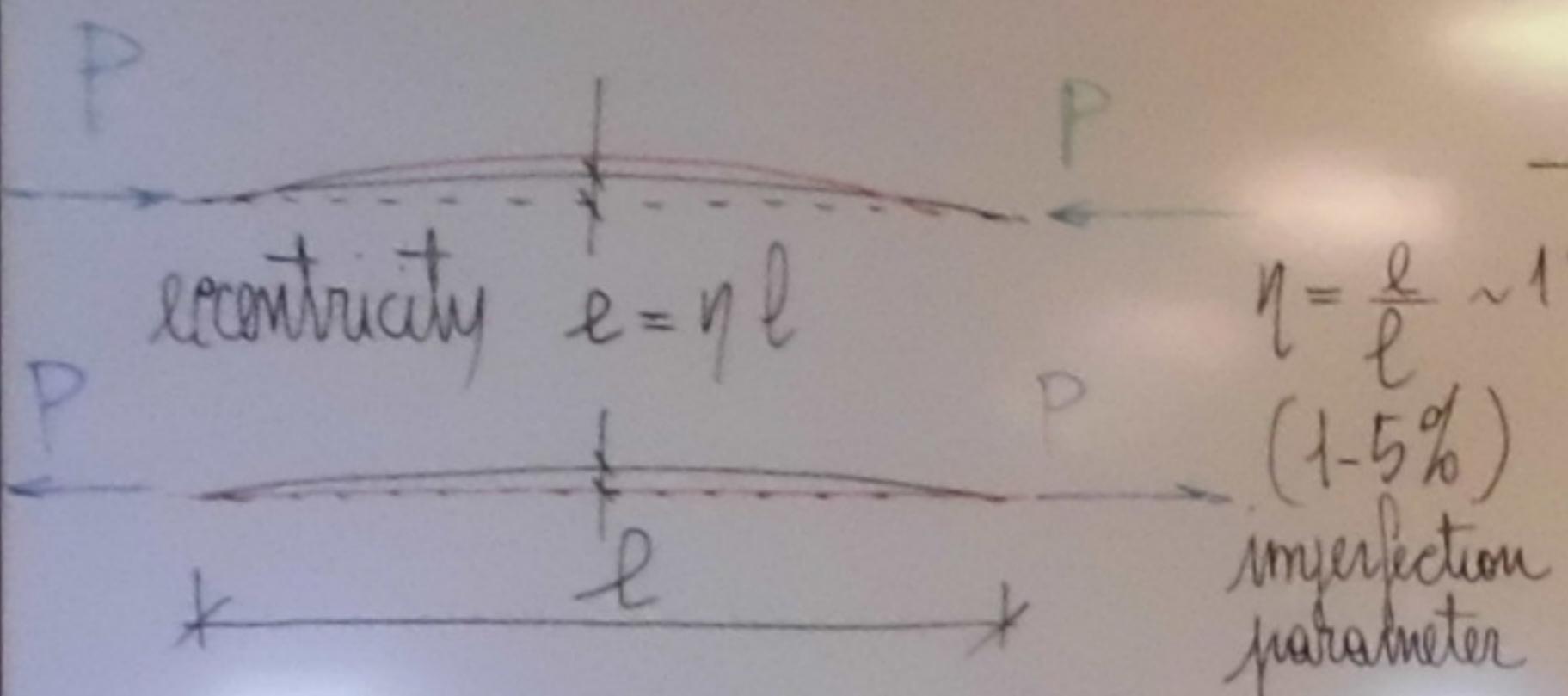


Instability analysis of real systems



- Geometrical (or other) imperfections in real systems (for instance eccentricities) may be prone to produce instability effects under compression-like loading

The question arises about how this may affect the stability phenomena, for instance in terms of allowable condition loading.

1-dof example

$$f = \eta f' \cos \theta$$

A hand-drawn diagram of a beam segment. The beam is shown in two configurations: its original straight configuration on the left, labeled 'A' at the bottom left, and its deformed configuration on the right, labeled 'varied or deformed configuration'. A vertical coordinate axis is indicated on the left. A horizontal force vector labeled 'P' is applied at the top of the beam. The angle between the original axis and the deformed axis is labeled 'θ'. The text 'also θ' is written above the beam.

$$P = p \frac{K}{l}$$

Rem

- for $\eta = 0$ (solution)
- for $\eta \neq 0$ (solution)

