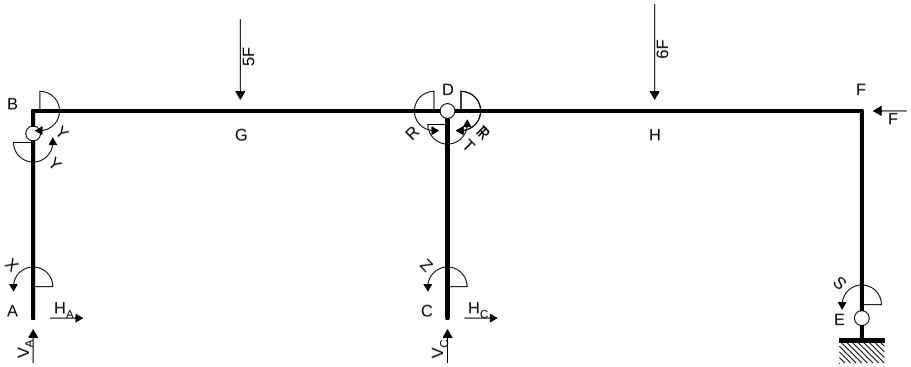


EQUAZIONI DI EQUILIBRIO

Rotazione intorno a E: aste EF FH HD DC DG GB BA

$-4V_Ab -2V_Cb = -Xb -Zb -Sb -22Fb$

Rotazione intorno a D: aste DC

$H_Cb = -Zb -Tb$

Rotazione intorno a D: aste DG GB BA

$H_Ab -2V_Ab = -Xb -Rb -5Fb$

Rotazione intorno a B: aste BA

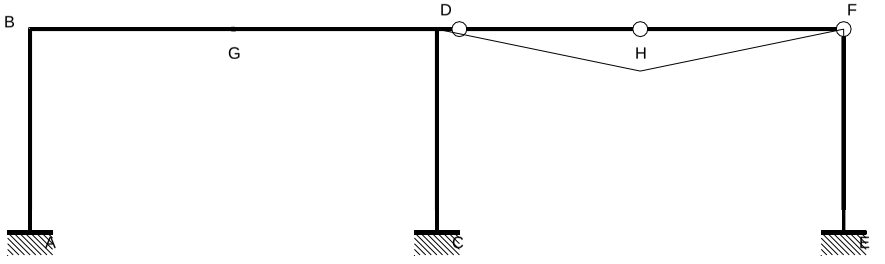
$H_Ab = -Xb -Yb$

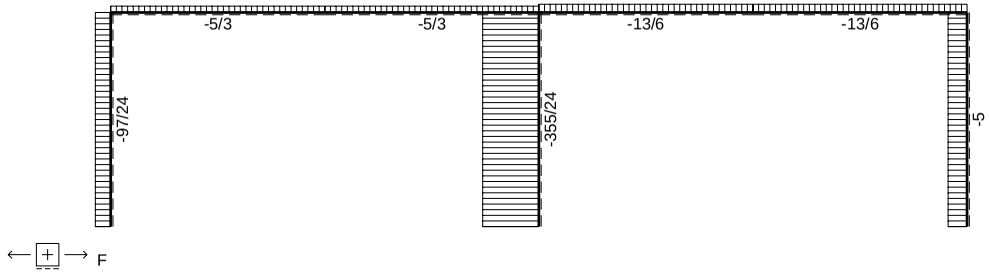
Matrice di equilibrio

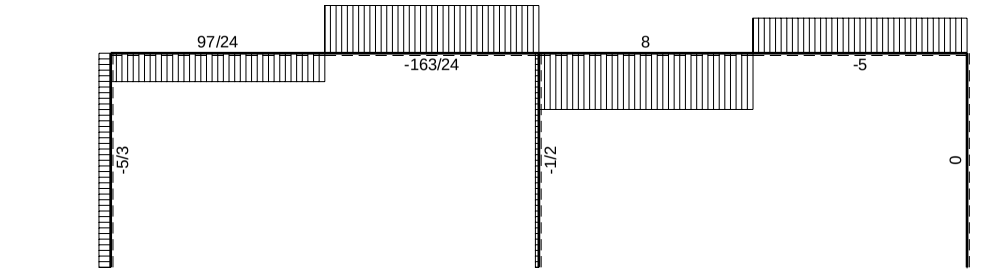
$$\begin{array}{c} \\ \varphi_{EF} \\ \varphi_{DC} \\ \varphi_{DG} \\ \varphi_{BA} \end{array} \begin{bmatrix} H_Ab & V_Ab & H_Cb & V_Cb \\ 0 & -4 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 1 & -2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1 & 0 & -1 & 0 & -1 & 0 & -22 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & -1 & -5 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

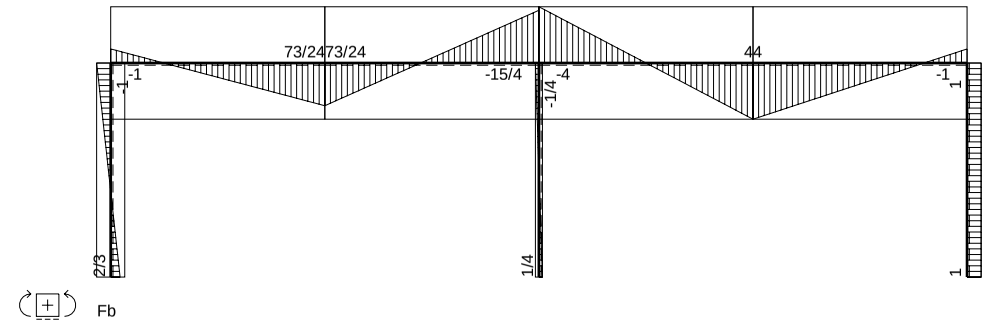
Soluzione del sistema

$$\begin{bmatrix} V_Ab \\ H_Cb \\ H_Ab \\ V_Cb \end{bmatrix} = \begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ 0 & -1/2 & 0 & 0 & 0 & 1/2 & 5/2 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1 & 1/2 & 0 & 1/2 & -1 & 6 \end{bmatrix}$$









PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	$\ge$	H_{14}
D_2	H_{21}	H_{22}	H_{23}	$\ge$	H_{24}
D_3	H_{31}	H_{32}	H_{33}	$\ge$	H_{34}
D_4	H_{41}	H_{42}	H_{43}	$\ge$	H_{44}
D_5	H_{51}	H_{52}	H_{53}	$\ge$	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{CD}$ $T=W_{DC}$ $S=W_{EF}$ $R=W_{DG}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	1	0	0	0	0	0	0	≤	1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	1	0	0	0	0	0	≤	1
W_{CD}-	0	0	1	0	0	0	0	≥	-1/4
W_{CD}+	0	0	1	0	0	0	0	≤	1/4
W_{DC}-	0	0	0	1	0	0	0	≥	-1/4
W_{DC}+	0	0	0	1	0	0	0	≤	1/4
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	1	0	0	≤	1
W_{FE}-	-1	-1	-1	-1	-1	0	-1	≥	-1
W_{FE}+	-1	-1	-1	-1	-1	0	-1	≤	1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	-1/2	0	0	0	-1/2	-5/2	≤	4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	1	0	≤	4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	-1	0	-1	0	≤	4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-7/2	≥	-4
W_{HF}+	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-7/2	≤	4
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	0	≥	-1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	≥	-1
W_{CD}-	0	0	1	0	0	0	0	≥	-1/4
W_{CD}+	0	0	-1	0	0	0	0	≥	-1/4
W_{DC}-	0	0	0	1	0	0	0	≥	-1/4
W_{DC}+	0	0	0	-1	0	0	0	≥	-1/4
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	≥	-1
W_{FE}-	-1	-1	-1	-1	-1	0	-1	≥	-1
W_{FE}+	1	1	1	1	1	0	1	≥	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	5/2	≥	-4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	≥	-4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	≥	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-7/2	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	7/2	≥	-4
Max	0	0	0	0	0	0	1	=	0

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Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1
W_{CD}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-1/4
W_{CD}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-1/4
W_{DC}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-1/4
W_{DC}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-1/4
W_{EF}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1
W_{FE}-	-1	-1	-1	-1	-1	0	1	1	1	1	1	0	-1	≥	-1
W_{FE}+	1	1	1	1	1	0	-1	-1	-1	-1	-1	0	1	≤	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	0	1/2	0	0	0	1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	0	-1/2	0	0	0	-1/2	5/2	≤	-4
W_{DG}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-4
W_{DH}-	0	0	0	-1	0	-1	0	0	0	1	0	1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	0	0	-1	0	-1	0	≤	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	1/2	1/2	1/2	1	1/2	1/2	-7/2	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	-1/2	-1/2	-1/2	-1	-1/2	-1/2	7/2	≤	-4
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.11.05.11 08.06.11

Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-1/4
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-1/4
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-1/4
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-1/4
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	-1	-1	-1	-1	-1	0	-1	5	≥	-1
φ_{FE}+	1	1	1	1	1	0	1	-5	≥	-1
φ_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	1	≥	-4
φ_{GD}+	0	1/2	0	0	0	1/2	5/2	-1	≥	-4
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-7/2	7/2	≥	-4
φ_{HF}+	1/2	1/2	1/2	1	1/2	1/2	7/2	-7/2	≥	-4
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 11-7

	X	Y	Z	T	S	R	φ_{FE}-	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-1/4
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-1/4
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-1/4
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-1/4
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
αbF	-1	-1	-1	-1	-1	0	-1	5	≥	-1
φ_{FE}+	0	0	0	0	0	0	-1	0	≥	-2
φ_{GD}-	5/2	2	5/2	5/2	5/2	-1/2	5/2	-23/2	≥	-3/2
φ_{GD}+	-5/2	-2	-5/2	-5/2	-5/2	1/2	-5/2	23/2	≥	-13/2
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	3	3	3	5/2	3	-1/2	7/2	-14	≥	-1/2
φ_{HF}+	-3	-3	-3	-5/2	-3	1/2	-7/2	14	≥	-15/2
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-1	-1	-1	-1	-1	0	-1	5	=	-1

Scambio pivotale 19-8

	X	Y	Z	T	S	R	φ_{FE}-	φ_{HF}-		Fb
φ_{AB}-	11/14	-3/14	-3/14	-5/28	-3/14	1/28	-1/4	1/14	≥	-27/28
φ_{AB}+	-11/14	3/14	3/14	5/28	3/14	-1/28	1/4	-1/14	≥	-29/28
φ_{BA}-	-3/14	11/14	-3/14	-5/28	-3/14	1/28	-1/4	1/14	≥	-27/28
φ_{BA}+	3/14	-11/14	3/14	5/28	3/14	-1/28	1/4	-1/14	≥	-29/28
φ_{CD}-	-3/14	-3/14	11/14	-5/28	-3/14	1/28	-1/4	1/14	≥	-3/14
φ_{CD}+	3/14	3/14	-11/14	5/28	3/14	-1/28	1/4	-1/14	≥	-2/7
φ_{DC}-	-3/14	-3/14	-3/14	23/28	-3/14	1/28	-1/4	1/14	≥	-3/14
φ_{DC}+	3/14	3/14	3/14	-23/28	3/14	-1/28	1/4	-1/14	≥	-2/7
φ_{EF}-	-3/14	-3/14	-3/14	-5/28	11/14	1/28	-1/4	1/14	≥	-27/28
φ_{EF}+	3/14	3/14	3/14	5/28	-11/14	-1/28	1/4	-1/14	≥	-29/28
αbF	1/14	1/14	1/14	-3/28	1/14	-5/28	1/4	-5/14	≥	-33/28
φ_{FE}+	0	0	0	0	0	0	-1	0	≥	-2
φ_{GD}-	1/28	-13/28	1/28	25/56	1/28	-5/56	-3/8	23/28	≥	-61/56
φ_{GD}+	-1/28	13/28	-1/28	-25/56	-1/28	5/56	3/8	-23/28	≥	-387/56
φ_{DG}-	-3/14	-3/14	-3/14	-5/28	-3/14	29/28	-1/4	1/14	≥	-111/28
φ_{DG}+	3/14	3/14	3/14	5/28	3/14	-29/28	1/4	-1/14	≥	-113/28
φ_{DH}-	3/7	3/7	3/7	-9/14	3/7	-15/14	1/2	-1/7	≥	-57/14
φ_{DH}+	-3/7	-3/7	-3/7	9/14	-3/7	15/14	-1/2	1/7	≥	-55/14
X-	3/14	3/14	3/14	5/28	3/14	-1/28	1/4	-1/14	≥	-1/28
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	-3/14	-3/14	-3/14	-5/28	-3/14	1/28	-1/4	1/14	≥	-111/28
Max	1/14	1/14	1/14	-3/28	1/14	-5/28	1/4	-5/14	=	-33/28

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Scambio pivotale 5-7

	X	Y	Z	T	S	R	φ_{CD}-	φ_{HF}-		Fb
φ_{AB}-	1	0	-1	0	0	0	1	0	≥	-3/4
φ_{AB}+	-1	0	1	0	0	0	-1	0	≥	-5/4
φ_{BA}-	0	1	-1	0	0	0	1	0	≥	-3/4
φ_{BA}+	0	-1	1	0	0	0	-1	0	≥	-5/4
φ_{FE}-	-6/7	-6/7	22/7	-5/7	-6/7	1/7	-4	2/7	≥	-6/7
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-1/2
φ_{DC}-	0	0	-1	1	0	0	1	0	≥	0
φ_{DC}+	0	0	1	-1	0	0	-1	0	≥	-1/2
φ_{EF}-	0	0	-1	0	1	0	1	0	≥	-3/4
φ_{EF}+	0	0	1	0	-1	0	-1	0	≥	-5/4
αbF	-1/7	-1/7	6/7	-2/7	-1/7	-1/7	-1	-2/7	≥	-39/28
φ_{FE}+	6/7	6/7	-22/7	5/7	6/7	-1/7	4	-2/7	≥	-8/7
φ_{GD}-	5/14	-1/7	-8/7	5/7	5/14	-1/7	3/2	5/7	≥	-43/56
φ_{GD}+	-5/14	1/7	8/7	-5/7	-5/14	1/7	-3/2	-5/7	≥	-405/56
φ_{DG}-	0	0	-1	0	0	1	1	0	≥	-15/4
φ_{DG}+	0	0	1	0	0	-1	-1	0	≥	-17/4
φ_{DH}-	0	0	2	-1	0	-1	-2	0	≥	-9/2
φ_{DH}+	0	0	-2	1	0	1	2	0	≥	-7/2
X-	0	0	1	0	0	0	-1	0	≥	-1/4
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	-1	0	0	0	1	0	≥	-15/4
Max	-1/7	-1/7	6/7	-2/7	-1/7	-1/7	-1	-2/7	=	-39/28

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.11.05.11 08.06.11

Scambio pivotale 7-3

	X	Y	φ_{DC}-	T	S	R	φ_{CD}-	φ_{HF}-		Fb
φ_{AB}-	1	0	1	-1	0	0	0	0	≥	-3/4
φ_{AB}+	-1	0	-1	1	0	0	0	0	≥	-5/4
φ_{BA}-	0	1	1	-1	0	0	0	0	≥	-3/4
φ_{BA}+	0	-1	-1	1	0	0	0	0	≥	-5/4
φ_{FE}-	-6/7	-6/7	-22/7	17/7	-6/7	1/7	-6/7	2/7	≥	-6/7
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-1/2
Z	0	0	-1	1	0	0	1	0	≥	0
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-1/2
φ_{EF}-	0	0	1	-1	1	0	0	0	≥	-3/4
φ_{EF}+	0	0	-1	1	-1	0	0	0	≥	-5/4
αbF	-1/7	-1/7	-6/7	4/7	-1/7	-1/7	-1/7	-2/7	≥	-39/28
φ_{FE}+	6/7	6/7	22/7	-17/7	6/7	-1/7	6/7	-2/7	≥	-8/7
φ_{GD}-	5/14	-1/7	8/7	-3/7	5/14	-1/7	5/14	5/7	≥	-43/56
φ_{GD}+	-5/14	1/7	-8/7	3/7	-5/14	1/7	-5/14	-5/7	≥	-405/56
φ_{DG}-	0	0	1	-1	0	1	0	0	≥	-15/4
φ_{DG}+	0	0	-1	1	0	-1	0	0	≥	-17/4
φ_{DH}-	0	0	-2	1	0	-1	0	0	≥	-9/2
φ_{DH}+	0	0	2	-1	0	1	0	0	≥	-7/2
X-	0	0	-1	1	0	0	0	0	≥	-1/4
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	1	-1	0	0	0	0	≥	-15/4
Max	-1/7	-1/7	-6/7	4/7	-1/7	-1/7	-1/7	-2/7	=	-39/28

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Scambio pivotale 12-4

	X	Y	φ_{DC}-	φ_{FE}+	S	R	φ_{CD}-	φ_{HF}-		Fb
φ_{AB}-	11/17	-6/17	-5/17	7/17	-6/17	1/17	-6/17	2/17	≥	-19/68
φ_{AB}+	-11/17	6/17	5/17	-7/17	6/17	-1/17	6/17	-2/17	≥	-117/68
φ_{BA}-	-6/17	11/17	-5/17	7/17	-6/17	1/17	-6/17	2/17	≥	-19/68
φ_{BA}+	6/17	-11/17	5/17	-7/17	6/17	-1/17	6/17	-2/17	≥	-117/68
φ_{FE}-	0	0	0	-1	0	0	0	0	≥	-2
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-1/2
Z	6/17	6/17	5/17	-7/17	6/17	-1/17	23/17	-2/17	≥	-8/17
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-1/2
φ_{EF}-	-6/17	-6/17	-5/17	7/17	11/17	1/17	-6/17	2/17	≥	-19/68
φ_{EF}+	6/17	6/17	5/17	-7/17	-11/17	-1/17	6/17	-2/17	≥	-117/68
αbF	1/17	1/17	-2/17	-4/17	1/17	-3/17	1/17	-6/17	≥	-113/68
T	6/17	6/17	22/17	-7/17	6/17	-1/17	6/17	-2/17	≥	-8/17
φ_{GD}-	7/34	-5/17	10/17	3/17	7/34	-2/17	7/34	13/17	≥	-77/136
φ_{GD}+	-7/34	5/17	-10/17	-3/17	-7/34	2/17	-7/34	-13/17	≥	-1011/136
φ_{DG}-	-6/17	-6/17	-5/17	7/17	-6/17	18/17	-6/17	2/17	≥	-223/68
φ_{DG}+	6/17	6/17	5/17	-7/17	6/17	-18/17	6/17	-2/17	≥	-321/68
φ_{DH}-	6/17	6/17	-12/17	-7/17	6/17	-18/17	6/17	-2/17	≥	-169/34
φ_{DH}+	-6/17	-6/17	12/17	7/17	-6/17	18/17	-6/17	2/17	≥	-103/34
X-	6/17	6/17	5/17	-7/17	6/17	-1/17	6/17	-2/17	≥	-49/68
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	-6/17	-6/17	-5/17	7/17	-6/17	1/17	-6/17	2/17	≥	-223/68
Max	1/17	1/17	-2/17	-4/17	1/17	-3/17	1/17	-6/17	=	-113/68

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Scambio pivotale 3-1

	φ_{BA}-	Y	φ_{DC}-	φ_{FE}+	S	R	φ_{CD}-	φ_{HF}-		Fb
φ_{AB}-	-11/6	5/6	-5/6	7/6	-1	1/6	-1	1/3	≥	-19/24
φ_{AB}+	11/6	-5/6	5/6	-7/6	1	-1/6	1	-1/3	≥	-29/24
X	-17/6	11/6	-5/6	7/6	-1	1/6	-1	1/3	≥	-19/24
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
φ_{FE}-	0	0	0	-1	0	0	0	0	≥	-2
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-1/2
Z	-1	1	0	0	0	0	1	0	≥	-3/4
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-1/2
φ_{EF}-	1	-1	0	0	1	0	0	0	≥	0
φ_{EF}+	-1	1	0	0	-1	0	0	0	≥	-2
αbF	-1/6	1/6	-1/6	-1/6	0	-1/6	0	-1/3	≥	-41/24
T	-1	1	1	0	0	0	0	0	≥	-3/4
φ_{GD}-	-7/12	1/12	5/12	5/12	0	-1/12	0	5/6	≥	-35/48
φ_{GD}+	7/12	-1/12	-5/12	-5/12	0	1/12	0	-5/6	≥	-349/48
φ_{DG}-	1	-1	0	0	0	1	0	0	≥	-3
φ_{DG}+	-1	1	0	0	0	-1	0	0	≥	-5
φ_{DH}-	-1	1	-1	0	0	-1	0	0	≥	-21/4
φ_{DH}+	1	-1	1	0	0	1	0	0	≥	-11/4
X-	-1	1	0	0	0	0	0	0	≥	-1
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	1	-1	0	0	0	0	0	0	≥	-3
Max	-1/6	1/6	-1/6	-1/6	0	-1/6	0	-1/3	=	-41/24

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Scambio pivotale 9-2

	φ_{BA}-	φ_{EF}-	φ_{DC}-	φ_{FE}+	S	R	φ_{CD}-	φ_{HF}-		Fb
φ_{AB}-	-1	-5/6	-5/6	7/6	-1/6	1/6	-1	1/3	≥	-19/24
φ_{AB}+	1	5/6	5/6	-7/6	1/6	-1/6	1	-1/3	≥	-29/24
X	-1	-11/6	-5/6	7/6	5/6	1/6	-1	1/3	≥	-19/24
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
φ_{FE}-	0	0	0	-1	0	0	0	0	≥	-2
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-1/2
Z	0	-1	0	0	1	0	1	0	≥	-3/4
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-1/2
Y	1	-1	0	0	1	0	0	0	≥	0
φ_{EF}+	0	-1	0	0	0	0	0	0	≥	-2
αbF	0	-1/6	-1/6	-1/6	1/6	-1/6	0	-1/3	≥	-41/24
T	0	-1	1	0	1	0	0	0	≥	-3/4
φ_{GD}-	-1/2	-1/12	5/12	5/12	1/12	-1/12	0	5/6	≥	-35/48
φ_{GD}+	1/2	1/12	-5/12	-5/12	-1/12	1/12	0	-5/6	≥	-349/48
φ_{DG}-	0	1	0	0	-1	1	0	0	≥	-3
φ_{DG}+	0	-1	0	0	1	-1	0	0	≥	-5
φ_{DH}-	0	-1	-1	0	1	-1	0	0	≥	-21/4
φ_{DH}+	0	1	1	0	-1	1	0	0	≥	-11/4
X-	0	-1	0	0	1	0	0	0	≥	-1
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	1	0	0	-1	0	0	0	≥	-3
Max	0	-1/6	-1/6	-1/6	1/6	-1/6	0	-1/3	=	-41/24

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Scambio pivotale 18-5

	φ_{BA}-	φ_{EF}-	φ_{DC}-	φ_{FE}+	φ_{DH}+	R	φ_{CD}-	φ_{HF}-		Fb
φ_{AB}-	-1	-1	-1	7/6	1/6	0	-1	1/3	≥	-1/3
φ_{AB}+	1	1	1	-7/6	-1/6	0	1	-1/3	≥	-5/3
X	-1	-1	0	7/6	-5/6	1	-1	1/3	≥	-37/12
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
φ_{FE}-	0	0	0	-1	0	0	0	0	≥	-2
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-1/2
Z	0	0	1	0	-1	1	1	0	≥	-7/2
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-1/2
Y	1	0	1	0	-1	1	0	0	≥	-11/4
φ_{EF}+	0	-1	0	0	0	0	0	0	≥	-2
αbF	0	0	0	-1/6	-1/6	0	0	-1/3	≥	-13/6
T	0	0	2	0	-1	1	0	0	≥	-7/2
φ_{GD}-	-1/2	0	1/2	5/12	-1/12	0	0	5/6	≥	-23/24
φ_{GD}+	1/2	0	-1/2	-5/12	1/12	0	0	-5/6	≥	-169/24
φ_{DG}-	0	0	-1	0	1	0	0	0	≥	-1/4
φ_{DG}+	0	0	1	0	-1	0	0	0	≥	-31/4
φ_{DH}-	0	0	0	0	-1	0	0	0	≥	-8
S	0	1	1	0	-1	1	0	0	≥	-11/4
X-	0	0	1	0	-1	1	0	0	≥	-15/4
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	-1	0	1	-1	0	0	≥	-1/4
Max	0	0	0	-1/6	-1/6	0	0	-1/3	=	-13/6

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Tableau finale

	φ_{BA}-	φ_{EF}-	φ_{DC}-	φ_{FE}+	φ_{DH}+	R	φ_{CD}-	φ_{HF}-		Fb
φ_{AB}-	-1	-1	-1	7/6	1/6	0	-1	1/3	≥	-1/3
φ_{AB}+	1	1	1	-7/6	-1/6	0	1	-1/3	≥	-5/3
X	-1	-1	0	7/6	-5/6	1	-1	1/3	≥	-37/12
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
φ_{FE}-	0	0	0	-1	0	0	0	0	≥	-2
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-1/2
Z	0	0	1	0	-1	1	1	0	≥	-7/2
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-1/2
Y	1	0	1	0	-1	1	0	0	≥	-11/4
φ_{EF}+	0	-1	0	0	0	0	0	0	≥	-2
αbF	0	0	0	-1/6	-1/6	0	0	-1/3	≥	-13/6
T	0	0	2	0	-1	1	0	0	≥	-7/2
φ_{GD}-	-1/2	0	1/2	5/12	-1/12	0	0	5/6	≥	-23/24
φ_{GD}+	1/2	0	-1/2	-5/12	1/12	0	0	-5/6	≥	-169/24
φ_{DG}-	0	0	-1	0	1	0	0	0	≥	-1/4
φ_{DG}+	0	0	1	0	-1	0	0	0	≥	-31/4
φ_{DH}-	0	0	0	0	-1	0	0	0	≥	-8
S	0	1	1	0	-1	1	0	0	≥	-11/4
X-	0	0	1	0	-1	1	0	0	≥	-15/4
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	-1	0	1	-1	0	0	≥	-1/4
Max	0	0	0	-1/6	-1/6	0	0	-1/3	=	-13/6

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Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	0
φ_{AB}+	0	0	0	0	0	0	0	0	≥	0
φ_{BA}-	0	0	0	0	0	0	0	0	≥	0
φ_{BA}+	0	0	0	0	0	0	0	0	≥	0
φ_{CD}-	0	0	0	0	0	0	0	0	≥	0
φ_{CD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DC}-	0	0	0	0	0	0	0	0	≥	0
φ_{DC}+	0	0	0	0	0	0	0	0	≥	0
φ_{EF}-	0	0	0	0	0	0	0	0	≥	0
φ_{EF}+	0	0	0	0	0	0	0	0	≥	0
φ_{FE}-	0	0	0	0	0	0	0	0	≥	0
φ_{FE}+	0	0	0	0	0	0	0	0	≥	1/6
φ_{GD}-	0	0	0	0	0	0	0	0	≥	0
φ_{GD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DG}-	0	0	0	0	0	0	0	0	≥	0
φ_{DG}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	1/6
φ_{HF}-	0	0	0	0	0	0	0	0	≥	1/3
φ_{HF}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	37/12	11/4	7/2	7/2	11/4	0	13/6	15/4	=	-13/6

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
-2/3	-1	-1/4	-1/4	-1	-15/4

Variabili soluzione differenza tra rotazioni

φ_{AB}	0
φ_{BA}	0
φ_{CD}	0
φ_{DC}	0
φ_{EF}	0
φ_{FE}	1/6
φ_{GD}	0
φ_{DG}	0
φ_{DH}	1/6
φ_{HF}	-1/3

REAZIONI Fattore di collasso = 13/6

H_A = 5/3F
V_A = 97/24F
W_A = -2/3Fb
H_C = 1/2F
V_C = 355/24F
W_C = -1/4Fb
H_E = 0
V_E = 5F
W_E = -Fb

H_{AB} = 5/3F	H_{CD} = 1/2F	H_{EF} = 0	H_{BG} = 5/3F	H_{GD} = 5/3F
V_{AB} = 97/24F	V_{CD} = 355/24F	V_{EF} = 5F	V_{BG} = 97/24F	V_{GD} = -163/24F
W_{AB} = -2/3Fb	W_{CD} = -1/4Fb	W_{EF} = -Fb	W_{BG} = Fb	W_{GD} = -73/24Fb
H_{BA} = -5/3F	H_{DC} = -1/2F	H_{FE} = 0	H_{GB} = -5/3F	H_{DG} = -5/3F
V_{BA} = -97/24F	V_{DC} = -355/24F	V_{FE} = -5F	V_{GB} = -97/24F	V_{DG} = 163/24F
W_{BA} = -Fb	W_{DC} = -1/4Fb	W_{FE} = Fb	W_{GB} = 73/24Fb	W_{DG} = -15/4Fb

H_{DH} = 13/6F	H_{HF} = 13/6F
V_{DH} = 8F	V_{HF} = -5F
W_{DH} = 4Fb	W_{HF} = -4Fb
H_{HD} = -13/6F	H_{FH} = -13/6F
V_{HD} = -8F	V_{FH} = 5F
W_{HD} = 4Fb	W_{FH} = -Fb

SPOSTAMENTI NODALI

u_{AAB} = 0	u_B = 0	u_{CCD} = 0	u_D = 0	u_{EEF} = 0	u_{FFE} = 0
v_{AAB} = 0	v_B = 0	v_{CCD} = 0	v_D = 0	v_{EEF} = 0	v_{FFE} = 0
φ_{AAB} = 0	φ_B = 0	φ_{CCD} = 0	φ_D = 0	φ_{EEF} = 0	φ_{FFH} = 0

u_G = 0	u_{HHD} = 0
v_G = 0	v_{HHD} = -1/6δ
φ_G = 0	φ_{HHD} = -1/6δ/b

SPOSTAMENTI RIGIDI DELLE ASTE

u_{AAB} = 0	u_{CCD} = 0	u_{EEF} = 0	u_{BBG} = 0	u_{GGD} = 0	u_{DDH} = 0
v_{AAB} = 0	v_{CCD} = 0	v_{EEF} = 0	v_{BBG} = 0	v_{GGD} = 0	v_{DDH} = 0
φ_{AAB} = 0	φ_{CCD} = 0	φ_{EEF} = 0	φ_{BBG} = 0	φ_{GGD} = 0	φ_{DDH} = -1/6δ/b

u_{HHF} = 0
v_{HHF} = -1/6δ
φ_{HHF} = 1/6δ/b

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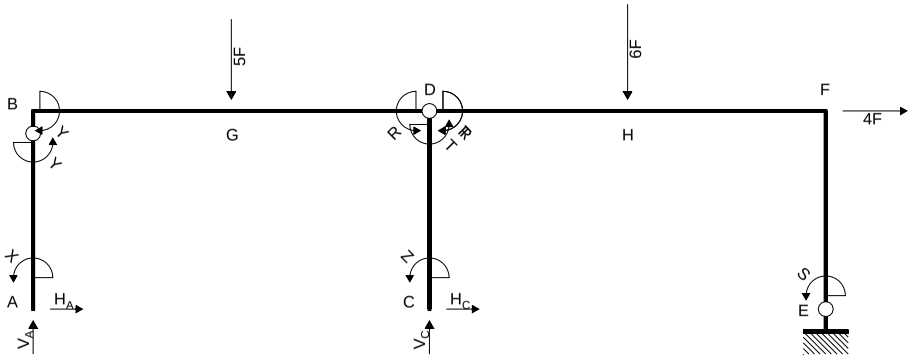


EQUAZIONI DI EQUILIBRIO

Rotazione intorno a E: aste EF FH HD DC DG GB BA

$-4V_Ab -2V_Cb = -Xb -Zb -Sb -17Fb$

Rotazione intorno a D: aste DC

$H_Cb = -Zb -Tb$

Rotazione intorno a D: aste DG GB BA

$H_Ab -2V_Ab = -Xb -Rb -5Fb$

Rotazione intorno a B: aste BA

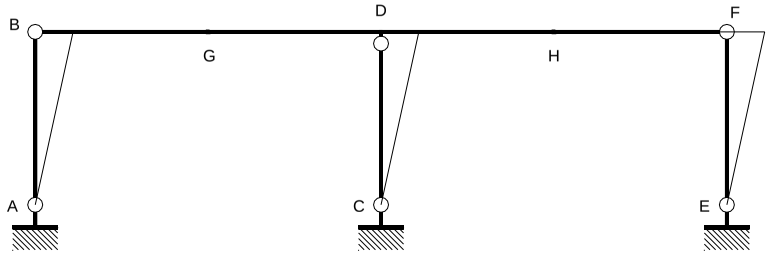
$H_Ab = -Xb -Yb$

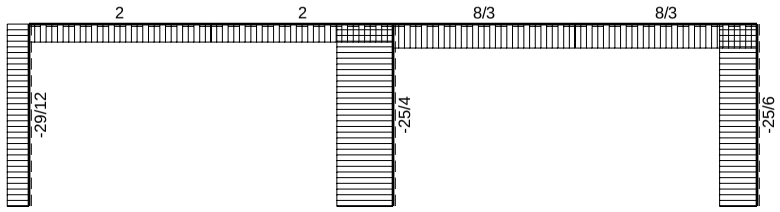
Matrice di equilibrio

$$\begin{array}{c} \\ \varphi_{EF} \\ \varphi_{DC} \\ \varphi_{DG} \\ \varphi_{BA} \end{array} \begin{bmatrix} H_Ab & V_Ab & H_Cb & V_Cb \\ 0 & -4 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 1 & -2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1 & 0 & -1 & 0 & -1 & 0 & -17 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & -1 & -5 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

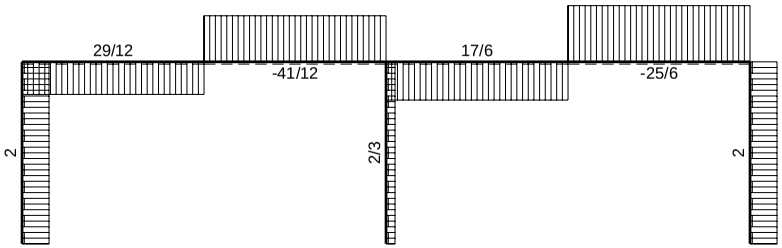
Soluzione del sistema

$$\begin{bmatrix} V_Ab \\ H_Cb \\ H_Ab \\ V_Cb \end{bmatrix} = \begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ 0 & -1/2 & 0 & 0 & 0 & 1/2 & 5/2 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1 & 1/2 & 0 & 1/2 & -1 & 7/2 \end{bmatrix}$$

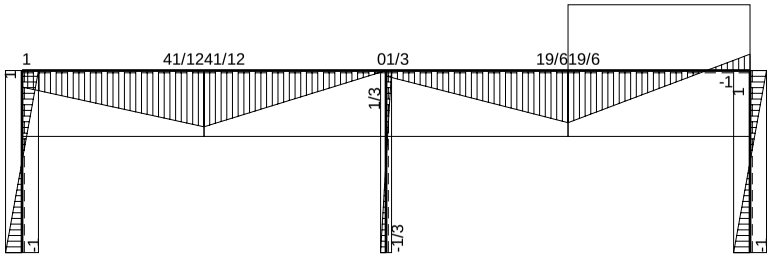




F



F



Fb

PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	≥	H_{14}
D_2	H_{21}	H_{22}	H_{23}	≥	H_{24}
D_3	H_{31}	H_{32}	H_{33}	≥	H_{34}
D_4	H_{41}	H_{42}	H_{43}	≥	H_{44}
D_5	H_{51}	H_{52}	H_{53}	≥	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{CD}$ $T=W_{DC}$ $S=W_{EF}$ $R=W_{DG}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	1	0	0	0	0	0	0	≤	1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	1	0	0	0	0	0	≤	1
W_{CD}-	0	0	1	0	0	0	0	≥	-1/3
W_{CD}+	0	0	1	0	0	0	0	≤	1/3
W_{DC}-	0	0	0	1	0	0	0	≥	-1/3
W_{DC}+	0	0	0	1	0	0	0	≤	1/3
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	1	0	0	≤	1
W_{FE}-	-1	-1	-1	-1	-1	0	4	≥	-1
W_{FE}+	-1	-1	-1	-1	-1	0	4	≤	1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	-1/2	0	0	0	-1/2	-5/2	≤	4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	1	0	≤	4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	-1	0	-1	0	≤	4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-1	≥	-4
W_{HF}+	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-1	≤	4
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	0	≥	-1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	≥	-1
W_{CD}-	0	0	1	0	0	0	0	≥	-1/3
W_{CD}+	0	0	-1	0	0	0	0	≥	-1/3
W_{DC}-	0	0	0	1	0	0	0	≥	-1/3
W_{DC}+	0	0	0	-1	0	0	0	≥	-1/3
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	≥	-1
W_{FE}-	-1	-1	-1	-1	-1	0	4	≥	-1
W_{FE}+	1	1	1	1	1	0	-4	≥	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	5/2	≥	-4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	≥	-4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	≥	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-1	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	1	≥	-4
Max	0	0	0	0	0	0	1	=	0

