

CM - Esempio

- Stato di sforzo alla de Saint Venant (DSV) \rightarrow particolare stato di

The diagram illustrates a beam element with two nodes, A and B. Node A is located at the left end, and node B is at the right end. At node A, there is a horizontal force F_x pointing to the right and a vertical force F_y pointing downwards. At node B, there is a vertical force F_y pointing downwards. The beam is represented by a rectangular frame with a central horizontal line. The left side of the frame is labeled with a green circle containing 'B' and a blue circle containing 'A'. The right side of the frame is labeled with a blue circle containing 'B'. A coordinate system is shown below the beam, with the horizontal axis labeled 'x' and the vertical axis labeled 'y'.

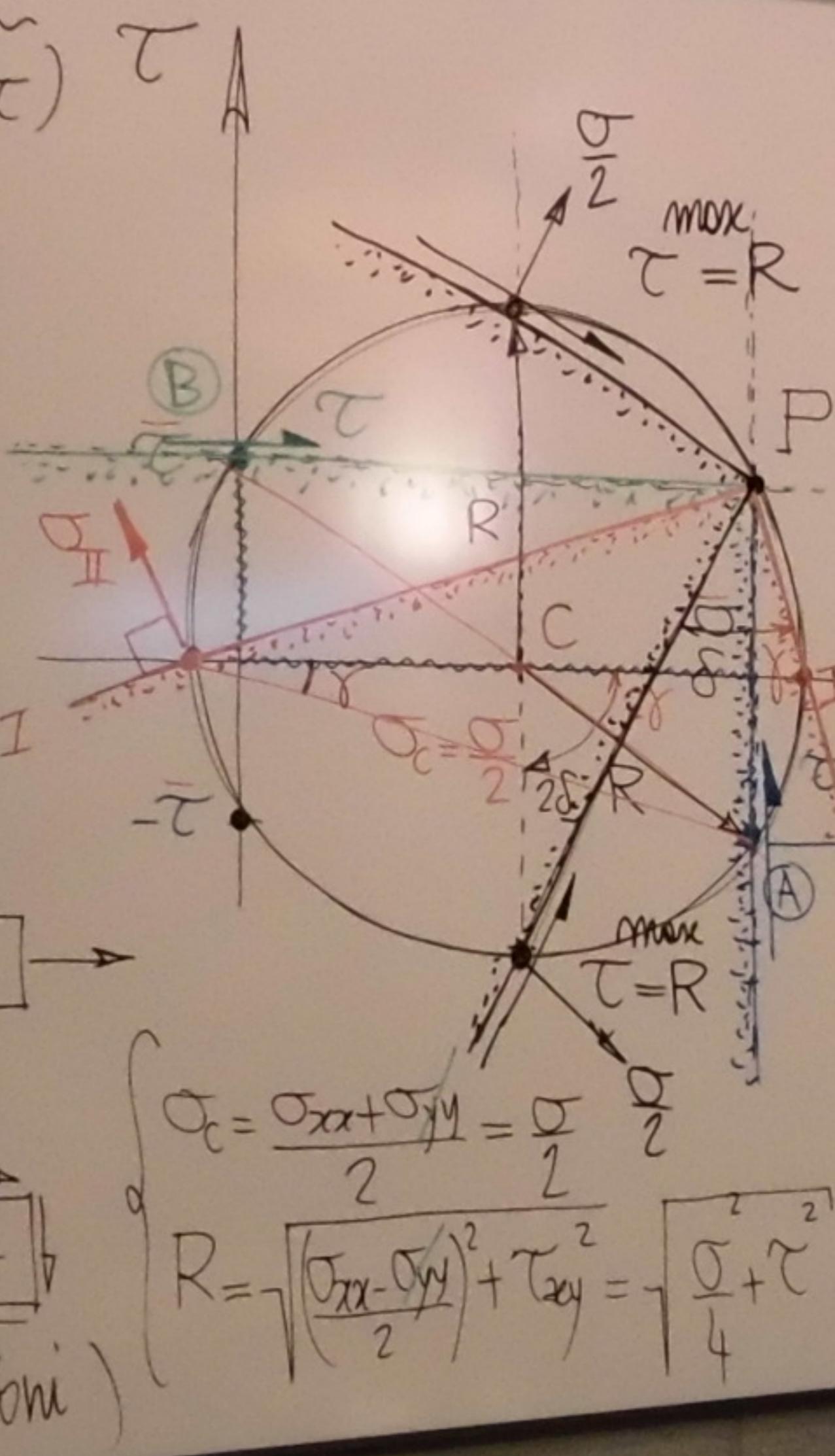
$\sigma_{xx} = \sigma$
 $\sigma_{yy} = 0$
 $\tau_{xy} = \tau$
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$\sigma_{zz} = \tau_{zx} = \tau_{zy} = 0$
 (sphärisches)

$$\sigma_{xy} = \tau_{xz} = \tau_{zy} = 0$$

(shear stress)

Piano de Molte



$$\sigma_c = \frac{\sigma_{xx} + \sigma_{yy}}{2} = \frac{\sigma}{2}$$

Dilarukanan

$$\tan 2\gamma = \frac{2}{\sigma}$$

\downarrow

$$\gamma = \frac{1}{2} \arctan \frac{2\tau}{\sigma}$$

$$B_{II} = b_c \hat{q}_{II}$$

$$= \frac{b}{2} q + \sqrt{\frac{b^2}{4} + r^2}$$

 NB (tens. princ. di sign)