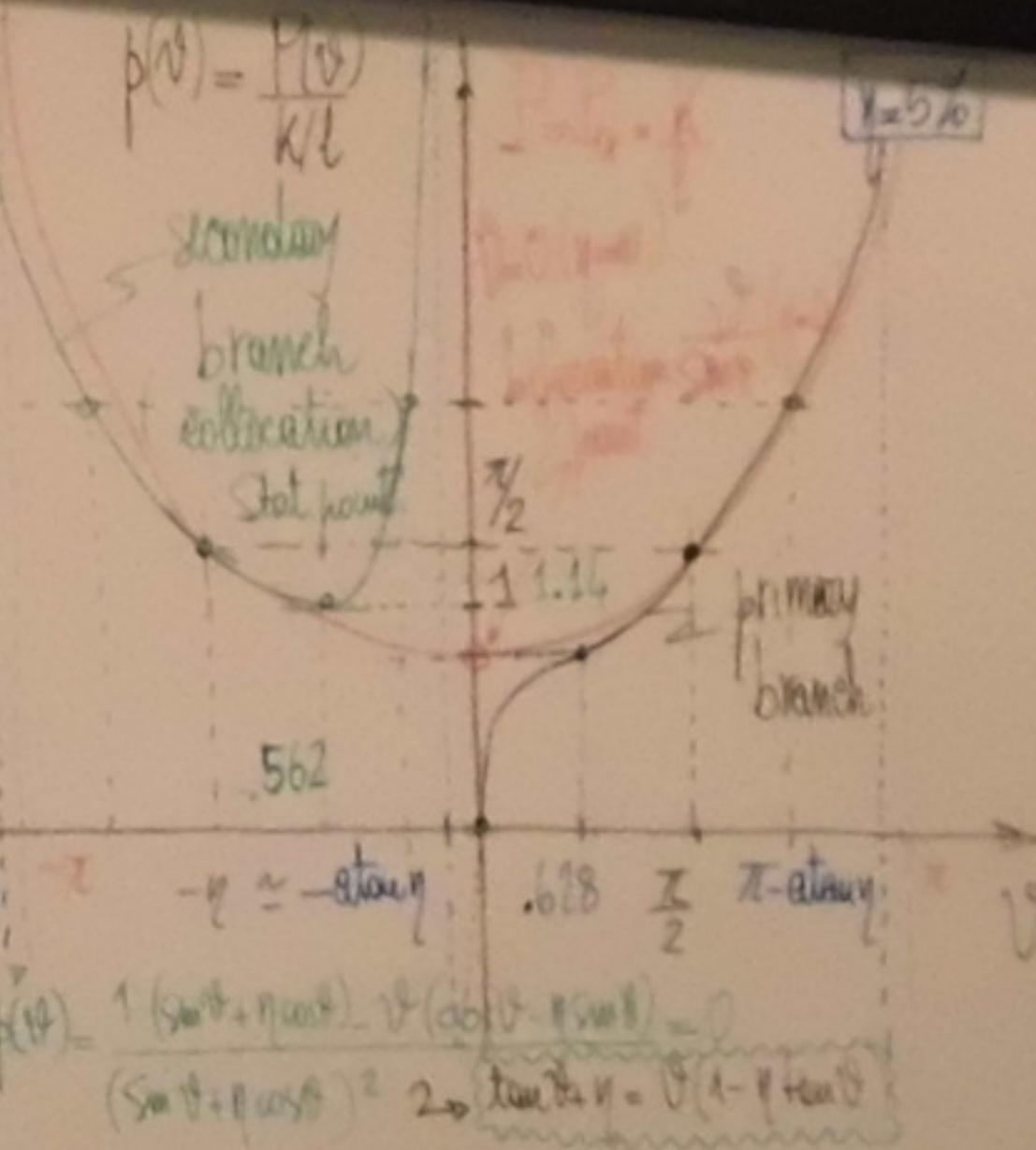
Remarks:

- for $\eta=0$ (ideal system), one gets back the original ideal solution $p(\vartheta) = \frac{V}{\sin \vartheta}$ (particular case)
 - for $\eta \neq 0$ (real system) $V=0$ is an equilibrium branch, only for $P < 0$
 - denominator vanishes for $\tan \vartheta = -\eta \Rightarrow \vartheta = -\pi - \arctan \eta = -\arctan \eta = \pi - \arctan \eta$ ($\eta < 0 \Rightarrow \arctan \eta \approx \eta \cdot \tan^{-1} 1$)
 - for $\vartheta = \pm \pi/2$, no matter the η , one has $p = \pi/2$
 - as η keeps small, the primary branch moves much near $V=0$. However it immediately leaves it as soon as $P > 0$
 - at $P=1$: $V = \sin \vartheta + \eta \cos \vartheta \rightarrow V \approx 0.678 \approx 36^\circ$
 - later on, the real system almost follows the bifurcated branch

