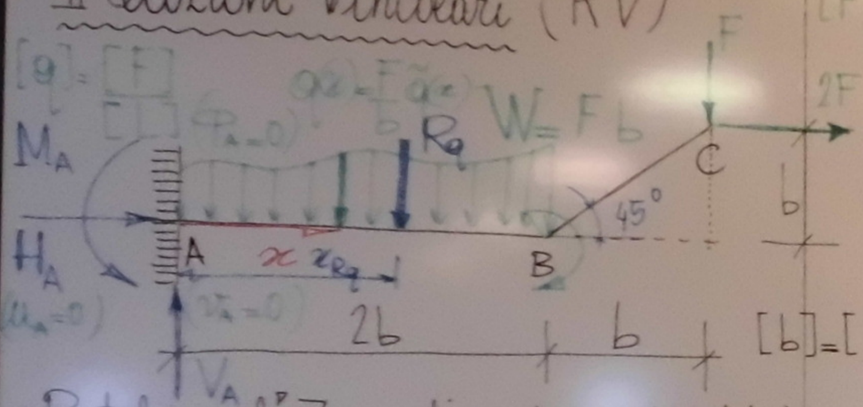
Reazioni Vincolari (RV)

$[q]=[F]$

$q(x)=F\frac{x}{b}$

$W=Fb$



$[F]=[F]$

$[W]=[F]\cdot[L]$

$[b]=[L]$

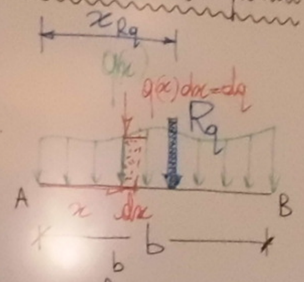
- Postuliamo l'∃nza di opportune azioni statiche, dette Reazioni Vincolari in corrispondenza di ogni gdv fornito (gdl rimosso)
- Equazioni cardinali della statica (piana):

$$\begin{cases} \mathbb{R}=\mathbb{0} \Rightarrow \begin{cases} \sum F_x=0 \Rightarrow H_A+2F=0 \Rightarrow H_A=-2F & \text{equil. alla traslaz. orizz.} \\ \sum F_y=0 \Rightarrow V_A-R_q-F=0 \Rightarrow V_A=R_q+F & \text{equil. alla traslaz. verticale} \end{cases} \\ M_O=\mathbb{0} \end{cases}$$

$$\sum_i M_{O_i}=0 \Rightarrow M_A-M_A^q-Fb-3Fb-2Fb=0$$

$$M_A=M_A^q+6Fb$$

Carico distribuito o ripartito



pto di applicaz.

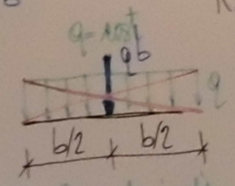
$M_A^q=R_q\,x_{R_q}$

(Th di Varignon)

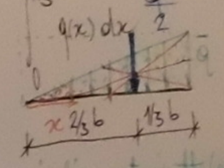
Risultante staticamente equiv. $R_q=\int_0^b q(x)\,dx$ $\quad [R_q]=[F]$

$$x_{R_q}=\frac{M_A^q}{R_q}=\frac{\int_0^b q(x)\,x\,dx}{\int_0^b q(x)\,dx}$$

esempi

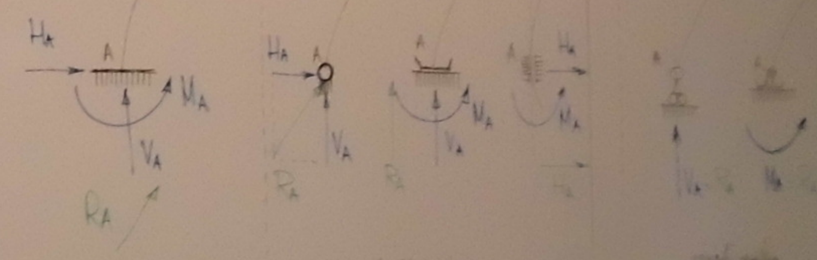


Carico con andamento rettangolare

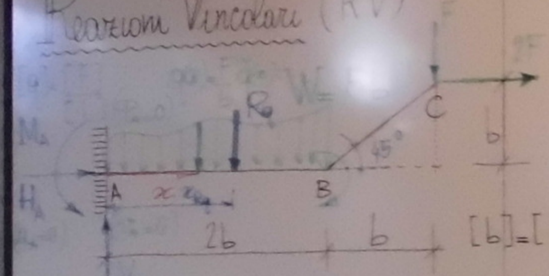


Carico triangolare rettangolo (vuoto): $q(x)=\bar{q}+\frac{\bar{q}}{b}x$

RV dei vincoli visti



Reazioni Vincolari (RV)



