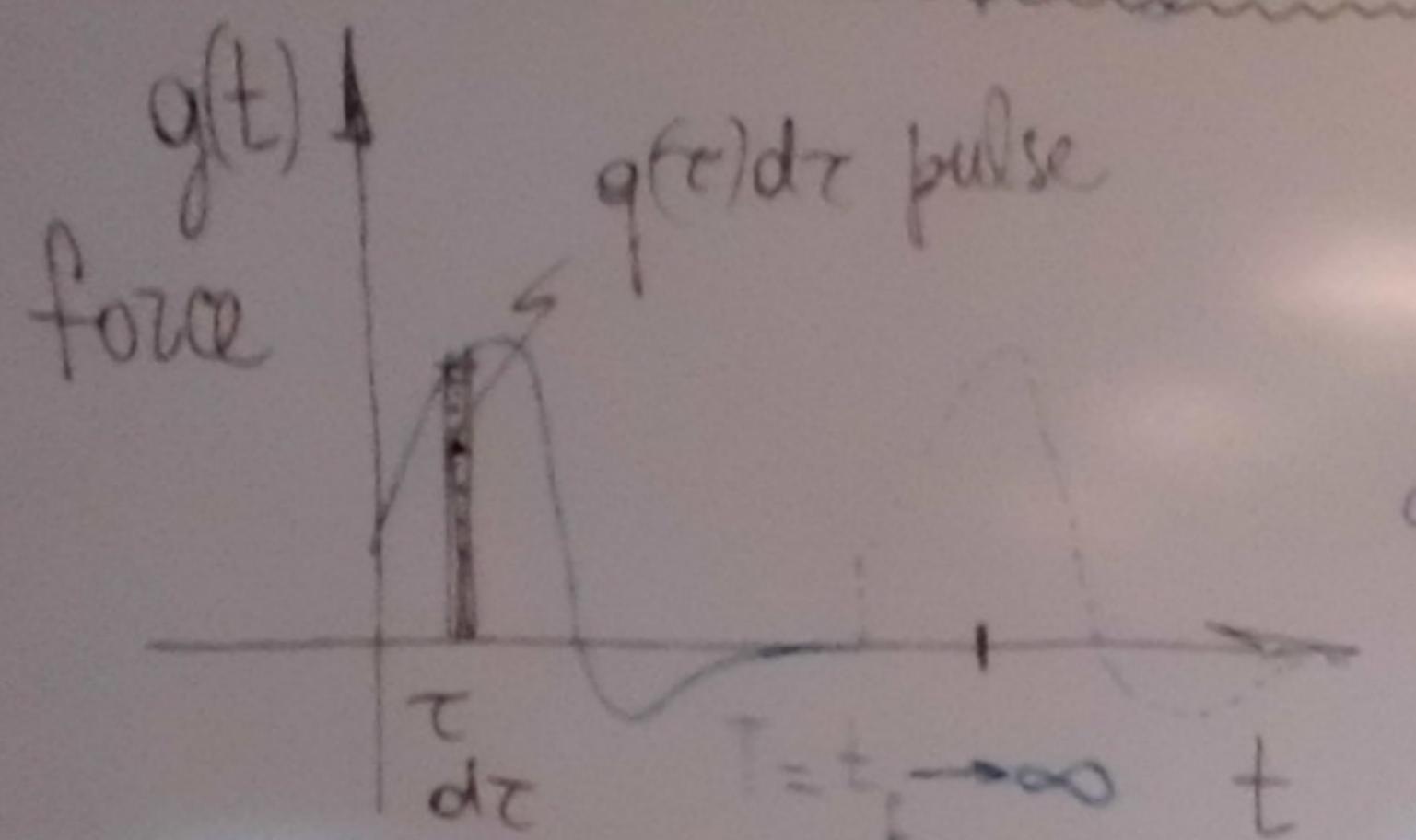


Response in the Frequency Domain



Property of Fourier transform:

$$F\left(\frac{d^n g(t)}{dt^n}\right) = (iw)^n F(g(t))$$

$$\text{based } \frac{d^n e^{iwt}}{dt^n} = (iw)^n e^{iwt}$$

$$\text{let. } iwt \cdot iwt \cdot \dots \cdot iwt \rightarrow (iw)^2 e^{iwt} \rightarrow \dots \rightarrow (iw)^n e^{iwt}$$

$$\text{thus. } \frac{d^n g(t)}{dt^n} = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{i\omega t} d\omega$$