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Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1
W_{CD}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-1/3
W_{CD}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-1/3
W_{DC}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-1/3
W_{DC}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-1/3
W_{EF}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1
W_{FE}-	-1	-1	-1	-1	-1	0	1	1	1	1	1	0	4	≥	-1
W_{FE}+	1	1	1	1	1	0	-1	-1	-1	-1	-1	0	-4	≤	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	0	1/2	0	0	0	1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	0	-1/2	0	0	0	-1/2	5/2	≤	-4
W_{DG}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-4
W_{DH}-	0	0	0	-1	0	-1	0	0	0	1	0	1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	0	0	-1	0	-1	0	≤	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	1/2	1/2	1/2	1	1/2	1/2	-1	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	-1/2	-1/2	-1/2	-1	-1/2	-1/2	1	≤	-4
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

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Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-1/3
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-1/3
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-1/3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-1/3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	-1	-1	-1	-1	-1	0	4	5	≥	-1
φ_{FE}+	1	1	1	1	1	0	-4	-5	≥	-1
φ_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	1	≥	-4
φ_{GD}+	0	1/2	0	0	0	1/2	5/2	-1	≥	-4
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-1	7/2	≥	-4
φ_{HF}+	1/2	1/2	1/2	1	1/2	1/2	1	-7/2	≥	-4
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 12-7

	X	Y	Z	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-1/3
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-1/3
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-1/3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-1/3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	1/4	1/4	1/4	1/4	1/4	0	-1/4	-5/4	≥	-1/4
φ_{GD}-	-5/8	-9/8	-5/8	-5/8	-5/8	-1/2	5/8	33/8	≥	-27/8
φ_{GD}+	5/8	9/8	5/8	5/8	5/8	1/2	-5/8	-33/8	≥	-37/8
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	-3/4	-3/4	-3/4	-5/4	-3/4	-1/2	1/4	19/4	≥	-15/4
φ_{HF}+	3/4	3/4	3/4	5/4	3/4	1/2	-1/4	-19/4	≥	-17/4
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	1/4	1/4	1/4	1/4	1/4	0	-1/4	-5/4	=	-1/4

Scambio pivotale 2-1

	φ_{AB}+	Y	Z	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-1/3
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-1/3
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-1/3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-1/3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/4	1/4	1/4	1/4	1/4	0	-1/4	-1	≥	-1/2
φ_{GD}-	5/8	-9/8	-5/8	-5/8	-5/8	-1/2	5/8	7/2	≥	-11/4
φ_{GD}+	-5/8	9/8	5/8	5/8	5/8	1/2	-5/8	-7/2	≥	-21/4
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	3/4	-3/4	-3/4	-5/4	-3/4	-1/2	1/4	4	≥	-3
φ_{HF}+	-3/4	3/4	3/4	5/4	3/4	1/2	-1/4	-4	≥	-5
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-1/4	1/4	1/4	1/4	1/4	0	-1/4	-1	=	-1/2

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Scambio pivotale 4-2

	φ_{AB}+	φ_{BA}+	Z	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	-1	0	0	0	0	0	0	≥	-2
Y	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-1/3
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-1/3
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-1/3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-1/3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/4	-1/4	1/4	1/4	1/4	0	-1/4	-3/4	≥	-3/4
φ_{GD}-	5/8	9/8	-5/8	-5/8	-5/8	-1/2	5/8	19/8	≥	-13/8
φ_{GD}+	-5/8	-9/8	5/8	5/8	5/8	1/2	-5/8	-19/8	≥	-51/8
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	3/4	3/4	-3/4	-5/4	-3/4	-1/2	1/4	13/4	≥	-9/4
φ_{HF}+	-3/4	-3/4	3/4	5/4	3/4	1/2	-1/4	-13/4	≥	-23/4
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-1/4	-1/4	1/4	1/4	1/4	0	-1/4	-3/4	=	-3/4

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Scambio pivotale 6-3

	φ_{AB}+	φ_{BA}+	φ_{CD}+	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	-1	0	0	0	0	0	0	≥	-2
Y	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-2/3
Z	0	0	-1	0	0	0	0	1	≥	-1/3
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-1/3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-1/3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/4	-1/4	-1/4	1/4	1/4	0	-1/4	-1/2	≥	-5/6
φ_{GD}-	5/8	9/8	5/8	-5/8	-5/8	-1/2	5/8	7/4	≥	-17/12
φ_{GD}+	-5/8	-9/8	-5/8	5/8	5/8	1/2	-5/8	-7/4	≥	-79/12
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	3/4	3/4	3/4	-5/4	-3/4	-1/2	1/4	5/2	≥	-2
φ_{HF}+	-3/4	-3/4	-3/4	5/4	3/4	1/2	-1/4	-5/2	≥	-6
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-1/4	-1/4	-1/4	1/4	1/4	0	-1/4	-1/2	=	-5/6

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Scambio pivotale 8-4

	φ_{AB}+	φ_{BA}+	φ_{CD}+	φ_{DC}+	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	-1	0	0	0	0	0	0	≥	-2
Y	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-2/3
Z	0	0	-1	0	0	0	0	1	≥	-1/3
φ_{DC}-	0	0	0	-1	0	0	0	0	≥	-2/3
T	0	0	0	-1	0	0	0	1	≥	-1/3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/4	-1/4	-1/4	-1/4	1/4	0	-1/4	-1/4	≥	-11/12
φ_{GD}-	5/8	9/8	5/8	5/8	-5/8	-1/2	5/8	9/8	≥	-29/24
φ_{GD}+	-5/8	-9/8	-5/8	-5/8	5/8	1/2	-5/8	-9/8	≥	-163/24
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	1	0	-1	0	1	≥	-11/3
φ_{DH}+	0	0	0	-1	0	1	0	-1	≥	-13/3
φ_{HF}-	3/4	3/4	3/4	5/4	-3/4	-1/2	1/4	5/4	≥	-19/12
φ_{HF}+	-3/4	-3/4	-3/4	-5/4	3/4	1/2	-1/4	-5/4	≥	-77/12
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-1/4	-1/4	-1/4	-1/4	1/4	0	-1/4	-1/4	=	-11/12

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Scambio pivotale 10-5

	φ_{AB}+	φ_{BA}+	φ_{CD}+	φ_{DC}+	φ_{EF}+	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	-1	0	0	0	0	0	0	≥	-2
Y	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-2/3
Z	0	0	-1	0	0	0	0	1	≥	-1/3
φ_{DC}-	0	0	0	-1	0	0	0	0	≥	-2/3
T	0	0	0	-1	0	0	0	1	≥	-1/3
φ_{EF}-	0	0	0	0	-1	0	0	0	≥	-2
S	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/4	-1/4	-1/4	-1/4	-1/4	0	-1/4	0	≥	-7/6
φ_{GD}-	5/8	9/8	5/8	5/8	5/8	-1/2	5/8	1/2	≥	-7/12
φ_{GD}+	-5/8	-9/8	-5/8	-5/8	-5/8	1/2	-5/8	-1/2	≥	-89/12
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	1	0	-1	0	1	≥	-11/3
φ_{DH}+	0	0	0	-1	0	1	0	-1	≥	-13/3
φ_{HF}-	3/4	3/4	3/4	5/4	3/4	-1/2	1/4	1/2	≥	-5/6
φ_{HF}+	-3/4	-3/4	-3/4	-5/4	-3/4	1/2	-1/4	-1/2	≥	-43/6
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-1/4	-1/4	-1/4	-1/4	-1/4	0	-1/4	0	=	-7/6

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Tableau finale

	φ_{AB}+	φ_{BA}+	φ_{CD}+	φ_{DC}+	φ_{EF}+	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	-1	0	0	0	0	0	0	≥	-2
Y	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-2/3
Z	0	0	-1	0	0	0	0	1	≥	-1/3
φ_{DC}-	0	0	0	-1	0	0	0	0	≥	-2/3
T	0	0	0	-1	0	0	0	1	≥	-1/3
φ_{EF}-	0	0	0	0	-1	0	0	0	≥	-2
S	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/4	-1/4	-1/4	-1/4	-1/4	0	-1/4	0	≥	-7/6
φ_{GD}-	5/8	9/8	5/8	5/8	5/8	-1/2	5/8	1/2	≥	-7/12
φ_{GD}+	-5/8	-9/8	-5/8	-5/8	-5/8	1/2	-5/8	-1/2	≥	-89/12
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	1	0	-1	0	1	≥	-11/3
φ_{DH}+	0	0	0	-1	0	1	0	-1	≥	-13/3
φ_{HF}-	3/4	3/4	3/4	5/4	3/4	-1/2	1/4	1/2	≥	-5/6
φ_{HF}+	-3/4	-3/4	-3/4	-5/4	-3/4	1/2	-1/4	-1/2	≥	-43/6
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-1/4	-1/4	-1/4	-1/4	-1/4	0	-1/4	0	=	-7/6

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Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	0
φ_{AB}+	0	0	0	0	0	0	0	0	≥	1/4
φ_{BA}-	0	0	0	0	0	0	0	0	≥	0
φ_{BA}+	0	0	0	0	0	0	0	0	≥	1/4
φ_{CD}-	0	0	0	0	0	0	0	0	≥	0
φ_{CD}+	0	0	0	0	0	0	0	0	≥	1/4
φ_{DC}-	0	0	0	0	0	0	0	0	≥	0
φ_{DC}+	0	0	0	0	0	0	0	0	≥	1/4
φ_{EF}-	0	0	0	0	0	0	0	0	≥	0
φ_{EF}+	0	0	0	0	0	0	0	0	≥	1/4
φ_{FE}-	0	0	0	0	0	0	0	0	≥	0
φ_{FE}+	0	0	0	0	0	0	0	0	≥	1/4
φ_{GD}-	0	0	0	0	0	0	0	0	≥	0
φ_{GD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DG}-	0	0	0	0	0	0	0	0	≥	0
φ_{DG}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	0
φ_{HF}-	0	0	0	0	0	0	0	0	≥	0
φ_{HF}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	1	1	1/3	1/3	1	0	7/6	0	=	-7/6

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
1	1	1/3	1/3	1	0

Variabili soluzione differenza tra rotazioni

φ_{AB}	1/4
φ_{BA}	1/4
φ_{CD}	1/4
φ_{DC}	1/4
φ_{EF}	1/4
φ_{FE}	1/4
φ_{GD}	0
φ_{DG}	0
φ_{DH}	0
φ_{HF}	0

REAZIONI Fattore di collasso = 7/6

H_A = -2F
V_A = 29/12F
W_A = Fb
H_C = -2/3F
V_C = 25/4F
W_C = 1/3Fb
H_E = -2F
V_E = 25/6F
W_E = Fb

H_{AB} = -2F	H_{CD} = -2/3F	H_{EF} = -2F	H_{BG} = -2F	H_{GD} = -2F
V_{AB} = 29/12F	V_{CD} = 25/4F	V_{EF} = 25/6F	V_{BG} = 29/12F	V_{GD} = -41/12F
W_{AB} = Fb	W_{CD} = 1/3Fb	W_{EF} = Fb	W_{BG} = -Fb	W_{GD} = -41/12Fb
H_{BA} = 2F	H_{DC} = 2/3F	H_{FE} = 2F	H_{GB} = 2F	H_{DG} = 2F
V_{BA} = -29/12F	V_{DC} = -25/4F	V_{FE} = -25/6F	V_{GB} = -29/12F	V_{DG} = 41/12F
W_{BA} = Fb	W_{DC} = 1/3Fb	W_{FE} = Fb	W_{GB} = 41/12Fb	W_{DG} = 0

H_{DH} = -8/3F	H_{HF} = -8/3F
V_{DH} = 17/6F	V_{HF} = -25/6F
W_{DH} = -1/3Fb	W_{HF} = -19/6Fb
H_{HD} = 8/3F	H_{FH} = 8/3F
V_{HD} = -17/6F	V_{FH} = 25/6F
W_{HD} = 19/6Fb	W_{FH} = -Fb

SPOSTAMENTI NODALI

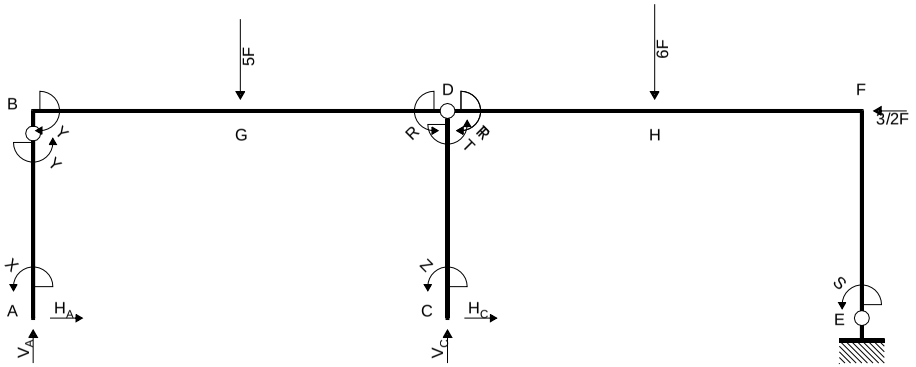
u_{AAB} = 0	u_{BBA} = 1/4δ	u_{CCD} = 0	u_D = 1/4δ	u_{EEF} = 0	u_{FFE} = 1/4δ
v_{AAB} = 0	v_{BBA} = 0	v_{CCD} = 0	v_D = 0	v_{EEF} = 0	v_{FFE} = 0
φ_{AAB} = -1/4δ/b	φ_{BBA} = -1/4δ/b	φ_{CCD} = -1/4δ/b	φ_D = -1/4δ/b	φ_{EEF} = -1/4δ/b	φ_{FFH} = -1/4δ/b

u_G = 1/4δ	u_H = 1/4δ
v_G = 0	v_H = 0
φ_G = 0	φ_H = 0

SPOSTAMENTI RIGIDI DELLE ASTE

u_{AAB} = 0	u_{CCD} = 0	u_{EEF} = 0	u_{BBG} = 1/4δ	u_{GGD} = 1/4δ	u_{DDH} = 1/4δ
v_{AAB} = 0	v_{CCD} = 0	v_{EEF} = 0	v_{BBG} = 0	v_{GGD} = 0	v_{DDH} = 0
φ_{AAB} = -1/4δ/b	φ_{CCD} = -1/4δ/b	φ_{EEF} = -1/4δ/b	φ_{BBG} = 0	φ_{GGD} = 0	φ_{DDH} = 0

u_{HHF} = 1/4δ
v_{HHF} = 0
φ_{HHF} = 0



EQUAZIONI DI EQUILIBRIO

Rotazione intorno a E: aste EF FH HD DC DG GB BA

$-4V_Ab -2V_Cb = -Xb -Zb -Sb -45/2Fb$

Rotazione intorno a D: aste DC

$H_Cb = -Zb -Tb$

Rotazione intorno a D: aste DG GB BA

$H_Ab -2V_Ab = -Xb -Rb -5Fb$

Rotazione intorno a B: aste BA

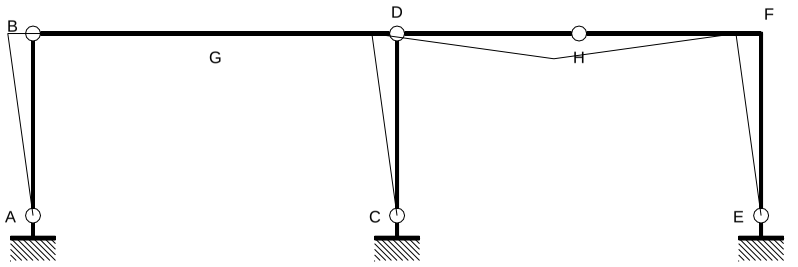
$H_Ab = -Xb -Yb$

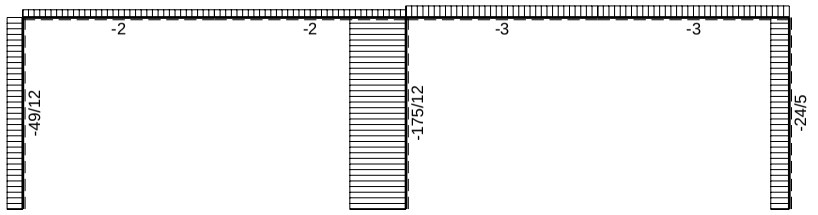
Matrice di equilibrio

	H_Ab	V_Ab	H_Cb	V_Cb		Xb	Yb	Zb	Tb	Sb	Rb	Fb
φ_{EF}	0	-4	0	-2	=	-1	0	-1	0	-1	0	-45/2
φ_{DC}	0	0	1	0		0	0	-1	-1	0	0	0
φ_{DG}	1	-2	0	0		-1	0	0	0	0	-1	-5
φ_{BA}	1	0	0	0		-1	-1	0	0	0	0	0

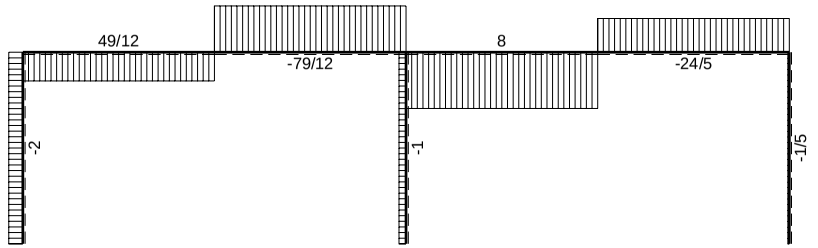
Soluzione del sistema

		Xb	Yb	Zb	Tb	Sb	Rb	Fb
V_Ab	=	0	-1/2	0	0	0	1/2	5/2
H_Cb		0	0	-1	-1	0	0	0
H_Ab		-1	-1	0	0	0	0	0
V_Cb		1/2	1	1/2	0	1/2	-1	25/4

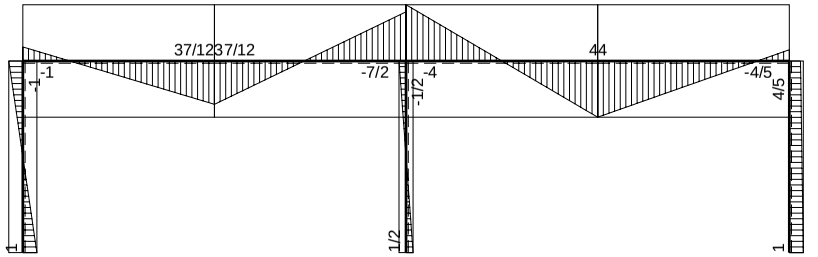




F



F



Fb

PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	$\ge$	H_{14}
D_2	H_{21}	H_{22}	H_{23}	$\ge$	H_{24}
D_3	H_{31}	H_{32}	H_{33}	$\ge$	H_{34}
D_4	H_{41}	H_{42}	H_{43}	$\ge$	H_{44}
D_5	H_{51}	H_{52}	H_{53}	$\ge$	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{CD}$ $T=W_{DC}$ $S=W_{EF}$ $R=W_{DG}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	1	0	0	0	0	0	0	≤	1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	1	0	0	0	0	0	≤	1
W_{CD}-	0	0	1	0	0	0	0	≥	-1/2
W_{CD}+	0	0	1	0	0	0	0	≤	1/2
W_{DC}-	0	0	0	1	0	0	0	≥	-1/2
W_{DC}+	0	0	0	1	0	0	0	≤	1/2
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	1	0	0	≤	1
W_{FE}-	-1	-1	-1	-1	-1	0	-3/2	≥	-1
W_{FE}+	-1	-1	-1	-1	-1	0	-3/2	≤	1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	-1/2	0	0	0	-1/2	-5/2	≤	4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	1	0	≤	4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	-1	0	-1	0	≤	4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-15/4	≥	-4
W_{HF}+	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-15/4	≤	4
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	0	≥	-1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	≥	-1
W_{CD}-	0	0	1	0	0	0	0	≥	-1/2
W_{CD}+	0	0	-1	0	0	0	0	≥	-1/2
W_{DC}-	0	0	0	1	0	0	0	≥	-1/2
W_{DC}+	0	0	0	-1	0	0	0	≥	-1/2
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	≥	-1
W_{FE}-	-1	-1	-1	-1	-1	0	-3/2	≥	-1
W_{FE}+	1	1	1	1	1	0	3/2	≥	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	5/2	≥	-4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	≥	-4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	≥	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-15/4	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	15/4	≥	-4
Max	0	0	0	0	0	0	1	=	0

PROCEDIMENTO E RISULTATI Nome: AL5.003

Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1
W_{CD}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-1/2
W_{CD}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-1/2
W_{DC}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-1/2
W_{DC}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-1/2
W_{EF}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1
W_{FE}-	-1	-1	-1	-1	-1	0	1	1	1	1	1	0	-3/2	≥	-1
W_{FE}+	1	1	1	1	1	0	-1	-1	-1	-1	-1	0	3/2	≤	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	0	1/2	0	0	0	1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	0	-1/2	0	0	0	-1/2	5/2	≤	-4
W_{DG}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-4
W_{DH}-	0	0	0	-1	0	-1	0	0	0	1	0	1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	0	0	-1	0	-1	0	≤	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	1/2	1/2	1/2	1	1/2	1/2	-15/4	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	-1/2	-1/2	-1/2	-1	-1/2	-1/2	15/4	≤	-4
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

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Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-1/2
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-1/2
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-1/2
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-1/2
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	-1	-1	-1	-1	-1	0	-3/2	5	≥	-1
φ_{FE}+	1	1	1	1	1	0	3/2	-5	≥	-1
φ_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	1	≥	-4
φ_{GD}+	0	1/2	0	0	0	1/2	5/2	-1	≥	-4
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-15/4	7/2	≥	-4
φ_{HF}+	1/2	1/2	1/2	1	1/2	1/2	15/4	-7/2	≥	-4
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 11-7

	X	Y	Z	T	S	R	φ_{FE}-	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-1/2
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-1/2
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-1/2
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-1/2
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
αbF	-2/3	-2/3	-2/3	-2/3	-2/3	0	-2/3	10/3	≥	-2/3
φ_{FE}+	0	0	0	0	0	0	-1	0	≥	-2
φ_{GD}-	5/3	7/6	5/3	5/3	5/3	-1/2	5/3	-22/3	≥	-7/3
φ_{GD}+	-5/3	-7/6	-5/3	-5/3	-5/3	1/2	-5/3	22/3	≥	-17/3
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	2	2	2	3/2	2	-1/2	5/2	-9	≥	-3/2
φ_{HF}+	-2	-2	-2	-3/2	-2	1/2	-5/2	9	≥	-13/2
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-2/3	-2/3	-2/3	-2/3	-2/3	0	-2/3	10/3	=	-2/3

Scambio pivotale 19-8

	X	Y	Z	T	S	R	φ_{FE}-	φ_{HF}-		Fb
φ_{AB}-	7/9	-2/9	-2/9	-1/6	-2/9	1/18	-5/18	1/9	≥	-5/6
φ_{AB}+	-7/9	2/9	2/9	1/6	2/9	-1/18	5/18	-1/9	≥	-7/6
φ_{BA}-	-2/9	7/9	-2/9	-1/6	-2/9	1/18	-5/18	1/9	≥	-5/6
φ_{BA}+	2/9	-7/9	2/9	1/6	2/9	-1/18	5/18	-1/9	≥	-7/6
φ_{CD}-	-2/9	-2/9	7/9	-1/6	-2/9	1/18	-5/18	1/9	≥	-1/3
φ_{CD}+	2/9	2/9	-7/9	1/6	2/9	-1/18	5/18	-1/9	≥	-2/3
φ_{DC}-	-2/9	-2/9	-2/9	5/6	-2/9	1/18	-5/18	1/9	≥	-1/3
φ_{DC}+	2/9	2/9	2/9	-5/6	2/9	-1/18	5/18	-1/9	≥	-2/3
φ_{EF}-	-2/9	-2/9	-2/9	-1/6	7/9	1/18	-5/18	1/9	≥	-5/6
φ_{EF}+	2/9	2/9	2/9	1/6	-7/9	-1/18	5/18	-1/9	≥	-7/6
αbF	2/27	2/27	2/27	-1/9	2/27	-5/27	7/27	-10/27	≥	-11/9
φ_{FE}+	0	0	0	0	0	0	-1	0	≥	-2
φ_{GD}-	1/27	-25/54	1/27	4/9	1/27	-5/54	-10/27	22/27	≥	-10/9
φ_{GD}+	-1/27	25/54	-1/27	-4/9	-1/27	5/54	10/27	-22/27	≥	-62/9
φ_{DG}-	-2/9	-2/9	-2/9	-1/6	-2/9	19/18	-5/18	1/9	≥	-23/6
φ_{DG}+	2/9	2/9	2/9	1/6	2/9	-19/18	5/18	-1/9	≥	-25/6
φ_{DH}-	4/9	4/9	4/9	-2/3	4/9	-10/9	5/9	-2/9	≥	-13/3
φ_{DH}+	-4/9	-4/9	-4/9	2/3	-4/9	10/9	-5/9	2/9	≥	-11/3
X-	2/9	2/9	2/9	1/6	2/9	-1/18	5/18	-1/9	≥	-1/6
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	-2/9	-2/9	-2/9	-1/6	-2/9	1/18	-5/18	1/9	≥	-23/6
Max	2/27	2/27	2/27	-1/9	2/27	-5/27	7/27	-10/27	=	-11/9

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Scambio pivotale 5-7

	X	Y	Z	T	S	R	φ_{CD}-	φ_{HF}-		Fb
φ_{AB}-	1	0	-1	0	0	0	1	0	≥	-1/2
φ_{AB}+	-1	0	1	0	0	0	-1	0	≥	-3/2
φ_{BA}-	0	1	-1	0	0	0	1	0	≥	-1/2
φ_{BA}+	0	-1	1	0	0	0	-1	0	≥	-3/2
φ_{FE}-	-4/5	-4/5	14/5	-3/5	-4/5	1/5	-18/5	2/5	≥	-6/5
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-1
φ_{DC}-	0	0	-1	1	0	0	1	0	≥	0
φ_{DC}+	0	0	1	-1	0	0	-1	0	≥	-1
φ_{EF}-	0	0	-1	0	1	0	1	0	≥	-1/2
φ_{EF}+	0	0	1	0	-1	0	-1	0	≥	-3/2
αbF	-2/15	-2/15	4/5	-4/15	-2/15	-2/15	-14/15	-4/15	≥	-23/15
φ_{FE}+	4/5	4/5	-14/5	3/5	4/5	-1/5	18/5	-2/5	≥	-4/5
φ_{GD}-	1/3	-1/6	-1	2/3	1/3	-1/6	4/3	2/3	≥	-2/3
φ_{GD}+	-1/3	1/6	1	-2/3	-1/3	1/6	-4/3	-2/3	≥	-22/3
φ_{DG}-	0	0	-1	0	0	1	1	0	≥	-7/2
φ_{DG}+	0	0	1	0	0	-1	-1	0	≥	-9/2
φ_{DH}-	0	0	2	-1	0	-1	-2	0	≥	-5
φ_{DH}+	0	0	-2	1	0	1	2	0	≥	-3
X-	0	0	1	0	0	0	-1	0	≥	-1/2
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	-1	0	0	0	1	0	≥	-7/2
Max	-2/15	-2/15	4/5	-4/15	-2/15	-2/15	-14/15	-4/15	=	-23/15

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Scambio pivotale 7-3

	X	Y	φ_{DC}-	T	S	R	φ_{CD}-	φ_{HF}-		Fb
φ_{AB}-	1	0	1	-1	0	0	0	0	≥	-1/2
φ_{AB}+	-1	0	-1	1	0	0	0	0	≥	-3/2
φ_{BA}-	0	1	1	-1	0	0	0	0	≥	-1/2
φ_{BA}+	0	-1	-1	1	0	0	0	0	≥	-3/2
φ_{FE}-	-4/5	-4/5	-14/5	11/5	-4/5	1/5	-4/5	2/5	≥	-6/5
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-1
Z	0	0	-1	1	0	0	1	0	≥	0
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-1
φ_{EF}-	0	0	1	-1	1	0	0	0	≥	-1/2
φ_{EF}+	0	0	-1	1	-1	0	0	0	≥	-3/2
αbF	-2/15	-2/15	-4/5	8/15	-2/15	-2/15	-2/15	-4/15	≥	-23/15
φ_{FE}+	4/5	4/5	14/5	-11/5	4/5	-1/5	4/5	-2/5	≥	-4/5
φ_{GD}-	1/3	-1/6	1	-1/3	1/3	-1/6	1/3	2/3	≥	-2/3
φ_{GD}+	-1/3	1/6	-1	1/3	-1/3	1/6	-1/3	-2/3	≥	-22/3
φ_{DG}-	0	0	1	-1	0	1	0	0	≥	-7/2
φ_{DG}+	0	0	-1	1	0	-1	0	0	≥	-9/2
φ_{DH}-	0	0	-2	1	0	-1	0	0	≥	-5
φ_{DH}+	0	0	2	-1	0	1	0	0	≥	-3
X-	0	0	-1	1	0	0	0	0	≥	-1/2
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	1	-1	0	0	0	0	≥	-7/2
Max	-2/15	-2/15	-4/5	8/15	-2/15	-2/15	-2/15	-4/15	=	-23/15

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Scambio pivotale 12-4

	X	Y	φ_{DC}-	φ_{FE}+	S	R	φ_{CD}-	φ_{HF}-		Fb
φ_{AB}-	7/11	-4/11	-3/11	5/11	-4/11	1/11	-4/11	2/11	≥	-3/22
φ_{AB}+	-7/11	4/11	3/11	-5/11	4/11	-1/11	4/11	-2/11	≥	-41/22
φ_{BA}-	-4/11	7/11	-3/11	5/11	-4/11	1/11	-4/11	2/11	≥	-3/22
φ_{BA}+	4/11	-7/11	3/11	-5/11	4/11	-1/11	4/11	-2/11	≥	-41/22
φ_{FE}-	0	0	0	-1	0	0	0	0	≥	-2
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-1
Z	4/11	4/11	3/11	-5/11	4/11	-1/11	15/11	-2/11	≥	-4/11
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-1
φ_{EF}-	-4/11	-4/11	-3/11	5/11	7/11	1/11	-4/11	2/11	≥	-3/22
φ_{EF}+	4/11	4/11	3/11	-5/11	-7/11	-1/11	4/11	-2/11	≥	-41/22
αbF	2/33	2/33	-4/33	-8/33	2/33	-2/11	2/33	-4/11	≥	-19/11
T	4/11	4/11	14/11	-5/11	4/11	-1/11	4/11	-2/11	≥	-4/11
φ_{GD}-	7/33	-19/66	19/33	5/33	7/33	-3/22	7/33	8/11	≥	-6/11
φ_{GD}+	-7/33	19/66	-19/33	-5/33	-7/33	3/22	-7/33	-8/11	≥	-82/11
φ_{DG}-	-4/11	-4/11	-3/11	5/11	-4/11	12/11	-4/11	2/11	≥	-69/22
φ_{DG}+	4/11	4/11	3/11	-5/11	4/11	-12/11	4/11	-2/11	≥	-107/22
φ_{DH}-	4/11	4/11	-8/11	-5/11	4/11	-12/11	4/11	-2/11	≥	-59/11
φ_{DH}+	-4/11	-4/11	8/11	5/11	-4/11	12/11	-4/11	2/11	≥	-29/11
X-	4/11	4/11	3/11	-5/11	4/11	-1/11	4/11	-2/11	≥	-19/22
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	-4/11	-4/11	-3/11	5/11	-4/11	1/11	-4/11	2/11	≥	-69/22
Max	2/33	2/33	-4/33	-8/33	2/33	-2/11	2/33	-4/11	=	-19/11

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Scambio pivotale 3-1

	φ_{BA}-	Y	φ_{DC}-	φ_{FE}+	S	R	φ_{CD}-	φ_{HF}-		Fb
φ_{AB}-	-7/4	3/4	-3/4	5/4	-1	1/4	-1	1/2	≥	-3/8
φ_{AB}+	7/4	-3/4	3/4	-5/4	1	-1/4	1	-1/2	≥	-13/8
X	-11/4	7/4	-3/4	5/4	-1	1/4	-1	1/2	≥	-3/8
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
φ_{FE}-	0	0	0	-1	0	0	0	0	≥	-2
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-1
Z	-1	1	0	0	0	0	1	0	≥	-1/2
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-1
φ_{EF}-	1	-1	0	0	1	0	0	0	≥	0
φ_{EF}+	-1	1	0	0	-1	0	0	0	≥	-2
αbF	-1/6	1/6	-1/6	-1/6	0	-1/6	0	-1/3	≥	-7/4
T	-1	1	1	0	0	0	0	0	≥	-1/2
φ_{GD}-	-7/12	1/12	5/12	5/12	0	-1/12	0	5/6	≥	-5/8
φ_{GD}+	7/12	-1/12	-5/12	-5/12	0	1/12	0	-5/6	≥	-59/8
φ_{DG}-	1	-1	0	0	0	1	0	0	≥	-3
φ_{DG}+	-1	1	0	0	0	-1	0	0	≥	-5
φ_{DH}-	-1	1	-1	0	0	-1	0	0	≥	-11/2
φ_{DH}+	1	-1	1	0	0	1	0	0	≥	-5/2
X-	-1	1	0	0	0	0	0	0	≥	-1
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	1	-1	0	0	0	0	0	0	≥	-3
Max	-1/6	1/6	-1/6	-1/6	0	-1/6	0	-1/3	=	-7/4

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Scambio pivotale 9-2

	φ_{BA}-	φ_{EF}-	φ_{DC}-	φ_{FE}+	S	R	φ_{CD}-	φ_{HF}-		Fb
φ_{AB}-	-1	-3/4	-3/4	5/4	-1/4	1/4	-1	1/2	≥	-3/8
φ_{AB}+	1	3/4	3/4	-5/4	1/4	-1/4	1	-1/2	≥	-13/8
X	-1	-7/4	-3/4	5/4	3/4	1/4	-1	1/2	≥	-3/8
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
φ_{FE}-	0	0	0	-1	0	0	0	0	≥	-2
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-1
Z	0	-1	0	0	1	0	1	0	≥	-1/2
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-1
Y	1	-1	0	0	1	0	0	0	≥	0
φ_{EF}+	0	-1	0	0	0	0	0	0	≥	-2
αbF	0	-1/6	-1/6	-1/6	1/6	-1/6	0	-1/3	≥	-7/4
T	0	-1	1	0	1	0	0	0	≥	-1/2
φ_{GD}-	-1/2	-1/12	5/12	5/12	1/12	-1/12	0	5/6	≥	-5/8
φ_{GD}+	1/2	1/12	-5/12	-5/12	-1/12	1/12	0	-5/6	≥	-59/8
φ_{DG}-	0	1	0	0	-1	1	0	0	≥	-3
φ_{DG}+	0	-1	0	0	1	-1	0	0	≥	-5
φ_{DH}-	0	-1	-1	0	1	-1	0	0	≥	-11/2
φ_{DH}+	0	1	1	0	-1	1	0	0	≥	-5/2
X-	0	-1	0	0	1	0	0	0	≥	-1
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	1	0	0	-1	0	0	0	≥	-3
Max	0	-1/6	-1/6	-1/6	1/6	-1/6	0	-1/3	=	-7/4

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Scambio pivotale 1-5

	φ_{BA}-	φ_{EF}-	φ_{DC}-	φ_{FE}+	φ_{AB}-	R	φ_{CD}-	φ_{HF}-		Fb
S	-4	-3	-3	5	-4	1	-4	2	≥	-3/2
φ_{AB}+	0	0	0	0	-1	0	0	0	≥	-2
X	-4	-4	-3	5	-3	1	-4	2	≥	-3/2
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
φ_{FE}-	0	0	0	-1	0	0	0	0	≥	-2
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-1
Z	-4	-4	-3	5	-4	1	-3	2	≥	-2
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-1
Y	-3	-4	-3	5	-4	1	-4	2	≥	-3/2
φ_{EF}+	0	-1	0	0	0	0	0	0	≥	-2
αbF	-2/3	-2/3	-2/3	2/3	-2/3	0	-2/3	0	≥	-2
T	-4	-4	-2	5	-4	1	-4	2	≥	-2
φ_{GD}-	-5/6	-1/3	1/6	5/6	-1/3	0	-1/3	1	≥	-3/4
φ_{GD}+	5/6	1/3	-1/6	-5/6	1/3	0	1/3	-1	≥	-29/4
φ_{DG}-	4	4	3	-5	4	0	4	-2	≥	-3/2
φ_{DG}+	-4	-4	-3	5	-4	0	-4	2	≥	-13/2
φ_{DH}-	-4	-4	-4	5	-4	0	-4	2	≥	-7
φ_{DH}+	4	4	4	-5	4	0	4	-2	≥	-1
X-	-4	-4	-3	5	-4	1	-4	2	≥	-5/2
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	4	4	3	-5	4	-1	4	-2	≥	-3/2
Max	-2/3	-2/3	-2/3	2/3	-2/3	0	-2/3	0	=	-2

