si ottiene il coefficiente pivotale H_{po}
- 4 Si scambia la variabile primale P_a con la duale D_a .
- 5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
- 6 Si ridefiniscono i coefficienti della colonna pivot $q: H_{ig} = H_{ng} H_{ig}$, escluso il pivot H_{ng} .
- 7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ii} = H_{ii} H_{ia} H_{pi}$
- 8 Si ridefiniscono i coefficienti della riga pivot $p: H_{pj} = -H_{pq} H_{pi}$, escluso il pivot H_{pq} .
- Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.

Giunti a questo punto, si individua la soluzione.

Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_i presenti sulla colonna di sinistra.

Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare *m*=6,*n*=4

SOLUZIONE DEL SIMPLESSO X=WAR Y=WRA Z=WRC T=WCD S=WDE R=WEG

Tableau con variabili non vincolate in segno

i ableau con variabili non vincolate in segno									
	[X	Υ	Z	Т	S	R	αbF		[Fb]
W _{AB} -	1	0	0	0	0	0	0	≥	-2
W_{AB} +	1	0	0	0	0	0	0	≤	2
W_{BA} -	0	1	0	0	0	0	0	≥	-2
W_{BA} +	0	1	0	0	0	0	0	≤	2
W_{BC} -	0	0	1	0	0	0	0	≥	-3/2
W _{BC} +	0	0	1	0	0	0	0	≤	3/2
W _{CD} -	0	0	0	1	0	0	0	≥	-3/2
W _{CD} +	0	0	0	1	0	0	0	≤	3/2
W_{DC} -	0	0	1	-2	0	0	-12	≥	-3/2
W_{DC} +	0	0	1	-2	0	0	-12	≤	3/2
W_{DE} -	0	0	0	0	1	0	0	≥	-2
W_{DE} +	0	0	0	0	1	0	0	≤	2
W_{ED} -	-1	-1	0	0	-1	0	9	≥	-2
W_{ED} +	-1	-1	0	0	-1	0	9	≤	2
W_{BF} -	0	-1	-1	0	0	0	0	≥	-2
W_{BF} +	0	-1	-1	0	0	0	0	≤	2
W_{FG} -	0	0	0	0	0	1	0	≥	-3/2
W_{FG} +	0	0	0	0	0	1	0	≤	3/2
W_{GH} -	0	1/2	1	-1	1/2	1	-7	≥	-3/2
W_{GH} +	0	1/2	1	-1	1/2	1	-7	≤	3/2
W_{HG} -	0	-1	-2	2	-1	-1	6	≥	-3/2
W_{HG} +	0	-1	-2	2	-1	-1	6	≤	3/2
W_{DH} -	0	0	-1	2	-1	0	12	≥	-2
W_{DH} +	0	0	-1	2	-1	0	12	≤	2
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	[X	Υ	Z	Т	S	R	αbF]		[Fb]
W_{AB} -	1	0	0	0	0	0	0	≥	-2
W_{AB} +	-1	0	0	0	0	0	0	≥	-2
W_{BA} -	0	1	0	0	0	0	0	≥	-2
W_{BA} +	0	-1	0	0	0	0	0	≥	-2
W_{BC} -	0	0	1	0	0	0	0	≥	-3/2
W_{BC} +	0	0	-1	0	0	0	0	≥	-3/2
W _{CD} -	0	0	0	1	0	0	0	≥	-3/2
W _{CD} +	0	0	0	-1	0	0	0	≥	-3/2
W _{DC} -	0	0	1	-2	0	0	-12	≥	-3/2
W_{DC} +	0	0	-1	2	0	0	12	≥	-3/2
W_{DE} -	0	0	0	0	1	0	0	≥	-2
W_{DE} +	0	0	0	0	-1	0	0	≥	-2
W_{ED} -	-1	-1	0	0	-1	0	9	≥	-2
W_{ED} +	1	1	0	0	1	0	-9	≥	-2
W_{BF} -	0	-1	-1	0	0	0	0	≥	-2
W_{BF} +	0	1	1	0	0	0	0	≥	-2
W_{FG} -	0	0	0	0	0	1	0	≥	-3/2
W_{FG} +	0	0	0	0	0	-1	0	≥	-3/2
W_{GH} -	0	1/2	1	-1	1/2	1	-7	≥	-3/2
W_{GH} +	0	-1/2	-1	1	-1/2	-1	7	≥	-3/2
$W_{\rm HG}$ -	0	-1	-2	2	-1	-1	6	≥	-3/2
W_{HG} +	0	1	2	-2	1	1	-6	≥	-3/2
W_{DH} -	0	0	-1	2	-1	0	12	≥	-2
W_{DH} +	0	0	1	-2	1	0	-12	≥	-2
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili vincolate in segno															
	[X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		[Fb]
W_{AB} -	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-2 ⁻
W_{AB} +	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-2
W_{AB} + W_{BA} -	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-2
W_{BA} +	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-2