Pearlin
force
(T)

$$\omega T = 2\pi$$

Fourier series:

$$g(t) = \sum_{n=-\infty}^{+\infty} C_n e^{i\omega_n t} = \sum_{n=-\infty}^{+\infty} \frac{\Delta\omega}{2\pi} \int_{-\infty}^{t+T/2} g(t') e^{-i\omega_n t'} dt' e^{i\omega_n t}$$

$$\omega_n = n\omega = n\frac{2\pi}{T} = n\Delta\omega$$

$$\begin{aligned} \text{discrete frequency instances} \\ \omega_n &= \frac{1}{\Delta\omega} \\ \Delta\omega &= \frac{2\pi}{T} \end{aligned}$$

frequency axis

real axis

ω

$$= \frac{1}{2\pi} \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} g(t) e^{-i\omega t} dt e^{i\omega t} dw$$

the frequency domain Frequency response function

Eq of motion:

$$F(m\ddot{u} + c\dot{u} + Ku = g(t))$$

$$[m(\omega)^2 + c\omega + K] U(\omega) = G(\omega)$$

$$H(\omega) = \frac{1}{[K - m\omega^2 + c\omega]} U(\omega) = G(\omega)$$

$$\frac{1}{K - m\omega^2 + c\omega}$$

$$H(\omega) = H(\omega) \cdot G(\omega)$$

$$U(\omega) = H(\omega) \cdot G(\omega)$$

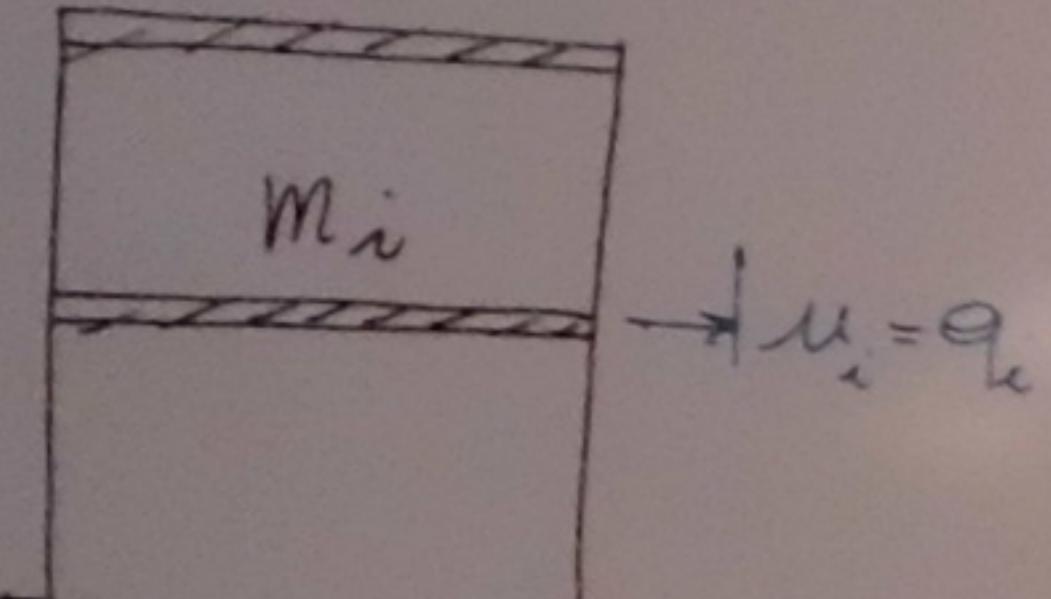
$$H(\omega) = \frac{1}{(A)}$$

$$\int g(t) dt < \infty$$

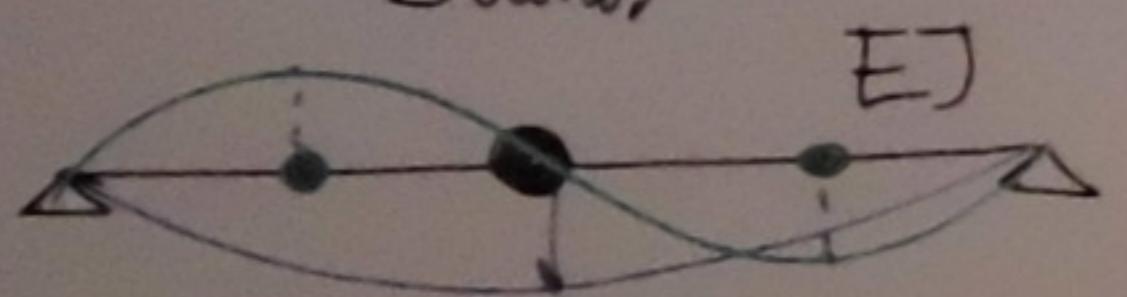
$$H(\omega) = \frac{1}{(A)}$$

$$H(\omega) = \frac{1$$

Multiple degree of freedom systems (MDOF) $\mathbf{q}(t)$



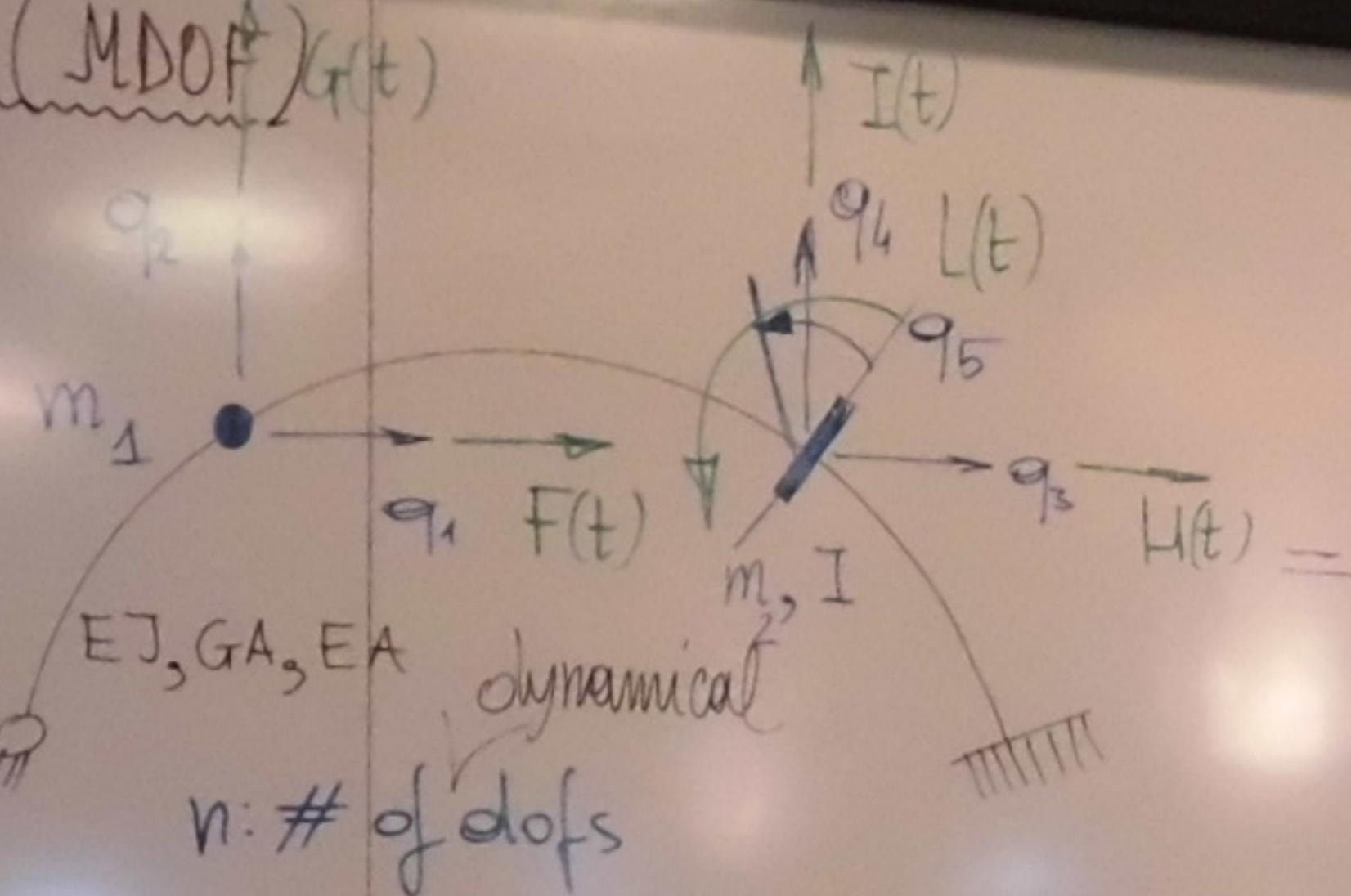
beams



Continuous system \rightarrow discrete system

discretization (with lumped masses)