Static approach

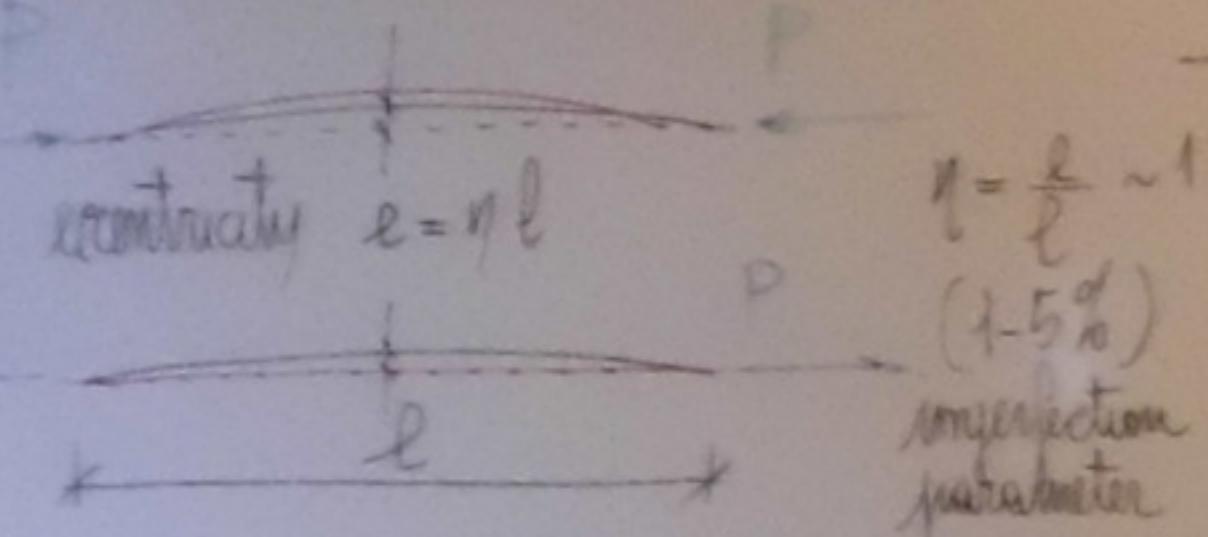
$$P(l \sin \theta + e \cos \theta) = K \vartheta \quad \text{pathas} \\ \frac{dl}{k} (\sin \theta + \eta \cos \theta) = \vartheta \quad (\text{equil cond}) \\ P(\theta) = -\frac{\vartheta}{\eta} \quad \begin{array}{l} p > p_s = 1/4 \\ (p > 1) \end{array}$$

equilibrium configurations (through geometric non-linearity or P-Δ effect)



Q - the valuation of paper on the
actual system. Because fundamental
importance is given to the
current of money can lead to the
value of a bank account which
based on the actual bank's situation
and the demand of the
individual who is holding it.

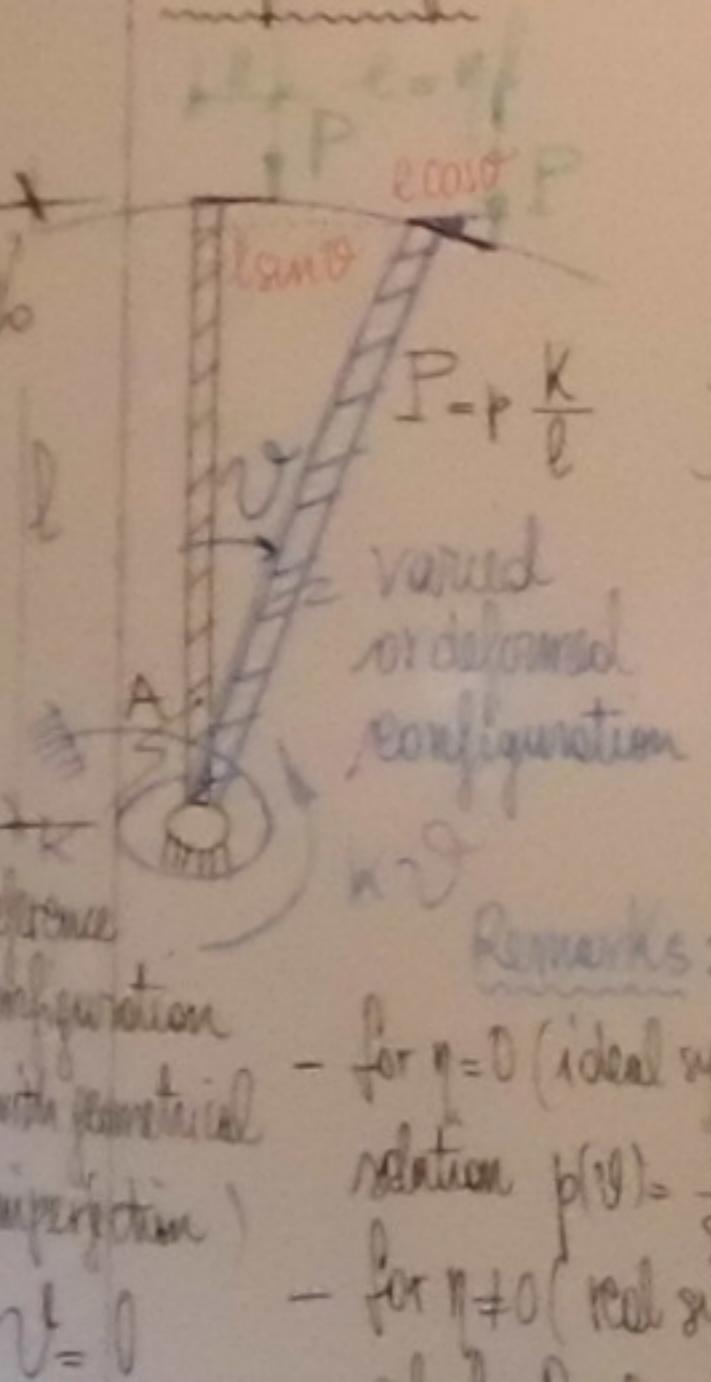
Instability analysis of real systems



- Geometrical (or other) imperfections in real systems (for instance eccentricities) may be prone to produce instability effects under compression-like loading.

Question: what happens if the eccentricity increases in time?