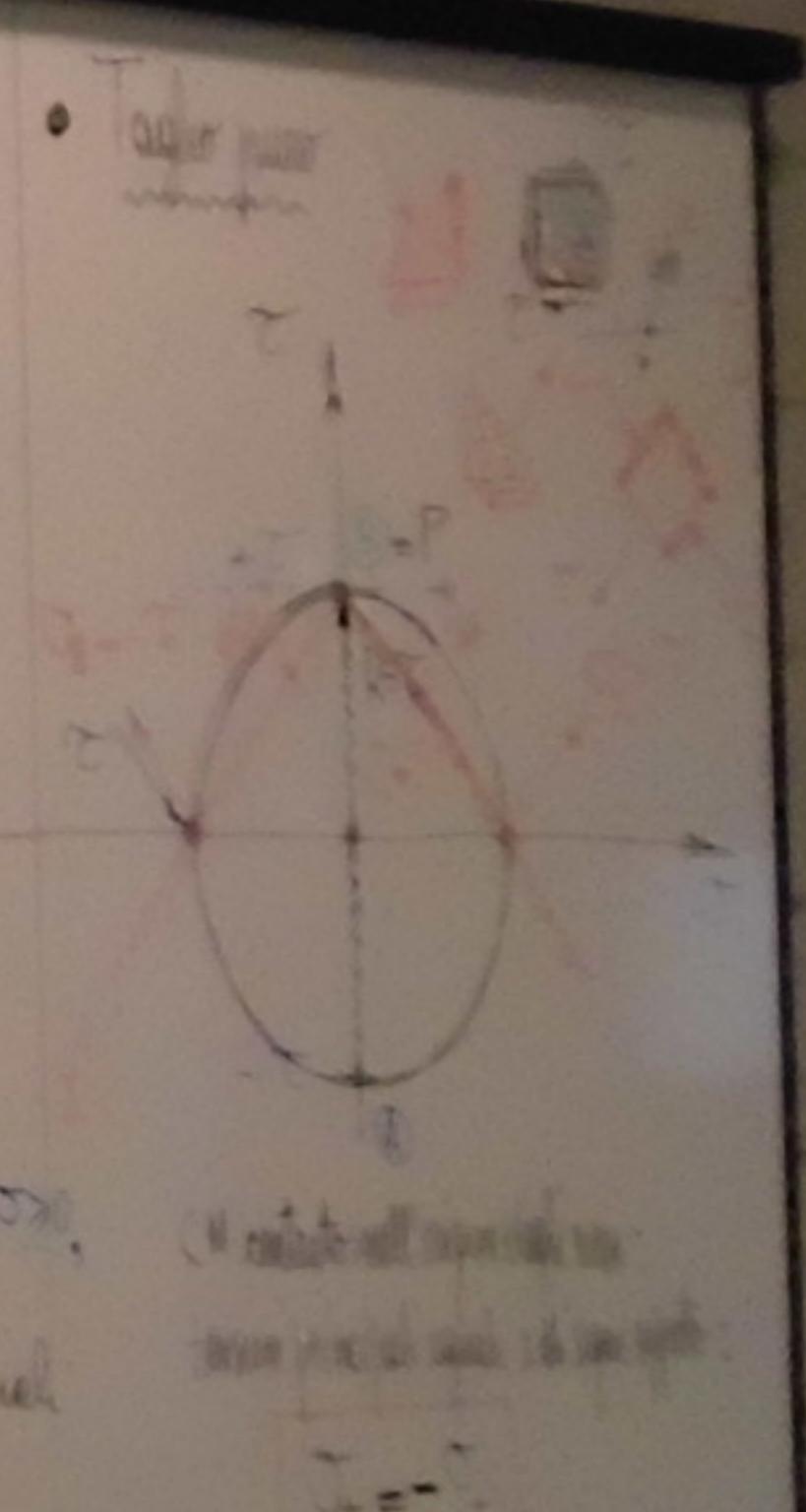
$$\sigma_I \cdot \sigma_{II} = \sigma_c^2 - R^2$$

$$= \frac{\sigma^2}{4} - \frac{\sigma^2}{4} - T^2$$

$$= -T \leq 0$$

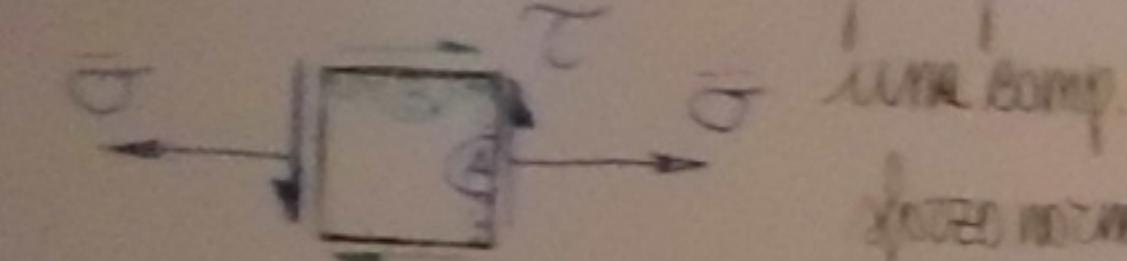
- Tabone / ammissione monastica

- Implicazioni nell'ambito della resistenza di materiali
es. resistente a trazione \rightarrow NO CM con $T > 0$



CM - Esercizi

- Stato di spazio alla de Saint Venant (DSV) = particolare stato di spazio piano con una componente di spazio normale nulla



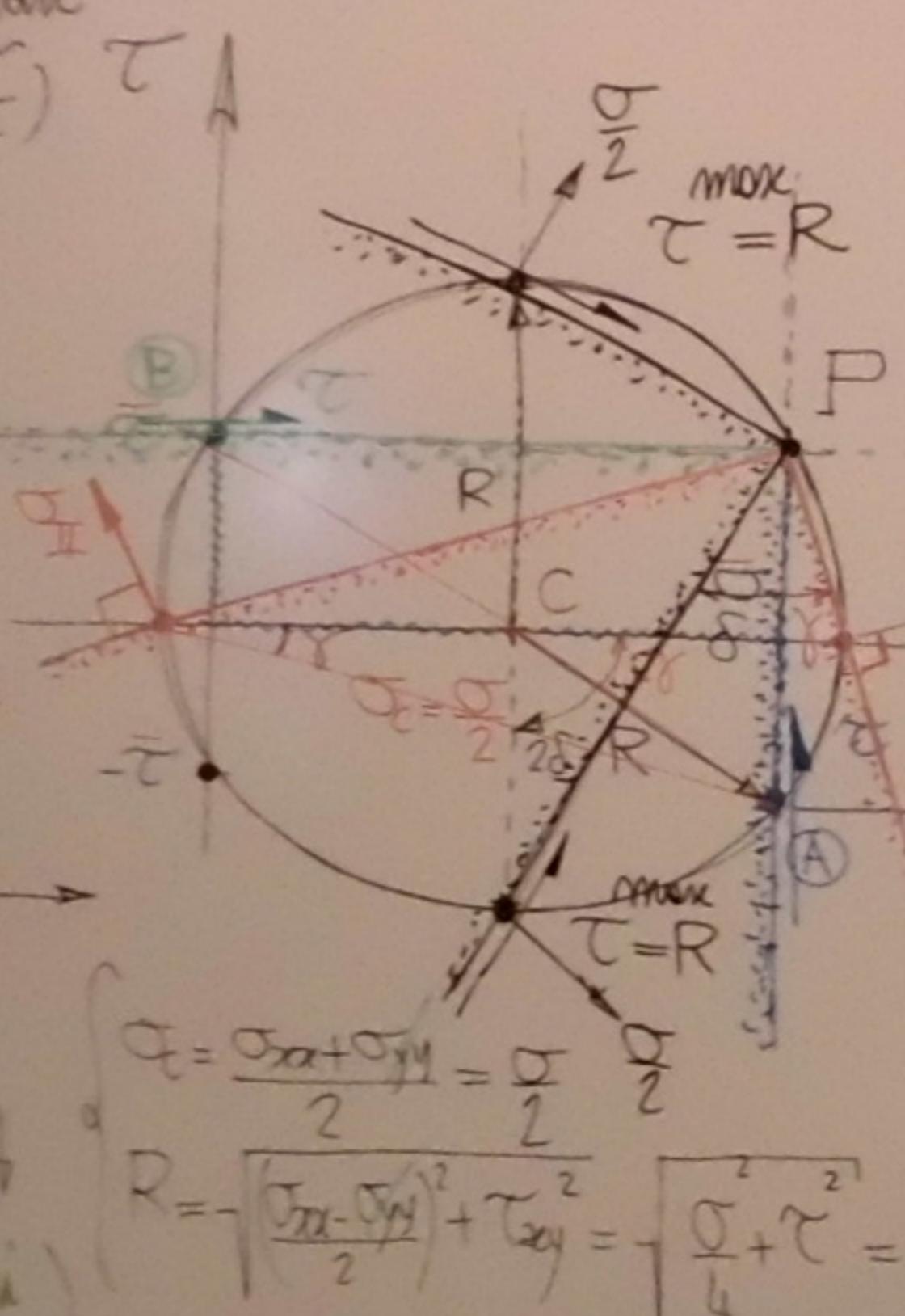
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(spazio piano)