$[b]=[L]$

Postuliamo l'∃ne di opportune azioni statiche, dette Reazioni Vincolari in corrispondenza di ogni gdv fornito (gdv ammesso)

Equazioni cardinali della statica (piane):

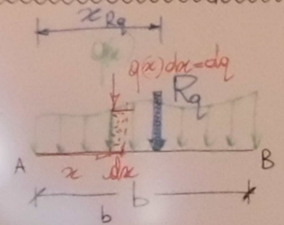
$R=0 \Rightarrow$ equil. alla traslaz. orizz. $\sum F_x = 0 \Rightarrow H_A + F = 0 \Rightarrow H_A = -F$

equil. alla traslaz. verticale $\sum F_y = 0 \Rightarrow V_A - R_q - F = 0 \Rightarrow V_A = R_q + F$

$M_O = 0$ equil. alla rotazione rispetto pol A $\sum M_O = 0 \Rightarrow M_A - M_A^q - Fb - 3Fb - 2Fb = 0$

$M_A = M_A^q + 6Fb$

Carico distribuito o ripartito

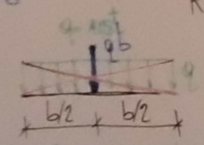


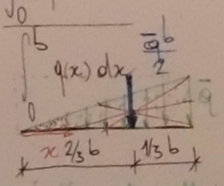
pto di applicaz. $M_A^q = R_q \cdot x$ (Th di Varignon)