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Scambio pivotale 18-4

	φ_{BA}-	φ_{EF}-	φ_{DC}-	φ_{DH}+	φ_{AB}-	R	φ_{CD}-	φ_{HF}-		Fb
S	0	1	1	-1	0	1	0	0	≥	-5/2
φ_{AB}+	0	0	0	0	-1	0	0	0	≥	-2
X	0	0	1	-1	1	1	0	0	≥	-5/2
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
φ_{FE}-	-4/5	-4/5	-4/5	1/5	-4/5	0	-4/5	2/5	≥	-9/5
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-1
Z	0	0	1	-1	0	1	1	0	≥	-3
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-1
Y	1	0	1	-1	0	1	0	0	≥	-5/2
φ_{EF}+	0	-1	0	0	0	0	0	0	≥	-2
αbF	-2/15	-2/15	-2/15	-2/15	-2/15	0	-2/15	-4/15	≥	-32/15
T	0	0	2	-1	0	1	0	0	≥	-3
φ_{GD}-	-1/6	1/3	5/6	-1/6	1/3	0	1/3	2/3	≥	-11/12
φ_{GD}+	1/6	-1/3	-5/6	1/6	-1/3	0	-1/3	-2/3	≥	-85/12
φ_{DG}-	0	0	-1	1	0	0	0	0	≥	-1/2
φ_{DG}+	0	0	1	-1	0	0	0	0	≥	-15/2
φ_{DH}-	0	0	0	-1	0	0	0	0	≥	-8
φ_{FE}+	4/5	4/5	4/5	-1/5	4/5	0	4/5	-2/5	≥	-1/5
X-	0	0	1	-1	0	1	0	0	≥	-7/2
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	-1	1	0	-1	0	0	≥	-1/2
Max	-2/15	-2/15	-2/15	-2/15	-2/15	0	-2/15	-4/15	=	-32/15

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Tableau finale

	φ_{BA}-	φ_{EF}-	φ_{DC}-	φ_{DH}+	φ_{AB}-	R	φ_{CD}-	φ_{HF}-		Fb
S	0	1	1	-1	0	1	0	0	≥	-5/2
φ_{AB}+	0	0	0	0	-1	0	0	0	≥	-2
X	0	0	1	-1	1	1	0	0	≥	-5/2
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
φ_{FE}-	-4/5	-4/5	-4/5	1/5	-4/5	0	-4/5	2/5	≥	-9/5
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-1
Z	0	0	1	-1	0	1	1	0	≥	-3
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-1
Y	1	0	1	-1	0	1	0	0	≥	-5/2
φ_{EF}+	0	-1	0	0	0	0	0	0	≥	-2
αbF	-2/15	-2/15	-2/15	-2/15	-2/15	0	-2/15	-4/15	≥	-32/15
T	0	0	2	-1	0	1	0	0	≥	-3
φ_{GD}-	-1/6	1/3	5/6	-1/6	1/3	0	1/3	2/3	≥	-11/12
φ_{GD}+	1/6	-1/3	-5/6	1/6	-1/3	0	-1/3	-2/3	≥	-85/12
φ_{DG}-	0	0	-1	1	0	0	0	0	≥	-1/2
φ_{DG}+	0	0	1	-1	0	0	0	0	≥	-15/2
φ_{DH}-	0	0	0	-1	0	0	0	0	≥	-8
φ_{FE}+	4/5	4/5	4/5	-1/5	4/5	0	4/5	-2/5	≥	-1/5
X-	0	0	1	-1	0	1	0	0	≥	-7/2
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	-1	1	0	-1	0	0	≥	-1/2
Max	-2/15	-2/15	-2/15	-2/15	-2/15	0	-2/15	-4/15	=	-32/15

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Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	2/15
φ_{AB}+	0	0	0	0	0	0	0	0	≥	0
φ_{BA}-	0	0	0	0	0	0	0	0	≥	2/15
φ_{BA}+	0	0	0	0	0	0	0	0	≥	0
φ_{CD}-	0	0	0	0	0	0	0	0	≥	2/15
φ_{CD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DC}-	0	0	0	0	0	0	0	0	≥	2/15
φ_{DC}+	0	0	0	0	0	0	0	0	≥	0
φ_{EF}-	0	0	0	0	0	0	0	0	≥	2/15
φ_{EF}+	0	0	0	0	0	0	0	0	≥	0
φ_{FE}-	0	0	0	0	0	0	0	0	≥	0
φ_{FE}+	0	0	0	0	0	0	0	0	≥	0
φ_{GD}-	0	0	0	0	0	0	0	0	≥	0
φ_{GD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DG}-	0	0	0	0	0	0	0	0	≥	0
φ_{DG}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	2/15
φ_{HF}-	0	0	0	0	0	0	0	0	≥	4/15
φ_{HF}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	5/2	5/2	3	3	5/2	0	32/15	7/2	=	-32/15

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
-1	-1	-1/2	-1/2	-1	-7/2

Variabili soluzione differenza tra rotazioni

φ_{AB}	-2/15
φ_{BA}	-2/15
φ_{CD}	-2/15
φ_{DC}	-2/15
φ_{EF}	-2/15
φ_{FE}	0
φ_{GD}	0
φ_{DG}	0
φ_{DH}	2/15
φ_{HF}	-4/15

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REAZIONI Fattore di collasso = 32/15

$H_A = 2F$
$V_A = 49/12F$
$W_A = -Fb$
$H_C = F$
$V_C = 175/12F$
$W_C = -1/2Fb$
$H_E = 1/5F$
$V_E = 24/5F$
$W_E = -Fb$

$H_{AB} = 2F$	$H_{CD} = F$	$H_{EF} = 1/5F$	$H_{BG} = 2F$	$H_{GD} = 2F$
$V_{AB} = 49/12F$	$V_{CD} = 175/12F$	$V_{EF} = 24/5F$	$V_{BG} = 49/12F$	$V_{GD} = -79/12F$
$W_{AB} = -Fb$	$W_{CD} = -1/2Fb$	$W_{EF} = -Fb$	$W_{BG} = Fb$	$W_{GD} = -37/12Fb$
$H_{BA} = -2F$	$H_{DC} = -F$	$H_{FE} = -1/5F$	$H_{GB} = -2F$	$H_{DG} = -2F$
$V_{BA} = -49/12F$	$V_{DC} = -175/12F$	$V_{FE} = -24/5F$	$V_{GB} = -49/12F$	$V_{DG} = 79/12F$
$W_{BA} = -Fb$	$W_{DC} = -1/2Fb$	$W_{FE} = 4/5Fb$	$W_{GB} = 37/12Fb$	$W_{DG} = -7/2Fb$

$H_{DH} = 3F$	$H_{HF} = 3F$
$V_{DH} = 8F$	$V_{HF} = -24/5F$
$W_{DH} = 4Fb$	$W_{HF} = -4Fb$
$H_{HD} = -3F$	$H_{FH} = -3F$
$V_{HD} = -8F$	$V_{FH} = 24/5F$
$W_{HD} = 4Fb$	$W_{FH} = -4/5Fb$

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_{BBA} = -2/15\delta$	$u_{CCD} = 0$	$u_{DDC} = -2/15\delta$	$u_{EEF} = 0$
$v_{AAB} = 0$	$v_{BBA} = 0$	$v_{CCD} = 0$	$v_{DDC} = 0$	$v_{EEF} = 0$
$\varphi_{AAB} = 2/15\delta/b$	$\varphi_{BBA} = 2/15\delta/b$	$\varphi_{CCD} = 2/15\delta/b$	$\varphi_{DDC} = 2/15\delta/b$	$\varphi_{EEF} = 2/15\delta/b$

$u_F = -2/15\delta$	$u_G = -2/15\delta$	$u_{HHD} = -2/15\delta$
$v_F = 0$	$v_G = 0$	$v_{HHD} = -2/15\delta$
$\varphi_F = 2/15\delta/b$	$\varphi_G = 0$	$\varphi_{HHD} = -2/15\delta/b$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$	$u_{CCD} = 0$	$u_{EEF} = 0$	$u_{BBG} = -2/15\delta$	$u_{GGD} = -2/15\delta$
$v_{AAB} = 0$	$v_{CCD} = 0$	$v_{EEF} = 0$	$v_{BBG} = 0$	$v_{GGD} = 0$
$\varphi_{AAB} = 2/15\delta/b$	$\varphi_{CCD} = 2/15\delta/b$	$\varphi_{EEF} = 2/15\delta/b$	$\varphi_{BBG} = 0$	$\varphi_{GGD} = 0$

$u_{DDH} = -2/15\delta$	$u_{HHF} = -2/15\delta$
$v_{DDH} = 0$	$v_{HHF} = -2/15\delta$
$\varphi_{DDH} = -2/15\delta/b$	$\varphi_{HHF} = 2/15\delta/b$

AL5.003

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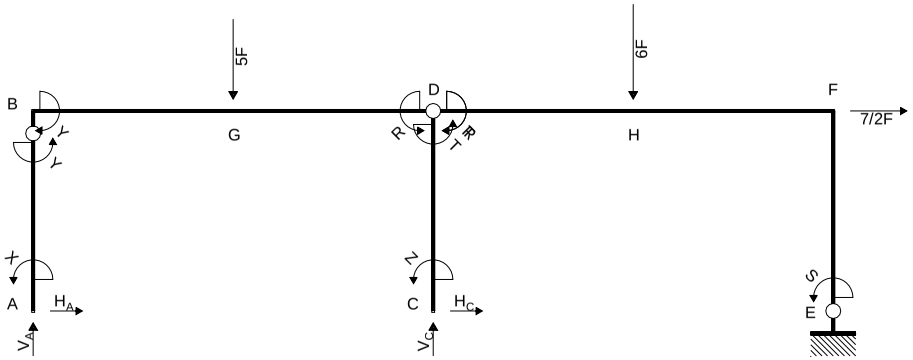


EQUAZIONI DI EQUILIBRIO

Rotazione intorno a E: aste EF FH HD DC DG GB BA

$-4V_Ab -2V_Cb = -Xb -Zb -Sb -35/2Fb$

Rotazione intorno a D: aste DC

$H_Cb = -Zb -Tb$

Rotazione intorno a D: aste DG GB BA

$H_Ab -2V_Ab = -Xb -Rb -5Fb$

Rotazione intorno a B: aste BA

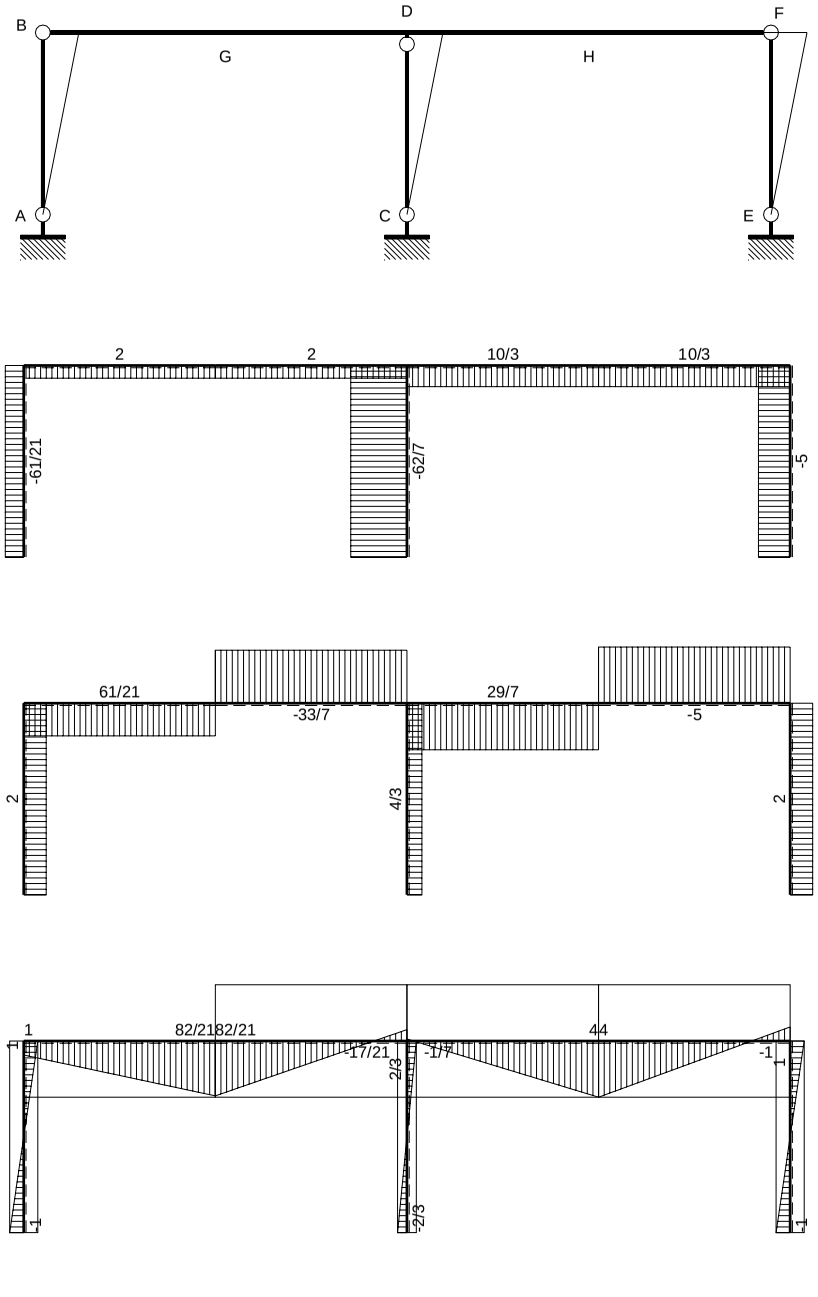
$H_Ab = -Xb -Yb$

Matrice di equilibrio

	H_Ab	V_Ab	H_Cb	V_Cb		Xb	Yb	Zb	Tb	Sb	Rb	Fb
φ_{EF}	0	-4	0	-2	=	-1	0	-1	0	-1	0	-35/2
φ_{DC}	0	0	1	0		0	0	-1	-1	0	0	0
φ_{DG}	1	-2	0	0		-1	0	0	0	0	-1	-5
φ_{BA}	1	0	0	0		-1	-1	0	0	0	0	0

Soluzione del sistema

	Xb	Yb	Zb	Tb	Sb	Rb	Fb
V_Ab	0	-1/2	0	0	0	1/2	5/2
H_Cb	0	0	-1	-1	0	0	0
H_Ab	-1	-1	0	0	0	0	0
V_Cb	1/2	1	1/2	0	1/2	-1	15/4



PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq} = 1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	$\ge$	H_{14}
D_2	H_{21}	H_{22}	H_{23}	$\ge$	H_{24}
D_3	H_{31}	H_{32}	H_{33}	$\ge$	H_{34}
D_4	H_{41}	H_{42}	H_{43}	$\ge$	H_{44}
D_5	H_{51}	H_{52}	H_{53}	$\ge$	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{CD}$ $T=W_{DC}$ $S=W_{EF}$ $R=W_{DG}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	1	0	0	0	0	0	0	≤	1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	1	0	0	0	0	0	≤	1
W_{CD}-	0	0	1	0	0	0	0	≥	-2/3
W_{CD}+	0	0	1	0	0	0	0	≤	2/3
W_{DC}-	0	0	0	1	0	0	0	≥	-2/3
W_{DC}+	0	0	0	1	0	0	0	≤	2/3
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	1	0	0	≤	1
W_{FE}-	-1	-1	-1	-1	-1	0	7/2	≥	-1
W_{FE}+	-1	-1	-1	-1	-1	0	7/2	≤	1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	-1/2	0	0	0	-1/2	-5/2	≤	4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	1	0	≤	4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	-1	0	-1	0	≤	4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-5/4	≥	-4
W_{HF}+	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-5/4	≤	4
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	0	≥	-1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	≥	-1
W_{CD}-	0	0	1	0	0	0	0	≥	-2/3
W_{CD}+	0	0	-1	0	0	0	0	≥	-2/3
W_{DC}-	0	0	0	1	0	0	0	≥	-2/3
W_{DC}+	0	0	0	-1	0	0	0	≥	-2/3
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	≥	-1
W_{FE}-	-1	-1	-1	-1	-1	0	7/2	≥	-1
W_{FE}+	1	1	1	1	1	0	-7/2	≥	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	5/2	≥	-4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	≥	-4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	≥	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-5/4	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	5/4	≥	-4
Max	0	0	0	0	0	0	1	=	0

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Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1
W_{CD}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-2/3
W_{CD}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-2/3
W_{DC}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-2/3
W_{DC}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-2/3
W_{EF}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1
W_{FE}-	-1	-1	-1	-1	-1	0	1	1	1	1	1	0	7/2	≥	-1
W_{FE}+	1	1	1	1	1	0	-1	-1	-1	-1	-1	0	-7/2	≤	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	0	1/2	0	0	0	1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	0	-1/2	0	0	0	-1/2	5/2	≤	-4
W_{DG}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-4
W_{DH}-	0	0	0	-1	0	-1	0	0	0	1	0	1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	0	0	-1	0	-1	0	≤	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	1/2	1/2	1/2	1	1/2	1/2	-5/4	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	-1/2	-1/2	-1/2	-1	-1/2	-1/2	5/4	≤	-4
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

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Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-2/3
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-2/3
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-2/3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-2/3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	-1	-1	-1	-1	-1	0	7/2	5	≥	-1
φ_{FE}+	1	1	1	1	1	0	-7/2	-5	≥	-1
φ_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	1	≥	-4
φ_{GD}+	0	1/2	0	0	0	1/2	5/2	-1	≥	-4
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-5/4	7/2	≥	-4
φ_{HF}+	1/2	1/2	1/2	1	1/2	1/2	5/4	-7/2	≥	-4
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 12-7

	X	Y	Z	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-2/3
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-2/3
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-2/3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-2/3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	2/7	2/7	2/7	2/7	2/7	0	-2/7	-10/7	≥	-2/7
φ_{GD}-	-5/7	-17/14	-5/7	-5/7	-5/7	-1/2	5/7	32/7	≥	-23/7
φ_{GD}+	5/7	17/14	5/7	5/7	5/7	1/2	-5/7	-32/7	≥	-33/7
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	-6/7	-6/7	-6/7	-19/14	-6/7	-1/2	5/14	37/7	≥	-51/14
φ_{HF}+	6/7	6/7	6/7	19/14	6/7	1/2	-5/14	-37/7	≥	-61/14
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	2/7	2/7	2/7	2/7	2/7	0	-2/7	-10/7	=	-2/7

Scambio pivotale 2-1

	φ_{AB}+	Y	Z	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-2/3
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-2/3
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-2/3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-2/3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-2/7	2/7	2/7	2/7	2/7	0	-2/7	-8/7	≥	-4/7
φ_{GD}-	5/7	-17/14	-5/7	-5/7	-5/7	-1/2	5/7	27/7	≥	-18/7
φ_{GD}+	-5/7	17/14	5/7	5/7	5/7	1/2	-5/7	-27/7	≥	-38/7
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	6/7	-6/7	-6/7	-19/14	-6/7	-1/2	5/14	31/7	≥	-39/14
φ_{HF}+	-6/7	6/7	6/7	19/14	6/7	1/2	-5/14	-31/7	≥	-73/14
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-2/7	2/7	2/7	2/7	2/7	0	-2/7	-8/7	=	-4/7

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Scambio pivotale 4-2

	φ_{AB}+	φ_{BA}+	Z	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	-1	0	0	0	0	0	0	≥	-2
Y	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-2/3
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-2/3
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-2/3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-2/3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-2/7	-2/7	2/7	2/7	2/7	0	-2/7	-6/7	≥	-6/7
φ_{GD}-	5/7	17/14	-5/7	-5/7	-5/7	-1/2	5/7	37/14	≥	-19/14
φ_{GD}+	-5/7	-17/14	5/7	5/7	5/7	1/2	-5/7	-37/14	≥	-93/14
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	6/7	6/7	-6/7	-19/14	-6/7	-1/2	5/14	25/7	≥	-27/14
φ_{HF}+	-6/7	-6/7	6/7	19/14	6/7	1/2	-5/14	-25/7	≥	-85/14
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-2/7	-2/7	2/7	2/7	2/7	0	-2/7	-6/7	=	-6/7

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Scambio pivotale 6-3

	φ_{AB}+	φ_{BA}+	φ_{CD}+	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	-1	0	0	0	0	0	0	≥	-2
Y	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-4/3
Z	0	0	-1	0	0	0	0	1	≥	-2/3
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-2/3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-2/3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-2/7	-2/7	-2/7	2/7	2/7	0	-2/7	-4/7	≥	-22/21
φ_{GD}-	5/7	17/14	5/7	-5/7	-5/7	-1/2	5/7	27/14	≥	-37/42
φ_{GD}+	-5/7	-17/14	-5/7	5/7	5/7	1/2	-5/7	-27/14	≥	-299/42
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	6/7	6/7	6/7	-19/14	-6/7	-1/2	5/14	19/7	≥	-19/14
φ_{HF}+	-6/7	-6/7	-6/7	19/14	6/7	1/2	-5/14	-19/7	≥	-93/14
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-2/7	-2/7	-2/7	2/7	2/7	0	-2/7	-4/7	=	-22/21

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Scambio pivotale 8-4

	φ_{AB}+	φ_{BA}+	φ_{CD}+	φ_{DC}+	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	-1	0	0	0	0	0	0	≥	-2
Y	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-4/3
Z	0	0	-1	0	0	0	0	1	≥	-2/3
φ_{DC}-	0	0	0	-1	0	0	0	0	≥	-4/3
T	0	0	0	-1	0	0	0	1	≥	-2/3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-2/7	-2/7	-2/7	-2/7	2/7	0	-2/7	-2/7	≥	-26/21
φ_{GD}-	5/7	17/14	5/7	5/7	-5/7	-1/2	5/7	17/14	≥	-17/42
φ_{GD}+	-5/7	-17/14	-5/7	-5/7	5/7	1/2	-5/7	-17/14	≥	-319/42
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	1	0	-1	0	1	≥	-10/3
φ_{DH}+	0	0	0	-1	0	1	0	-1	≥	-14/3
φ_{HF}-	6/7	6/7	6/7	19/14	-6/7	-1/2	5/14	19/14	≥	-19/42
φ_{HF}+	-6/7	-6/7	-6/7	-19/14	6/7	1/2	-5/14	-19/14	≥	-317/42
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-2/7	-2/7	-2/7	-2/7	2/7	0	-2/7	-2/7	=	-26/21

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Scambio pivotale 19-5

	φ_{AB}+	φ_{BA}+	φ_{CD}+	φ_{DC}+	φ_{HF}-	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	-1	0	0	0	0	0	0	≥	-2
Y	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-4/3
Z	0	0	-1	0	0	0	0	1	≥	-2/3
φ_{DC}-	0	0	0	-1	0	0	0	0	≥	-4/3
T	0	0	0	-1	0	0	0	1	≥	-2/3
φ_{EF}-	1	1	1	19/12	-7/6	-7/12	5/12	7/12	≥	-55/36
φ_{EF}+	-1	-1	-1	-19/12	7/6	7/12	-5/12	-7/12	≥	-17/36
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	0	0	0	1/6	-1/3	-1/6	-1/6	1/6	≥	-25/18
φ_{GD}-	0	1/2	0	-5/12	5/6	-1/12	5/12	1/12	≥	-1/36
φ_{GD}+	0	-1/2	0	5/12	-5/6	1/12	-5/12	-1/12	≥	-287/36
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	1	0	-1	0	1	≥	-10/3
φ_{DH}+	0	0	0	-1	0	1	0	-1	≥	-14/3
S	1	1	1	19/12	-7/6	-7/12	5/12	19/12	≥	-19/36
φ_{HF}+	0	0	0	0	-1	0	0	0	≥	-8
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	0	0	0	1/6	-1/3	-1/6	-1/6	1/6	=	-25/18

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Scambio pivotale 13-4

	φ_{AB}+	φ_{BA}+	φ_{CD}+	φ_{GD}-	φ_{HF}-	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	-1	0	0	0	0	0	0	≥	-2
Y	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-4/3
Z	0	0	-1	0	0	0	0	1	≥	-2/3
φ_{DC}-	0	-6/5	0	12/5	-2	1/5	-1	-1/5	≥	-19/15
T	0	-6/5	0	12/5	-2	1/5	-1	4/5	≥	-3/5
φ_{EF}-	1	29/10	1	-19/5	2	-9/10	2	9/10	≥	-49/30
φ_{EF}+	-1	-29/10	-1	19/5	-2	9/10	-2	-9/10	≥	-11/30
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	0	1/5	0	-2/5	0	-1/5	0	1/5	≥	-7/5
φ_{DC}+	0	6/5	0	-12/5	2	-1/5	1	1/5	≥	-1/15
φ_{GD}+	0	0	0	-1	0	0	0	0	≥	-8
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	6/5	0	-12/5	2	-6/5	1	6/5	≥	-17/5
φ_{DH}+	0	-6/5	0	12/5	-2	6/5	-1	-6/5	≥	-23/5
S	1	29/10	1	-19/5	2	-9/10	2	19/10	≥	-19/30
φ_{HF}+	0	0	0	0	-1	0	0	0	≥	-8
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	0	1/5	0	-2/5	0	-1/5	0	1/5	=	-7/5

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Scambio pivotale 10-2

	φ_{AB}+	φ_{EF}+	φ_{CD}+	φ_{GD}-	φ_{HF}-	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	10/29	10/29	10/29	-38/29	20/29	-9/29	20/29	9/29	≥	-163/87
Y	10/29	10/29	10/29	-38/29	20/29	-9/29	20/29	38/29	≥	-76/87
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-4/3
Z	0	0	-1	0	0	0	0	1	≥	-2/3
φ_{DC}-	12/29	12/29	12/29	24/29	-34/29	-5/29	-5/29	5/29	≥	-97/87
T	12/29	12/29	12/29	24/29	-34/29	-5/29	-5/29	34/29	≥	-13/29
φ_{EF}-	0	-1	0	0	0	0	0	0	≥	-2
φ_{BA}+	-10/29	-10/29	-10/29	38/29	-20/29	9/29	-20/29	-9/29	≥	-11/87
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-2/29	-2/29	-2/29	-4/29	-4/29	-4/29	-4/29	4/29	≥	-124/87
φ_{DC}+	-12/29	-12/29	-12/29	-24/29	34/29	5/29	5/29	-5/29	≥	-19/87
φ_{GD}+	0	0	0	-1	0	0	0	0	≥	-8
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	-12/29	-12/29	-12/29	-24/29	34/29	-24/29	5/29	24/29	≥	-103/29
φ_{DH}+	12/29	12/29	12/29	24/29	-34/29	24/29	-5/29	-24/29	≥	-129/29
S	0	-1	0	0	0	0	0	1	≥	-1
φ_{HF}+	0	0	0	0	-1	0	0	0	≥	-8
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-2/29	-2/29	-2/29	-4/29	-4/29	-4/29	-4/29	4/29	=	-124/87

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Scambio pivotale 10-8

	φ_{AB}+	φ_{EF}+	φ_{CD}+	φ_{GD}-	φ_{HF}-	R	φ_{FE}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-19/9	-10/9	-10/9	38/9	-20/9	1	-20/9	-29/9	≥	-38/27
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
Y	-10/9	-10/9	-10/9	38/9	-20/9	1	-20/9	-38/9	≥	-38/27
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-4/3
Z	-10/9	-10/9	-19/9	38/9	-20/9	1	-20/9	-29/9	≥	-29/27
φ_{DC}-	2/9	2/9	2/9	14/9	-14/9	0	-5/9	-5/9	≥	-32/27
T	-8/9	-8/9	-8/9	52/9	-34/9	1	-25/9	-34/9	≥	-25/27
φ_{EF}-	0	-1	0	0	0	0	0	0	≥	-2
X-	-10/9	-10/9	-10/9	38/9	-20/9	1	-20/9	-29/9	≥	-11/27
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-2/9	-2/9	-2/9	4/9	-4/9	0	-4/9	-4/9	≥	-40/27
φ_{DC}+	-2/9	-2/9	-2/9	-14/9	14/9	0	5/9	5/9	≥	-4/27
φ_{GD}+	0	0	0	-1	0	0	0	0	≥	-8
φ_{DG}-	10/9	10/9	10/9	-38/9	20/9	0	20/9	29/9	≥	-97/27
φ_{DG}+	-10/9	-10/9	-10/9	38/9	-20/9	0	-20/9	-29/9	≥	-119/27
φ_{DH}-	-4/3	-4/3	-4/3	8/3	-2/3	0	-5/3	-8/3	≥	-35/9
φ_{DH}+	4/3	4/3	4/3	-8/3	2/3	0	5/3	8/3	≥	-37/9
S	-10/9	-19/9	-10/9	38/9	-20/9	1	-20/9	-29/9	≥	-38/27
φ_{HF}+	0	0	0	0	-1	0	0	0	≥	-8
L_X	10/9	10/9	10/9	-38/9	20/9	-1	20/9	29/9	≥	-97/27
Max	-2/9	-2/9	-2/9	4/9	-4/9	0	-4/9	-4/9	=	-40/27

Scambio pivotale 13-4

	φ_{AB}+	φ_{EF}+	φ_{CD}+	φ_{DC}+	φ_{HF}-	R	φ_{FE}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-19/7	-12/7	-12/7	-19/7	2	1	-5/7	-12/7	≥	-38/21
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
Y	-12/7	-12/7	-12/7	-19/7	2	1	-5/7	-19/7	≥	-38/21
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-4/3
Z	-12/7	-12/7	-19/7	-19/7	2	1	-5/7	-12/7	≥	-31/21
φ_{DC}-	0	0	0	-1	0	0	0	0	≥	-4/3
T	-12/7	-12/7	-12/7	-26/7	2	1	-5/7	-12/7	≥	-31/21
φ_{EF}-	0	-1	0	0	0	0	0	0	≥	-2
X-	-12/7	-12/7	-12/7	-19/7	2	1	-5/7	-12/7	≥	-17/21
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-2/7	-2/7	-2/7	-2/7	0	0	-2/7	-2/7	≥	-32/21
φ_{GD}-	-1/7	-1/7	-1/7	-9/14	1	0	5/14	5/14	≥	-2/21
φ_{GD}+	1/7	1/7	1/7	9/14	-1	0	-5/14	-5/14	≥	-166/21
φ_{DG}-	12/7	12/7	12/7	19/7	-2	0	5/7	12/7	≥	-67/21
φ_{DG}+	-12/7	-12/7	-12/7	-19/7	2	0	-5/7	-12/7	≥	-101/21
φ_{DH}-	-12/7	-12/7	-12/7	-12/7	2	0	-5/7	-12/7	≥	-29/7
φ_{DH}+	12/7	12/7	12/7	12/7	-2	0	5/7	12/7	≥	-27/7
S	-12/7	-19/7	-12/7	-19/7	2	1	-5/7	-12/7	≥	-38/21
φ_{HF}+	0	0	0	0	-1	0	0	0	≥	-8
L_X	12/7	12/7	12/7	19/7	-2	-1	5/7	12/7	≥	-67/21
Max	-2/7	-2/7	-2/7	-2/7	0	0	-2/7	-2/7	=	-32/21

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Tableau finale

	φ_{AB}+	φ_{EF}+	φ_{CD}+	φ_{DC}+	φ_{HF}-	R	φ_{FE}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-19/7	-12/7	-12/7	-19/7	2	1	-5/7	-12/7	≥	-38/21
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
Y	-12/7	-12/7	-12/7	-19/7	2	1	-5/7	-19/7	≥	-38/21
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-4/3
Z	-12/7	-12/7	-19/7	-19/7	2	1	-5/7	-12/7	≥	-31/21
φ_{DC}-	0	0	0	-1	0	0	0	0	≥	-4/3
T	-12/7	-12/7	-12/7	-26/7	2	1	-5/7	-12/7	≥	-31/21
φ_{EF}-	0	-1	0	0	0	0	0	0	≥	-2
X-	-12/7	-12/7	-12/7	-19/7	2	1	-5/7	-12/7	≥	-17/21
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-2/7	-2/7	-2/7	-2/7	0	0	-2/7	-2/7	≥	-32/21
φ_{GD}-	-1/7	-1/7	-1/7	-9/14	1	0	5/14	5/14	≥	-2/21
φ_{GD}+	1/7	1/7	1/7	9/14	-1	0	-5/14	-5/14	≥	-166/21
φ_{DG}-	12/7	12/7	12/7	19/7	-2	0	5/7	12/7	≥	-67/21
φ_{DG}+	-12/7	-12/7	-12/7	-19/7	2	0	-5/7	-12/7	≥	-101/21
φ_{DH}-	-12/7	-12/7	-12/7	-12/7	2	0	-5/7	-12/7	≥	-29/7
φ_{DH}+	12/7	12/7	12/7	12/7	-2	0	5/7	12/7	≥	-27/7
S	-12/7	-19/7	-12/7	-19/7	2	1	-5/7	-12/7	≥	-38/21
φ_{HF}+	0	0	0	0	-1	0	0	0	≥	-8
L_X	12/7	12/7	12/7	19/7	-2	-1	5/7	12/7	≥	-67/21
Max	-2/7	-2/7	-2/7	-2/7	0	0	-2/7	-2/7	=	-32/21

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Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	0
φ_{AB}+	0	0	0	0	0	0	0	0	≥	2/7
φ_{BA}-	0	0	0	0	0	0	0	0	≥	0
φ_{BA}+	0	0	0	0	0	0	0	0	≥	2/7
φ_{CD}-	0	0	0	0	0	0	0	0	≥	0
φ_{CD}+	0	0	0	0	0	0	0	0	≥	2/7
φ_{DC}-	0	0	0	0	0	0	0	0	≥	0
φ_{DC}+	0	0	0	0	0	0	0	0	≥	2/7
φ_{EF}-	0	0	0	0	0	0	0	0	≥	0
φ_{EF}+	0	0	0	0	0	0	0	0	≥	2/7
φ_{FE}-	0	0	0	0	0	0	0	0	≥	0
φ_{FE}+	0	0	0	0	0	0	0	0	≥	2/7
φ_{GD}-	0	0	0	0	0	0	0	0	≥	0
φ_{GD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DG}-	0	0	0	0	0	0	0	0	≥	0
φ_{DG}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	0
φ_{HF}-	0	0	0	0	0	0	0	0	≥	0
φ_{HF}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	38/21	38/21	31/21	31/21	38/21	0	32/21	17/21	=	-32/21

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
1	1	2/3	2/3	1	-17/21

Variabili soluzione differenza tra rotazioni

φ_{AB}	2/7
φ_{BA}	2/7
φ_{CD}	2/7
φ_{DC}	2/7
φ_{EF}	2/7
φ_{FE}	2/7
φ_{GD}	0
φ_{DG}	0
φ_{DH}	0
φ_{HF}	0

REAZIONI Fattore di collasso = 32/21

$H_A = -2F$
$V_A = 61/21F$
$W_A = Fb$
$H_C = -4/3F$
$V_C = 62/7F$
$W_C = 2/3Fb$
$H_E = -2F$
$V_E = 5F$
$W_E = Fb$

$H_{AB} = -2F$	$H_{CD} = -4/3F$	$H_{EF} = -2F$	$H_{BG} = -2F$	$H_{GD} = -2F$
$V_{AB} = 61/21F$	$V_{CD} = 62/7F$	$V_{EF} = 5F$	$V_{BG} = 61/21F$	$V_{GD} = -33/7F$
$W_{AB} = Fb$	$W_{CD} = 2/3Fb$	$W_{EF} = Fb$	$W_{BG} = -Fb$	$W_{GD} = -82/21Fb$
$H_{BA} = 2F$	$H_{DC} = 4/3F$	$H_{FE} = 2F$	$H_{GB} = 2F$	$H_{DG} = 2F$
$V_{BA} = -61/21F$	$V_{DC} = -62/7F$	$V_{FE} = -5F$	$V_{GB} = -61/21F$	$V_{DG} = 33/7F$
$W_{BA} = Fb$	$W_{DC} = 2/3Fb$	$W_{FE} = Fb$	$W_{GB} = 82/21Fb$	$W_{DG} = -17/21Fb$

$H_{DH} = -10/3F$	$H_{HF} = -10/3F$
$V_{DH} = 29/7F$	$V_{HF} = -5F$
$W_{DH} = 1/7Fb$	$W_{HF} = -4Fb$
$H_{HD} = 10/3F$	$H_{FH} = 10/3F$
$V_{HD} = -29/7F$	$V_{FH} = 5F$
$W_{HD} = 4Fb$	$W_{FH} = -Fb$