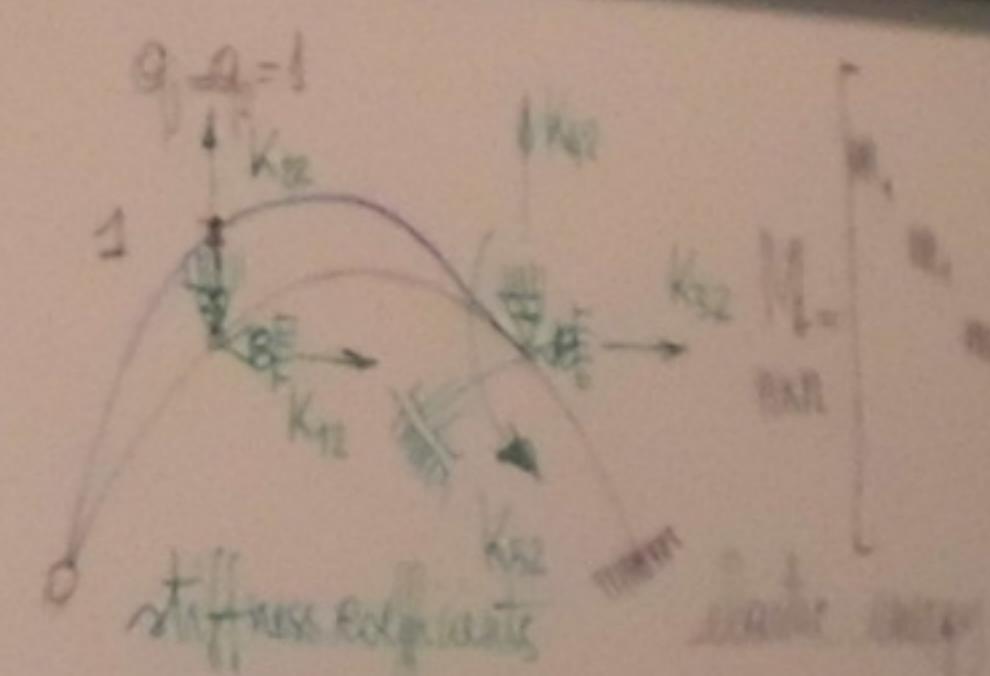
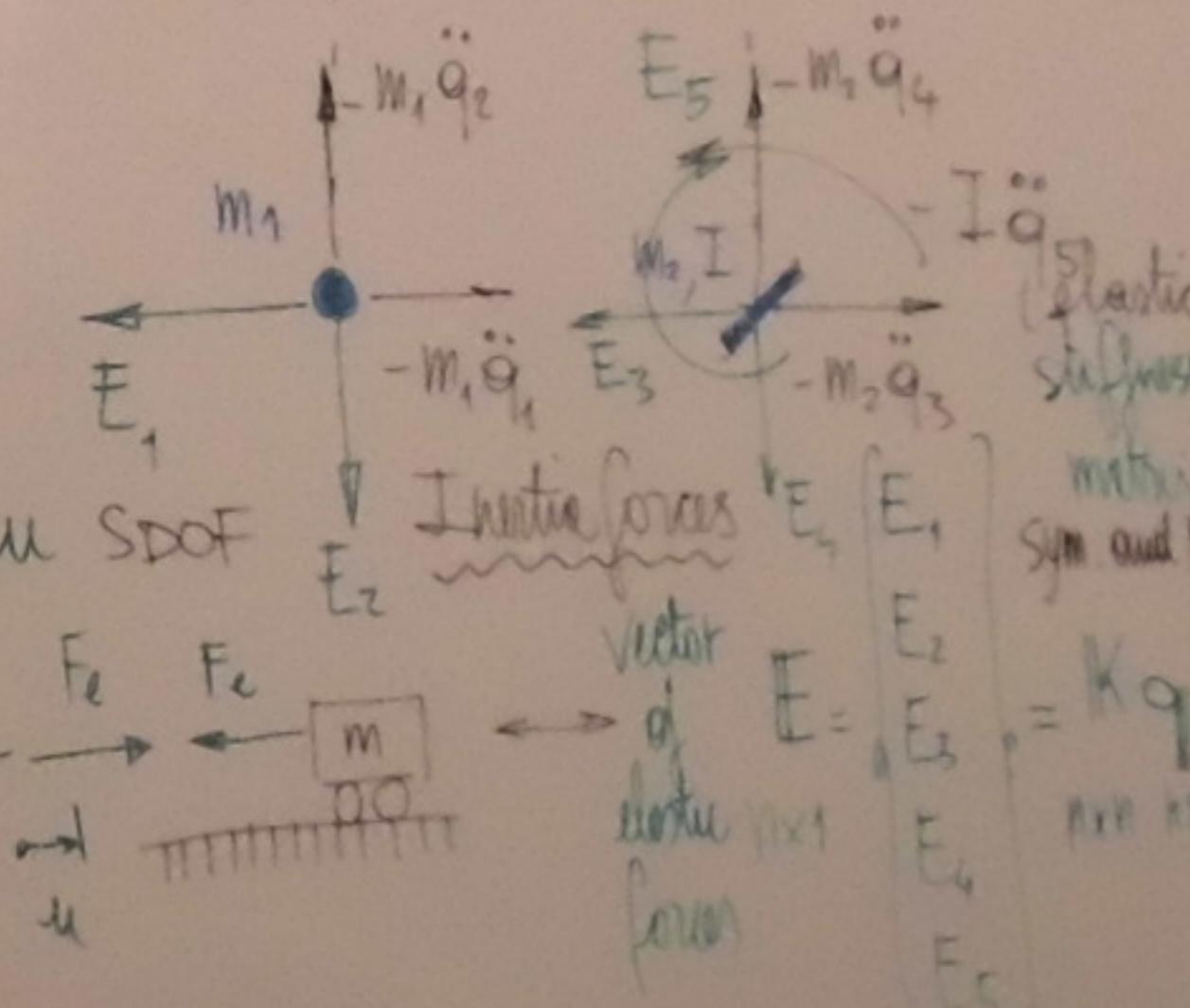
finite dofs