1-dof example



Static approach

$$P_0 \sin(\theta) + e \cos(\theta) = K\theta$$

$$P_0 (\sin(\theta) + \eta \cos(\theta)) = \theta$$

$$\frac{P_0}{K} = \frac{\theta}{\sin(\theta) + \eta \cos(\theta)}$$

equilibrium configurations (through geometric non-linearity or P-Δ effect)

Remarks:

- for $\eta=0$ (ideal system), one gets back the original ideal relation $p(\theta) = \frac{\theta}{\sin(\theta)}$ (particular case)
- for $\eta \neq 0$ (real system) $\theta=0$ is an equilibrium only for $P=0$
- denominator vanishes for $\tan(\theta) = -\eta \Rightarrow \theta = -\pi/2 + \eta$

Map of equilibrium paths (equil config)

($n > 1$) post-critical

$$p(\theta) = \frac{P(\theta)}{K/l}$$

secondary branch
collimation

total point

5.62

Stat p(θ)=

1 (sinθ+ηcosθ)

θ (θsinθ-ηsinθ)

(sinθ+ηcosθ)^2

2 tanθ + η = θ(1-η+ηθ)

$$P = P_{cr} = \frac{K}{l}$$

$\theta = 0 (\eta = 0)$

bifurcation point

$\eta/2$

14.34

5.62

primary branch

$$P = P_{cr} = \frac{K}{l}$$

$\theta = 0 (\eta = 0)$

bifurcation point

$\eta/2$

14.34

5.62

primary branch

- bifurcation point no longer appears as $\eta \neq 0$ (real system) and system undergoes a continuous transition from the state $(\theta=0, P=0)$ on the side of $\theta=\eta l$, towards the bifurcated branch of the ideal sys.

- the evaluation of $P=P_{cr}$ in the ideal system keeps of fundamental importance in quantifying the amount of compression load that may be allowed in a real system based on the accepted level of deformation and corresponding load P . P_{cr} in a real system may be stated as $P_{cr}^{real} = f \cdot P_{cr}^{ideal}$ where f is a safety factor.

- Analysis under "geometrically small" displacements

$|\theta| \ll 1 \Rightarrow \sin \theta \approx \theta; \cos \theta \approx 1$

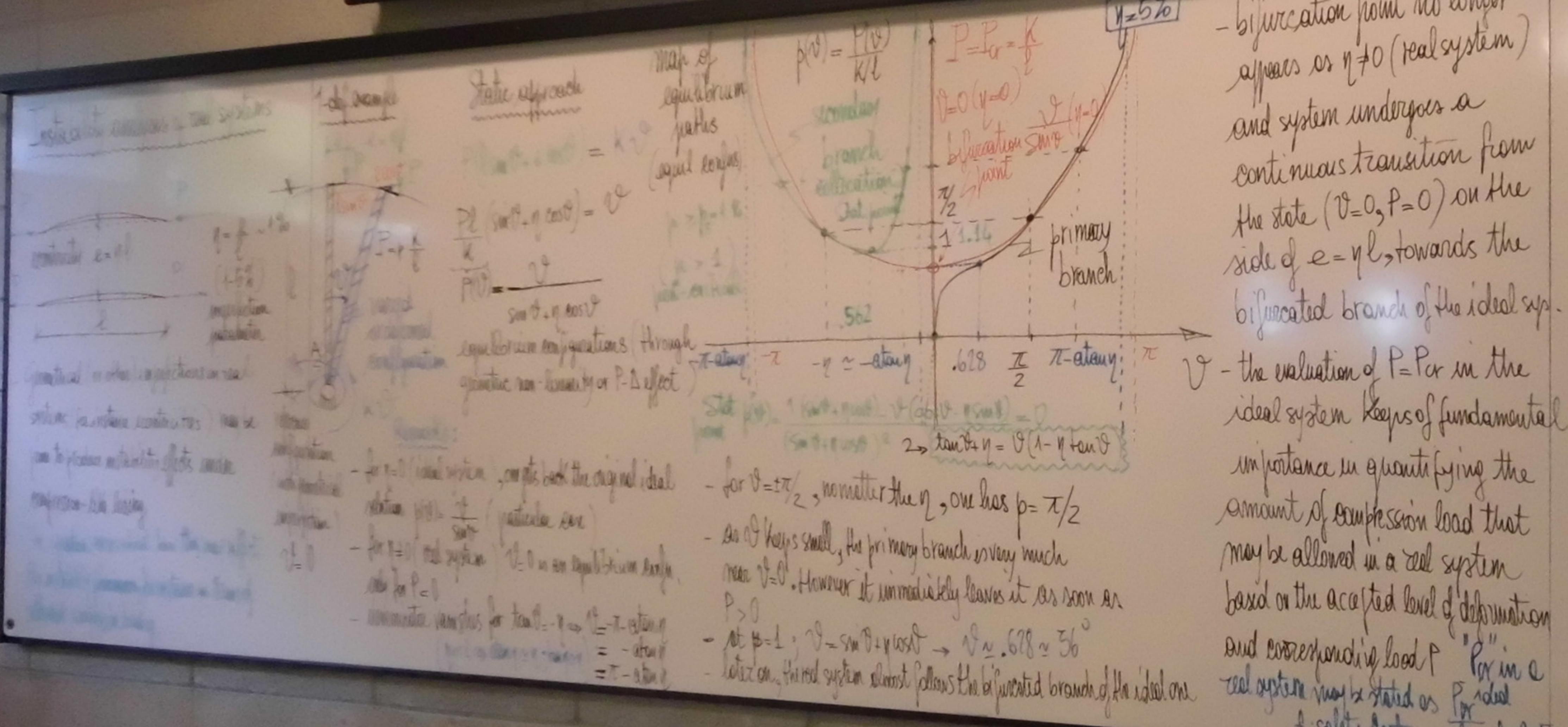
$$P(\theta + \eta) = K\theta$$

$$\frac{P(\theta + \eta)}{K} = \frac{\theta}{\theta + \eta} = \frac{1}{1 + \eta/\theta}$$