Risultante staticamente equiv. $R_q = \int_0^b q(x)\,dx \quad [R_q]=[F]$

$$x_{R_q} = \frac{M_A^q}{R_q} = \frac{\int_0^b q(x)\,x\,dx}{\int_0^b q(x)\,dx}$$

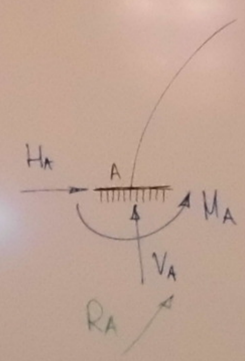
esempi



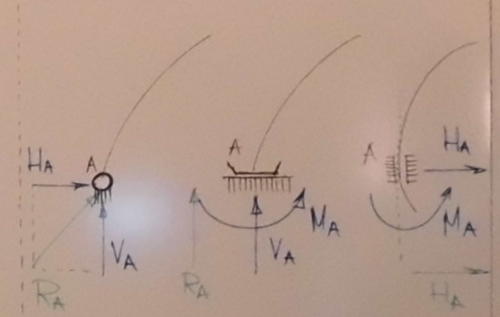


Carico linearmente ripartito (q(x)=0): $q(x) = q_0 + \frac{\bar{q}}{b}x$

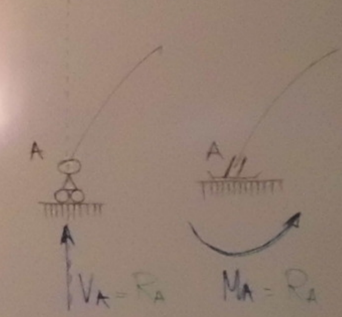
RV dei vincoli visti



vincolo triplo



vincoli doppi



vincoli semplici

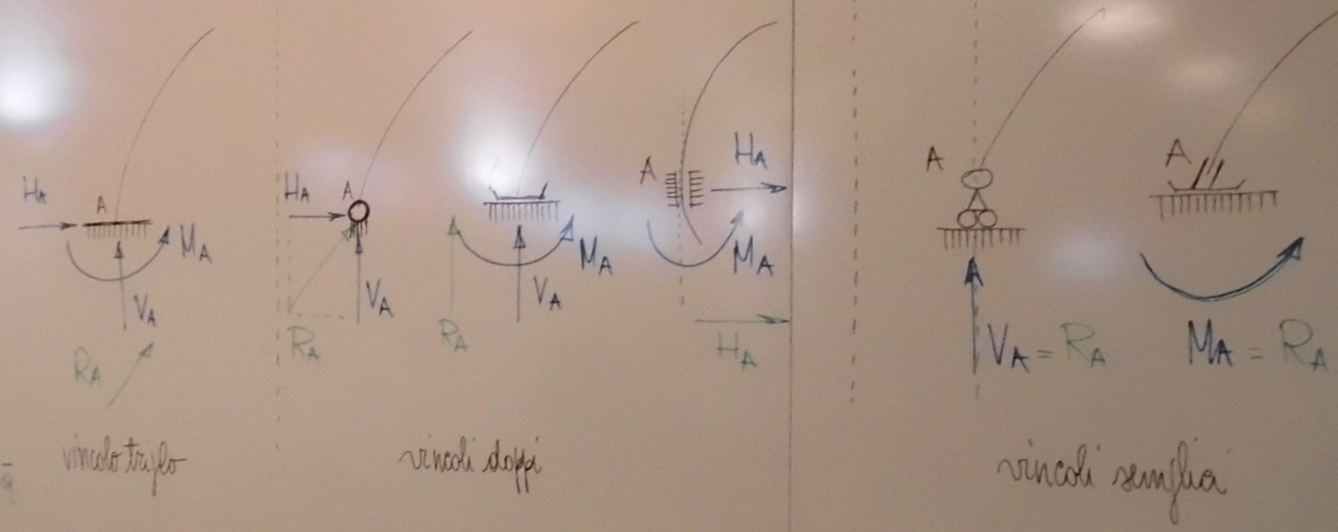
carico distribuito o ripartito

RV dei vincoli visti

$M_A^q = R_q \, x_{R_q}$

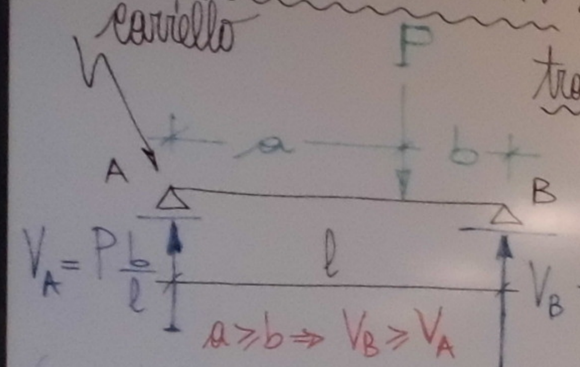
$R_q = \int_0^b q(x)\,dx \qquad [R_q]=[F]$

$x_{R_q} = \frac{M_A^q}{R_q} = \frac{\int_0^b q(x)\,x\,dx}{\int_0^b q(x)\,dx}$



Esempi di calcolo delle RV:

carrello



$V_A = P\frac{b}{l}$ $\qquad$ $V_B = P\frac{a}{l}$

$a \geq b \Rightarrow V_B \geq V_A$

(con cerniera in A $\Rightarrow H_A = 0$, ininfluente)

$$\begin{cases} \sum_i M_{B_i} = 0 \Rightarrow -V_A l + Pb = 0 \\ \sum_i M_{A_i} = 0 \Rightarrow V_B l - Pa = 0 \end{cases}$$

due eq.ni di equil. alla rotazione rispetto a p.ti ≠

$V_A = P\frac{b}{l}$

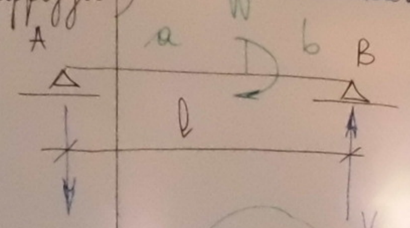
$V_B = P\frac{a}{l}$

Check

$\sum_i F_{y_i} = 0 \Rightarrow V_A + V_B = P \Rightarrow P\frac{b}{l} + P\frac{a}{l} = P\frac{a+b}{l} = P$ OK

trave semplicemente appoggiata (appoggio-appoggio)