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(spazio piano)

Piano di Mohr



$$\tan 2\theta = \frac{\tau}{\frac{\sigma}{2}}$$

$$R = \frac{1}{2} \sqrt{\sigma^2 + \tau^2}$$

$$\begin{aligned} \sigma_I, \sigma_{II} &= \sigma_c \pm R \\ &= \frac{\sigma}{2} \pm \sqrt{\frac{\sigma^2}{4} + \tau^2} \\ \text{NB (trans. princ. di segno)} \quad \sigma_I \cdot \sigma_{II} &= \sigma_c^2 - R^2 \text{ opposto} \\ &= \frac{\sigma^2}{4} - \frac{\sigma^2}{4} - \tau^2 \\ &= -\tau^2 \leq 0 \end{aligned}$$

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \tau_{xy}^2} = \sqrt{\frac{\sigma^2}{4} + \tau^2} = \frac{1}{2}\sqrt{\sigma^2 + 4\tau^2}$$

$$\text{Dir. con } \tau_{\max} : \delta = \frac{\pi}{4} - \theta$$

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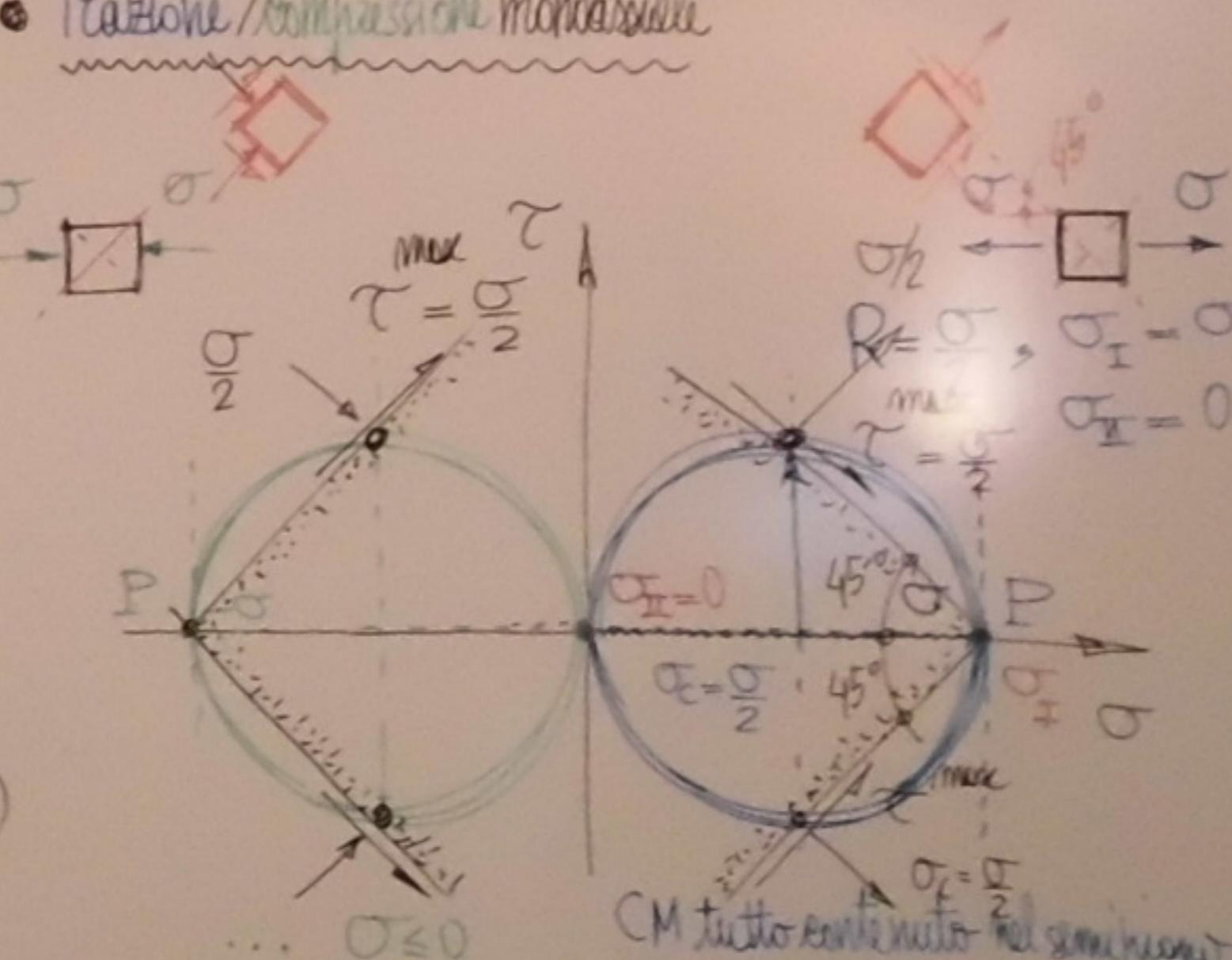
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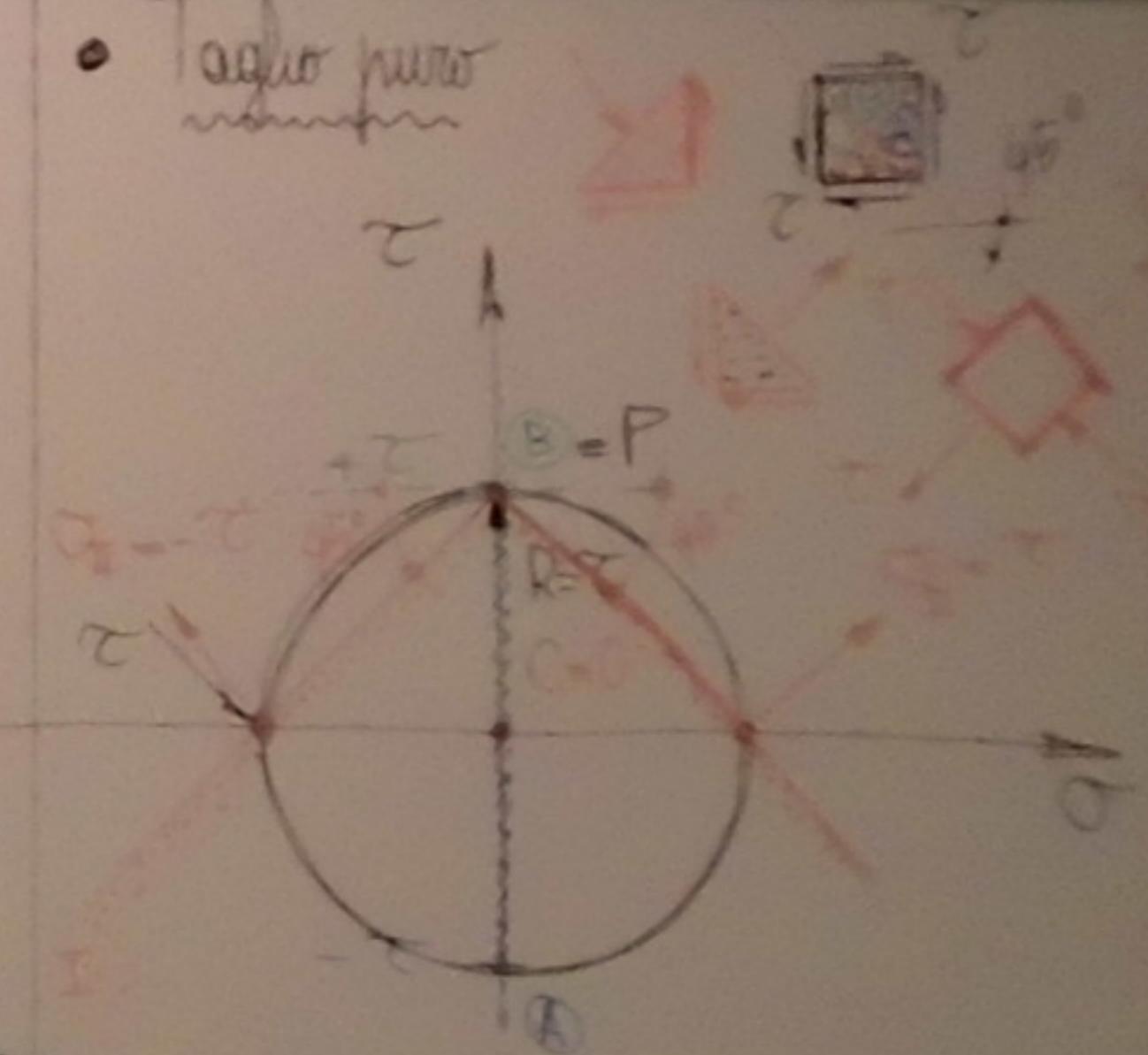
• Trazione/compressione monassiale



