

Università degli studi di Bergamo
Scuola di Ingegneria (Dalmine)

CCS Ingegneria Edile

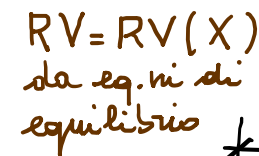
LM-24 Ingegneria delle Costruzioni Edili

Complementi di Scienza delle Costruzioni
(ICAR/08 - SdC; 6 CFU)

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LEZIONE 09


$$y_i(x_i) \underset{|y_i| \ll 1}{\approx} \chi_i(x_i) = \underbrace{\chi_{ie}(x_i)}_{M_i(x_i)} + \underbrace{\chi_{it}(x_i)}_{V_i(x_i)}$$

$$EJ \cdot y_i''(x_i) = \frac{M_i(x_i)}{y_i(x_i)} + EJ \cdot \psi_t(x_i)$$

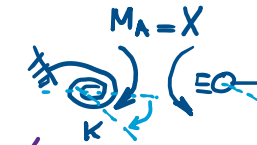

Tratto 1 :

Tratto 2:

$$EJ y_2(x_2) = \underbrace{B_1 x_2 + B_2}_{\text{Integration Constant}}$$

$$A_1, A_2, B_1, B_2; X$$

Scrittura e imposizione delle condizioni al contorno (c.c.) [in numero pari a $n_{cc} = 2n + 1 = 5$]

- $y_1(0) = 0$ *cerniera in A*
 - $y_1'(0) = -\frac{X}{K}$ *molla elastica rotazionale in A* 
 - $y_1(b) = -\delta + \epsilon_t b = 0$ *carrello in C con cedimento $\delta + (\Delta l_t^{BC} = \epsilon_t b)$*
 - $y_1'(b) = y_2'(b)$ *continuità alla rotazione in B* 
 - $y_2(b) = 0$ *cerniera in A + invest. assiale di AB ($EA_{AB} \rightarrow \infty$)*
- spost. assolute*
 \downarrow
B è nodo fisso ($u_B = v_B = 0$)

$EJ y_1(0) = 0 \Rightarrow A_2 = 0$

$EJ y_1'(0) = A_1 = -\frac{X}{K} EJ = -\frac{X}{\beta \frac{EI}{L}} = -\frac{L}{\beta} X \Rightarrow A_1 = A_1(X) = -\frac{L}{\beta} X \rightarrow A_1 = -\frac{L}{\beta} \frac{13}{8} \frac{FL}{3+\beta} = -\frac{13}{8} \frac{1}{3+\beta} FL^2 = A_1$

$EJ y_1(b) = -\frac{F}{b} \frac{b^4}{24} + \left(\frac{F}{2} + \frac{X}{b}\right) \frac{b^3}{6} + (Fb - X) \frac{b^2}{2} + A_1 b + A_2 = 0 \Rightarrow \left(\frac{1}{6} - \frac{3}{2} \frac{1}{\beta} - \frac{1}{\beta}\right) X b^2 = \left(\frac{1}{24} - \frac{1}{12} - \frac{1}{2}\right) FL^2$

$EJ y_1'(b) = -\frac{F}{b} \frac{b^3}{6} + \left(\frac{F}{2} + \frac{X}{b}\right) \frac{b^2}{2} + (Fb - X)b + A_1 = EJ y_2'(b) = B_1$

$EJ y_2(b) = B_1 b + B_2 = 0 \Rightarrow B_2 = -B_1 b \rightarrow u_c = \phi_B b$
($v_c = -\delta$)

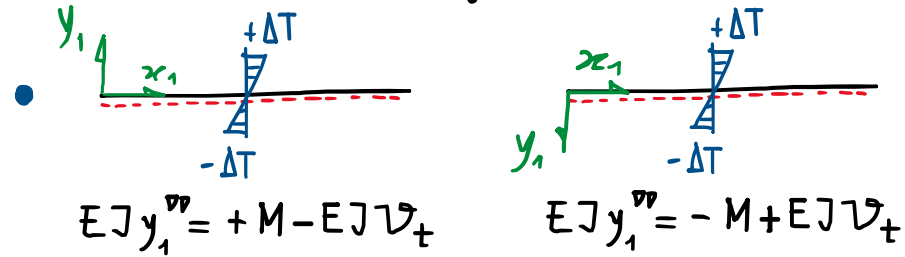
$\phi_B \geq |\phi_A|$
 $\phi_B = \frac{B_1}{EJ} = \frac{13}{8} \frac{1+\beta/6}{3+\beta} \frac{FL^2}{EJ}$