$$P(\theta) = \frac{P(\theta + \eta)}{1 + \eta/\theta}$$

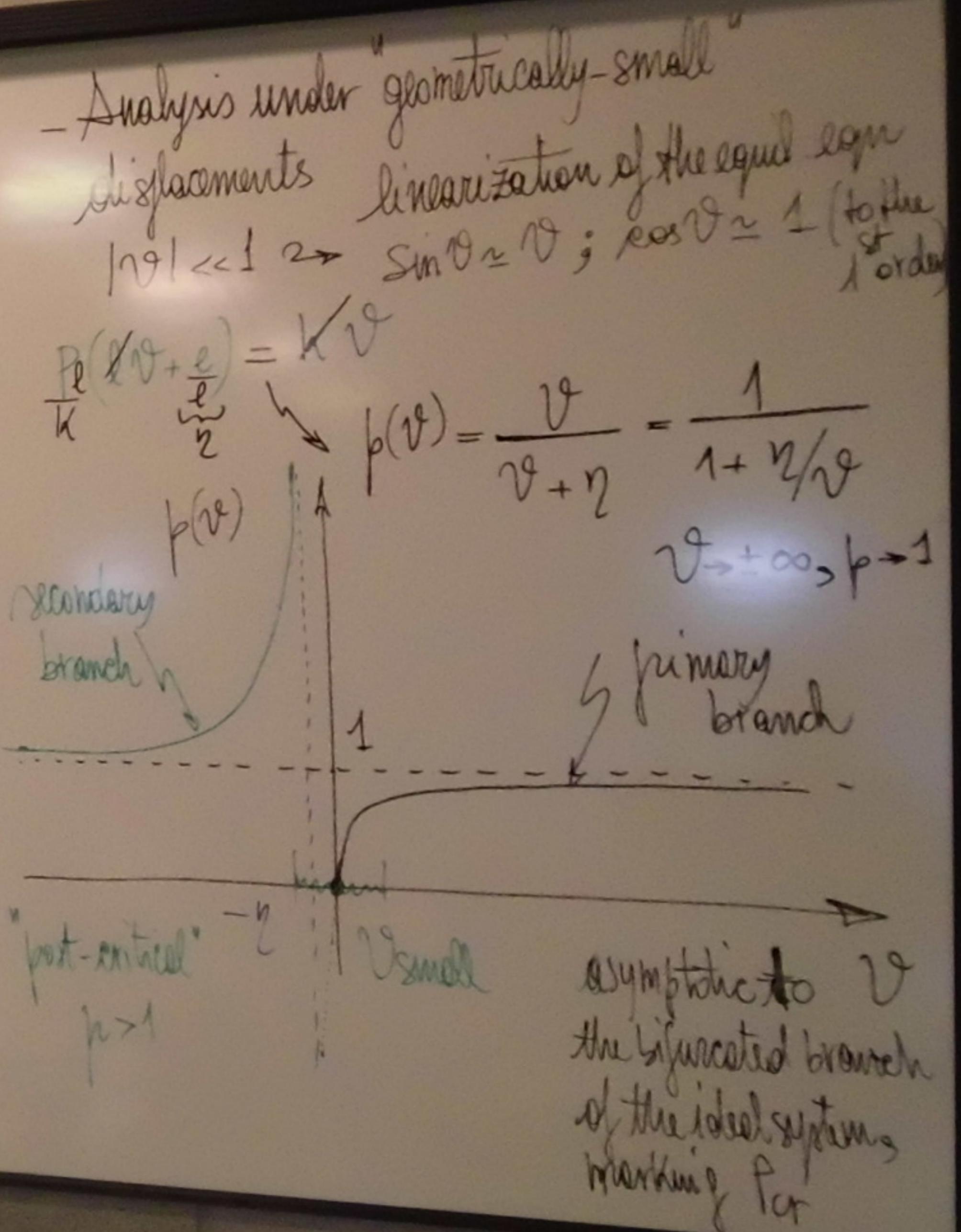
$$P(\theta) = \frac{P(\theta)}{1 + \eta/\theta}$$

$$P(\theta)$$



- bifurcation point no longer appears as $\eta \neq 0$ (real system) and system undergoes a continuous transition from the state $(\theta=0, P=0)$ on the side of $\theta=\eta l$, towards the bifurcated branch of the ideal sys.