CM tutto contenuto nel semipiano $\sigma \geq 0$, tangente all'asse τ nell'origine

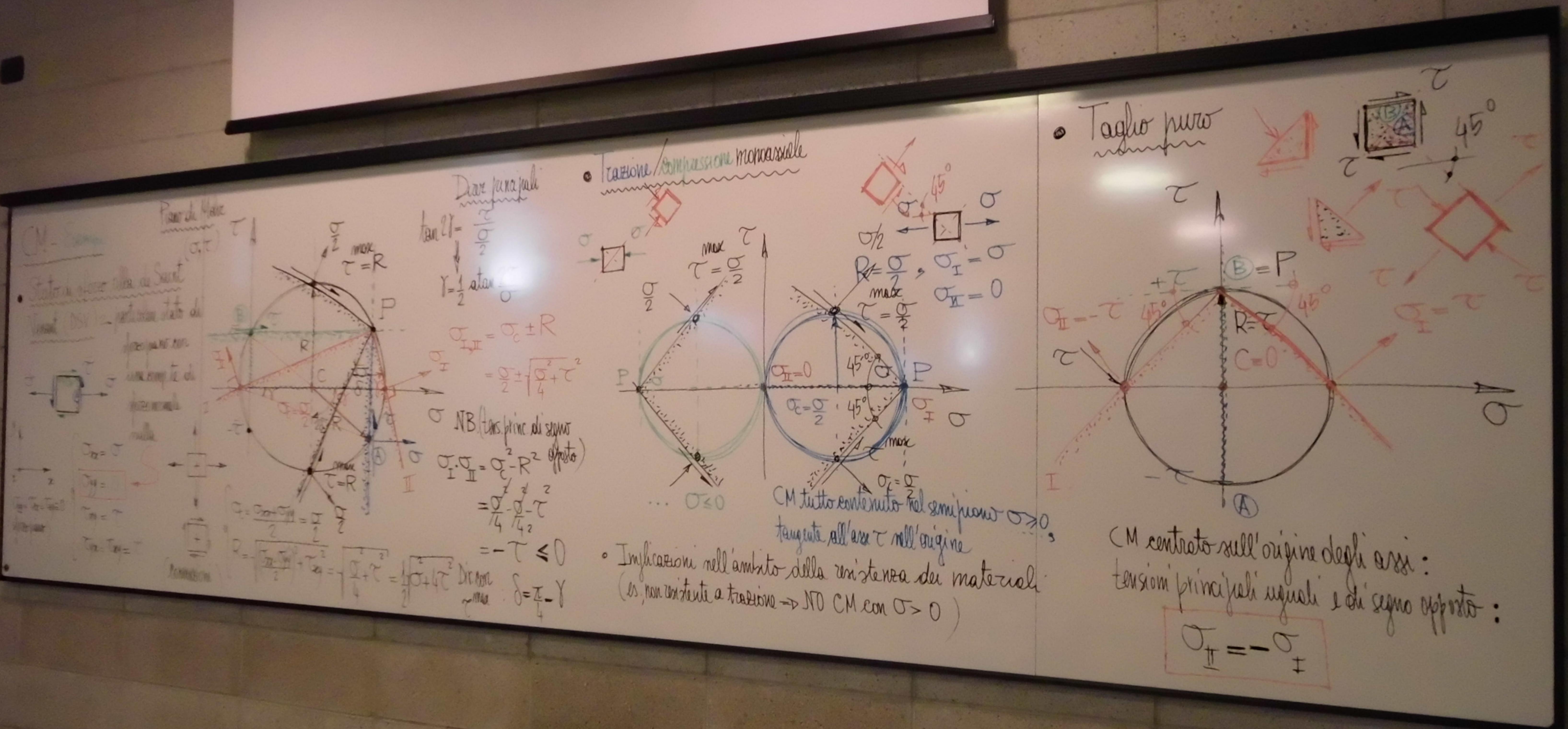
• Implicazioni nell'ambito della resistenza dei materiali
(es. non resistente a trazione \Rightarrow NO CM con $\sigma > 0$)