VV _{AB} -	1	U	U	U	U	U	-1	U	U	U	U	U	U	-	-2	
W _{AB} +	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-2	
W _{BA} -	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-2	
W _{BA} +	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-2	
W _{BC} -	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2	
W _{BC} +	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2	
W _{CD} -	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2	
W _{CD} +	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2	
W _{DC} -	0	0	1	-2	0	0	0	0	-1	2	0	0	-12	≥	-3/2	
W _{DC} +	0	0	-1	2	0	0	0	0	1	-2	0	0	12	≤	-3/2	
W _{DE} -	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-2	
W _{DF} +	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-2	
W _{ED} -	-1	-1	0	0	-1	0	1	1	0	0	1	0	9	≥	-2	
W _{ED} +	1	1	0	0	1	0	-1	-1	0	0	-1	0	-9	≤	-2	
W _{BF} -	0	-1	-1	0	0	0	0	1	1	0	0	0	0	≥	-2	
W _{BF} +	0	1	1	0	0	0	0	-1	-1	0	0	0	0	≤	-2	
W _{FG} -	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-3/2	
W _{FG} +	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-3/2	
W _{GH} -	0	1/2	1	-1	1/2	1	0	-1/2	-1	1	-1/2	-1	-7	≥	-3/2	
W _{GH} +	0	-1/2	-1	1	-1/2	-1	0	1/2	1	-1	1/2	1	7	≤	-3/2	
W _{HG} -	0	-1	-2	2	-1	-1	0	1	2	-2	1	1	6	≥	-3/2	
W _{HG} +	0	1	2	-2	1	1	0	-1	-2	2	-1	-1	-6	≤	-3/2	
W _{DH} -	0	0	-1	2	-1	0	0	0	1	-2	1	0	12	≥	-2	
W _{DH} +	0	0	1	-2	1	0	0	0	-1	2	-1	0	-12	≤	-2	
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	[0]	ı

Tableau a variabili negative su X- e limitate

	X	Υ	Z	T	S	R	αbF	X- [[Fb]
$\phi_{AB}\text{-}$	1	0	0	0	0	0	0	-1	≥	-2
ϕ_{AB} +	-1	0	0	0	0	0	0	1	≥	-2
$\phi_{\text{BA}}\text{-}$	0	1	0	0	0	0	0	-1	≥	-2
$\phi_{\text{BA}} \textbf{+}$	0	-1	0	0	0	0	0	1	≥	-2
$\phi_{\text{BC}}\text{-}$	0	0	1	0	0	0	0	-1	≥	-3/2
$\phi_{\text{BC}} \textbf{+}$	0	0	-1	0	0	0	0	1	≥	-3/2
ϕ_{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2
$\phi_{\text{CD}} \textbf{+}$	0	0	0	-1	0	0	0	1	≥	-3/2
$\phi_{\text{DC}}\text{-}$	0	0	1	-2	0	0	-12	1	≥	-3/2
$\phi_{\text{DC}} \textbf{+}$	0	0	-1	2	0	0	12	-1	≥	-3/2
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	≥	-2
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	≥	-2
$\phi_{\text{ED}}\text{-}$	-1	-1	0	0	-1	0	9	3	≥	-2
$\phi_{\text{ED}} \textbf{+}$	1	1	0	0	1	0	-9	-3	≥	-2
$\phi_{\text{BF}}\text{-}$	0	-1	-1	0	0	0	0	2	≥	-2
$\phi_{\text{BF}}\text{+}$	0	1	1	0	0	0	0	-2	≥	-2
$\phi_{\text{FG}}\text{-}$	0	0	0	0	0	1	0	-1	≥	-3/2
$\phi_{\text{FG}} \textbf{+}$	0	0	0	0	0	-1	0	1	≥	-3/2
$\phi_{\text{GH}}\text{-}$	0	1/2	1	-1	1/2	1	-7	-2	≥	-3/2
$\phi_{\text{GH}} \textbf{+}$	0	-1/2	-1	1	-1/2	-1	7	2	≥	-3/2
$\phi_{\text{HG}}\text{-}$	0	-1	-2	2	-1	-1	6	3	≥	-3/2
$\phi_{\text{HG}} \textbf{+}$	0	1	2	-2	1	1	-6	-3	≥	-3/2
$\phi_{\text{DH}}\text{-}$	0	0	-1	2	-1	0	12	0	≥	-2
$\phi_{\text{DH}} \textbf{+}$	0	0	1	-2	1	0	-12	0	≥	-2
L_{X}	0	0	0	0	0	0	0	-1	≥	-2
Max	0	0	0	0	0	0	1	0	=	0