SPOSTAMENTI NODALI

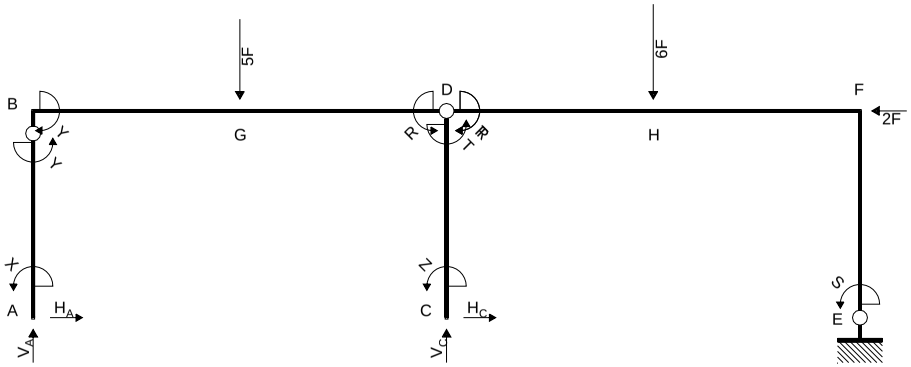
$u_{AAB} = 0$	$u_{BBA} = 2/7\delta$	$u_{CCD} = 0$	$u_D = 2/7\delta$	$u_{EEF} = 0$	$u_{FFE} = 2/7\delta$
$v_{AAB} = 0$	$v_{BBA} = 0$	$v_{CCD} = 0$	$v_D = 0$	$v_{EEF} = 0$	$v_{FFE} = 0$
$\varphi_{AAB} = -2/7\delta/b$	$\varphi_{BBA} = -2/7\delta/b$	$\varphi_{CCD} = -2/7\delta/b$	$\varphi_D = -2/7\delta/b$	$\varphi_{EEF} = -2/7\delta/b$	$\varphi_{FFH} = -2/7\delta/b$

$u_G = 2/7\delta$	$u_H = 2/7\delta$
$v_G = 0$	$v_H = 0$
$\varphi_G = 0$	$\varphi_H = 0$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$	$u_{CCD} = 0$	$u_{EEF} = 0$	$u_{BBG} = 2/7\delta$	$u_{GGD} = 2/7\delta$	$u_{DDH} = 2/7\delta$
$v_{AAB} = 0$	$v_{CCD} = 0$	$v_{EEF} = 0$	$v_{BBG} = 0$	$v_{GGD} = 0$	$v_{DDH} = 0$
$\varphi_{AAB} = -2/7\delta/b$	$\varphi_{CCD} = -2/7\delta/b$	$\varphi_{EEF} = -2/7\delta/b$	$\varphi_{BBG} = 0$	$\varphi_{GGD} = 0$	$\varphi_{DDH} = 0$

$u_{HHF} = 2/7\delta$
$v_{HHF} = 0$
$\varphi_{HHF} = 0$



EQUAZIONI DI EQUILIBRIO

Rotazione intorno a E: aste EF FH HD DC DG GB BA

$-4V_Ab -2V_Cb = -Xb -Zb -Sb -23Fb$

Rotazione intorno a D: aste DC

$H_Cb = -Zb -Tb$

Rotazione intorno a D: aste DG GB BA

$H_Ab -2V_Ab = -Xb -Rb -5Fb$

Rotazione intorno a B: aste BA

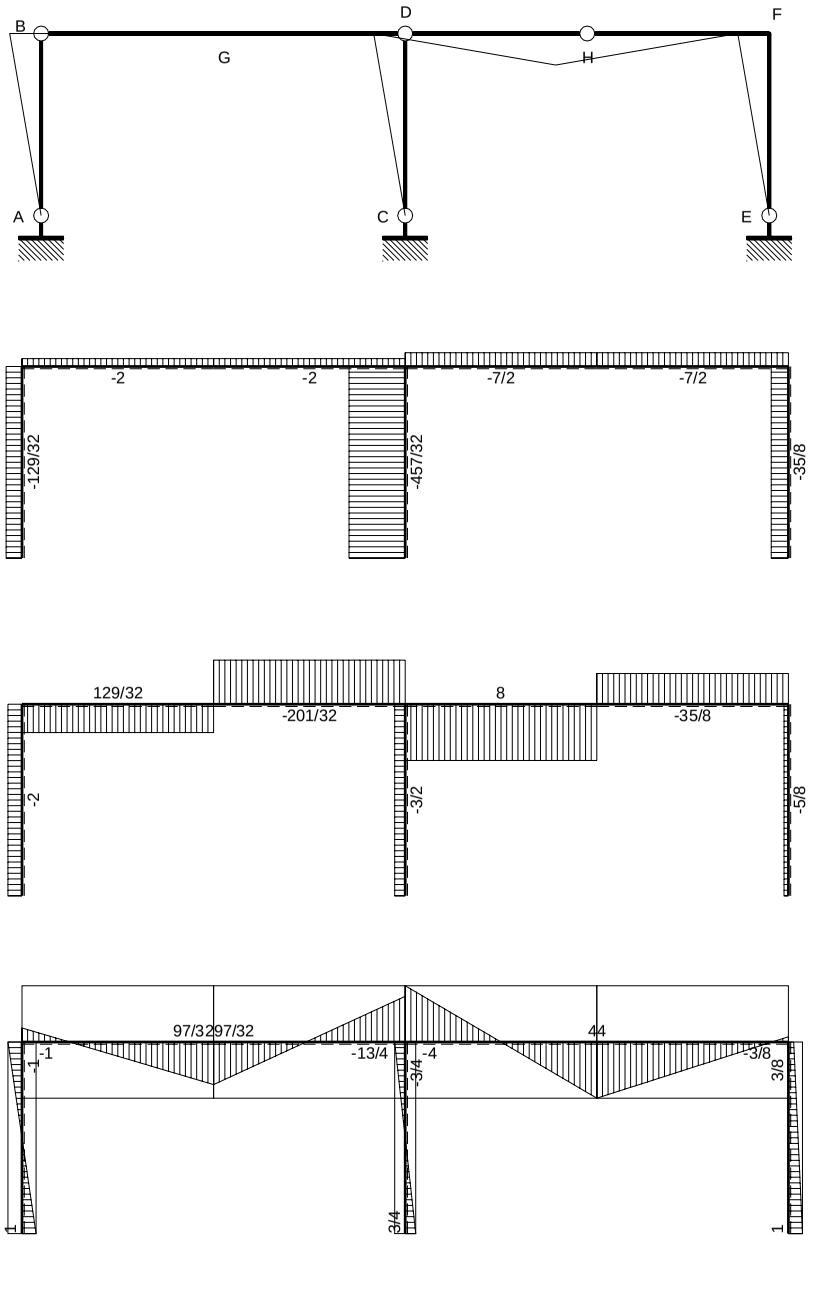
$H_Ab = -Xb -Yb$

Matrice di equilibrio

$$\begin{array}{c} \\ \varphi_{EF} \\ \varphi_{DC} \\ \varphi_{DG} \\ \varphi_{BA} \end{array} \begin{bmatrix} H_Ab & V_Ab & H_Cb & V_Cb \\ 0 & -4 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 1 & -2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1 & 0 & -1 & 0 & -1 & 0 & -23 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & -1 & -5 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Soluzione del sistema

$$\begin{bmatrix} V_Ab \\ H_Cb \\ H_Ab \\ V_Cb \end{bmatrix} = \begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ 0 & -1/2 & 0 & 0 & 0 & 1/2 & 5/2 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1 & 1/2 & 0 & 1/2 & -1 & 13/2 \end{bmatrix}$$



PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq} = 1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	$\ge$	H_{14}
D_2	H_{21}	H_{22}	H_{23}	$\ge$	H_{24}
D_3	H_{31}	H_{32}	H_{33}	$\ge$	H_{34}
D_4	H_{41}	H_{42}	H_{43}	$\ge$	H_{44}
D_5	H_{51}	H_{52}	H_{53}	$\ge$	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{CD}$ $T=W_{DC}$ $S=W_{EF}$ $R=W_{DG}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	1	0	0	0	0	0	0	≤	1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	1	0	0	0	0	0	≤	1
W_{CD}-	0	0	1	0	0	0	0	≥	-3/4
W_{CD}+	0	0	1	0	0	0	0	≤	3/4
W_{DC}-	0	0	0	1	0	0	0	≥	-3/4
W_{DC}+	0	0	0	1	0	0	0	≤	3/4
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	1	0	0	≤	1
W_{FE}-	-1	-1	-1	-1	-1	0	-2	≥	-1
W_{FE}+	-1	-1	-1	-1	-1	0	-2	≤	1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	-1/2	0	0	0	-1/2	-5/2	≤	4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	1	0	≤	4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	-1	0	-1	0	≤	4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-4	≥	-4
W_{HF}+	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-4	≤	4
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	0	≥	-1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	≥	-1
W_{CD}-	0	0	1	0	0	0	0	≥	-3/4
W_{CD}+	0	0	-1	0	0	0	0	≥	-3/4
W_{DC}-	0	0	0	1	0	0	0	≥	-3/4
W_{DC}+	0	0	0	-1	0	0	0	≥	-3/4
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	≥	-1
W_{FE}-	-1	-1	-1	-1	-1	0	-2	≥	-1
W_{FE}+	1	1	1	1	1	0	2	≥	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	5/2	≥	-4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	≥	-4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	≥	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-4	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	4	≥	-4
Max	0	0	0	0	0	0	1	=	0

PROCEDIMENTO E RISULTATI Nome: AL5.005

Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1
W_{CD}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/4
W_{CD}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/4
W_{DC}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/4
W_{DC}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/4
W_{EF}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1
W_{FE}-	-1	-1	-1	-1	-1	0	1	1	1	1	1	0	-2	≥	-1
W_{FE}+	1	1	1	1	1	0	-1	-1	-1	-1	-1	0	2	≤	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	0	1/2	0	0	0	1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	0	-1/2	0	0	0	-1/2	5/2	≤	-4
W_{DG}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-4
W_{DH}-	0	0	0	-1	0	-1	0	0	0	1	0	1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	0	0	-1	0	-1	0	≤	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	1/2	1/2	1/2	1	1/2	1/2	-4	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	-1/2	-1/2	-1/2	-1	-1/2	-1/2	4	≤	-4
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.11.05.11 08.06.11

Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-3/4
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-3/4
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-3/4
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-3/4
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	-1	-1	-1	-1	-1	0	-2	5	≥	-1
φ_{FE}+	1	1	1	1	1	0	2	-5	≥	-1
φ_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	1	≥	-4
φ_{GD}+	0	1/2	0	0	0	1/2	5/2	-1	≥	-4
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-4	7/2	≥	-4
φ_{HF}+	1/2	1/2	1/2	1	1/2	1/2	4	-7/2	≥	-4
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 11-7

	X	Y	Z	T	S	R	φ_{FE}-	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-3/4
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-3/4
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-3/4
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-3/4
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
αbF	-1/2	-1/2	-1/2	-1/2	-1/2	0	-1/2	5/2	≥	-1/2
φ_{FE}+	0	0	0	0	0	0	-1	0	≥	-2
φ_{GD}-	5/4	3/4	5/4	5/4	5/4	-1/2	5/4	-21/4	≥	-11/4
φ_{GD}+	-5/4	-3/4	-5/4	-5/4	-5/4	1/2	-5/4	21/4	≥	-21/4
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	3/2	3/2	3/2	1	3/2	-1/2	2	-13/2	≥	-2
φ_{HF}+	-3/2	-3/2	-3/2	-1	-3/2	1/2	-2	13/2	≥	-6
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-1/2	-1/2	-1/2	-1/2	-1/2	0	-1/2	5/2	=	-1/2

Scambio pivotale 19-8

	X	Y	Z	T	S	R	φ_{FE}-	φ_{HF}-		Fb
φ_{AB}-	10/13	-3/13	-3/13	-2/13	-3/13	1/13	-4/13	2/13	≥	-9/13
φ_{AB}+	-10/13	3/13	3/13	2/13	3/13	-1/13	4/13	-2/13	≥	-17/13
φ_{BA}-	-3/13	10/13	-3/13	-2/13	-3/13	1/13	-4/13	2/13	≥	-9/13
φ_{BA}+	3/13	-10/13	3/13	2/13	3/13	-1/13	4/13	-2/13	≥	-17/13
φ_{CD}-	-3/13	-3/13	10/13	-2/13	-3/13	1/13	-4/13	2/13	≥	-23/52
φ_{CD}+	3/13	3/13	-10/13	2/13	3/13	-1/13	4/13	-2/13	≥	-55/52
φ_{DC}-	-3/13	-3/13	-3/13	11/13	-3/13	1/13	-4/13	2/13	≥	-23/52
φ_{DC}+	3/13	3/13	3/13	-11/13	3/13	-1/13	4/13	-2/13	≥	-55/52
φ_{EF}-	-3/13	-3/13	-3/13	-2/13	10/13	1/13	-4/13	2/13	≥	-9/13
φ_{EF}+	3/13	3/13	3/13	2/13	-10/13	-1/13	4/13	-2/13	≥	-17/13
αbF	1/13	1/13	1/13	-3/26	1/13	-5/26	7/26	-5/13	≥	-33/26
φ_{FE}+	0	0	0	0	0	0	-1	0	≥	-2
φ_{GD}-	1/26	-6/13	1/26	23/52	1/26	-5/52	-19/52	21/26	≥	-59/52
φ_{GD}+	-1/26	6/13	-1/26	-23/52	-1/26	5/52	19/52	-21/26	≥	-357/52
φ_{DG}-	-3/13	-3/13	-3/13	-2/13	-3/13	14/13	-4/13	2/13	≥	-48/13
φ_{DG}+	3/13	3/13	3/13	2/13	3/13	-14/13	4/13	-2/13	≥	-56/13
φ_{DH}-	6/13	6/13	6/13	-9/13	6/13	-15/13	8/13	-4/13	≥	-60/13
φ_{DH}+	-6/13	-6/13	-6/13	9/13	-6/13	15/13	-8/13	4/13	≥	-44/13
X-	3/13	3/13	3/13	2/13	3/13	-1/13	4/13	-2/13	≥	-4/13
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	-3/13	-3/13	-3/13	-2/13	-3/13	1/13	-4/13	2/13	≥	-48/13
Max	1/13	1/13	1/13	-3/26	1/13	-5/26	7/26	-5/13	=	-33/26

PROCEDIMENTO E RISULTATI Nome: AL5.005

Scambio pivotale 5-7

	X	Y	Z	T	S	R	φ_{CD}-	φ_{HF}-		Fb
φ_{AB}-	1	0	-1	0	0	0	1	0	≥	-1/4
φ_{AB}+	-1	0	1	0	0	0	-1	0	≥	-7/4
φ_{BA}-	0	1	-1	0	0	0	1	0	≥	-1/4
φ_{BA}+	0	-1	1	0	0	0	-1	0	≥	-7/4
φ_{FE}-	-3/4	-3/4	5/2	-1/2	-3/4	1/4	-13/4	1/2	≥	-23/16
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-3/2
φ_{DC}-	0	0	-1	1	0	0	1	0	≥	0
φ_{DC}+	0	0	1	-1	0	0	-1	0	≥	-3/2
φ_{EF}-	0	0	-1	0	1	0	1	0	≥	-1/4
φ_{EF}+	0	0	1	0	-1	0	-1	0	≥	-7/4
αbF	-1/8	-1/8	3/4	-1/4	-1/8	-1/8	-7/8	-1/4	≥	-53/32
φ_{FE}+	3/4	3/4	-5/2	1/2	3/4	-1/4	13/4	-1/2	≥	-9/16
φ_{GD}-	5/16	-3/16	-7/8	5/8	5/16	-3/16	19/16	5/8	≥	-39/64
φ_{GD}+	-5/16	3/16	7/8	-5/8	-5/16	3/16	-19/16	-5/8	≥	-473/64
φ_{DG}-	0	0	-1	0	0	1	1	0	≥	-13/4
φ_{DG}+	0	0	1	0	0	-1	-1	0	≥	-19/4
φ_{DH}-	0	0	2	-1	0	-1	-2	0	≥	-11/2
φ_{DH}+	0	0	-2	1	0	1	2	0	≥	-5/2
X-	0	0	1	0	0	0	-1	0	≥	-3/4
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	-1	0	0	0	1	0	≥	-13/4
Max	-1/8	-1/8	3/4	-1/4	-1/8	-1/8	-7/8	-1/4	=	-53/32

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Scambio pivotale 7-3

	X	Y	φ_{DC}-	T	S	R	φ_{CD}-	φ_{HF}-		Fb
φ_{AB}-	1	0	1	-1	0	0	0	0	≥	-1/4
φ_{AB}+	-1	0	-1	1	0	0	0	0	≥	-7/4
φ_{BA}-	0	1	1	-1	0	0	0	0	≥	-1/4
φ_{BA}+	0	-1	-1	1	0	0	0	0	≥	-7/4
φ_{FE}-	-3/4	-3/4	-5/2	2	-3/4	1/4	-3/4	1/2	≥	-23/16
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-3/2
Z	0	0	-1	1	0	0	1	0	≥	0
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-3/2
φ_{EF}-	0	0	1	-1	1	0	0	0	≥	-1/4
φ_{EF}+	0	0	-1	1	-1	0	0	0	≥	-7/4
αbF	-1/8	-1/8	-3/4	1/2	-1/8	-1/8	-1/8	-1/4	≥	-53/32
φ_{FE}+	3/4	3/4	5/2	-2	3/4	-1/4	3/4	-1/2	≥	-9/16
φ_{GD}-	5/16	-3/16	7/8	-1/4	5/16	-3/16	5/16	5/8	≥	-39/64
φ_{GD}+	-5/16	3/16	-7/8	1/4	-5/16	3/16	-5/16	-5/8	≥	-473/64
φ_{DG}-	0	0	1	-1	0	1	0	0	≥	-13/4
φ_{DG}+	0	0	-1	1	0	-1	0	0	≥	-19/4
φ_{DH}-	0	0	-2	1	0	-1	0	0	≥	-11/2
φ_{DH}+	0	0	2	-1	0	1	0	0	≥	-5/2
X-	0	0	-1	1	0	0	0	0	≥	-3/4
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	1	-1	0	0	0	0	≥	-13/4
Max	-1/8	-1/8	-3/4	1/2	-1/8	-1/8	-1/8	-1/4	=	-53/32

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Scambio pivotale 1-4

	X	Y	φ_{DC}-	φ_{AB}-	S	R	φ_{CD}-	φ_{HF}-		Fb
T	1	0	1	-1	0	0	0	0	≥	-1/4
φ_{AB}+	0	0	0	-1	0	0	0	0	≥	-2
φ_{BA}-	-1	1	0	1	0	0	0	0	≥	0
φ_{BA}+	1	-1	0	-1	0	0	0	0	≥	-2
φ_{FE}-	5/4	-3/4	-1/2	-2	-3/4	1/4	-3/4	1/2	≥	-31/16
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-3/2
Z	1	0	0	-1	0	0	1	0	≥	-1/4
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-3/2
φ_{EF}-	-1	0	0	1	1	0	0	0	≥	0
φ_{EF}+	1	0	0	-1	-1	0	0	0	≥	-2
αbF	3/8	-1/8	-1/4	-1/2	-1/8	-1/8	-1/8	-1/4	≥	-57/32
φ_{FE}+	-5/4	3/4	1/2	2	3/4	-1/4	3/4	-1/2	≥	-1/16
φ_{GD}-	1/16	-3/16	5/8	1/4	5/16	-3/16	5/16	5/8	≥	-35/64
φ_{GD}+	-1/16	3/16	-5/8	-1/4	-5/16	3/16	-5/16	-5/8	≥	-477/64
φ_{DG}-	-1	0	0	1	0	1	0	0	≥	-3
φ_{DG}+	1	0	0	-1	0	-1	0	0	≥	-5
φ_{DH}-	1	0	-1	-1	0	-1	0	0	≥	-23/4
φ_{DH}+	-1	0	1	1	0	1	0	0	≥	-9/4
X-	1	0	0	-1	0	0	0	0	≥	-1
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	-1	0	0	1	0	0	0	0	≥	-3
Max	3/8	-1/8	-1/4	-1/2	-1/8	-1/8	-1/8	-1/4	=	-57/32

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Scambio pivotale 3-1

	φ_{BA}-	Y	φ_{DC}-	φ_{AB}-	S	R	φ_{CD}-	φ_{HF}-		Fb
T	-1	1	1	0	0	0	0	0	≥	-1/4
φ_{AB}+	0	0	0	-1	0	0	0	0	≥	-2
X	-1	1	0	1	0	0	0	0	≥	0
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
φ_{FE}-	-5/4	1/2	-1/2	-3/4	-3/4	1/4	-3/4	1/2	≥	-31/16
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-3/2
Z	-1	1	0	0	0	0	1	0	≥	-1/4
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-3/2
φ_{EF}-	1	-1	0	0	1	0	0	0	≥	0
φ_{EF}+	-1	1	0	0	-1	0	0	0	≥	-2
αbF	-3/8	1/4	-1/4	-1/8	-1/8	-1/8	-1/8	-1/4	≥	-57/32
φ_{FE}+	5/4	-1/2	1/2	3/4	3/4	-1/4	3/4	-1/2	≥	-1/16
φ_{GD}-	-1/16	-1/8	5/8	5/16	5/16	-3/16	5/16	5/8	≥	-35/64
φ_{GD}+	1/16	1/8	-5/8	-5/16	-5/16	3/16	-5/16	-5/8	≥	-477/64
φ_{DG}-	1	-1	0	0	0	1	0	0	≥	-3
φ_{DG}+	-1	1	0	0	0	-1	0	0	≥	-5
φ_{DH}-	-1	1	-1	0	0	-1	0	0	≥	-23/4
φ_{DH}+	1	-1	1	0	0	1	0	0	≥	-9/4
X-	-1	1	0	0	0	0	0	0	≥	-1
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	1	-1	0	0	0	0	0	0	≥	-3
Max	-3/8	1/4	-1/4	-1/8	-1/8	-1/8	-1/8	-1/4	=	-57/32

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Scambio pivotale 9-2

	φ_{BA}-	φ_{EF}-	φ_{DC}-	φ_{AB}-	S	R	φ_{CD}-	φ_{HF}-		Fb
T	0	-1	1	0	1	0	0	0	≥	-1/4
φ_{AB}+	0	0	0	-1	0	0	0	0	≥	-2
X	0	-1	0	1	1	0	0	0	≥	0
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
φ_{FE}-	-3/4	-1/2	-1/2	-3/4	-1/4	1/4	-3/4	1/2	≥	-31/16
φ_{CD}+	0	0	0	0	0	0	-1	0	≥	-3/2
Z	0	-1	0	0	1	0	1	0	≥	-1/4
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-3/2
Y	1	-1	0	0	1	0	0	0	≥	0
φ_{EF}+	0	-1	0	0	0	0	0	0	≥	-2
αbF	-1/8	-1/4	-1/4	-1/8	1/8	-1/8	-1/8	-1/4	≥	-57/32
φ_{FE}+	3/4	1/2	1/2	3/4	1/4	-1/4	3/4	-1/2	≥	-1/16
φ_{GD}-	-3/16	1/8	5/8	5/16	3/16	-3/16	5/16	5/8	≥	-35/64
φ_{GD}+	3/16	-1/8	-5/8	-5/16	-3/16	3/16	-5/16	-5/8	≥	-477/64
φ_{DG}-	0	1	0	0	-1	1	0	0	≥	-3
φ_{DG}+	0	-1	0	0	1	-1	0	0	≥	-5
φ_{DH}-	0	-1	-1	0	1	-1	0	0	≥	-23/4
φ_{DH}+	0	1	1	0	-1	1	0	0	≥	-9/4
X-	0	-1	0	0	1	0	0	0	≥	-1
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	1	0	0	-1	0	0	0	≥	-3
Max	-1/8	-1/4	-1/4	-1/8	1/8	-1/8	-1/8	-1/4	=	-57/32

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Scambio pivotale 18-5

	φ_{BA}^-	φ_{EF}^-	φ_{DC}^-	φ_{AB}^-	φ_{DH}^+	R	φ_{CD}^-	φ_{HF}^-		Fb
T	0	0	2	0	-1	1	0	0	≥	-5/2
φ_{AB}^+	0	0	0	-1	0	0	0	0	≥	-2
X	0	0	1	1	-1	1	0	0	≥	-9/4
φ_{BA}^+	-1	0	0	0	0	0	0	0	≥	-2
φ_{FE}^-	-3/4	-3/4	-3/4	-3/4	1/4	0	-3/4	1/2	≥	-11/8
φ_{CD}^+	0	0	0	0	0	0	-1	0	≥	-3/2
Z	0	0	1	0	-1	1	1	0	≥	-5/2
φ_{DC}^+	0	0	-1	0	0	0	0	0	≥	-3/2
Y	1	0	1	0	-1	1	0	0	≥	-9/4
φ_{EF}^+	0	-1	0	0	0	0	0	0	≥	-2
αbF	-1/8	-1/8	-1/8	-1/8	-1/8	0	-1/8	-1/4	≥	-33/16
φ_{FE}^+	3/4	3/4	3/4	3/4	-1/4	0	3/4	-1/2	≥	-5/8
φ_{GD}^-	-3/16	5/16	13/16	5/16	-3/16	0	5/16	5/8	≥	-31/32
φ_{GD}^+	3/16	-5/16	-13/16	-5/16	3/16	0	-5/16	-5/8	≥	-225/32
φ_{DG}^-	0	0	-1	0	1	0	0	0	≥	-3/4
φ_{DG}^+	0	0	1	0	-1	0	0	0	≥	-29/4
φ_{DH}^-	0	0	0	0	-1	0	0	0	≥	-8
S	0	1	1	0	-1	1	0	0	≥	-9/4
X-	0	0	1	0	-1	1	0	0	≥	-13/4
φ_{HF}^+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	-1	0	1	-1	0	0	≥	-3/4
Max	-1/8	-1/8	-1/8	-1/8	-1/8	0	-1/8	-1/4	=	-33/16

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Tableau finale

	φ_{BA}^-	φ_{EF}^-	φ_{DC}^-	φ_{AB}^-	φ_{DH}^+	R	φ_{CD}^-	φ_{HF}^-		Fb
T	0	0	2	0	-1	1	0	0	≥	-5/2
φ_{AB}^+	0	0	0	-1	0	0	0	0	≥	-2
X	0	0	1	1	-1	1	0	0	≥	-9/4
φ_{BA}^+	-1	0	0	0	0	0	0	0	≥	-2
φ_{FE}^-	-3/4	-3/4	-3/4	-3/4	1/4	0	-3/4	1/2	≥	-11/8
φ_{CD}^+	0	0	0	0	0	0	-1	0	≥	-3/2
Z	0	0	1	0	-1	1	1	0	≥	-5/2
φ_{DC}^+	0	0	-1	0	0	0	0	0	≥	-3/2
Y	1	0	1	0	-1	1	0	0	≥	-9/4
φ_{EF}^+	0	-1	0	0	0	0	0	0	≥	-2
αbF	-1/8	-1/8	-1/8	-1/8	-1/8	0	-1/8	-1/4	≥	-33/16
φ_{FE}^+	3/4	3/4	3/4	3/4	-1/4	0	3/4	-1/2	≥	-5/8
φ_{GD}^-	-3/16	5/16	13/16	5/16	-3/16	0	5/16	5/8	≥	-31/32
φ_{GD}^+	3/16	-5/16	-13/16	-5/16	3/16	0	-5/16	-5/8	≥	-225/32
φ_{DG}^-	0	0	-1	0	1	0	0	0	≥	-3/4
φ_{DG}^+	0	0	1	0	-1	0	0	0	≥	-29/4
φ_{DH}^-	0	0	0	0	-1	0	0	0	≥	-8
S	0	1	1	0	-1	1	0	0	≥	-9/4
X-	0	0	1	0	-1	1	0	0	≥	-13/4
φ_{HF}^+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	-1	0	1	-1	0	0	≥	-3/4
Max	-1/8	-1/8	-1/8	-1/8	-1/8	0	-1/8	-1/4	=	-33/16

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Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	1/8
φ_{AB}+	0	0	0	0	0	0	0	0	≥	0
φ_{BA}-	0	0	0	0	0	0	0	0	≥	1/8
φ_{BA}+	0	0	0	0	0	0	0	0	≥	0
φ_{CD}-	0	0	0	0	0	0	0	0	≥	1/8
φ_{CD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DC}-	0	0	0	0	0	0	0	0	≥	1/8
φ_{DC}+	0	0	0	0	0	0	0	0	≥	0
φ_{EF}-	0	0	0	0	0	0	0	0	≥	1/8
φ_{EF}+	0	0	0	0	0	0	0	0	≥	0
φ_{FE}-	0	0	0	0	0	0	0	0	≥	0
φ_{FE}+	0	0	0	0	0	0	0	0	≥	0
φ_{GD}-	0	0	0	0	0	0	0	0	≥	0
φ_{GD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DG}-	0	0	0	0	0	0	0	0	≥	0
φ_{DG}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	1/8
φ_{HF}-	0	0	0	0	0	0	0	0	≥	1/4
φ_{HF}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	9/4	9/4	5/2	5/2	9/4	0	33/16	13/4	=	-33/16

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
-1	-1	-3/4	-3/4	-1	-13/4

Variabili soluzione differenza tra rotazioni

φ_{AB}	-1/8
φ_{BA}	-1/8
φ_{CD}	-1/8
φ_{DC}	-1/8
φ_{EF}	-1/8
φ_{FE}	0
φ_{GD}	0
φ_{DG}	0
φ_{DH}	1/8
φ_{HF}	-1/4

REAZIONI Fattore di collasso = 33/16

$H_A = 2F$
$V_A = 129/32F$
$W_A = -Fb$
$H_C = 3/2F$
$V_C = 457/32F$
$W_C = -3/4Fb$
$H_E = 5/8F$
$V_E = 35/8F$
$W_E = -Fb$

$H_{AB} = 2F$	$H_{CD} = 3/2F$	$H_{EF} = 5/8F$	$H_{BG} = 2F$	$H_{GD} = 2F$
$V_{AB} = 129/32F$	$V_{CD} = 457/32F$	$V_{EF} = 35/8F$	$V_{BG} = 129/32F$	$V_{GD} = -201/32F$
$W_{AB} = -Fb$	$W_{CD} = -3/4Fb$	$W_{EF} = -Fb$	$W_{BG} = Fb$	$W_{GD} = -97/32Fb$
$H_{BA} = -2F$	$H_{DC} = -3/2F$	$H_{FE} = -5/8F$	$H_{GB} = -2F$	$H_{DG} = -2F$
$V_{BA} = -129/32F$	$V_{DC} = -457/32F$	$V_{FE} = -35/8F$	$V_{GB} = -129/32F$	$V_{DG} = 201/32F$
$W_{BA} = -Fb$	$W_{DC} = -3/4Fb$	$W_{FE} = 3/8Fb$	$W_{GB} = 97/32Fb$	$W_{DG} = -13/4Fb$

$H_{DH} = 7/2F$	$H_{HF} = 7/2F$
$V_{DH} = 8F$	$V_{HF} = -35/8F$
$W_{DH} = 4Fb$	$W_{HF} = -4Fb$
$H_{HD} = -7/2F$	$H_{FH} = -7/2F$
$V_{HD} = -8F$	$V_{FH} = 35/8F$
$W_{HD} = 4Fb$	$W_{FH} = -3/8Fb$

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_{BBA} = -1/8\delta$	$u_{CCD} = 0$	$u_{DDC} = -1/8\delta$	$u_{EEF} = 0$	$u_F = -1/8\delta$
$v_{AAB} = 0$	$v_{BBA} = 0$	$v_{CCD} = 0$	$v_{DDC} = 0$	$v_{EEF} = 0$	$v_F = 0$
$\varphi_{AAB} = 1/8\delta/b$	$\varphi_{BBA} = 1/8\delta/b$	$\varphi_{CCD} = 1/8\delta/b$	$\varphi_{DDC} = 1/8\delta/b$	$\varphi_{EEF} = 1/8\delta/b$	$\varphi_F = 1/8\delta/b$

$u_G = -1/8\delta$	$u_{HHD} = -1/8\delta$
$v_G = 0$	$v_{HHD} = -1/8\delta$
$\varphi_G = 0$	$\varphi_{HHD} = -1/8\delta/b$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$	$u_{CCD} = 0$	$u_{EEF} = 0$	$u_{BBG} = -1/8\delta$	$u_{GGD} = -1/8\delta$	$u_{DDH} = -1/8\delta$
$v_{AAB} = 0$	$v_{CCD} = 0$	$v_{EEF} = 0$	$v_{BBG} = 0$	$v_{GGD} = 0$	$v_{DDH} = 0$
$\varphi_{AAB} = 1/8\delta/b$	$\varphi_{CCD} = 1/8\delta/b$	$\varphi_{EEF} = 1/8\delta/b$	$\varphi_{BBG} = 0$	$\varphi_{GGD} = 0$	$\varphi_{DDH} = -1/8\delta/b$

$u_{HHF} = -1/8\delta$
$v_{HHF} = -1/8\delta$
$\varphi_{HHF} = 1/8\delta/b$

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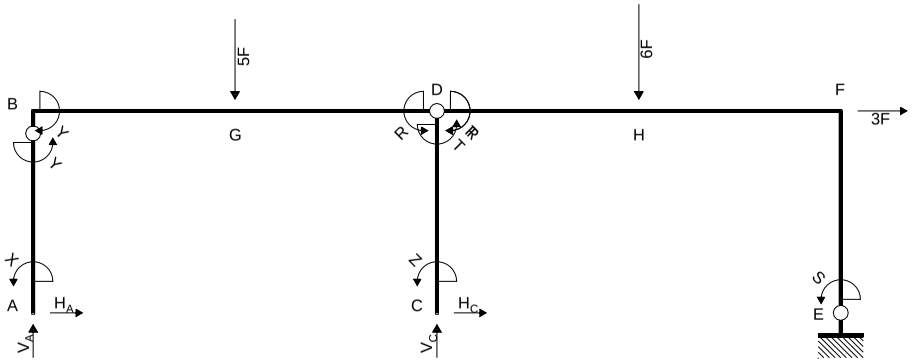


EQUAZIONI DI EQUILIBRIO

Rotazione intorno a E: aste EF FH HD DC DG GB BA

$-4V_Ab -2V_Cb = -Xb -Zb -Sb -18Fb$

Rotazione intorno a D: aste DC

$H_Cb = -Zb -Tb$

Rotazione intorno a D: aste DG GB BA

$H_Ab -2V_Ab = -Xb -Rb -5Fb$

Rotazione intorno a B: aste BA

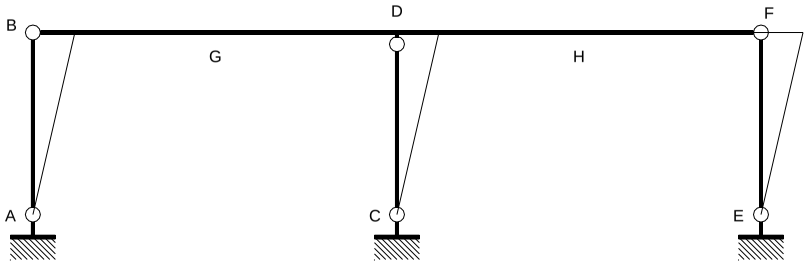
$H_Ab = -Xb -Yb$

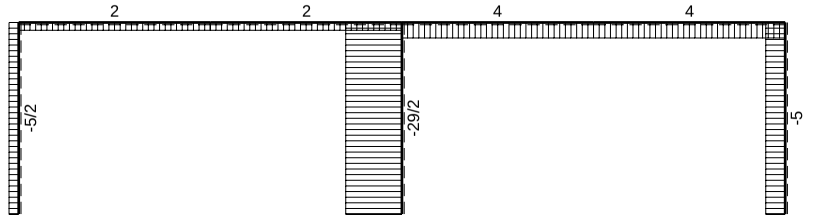
Matrice di equilibrio

$$\begin{array}{c} \\ \varphi_{EF} \\ \varphi_{DC} \\ \varphi_{DG} \\ \varphi_{BA} \end{array} \begin{bmatrix} H_Ab & V_Ab & H_Cb & V_Cb \\ 0 & -4 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 1 & -2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1 & 0 & -1 & 0 & -1 & 0 & -18 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & -1 & -5 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