• Taglio puro

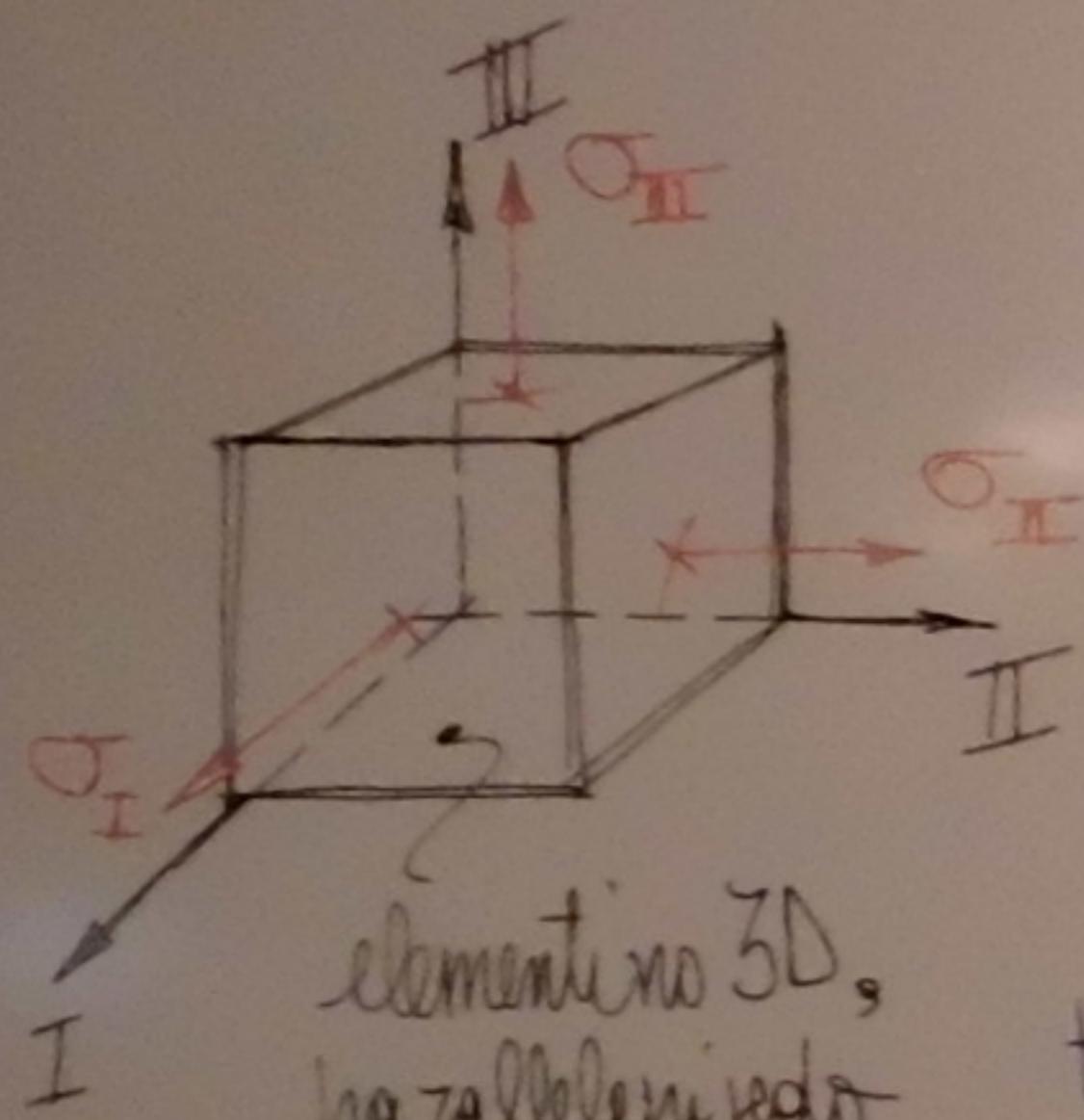


CM contenuto nell'origine degli assi:
tensioni principali uguali e di segno opposto:

$$\sigma_I = -\sigma_{II}$$



Cerchi e Arbolo di Mohr



elementino 3D,
parallelepipedo
ritagliato secondo le
tecniche principale

$$[\sigma] = \begin{bmatrix} \sigma_I & & \\ & \sigma_{II} & \\ & & \sigma_{III} \end{bmatrix}$$

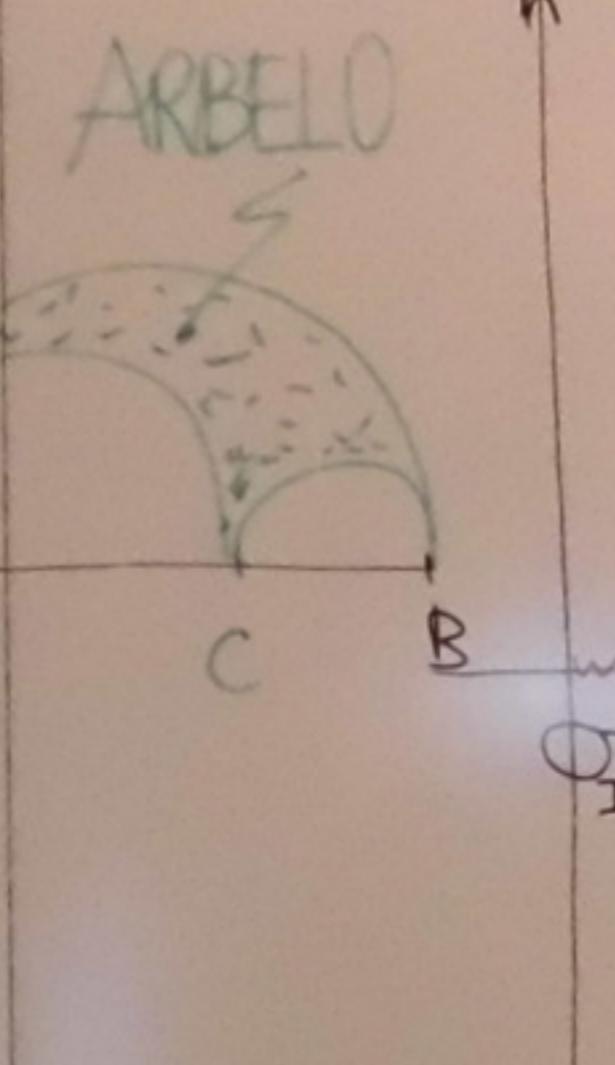
$$\begin{aligned} t_n &= \sigma_n + \tau_n \\ t_n &= \sigma_n^2 + \tau_n^2 \\ \tau &= t_n - \sigma_n \end{aligned}$$