Scam	bio pivo	otale 9-7	,								
	[X	Υ	Z	Т	S	R	ϕ_{DC} -	X-]		[Fb]	
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	[-2]	
$\phi_{AB}\textbf{+}$	-1	0	0	0	0	0	0	1	≥	-2	
$\phi_{\text{BA}}\text{-}$	0	1	0	0	0	0	0	-1	≥	-2	
ϕ_{BA} +	0	-1	0	0	0	0	0	1	≥	-2	
ϕ_{BC} -	0	0	1	0	0	0	0	-1	≥	-3/2	
$\phi_{BC}\text{+}$	0	0	-1	0	0	0	0	1	≥	-3/2	
$\phi_{\text{CD}}\text{-}$	0	0	0	1	0	0	0	-1	≥	-3/2	
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2	
αbF	0	0	1/12	-1/6	0	0	-1/12	1/12	≥	-1/8	
$\phi_{\text{DC}}\text{+}$	0	0	0	0	0	0	-1	0	≥	-3	
ϕ_{DE} -	0	0	0	0	1	0	0	-1	≥	-2	
$\phi_{\text{DE}}\text{+}$	0	0	0	0	-1	0	0	1	≥	-2	
$\phi_{\text{ED}}\text{-}$	-1	-1	3/4	-3/2	-1	0	-3/4	15/4	≥	-25/8	
φ _{ED} +	1	1	-3/4	3/2	1	0	3/4	-15/4	≥	-7/8	
φ _{BF} -	0	-1	-1	0	0	0	0	2	≥	-2	
ϕ_{BF} +	0	1	1	0	0	0	0	-2	≥	-2	
ϕ_{FG} -	0	0	0	0	0	1	0	-1	≥	-3/2	
ϕ_{FG} +	0	0	0	0	0	-1	0	1	≥	-3/2	
ϕ_{GH} -	0	1/2	5/12	1/6	1/2	1	7/12	-31/12	≥	-5/8	
ϕ_{GH} +	0	-1/2	-5/12	-1/6	-1/2	-1	-7/12	31/12	≥	-19/8	
ϕ_{HG} -	0	-1	-3/2	1	-1	-1	-1/2	7/2	≥	-9/4	
ϕ_{HG} +	0	1	3/2	-1	1	1	1/2	-7/2	≥	-3/4	
ϕ_{DH} -	0	0	0	0	-1	0	-1	1	≥	-7/2	
ϕ_{DH} +	0	0	0	0	1	0	1	-1	≥	-1/2	
L _X	0	0	0	0	0	0	0	-1	≥	-2	
Max	0	0	1/12	-1/6	0	0	-1/12	1/12	=	-1/8	

Scambio pivotale 14-3

			-							
	X	Υ	$\phi_{\text{ED}} \textbf{+}$	Т	S	R	ϕ_{DC} -	X-		[Fb]
ϕ_{AB} -	1	0	0	0	0	0	0	-1	≥	[-2]
ϕ_{AB} +	-1	0	0	0	0	0	0	1	≥	-2
$\phi_{\text{BA}}\text{-}$	0	1	0	0	0	0	0	-1	≥	-2
ϕ_{BA} +	0	-1	0	0	0	0	0	1	≥	-2
ϕ_{BC} -	4/3	4/3	-4/3	2	4/3	0	1	-6	≥	-8/3
ϕ_{BC} +	-4/3	-4/3	4/3	-2	-4/3	0	-1	6	≥	-1/3
ϕ_{CD} -	0	0	0	1	0	0	0	-1	≥	-3/2
ϕ_{CD} +	0	0	0	-1	0	0	0	1	≥	-3/2
αbF	1/9	1/9	-1/9	0	1/9	0	0	-1/3	≥	-2/9
$\phi_{\text{DC}}\text{+}$	0	0	0	0	0	0	-1	0	≥	-3
ϕ_{DE} -	0	0	0	0	1	0	0	-1	≥	-2
ϕ_{DE} +	0	0	0	0	-1	0	0	1	≥	-2
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4
Z	4/3	4/3	-4/3	2	4/3	0	1	-5	≥	-7/6
ϕ_{BF} -	-4/3	-7/3	4/3	-2	-4/3	0	-1	7	≥	-5/6
ϕ_{BF} +	4/3	7/3	-4/3	2	4/3	0	1	-7	≥	-19/6
ϕ_{FG} -	0	0	0	0	0	1	0	-1	≥	-3/2
$\phi_{\text{FG}}\text{+}$	0	0	0	0	0	-1	0	1	≥	-3/2
$\phi_{\text{GH}}\text{-}$	5/9	19/18	-5/9	1	19/18	1	1	-14/3	≥	-10/9
ϕ_{GH} +	-5/9	-19/18	5/9	-1	-19/18	-1	-1	14/3	≥	-17/9
$\phi_{\text{HG}}\text{-}$	-2	-3	2	-2	-3	-1	-2	11	≥	-1/2
$\phi_{\text{HG}}\text{+}$	2	3	-2	2	3	1	2	-11	≥	-5/2
ϕ_{DH} -	0	0	0	0	-1	0	-1	1	≥	-7/2
$\phi_{\text{DH}}\text{+}$	0	0	0	0	1	0	1	-1	≥	-1/2
L_{X}	0	0	0	0	0	0	0	-1	≥	-2
Max	1/9	1/9	-1/9	0	1/9	0	0	-1/3	=	2/9