Vector of degrees of freedom
 $\mathbf{q} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \end{bmatrix}$

Force vector (external loading actions)
 $\mathbf{F}_{\text{ext}} = \begin{bmatrix} F \\ G \\ H \\ I \\ L \end{bmatrix}$

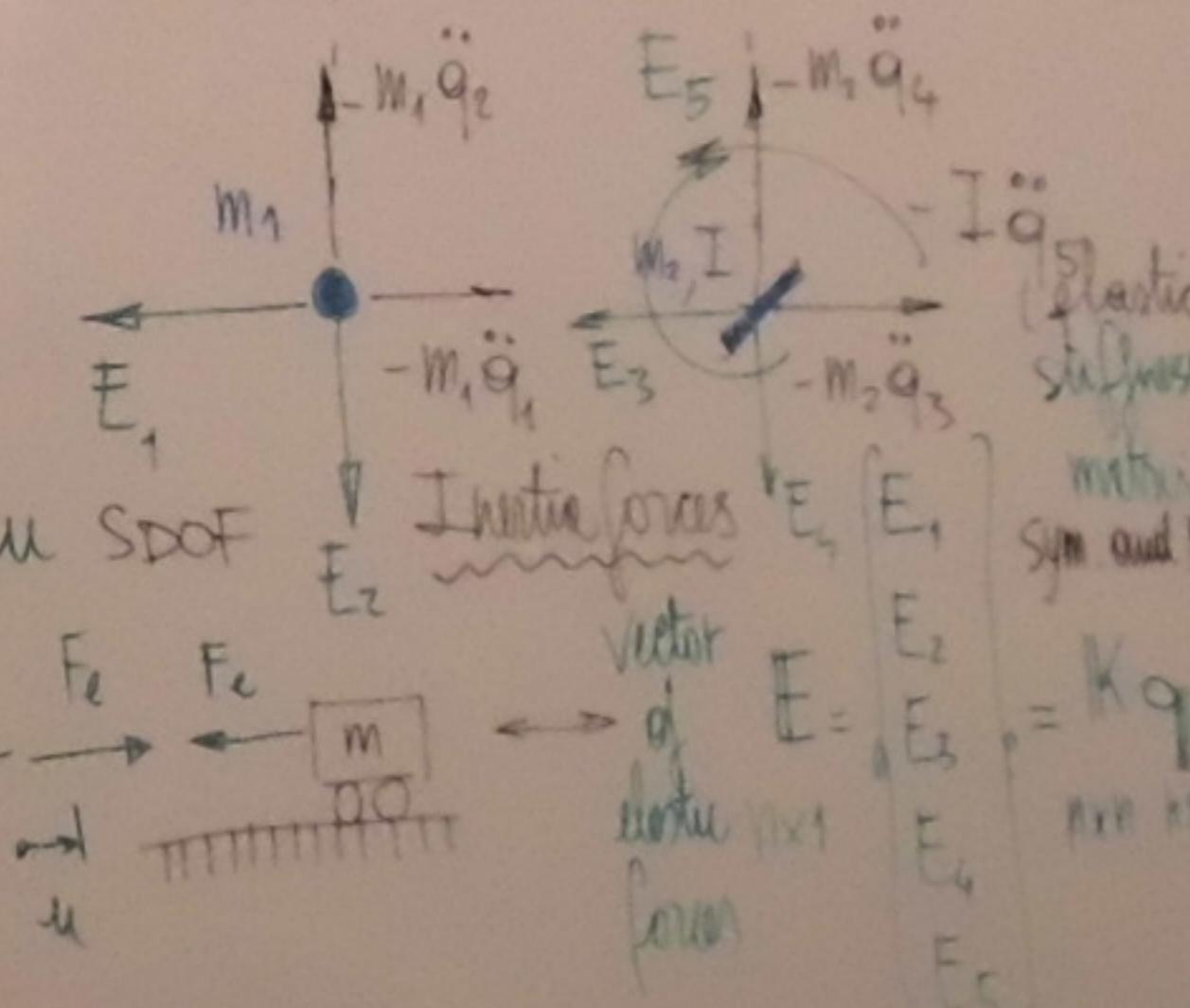