T definita
in modo
in 3D ($\tau \geq 0$)

sforzo 3D

$$\sigma_I \geq \sigma_{II} \geq \sigma_{III}$$

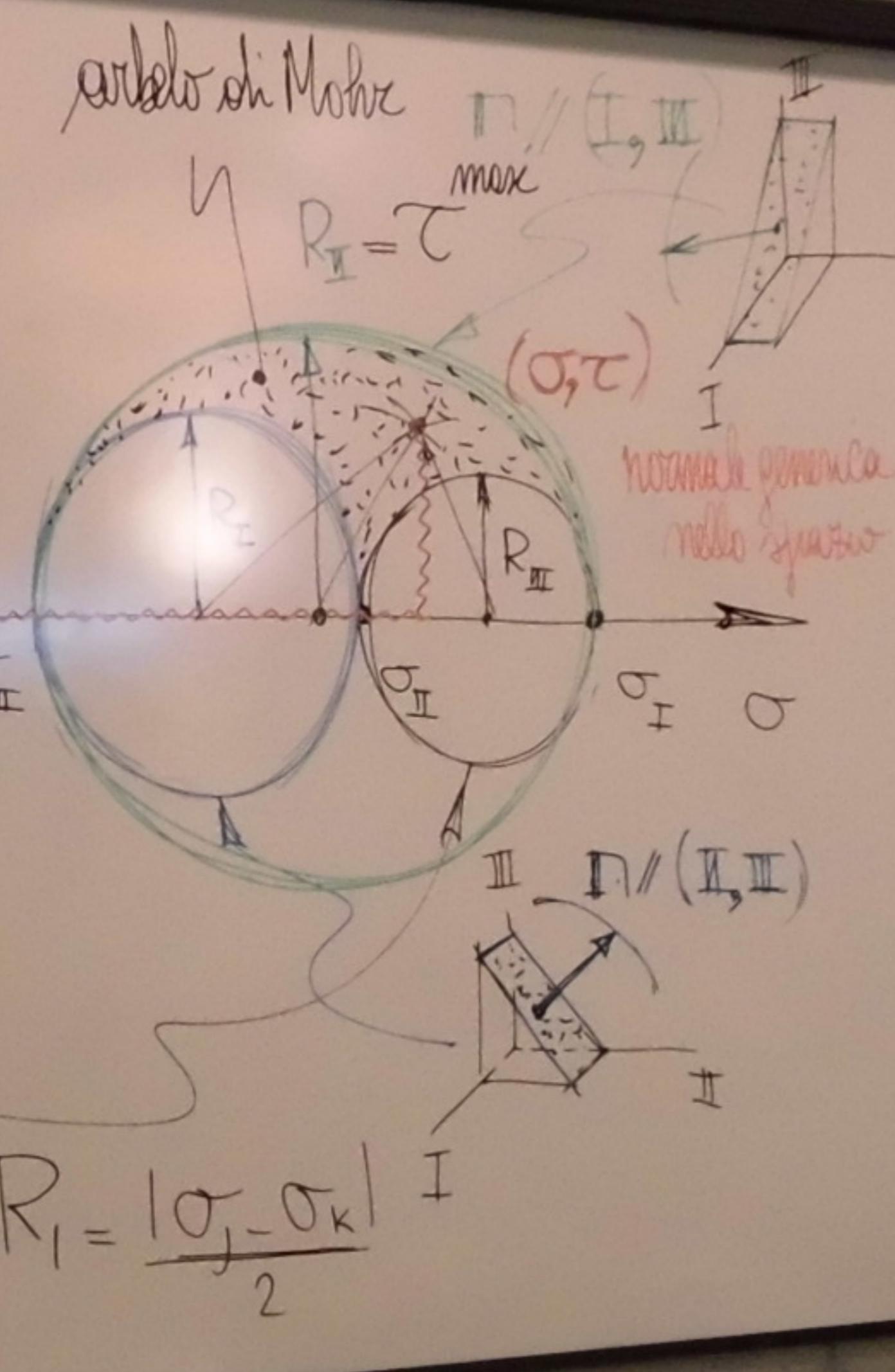
ARBELLO



$\tau \parallel (I, II)$

$\tau \parallel (I, III)$

$\tau \parallel (II, III)$



Resistenza del materiale

$$\tau = \max R_i$$

(raggio del CM più grande)

$$\sigma = \sigma_I, \sigma = \sigma_{III}$$

(su CM più
grande)

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(su CM più
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$$\sigma = \sigma_I, \sigma = \sigma_{III}$$

il CM più grande rappresenta
l'entità dello stato di

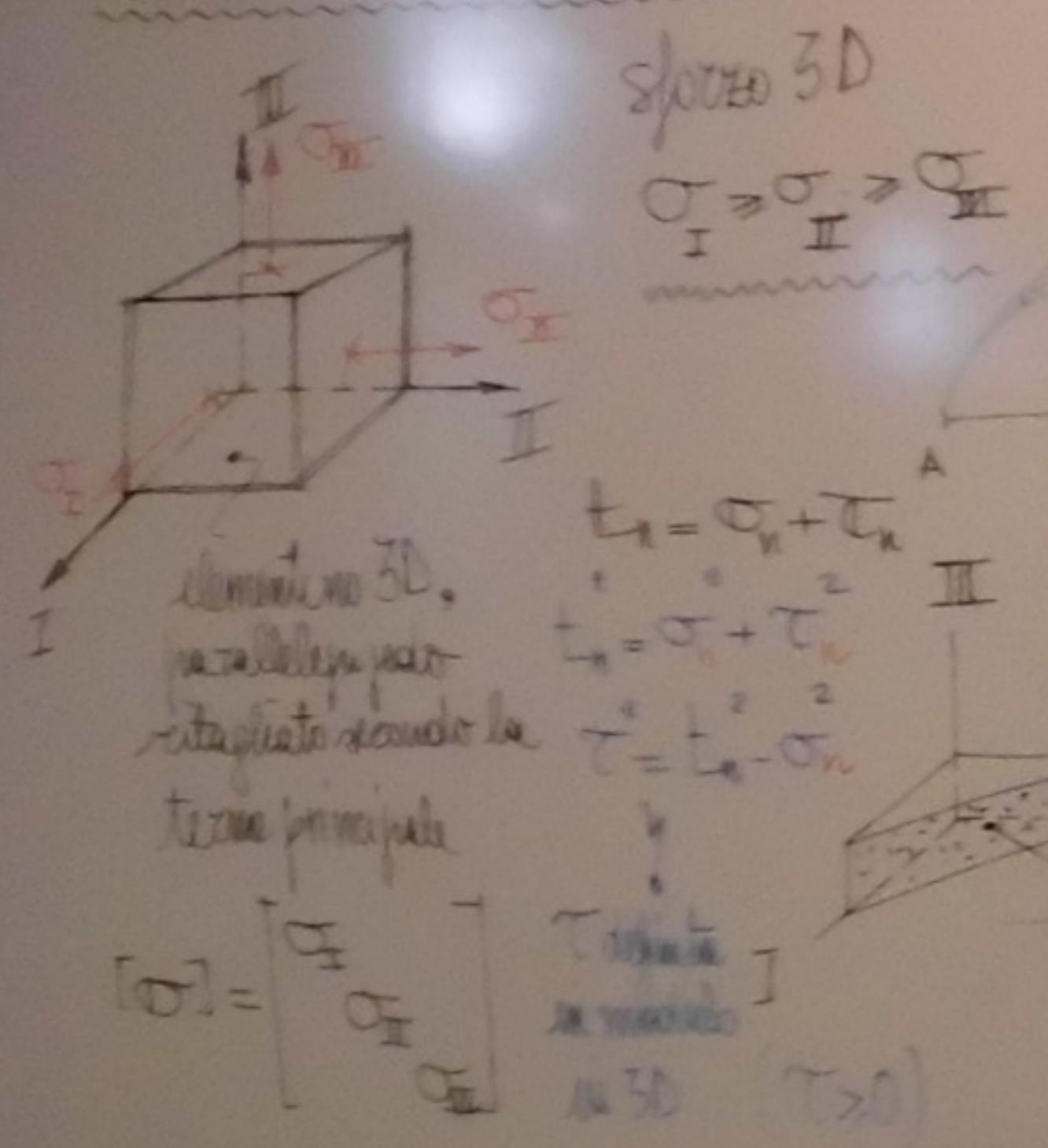
sforzo ai fini della
resistenza del materiale