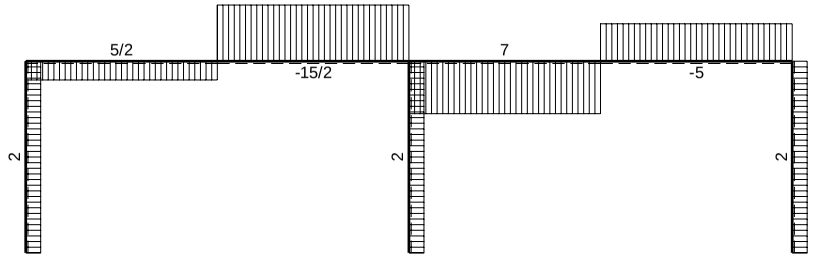
Soluzione del sistema

$$\begin{bmatrix} V_Ab \\ H_Cb \\ H_Ab \\ V_Cb \end{bmatrix} = \begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ 0 & -1/2 & 0 & 0 & 0 & 1/2 & 5/2 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1 & 1/2 & 0 & 1/2 & -1 & 4 \end{bmatrix}$$

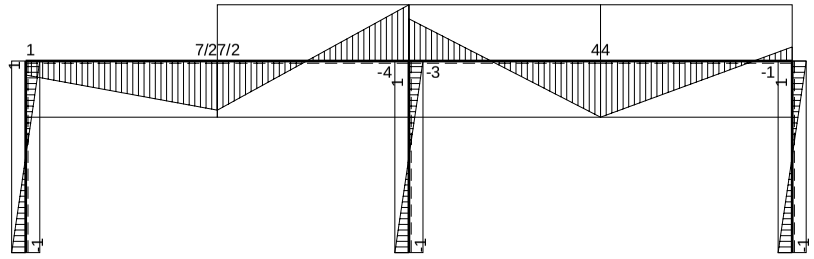




F



F



Fb

PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq} = 1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	$\ge$	H_{14}
D_2	H_{21}	H_{22}	H_{23}	$\ge$	H_{24}
D_3	H_{31}	H_{32}	H_{33}	$\ge$	H_{34}
D_4	H_{41}	H_{42}	H_{43}	$\ge$	H_{44}
D_5	H_{51}	H_{52}	H_{53}	$\ge$	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{CD}$ $T=W_{DC}$ $S=W_{EF}$ $R=W_{DG}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	1	0	0	0	0	0	0	≤	1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	1	0	0	0	0	0	≤	1
W_{CD}-	0	0	1	0	0	0	0	≥	-1
W_{CD}+	0	0	1	0	0	0	0	≤	1
W_{DC}-	0	0	0	1	0	0	0	≥	-1
W_{DC}+	0	0	0	1	0	0	0	≤	1
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	1	0	0	≤	1
W_{FE}-	-1	-1	-1	-1	-1	0	3	≥	-1
W_{FE}+	-1	-1	-1	-1	-1	0	3	≤	1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	-1/2	0	0	0	-1/2	-5/2	≤	4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	1	0	≤	4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	-1	0	-1	0	≤	4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-3/2	≥	-4
W_{HF}+	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-3/2	≤	4
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	0	≥	-1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	≥	-1
W_{CD}-	0	0	1	0	0	0	0	≥	-1
W_{CD}+	0	0	-1	0	0	0	0	≥	-1
W_{DC}-	0	0	0	1	0	0	0	≥	-1
W_{DC}+	0	0	0	-1	0	0	0	≥	-1
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	≥	-1
W_{FE}-	-1	-1	-1	-1	-1	0	3	≥	-1
W_{FE}+	1	1	1	1	1	0	-3	≥	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	5/2	≥	-4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	≥	-4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	≥	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-3/2	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	3/2	≥	-4
Max	0	0	0	0	0	0	1	=	0

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Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1
W_{CD}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-1
W_{CD}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-1
W_{DC}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-1
W_{DC}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-1
W_{EF}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1
W_{FE}-	-1	-1	-1	-1	-1	0	1	1	1	1	1	0	3	≥	-1
W_{FE}+	1	1	1	1	1	0	-1	-1	-1	-1	-1	0	-3	≤	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	0	1/2	0	0	0	1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	0	-1/2	0	0	0	-1/2	5/2	≤	-4
W_{DG}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-4
W_{DH}-	0	0	0	-1	0	-1	0	0	0	1	0	1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	0	0	-1	0	-1	0	≤	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	1/2	1/2	1/2	1	1/2	1/2	-3/2	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	-1/2	-1/2	-1/2	-1	-1/2	-1/2	3/2	≤	-4
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

@ Adolfo Zavelani Rossi, Politecnico di Milano, vers.11.05.11 08.06.11

Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-1
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-1
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-1
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-1
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	-1	-1	-1	-1	-1	0	3	5	≥	-1
φ_{FE}+	1	1	1	1	1	0	-3	-5	≥	-1
φ_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	1	≥	-4
φ_{GD}+	0	1/2	0	0	0	1/2	5/2	-1	≥	-4
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-3/2	7/2	≥	-4
φ_{HF}+	1/2	1/2	1/2	1	1/2	1/2	3/2	-7/2	≥	-4
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 12-7

	X	Y	Z	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-1
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-1
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-1
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-1
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	1/3	1/3	1/3	1/3	1/3	0	-1/3	-5/3	≥	-1/3
φ_{GD}-	-5/6	-4/3	-5/6	-5/6	-5/6	-1/2	5/6	31/6	≥	-19/6
φ_{GD}+	5/6	4/3	5/6	5/6	5/6	1/2	-5/6	-31/6	≥	-29/6
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	-1	-1	-1	-3/2	-1	-1/2	1/2	6	≥	-7/2
φ_{HF}+	1	1	1	3/2	1	1/2	-1/2	-6	≥	-9/2
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	1/3	1/3	1/3	1/3	1/3	0	-1/3	-5/3	=	-1/3

Scambio pivotale 2-1

	φ_{AB}+	Y	Z	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-1
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-1
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-1
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-1
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/3	1/3	1/3	1/3	1/3	0	-1/3	-4/3	≥	-2/3
φ_{GD}-	5/6	-4/3	-5/6	-5/6	-5/6	-1/2	5/6	13/3	≥	-7/3
φ_{GD}+	-5/6	4/3	5/6	5/6	5/6	1/2	-5/6	-13/3	≥	-17/3
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	1	-1	-1	-3/2	-1	-1/2	1/2	5	≥	-5/2
φ_{HF}+	-1	1	1	3/2	1	1/2	-1/2	-5	≥	-11/2
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-1/3	1/3	1/3	1/3	1/3	0	-1/3	-4/3	=	-2/3

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Scambio pivotale 4-2

	φ_{AB}+	φ_{BA}+	Z	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	-1	0	0	0	0	0	0	≥	-2
Y	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-1
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-1
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-1
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-1
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/3	-1/3	1/3	1/3	1/3	0	-1/3	-1	≥	-1
φ_{GD}-	5/6	4/3	-5/6	-5/6	-5/6	-1/2	5/6	3	≥	-1
φ_{GD}+	-5/6	-4/3	5/6	5/6	5/6	1/2	-5/6	-3	≥	-7
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	1	1	-1	-3/2	-1	-1/2	1/2	4	≥	-3/2
φ_{HF}+	-1	-1	1	3/2	1	1/2	-1/2	-4	≥	-13/2
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-1/3	-1/3	1/3	1/3	1/3	0	-1/3	-1	=	-1

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Scambio pivotale 6-3

	φ_{AB}+	φ_{BA}+	φ_{CD}+	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	-1	0	0	0	0	0	0	≥	-2
Y	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-2
Z	0	0	-1	0	0	0	0	1	≥	-1
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-1
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-1
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/3	-1/3	-1/3	1/3	1/3	0	-1/3	-2/3	≥	-4/3
φ_{GD}-	5/6	4/3	5/6	-5/6	-5/6	-1/2	5/6	13/6	≥	-1/6
φ_{GD}+	-5/6	-4/3	-5/6	5/6	5/6	1/2	-5/6	-13/6	≥	-47/6
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	1	1	1	-3/2	-1	-1/2	1/2	3	≥	-1/2
φ_{HF}+	-1	-1	-1	3/2	1	1/2	-1/2	-3	≥	-15/2
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-1/3	-1/3	-1/3	1/3	1/3	0	-1/3	-2/3	=	-4/3

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Scambio pivotale 13-4

	φ_{AB}+	φ_{BA}+	φ_{CD}+	φ_{GD}-	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	-1	0	0	0	0	0	0	≥	-2
Y	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-2
Z	0	0	-1	0	0	0	0	1	≥	-1
φ_{DC}-	1	8/5	1	-6/5	-1	-3/5	1	8/5	≥	-6/5
φ_{DC}+	-1	-8/5	-1	6/5	1	3/5	-1	-8/5	≥	-4/5
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	0	1/5	0	-2/5	0	-1/5	0	1/5	≥	-7/5
T	1	8/5	1	-6/5	-1	-3/5	1	13/5	≥	-1/5
φ_{GD}+	0	0	0	-1	0	0	0	0	≥	-8
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	-1	-8/5	-1	6/5	1	-2/5	-1	-3/5	≥	-19/5
φ_{DH}+	1	8/5	1	-6/5	-1	2/5	1	3/5	≥	-21/5
φ_{HF}-	-1/2	-7/5	-1/2	9/5	1/2	2/5	-1	-9/10	≥	-1/5
φ_{HF}+	1/2	7/5	1/2	-9/5	-1/2	-2/5	1	9/10	≥	-39/5
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	0	1/5	0	-2/5	0	-1/5	0	1/5	=	-7/5

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Scambio pivotale 19-2

	φ_{AB}+	φ_{HF}-	φ_{CD}+	φ_{GD}-	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	5/14	5/7	5/14	-9/7	-5/14	-2/7	5/7	9/14	≥	-13/7
Y	5/14	5/7	5/14	-9/7	-5/14	-2/7	5/7	23/14	≥	-6/7
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-2
Z	0	0	-1	0	0	0	0	1	≥	-1
φ_{DC}-	3/7	-8/7	3/7	6/7	-3/7	-1/7	-1/7	4/7	≥	-10/7
φ_{DC}+	-3/7	8/7	-3/7	-6/7	3/7	1/7	1/7	-4/7	≥	-4/7
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/14	-1/7	-1/14	-1/7	1/14	-1/7	-1/7	1/14	≥	-10/7
T	3/7	-8/7	3/7	6/7	-3/7	-1/7	-1/7	11/7	≥	-3/7
φ_{GD}+	0	0	0	-1	0	0	0	0	≥	-8
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	-3/7	8/7	-3/7	-6/7	3/7	-6/7	1/7	3/7	≥	-25/7
φ_{DH}+	3/7	-8/7	3/7	6/7	-3/7	6/7	-1/7	-3/7	≥	-31/7
φ_{BA}+	-5/14	-5/7	-5/14	9/7	5/14	2/7	-5/7	-9/14	≥	-1/7
φ_{HF}+	0	-1	0	0	0	0	0	0	≥	-8
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-1/14	-1/7	-1/14	-1/7	1/14	-1/7	-1/7	1/14	=	-10/7

PROCEDIMENTO E RISULTATI Nome: AL5.006

Scambio pivotale 10-5

	φ_{AB}+	φ_{HF}-	φ_{CD}+	φ_{GD}-	φ_{EF}+	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	5/14	5/7	5/14	-9/7	5/14	-2/7	5/7	2/7	≥	-3/2
Y	5/14	5/7	5/14	-9/7	5/14	-2/7	5/7	9/7	≥	-1/2
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-2
Z	0	0	-1	0	0	0	0	1	≥	-1
φ_{DC}-	3/7	-8/7	3/7	6/7	3/7	-1/7	-1/7	1/7	≥	-1
φ_{DC}+	-3/7	8/7	-3/7	-6/7	-3/7	1/7	1/7	-1/7	≥	-1
φ_{EF}-	0	0	0	0	-1	0	0	0	≥	-2
S	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/14	-1/7	-1/14	-1/7	-1/14	-1/7	-1/7	1/7	≥	-3/2
T	3/7	-8/7	3/7	6/7	3/7	-1/7	-1/7	8/7	≥	0
φ_{GD}+	0	0	0	-1	0	0	0	0	≥	-8
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	-3/7	8/7	-3/7	-6/7	-3/7	-6/7	1/7	6/7	≥	-4
φ_{DH}+	3/7	-8/7	3/7	6/7	3/7	6/7	-1/7	-6/7	≥	-4
φ_{BA}+	-5/14	-5/7	-5/14	9/7	-5/14	2/7	-5/7	-2/7	≥	-1/2
φ_{HF}+	0	-1	0	0	0	0	0	0	≥	-8
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-1/14	-1/7	-1/14	-1/7	-1/14	-1/7	-1/7	1/7	=	-3/2

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Scambio pivotale 19-8

	φ_{AB}+	φ_{HF}-	φ_{CD}+	φ_{GD}-	φ_{EF}+	R	φ_{FE}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-9/4	-5/2	-5/4	9/2	-5/4	1	-5/2	-7/2	≥	-11/4
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
Y	-5/4	-5/2	-5/4	9/2	-5/4	1	-5/2	-9/2	≥	-11/4
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-2
Z	-5/4	-5/2	-9/4	9/2	-5/4	1	-5/2	-7/2	≥	-11/4
φ_{DC}-	1/4	-3/2	1/4	3/2	1/4	0	-1/2	-1/2	≥	-5/4
φ_{DC}+	-1/4	3/2	-1/4	-3/2	-1/4	0	1/2	1/2	≥	-3/4
φ_{EF}-	0	0	0	0	-1	0	0	0	≥	-2
S	-5/4	-5/2	-5/4	9/2	-9/4	1	-5/2	-7/2	≥	-11/4
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/4	-1/2	-1/4	1/2	-1/4	0	-1/2	-1/2	≥	-7/4
T	-1	-4	-1	6	-1	1	-3	-4	≥	-2
φ_{GD}+	0	0	0	-1	0	0	0	0	≥	-8
φ_{DG}-	5/4	5/2	5/4	-9/2	5/4	0	5/2	7/2	≥	-9/4
φ_{DG}+	-5/4	-5/2	-5/4	9/2	-5/4	0	-5/2	-7/2	≥	-23/4
φ_{DH}-	-3/2	-1	-3/2	3	-3/2	0	-2	-3	≥	-11/2
φ_{DH}+	3/2	1	3/2	-3	3/2	0	2	3	≥	-5/2
X-	-5/4	-5/2	-5/4	9/2	-5/4	1	-5/2	-7/2	≥	-7/4
φ_{HF}+	0	-1	0	0	0	0	0	0	≥	-8
L_X	5/4	5/2	5/4	-9/2	5/4	-1	5/2	7/2	≥	-9/4
Max	-1/4	-1/2	-1/4	1/2	-1/4	0	-1/2	-1/2	=	-7/4

PROCEDIMENTO E RISULTATI Nome: AL5.006

Scambio pivotale 8-4

	φ_{AB}+	φ_{HF}-	φ_{CD}+	φ_{DC}+	φ_{EF}+	R	φ_{FE}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-3	2	-2	-3	-2	1	-1	-2	≥	-5
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
Y	-2	2	-2	-3	-2	1	-1	-3	≥	-5
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-2
Z	-2	2	-3	-3	-2	1	-1	-2	≥	-5
φ_{DC}-	0	0	0	-1	0	0	0	0	≥	-2
φ_{GD}-	-1/6	1	-1/6	-2/3	-1/6	0	1/3	1/3	≥	-1/2
φ_{EF}-	0	0	0	0	-1	0	0	0	≥	-2
S	-2	2	-2	-3	-3	1	-1	-2	≥	-5
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/3	0	-1/3	-1/3	-1/3	0	-1/3	-1/3	≥	-2
T	-2	2	-2	-4	-2	1	-1	-2	≥	-5
φ_{GD}+	1/6	-1	1/6	2/3	1/6	0	-1/3	-1/3	≥	-15/2
φ_{DG}-	2	-2	2	3	2	0	1	2	≥	0
φ_{DG}+	-2	2	-2	-3	-2	0	-1	-2	≥	-8
φ_{DH}-	-2	2	-2	-2	-2	0	-1	-2	≥	-7
φ_{DH}+	2	-2	2	2	2	0	1	2	≥	-1
X-	-2	2	-2	-3	-2	1	-1	-2	≥	-4
φ_{HF}+	0	-1	0	0	0	0	0	0	≥	-8
L_X	2	-2	2	3	2	-1	1	2	≥	0
Max	-1/3	0	-1/3	-1/3	-1/3	0	-1/3	-1/3	=	-2

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Tableau finale

	φ_{AB}+	φ_{HF}-	φ_{CD}+	φ_{DC}+	φ_{EF}+	R	φ_{FE}+	φ_{BA}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-3	2	-2	-3	-2	1	-1	-2	≥	-5
φ_{BA}-	0	0	0	0	0	0	0	-1	≥	-2
Y	-2	2	-2	-3	-2	1	-1	-3	≥	-5
φ_{CD}-	0	0	-1	0	0	0	0	0	≥	-2
Z	-2	2	-3	-3	-2	1	-1	-2	≥	-5
φ_{DC}-	0	0	0	-1	0	0	0	0	≥	-2
φ_{GD}-	-1/6	1	-1/6	-2/3	-1/6	0	1/3	1/3	≥	-1/2
φ_{EF}-	0	0	0	0	-1	0	0	0	≥	-2
S	-2	2	-2	-3	-3	1	-1	-2	≥	-5
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/3	0	-1/3	-1/3	-1/3	0	-1/3	-1/3	≥	-2
T	-2	2	-2	-4	-2	1	-1	-2	≥	-5
φ_{GD}+	1/6	-1	1/6	2/3	1/6	0	-1/3	-1/3	≥	-15/2
φ_{DG}-	2	-2	2	3	2	0	1	2	≥	0
φ_{DG}+	-2	2	-2	-3	-2	0	-1	-2	≥	-8
φ_{DH}-	-2	2	-2	-2	-2	0	-1	-2	≥	-7
φ_{DH}+	2	-2	2	2	2	0	1	2	≥	-1
X-	-2	2	-2	-3	-2	1	-1	-2	≥	-4
φ_{HF}+	0	-1	0	0	0	0	0	0	≥	-8
L_X	2	-2	2	3	2	-1	1	2	≥	0
Max	-1/3	0	-1/3	-1/3	-1/3	0	-1/3	-1/3	=	-2

Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	0
φ_{AB}+	0	0	0	0	0	0	0	0	≥	1/3
φ_{BA}-	0	0	0	0	0	0	0	0	≥	0
φ_{BA}+	0	0	0	0	0	0	0	0	≥	1/3
φ_{CD}-	0	0	0	0	0	0	0	0	≥	0
φ_{CD}+	0	0	0	0	0	0	0	0	≥	1/3
φ_{DC}-	0	0	0	0	0	0	0	0	≥	0
φ_{DC}+	0	0	0	0	0	0	0	0	≥	1/3
φ_{EF}-	0	0	0	0	0	0	0	0	≥	0
φ_{EF}+	0	0	0	0	0	0	0	0	≥	1/3
φ_{FE}-	0	0	0	0	0	0	0	0	≥	0
φ_{FE}+	0	0	0	0	0	0	0	0	≥	1/3
φ_{GD}-	0	0	0	0	0	0	0	0	≥	0
φ_{GD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DG}-	0	0	0	0	0	0	0	0	≥	0
φ_{DG}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	0
φ_{HF}-	0	0	0	0	0	0	0	0	≥	0
φ_{HF}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	5	5	5	5	5	0	2	4	=	-2

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
1	1	1	1	1	-4

Variabili soluzione differenza tra rotazioni

φ_{AB}	1/3
φ_{BA}	1/3
φ_{CD}	1/3
φ_{DC}	1/3
φ_{EF}	1/3
φ_{FE}	1/3
φ_{GD}	0
φ_{DG}	0
φ_{DH}	0
φ_{HF}	0

REAZIONI Fattore di collasso = 2

$H_A = -2F$
$V_A = 5/2F$
$W_A = Fb$
$H_C = -2F$
$V_C = 29/2F$
$W_C = Fb$
$H_E = -2F$
$V_E = 5F$
$W_E = Fb$

$H_{AB} = -2F$	$H_{CD} = -2F$	$H_{EF} = -2F$	$H_{BG} = -2F$	$H_{GD} = -2F$	$H_{DH} = -4F$
$V_{AB} = 5/2F$	$V_{CD} = 29/2F$	$V_{EF} = 5F$	$V_{BG} = 5/2F$	$V_{GD} = -15/2F$	$V_{DH} = 7F$
$W_{AB} = Fb$	$W_{CD} = Fb$	$W_{EF} = Fb$	$W_{BG} = -Fb$	$W_{GD} = -7/2Fb$	$W_{DH} = 3Fb$
$H_{BA} = 2F$	$H_{DC} = 2F$	$H_{FE} = 2F$	$H_{GB} = 2F$	$H_{DG} = 2F$	$H_{HD} = 4F$
$V_{BA} = -5/2F$	$V_{DC} = -29/2F$	$V_{FE} = -5F$	$V_{GB} = -5/2F$	$V_{DG} = 15/2F$	$V_{HD} = -7F$
$W_{BA} = Fb$	$W_{DC} = Fb$	$W_{FE} = Fb$	$W_{GB} = 7/2Fb$	$W_{DG} = -4Fb$	$W_{HD} = 4Fb$

$H_{HF} = -4F$
$V_{HF} = -5F$
$W_{HF} = -4Fb$
$H_{FH} = 4F$
$V_{FH} = 5F$
$W_{FH} = -Fb$

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_{BBA} = 1/3\delta$	$u_{CCD} = 0$	$u_D = 1/3\delta$	$u_{EEF} = 0$	$u_{FFE} = 1/3\delta$
$v_{AAB} = 0$	$v_{BBA} = 0$	$v_{CCD} = 0$	$v_D = 0$	$v_{EEF} = 0$	$v_{FFE} = 0$
$\varphi_{AAB} = -1/3\delta/b$	$\varphi_{BBA} = -1/3\delta/b$	$\varphi_{CCD} = -1/3\delta/b$	$\varphi_D = -1/3\delta/b$	$\varphi_{EEF} = -1/3\delta/b$	$\varphi_{FFH} = -1/3\delta/b$

$u_G = 1/3\delta$	$u_H = 1/3\delta$
$v_G = 0$	$v_H = 0$
$\varphi_G = 0$	$\varphi_H = 0$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$	$u_{CCD} = 0$	$u_{EEF} = 0$	$u_{BBG} = 1/3\delta$	$u_{GGD} = 1/3\delta$	$u_{DDH} = 1/3\delta$
$v_{AAB} = 0$	$v_{CCD} = 0$	$v_{EEF} = 0$	$v_{BBG} = 0$	$v_{GGD} = 0$	$v_{DDH} = 0$
$\varphi_{AAB} = -1/3\delta/b$	$\varphi_{CCD} = -1/3\delta/b$	$\varphi_{EEF} = -1/3\delta/b$	$\varphi_{BBG} = 0$	$\varphi_{GGD} = 0$	$\varphi_{DDH} = 0$

$u_{HHF} = 1/3\delta$
$v_{HHF} = 0$
$\varphi_{HHF} = 0$

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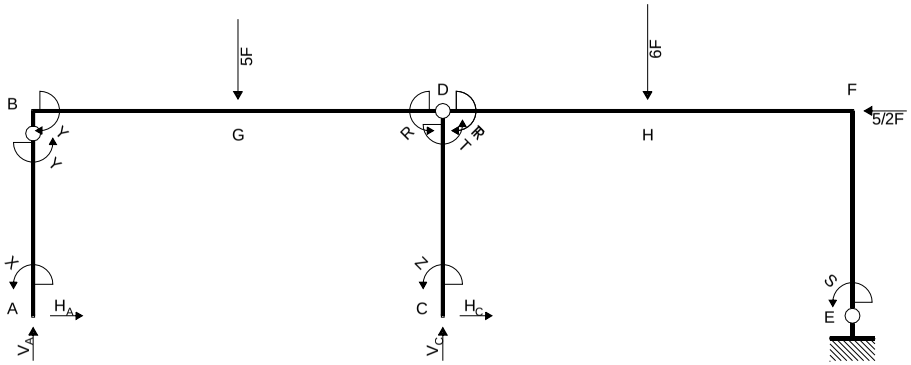


EQUAZIONI DI EQUILIBRIO

Rotazione intorno a E: aste EF FH HD DC DG GB BA

$-4V_Ab -2V_Cb = -Xb -Zb -Sb -47/2Fb$

Rotazione intorno a D: aste DC

$H_Cb = -Zb -Tb$

Rotazione intorno a D: aste DG GB BA

$H_Ab -2V_Ab = -Xb -Rb -5Fb$

Rotazione intorno a B: aste BA

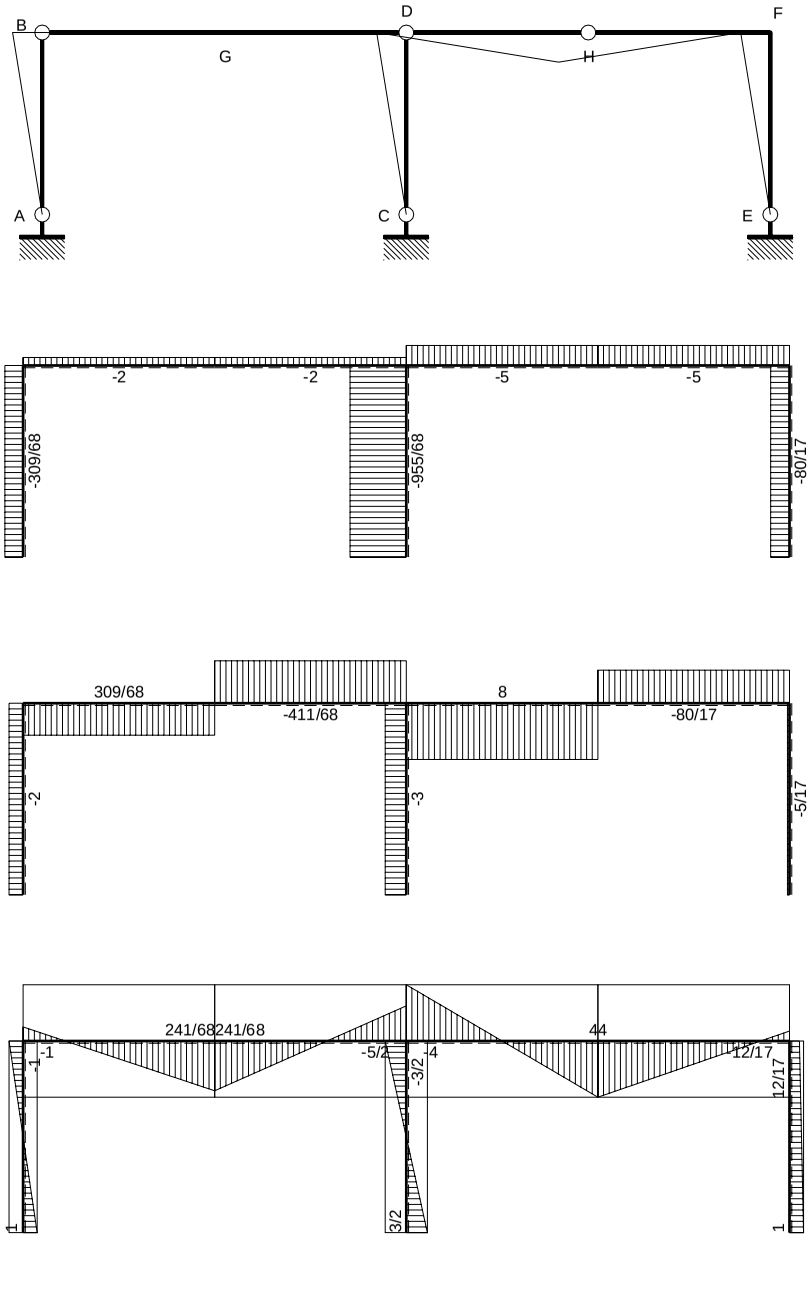
$H_Ab = -Xb -Yb$

Matrice di equilibrio

	H_Ab	V_Ab	H_Cb	V_Cb		Xb	Yb	Zb	Tb	Sb	Rb	Fb
φ_{EF}	0	-4	0	-2	=	-1	0	-1	0	-1	0	-47/2
φ_{DC}	0	0	1	0		0	0	-1	-1	0	0	0
φ_{DG}	1	-2	0	0		-1	0	0	0	0	-1	-5
φ_{BA}	1	0	0	0		-1	-1	0	0	0	0	0

Soluzione del sistema

	Xb	Yb	Zb	Tb	Sb	Rb	Fb
V_Ab	0	-1/2	0	0	0	1/2	5/2
H_Cb	0	0	-1	-1	0	0	0
H_Ab	-1	-1	0	0	0	0	0
V_Cb	1/2	1	1/2	0	1/2	-1	27/4



PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	≥	H_{14}
D_2	H_{21}	H_{22}	H_{23}	≥	H_{24}
D_3	H_{31}	H_{32}	H_{33}	≥	H_{34}
D_4	H_{41}	H_{42}	H_{43}	≥	H_{44}
D_5	H_{51}	H_{52}	H_{53}	≥	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{CD}$ $T=W_{DC}$ $S=W_{EF}$ $R=W_{DG}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	1	0	0	0	0	0	0	≤	1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	1	0	0	0	0	0	≤	1
W_{CD}-	0	0	1	0	0	0	0	≥	-3/2
W_{CD}+	0	0	1	0	0	0	0	≤	3/2
W_{DC}-	0	0	0	1	0	0	0	≥	-3/2
W_{DC}+	0	0	0	1	0	0	0	≤	3/2
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	1	0	0	≤	1
W_{FE}-	-1	-1	-1	-1	-1	0	-5/2	≥	-1
W_{FE}+	-1	-1	-1	-1	-1	0	-5/2	≤	1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	-1/2	0	0	0	-1/2	-5/2	≤	4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	1	0	≤	4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	-1	0	-1	0	≤	4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-17/4	≥	-4
W_{HF}+	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-17/4	≤	4
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	0	≥	-1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	≥	-1
W_{CD}-	0	0	1	0	0	0	0	≥	-3/2
W_{CD}+	0	0	-1	0	0	0	0	≥	-3/2
W_{DC}-	0	0	0	1	0	0	0	≥	-3/2
W_{DC}+	0	0	0	-1	0	0	0	≥	-3/2
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	≥	-1
W_{FE}-	-1	-1	-1	-1	-1	0	-5/2	≥	-1
W_{FE}+	1	1	1	1	1	0	5/2	≥	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	5/2	≥	-4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	≥	-4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	≥	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-17/4	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	17/4	≥	-4
Max	0	0	0	0	0	0	1	=	0

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Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1
W_{CD}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3/2
W_{CD}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3/2
W_{DC}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3/2
W_{DC}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3/2
W_{EF}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1
W_{FE}-	-1	-1	-1	-1	-1	0	1	1	1	1	1	0	-5/2	≥	-1
W_{FE}+	1	1	1	1	1	0	-1	-1	-1	-1	-1	0	5/2	≤	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	0	1/2	0	0	0	1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	0	-1/2	0	0	0	-1/2	5/2	≤	-4
W_{DG}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-4
W_{DH}-	0	0	0	-1	0	-1	0	0	0	1	0	1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	0	0	-1	0	-1	0	≤	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	1/2	1/2	1/2	1	1/2	1/2	-17/4	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	-1/2	-1/2	-1/2	-1	-1/2	-1/2	17/4	≤	-4
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

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Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	-1	-1	-1	-1	-1	0	-5/2	5	≥	-1
φ_{FE}+	1	1	1	1	1	0	5/2	-5	≥	-1
φ_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	1	≥	-4
φ_{GD}+	0	1/2	0	0	0	1/2	5/2	-1	≥	-4
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-17/4	7/2	≥	-4
φ_{HF}+	1/2	1/2	1/2	1	1/2	1/2	17/4	-7/2	≥	-4
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 11-7

	X	Y	Z	T	S	R	φ_{FE}-	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-3/2
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-3/2
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-3/2
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-3/2
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
αbF	-2/5	-2/5	-2/5	-2/5	-2/5	0	-2/5	2	≥	-2/5
φ_{FE}+	0	0	0	0	0	0	-1	0	≥	-2
φ_{GD}-	1	1/2	1	1	1	-1/2	1	-4	≥	-3
φ_{GD}+	-1	-1/2	-1	-1	-1	1/2	-1	4	≥	-5
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	6/5	6/5	6/5	7/10	6/5	-1/2	17/10	-5	≥	-23/10
φ_{HF}+	-6/5	-6/5	-6/5	-7/10	-6/5	1/2	-17/10	5	≥	-57/10
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-2/5	-2/5	-2/5	-2/5	-2/5	0	-2/5	2	=	-2/5

Scambio pivotale 19-8

	X	Y	Z	T	S	R	φ_{FE}-	φ_{HF}-		Fb
φ_{AB}-	19/25	-6/25	-6/25	-7/50	-6/25	1/10	-17/50	1/5	≥	-27/50
φ_{AB}+	-19/25	6/25	6/25	7/50	6/25	-1/10	17/50	-1/5	≥	-73/50
φ_{BA}-	-6/25	19/25	-6/25	-7/50	-6/25	1/10	-17/50	1/5	≥	-27/50
φ_{BA}+	6/25	-19/25	6/25	7/50	6/25	-1/10	17/50	-1/5	≥	-73/50
φ_{CD}-	-6/25	-6/25	19/25	-7/50	-6/25	1/10	-17/50	1/5	≥	-26/25
φ_{CD}+	6/25	6/25	-19/25	7/50	6/25	-1/10	17/50	-1/5	≥	-49/25
φ_{DC}-	-6/25	-6/25	-6/25	43/50	-6/25	1/10	-17/50	1/5	≥	-26/25
φ_{DC}+	6/25	6/25	6/25	-43/50	6/25	-1/10	17/50	-1/5	≥	-49/25
φ_{EF}-	-6/25	-6/25	-6/25	-7/50	19/25	1/10	-17/50	1/5	≥	-27/50
φ_{EF}+	6/25	6/25	6/25	7/50	-19/25	-1/10	17/50	-1/5	≥	-73/50
αbF	2/25	2/25	2/25	-3/25	2/25	-1/5	7/25	-2/5	≥	-33/25
φ_{FE}+	0	0	0	0	0	0	-1	0	≥	-2
φ_{GD}-	1/25	-23/50	1/25	11/25	1/25	-1/10	-9/25	4/5	≥	-29/25
φ_{GD}+	-1/25	23/50	-1/25	-11/25	-1/25	1/10	9/25	-4/5	≥	-171/25
φ_{DG}-	-6/25	-6/25	-6/25	-7/50	-6/25	11/10	-17/50	1/5	≥	-177/50
φ_{DG}+	6/25	6/25	6/25	7/50	6/25	-11/10	17/50	-1/5	≥	-223/50
φ_{DH}-	12/25	12/25	12/25	-18/25	12/25	-6/5	17/25	-2/5	≥	-123/25
φ_{DH}+	-12/25	-12/25	-12/25	18/25	-12/25	6/5	-17/25	2/5	≥	-77/25
X-	6/25	6/25	6/25	7/50	6/25	-1/10	17/50	-1/5	≥	-23/50
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	-6/25	-6/25	-6/25	-7/50	-6/25	1/10	-17/50	1/5	≥	-177/50
Max	2/25	2/25	2/25	-3/25	2/25	-1/5	7/25	-2/5	=	-33/25

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Scambio pivotale 1-7

	X	Y	Z	T	S	R	φ_{AB}-	φ_{HF}-		Fb
φ_{FE}-	38/17	-12/17	-12/17	-7/17	-12/17	5/17	-50/17	10/17	≥	-27/17
φ_{AB}+	0	0	0	0	0	0	-1	0	≥	-2
φ_{BA}-	-1	1	0	0	0	0	1	0	≥	0
φ_{BA}+	1	-1	0	0	0	0	-1	0	≥	-2
φ_{CD}-	-1	0	1	0	0	0	1	0	≥	-1/2
φ_{CD}+	1	0	-1	0	0	0	-1	0	≥	-5/2
φ_{DC}-	-1	0	0	1	0	0	1	0	≥	-1/2
φ_{DC}+	1	0	0	-1	0	0	-1	0	≥	-5/2
φ_{EF}-	-1	0	0	0	1	0	1	0	≥	0
φ_{EF}+	1	0	0	0	-1	0	-1	0	≥	-2
αbF	12/17	-2/17	-2/17	-4/17	-2/17	-2/17	-14/17	-4/17	≥	-30/17
φ_{FE}+	-38/17	12/17	12/17	7/17	12/17	-5/17	50/17	-10/17	≥	-7/17
φ_{GD}-	-13/17	-7/34	5/17	10/17	5/17	-7/34	18/17	10/17	≥	-10/17
φ_{GD}+	13/17	7/34	-5/17	-10/17	-5/17	7/34	-18/17	-10/17	≥	-126/17
φ_{DG}-	-1	0	0	0	0	1	1	0	≥	-3
φ_{DG}+	1	0	0	0	0	-1	-1	0	≥	-5
φ_{DH}-	2	0	0	-1	0	-1	-2	0	≥	-6
φ_{DH}+	-2	0	0	1	0	1	2	0	≥	-2
X-	1	0	0	0	0	0	-1	0	≥	-1
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	-1	0	0	0	0	0	1	0	≥	-3
Max	12/17	-2/17	-2/17	-4/17	-2/17	-2/17	-14/17	-4/17	=	-30/17