Scambio pivotale 6-1

Ooaiii	DIO PIV	olaio (
	φ_{BC} +	Υ	$\phi_{\text{ED}}\text{+}$	Т	S	R	ϕ_{DC}	X-]		[Fb]
ϕ_{AB} -	-3/4	-1	1	-3/2	-1	0	-3/4	7/2	\geq	-9/4
ϕ_{AB} +	3/4	1	-1	3/2	1	0	3/4	-7/2	\geq	-7/4
$\phi_{\text{BA}}\text{-}$	0	1	0	0	0	0	0	-1	\geq	-2
$\phi_{\text{BA}}\text{+}$	0	-1	0	0	0	0	0	1	\geq	-2
ϕ_{BC} -	-1	0	0	0	0	0	0	0	\geq	-3
Χ	-3/4	-1	1	-3/2	-1	0	-3/4	9/2	\geq	-1/4
ϕ_{CD}	0	0	0	1	0	0	0	-1	\geq	-3/2
ϕ_{CD} +	0	0	0	-1	0	0	0	1	\geq	-3/2
$\alpha b F$	-1/12	0	0	-1/6	0	0	-1/12	1/6	\geq	-1/4
ϕ_{DC} +	0	0	0	0	0	0	-1	0	\geq	-3
$\phi_{\text{DE}}\text{-}$	0	0	0	0	1	0	0	-1	\geq	-2
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	-1	0	0	1	\geq	-2
$\phi_{\text{ED}}\text{-}$	0	0	-1	0	0	0	0	0	\geq	-4
Z	-1	0	0	0	0	0	0	1	\geq	-3/2
$\phi_{\text{BF}}\text{-}$	1	-1	0	0	0	0	0	1	\geq	-1/2
$\phi_{\text{BF}}\text{+}$	-1	1	0	0	0	0	0	-1	\geq	-7/2
$\phi_{\text{FG}}\text{-}$	0	0	0	0	0	1	0	-1	\geq	-3/2
$\phi_{\text{FG}}\text{+}$	0	0	0	0	0	-1	0	1	\geq	-3/2
ϕ_{GH} -	-5/12	1/2	0	1/6	1/2	1	7/12	-13/6	\geq	-5/4
$\phi_{\text{GH}} \textbf{+}$	5/12	-1/2	0	-1/6	-1/2	-1	-7/12	13/6	\geq	-7/4
$\phi_{\text{HG}}\text{-}$	3/2	-1	0	1	-1	-1	-1/2	2	\geq	0
$\phi_{\text{HG}} \textbf{+}$	-3/2	1	0	-1	1	1	1/2	-2	\geq	-3
$\phi_{\text{DH}}\text{-}$	0	0	0	0	-1	0	-1	1	\geq	-7/2
$\phi_{\text{DH}} \textbf{+}$	0	0	0	0	1	0	1	-1	\geq	-1/2
L_{X}	0	0	0	0	0	0	0	-1	\geq	-2
Max	-1/12	0	0	-1/6	0	0	-1/12	1/6	=	-1/4