$$\tau \leq \tau_{\max}$$

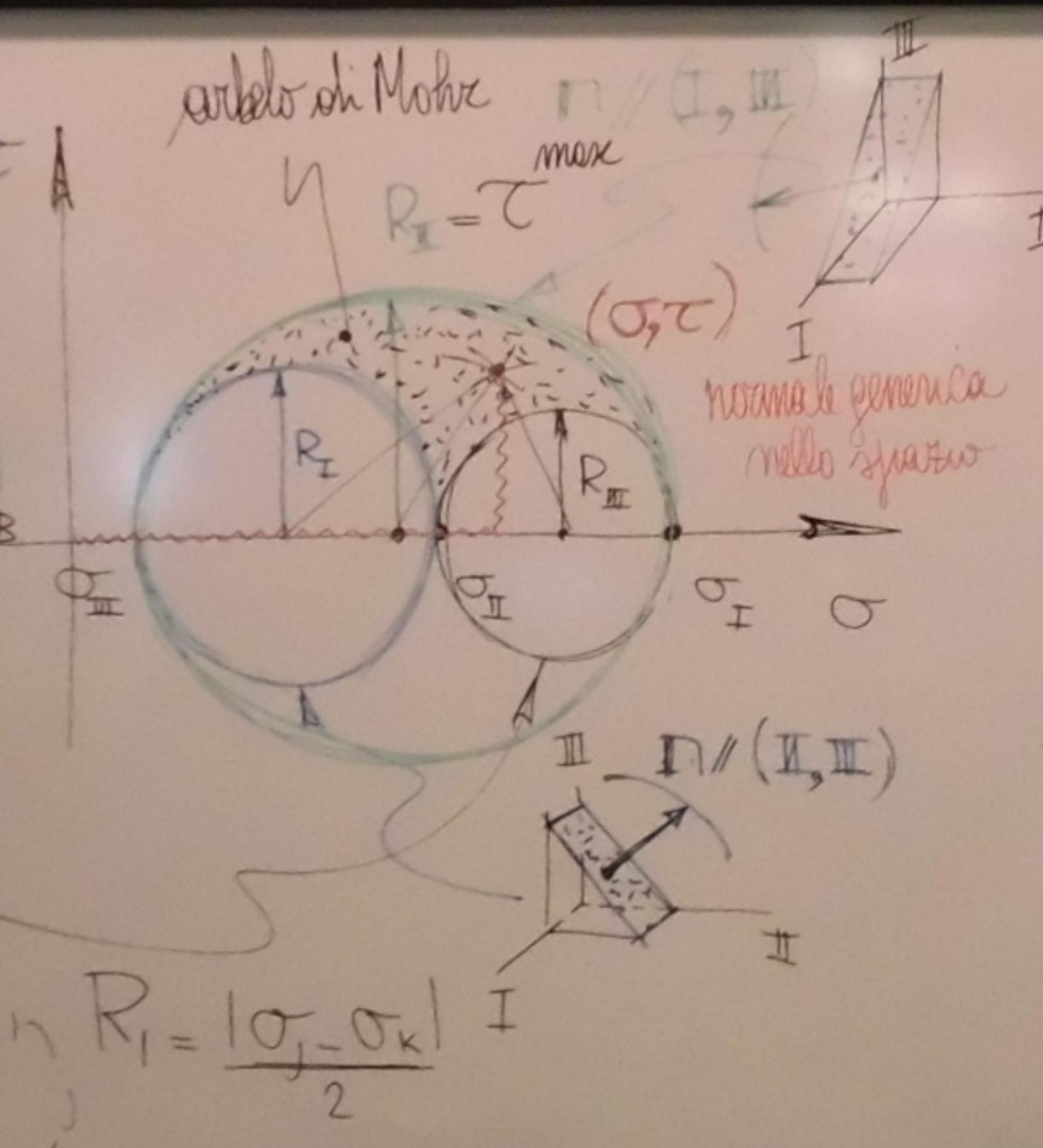
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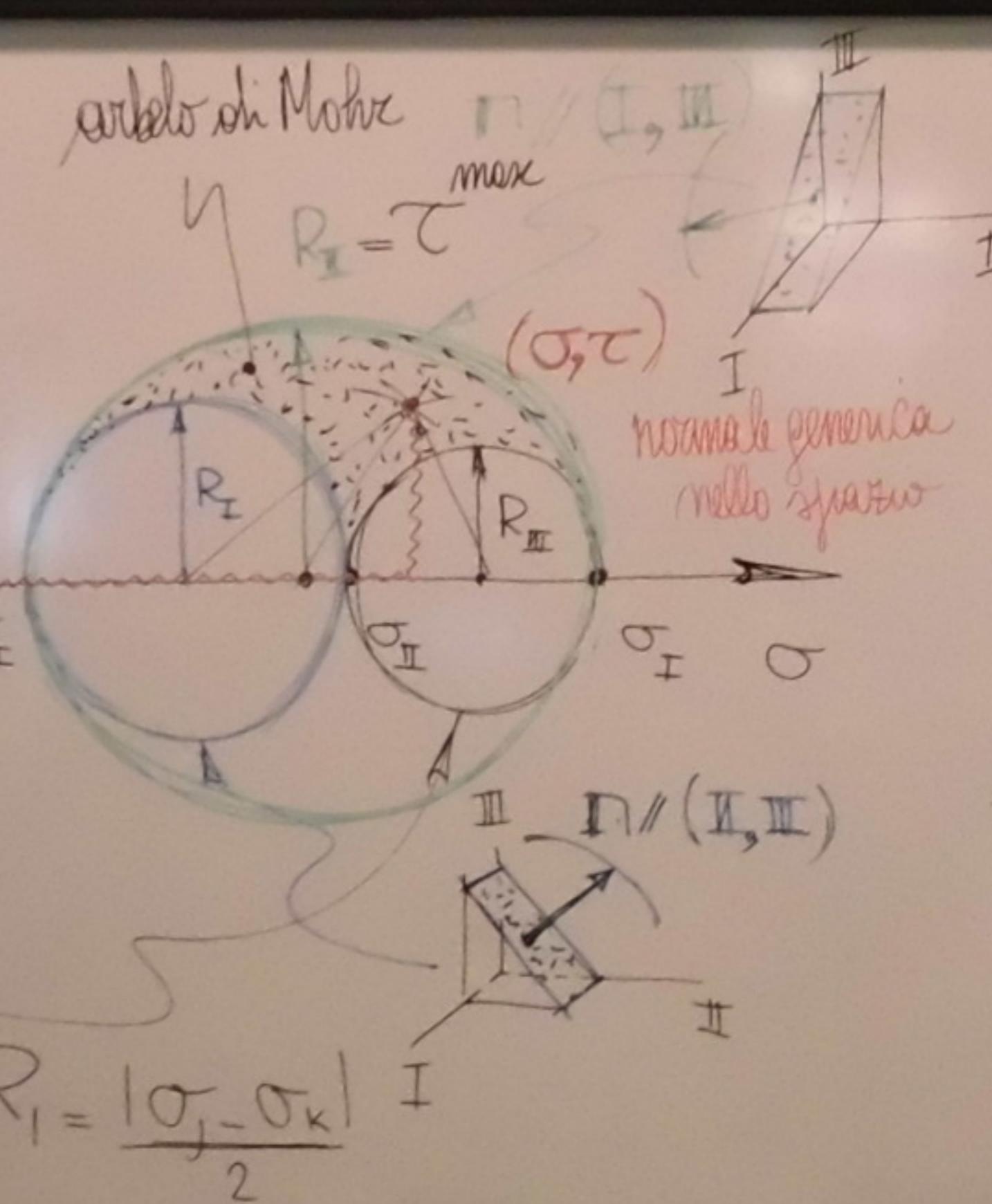
Cerchi e Arco di Mohr