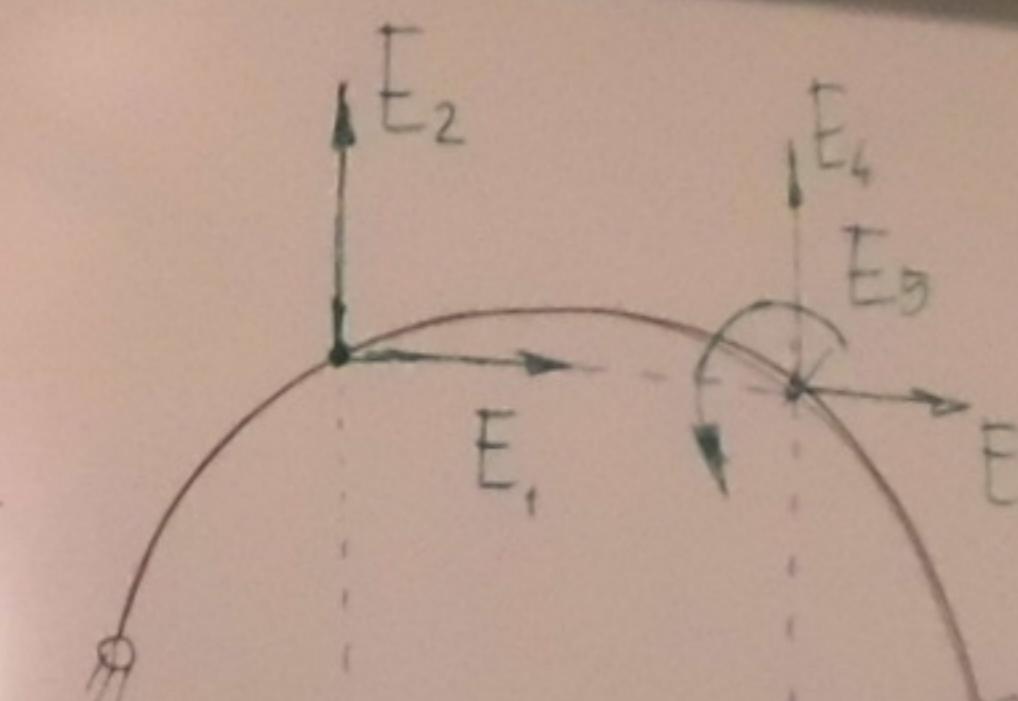
$$\mathbf{F}_e = \mathbf{K} \mathbf{u} \quad \text{SDOF}$$



$$\mathbf{K} = \mathbf{K}_{11} > \mathbf{K}_{12}, \mathbf{K}_{22}$$

$$\ddot{\mathbf{E}} = \{ \ddot{\mathbf{E}}_1, \dots, \ddot{\mathbf{E}}_n \} \quad \ddot{\mathbf{E}} > 0$$