Scambio pivotale 2-8

Ooaiii	5.0 p. vo.	.a.o _ c						_		
	φ _{BC} +	Υ	ϕ_{ED} +	Т	S	R	ϕ_{DC} -	ϕ_{AB} +		[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	-4
X-	3/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1/2
$\phi_{\text{BA}}\text{-}$	-3/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2
ϕ_{BA} +	3/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-5/2
ϕ_{BC} -	-1	0	0	0	0	0	0	0	≥	-3
Χ	3/14	2/7	-2/7	3/7	2/7	0	3/14	-9/7	≥	-5/2
φ _{CD} -	-3/14	-2/7	2/7	4/7	-2/7	0	-3/14	2/7	≥	-1
ϕ_{CD} +	3/14	2/7	-2/7	-4/7	2/7	0	3/14	-2/7	≥	-2
$\alpha b F$	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	≥	-1/3
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3
ϕ_{DE} -	-3/14	-2/7	2/7	-3/7	5/7	0	-3/14	2/7	≥	-3/2
ϕ_{DE} +	3/14	2/7	-2/7	3/7	-5/7	0	3/14	-2/7	≥	-5/2
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4
Z	-11/14	2/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-2
$\phi_{\text{BF}}\text{-}$	17/14	-5/7	-2/7	3/7	2/7	0	3/14	-2/7	≥	-1
ϕ_{BF} +	-17/14	5/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3
ϕ_{FG} -	-3/14	-2/7	2/7	-3/7	-2/7	1	-3/14	2/7	≥	-1
ϕ_{FG} +	3/14	2/7	-2/7	3/7	2/7	-1	3/14	-2/7	≥	-2
φ _{GH} -	-37/42	-5/42	13/21	-16/21	-5/42	1	5/42	13/21	≥	-1/6
ϕ_{GH} +	37/42	5/42	-13/21	16/21	5/42	-1	-5/42	-13/21	≥	-17/6
φ _{HG} -	27/14	-3/7	-4/7	13/7	-3/7	-1	-1/14	-4/7	≥	-1
ϕ_{HG} +	-27/14	3/7	4/7	-13/7	3/7	1	1/14	4/7	≥	-2
φ _{DH} -	3/14	2/7	-2/7	3/7	-5/7	0	-11/14	-2/7	≥	-4
φ _{DH} +	-3/14	-2/7	2/7	-3/7	5/7	0	11/14	2/7	≥	0
L_{x}	-3/14	-2/7	2/7	-3/7	-2/7	0	-3/14	2/7	≥	-3/2
Max	-1/21	1/21	-1/21	-2/21	1/21	0	-1/21	-1/21	=	-1/3

Scambio pivotale 24-2

	φ _{BC} +	ϕ_{DH} +	ϕ_{ED} +	Т	S	R	ϕ_{DC}	φ _{AB} +]	[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	-1] ≥	-4
X-	0	-1	0	0	1	0	1	0	≥	-1/2
ϕ_{BA} -	-3/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3/2
ϕ_{BA} +	3/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-5/2
ϕ_{BC} -	-1	0	0	0	0	0	0	0	≥	-3
Χ	0	-1	0	0	1	0	1	-1	≥	-5/2
$\phi_{\text{CD}}\text{-}$	0	1	0	1	-1	0	-1	0	≥	-1
$\phi_{\text{CD}}\text{+}$	0	-1	0	-1	1	0	1	0	≥	-2
αbF	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	≥	-1/3
$\phi_{\text{DC}}\text{+}$	0	0	0	0	0	0	-1	0	≥	-3
ϕ_{DE} -	0	1	0	0	0	0	-1	0	≥	-3/2
ϕ_{DE} +	0	-1	0	0	0	0	1	0	≥	-5/2
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4
Z	-1	-1	0	0	1	0	1	0	≥	-2
$\phi_{\text{BF}}\text{-}$	7/4	5/2	-1	3/2	-3/2	0	-7/4	-1	≥	-1
ϕ_{BF} +	-7/4	-5/2	1	-3/2	3/2	0	7/4	1	≥	-3
ϕ_{FG} -	0	1	0	0	-1	1	-1	0	≥	-1
$\phi_{\text{FG}}\text{+}$	0	-1	0	0	1	-1	1	0	≥	-2
$\phi_{\text{GH}}\text{-}$	-19/24	5/12	1/2	-7/12	-5/12	1	-5/24	1/2	≥	-1/6
$\phi_{\text{GH}} \textbf{+}$	19/24	-5/12	-1/2	7/12	5/12	-1	5/24	-1/2	≥	-17/6
$\phi_{\text{HG}}\text{-}$	9/4	3/2	-1	5/2	-3/2	-1	-5/4	-1	≥	-1
ϕ_{HG} +	-9/4	-3/2	1	-5/2	3/2	1	5/4	1	≥	-2
$\phi_{\text{DH}}\text{-}$	0	-1	0	0	0	0	0	0	≥	-4
Υ	-3/4	-7/2	1	-3/2	5/2	0	11/4	1	≥	0
L_{X}	0	1	0	0	-1	0	-1	0	≥	-3/2
Max	-1/12	-1/6	0	-1/6	1/6	0	1/12	0	=	1/3 _