(labile alla traslaz. orizzontale: IPOstatica per carichi verticali)

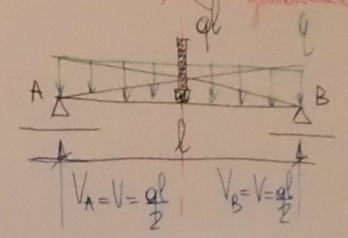


$V_A = V = \frac{W}{l}$ $\qquad$ $\forall a, b$ $\qquad$ $V_B = V = \frac{W}{l}$

$$\begin{cases} ② \sum_i M_{\substack{A_i \\ B_i \\ O_i}} = 0 \Rightarrow V \cdot l = W \\ ① \sum_i F_{y_i} = 0 \Rightarrow V_A = V_B = V \end{cases}$$

$V_A = V_B = V$ formano coppia di forze uguale e contraria alla coppia W applicata

asse di simmetria



$V_A = V = \frac{ql}{2}$ $\qquad$ $V_B = V = \frac{ql}{2}$

carico uniformemente distr.

$\sum_i M_{A_i} = 0 \Rightarrow V_B l - ql\frac{l}{2} = 0$

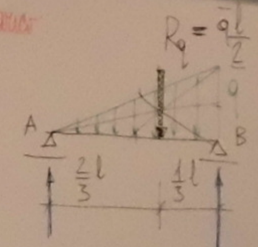
$\sum_i M_{B_i} = 0 \Rightarrow -V_A l + ql\frac{l}{2} = 0$

$\left(\sum_i F_{y_i} = 0 \Rightarrow V_A + V_B = ql\right)$

$\frac{ql}{2} + \frac{ql}{2} = ql$

infatti per simmetria: $V_A = V_B = V$ $\Rightarrow 2V = ql \Rightarrow V = \frac{ql}{2}$

Equil. alla trasl. vert.



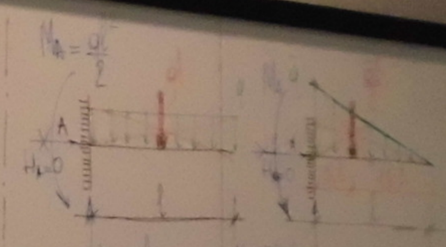
$V_A = R_q \frac{\frac{1}{3}l}{l} = \frac{ql}{6}$ (da formule 1° caso) $\qquad$ $V_B = R_q \frac{\frac{2}{3}l}{l} = \frac{ql}{3}$

carico linearmente distribuito

Check $\sum_i F_{y_i} = 0$

$V_A + V_B \overset{?}{=} \frac{ql}{2}$

$\frac{ql}{6} + \frac{ql}{3} = \frac{3ql}{6} = \frac{ql}{2}$ OK

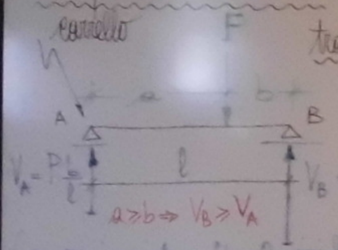


$V_A = R_q = \int_0^l q\,dx = q\int_0^l dx = ql$

$M_A = M_A^q = \int_0^l q\,x\,dx = q\int_0^l x\,dx = \frac{ql^2}{2}$

Esempi di calcolo delle RV:

carrello

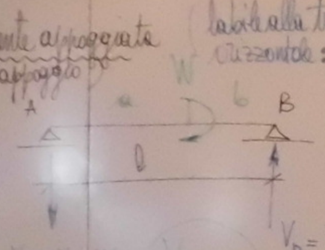


$V_A = P\frac{b}{l}$ $\quad V_B = P\frac{a}{l}$

$a \geq b \Rightarrow V_B \geq V_A$

con estremo in A $\rightarrow H_A = 0$, confluente

trave semplicemente appoggiata (appoggio-appoggio)



$V_A = V = \frac{W}{l}$ $\quad (\forall a, b)$ $\quad V_B = V = \frac{W}{l}$

labile alla traslaz. orizzontale: l'isostatica per carichi verticali

$$\begin{cases} \sum_i M_{B_i} = 0 \Rightarrow -V_A l + Pb = 0 \\ \sum_i M_{A_i} = 0 \end{cases}$$