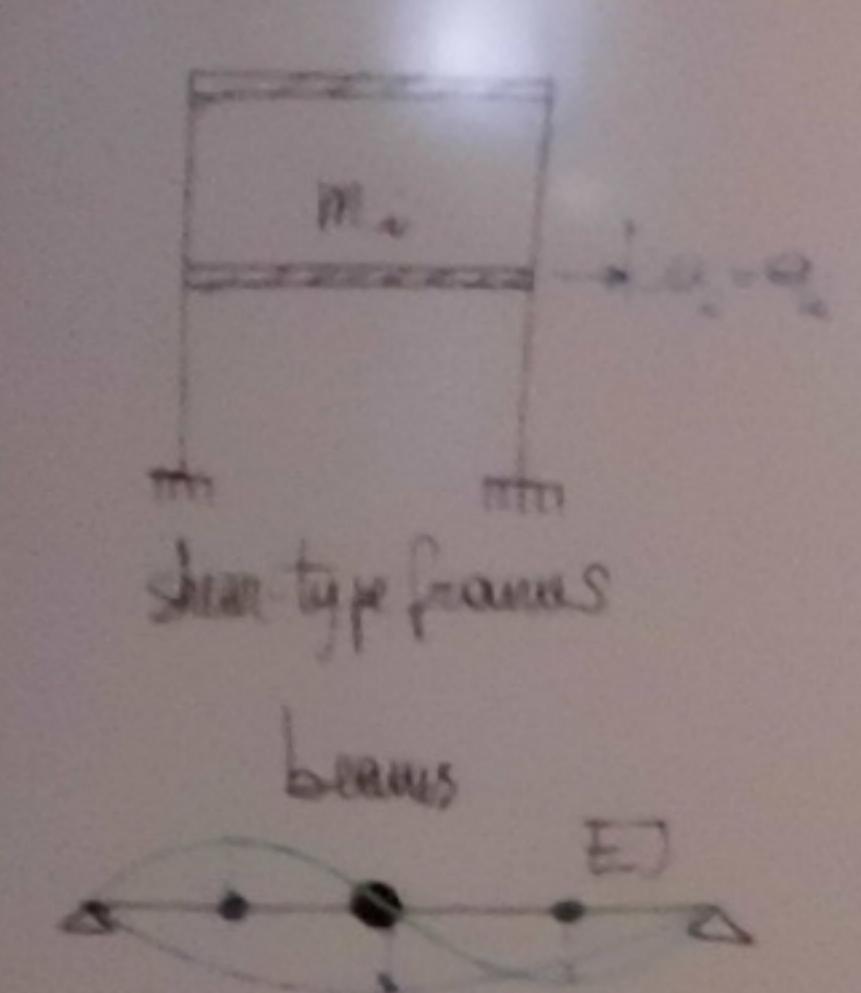
$$= \frac{1}{2} \mathbf{q}^T \mathbf{K} \mathbf{q} + \mathbf{L}^T \mathbf{E}$$



$$\mathbf{K} = \mathbf{K}_{11} > \mathbf{K}_{12}, \mathbf{K}_{22}$$

$$= \frac{1}{2} \mathbf{q}^T \mathbf{K} \mathbf{q} + \mathbf{L}^T \mathbf{E}$$

Multiple degree of freedom systems (MDOF)