Scambio pivotale 19-5

	[φ _{BC} +	φ _{DH} +	φ _{ED} +	Т	φ _{GH} -	R	φ _{DC} -	φ _{AB} + _		[Fb]	
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	[-4]	
X-	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-9/10	
ϕ_{BA} -	-18/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-21/10	
ϕ_{BA} +	18/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-19/10	
ϕ_{BC} -	-1	0	0	0	0	0	0	0	≥	-3	
Χ	-19/10	0	6/5	-7/5	-12/5	12/5	1/2	1/5	≥	-29/10	
φ _{CD} -	19/10	0	-6/5	12/5	12/5	-12/5	-1/2	-6/5	≥	-3/5	
ϕ_{CD} +	-19/10	0	6/5	-12/5	-12/5	12/5	1/2	6/5	≥	-12/5	
αbF	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	≥	-2/5	
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3	
ϕ_{DE} -	0	1	0	0	0	0	-1	0	≥	-3/2	
ϕ_{DE} +	0	-1	0	0	0	0	1	0	≥	-5/2	
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4	
Z	-29/10	0	6/5	-7/5	-12/5	12/5	1/2	6/5	≥	-12/5	
ϕ_{BF} -	23/5	1	-14/5	18/5	18/5	-18/5	-1	-14/5	≥	-2/5	
ϕ_{BF} +	-23/5	-1	14/5	-18/5	-18/5	18/5	1	14/5	≥	-18/5	
ϕ_{FG} -	19/10	0	-6/5	7/5	12/5	-7/5	-1/2	-6/5	≥	-3/5	
ϕ_{FG} +	-19/10	0	6/5	-7/5	-12/5	7/5	1/2	6/5	≥	-12/5	
S	-19/10	1	6/5	-7/5	-12/5	12/5	-1/2	6/5	≥	-2/5	
ϕ_{GH} +	0	0	0	0	-1	0	0	0	≥	-3	
ϕ_{HG} -	51/10	0	-14/5	23/5	18/5	-23/5	-1/2	-14/5	≥	-2/5	
φ _{HG} +	-51/10	0	14/5	-23/5	-18/5	23/5	1/2	14/5	≥	-13/5	
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-4	
Υ	-11/2	-1	4	-5	-6	6	3/2	4	≥	-1	
L_{x}	19/10	0	-6/5	7/5	12/5	-12/5	-1/2	-6/5	≥	-11/10	
Max	-2/5	0	1/5	-2/5	-2/5	2/5	0	1/5	=	2/5]	

Scambio pivotale 21-6

	[φ _{BC} +	ϕ_{DH} +	ϕ_{ED} +	Т	φ _{GH} -	φ _{HG} -	ϕ_{DC} -	ϕ_{AB} +]		[Fb]
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	-4
X-	35/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	≥	-51/46
ϕ_{BA} -	9/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	≥	-111/46
ϕ_{BA} +	-9/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	≥	-73/46
ϕ_{BC} -	-1	0	0	0	0	0	0	0	≥	-3
X	35/46	0	-6/23	1	-12/23	-12/23	11/46	-29/23	≥	-143/46
ϕ_{CD} -	-35/46	0	6/23	0	12/23	12/23	-11/46	6/23	≥	-9/23
ϕ_{CD} +	35/46	0	-6/23	0	-12/23	-12/23	11/46	-6/23	≥	-60/23
αbF	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	≥	-10/23
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3
$\phi_{\text{DE}}\text{-}$	0	1	0	0	0	0	-1	0	≥	-3/2
$\phi_{\text{DE}} \textbf{+}$	0	-1	0	0	0	0	1	0	≥	-5/2
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4
Z	-11/46	0	-6/23	1	-12/23	-12/23	11/46	-6/23	≥	-60/23
ϕ_{BF} -	14/23	1	-14/23	0	18/23	18/23	-14/23	-14/23	≥	-2/23
ϕ_{BF} +	-14/23	-1	14/23	0	-18/23	-18/23	14/23	14/23	≥	-90/23
ϕ_{FG} -	8/23	0	-8/23	0	30/23	7/23	-8/23	-8/23	≥	-11/23
ϕ_{FG} +	-8/23	0	8/23	0	-30/23	-7/23	8/23	8/23	≥	-58/23
S	35/46	1	-6/23	1	-12/23	-12/23	-35/46	-6/23	≥	-14/23
ϕ_{GH} +	0	0	0	0	-1	0	0	0	≥	-3
R	51/46	0	-14/23	1	18/23	-5/23	-5/46	-14/23	≥	-2/23
ϕ_{HG} +	0	0	0	0	0	-1	0	0	≥	-3
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-4
Υ	53/46	-1	8/23	1	-30/23	-30/23	39/46	8/23	≥	-35/23
L_{x}	-35/46	0	6/23	-1	12/23	12/23	-11/46	6/23	≥	-41/46
Max	1/23	0	-1/23	0	-2/23	-2/23	-1/23	-1/23	=	-10/23