$B_2 = -\frac{13}{48} \frac{6+\beta}{3+\beta} FL^3$ $B_1 = \frac{13}{48} \frac{6+\beta}{3+\beta} FL^2$

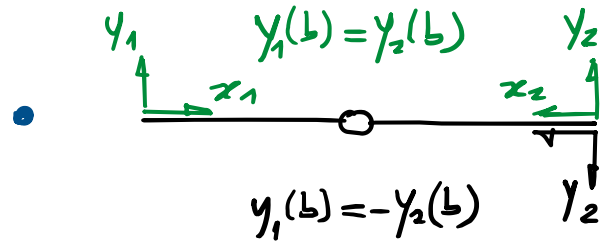
$X = \frac{13}{8} \frac{\beta}{3+\beta} FL$ $\beta \rightarrow 0, X \rightarrow 0$
 $\beta \rightarrow \infty, X \rightarrow \frac{13}{24} FL$

- LE finali, sost. A_1, A_2, B_1, B_2 e X e tracciamento della deformata qualitativa.

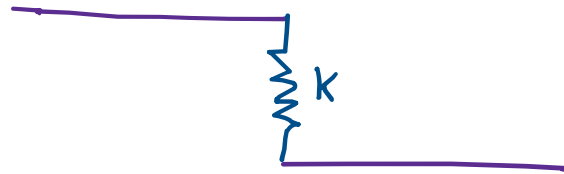
- Attenzione ai segni su LE e c.c.!



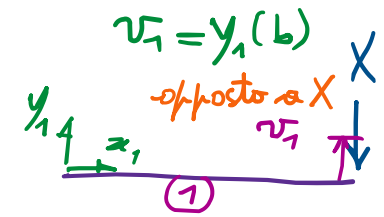
curvature elastiche e termiche



molla relativa
elongazionale



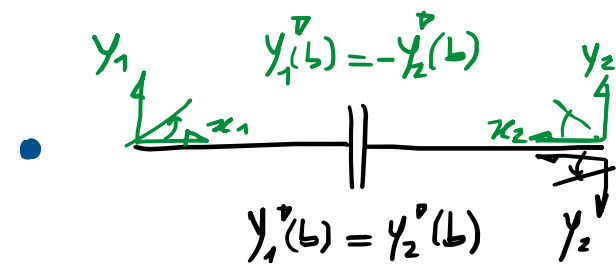
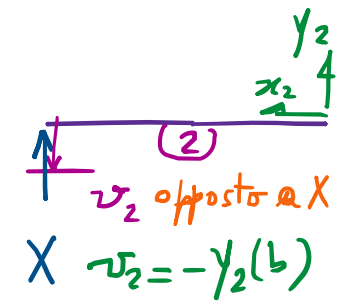
letto nel riferim. locale



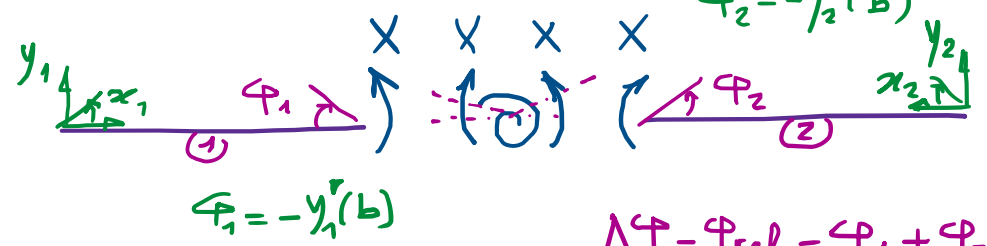
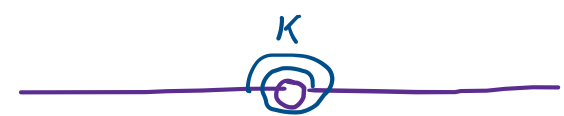
$$y_1(l) - y_2(l) = \frac{X}{K}$$



spost. relative
 $\Delta l = v_{rel} = v_1 + v_2$
 $= \frac{X}{K}$ legge di Hooke



rotazionale



$$-y_1'(l) - y_2'(l) = \frac{X}{K}$$

$\phi_2 = -y_2'(l)$
 $\Delta \phi = \phi_{rel} = \phi_1 + \phi_2$
 $= \frac{X}{K}$