- the evaluation of $P=P_{cr}$ in the ideal system keeps of fundamental importance in quantifying the amount of compression load that may be allowed in a real system based on the accepted level of deformation and corresponding load P in a real system may be stated as P_{cr}^{ideal} f safety factor F $f \approx 2.5$



Energy approach

Total Potential Energy $V(\theta) = \frac{1}{2} K \theta^2 - P \cdot v(\theta)$

$$V_f = -L_{\text{def}} \\ = K \left(\frac{1}{2} \theta^2 - p f \cos \theta + \eta \sin \theta \right)$$

$$V(\theta) = K \left(\theta - p (\sin \theta + \eta \cos \theta) \right) = 0 \quad \text{S.C.}$$

$\theta = p (\sin \theta + \eta \cos \theta)$ equil. config.
as determined from the static approach

$$V''(\theta) = K (1 - p (\cos \theta - \eta \sin \theta))$$

In the equil. conf. where

$$p(\theta) = \frac{1}{\sin \theta + \eta \cos \theta} \\ = K \left(1 - \frac{\cos \theta - \eta \sin \theta}{\sin^2 \theta + \eta \cos^2 \theta} \right)$$

$$= 0 \Rightarrow \tan \theta + \eta = V (1 - \eta \tan \theta)$$

1-dof example

$$P = \eta f$$

$$\theta = \pi/2$$

$$P = p \frac{K}{l}$$

$$p = l/K$$

$$\theta = \pi/2$$

$$V(\theta) = K \theta$$

$$V''(\theta) = K$$

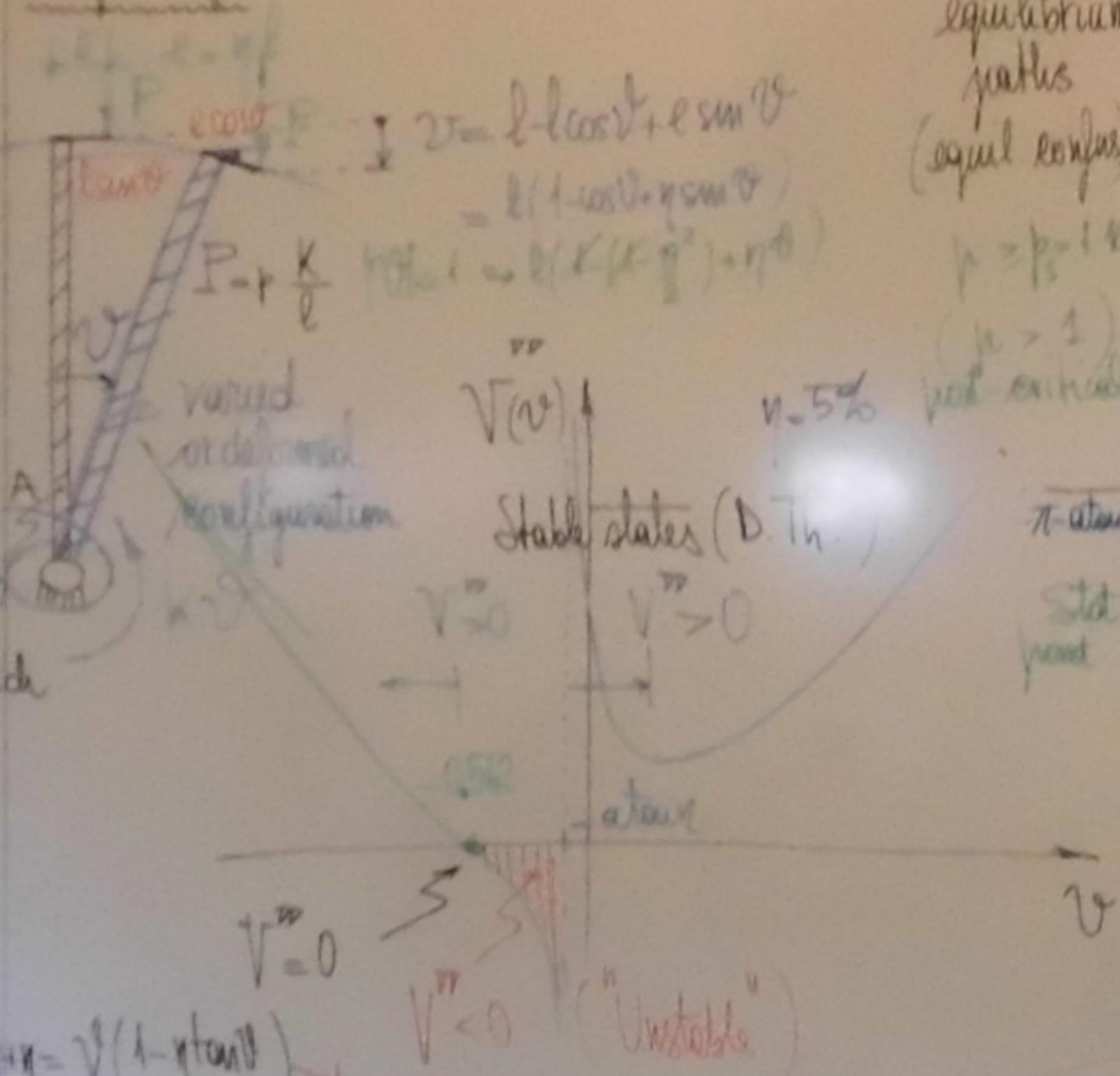
Entzündung

$$\text{Total Potential Energy } V(\theta) = \underbrace{\frac{1}{2} \dot{\theta}^2}_{V_T} - \underbrace{F_r r(\theta)}_{V_U}$$