Scambio pivotale 7-1

Coambio pivotaio i i											
	φ _{CD} -	$\phi_{\text{DH}} \textbf{+}$	ϕ_{ED} +	Т	ϕ_{GH} -	ϕ_{HG} -	ϕ_{DC} -	φ_{AB} +		[Fb]	
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	[-4]	
X-	-1	0	0	1	0	0	0	0	≥	-3/2	
ϕ_{BA} -	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	≥	-183/70	
ϕ_{BA} +	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	≥	-97/70	
ϕ_{BC} -	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	≥	-87/35	
Χ	-1	0	0	1	0	0	0	-1	≥	-7/2	
ϕ_{BC} +	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	≥	-18/35	
ϕ_{CD} +	-1	0	0	0	0	0	0	0	≥	-3	
αbF	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	≥	-16/35	
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3	
ϕ_{DE} -	0	1	0	0	0	0	-1	0	≥	-3/2	
ϕ_{DE} +	0	-1	0	0	0	0	1	0	≥	-5/2	
φ _{ED} -	0	0	-1	0	0	0	0	0	≥	-4	
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	≥	-87/35	
ϕ_{BF} -	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5	
ϕ_{BF} +	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	≥	-18/5	
φ _{FG} -	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	≥	-23/35	
ϕ_{FG} +	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	≥	-82/35	
S	-1	1	0	1	0	0	-1	0	≥	-1	
ϕ_{GH} +	0	0	0	0	-1	0	0	0	≥	-3	
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	≥	-23/35	
ϕ_{HG} +	0	0	0	0	0	-1	0	0	≥	-3	
φ _{DH} -	0	-1	0	0	0	0	0	0	≥	-4	
Υ	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35	
L_{x}	1	0	0	-1	0	0	0	0	≥	-1/2	
Max	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	-16/35	

Tableau finale

	[φ _{CD} -	ϕ_{DH} +	ϕ_{ED} +	Т	φ _{GH} -	φ _{HG} -	φ _{DC} -	ϕ_{AB} +		[Fb]	
ϕ_{AB} -	0	0	0	0	0	0	0	-1	≥	「 -4]	
X-	-1	0	0	1	0	0	0	0	≥	-3/2	
ϕ_{BA} -	-18/35	-1	26/35	0	-18/35	-18/35	17/35	26/35	≥	-183/70	
ϕ_{BA} +	18/35	1	-26/35	0	18/35	18/35	-17/35	-26/35	≥	-97/70	
ϕ_{BC} -	46/35	0	-12/35	0	-24/35	-24/35	11/35	-12/35	≥	-87/35	
X	-1	0	0	1	0	0	0	-1	≥	-7/2	
ϕ_{BC} +	-46/35	0	12/35	0	24/35	24/35	-11/35	12/35	≥	-18/35	
ϕ_{CD} +	-1	0	0	0	0	0	0	0	≥	-3	
αbF	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	≥	-16/35	
ϕ_{DC} +	0	0	0	0	0	0	-1	0	≥	-3	
ϕ_{DE} -	0	1	0	0	0	0	-1	0	≥	-3/2	
ϕ_{DE} +	0	-1	0	0	0	0	1	0	≥	-5/2	
ϕ_{ED} -	0	0	-1	0	0	0	0	0	≥	-4	
Z	11/35	0	-12/35	1	-24/35	-24/35	11/35	-12/35	≥	-87/35	
ϕ_{BF} -	-4/5	1	-2/5	0	6/5	6/5	-4/5	-2/5	≥	-2/5	
ϕ_{BF} +	4/5	-1	2/5	0	-6/5	-6/5	4/5	2/5	≥	-18/5	
ϕ_{FG} -	-16/35	0	-8/35	0	54/35	19/35	-16/35	-8/35	≥	-23/35	
ϕ_{FG} +	16/35	0	8/35	0	-54/35	-19/35	16/35	8/35	≥	-82/35	
S	-1	1	0	1	0	0	-1	0	≥	-1	
ϕ_{GH} +	0	0	0	0	-1	0	0	0	≥	-3	
R	-51/35	0	-8/35	1	54/35	19/35	-16/35	-8/35	≥	-23/35	
ϕ_{HG} +	0	0	0	0	0	-1	0	0	≥	-3	
ϕ_{DH} -	0	-1	0	0	0	0	0	0	≥	-4	
Υ	-53/35	-1	26/35	1	-18/35	-18/35	17/35	26/35	≥	-74/35	
L_{x}	1	0	0	-1	0	0	0	0	≥	-1/2	
Max	-2/35	0	-1/35	0	-2/35	-2/35	-2/35	-1/35	=	16/35]	