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Scambio pivotale 3-1

	φ_{BA}-	Y	Z	T	S	R	φ_{AB}-	φ_{HF}-		Fb
φ_{FE}-	-38/17	26/17	-12/17	-7/17	-12/17	5/17	-12/17	10/17	≥	-27/17
φ_{AB}+	0	0	0	0	0	0	-1	0	≥	-2
X	-1	1	0	0	0	0	1	0	≥	0
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
φ_{CD}-	1	-1	1	0	0	0	0	0	≥	-1/2
φ_{CD}+	-1	1	-1	0	0	0	0	0	≥	-5/2
φ_{DC}-	1	-1	0	1	0	0	0	0	≥	-1/2
φ_{DC}+	-1	1	0	-1	0	0	0	0	≥	-5/2
φ_{EF}-	1	-1	0	0	1	0	0	0	≥	0
φ_{EF}+	-1	1	0	0	-1	0	0	0	≥	-2
αbF	-12/17	10/17	-2/17	-4/17	-2/17	-2/17	-2/17	-4/17	≥	-30/17
φ_{FE}+	38/17	-26/17	12/17	7/17	12/17	-5/17	12/17	-10/17	≥	-7/17
φ_{GD}-	13/17	-33/34	5/17	10/17	5/17	-7/34	5/17	10/17	≥	-10/17
φ_{GD}+	-13/17	33/34	-5/17	-10/17	-5/17	7/34	-5/17	-10/17	≥	-126/17
φ_{DG}-	1	-1	0	0	0	1	0	0	≥	-3
φ_{DG}+	-1	1	0	0	0	-1	0	0	≥	-5
φ_{DH}-	-2	2	0	-1	0	-1	0	0	≥	-6
φ_{DH}+	2	-2	0	1	0	1	0	0	≥	-2
X-	-1	1	0	0	0	0	0	0	≥	-1
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	1	-1	0	0	0	0	0	0	≥	-3
Max	-12/17	10/17	-2/17	-4/17	-2/17	-2/17	-2/17	-4/17	=	-30/17

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Scambio pivotale 9-2

	φ_{BA}-	φ_{EF}-	Z	T	S	R	φ_{AB}-	φ_{HF}-		Fb
φ_{FE}-	-12/17	-26/17	-12/17	-7/17	14/17	5/17	-12/17	10/17	≥	-27/17
φ_{AB}+	0	0	0	0	0	0	-1	0	≥	-2
X	0	-1	0	0	1	0	1	0	≥	0
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
φ_{CD}-	0	1	1	0	-1	0	0	0	≥	-1/2
φ_{CD}+	0	-1	-1	0	1	0	0	0	≥	-5/2
φ_{DC}-	0	1	0	1	-1	0	0	0	≥	-1/2
φ_{DC}+	0	-1	0	-1	1	0	0	0	≥	-5/2
Y	1	-1	0	0	1	0	0	0	≥	0
φ_{EF}+	0	-1	0	0	0	0	0	0	≥	-2
αbF	-2/17	-10/17	-2/17	-4/17	8/17	-2/17	-2/17	-4/17	≥	-30/17
φ_{FE}+	12/17	26/17	12/17	7/17	-14/17	-5/17	12/17	-10/17	≥	-7/17
φ_{GD}-	-7/34	33/34	5/17	10/17	-23/34	-7/34	5/17	10/17	≥	-10/17
φ_{GD}+	7/34	-33/34	-5/17	-10/17	23/34	7/34	-5/17	-10/17	≥	-126/17
φ_{DG}-	0	1	0	0	-1	1	0	0	≥	-3
φ_{DG}+	0	-1	0	0	1	-1	0	0	≥	-5
φ_{DH}-	0	-2	0	-1	2	-1	0	0	≥	-6
φ_{DH}+	0	2	0	1	-2	1	0	0	≥	-2
X-	0	-1	0	0	1	0	0	0	≥	-1
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	1	0	0	-1	0	0	0	≥	-3
Max	-2/17	-10/17	-2/17	-4/17	8/17	-2/17	-2/17	-4/17	=	-30/17

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Scambio pivotale 5-5

	φ_{BA}-	φ_{EF}-	Z	T	φ_{CD}-	R	φ_{AB}-	φ_{HF}-		Fb
φ_{FE}-	-12/17	-12/17	2/17	-7/17	-14/17	5/17	-12/17	10/17	≥	-2
φ_{AB}+	0	0	0	0	0	0	-1	0	≥	-2
X	0	0	1	0	-1	0	1	0	≥	-1/2
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
S	0	1	1	0	-1	0	0	0	≥	-1/2
φ_{CD}+	0	0	0	0	-1	0	0	0	≥	-3
φ_{DC}-	0	0	-1	1	1	0	0	0	≥	0
φ_{DC}+	0	0	1	-1	-1	0	0	0	≥	-3
Y	1	0	1	0	-1	0	0	0	≥	-1/2
φ_{EF}+	0	-1	0	0	0	0	0	0	≥	-2
αbF	-2/17	-2/17	6/17	-4/17	-8/17	-2/17	-2/17	-4/17	≥	-2
φ_{FE}+	12/17	12/17	-2/17	7/17	14/17	-5/17	12/17	-10/17	≥	0
φ_{GD}-	-7/34	5/17	-13/34	10/17	23/34	-7/34	5/17	10/17	≥	-1/4
φ_{GD}+	7/34	-5/17	13/34	-10/17	-23/34	7/34	-5/17	-10/17	≥	-31/4
φ_{DG}-	0	0	-1	0	1	1	0	0	≥	-5/2
φ_{DG}+	0	0	1	0	-1	-1	0	0	≥	-11/2
φ_{DH}-	0	0	2	-1	-2	-1	0	0	≥	-7
φ_{DH}+	0	0	-2	1	2	1	0	0	≥	-1
X-	0	0	1	0	-1	0	0	0	≥	-3/2
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	-1	0	1	0	0	0	≥	-5/2
Max	-2/17	-2/17	6/17	-4/17	-8/17	-2/17	-2/17	-4/17	=	-2

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Scambio pivotale 7-3

	φ_{BA}-	φ_{EF}-	φ_{DC}-	T	φ_{CD}-	R	φ_{AB}-	φ_{HF}-		Fb
φ_{FE}-	-12/17	-12/17	-2/17	-5/17	-12/17	5/17	-12/17	10/17	≥	-2
φ_{AB}+	0	0	0	0	0	0	-1	0	≥	-2
X	0	0	-1	1	0	0	1	0	≥	-1/2
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
S	0	1	-1	1	0	0	0	0	≥	-1/2
φ_{CD}+	0	0	0	0	-1	0	0	0	≥	-3
Z	0	0	-1	1	1	0	0	0	≥	0
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-3
Y	1	0	-1	1	0	0	0	0	≥	-1/2
φ_{EF}+	0	-1	0	0	0	0	0	0	≥	-2
αbF	-2/17	-2/17	-6/17	2/17	-2/17	-2/17	-2/17	-4/17	≥	-2
φ_{FE}+	12/17	12/17	2/17	5/17	12/17	-5/17	12/17	-10/17	≥	0
φ_{GD}-	-7/34	5/17	13/34	7/34	5/17	-7/34	5/17	10/17	≥	-1/4
φ_{GD}+	7/34	-5/17	-13/34	-7/34	-5/17	7/34	-5/17	-10/17	≥	-31/4
φ_{DG}-	0	0	1	-1	0	1	0	0	≥	-5/2
φ_{DG}+	0	0	-1	1	0	-1	0	0	≥	-11/2
φ_{DH}-	0	0	-2	1	0	-1	0	0	≥	-7
φ_{DH}+	0	0	2	-1	0	1	0	0	≥	-1
X-	0	0	-1	1	0	0	0	0	≥	-3/2
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	1	-1	0	0	0	0	≥	-5/2
Max	-2/17	-2/17	-6/17	2/17	-2/17	-2/17	-2/17	-4/17	=	-2

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Scambio pivotale 18-4

	φ_{BA}-	φ_{EF}-	φ_{DC}-	φ_{DH}+	φ_{CD}-	R	φ_{AB}-	φ_{HF}-		Fb
φ_{FE}-	-12/17	-12/17	-12/17	5/17	-12/17	0	-12/17	10/17	≥	-29/17
φ_{AB}+	0	0	0	0	0	0	-1	0	≥	-2
X	0	0	1	-1	0	1	1	0	≥	-3/2
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
S	0	1	1	-1	0	1	0	0	≥	-3/2
φ_{CD}+	0	0	0	0	-1	0	0	0	≥	-3
Z	0	0	1	-1	1	1	0	0	≥	-1
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-3
Y	1	0	1	-1	0	1	0	0	≥	-3/2
φ_{EF}+	0	-1	0	0	0	0	0	0	≥	-2
αbF	-2/17	-2/17	-2/17	-2/17	-2/17	0	-2/17	-4/17	≥	-36/17
φ_{FE}+	12/17	12/17	12/17	-5/17	12/17	0	12/17	-10/17	≥	-5/17
φ_{GD}-	-7/34	5/17	27/34	-7/34	5/17	0	5/17	10/17	≥	-31/68
φ_{GD}+	7/34	-5/17	-27/34	7/34	-5/17	0	-5/17	-10/17	≥	-513/68
φ_{DG}-	0	0	-1	1	0	0	0	0	≥	-3/2
φ_{DG}+	0	0	1	-1	0	0	0	0	≥	-13/2
φ_{DH}-	0	0	0	-1	0	0	0	0	≥	-8
T	0	0	2	-1	0	1	0	0	≥	-1
X-	0	0	1	-1	0	1	0	0	≥	-5/2
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	-1	1	0	-1	0	0	≥	-3/2
Max	-2/17	-2/17	-2/17	-2/17	-2/17	0	-2/17	-4/17	=	-36/17

PROCEDIMENTO E RISULTATI Nome: AL5.007

Tableau finale

	φ_{BA}-	φ_{EF}-	φ_{DC}-	φ_{DH}+	φ_{CD}-	R	φ_{AB}-	φ_{HF}-		Fb
φ_{FE}-	-12/17	-12/17	-12/17	5/17	-12/17	0	-12/17	10/17	≥	-29/17
φ_{AB}+	0	0	0	0	0	0	-1	0	≥	-2
X	0	0	1	-1	0	1	1	0	≥	-3/2
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
S	0	1	1	-1	0	1	0	0	≥	-3/2
φ_{CD}+	0	0	0	0	-1	0	0	0	≥	-3
Z	0	0	1	-1	1	1	0	0	≥	-1
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-3
Y	1	0	1	-1	0	1	0	0	≥	-3/2
φ_{EF}+	0	-1	0	0	0	0	0	0	≥	-2
αbF	-2/17	-2/17	-2/17	-2/17	-2/17	0	-2/17	-4/17	≥	-36/17
φ_{FE}+	12/17	12/17	12/17	-5/17	12/17	0	12/17	-10/17	≥	-5/17
φ_{GD}-	-7/34	5/17	27/34	-7/34	5/17	0	5/17	10/17	≥	-31/68
φ_{GD}+	7/34	-5/17	-27/34	7/34	-5/17	0	-5/17	-10/17	≥	-513/68
φ_{DG}-	0	0	-1	1	0	0	0	0	≥	-3/2
φ_{DG}+	0	0	1	-1	0	0	0	0	≥	-13/2
φ_{DH}-	0	0	0	-1	0	0	0	0	≥	-8
T	0	0	2	-1	0	1	0	0	≥	-1
X-	0	0	1	-1	0	1	0	0	≥	-5/2
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	-1	1	0	-1	0	0	≥	-3/2
Max	-2/17	-2/17	-2/17	-2/17	-2/17	0	-2/17	-4/17	=	-36/17

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Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	2/17
φ_{AB}+	0	0	0	0	0	0	0	0	≥	0
φ_{BA}-	0	0	0	0	0	0	0	0	≥	2/17
φ_{BA}+	0	0	0	0	0	0	0	0	≥	0
φ_{CD}-	0	0	0	0	0	0	0	0	≥	2/17
φ_{CD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DC}-	0	0	0	0	0	0	0	0	≥	2/17
φ_{DC}+	0	0	0	0	0	0	0	0	≥	0
φ_{EF}-	0	0	0	0	0	0	0	0	≥	2/17
φ_{EF}+	0	0	0	0	0	0	0	0	≥	0
φ_{FE}-	0	0	0	0	0	0	0	0	≥	0
φ_{FE}+	0	0	0	0	0	0	0	0	≥	0
φ_{GD}-	0	0	0	0	0	0	0	0	≥	0
φ_{GD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DG}-	0	0	0	0	0	0	0	0	≥	0
φ_{DG}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	2/17
φ_{HF}-	0	0	0	0	0	0	0	0	≥	4/17
φ_{HF}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	3/2	3/2	1	1	3/2	0	36/17	5/2	=	-36/17

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
-1	-1	-3/2	-3/2	-1	-5/2

Variabili soluzione differenza tra rotazioni

φ_{AB}	-2/17
φ_{BA}	-2/17
φ_{CD}	-2/17
φ_{DC}	-2/17
φ_{EF}	-2/17
φ_{FE}	0
φ_{GD}	0
φ_{DG}	0
φ_{DH}	2/17
φ_{HF}	-4/17

PROCEDIMENTO E RISULTATI Nome: AL5.007

REAZIONI Fattore di collasso = 36/17

$H_A = 2F$
$V_A = 309/68F$
$W_A = -Fb$
$H_C = 3F$
$V_C = 955/68F$
$W_C = -3/2Fb$
$H_E = 5/17F$
$V_E = 80/17F$
$W_E = -Fb$

$H_{AB} = 2F$	$H_{CD} = 3F$	$H_{EF} = 5/17F$	$H_{BG} = 2F$	$H_{GD} = 2F$
$V_{AB} = 309/68F$	$V_{CD} = 955/68F$	$V_{EF} = 80/17F$	$V_{BG} = 309/68F$	$V_{GD} = -411/68F$
$W_{AB} = -Fb$	$W_{CD} = -3/2Fb$	$W_{EF} = -Fb$	$W_{BG} = Fb$	$W_{GD} = -241/68Fb$
$H_{BA} = -2F$	$H_{DC} = -3F$	$H_{FE} = -5/17F$	$H_{GB} = -2F$	$H_{DG} = -2F$
$V_{BA} = -309/68F$	$V_{DC} = -955/68F$	$V_{FE} = -80/17F$	$V_{GB} = -309/68F$	$V_{DG} = 411/68F$
$W_{BA} = -Fb$	$W_{DC} = -3/2Fb$	$W_{FE} = 12/17Fb$	$W_{GB} = 241/68Fb$	$W_{DG} = -5/2Fb$

$H_{DH} = 5F$	$H_{HF} = 5F$
$V_{DH} = 8F$	$V_{HF} = -80/17F$
$W_{DH} = 4Fb$	$W_{HF} = -4Fb$
$H_{HD} = -5F$	$H_{FH} = -5F$
$V_{HD} = -8F$	$V_{FH} = 80/17F$
$W_{HD} = 4Fb$	$W_{FH} = -12/17Fb$

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_{BBA} = -2/17\delta$	$u_{CCD} = 0$	$u_{DDC} = -2/17\delta$	$u_{EEF} = 0$
$v_{AAB} = 0$	$v_{BBA} = 0$	$v_{CCD} = 0$	$v_{DDC} = 0$	$v_{EEF} = 0$
$\varphi_{AAB} = 2/17\delta/b$	$\varphi_{BBA} = 2/17\delta/b$	$\varphi_{CCD} = 2/17\delta/b$	$\varphi_{DDC} = 2/17\delta/b$	$\varphi_{EEF} = 2/17\delta/b$

$u_F = -2/17\delta$	$u_G = -2/17\delta$	$u_{HHD} = -2/17\delta$
$v_F = 0$	$v_G = 0$	$v_{HHD} = -2/17\delta$
$\varphi_F = 2/17\delta/b$	$\varphi_G = 0$	$\varphi_{HHD} = -2/17\delta/b$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$	$u_{CCD} = 0$	$u_{EEF} = 0$	$u_{BBG} = -2/17\delta$	$u_{GGD} = -2/17\delta$
$v_{AAB} = 0$	$v_{CCD} = 0$	$v_{EEF} = 0$	$v_{BBG} = 0$	$v_{GGD} = 0$
$\varphi_{AAB} = 2/17\delta/b$	$\varphi_{CCD} = 2/17\delta/b$	$\varphi_{EEF} = 2/17\delta/b$	$\varphi_{BBG} = 0$	$\varphi_{GGD} = 0$

$u_{DDH} = -2/17\delta$	$u_{HHF} = -2/17\delta$
$v_{DDH} = 0$	$v_{HHF} = -2/17\delta$
$\varphi_{DDH} = -2/17\delta/b$	$\varphi_{HHF} = 2/17\delta/b$

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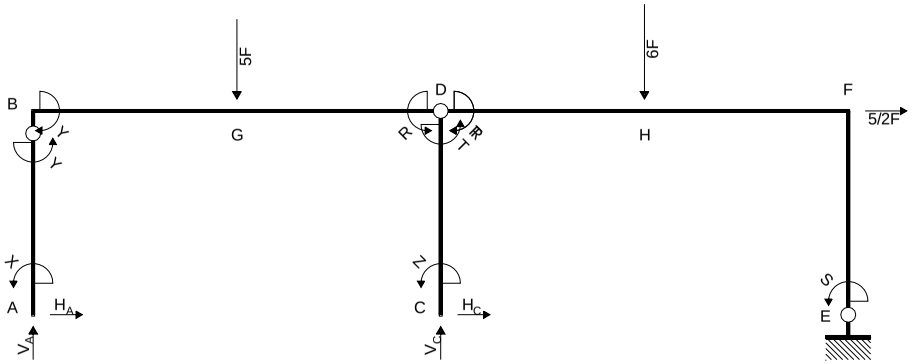


EQUAZIONI DI EQUILIBRIO

Rotazione intorno a E: aste EF FH HD DC DG GB BA

$-4V_Ab -2V_Cb = -Xb -Zb -Sb -37/2Fb$

Rotazione intorno a D: aste DC

$H_Cb = -Zb -Tb$

Rotazione intorno a D: aste DG GB BA

$H_Ab -2V_Ab = -Xb -Rb -5Fb$

Rotazione intorno a B: aste BA

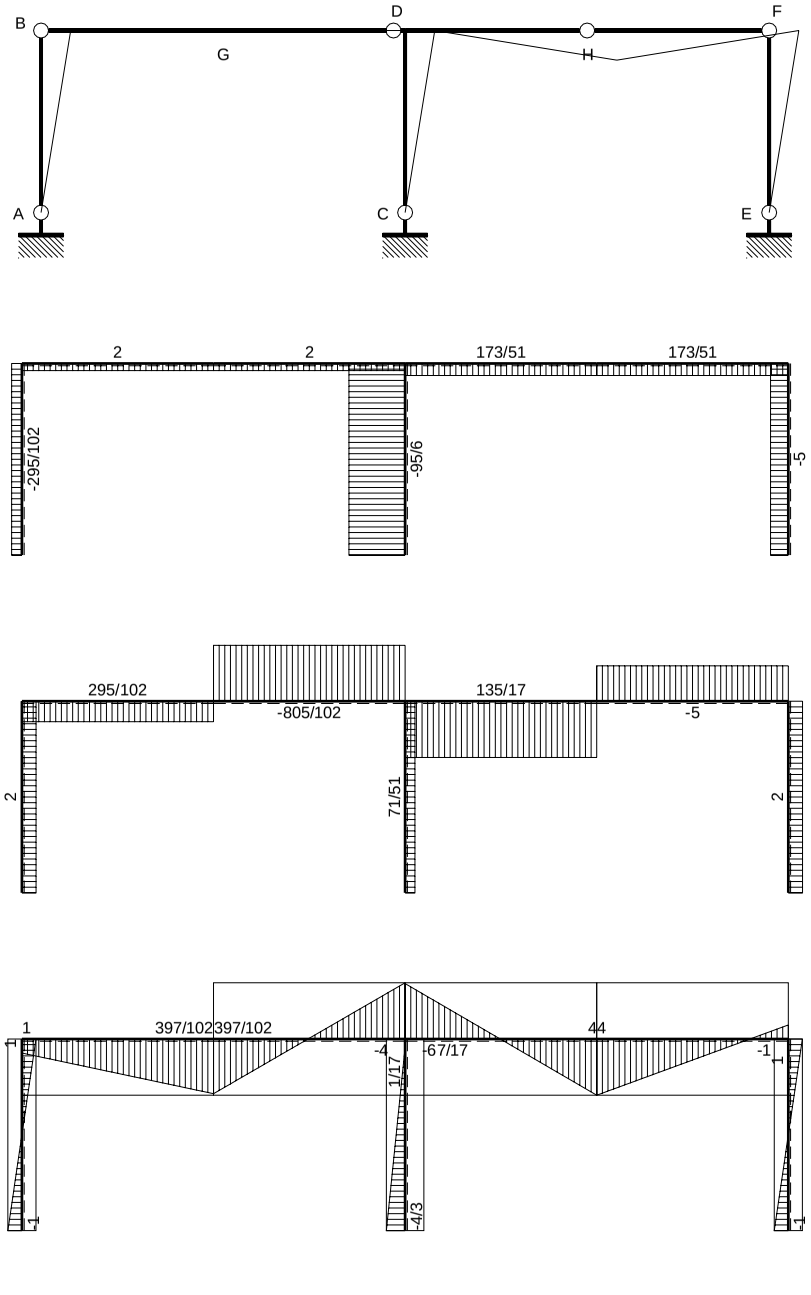
$H_Ab = -Xb -Yb$

Matrice di equilibrio

	H_Ab	V_Ab	H_Cb	V_Cb		Xb	Yb	Zb	Tb	Sb	Rb	Fb
φ_{EF}	0	-4	0	-2		-1	0	-1	0	-1	0	-37/2
φ_{DC}	0	0	1	0	=	0	0	-1	-1	0	0	0
φ_{DG}	1	-2	0	0		-1	0	0	0	0	-1	-5
φ_{BA}	1	0	0	0		-1	-1	0	0	0	0	0

Soluzione del sistema

		Xb	Yb	Zb	Tb	Sb	Rb	Fb
V_Ab		0	-1/2	0	0	0	1/2	5/2
H_Cb	=	0	0	-1	-1	0	0	0
H_Ab		-1	-1	0	0	0	0	0
V_Cb		1/2	1	1/2	0	1/2	-1	17/4



PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq}=1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	≥	H_{14}
D_2	H_{21}	H_{22}	H_{23}	≥	H_{24}
D_3	H_{31}	H_{32}	H_{33}	≥	H_{34}
D_4	H_{41}	H_{42}	H_{43}	≥	H_{44}
D_5	H_{51}	H_{52}	H_{53}	≥	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{CD}$ $T=W_{DC}$ $S=W_{EF}$ $R=W_{DG}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	1	0	0	0	0	0	0	≤	1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	1	0	0	0	0	0	≤	1
W_{CD}-	0	0	1	0	0	0	0	≥	-4/3
W_{CD}+	0	0	1	0	0	0	0	≤	4/3
W_{DC}-	0	0	0	1	0	0	0	≥	-4/3
W_{DC}+	0	0	0	1	0	0	0	≤	4/3
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	1	0	0	≤	1
W_{FE}-	-1	-1	-1	-1	-1	0	5/2	≥	-1
W_{FE}+	-1	-1	-1	-1	-1	0	5/2	≤	1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	-1/2	0	0	0	-1/2	-5/2	≤	4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	1	0	≤	4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	-1	0	-1	0	≤	4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-7/4	≥	-4
W_{HF}+	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-7/4	≤	4
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	0	≥	-1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	≥	-1
W_{CD}-	0	0	1	0	0	0	0	≥	-4/3
W_{CD}+	0	0	-1	0	0	0	0	≥	-4/3
W_{DC}-	0	0	0	1	0	0	0	≥	-4/3
W_{DC}+	0	0	0	-1	0	0	0	≥	-4/3
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	≥	-1
W_{FE}-	-1	-1	-1	-1	-1	0	5/2	≥	-1
W_{FE}+	1	1	1	1	1	0	-5/2	≥	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	5/2	≥	-4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	≥	-4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	≥	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-7/4	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	7/4	≥	-4
Max	0	0	0	0	0	0	1	=	0

PROCEDIMENTO E RISULTATI Nome: AL5.008

Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1
W_{CD}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-4/3
W_{CD}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-4/3
W_{DC}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-4/3
W_{DC}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-4/3
W_{EF}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1
W_{FE}-	-1	-1	-1	-1	-1	0	1	1	1	1	1	0	5/2	≥	-1
W_{FE}+	1	1	1	1	1	0	-1	-1	-1	-1	-1	0	-5/2	≤	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	0	1/2	0	0	0	1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	0	-1/2	0	0	0	-1/2	5/2	≤	-4
W_{DG}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-4
W_{DH}-	0	0	0	-1	0	-1	0	0	0	1	0	1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	0	0	-1	0	-1	0	≤	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	1/2	1/2	1/2	1	1/2	1/2	-7/4	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	-1/2	-1/2	-1/2	-1	-1/2	-1/2	7/4	≤	-4
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

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Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-4/3
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-4/3
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-4/3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-4/3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	-1	-1	-1	-1	-1	0	5/2	5	≥	-1
φ_{FE}+	1	1	1	1	1	0	-5/2	-5	≥	-1
φ_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	1	≥	-4
φ_{GD}+	0	1/2	0	0	0	1/2	5/2	-1	≥	-4
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-7/4	7/2	≥	-4
φ_{HF}+	1/2	1/2	1/2	1	1/2	1/2	7/4	-7/2	≥	-4
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 12-7

	X	Y	Z	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-4/3
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-4/3
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-4/3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-4/3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	2/5	2/5	2/5	2/5	2/5	0	-2/5	-2	≥	-2/5
φ_{GD}-	-1	-3/2	-1	-1	-1	-1/2	1	6	≥	-3
φ_{GD}+	1	3/2	1	1	1	1/2	-1	-6	≥	-5
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	-6/5	-6/5	-6/5	-17/10	-6/5	-1/2	7/10	7	≥	-33/10
φ_{HF}+	6/5	6/5	6/5	17/10	6/5	1/2	-7/10	-7	≥	-47/10
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	2/5	2/5	2/5	2/5	2/5	0	-2/5	-2	=	-2/5

Scambio pivotale 2-1

	φ_{AB}+	Y	Z	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-4/3
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-4/3
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-4/3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-4/3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-2/5	2/5	2/5	2/5	2/5	0	-2/5	-8/5	≥	-4/5
φ_{GD}-	1	-3/2	-1	-1	-1	-1/2	1	5	≥	-2
φ_{GD}+	-1	3/2	1	1	1	1/2	-1	-5	≥	-6
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	6/5	-6/5	-6/5	-17/10	-6/5	-1/2	7/10	29/5	≥	-21/10
φ_{HF}+	-6/5	6/5	6/5	17/10	6/5	1/2	-7/10	-29/5	≥	-59/10
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-2/5	2/5	2/5	2/5	2/5	0	-2/5	-8/5	=	-4/5

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Scambio pivotale 4-2

	φ_{AB}+	φ_{BA}+	Z	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	-1	0	0	0	0	0	0	≥	-2
Y	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-4/3
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-4/3
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-4/3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-4/3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-2/5	-2/5	2/5	2/5	2/5	0	-2/5	-6/5	≥	-6/5
φ_{GD}-	1	3/2	-1	-1	-1	-1/2	1	7/2	≥	-1/2
φ_{GD}+	-1	-3/2	1	1	1	1/2	-1	-7/2	≥	-15/2
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	6/5	6/5	-6/5	-17/10	-6/5	-1/2	7/10	23/5	≥	-9/10
φ_{HF}+	-6/5	-6/5	6/5	17/10	6/5	1/2	-7/10	-23/5	≥	-71/10
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-2/5	-2/5	2/5	2/5	2/5	0	-2/5	-6/5	=	-6/5

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Scambio pivotale 13-3

	φ_{AB}+	φ_{BA}+	φ_{GD}-	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	-1	0	0	0	0	0	0	≥	-2
Y	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	1	3/2	-1	-1	-1	-1/2	1	5/2	≥	-11/6
φ_{CD}+	-1	-3/2	1	1	1	1/2	-1	-5/2	≥	-5/6
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-4/3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-4/3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	0	1/5	-2/5	0	0	-1/5	0	1/5	≥	-7/5
Z	1	3/2	-1	-1	-1	-1/2	1	7/2	≥	-1/2
φ_{GD}+	0	0	-1	0	0	0	0	0	≥	-8
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	0	-3/5	6/5	-1/2	0	1/10	-1/2	2/5	≥	-3/10
φ_{HF}+	0	3/5	-6/5	1/2	0	-1/10	1/2	-2/5	≥	-77/10
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	0	1/5	-2/5	0	0	-1/5	0	1/5	=	-7/5

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Scambio pivotale 19-2

	φ_{AB}+	φ_{HF}-	φ_{GD}-	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	5/3	-2	5/6	0	-1/6	5/6	-2/3	≥	-3/2
Y	0	5/3	-2	5/6	0	-1/6	5/6	1/3	≥	-1/2
φ_{CD}-	1	-5/2	2	-9/4	-1	-1/4	-1/4	7/2	≥	-31/12
φ_{CD}+	-1	5/2	-2	9/4	1	1/4	1/4	-7/2	≥	-1/12
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-4/3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-4/3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	0	-1/3	0	-1/6	0	-1/6	-1/6	1/3	≥	-3/2
Z	1	-5/2	2	-9/4	-1	-1/4	-1/4	9/2	≥	-5/4
φ_{GD}+	0	0	-1	0	0	0	0	0	≥	-8
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{BA}+	0	-5/3	2	-5/6	0	1/6	-5/6	2/3	≥	-1/2
φ_{HF}+	0	-1	0	0	0	0	0	0	≥	-8
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	0	-1/3	0	-1/6	0	-1/6	-1/6	1/3	=	-3/2

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Scambio pivotale 6-8

	φ_{AB}+	φ_{HF}-	φ_{GD}-	T	S	R	φ_{FE}+	φ_{CD}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-9/7	5/7	-4/7	9/14	2/7	1/14	1/14	-2/7	≥	-43/42
φ_{BA}-	4/21	25/21	-34/21	17/42	-4/21	-3/14	11/14	4/21	≥	-187/126
Y	-2/21	40/21	-46/21	22/21	2/21	-1/7	6/7	-2/21	≥	-32/63
φ_{CD}-	0	0	0	0	0	0	0	-1	≥	-8/3
X-	-2/7	5/7	-4/7	9/14	2/7	1/14	1/14	-2/7	≥	-1/42
φ_{DC}-	2/7	-5/7	4/7	5/14	-2/7	-1/14	-1/14	2/7	≥	-55/42
φ_{DC}+	-2/7	5/7	-4/7	-5/14	2/7	1/14	1/14	-2/7	≥	-19/14
φ_{EF}-	2/7	-5/7	4/7	-9/14	5/7	-1/14	-1/14	2/7	≥	-41/42
φ_{EF}+	-2/7	5/7	-4/7	9/14	-5/7	1/14	1/14	-2/7	≥	-43/42
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-2/21	-2/21	-4/21	1/21	2/21	-1/7	-1/7	-2/21	≥	-95/63
Z	-2/7	5/7	-4/7	9/14	2/7	1/14	1/14	-9/7	≥	-19/14
φ_{GD}+	0	0	-1	0	0	0	0	0	≥	-8
φ_{DG}-	2/7	-5/7	4/7	-9/14	-2/7	13/14	-1/14	2/7	≥	-167/42
φ_{DG}+	-2/7	5/7	-4/7	9/14	2/7	-13/14	1/14	-2/7	≥	-169/42
φ_{DH}-	-4/7	10/7	-8/7	2/7	4/7	-6/7	1/7	-4/7	≥	-85/21
φ_{DH}+	4/7	-10/7	8/7	-2/7	-4/7	6/7	-1/7	4/7	≥	-83/21
φ_{BA}+	-4/21	-25/21	34/21	-17/42	4/21	3/14	-11/14	-4/21	≥	-65/126
φ_{HF}+	0	-1	0	0	0	0	0	0	≥	-8
L_X	2/7	-5/7	4/7	-9/14	-2/7	-1/14	-1/14	2/7	≥	-167/42
Max	-2/21	-2/21	-4/21	1/21	2/21	-1/7	-1/7	-2/21	=	-95/63

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Scambio pivotale 10-5

	φ_{AB}+	φ_{HF}-	φ_{GD}-	T	φ_{EF}+	R	φ_{FE}+	φ_{CD}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-7/5	1	-4/5	9/10	-2/5	1/10	1/10	-2/5	≥	-43/30
φ_{BA}-	4/15	1	-22/15	7/30	4/15	-7/30	23/30	4/15	≥	-109/90
Y	-2/15	2	-34/15	17/15	-2/15	-2/15	13/15	-2/15	≥	-29/45
φ_{CD}-	0	0	0	0	0	0	0	-1	≥	-8/3
X-	-2/5	1	-4/5	9/10	-2/5	1/10	1/10	-2/5	≥	-13/30
φ_{DC}-	2/5	-1	4/5	1/10	2/5	-1/10	-1/10	2/5	≥	-9/10
φ_{DC}+	-2/5	1	-4/5	-1/10	-2/5	1/10	1/10	-2/5	≥	-53/30
φ_{EF}-	0	0	0	0	-1	0	0	0	≥	-2
S	-2/5	1	-4/5	9/10	-7/5	1/10	1/10	-2/5	≥	-43/30
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-2/15	0	-4/15	2/15	-2/15	-2/15	-2/15	-2/15	≥	-74/45
Z	-2/5	1	-4/5	9/10	-2/5	1/10	1/10	-7/5	≥	-53/30
φ_{GD}+	0	0	-1	0	0	0	0	0	≥	-8
φ_{DG}-	2/5	-1	4/5	-9/10	2/5	9/10	-1/10	2/5	≥	-107/30
φ_{DG}+	-2/5	1	-4/5	9/10	-2/5	-9/10	1/10	-2/5	≥	-133/30
φ_{DH}-	-4/5	2	-8/5	4/5	-4/5	-4/5	1/5	-4/5	≥	-73/15
φ_{DH}+	4/5	-2	8/5	-4/5	4/5	4/5	-1/5	4/5	≥	-47/15
φ_{BA}+	-4/15	-1	22/15	-7/30	-4/15	7/30	-23/30	-4/15	≥	-71/90
φ_{HF}+	0	-1	0	0	0	0	0	0	≥	-8
L_X	2/5	-1	4/5	-9/10	2/5	-1/10	-1/10	2/5	≥	-107/30
Max	-2/15	0	-4/15	2/15	-2/15	-2/15	-2/15	-2/15	=	-74/45

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Scambio pivotale 19-4

	φ_{AB}+	φ_{HF}-	φ_{GD}-	φ_{BA}+	φ_{EF}+	R	φ_{FE}+	φ_{CD}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-17/7	-20/7	34/7	-27/7	-10/7	1	-20/7	-10/7	≥	-94/21
φ_{BA}-	0	0	0	-1	0	0	0	0	≥	-2
Y	-10/7	-20/7	34/7	-34/7	-10/7	1	-20/7	-10/7	≥	-94/21
φ_{CD}-	0	0	0	0	0	0	0	-1	≥	-8/3
X-	-10/7	-20/7	34/7	-27/7	-10/7	1	-20/7	-10/7	≥	-73/21
φ_{DC}-	2/7	-10/7	10/7	-3/7	2/7	0	-3/7	2/7	≥	-26/21
φ_{DC}+	-2/7	10/7	-10/7	3/7	-2/7	0	3/7	-2/7	≥	-10/7
φ_{EF}-	0	0	0	0	-1	0	0	0	≥	-2
S	-10/7	-20/7	34/7	-27/7	-17/7	1	-20/7	-10/7	≥	-94/21
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-2/7	-4/7	4/7	-4/7	-2/7	0	-4/7	-2/7	≥	-44/21
Z	-10/7	-20/7	34/7	-27/7	-10/7	1	-20/7	-17/7	≥	-101/21
φ_{GD}+	0	0	-1	0	0	0	0	0	≥	-8
φ_{DG}-	10/7	20/7	-34/7	27/7	10/7	0	20/7	10/7	≥	-11/21
φ_{DG}+	-10/7	-20/7	34/7	-27/7	-10/7	0	-20/7	-10/7	≥	-157/21
φ_{DH}-	-12/7	-10/7	24/7	-24/7	-12/7	0	-17/7	-12/7	≥	-53/7
φ_{DH}+	12/7	10/7	-24/7	24/7	12/7	0	17/7	12/7	≥	-3/7
T	-8/7	-30/7	44/7	-30/7	-8/7	1	-23/7	-8/7	≥	-71/21
φ_{HF}+	0	-1	0	0	0	0	0	0	≥	-8
L_X	10/7	20/7	-34/7	27/7	10/7	-1	20/7	10/7	≥	-11/21
Max	-2/7	-4/7	4/7	-4/7	-2/7	0	-4/7	-2/7	=	-44/21