$$V = \rho (\sin \theta + i \cos \theta) \quad \text{equal angles.}$$

as determined from the static approach

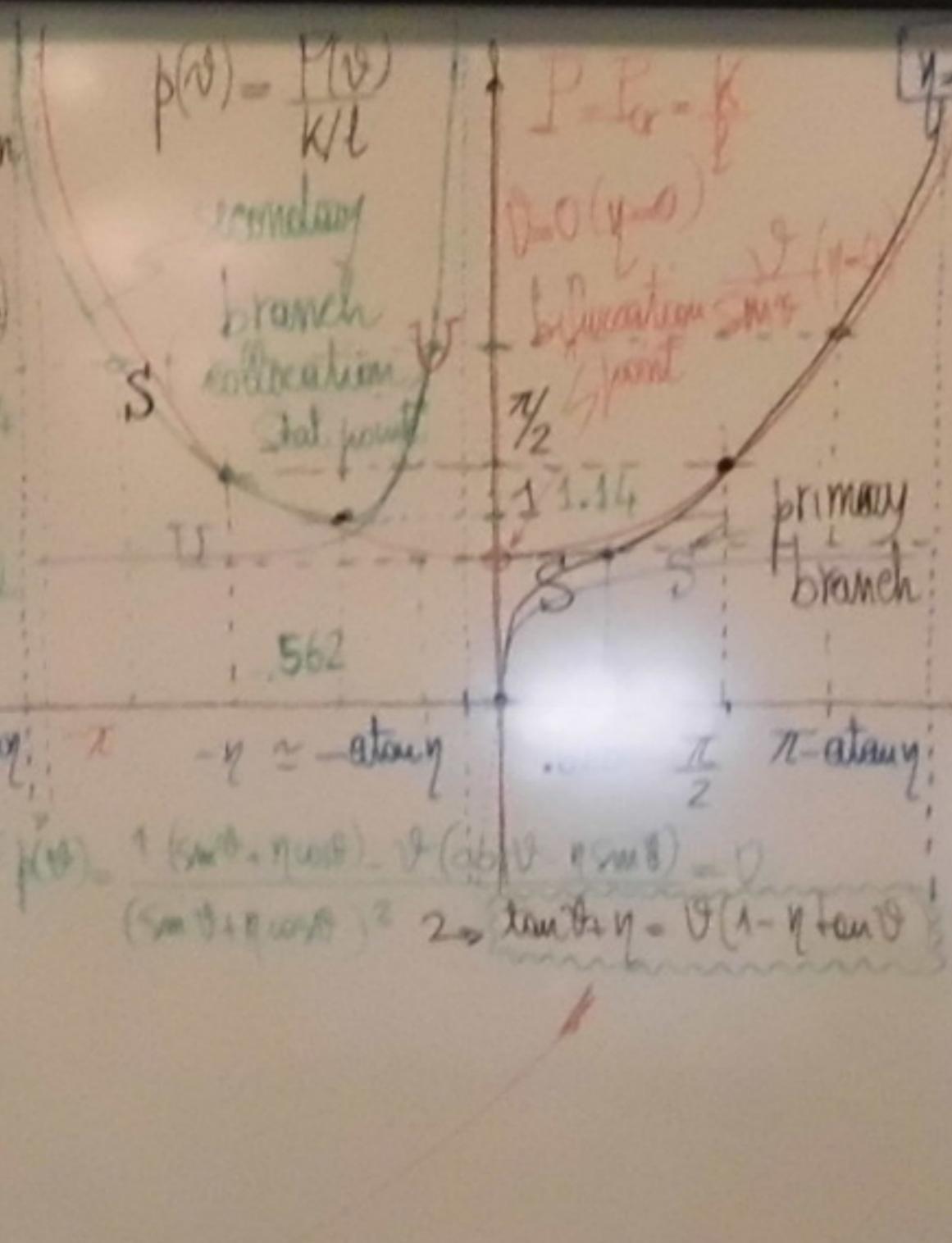
1-dot example



Map of
equilibrium
paths
(equal regions)

$$p(v) = \frac{P(v)}{k+1}$$

lunula
branch
breakin
g



$$V(v) \approx V_2(v) = 4K_0^2 P v$$

$$\sin \theta \approx v$$

$$\cos \theta \approx 1 - \frac{\theta^2}{2} = \frac{1}{2}K\theta^2 - Pl \left(\frac{\theta^2}{2} + q\theta \right)$$

$$V_2(\vartheta) = K(\vartheta - p(\vartheta + n)) = 0$$

$$\vartheta = p(\vartheta + n)$$

$$1-p \begin{cases} > 0 & p < 1 \text{ STABLE} \\ = 0 & p = 1 \\ < 0 & p > 1 \end{cases}$$

- Analysis under Governmental
Departments Organization of the
not less than 50% ; and the

$$\checkmark \quad \frac{d}{dt} \left(\frac{v}{v+\eta} \right) = \frac{v}{(v+\eta)^2} = \frac{1}{1+v/\eta}$$