$V_A = P\frac{b}{l}$

due sforzi di equil. alla rotazione rispetto a p.ti $\neq$

$\sum_i M_{A_i} = 0 \Rightarrow V_B l - Pa = 0$

$V_B = P\frac{a}{l}$

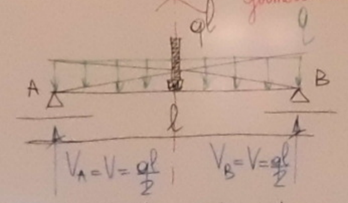
Check

$\sum_i F_{y_i} = 0 \Rightarrow V_A + V_B = P \Rightarrow Pb + Pa = P\frac{a+b}{l} = P$

$$\begin{cases} ② \sum_i M_{A_i, B_i, O_i} = 0 \Rightarrow V \cdot l = W \\ ① \sum_i F_{y_i} = 0 \Rightarrow V_A = V_B = V \end{cases}$$

$V_A = V_B = V$ formano coppia di forze uguali e contrarie alla coppia W applicata

asse di simmetria geometrica e di carico



$V_A = V = \frac{ql}{2}$ $\quad V_B = V = \frac{ql}{2}$

carico uniformemente distr.

$\sum_i M_{A_i} = 0 \Rightarrow V_B l - ql\frac{l}{2} = 0$

$\sum_i M_{B_i} = 0 \Rightarrow -V_A l + ql\frac{l}{2} = 0$

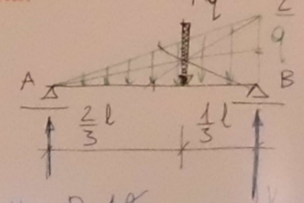
$\left(\sum_i F_{y_i} = 0 \Rightarrow V_A + V_B = ql\right)$

$\frac{ql}{2} + \frac{ql}{2} \overset{\checkmark}{=} ql$

infatti per simmetria: $V_A = V_B = V$ $\Rightarrow 2V = ql \Rightarrow V = \frac{ql}{2}$

Equil. alla trasl. vert.

$R_q = \frac{ql}{2}$



$V_A = R_q \frac{1}{3}\frac{l}{l} = \frac{ql}{6}$ (da formule 1° caso)

$V_B = R_q \frac{2}{3}\frac{l}{l} = \frac{ql}{3}$

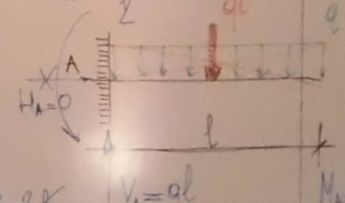
carico linearmente distribuito

Check: $\sum_i F_{y_i} = 0$

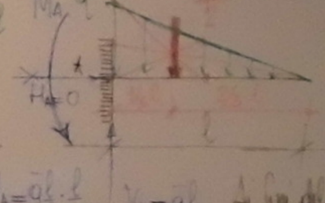
$V_A + V_B \overset{?}{=} q\frac{l}{2}$

$\frac{ql}{6} + \frac{2ql}{3} = \frac{3}{6}ql \overset{\checkmark}{=} \frac{ql}{2}$ OK

$M_A = \frac{ql^2}{2}$



$V_A = ql$



$M_A = \frac{ql}{2}\cdot\frac{l}{3} = \frac{ql^2}{6}$ $\quad V_A = \frac{ql}{2}$

trave a mensola (asta incastrata)

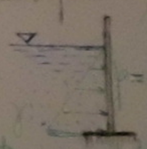
carico uniforme

$V_A = R_q = \int_0^l q\,dx = q[x]_0^l = ql$

$M_A = M_A^q = \int_0^{ql} q\,x\,dx = q\left[\frac{x^2}{2}\right]_0^l = \frac{ql^2}{2}$

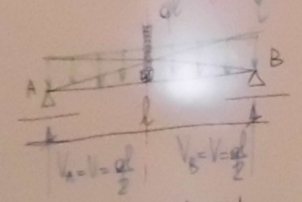
carico lineare

in spinta idrostatica su parete



A fine di calcolo delle RV è sufficiente considerare il risultante del carico distr., del tutto equivalente ai fini statici

asse di simmetria geometrica e di carico



carico uniformemente distr.