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Scambio pivotale 15-3

	φ_{AB}+	φ_{HF}-	φ_{DG}-	φ_{BA}+	φ_{EF}+	R	φ_{FE}+	φ_{CD}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	-1	0	0	1	0	0	≥	-5
φ_{BA}-	0	0	0	-1	0	0	0	0	≥	-2
Y	0	0	-1	-1	0	1	0	0	≥	-5
φ_{CD}-	0	0	0	0	0	0	0	-1	≥	-8/3
X-	0	0	-1	0	0	1	0	0	≥	-4
φ_{DC}-	12/17	-10/17	-5/17	12/17	12/17	0	7/17	12/17	≥	-71/51
φ_{DC}+	-12/17	10/17	5/17	-12/17	-12/17	0	-7/17	-12/17	≥	-65/51
φ_{EF}-	0	0	0	0	-1	0	0	0	≥	-2
S	0	0	-1	0	-1	1	0	0	≥	-5
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-2/17	-4/17	-2/17	-2/17	-2/17	0	-4/17	-2/17	≥	-110/51
Z	0	0	-1	0	0	1	0	-1	≥	-16/3
φ_{GD}+	-5/17	-10/17	7/34	-27/34	-5/17	0	-10/17	-5/17	≥	-805/102
φ_{GD}-	5/17	10/17	-7/34	27/34	5/17	0	10/17	5/17	≥	-11/102
φ_{DG}+	0	0	-1	0	0	0	0	0	≥	-8
φ_{DH}-	-12/17	10/17	-12/17	-12/17	-12/17	0	-7/17	-12/17	≥	-135/17
φ_{DH}+	12/17	-10/17	12/17	12/17	12/17	0	7/17	12/17	≥	-1/17
T	12/17	-10/17	-22/17	12/17	12/17	1	7/17	12/17	≥	-69/17
φ_{HF}+	0	-1	0	0	0	0	0	0	≥	-8
L_X	0	0	1	0	0	-1	0	0	≥	0
Max	-2/17	-4/17	-2/17	-2/17	-2/17	0	-4/17	-2/17	=	-110/51

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Tableau finale

	φ_{AB}+	φ_{HF}-	φ_{DG}-	φ_{BA}+	φ_{EF}+	R	φ_{FE}+	φ_{CD}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	-1	0	0	1	0	0	≥	-5
φ_{BA}-	0	0	0	-1	0	0	0	0	≥	-2
Y	0	0	-1	-1	0	1	0	0	≥	-5
φ_{CD}-	0	0	0	0	0	0	0	-1	≥	-8/3
X-	0	0	-1	0	0	1	0	0	≥	-4
φ_{DC}-	12/17	-10/17	-5/17	12/17	12/17	0	7/17	12/17	≥	-71/51
φ_{DC}+	-12/17	10/17	5/17	-12/17	-12/17	0	-7/17	-12/17	≥	-65/51
φ_{EF}-	0	0	0	0	-1	0	0	0	≥	-2
S	0	0	-1	0	-1	1	0	0	≥	-5
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-2/17	-4/17	-2/17	-2/17	-2/17	0	-4/17	-2/17	≥	-110/51
Z	0	0	-1	0	0	1	0	-1	≥	-16/3
φ_{GD}+	-5/17	-10/17	7/34	-27/34	-5/17	0	-10/17	-5/17	≥	-805/102
φ_{GD}-	5/17	10/17	-7/34	27/34	5/17	0	10/17	5/17	≥	-11/102
φ_{DG}+	0	0	-1	0	0	0	0	0	≥	-8
φ_{DH}-	-12/17	10/17	-12/17	-12/17	-12/17	0	-7/17	-12/17	≥	-135/17
φ_{DH}+	12/17	-10/17	12/17	12/17	12/17	0	7/17	12/17	≥	-1/17
T	12/17	-10/17	-22/17	12/17	12/17	1	7/17	12/17	≥	-69/17
φ_{HF}+	0	-1	0	0	0	0	0	0	≥	-8
L_X	0	0	1	0	0	-1	0	0	≥	0
Max	-2/17	-4/17	-2/17	-2/17	-2/17	0	-4/17	-2/17	=	-110/51

Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	0
φ_{AB}+	0	0	0	0	0	0	0	0	≥	2/17
φ_{BA}-	0	0	0	0	0	0	0	0	≥	0
φ_{BA}+	0	0	0	0	0	0	0	0	≥	2/17
φ_{CD}-	0	0	0	0	0	0	0	0	≥	0
φ_{CD}+	0	0	0	0	0	0	0	0	≥	2/17
φ_{DC}-	0	0	0	0	0	0	0	0	≥	0
φ_{DC}+	0	0	0	0	0	0	0	0	≥	0
φ_{EF}-	0	0	0	0	0	0	0	0	≥	0
φ_{EF}+	0	0	0	0	0	0	0	0	≥	2/17
φ_{FE}-	0	0	0	0	0	0	0	0	≥	0
φ_{FE}+	0	0	0	0	0	0	0	0	≥	4/17
φ_{GD}-	0	0	0	0	0	0	0	0	≥	0
φ_{GD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DG}-	0	0	0	0	0	0	0	0	≥	2/17
φ_{DG}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	0
φ_{HF}-	0	0	0	0	0	0	0	0	≥	4/17
φ_{HF}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	5	5	16/3	69/17	5	0	110/51	4	=	-110/51

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
1	1	4/3	1/17	1	-4

Variabili soluzione differenza tra rotazioni

φ_{AB}	2/17
φ_{BA}	2/17
φ_{CD}	2/17
φ_{DC}	0
φ_{EF}	2/17
φ_{FE}	4/17
φ_{GD}	0
φ_{DG}	-2/17
φ_{DH}	0
φ_{HF}	-4/17

REAZIONI Fattore di collasso = 110/51

H_A = -2F
V_A = 295/102F
W_A = Fb
H_C = -71/51F
V_C = 95/6F
W_C = 4/3Fb
H_E = -2F
V_E = 5F
W_E = Fb

H_{AB} = -2F	H_{CD} = -71/51F	H_{EF} = -2F	H_{BG} = -2F
V_{AB} = 295/102F	V_{CD} = 95/6F	V_{EF} = 5F	V_{BG} = 295/102F
W_{AB} = Fb	W_{CD} = 4/3Fb	W_{EF} = Fb	W_{BG} = -Fb
H_{BA} = 2F	H_{DC} = 71/51F	H_{FE} = 2F	H_{GB} = 2F
V_{BA} = -295/102F	V_{DC} = -95/6F	V_{FE} = -5F	V_{GB} = -295/102F
W_{BA} = Fb	W_{DC} = 1/17Fb	W_{FE} = Fb	W_{GB} = 397/102Fb

H_{GD} = -2F	H_{DH} = -173/51F	H_{HF} = -173/51F
V_{GD} = -805/102F	V_{DH} = 135/17F	V_{HF} = -5F
W_{GD} = -397/102Fb	W_{DH} = 67/17Fb	W_{HF} = -4Fb
H_{DG} = 2F	H_{HD} = 173/51F	H_{FH} = 173/51F
V_{DG} = 805/102F	V_{HD} = -135/17F	V_{FH} = 5F
W_{DG} = -4Fb	W_{HD} = 4Fb	W_{FH} = -Fb

SPOSTAMENTI NODALI

u_{AAB} = 0	u_{BBA} = 2/17δ	u_{CCD} = 0	u_D = 2/17δ	u_{EEF} = 0
v_{AAB} = 0	v_{BBA} = 0	v_{CCD} = 0	v_D = 0	v_{EEF} = 0
φ_{AAB} = -2/17δ/b	φ_{BBA} = -2/17δ/b	φ_{CCD} = -2/17δ/b	φ_D = -2/17δ/b	φ_{EEF} = -2/17δ/b

u_{FFE} = 2/17δ	u_G = 2/17δ	u_{HHD} = 2/17δ
v_{FFE} = 0	v_G = 0	v_{HHD} = -2/17δ
φ_{FFH} = -2/17δ/b	φ_G = 0	φ_{HHD} = -2/17δ/b

SPOSTAMENTI RIGIDI DELLE ASTE

u_{AAB} = 0	u_{CCD} = 0	u_{EEF} = 0	u_{BBG} = 2/17δ	u_{GGD} = 2/17δ
v_{AAB} = 0	v_{CCD} = 0	v_{EEF} = 0	v_{BBG} = 0	v_{GGD} = 0
φ_{AAB} = -2/17δ/b	φ_{CCD} = -2/17δ/b	φ_{EEF} = -2/17δ/b	φ_{BBG} = 0	φ_{GGD} = 0

u_{DDH} = 2/17δ	u_{HHF} = 2/17δ
v_{DDH} = 0	v_{HHF} = -2/17δ
φ_{DDH} = -2/17δ/b	φ_{HHF} = 2/17δ/b

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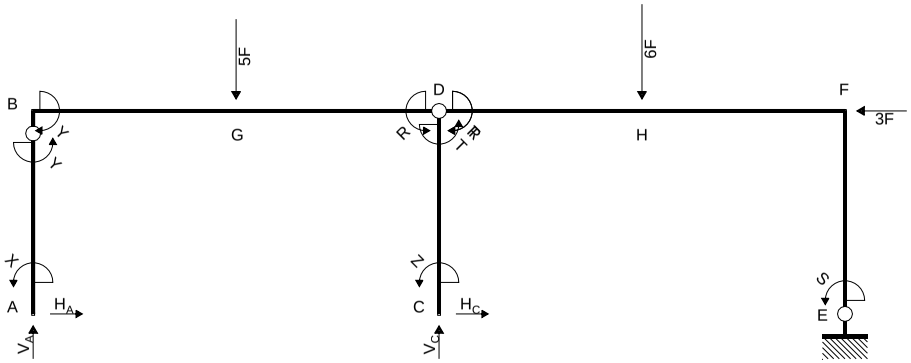


EQUAZIONI DI EQUILIBRIO

Rotazione intorno a E: aste EF FH HD DC DG GB BA

$-4V_Ab\ -2V_Cb = -Xb\ -Zb\ -Sb\ -24Fb$

Rotazione intorno a D: aste DC

$H_Cb = -Zb\ -Tb$

Rotazione intorno a D: aste DG GB BA

$H_Ab\ -2V_Ab = -Xb\ -Rb\ -5Fb$

Rotazione intorno a B: aste BA

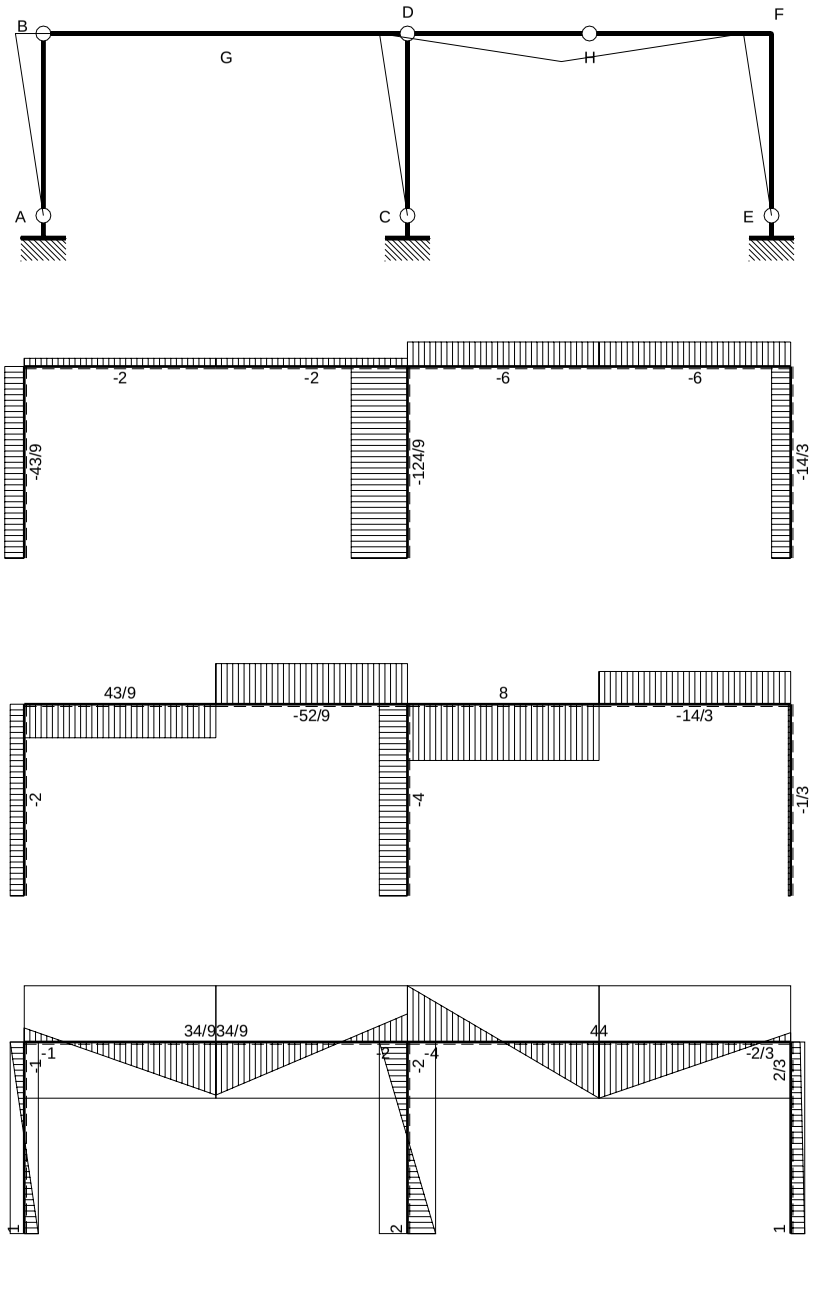
$H_Ab = -Xb\ -Yb$

Matrice di equilibrio

$$\begin{array}{c} \\ \varphi_{EF} \\ \varphi_{DC} \\ \varphi_{DG} \\ \varphi_{BA} \end{array} \begin{bmatrix} H_Ab & V_Ab & H_Cb & V_Cb \\ 0 & -4 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 1 & -2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1 & 0 & -1 & 0 & -1 & 0 & -24 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & -1 & -5 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Soluzione del sistema

$$\begin{bmatrix} V_Ab \\ H_Cb \\ H_Ab \\ V_Cb \end{bmatrix} = \begin{array}{c} \begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \end{bmatrix} \\ \begin{bmatrix} 0 & -1/2 & 0 & 0 & 0 & 1/2 & 5/2 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1 & 1/2 & 0 & 1/2 & -1 & 7 \end{bmatrix} \end{array}$$



PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq} = 1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	$\ge$	H_{14}
D_2	H_{21}	H_{22}	H_{23}	$\ge$	H_{24}
D_3	H_{31}	H_{32}	H_{33}	$\ge$	H_{34}
D_4	H_{41}	H_{42}	H_{43}	$\ge$	H_{44}
D_5	H_{51}	H_{52}	H_{53}	$\ge$	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{CD}$ $T=W_{DC}$ $S=W_{EF}$ $R=W_{DG}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	1	0	0	0	0	0	0	≤	1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	1	0	0	0	0	0	≤	1
W_{CD}-	0	0	1	0	0	0	0	≥	-2
W_{CD}+	0	0	1	0	0	0	0	≤	2
W_{DC}-	0	0	0	1	0	0	0	≥	-2
W_{DC}+	0	0	0	1	0	0	0	≤	2
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	1	0	0	≤	1
W_{FE}-	-1	-1	-1	-1	-1	0	-3	≥	-1
W_{FE}+	-1	-1	-1	-1	-1	0	-3	≤	1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	-1/2	0	0	0	-1/2	-5/2	≤	4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	1	0	≤	4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	-1	0	-1	0	≤	4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-9/2	≥	-4
W_{HF}+	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-9/2	≤	4
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	0	≥	-1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	≥	-1
W_{CD}-	0	0	1	0	0	0	0	≥	-2
W_{CD}+	0	0	-1	0	0	0	0	≥	-2
W_{DC}-	0	0	0	1	0	0	0	≥	-2
W_{DC}+	0	0	0	-1	0	0	0	≥	-2
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	≥	-1
W_{FE}-	-1	-1	-1	-1	-1	0	-3	≥	-1
W_{FE}+	1	1	1	1	1	0	3	≥	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	5/2	≥	-4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	≥	-4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	≥	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-9/2	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	9/2	≥	-4
Max	0	0	0	0	0	0	1	=	0

PROCEDIMENTO E RISULTATI Nome: AL5.009

Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1
W_{CD}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-2
W_{CD}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-2
W_{DC}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-2
W_{DC}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-2
W_{EF}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1
W_{FE}-	-1	-1	-1	-1	-1	0	1	1	1	1	1	0	-3	≥	-1
W_{FE}+	1	1	1	1	1	0	-1	-1	-1	-1	-1	0	3	≤	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	0	1/2	0	0	0	1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	0	-1/2	0	0	0	-1/2	5/2	≤	-4
W_{DG}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-4
W_{DH}-	0	0	0	-1	0	-1	0	0	0	1	0	1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	0	0	-1	0	-1	0	≤	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	1/2	1/2	1/2	1	1/2	1/2	-9/2	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	-1/2	-1/2	-1/2	-1	-1/2	-1/2	9/2	≤	-4
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

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Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-2
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-2
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-2
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-2
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	-1	-1	-1	-1	-1	0	-3	5	≥	-1
φ_{FE}+	1	1	1	1	1	0	3	-5	≥	-1
φ_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	1	≥	-4
φ_{GD}+	0	1/2	0	0	0	1/2	5/2	-1	≥	-4
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-9/2	7/2	≥	-4
φ_{HF}+	1/2	1/2	1/2	1	1/2	1/2	9/2	-7/2	≥	-4
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 11-7

	X	Y	Z	T	S	R	φ_{FE}-	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-2
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-2
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-2
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-2
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
αbF	-1/3	-1/3	-1/3	-1/3	-1/3	0	-1/3	5/3	≥	-1/3
φ_{FE}+	0	0	0	0	0	0	-1	0	≥	-2
φ_{GD}-	5/6	1/3	5/6	5/6	5/6	-1/2	5/6	-19/6	≥	-19/6
φ_{GD}+	-5/6	-1/3	-5/6	-5/6	-5/6	1/2	-5/6	19/6	≥	-29/6
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	1	1	1	1/2	1	-1/2	3/2	-4	≥	-5/2
φ_{HF}+	-1	-1	-1	-1/2	-1	1/2	-3/2	4	≥	-11/2
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-1/3	-1/3	-1/3	-1/3	-1/3	0	-1/3	5/3	=	-1/3

Scambio pivotale 19-8

	X	Y	Z	T	S	R	φ_{FE}-	φ_{HF}-		Fb
φ_{AB}-	3/4	-1/4	-1/4	-1/8	-1/4	1/8	-3/8	1/4	≥	-3/8
φ_{AB}+	-3/4	1/4	1/4	1/8	1/4	-1/8	3/8	-1/4	≥	-13/8
φ_{BA}-	-1/4	3/4	-1/4	-1/8	-1/4	1/8	-3/8	1/4	≥	-3/8
φ_{BA}+	1/4	-3/4	1/4	1/8	1/4	-1/8	3/8	-1/4	≥	-13/8
φ_{CD}-	-1/4	-1/4	3/4	-1/8	-1/4	1/8	-3/8	1/4	≥	-11/8
φ_{CD}+	1/4	1/4	-3/4	1/8	1/4	-1/8	3/8	-1/4	≥	-21/8
φ_{DC}-	-1/4	-1/4	-1/4	7/8	-1/4	1/8	-3/8	1/4	≥	-11/8
φ_{DC}+	1/4	1/4	1/4	-7/8	1/4	-1/8	3/8	-1/4	≥	-21/8
φ_{EF}-	-1/4	-1/4	-1/4	-1/8	3/4	1/8	-3/8	1/4	≥	-3/8
φ_{EF}+	1/4	1/4	1/4	1/8	-3/4	-1/8	3/8	-1/4	≥	-13/8
αbF	1/12	1/12	1/12	-1/8	1/12	-5/24	7/24	-5/12	≥	-11/8
φ_{FE}+	0	0	0	0	0	0	-1	0	≥	-2
φ_{GD}-	1/24	-11/24	1/24	7/16	1/24	-5/48	-17/48	19/24	≥	-19/16
φ_{GD}+	-1/24	11/24	-1/24	-7/16	-1/24	5/48	17/48	-19/24	≥	-109/16
φ_{DG}-	-1/4	-1/4	-1/4	-1/8	-1/4	9/8	-3/8	1/4	≥	-27/8
φ_{DG}+	1/4	1/4	1/4	1/8	1/4	-9/8	3/8	-1/4	≥	-37/8
φ_{DH}-	1/2	1/2	1/2	-3/4	1/2	-5/4	3/4	-1/2	≥	-21/4
φ_{DH}+	-1/2	-1/2	-1/2	3/4	-1/2	5/4	-3/4	1/2	≥	-11/4
X-	1/4	1/4	1/4	1/8	1/4	-1/8	3/8	-1/4	≥	-5/8
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	-1/4	-1/4	-1/4	-1/8	-1/4	1/8	-3/8	1/4	≥	-27/8
Max	1/12	1/12	1/12	-1/8	1/12	-5/24	7/24	-5/12	=	-11/8

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Scambio pivotale 1-7

	X	Y	Z	T	S	R	φ_{AB}-	φ_{HF}-		Fb
φ_{FE}-	2	-2/3	-2/3	-1/3	-2/3	1/3	-8/3	2/3	≥	-1
φ_{AB}+	0	0	0	0	0	0	-1	0	≥	-2
φ_{BA}-	-1	1	0	0	0	0	1	0	≥	0
φ_{BA}+	1	-1	0	0	0	0	-1	0	≥	-2
φ_{CD}-	-1	0	1	0	0	0	1	0	≥	-1
φ_{CD}+	1	0	-1	0	0	0	-1	0	≥	-3
φ_{DC}-	-1	0	0	1	0	0	1	0	≥	-1
φ_{DC}+	1	0	0	-1	0	0	-1	0	≥	-3
φ_{EF}-	-1	0	0	0	1	0	1	0	≥	0
φ_{EF}+	1	0	0	0	-1	0	-1	0	≥	-2
αbF	2/3	-1/9	-1/9	-2/9	-1/9	-1/9	-7/9	-2/9	≥	-5/3
φ_{FE}+	-2	2/3	2/3	1/3	2/3	-1/3	8/3	-2/3	≥	-1
φ_{GD}-	-2/3	-2/9	5/18	5/9	5/18	-2/9	17/18	5/9	≥	-5/6
φ_{GD}+	2/3	2/9	-5/18	-5/9	-5/18	2/9	-17/18	-5/9	≥	-43/6
φ_{DG}-	-1	0	0	0	0	1	1	0	≥	-3
φ_{DG}+	1	0	0	0	0	-1	-1	0	≥	-5
φ_{DH}-	2	0	0	-1	0	-1	-2	0	≥	-6
φ_{DH}+	-2	0	0	1	0	1	2	0	≥	-2
X-	1	0	0	0	0	0	-1	0	≥	-1
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	-1	0	0	0	0	0	1	0	≥	-3
Max	2/3	-1/9	-1/9	-2/9	-1/9	-1/9	-7/9	-2/9	=	-5/3

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Scambio pivotale 3-1

	φ_{BA}-	Y	Z	T	S	R	φ_{AB}-	φ_{HF}-		Fb
φ_{FE}-	-2	4/3	-2/3	-1/3	-2/3	1/3	-2/3	2/3	≥	-1
φ_{AB}+	0	0	0	0	0	0	-1	0	≥	-2
X	-1	1	0	0	0	0	1	0	≥	0
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
φ_{CD}-	1	-1	1	0	0	0	0	0	≥	-1
φ_{CD}+	-1	1	-1	0	0	0	0	0	≥	-3
φ_{DC}-	1	-1	0	1	0	0	0	0	≥	-1
φ_{DC}+	-1	1	0	-1	0	0	0	0	≥	-3
φ_{EF}-	1	-1	0	0	1	0	0	0	≥	0
φ_{EF}+	-1	1	0	0	-1	0	0	0	≥	-2
αbF	-2/3	5/9	-1/9	-2/9	-1/9	-1/9	-1/9	-2/9	≥	-5/3
φ_{FE}+	2	-4/3	2/3	1/3	2/3	-1/3	2/3	-2/3	≥	-1
φ_{GD}-	2/3	-8/9	5/18	5/9	5/18	-2/9	5/18	5/9	≥	-5/6
φ_{GD}+	-2/3	8/9	-5/18	-5/9	-5/18	2/9	-5/18	-5/9	≥	-43/6
φ_{DG}-	1	-1	0	0	0	1	0	0	≥	-3
φ_{DG}+	-1	1	0	0	0	-1	0	0	≥	-5
φ_{DH}-	-2	2	0	-1	0	-1	0	0	≥	-6
φ_{DH}+	2	-2	0	1	0	1	0	0	≥	-2
X-	-1	1	0	0	0	0	0	0	≥	-1
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	1	-1	0	0	0	0	0	0	≥	-3
Max	-2/3	5/9	-1/9	-2/9	-1/9	-1/9	-1/9	-2/9	=	-5/3

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Scambio pivotale 9-2

	φ_{BA}-	φ_{EF}-	Z	T	S	R	φ_{AB}-	φ_{HF}-		Fb
φ_{FE}-	-2/3	-4/3	-2/3	-1/3	2/3	1/3	-2/3	2/3	≥	-1
φ_{AB}+	0	0	0	0	0	0	-1	0	≥	-2
X	0	-1	0	0	1	0	1	0	≥	0
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
φ_{CD}-	0	1	1	0	-1	0	0	0	≥	-1
φ_{CD}+	0	-1	-1	0	1	0	0	0	≥	-3
φ_{DC}-	0	1	0	1	-1	0	0	0	≥	-1
φ_{DC}+	0	-1	0	-1	1	0	0	0	≥	-3
Y	1	-1	0	0	1	0	0	0	≥	0
φ_{EF}+	0	-1	0	0	0	0	0	0	≥	-2
αbF	-1/9	-5/9	-1/9	-2/9	4/9	-1/9	-1/9	-2/9	≥	-5/3
φ_{FE}+	2/3	4/3	2/3	1/3	-2/3	-1/3	2/3	-2/3	≥	-1
φ_{GD}-	-2/9	8/9	5/18	5/9	-11/18	-2/9	5/18	5/9	≥	-5/6
φ_{GD}+	2/9	-8/9	-5/18	-5/9	11/18	2/9	-5/18	-5/9	≥	-43/6
φ_{DG}-	0	1	0	0	-1	1	0	0	≥	-3
φ_{DG}+	0	-1	0	0	1	-1	0	0	≥	-5
φ_{DH}-	0	-2	0	-1	2	-1	0	0	≥	-6
φ_{DH}+	0	2	0	1	-2	1	0	0	≥	-2
X-	0	-1	0	0	1	0	0	0	≥	-1
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	1	0	0	-1	0	0	0	≥	-3
Max	-1/9	-5/9	-1/9	-2/9	4/9	-1/9	-1/9	-2/9	=	-5/3

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Scambio pivotale 5-5

	φ_{BA}-	φ_{EF}-	Z	T	φ_{CD}-	R	φ_{AB}-	φ_{HF}-		Fb
φ_{FE}-	-2/3	-2/3	0	-1/3	-2/3	1/3	-2/3	2/3	≥	-5/3
φ_{AB}+	0	0	0	0	0	0	-1	0	≥	-2
X	0	0	1	0	-1	0	1	0	≥	-1
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
S	0	1	1	0	-1	0	0	0	≥	-1
φ_{CD}+	0	0	0	0	-1	0	0	0	≥	-4
φ_{DC}-	0	0	-1	1	1	0	0	0	≥	0
φ_{DC}+	0	0	1	-1	-1	0	0	0	≥	-4
Y	1	0	1	0	-1	0	0	0	≥	-1
φ_{EF}+	0	-1	0	0	0	0	0	0	≥	-2
αbF	-1/9	-1/9	1/3	-2/9	-4/9	-1/9	-1/9	-2/9	≥	-19/9
φ_{FE}+	2/3	2/3	0	1/3	2/3	-1/3	2/3	-2/3	≥	-1/3
φ_{GD}-	-2/9	5/18	-1/3	5/9	11/18	-2/9	5/18	5/9	≥	-2/9
φ_{GD}+	2/9	-5/18	1/3	-5/9	-11/18	2/9	-5/18	-5/9	≥	-70/9
φ_{DG}-	0	0	-1	0	1	1	0	0	≥	-2
φ_{DG}+	0	0	1	0	-1	-1	0	0	≥	-6
φ_{DH}-	0	0	2	-1	-2	-1	0	0	≥	-8
φ_{DH}+	0	0	-2	1	2	1	0	0	≥	0
X-	0	0	1	0	-1	0	0	0	≥	-2
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	-1	0	1	0	0	0	≥	-2
Max	-1/9	-1/9	1/3	-2/9	-4/9	-1/9	-1/9	-2/9	=	-19/9

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Scambio pivotale 7-3

	φ_{BA}-	φ_{EF}-	φ_{DC}-	T	φ_{CD}-	R	φ_{AB}-	φ_{HF}-		Fb
φ_{FE}-	-2/3	-2/3	0	-1/3	-2/3	1/3	-2/3	2/3	≥	-5/3
φ_{AB}+	0	0	0	0	0	0	-1	0	≥	-2
X	0	0	-1	1	0	0	1	0	≥	-1
φ_{BA}+	-1	0	0	0	0	0	0	0	≥	-2
S	0	1	-1	1	0	0	0	0	≥	-1
φ_{CD}+	0	0	0	0	-1	0	0	0	≥	-4
Z	0	0	-1	1	1	0	0	0	≥	0
φ_{DC}+	0	0	-1	0	0	0	0	0	≥	-4
Y	1	0	-1	1	0	0	0	0	≥	-1
φ_{EF}+	0	-1	0	0	0	0	0	0	≥	-2
αbF	-1/9	-1/9	-1/3	1/9	-1/9	-1/9	-1/9	-2/9	≥	-19/9
φ_{FE}+	2/3	2/3	0	1/3	2/3	-1/3	2/3	-2/3	≥	-1/3
φ_{GD}-	-2/9	5/18	1/3	2/9	5/18	-2/9	5/18	5/9	≥	-2/9
φ_{GD}+	2/9	-5/18	-1/3	-2/9	-5/18	2/9	-5/18	-5/9	≥	-70/9
φ_{DG}-	0	0	1	-1	0	1	0	0	≥	-2
φ_{DG}+	0	0	-1	1	0	-1	0	0	≥	-6
φ_{DH}-	0	0	-2	1	0	-1	0	0	≥	-8
φ_{DH}+	0	0	2	-1	0	1	0	0	≥	0
X-	0	0	-1	1	0	0	0	0	≥	-2
φ_{HF}+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	1	-1	0	0	0	0	≥	-2
Max	-1/9	-1/9	-1/3	1/9	-1/9	-1/9	-1/9	-2/9	=	-19/9

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Scambio pivotale 18-4

	φ_{BA}^-	φ_{EF}^-	φ_{DC}^-	φ_{DH}^+	φ_{CD}^-	R	φ_{AB}^-	φ_{HF}^-		Fb
φ_{FE}^-	-2/3	-2/3	-2/3	1/3	-2/3	0	-2/3	2/3	≥	-5/3
φ_{AB}^+	0	0	0	0	0	0	-1	0	≥	-2
X	0	0	1	-1	0	1	1	0	≥	-1
φ_{BA}^+	-1	0	0	0	0	0	0	0	≥	-2
S	0	1	1	-1	0	1	0	0	≥	-1
φ_{CD}^+	0	0	0	0	-1	0	0	0	≥	-4
Z	0	0	1	-1	1	1	0	0	≥	0
φ_{DC}^+	0	0	-1	0	0	0	0	0	≥	-4
Y	1	0	1	-1	0	1	0	0	≥	-1
φ_{EF}^+	0	-1	0	0	0	0	0	0	≥	-2
αbF	-1/9	-1/9	-1/9	-1/9	-1/9	0	-1/9	-2/9	≥	-19/9
φ_{FE}^+	2/3	2/3	2/3	-1/3	2/3	0	2/3	-2/3	≥	-1/3
φ_{GD}^-	-2/9	5/18	7/9	-2/9	5/18	0	5/18	5/9	≥	-2/9
φ_{GD}^+	2/9	-5/18	-7/9	2/9	-5/18	0	-5/18	-5/9	≥	-70/9
φ_{DG}^-	0	0	-1	1	0	0	0	0	≥	-2
φ_{DG}^+	0	0	1	-1	0	0	0	0	≥	-6
φ_{DH}^-	0	0	0	-1	0	0	0	0	≥	-8
T	0	0	2	-1	0	1	0	0	≥	0
X-	0	0	1	-1	0	1	0	0	≥	-2
φ_{HF}^+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	-1	1	0	-1	0	0	≥	-2
Max	-1/9	-1/9	-1/9	-1/9	-1/9	0	-1/9	-2/9	=	-19/9

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Tableau finale

	φ_{BA}^-	φ_{EF}^-	φ_{DC}^-	φ_{DH}^+	φ_{CD}^-	R	φ_{AB}^-	φ_{HF}^-		Fb
φ_{FE}^-	-2/3	-2/3	-2/3	1/3	-2/3	0	-2/3	2/3	≥	-5/3
φ_{AB}^+	0	0	0	0	0	0	-1	0	≥	-2
X	0	0	1	-1	0	1	1	0	≥	-1
φ_{BA}^+	-1	0	0	0	0	0	0	0	≥	-2
S	0	1	1	-1	0	1	0	0	≥	-1
φ_{CD}^+	0	0	0	0	-1	0	0	0	≥	-4
Z	0	0	1	-1	1	1	0	0	≥	0
φ_{DC}^+	0	0	-1	0	0	0	0	0	≥	-4
Y	1	0	1	-1	0	1	0	0	≥	-1
φ_{EF}^+	0	-1	0	0	0	0	0	0	≥	-2
αbF	-1/9	-1/9	-1/9	-1/9	-1/9	0	-1/9	-2/9	≥	-19/9
φ_{FE}^+	2/3	2/3	2/3	-1/3	2/3	0	2/3	-2/3	≥	-1/3
φ_{GD}^-	-2/9	5/18	7/9	-2/9	5/18	0	5/18	5/9	≥	-2/9
φ_{GD}^+	2/9	-5/18	-7/9	2/9	-5/18	0	-5/18	-5/9	≥	-70/9
φ_{DG}^-	0	0	-1	1	0	0	0	0	≥	-2
φ_{DG}^+	0	0	1	-1	0	0	0	0	≥	-6
φ_{DH}^-	0	0	0	-1	0	0	0	0	≥	-8
T	0	0	2	-1	0	1	0	0	≥	0
X-	0	0	1	-1	0	1	0	0	≥	-2
φ_{HF}^+	0	0	0	0	0	0	0	-1	≥	-8
L_X	0	0	-1	1	0	-1	0	0	≥	-2
Max	-1/9	-1/9	-1/9	-1/9	-1/9	0	-1/9	-2/9	=	-19/9

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Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	1/9
φ_{AB}+	0	0	0	0	0	0	0	0	≥	0
φ_{BA}-	0	0	0	0	0	0	0	0	≥	1/9
φ_{BA}+	0	0	0	0	0	0	0	0	≥	0
φ_{CD}-	0	0	0	0	0	0	0	0	≥	1/9
φ_{CD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DC}-	0	0	0	0	0	0	0	0	≥	1/9
φ_{DC}+	0	0	0	0	0	0	0	0	≥	0
φ_{EF}-	0	0	0	0	0	0	0	0	≥	1/9
φ_{EF}+	0	0	0	0	0	0	0	0	≥	0
φ_{FE}-	0	0	0	0	0	0	0	0	≥	0
φ_{FE}+	0	0	0	0	0	0	0	0	≥	0
φ_{GD}-	0	0	0	0	0	0	0	0	≥	0
φ_{GD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DG}-	0	0	0	0	0	0	0	0	≥	0
φ_{DG}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	1/9
φ_{HF}-	0	0	0	0	0	0	0	0	≥	2/9
φ_{HF}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	1	1	0	0	1	0	19/9	2	=	-19/9

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
-1	-1	-2	-2	-1	-2

Variabili soluzione differenza tra rotazioni

φ_{AB}	-1/9
φ_{BA}	-1/9
φ_{CD}	-1/9
φ_{DC}	-1/9
φ_{EF}	-1/9
φ_{FE}	0
φ_{GD}	0
φ_{DG}	0
φ_{DH}	1/9
φ_{HF}	-2/9