Analysis under "geometrically-small" displacements
 linearization of the equilibrium eqn
 $|V| \ll 1 \Rightarrow \sin V \approx V; \cos V \approx 1$ (to the 1st order)

$$V(\theta) \approx V_2(\theta) = \frac{1}{2} k\theta - P\theta^2$$

$$\sin \theta \approx \theta^2 \approx \frac{1}{2} k\theta^2 - P\theta \left(\frac{\theta^2}{2} + \eta \theta \right)$$

$$V_2(\theta) = K(\theta - p(\theta + \eta)) = 0$$

$$\theta - p(\theta + \eta) = 0$$

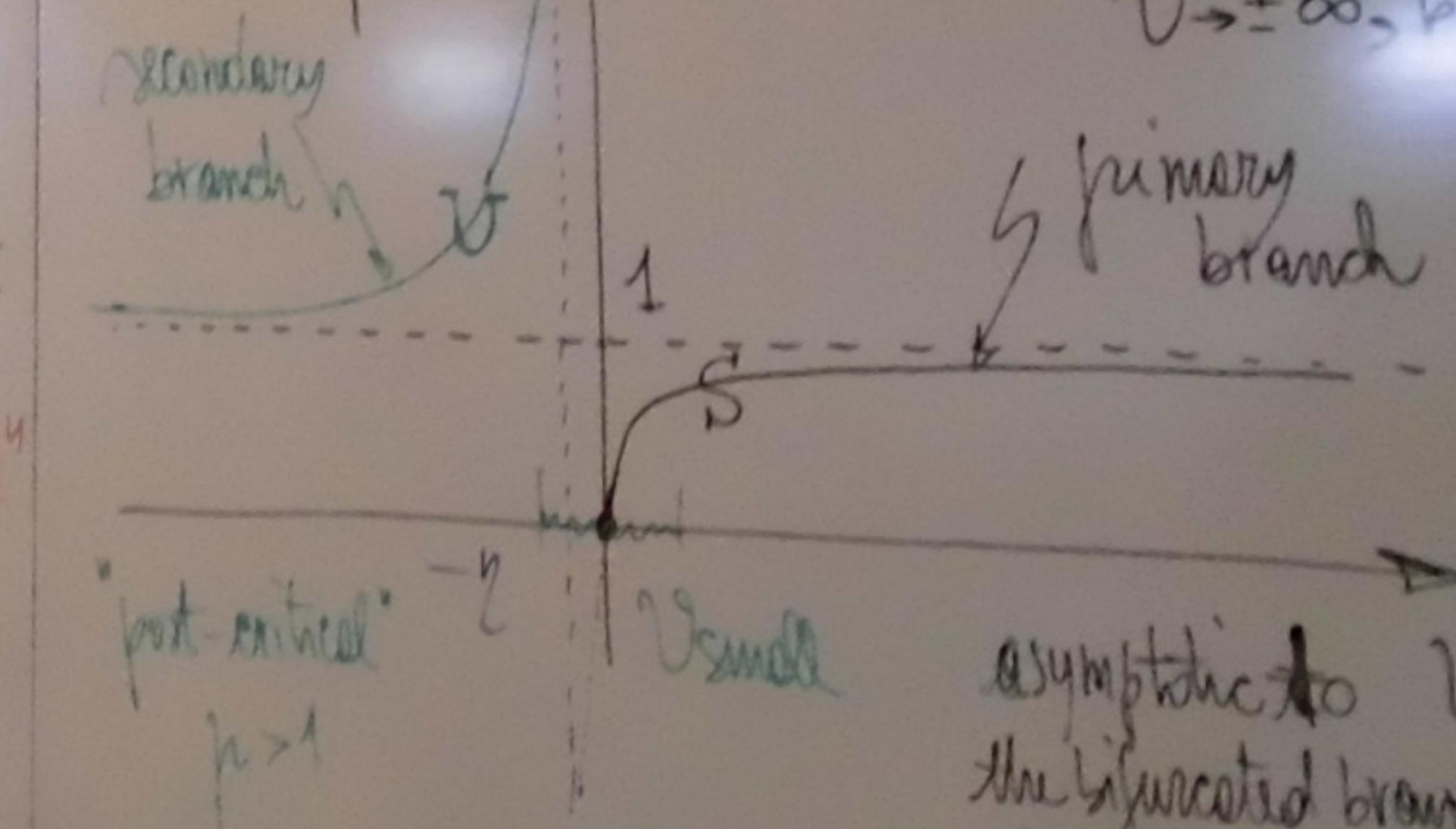
$$V \approx V_2(\theta) = K(1 - p)$$

$$\begin{cases} > 0 & p < 1 \text{ STABLE} \\ = 0 & (p = 1) \text{ "Unstable"} \\ < 0 & p > 1 \end{cases}$$

$$\frac{p(\theta + \eta)}{K} = \frac{\theta}{2}$$

$$p(\theta) = \frac{\theta}{\theta + \eta} = \frac{1}{1 + \eta/\theta}$$

$$\theta \rightarrow \infty, b \rightarrow 1$$



asymptotic to V
 the bifurcated branch
 of the ideal system,
 marking for