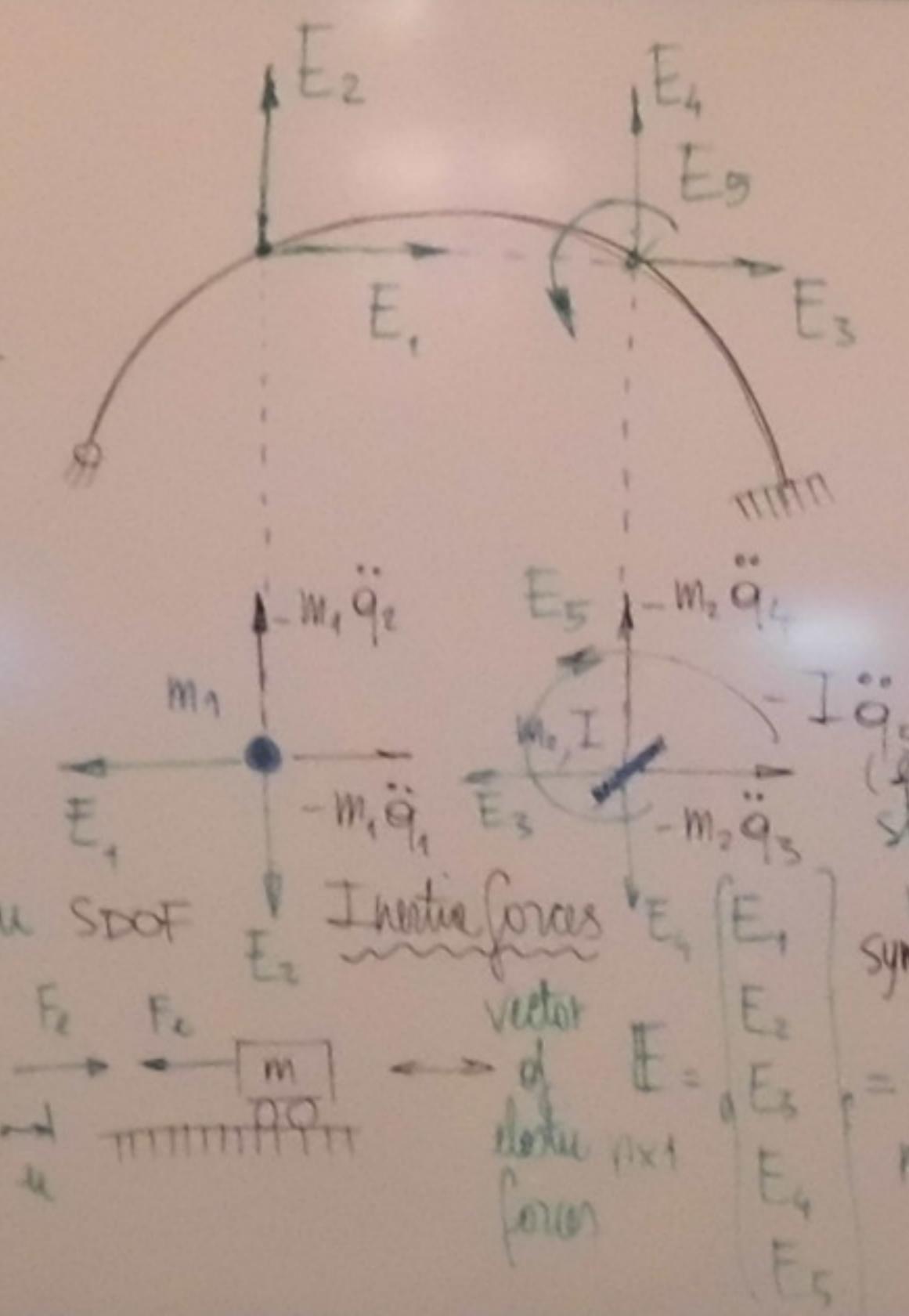


Conjunctive system → Disjunctive system

Chloridites (new genus) (see next page) 95

On days fine days

discrete	continuous	discrete	continuous	discrete	continuous
discrete	continuous	discrete	continuous	discrete	continuous
discrete	continuous	discrete	continuous	discrete	continuous
discrete	continuous	discrete	continuous	discrete	continuous
discrete	continuous	discrete	continuous	discrete	continuous



The diagram illustrates a single-degree-of-freedom system (SDOF) with mass m and stiffness K . The displacement u is shown as a horizontal vector. A vertical force F_e acts downwards at the center of mass. The system is subject to inertial forces due to acceleration \ddot{q}_1 and \ddot{q}_3 , represented by vectors $-m_1 \ddot{q}_1 E_3$ and $-m_2 \ddot{q}_3 E_5$ respectively. The total energy E is represented by a vector sum of five components E_1, E_2, E_3, E_4, E_5 . The energy components E_1, E_2, E_3, E_4, E_5 are defined as follows:

- E_1 : Potential Energy (PE) = $\frac{1}{2} K u^2$
- E_2 : Kinetic Energy (KE) = $\frac{1}{2} m \dot{u}^2$
- E_3 : Inertial Force Energy (IFE) = $-m_1 \ddot{q}_1 E_3$
- E_4 : Inertial Force Energy (IFE) = $-m_2 \ddot{q}_3 E_5$
- E_5 : Inertial Force Energy (IFE) = $-m_2 \ddot{q}_3 E_5$