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REAZIONI Fattore di collasso = 19/9

$H_A = 2F$
$V_A = 43/9F$
$W_A = -Fb$
$H_C = 4F$
$V_C = 124/9F$
$W_C = -2Fb$
$H_E = 1/3F$
$V_E = 14/3F$
$W_E = -Fb$

$H_{AB} = 2F$	$H_{CD} = 4F$	$H_{EF} = 1/3F$	$H_{BG} = 2F$	$H_{GD} = 2F$
$V_{AB} = 43/9F$	$V_{CD} = 124/9F$	$V_{EF} = 14/3F$	$V_{BG} = 43/9F$	$V_{GD} = -52/9F$
$W_{AB} = -Fb$	$W_{CD} = -2Fb$	$W_{EF} = -Fb$	$W_{BG} = Fb$	$W_{GD} = -34/9Fb$
$H_{BA} = -2F$	$H_{DC} = -4F$	$H_{FE} = -1/3F$	$H_{GB} = -2F$	$H_{DG} = -2F$
$V_{BA} = -43/9F$	$V_{DC} = -124/9F$	$V_{FE} = -14/3F$	$V_{GB} = -43/9F$	$V_{DG} = 52/9F$
$W_{BA} = -Fb$	$W_{DC} = -2Fb$	$W_{FE} = 2/3Fb$	$W_{GB} = 34/9Fb$	$W_{DG} = -2Fb$

$H_{DH} = 6F$	$H_{HF} = 6F$
$V_{DH} = 8F$	$V_{HF} = -14/3F$
$W_{DH} = 4Fb$	$W_{HF} = -4Fb$
$H_{HD} = -6F$	$H_{FH} = -6F$
$V_{HD} = -8F$	$V_{FH} = 14/3F$
$W_{HD} = 4Fb$	$W_{FH} = -2/3Fb$

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_{BBA} = -1/9\delta$	$u_{CCD} = 0$	$u_{DDC} = -1/9\delta$	$u_{EEF} = 0$	$u_F = -1/9\delta$
$v_{AAB} = 0$	$v_{BBA} = 0$	$v_{CCD} = 0$	$v_{DDC} = 0$	$v_{EEF} = 0$	$v_F = 0$
$\varphi_{AAB} = 1/9\delta/b$	$\varphi_{BBA} = 1/9\delta/b$	$\varphi_{CCD} = 1/9\delta/b$	$\varphi_{DDC} = 1/9\delta/b$	$\varphi_{EEF} = 1/9\delta/b$	$\varphi_F = 1/9\delta/b$

$u_G = -1/9\delta$	$u_{HHD} = -1/9\delta$
$v_G = 0$	$v_{HHD} = -1/9\delta$
$\varphi_G = 0$	$\varphi_{HHD} = -1/9\delta/b$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$	$u_{CCD} = 0$	$u_{EEF} = 0$	$u_{BBG} = -1/9\delta$	$u_{GGD} = -1/9\delta$	$u_{DDH} = -1/9\delta$
$v_{AAB} = 0$	$v_{CCD} = 0$	$v_{EEF} = 0$	$v_{BBG} = 0$	$v_{GGD} = 0$	$v_{DDH} = 0$
$\varphi_{AAB} = 1/9\delta/b$	$\varphi_{CCD} = 1/9\delta/b$	$\varphi_{EEF} = 1/9\delta/b$	$\varphi_{BBG} = 0$	$\varphi_{GGD} = 0$	$\varphi_{DDH} = -1/9\delta/b$

$u_{HHF} = -1/9\delta$
$v_{HHF} = -1/9\delta$
$\varphi_{HHF} = 1/9\delta/b$

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AL5.009

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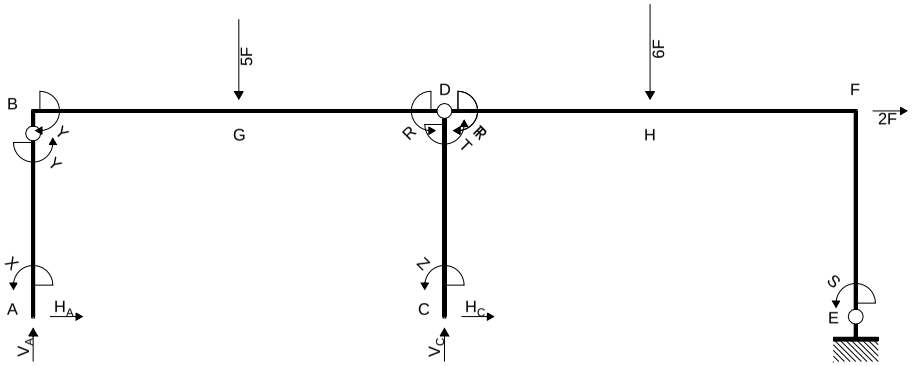


EQUAZIONI DI EQUILIBRIO

Rotazione intorno a E: aste EF FH HD DC DG GB BA

$-4V_Ab -2V_Cb = -Xb -Zb -Sb -19Fb$

Rotazione intorno a D: aste DC

$H_Cb = -Zb -Tb$

Rotazione intorno a D: aste DG GB BA

$H_Ab -2V_Ab = -Xb -Rb -5Fb$

Rotazione intorno a B: aste BA

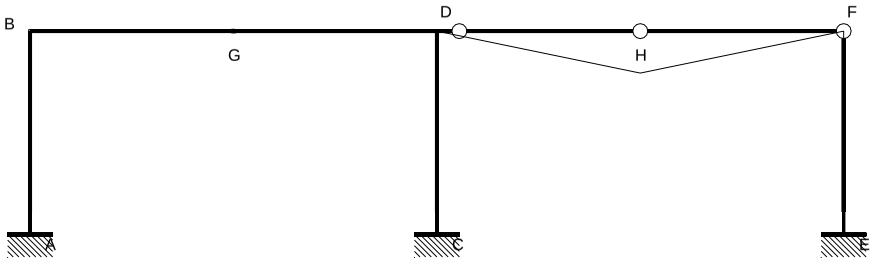
$H_Ab = -Xb -Yb$

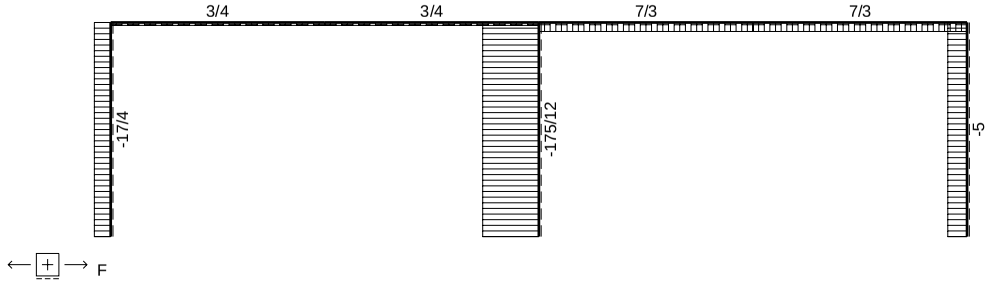
Matrice di equilibrio

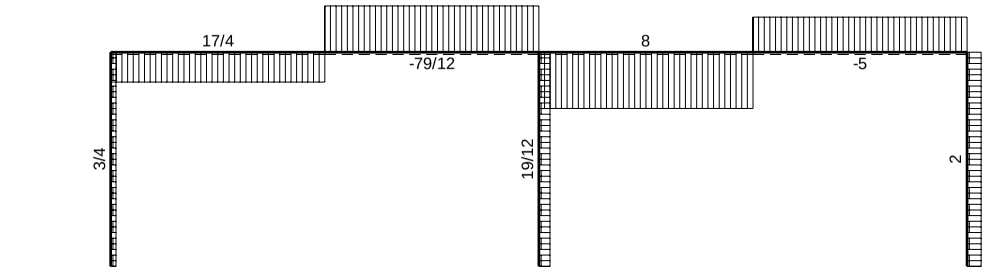
$$\begin{array}{c} \\ \varphi_{EF} \\ \varphi_{DC} \\ \varphi_{DG} \\ \varphi_{BA} \end{array} \begin{bmatrix} H_Ab & V_Ab & H_Cb & V_Cb \\ 0 & -4 & 0 & -2 \\ 0 & 0 & 1 & 0 \\ 1 & -2 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ -1 & 0 & -1 & 0 & -1 & 0 & -19 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & -1 & -5 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

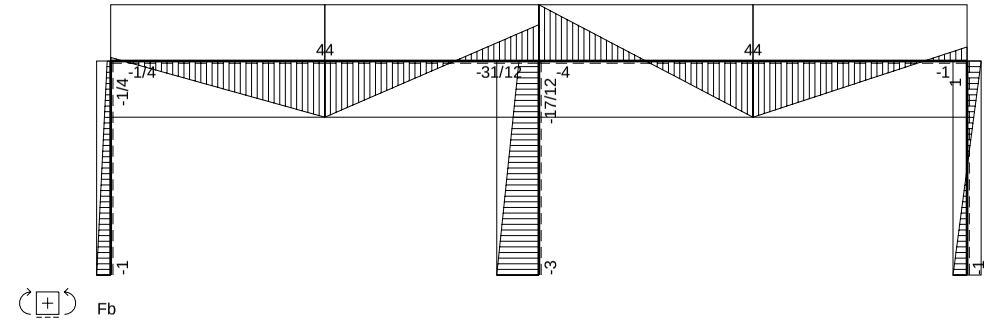
Soluzione del sistema

$$\begin{bmatrix} V_Ab \\ H_Cb \\ H_Ab \\ V_Cb \end{bmatrix} = \begin{bmatrix} Xb & Yb & Zb & Tb & Sb & Rb & Fb \\ 0 & -1/2 & 0 & 0 & 0 & 1/2 & 5/2 \\ 0 & 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 1 & 1/2 & 0 & 1/2 & -1 & 9/2 \end{bmatrix}$$









PROGRAMMAZIONE LINEARE

Sia H_{ij} la matrice del simplesso, con m righe e n colonne.
Siano P_j le variabili primali di riga e D_i le variabili duali di colonna, con $1 \le j < n$, $1 \le i < m$.
Siano a riga m i coefficienti della funzione obiettivo primale $max\ \Sigma_j H_{mj} P_j$, $1 \le j < n$.
Siano a colonna n i coefficienti della funzione obiettivo duale $min\ \Sigma_i H_{in} D_i$, $1 \le i < m$.
Sequenza di operazioni pivotali:
1 Sia q $(1 \le q < n)$ la colonna pivot con massimo valore H_{mj} in riga m.
2 Sia p $(1 \le p < m)$ la riga pivot di colonna q, a coefficiente negativo H_{iq}, che minimizza il rapporto H_{in}/H_{iq}.
3 Si ottiene il coefficiente pivotale H_{pq}.
4 Si scambia la variabile primale P_q con la duale D_p.
5 Si ridefinisce il coefficiente pivotale $H_{pq} = 1/H_{pq}$.
6 Si ridefiniscono i coefficienti della colonna pivot q: $H_{iq} = H_{pq} H_{iq}$, escluso il pivot H_{pq}.
7 Si ridefiniscono tutti i coefficienti della matrice, esclusa la riga p e la colonna q: $H_{ij} = H_{ij} - H_{iq} H_{pj}$.
8 Si ridefiniscono i coefficienti della riga pivot p: $H_{pj} = - H_{pq} H_{pj}$, escluso il pivot H_{pq}.
Si ripete il ciclo 1-8 sino a quando la funzione obiettivo di riga m ha solo coefficienti non-positivi.
Giunti a questo punto, si individua la soluzione.
Si hanno gli elementi non nulli del vettore soluzione primale, con segno cambiato, sulla colonna n dei termini noti, in corrispondenza delle variabili P_j presenti sulla colonna di sinistra.
Si hanno gli elementi non nulli del vettore soluzione duale, con segno cambiato, sulla riga m della funzione obiettivo, in corrispondenza delle variabili D_i presenti sulla colonna superiore.

Programmazione lineare $m=6, n=4$

	P_1	P_2	P_3		MIN
D_1	H_{11}	H_{12}	H_{13}	$\ge$	H_{14}
D_2	H_{21}	H_{22}	H_{23}	$\ge$	H_{24}
D_3	H_{31}	H_{32}	H_{33}	$\ge$	H_{34}
D_4	H_{41}	H_{42}	H_{43}	$\ge$	H_{44}
D_5	H_{51}	H_{52}	H_{53}	$\ge$	H_{54}
MAX	H_{61}	H_{62}	H_{63}	=	H_{64}

SOLUZIONE DEL SIMPLESSO $X=W_{AB}$ $Y=W_{BA}$ $Z=W_{CD}$ $T=W_{DC}$ $S=W_{EF}$ $R=W_{DG}$

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	1	0	0	0	0	0	0	≤	1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	1	0	0	0	0	0	≤	1
W_{CD}-	0	0	1	0	0	0	0	≥	-3
W_{CD}+	0	0	1	0	0	0	0	≤	3
W_{DC}-	0	0	0	1	0	0	0	≥	-3
W_{DC}+	0	0	0	1	0	0	0	≤	3
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	1	0	0	≤	1
W_{FE}-	-1	-1	-1	-1	-1	0	2	≥	-1
W_{FE}+	-1	-1	-1	-1	-1	0	2	≤	1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	-1/2	0	0	0	-1/2	-5/2	≤	4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	1	0	≤	4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	-1	0	-1	0	≤	4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-2	≥	-4
W_{HF}+	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-2	≤	4
Max	0	0	0	0	0	0	1	=	0

Tableau con variabili non vincolate in segno

	X	Y	Z	T	S	R	αbF		Fb
W_{AB}-	1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	0	≥	-1
W_{BA}-	0	1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	≥	-1
W_{CD}-	0	0	1	0	0	0	0	≥	-3
W_{CD}+	0	0	-1	0	0	0	0	≥	-3
W_{DC}-	0	0	0	1	0	0	0	≥	-3
W_{DC}+	0	0	0	-1	0	0	0	≥	-3
W_{EF}-	0	0	0	0	1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	≥	-1
W_{FE}-	-1	-1	-1	-1	-1	0	2	≥	-1
W_{FE}+	1	1	1	1	1	0	-2	≥	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	5/2	≥	-4
W_{DG}-	0	0	0	0	0	1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	≥	-4
W_{DH}-	0	0	0	-1	0	-1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	≥	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-2	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	2	≥	-4
Max	0	0	0	0	0	0	1	=	0

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Tableau con variabili vincolate in segno

	X+	Y+	Z+	T+	S+	R+	X-	Y-	Z-	T-	S-	R-	αbF		Fb
W_{AB}-	1	0	0	0	0	0	-1	0	0	0	0	0	0	≥	-1
W_{AB}+	-1	0	0	0	0	0	1	0	0	0	0	0	0	≤	-1
W_{BA}-	0	1	0	0	0	0	0	-1	0	0	0	0	0	≥	-1
W_{BA}+	0	-1	0	0	0	0	0	1	0	0	0	0	0	≤	-1
W_{CD}-	0	0	1	0	0	0	0	0	-1	0	0	0	0	≥	-3
W_{CD}+	0	0	-1	0	0	0	0	0	1	0	0	0	0	≤	-3
W_{DC}-	0	0	0	1	0	0	0	0	0	-1	0	0	0	≥	-3
W_{DC}+	0	0	0	-1	0	0	0	0	0	1	0	0	0	≤	-3
W_{EF}-	0	0	0	0	1	0	0	0	0	0	-1	0	0	≥	-1
W_{EF}+	0	0	0	0	-1	0	0	0	0	0	1	0	0	≤	-1
W_{FE}-	-1	-1	-1	-1	-1	0	1	1	1	1	1	0	2	≥	-1
W_{FE}+	1	1	1	1	1	0	-1	-1	-1	-1	-1	0	-2	≤	-1
W_{GD}-	0	-1/2	0	0	0	-1/2	0	1/2	0	0	0	1/2	-5/2	≥	-4
W_{GD}+	0	1/2	0	0	0	1/2	0	-1/2	0	0	0	-1/2	5/2	≤	-4
W_{DG}-	0	0	0	0	0	1	0	0	0	0	0	-1	0	≥	-4
W_{DG}+	0	0	0	0	0	-1	0	0	0	0	0	1	0	≤	-4
W_{DH}-	0	0	0	-1	0	-1	0	0	0	1	0	1	0	≥	-4
W_{DH}+	0	0	0	1	0	1	0	0	0	-1	0	-1	0	≤	-4
W_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	1/2	1/2	1/2	1	1/2	1/2	-2	≥	-4
W_{HF}+	1/2	1/2	1/2	1	1/2	1/2	-1/2	-1/2	-1/2	-1	-1/2	-1/2	2	≤	-4
Max	0	0	0	0	0	0	0	0	0	0	0	0	1	=	0

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Tableau a variabili negative su X- e limitate

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-3
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-3
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	-1	-1	-1	-1	-1	0	2	5	≥	-1
φ_{FE}+	1	1	1	1	1	0	-2	-5	≥	-1
φ_{GD}-	0	-1/2	0	0	0	-1/2	-5/2	1	≥	-4
φ_{GD}+	0	1/2	0	0	0	1/2	5/2	-1	≥	-4
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	-1/2	-1/2	-1/2	-1	-1/2	-1/2	-2	7/2	≥	-4
φ_{HF}+	1/2	1/2	1/2	1	1/2	1/2	2	-7/2	≥	-4
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	0	0	0	0	0	0	1	0	=	0

Scambio pivotale 12-7

	X	Y	Z	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	1	0	0	0	0	0	0	-1	≥	-1
φ_{AB}+	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-3
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-3
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	1/2	1/2	1/2	1/2	1/2	0	-1/2	-5/2	≥	-1/2
φ_{GD}-	-5/4	-7/4	-5/4	-5/4	-5/4	-1/2	5/4	29/4	≥	-11/4
φ_{GD}+	5/4	7/4	5/4	5/4	5/4	1/2	-5/4	-29/4	≥	-21/4
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	-3/2	-3/2	-3/2	-2	-3/2	-1/2	1	17/2	≥	-3
φ_{HF}+	3/2	3/2	3/2	2	3/2	1/2	-1	-17/2	≥	-5
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	1/2	1/2	1/2	1/2	1/2	0	-1/2	-5/2	=	-1/2

Scambio pivotale 2-1

	φ_{AB}+	Y	Z	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	1	0	0	0	0	0	-1	≥	-1
φ_{BA}+	0	-1	0	0	0	0	0	1	≥	-1
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-3
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-3
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/2	1/2	1/2	1/2	1/2	0	-1/2	-2	≥	-1
φ_{GD}-	5/4	-7/4	-5/4	-5/4	-5/4	-1/2	5/4	6	≥	-3/2
φ_{GD}+	-5/4	7/4	5/4	5/4	5/4	1/2	-5/4	-6	≥	-13/2
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	3/2	-3/2	-3/2	-2	-3/2	-1/2	1	7	≥	-3/2
φ_{HF}+	-3/2	3/2	3/2	2	3/2	1/2	-1	-7	≥	-13/2
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-1/2	1/2	1/2	1/2	1/2	0	-1/2	-2	=	-1

PROCEDIMENTO E RISULTATI Nome: AL5.010

Scambio pivotale 13-2

	φ_{AB}+	φ_{GD}-	Z	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	5/7	-4/7	-5/7	-5/7	-5/7	-2/7	5/7	17/7	≥	-13/7
φ_{BA}+	-5/7	4/7	5/7	5/7	5/7	2/7	-5/7	-17/7	≥	-1/7
φ_{CD}-	0	0	1	0	0	0	0	-1	≥	-3
φ_{CD}+	0	0	-1	0	0	0	0	1	≥	-3
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/7	-2/7	1/7	1/7	1/7	-1/7	-1/7	-2/7	≥	-10/7
Y	5/7	-4/7	-5/7	-5/7	-5/7	-2/7	5/7	24/7	≥	-6/7
φ_{GD}+	0	-1	0	0	0	0	0	0	≥	-8
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
φ_{HF}-	3/7	6/7	-3/7	-13/14	-3/7	-1/14	-1/14	13/7	≥	-3/14
φ_{HF}+	-3/7	-6/7	3/7	13/14	3/7	1/14	1/14	-13/7	≥	-109/14
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	-1/7	-2/7	1/7	1/7	1/7	-1/7	-1/7	-2/7	=	-10/7

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Scambio pivotale 19-3

	φ_{AB}+	φ_{GD}-	φ_{HF}-	T	S	R	φ_{FE}+	X-		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1	0	0	0	0	0	0	1	≥	-1
φ_{BA}-	0	-2	5/3	5/6	0	-1/6	5/6	-2/3	≥	-3/2
φ_{BA}+	0	2	-5/3	-5/6	0	1/6	-5/6	2/3	≥	-1/2
φ_{CD}-	1	2	-7/3	-13/6	-1	-1/6	-1/6	10/3	≥	-7/2
φ_{CD}+	-1	-2	7/3	13/6	1	1/6	1/6	-10/3	≥	-5/2
φ_{DC}-	0	0	0	1	0	0	0	-1	≥	-3
φ_{DC}+	0	0	0	-1	0	0	0	1	≥	-3
φ_{EF}-	0	0	0	0	1	0	0	-1	≥	-1
φ_{EF}+	0	0	0	0	-1	0	0	1	≥	-1
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	0	0	-1/3	-1/6	0	-1/6	-1/6	1/3	≥	-3/2
Y	0	-2	5/3	5/6	0	-1/6	5/6	1/3	≥	-1/2
φ_{GD}+	0	-1	0	0	0	0	0	0	≥	-8
φ_{DG}-	0	0	0	0	0	1	0	-1	≥	-4
φ_{DG}+	0	0	0	0	0	-1	0	1	≥	-4
φ_{DH}-	0	0	0	-1	0	-1	0	2	≥	-4
φ_{DH}+	0	0	0	1	0	1	0	-2	≥	-4
Z	1	2	-7/3	-13/6	-1	-1/6	-1/6	13/3	≥	-1/2
φ_{HF}+	0	0	-1	0	0	0	0	0	≥	-8
L_X	0	0	0	0	0	0	0	-1	≥	-4
Max	0	0	-1/3	-1/6	0	-1/6	-1/6	1/3	=	-3/2

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Scambio pivotale 6-8

	φ_{AB}+	φ_{GD}-	φ_{HF}-	T	S	R	φ_{FE}+	φ_{CD}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-13/10	-3/5	7/10	13/20	3/10	1/20	1/20	-3/10	≥	-7/4
φ_{BA}-	1/5	-8/5	6/5	2/5	-1/5	-1/5	4/5	1/5	≥	-1
φ_{BA}+	-1/5	8/5	-6/5	-2/5	1/5	1/5	-4/5	-1/5	≥	-1
φ_{CD}-	0	0	0	0	0	0	0	-1	≥	-6
X-	-3/10	-3/5	7/10	13/20	3/10	1/20	1/20	-3/10	≥	-3/4
φ_{DC}-	3/10	3/5	-7/10	7/20	-3/10	-1/20	-1/20	3/10	≥	-9/4
φ_{DC}+	-3/10	-3/5	7/10	-7/20	3/10	1/20	1/20	-3/10	≥	-15/4
φ_{EF}-	3/10	3/5	-7/10	-13/20	7/10	-1/20	-1/20	3/10	≥	-1/4
φ_{EF}+	-3/10	-3/5	7/10	13/20	-7/10	1/20	1/20	-3/10	≥	-7/4
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/10	-1/5	-1/10	1/20	1/10	-3/20	-3/20	-1/10	≥	-7/4
Y	-1/10	-11/5	19/10	21/20	1/10	-3/20	17/20	-1/10	≥	-3/4
φ_{GD}+	0	-1	0	0	0	0	0	0	≥	-8
φ_{DG}-	3/10	3/5	-7/10	-13/20	-3/10	19/20	-1/20	3/10	≥	-13/4
φ_{DG}+	-3/10	-3/5	7/10	13/20	3/10	-19/20	1/20	-3/10	≥	-19/4
φ_{DH}-	-3/5	-6/5	7/5	3/10	3/5	-9/10	1/10	-3/5	≥	-11/2
φ_{DH}+	3/5	6/5	-7/5	-3/10	-3/5	9/10	-1/10	3/5	≥	-5/2
Z	-3/10	-3/5	7/10	13/20	3/10	1/20	1/20	-13/10	≥	-15/4
φ_{HF}+	0	0	-1	0	0	0	0	0	≥	-8
L_X	3/10	3/5	-7/10	-13/20	-3/10	-1/20	-1/20	3/10	≥	-13/4
Max	-1/10	-1/5	-1/10	1/20	1/10	-3/20	-3/20	-1/10	=	-7/4

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Scambio pivotale 10-5

	φ_{AB}+	φ_{GD}-	φ_{HF}-	T	φ_{EF}+	R	φ_{FE}+	φ_{CD}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-10/7	-6/7	1	13/14	-3/7	1/14	1/14	-3/7	≥	-5/2
φ_{BA}-	2/7	-10/7	1	3/14	2/7	-3/14	11/14	2/7	≥	-1/2
φ_{BA}+	-2/7	10/7	-1	-3/14	-2/7	3/14	-11/14	-2/7	≥	-3/2
φ_{CD}-	0	0	0	0	0	0	0	-1	≥	-6
X-	-3/7	-6/7	1	13/14	-3/7	1/14	1/14	-3/7	≥	-3/2
φ_{DC}-	3/7	6/7	-1	1/14	3/7	-1/14	-1/14	3/7	≥	-3/2
φ_{DC}+	-3/7	-6/7	1	-1/14	-3/7	1/14	1/14	-3/7	≥	-9/2
φ_{EF}-	0	0	0	0	-1	0	0	0	≥	-2
S	-3/7	-6/7	1	13/14	-10/7	1/14	1/14	-3/7	≥	-5/2
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	-1/7	-2/7	0	1/7	-1/7	-1/7	-1/7	-1/7	≥	-2
Y	-1/7	-16/7	2	8/7	-1/7	-1/7	6/7	-1/7	≥	-1
φ_{GD}+	0	-1	0	0	0	0	0	0	≥	-8
φ_{DG}-	3/7	6/7	-1	-13/14	3/7	13/14	-1/14	3/7	≥	-5/2
φ_{DG}+	-3/7	-6/7	1	13/14	-3/7	-13/14	1/14	-3/7	≥	-11/2
φ_{DH}-	-6/7	-12/7	2	6/7	-6/7	-6/7	1/7	-6/7	≥	-7
φ_{DH}+	6/7	12/7	-2	-6/7	6/7	6/7	-1/7	6/7	≥	-1
Z	-3/7	-6/7	1	13/14	-3/7	1/14	1/14	-10/7	≥	-9/2
φ_{HF}+	0	0	-1	0	0	0	0	0	≥	-8
L_X	3/7	6/7	-1	-13/14	3/7	-1/14	-1/14	3/7	≥	-5/2
Max	-1/7	-2/7	0	1/7	-1/7	-1/7	-1/7	-1/7	=	-2

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Scambio pivotale 18-4

	φ_{AB}+	φ_{GD}-	φ_{HF}-	φ_{DH}+	φ_{EF}+	R	φ_{FE}+	φ_{CD}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1/2	1	-7/6	-13/12	1/2	1	-1/12	1/2	≥	-43/12
φ_{BA}-	1/2	-1	1/2	-1/4	1/2	0	3/4	1/2	≥	-3/4
φ_{BA}+	-1/2	1	-1/2	1/4	-1/2	0	-3/4	-1/2	≥	-5/4
φ_{CD}-	0	0	0	0	0	0	0	-1	≥	-6
X-	1/2	1	-7/6	-13/12	1/2	1	-1/12	1/2	≥	-31/12
φ_{DC}-	1/2	1	-7/6	-1/12	1/2	0	-1/12	1/2	≥	-19/12
φ_{DC}+	-1/2	-1	7/6	1/12	-1/2	0	1/12	-1/2	≥	-53/12
φ_{EF}-	0	0	0	0	-1	0	0	0	≥	-2
S	1/2	1	-7/6	-13/12	-1/2	1	-1/12	1/2	≥	-43/12
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	0	0	-1/3	-1/6	0	0	-1/6	0	≥	-13/6
Y	1	0	-2/3	-4/3	1	1	2/3	1	≥	-7/3
φ_{GD}+	0	-1	0	0	0	0	0	0	≥	-8
φ_{DG}-	-1/2	-1	7/6	13/12	-1/2	0	1/12	-1/2	≥	-17/12
φ_{DG}+	1/2	1	-7/6	-13/12	1/2	0	-1/12	1/2	≥	-79/12
φ_{DH}-	0	0	0	-1	0	0	0	0	≥	-8
T	1	2	-7/3	-7/6	1	1	-1/6	1	≥	-7/6
Z	1/2	1	-7/6	-13/12	1/2	1	-1/12	-1/2	≥	-67/12
φ_{HF}+	0	0	-1	0	0	0	0	0	≥	-8
L_X	-1/2	-1	7/6	13/12	-1/2	-1	1/12	-1/2	≥	-17/12
Max	0	0	-1/3	-1/6	0	0	-1/6	0	=	-13/6

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Tableau finale

	φ_{AB}+	φ_{GD}-	φ_{HF}-	φ_{DH}+	φ_{EF}+	R	φ_{FE}+	φ_{CD}+		Fb
φ_{AB}-	-1	0	0	0	0	0	0	0	≥	-2
X	-1/2	1	-7/6	-13/12	1/2	1	-1/12	1/2	≥	-43/12
φ_{BA}-	1/2	-1	1/2	-1/4	1/2	0	3/4	1/2	≥	-3/4
φ_{BA}+	-1/2	1	-1/2	1/4	-1/2	0	-3/4	-1/2	≥	-5/4
φ_{CD}-	0	0	0	0	0	0	0	-1	≥	-6
X-	1/2	1	-7/6	-13/12	1/2	1	-1/12	1/2	≥	-31/12
φ_{DC}-	1/2	1	-7/6	-1/12	1/2	0	-1/12	1/2	≥	-19/12
φ_{DC}+	-1/2	-1	7/6	1/12	-1/2	0	1/12	-1/2	≥	-53/12
φ_{EF}-	0	0	0	0	-1	0	0	0	≥	-2
S	1/2	1	-7/6	-13/12	-1/2	1	-1/12	1/2	≥	-43/12
φ_{FE}-	0	0	0	0	0	0	-1	0	≥	-2
αbF	0	0	-1/3	-1/6	0	0	-1/6	0	≥	-13/6
Y	1	0	-2/3	-4/3	1	1	2/3	1	≥	-7/3
φ_{GD}+	0	-1	0	0	0	0	0	0	≥	-8
φ_{DG}-	-1/2	-1	7/6	13/12	-1/2	0	1/12	-1/2	≥	-17/12
φ_{DG}+	1/2	1	-7/6	-13/12	1/2	0	-1/12	1/2	≥	-79/12
φ_{DH}-	0	0	0	-1	0	0	0	0	≥	-8
T	1	2	-7/3	-7/6	1	1	-1/6	1	≥	-7/6
Z	1/2	1	-7/6	-13/12	1/2	1	-1/12	-1/2	≥	-67/12
φ_{HF}+	0	0	-1	0	0	0	0	0	≥	-8
L_X	-1/2	-1	7/6	13/12	-1/2	-1	1/12	-1/2	≥	-17/12
Max	0	0	-1/3	-1/6	0	0	-1/6	0	=	-13/6

Vettori soluzione della programmazione lineare

	X	Y	Z	T	S	R	αbF	X-		Fb
φ_{AB}-	0	0	0	0	0	0	0	0	≥	0
φ_{AB}+	0	0	0	0	0	0	0	0	≥	0
φ_{BA}-	0	0	0	0	0	0	0	0	≥	0
φ_{BA}+	0	0	0	0	0	0	0	0	≥	0
φ_{CD}-	0	0	0	0	0	0	0	0	≥	0
φ_{CD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DC}-	0	0	0	0	0	0	0	0	≥	0
φ_{DC}+	0	0	0	0	0	0	0	0	≥	0
φ_{EF}-	0	0	0	0	0	0	0	0	≥	0
φ_{EF}+	0	0	0	0	0	0	0	0	≥	0
φ_{FE}-	0	0	0	0	0	0	0	0	≥	0
φ_{FE}+	0	0	0	0	0	0	0	0	≥	1/6
φ_{GD}-	0	0	0	0	0	0	0	0	≥	0
φ_{GD}+	0	0	0	0	0	0	0	0	≥	0
φ_{DG}-	0	0	0	0	0	0	0	0	≥	0
φ_{DG}+	0	0	0	0	0	0	0	0	≥	0
φ_{DH}-	0	0	0	0	0	0	0	0	≥	0
φ_{DH}+	0	0	0	0	0	0	0	0	≥	1/6
φ_{HF}-	0	0	0	0	0	0	0	0	≥	1/3
φ_{HF}+	0	0	0	0	0	0	0	0	≥	0
L_X	0	0	0	0	0	0	0	0	≥	0
Max	43/12	7/3	67/12	7/6	43/12	0	13/6	31/12	=	-13/6

Variabili soluzione dedotto il valore X-

X	Y	Z	T	S	R
1	-1/4	3	-17/12	1	-31/12

Variabili soluzione differenza tra rotazioni

φ_{AB}	0
φ_{BA}	0
φ_{CD}	0
φ_{DC}	0
φ_{EF}	0
φ_{FE}	1/6
φ_{GD}	0
φ_{DG}	0
φ_{DH}	1/6
φ_{HF}	-1/3

REAZIONI Fattore di collasso = 13/6

$H_A = -3/4F$
$V_A = 17/4F$
$W_A = Fb$
$H_C = -19/12F$
$V_C = 175/12F$
$W_C = 3Fb$
$H_E = -2F$
$V_E = 5F$
$W_E = Fb$

$H_{AB} = -3/4F$	$H_{CD} = -19/12F$	$H_{EF} = -2F$	$H_{BG} = -3/4F$	$H_{GD} = -3/4F$
$V_{AB} = 17/4F$	$V_{CD} = 175/12F$	$V_{EF} = 5F$	$V_{BG} = 17/4F$	$V_{GD} = -79/12F$
$W_{AB} = Fb$	$W_{CD} = 3Fb$	$W_{EF} = Fb$	$W_{BG} = 1/4Fb$	$W_{GD} = -4Fb$
$H_{BA} = 3/4F$	$H_{DC} = 19/12F$	$H_{FE} = 2F$	$H_{GB} = 3/4F$	$H_{DG} = 3/4F$
$V_{BA} = -17/4F$	$V_{DC} = -175/12F$	$V_{FE} = -5F$	$V_{GB} = -17/4F$	$V_{DG} = 79/12F$
$W_{BA} = -1/4Fb$	$W_{DC} = -17/12Fb$	$W_{FE} = Fb$	$W_{GB} = 4Fb$	$W_{DG} = -31/12Fb$

$H_{DH} = -7/3F$	$H_{HF} = -7/3F$
$V_{DH} = 8F$	$V_{HF} = -5F$
$W_{DH} = 4Fb$	$W_{HF} = -4Fb$
$H_{HD} = 7/3F$	$H_{FH} = 7/3F$
$V_{HD} = -8F$	$V_{FH} = 5F$
$W_{HD} = 4Fb$	$W_{FH} = -Fb$

SPOSTAMENTI NODALI

$u_{AAB} = 0$	$u_B = 0$	$u_{CCD} = 0$	$u_D = 0$	$u_{EEF} = 0$	$u_{FFE} = 0$
$v_{AAB} = 0$	$v_B = 0$	$v_{CCD} = 0$	$v_D = 0$	$v_{EEF} = 0$	$v_{FFE} = 0$
$\varphi_{AAB} = 0$	$\varphi_B = 0$	$\varphi_{CCD} = 0$	$\varphi_D = 0$	$\varphi_{EEF} = 0$	$\varphi_{FFH} = 0$

$u_G = 0$	$u_{HHD} = 0$
$v_G = 0$	$v_{HHD} = -1/6\delta$
$\varphi_G = 0$	$\varphi_{HHD} = -1/6\delta/b$

SPOSTAMENTI RIGIDI DELLE ASTE

$u_{AAB} = 0$	$u_{CCD} = 0$	$u_{EEF} = 0$	$u_{BBG} = 0$	$u_{GGD} = 0$	$u_{DDH} = 0$
$v_{AAB} = 0$	$v_{CCD} = 0$	$v_{EEF} = 0$	$v_{BBG} = 0$	$v_{GGD} = 0$	$v_{DDH} = 0$
$\varphi_{AAB} = 0$	$\varphi_{CCD} = 0$	$\varphi_{EEF} = 0$	$\varphi_{BBG} = 0$	$\varphi_{GGD} = 0$	$\varphi_{DDH} = -1/6\delta/b$

$u_{HHF} = 0$
$v_{HHF} = -1/6\delta$
$\varphi_{HHF} = 1/6\delta/b$

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