$\sum_i M_{A_i} = 0 \Rightarrow V_B \ell - q\ell\frac{\ell}{2} = 0$

$\sum_i M_{B_i} = 0 \Rightarrow -V_A \ell + q\ell\frac{\ell}{2} = 0$

$\sum_i F_{y_i} = 0 \Rightarrow V_A + V_B = q\ell$

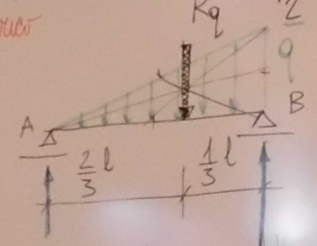
$\frac{q\ell}{2} + \frac{q\ell}{2} = q\ell$

per simmetria: $V_A = V_B = V$

$\Rightarrow 2V = q\ell \Rightarrow V = \frac{q\ell}{2}$

Sym. alla Pant. Vert.

$R_q = \frac{q\ell}{2}$



$V_A = R_q \frac{1}{3}\frac{\ell}{\ell} = \frac{q\ell}{6}$ (da formule 1° caso)

$V_B = R_q \frac{2}{3}\frac{\ell}{\ell} = \frac{q\ell}{3}$

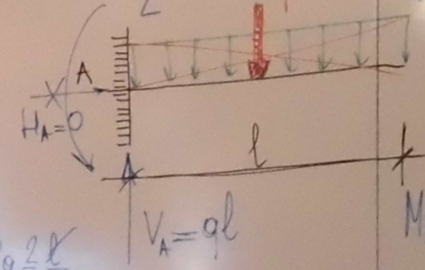
carico linearmente distribuito.

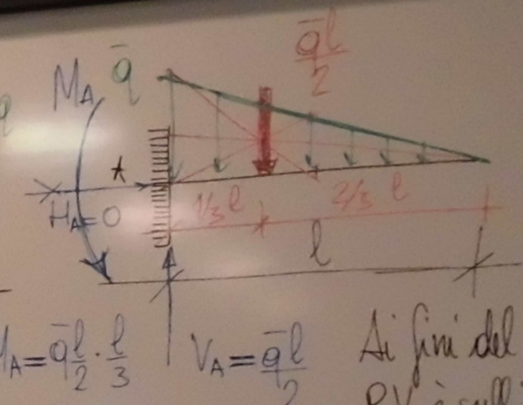
Check $\sum_i F_{y_i} = 0$

$V_A + V_B \overset{?}{=} \frac{q\ell}{2}$

$\frac{q\ell}{6} + \frac{2q\ell}{3} = \frac{3}{6}q\ell = \frac{q\ell}{2}$ OK

$M_A = \frac{q\ell^2}{2}$





$M_A = \frac{\bar{q}\ell}{2} \cdot \frac{\ell}{3} = \frac{\bar{q}\ell^2}{6}$

$V_A = \frac{\bar{q}\ell}{2}$

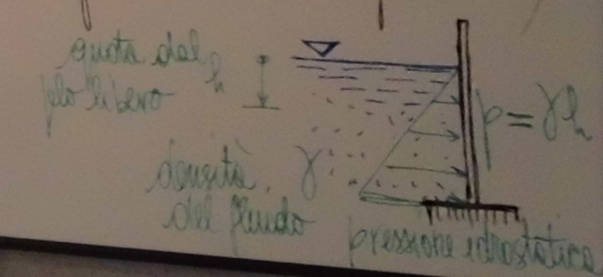
Ai fini del calcolo delle RV è sufficiente considerare il risultante del carico distr., del tutto equivalente ai fini statici

trave a mensola (asta incastrata)

carico uniforme

$V_A = R_q = \int_0^\ell q\,dx = q[x]_0^\ell = q\ell$

$M_A = M_A^q = \int_0^\ell q x\,dx = q\left[\frac{x^2}{2}\right]_0^\ell = \frac{q\ell^2}{2}$

carico lineare

(es. spinta idrostatica su paratia)