Vettori soluzione della programmazione lineare

	[X	Υ	Z	Т	S	R	$\alpha b F$	Χ-		[Fb]	
$\phi_{\text{AB}}\text{-}$	0	0	0	0	0	0	0	0	≥	[0]	
ϕ_{AB} +	0	0	0	0	0	0	0	0	≥	1/35	
ϕ_{BA} -	0	0	0	0	0	0	0	0	≥	0	
ϕ_{BA} +	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{BC}}\text{-}$	0	0	0	0	0	0	0	0	≥	0	
ϕ_{BC} +	0	0	0	0	0	0	0	0	≥	0	
ϕ_{CD} -	0	0	0	0	0	0	0	0	≥	2/35	
$\phi_{\text{CD}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{DC}}\text{-}$	0	0	0	0	0	0	0	0	≥	2/35	
$\phi_{\text{DC}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{DE}}\text{-}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{DE}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{ED}}\text{-}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{ED}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	1/35	
$\phi_{\text{BF}}\text{-}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{BF}}\text{+}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{FG}}\text{-}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{FG}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0	
ϕ_{GH} -	0	0	0	0	0	0	0	0	≥	2/35	
$\phi_{\text{GH}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0	
ϕ_{HG} -	0	0	0	0	0	0	0	0	≥	2/35	
$\phi_{\text{HG}} \textbf{+}$	0	0	0	0	0	0	0	0	≥	0	
$\phi_{\text{DH}}\text{-}$	0	0	0	0	0	0	0	0	≥	0	
ϕ_{DH} +	0	0	0	0	0	0	0	0	≥	0	
L_{X}	0	0	0	0	0	0	0	0	≥	0	
Max	7/2	74/35	87/35	0	1	23/35	16/35	3/2	=	-16/35	

Variabili soluzione dedotto il valore X-

Variabili soluzione differenza tra rotazioni

ϕ_{AB}	1/35
ϕ_{BA}	0
ϕ_{BC}	0
ϕ_{CD}	-2/35
ϕ_{DC}	-2/35
ϕ_{DE}	0
ϕ_{ED}	1/35
ϕ_{BF}	0
ϕ_{FG}	0
ϕ_{GH}	-2/35
ϕ_{HG}	-2/35
ϕ_{DH}	[0]

REAZIONI Fattore di collasso = 16/35

 $H_{\Lambda} = -61/70F$

 $V_{\Delta} = 11/7F$

 $W_A = 2Fb$

 $H_{r} = -1/2F$

 $V_E = 3F$

 $W_F = 2Fb$

$$\begin{array}{llll} H_{FG} = 7/6F & H_{GH} = 7/6F & H_{HD} = 7/6F \\ V_{FG} = 23/70F & V_{GH} = -3/2F & V_{HD} = -3/2F \\ W_{FG} = -59/70Fb & W_{GH} = -3/2Fb & W_{HD} = 3/2Fb \\ H_{GF} = -7/6F & H_{HG} = -7/6F & H_{DH} = -7/6F \\ V_{GF} = -23/70F & V_{HG} = 3/2F & V_{DH} = 3/2F \\ W_{GF} = 3/2Fb & W_{HG} = -3/2Fb & W_{DH} = 2Fb \end{array}$$

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_B = 3/35\delta$	$u_{CCB} = 3/35\delta$	$u_D = 3/35\delta$	$u_{EED} = 0$
$V_{AAB} = 0$	$V_B = 0$	$V_{CCB} = -2/35\delta$	$V_D = 0$	$V_{EED} = 0$
$\phi_{AAB} = -1/35\delta/b$	$\phi_{\rm p} = -1/35\delta/b$	$\phi_{CCR} = -1/35\delta/b$	$\phi_{D} = 1/35\delta/b$	$\phi_{E} = -1/35\delta/b$

$u_F = 6/35\delta$	$u_{GGF} = 6/35\delta$	$u_{HHG} = 6/35\delta$
$V_F = 0$	$V_{GGF} = -2/35\delta$	$V_{HHG} = 0$
$\varphi_F = -1/35\delta/b$	$\varphi_{GGF} = -1/35\delta/b$	$\varphi_{HHD} = 1/35\delta/b$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$	$u_{BBC} = 3/35\delta$	$u_{CCD} = 3/35\delta$	$u_{DDE} = 3/35\delta$	$u_{BBF} = 3/35\delta$
$V_{AAB} = 0$		$V_{CCD} = -2/35\delta$	$V_{DDE} = 0$	$V_{BBF} = 0$
$\phi_{AAB} = -1/35\delta/b$	$\phi_{BBC} = -1/35\delta/b$	$\varphi_{CCD} = 1/35\delta/b$	$\phi_{DDE} = -1/35\delta/b$	$\phi_{BBF} = -1/35\delta/b$

$$\begin{array}{lll} u_{FFG} = 6/35\delta & u_{GGH} = 6/35\delta & u_{HHD} = 6/35\delta \\ v_{FFG} = 0 & v_{GGH} = -2/35\delta & v_{HHD} = 0 \\ \phi_{FFG} = -1/35\delta/b & \phi_{GGH} = 1/35\delta/b & \phi_{HHD} = -1/35\delta/b \end{array}$$