ARCO



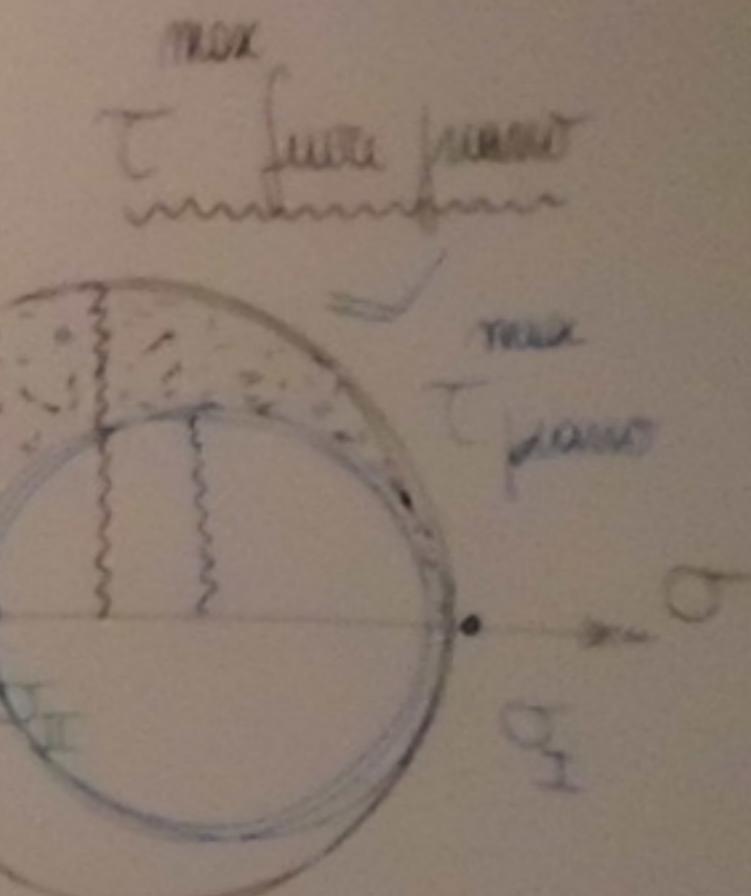
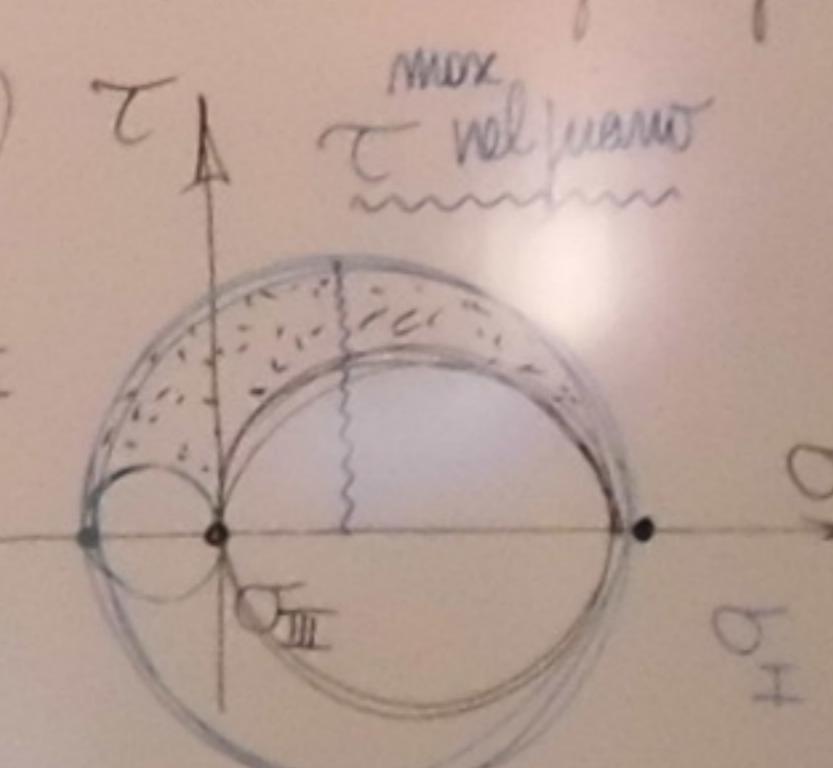
arco di Mohr



Resistenza del materiale

- $\tau = \max R_1$
(raggio del CM più grande)
 - Per stato di forza piano ($\sigma_{III} = 0$)
 τ_{\max} nel piano
 $\sigma = \sigma_I, \tau = \sigma_{II}$
(su CM più grande)
- il CM più grande rappresenta l'entità dello stato di tensioni principali nel piano di segno opposto $\sigma_I \cdot \sigma_{II} < 0$
(come per forza alla DSV)

VS.



tensioni principali nel piano della retta neutra $\sigma_I \cdot \sigma_{II} \geq 0$



Circo e Cerchio di Mohr

spazio

$\sigma_1 > \sigma_2$

verso

verso