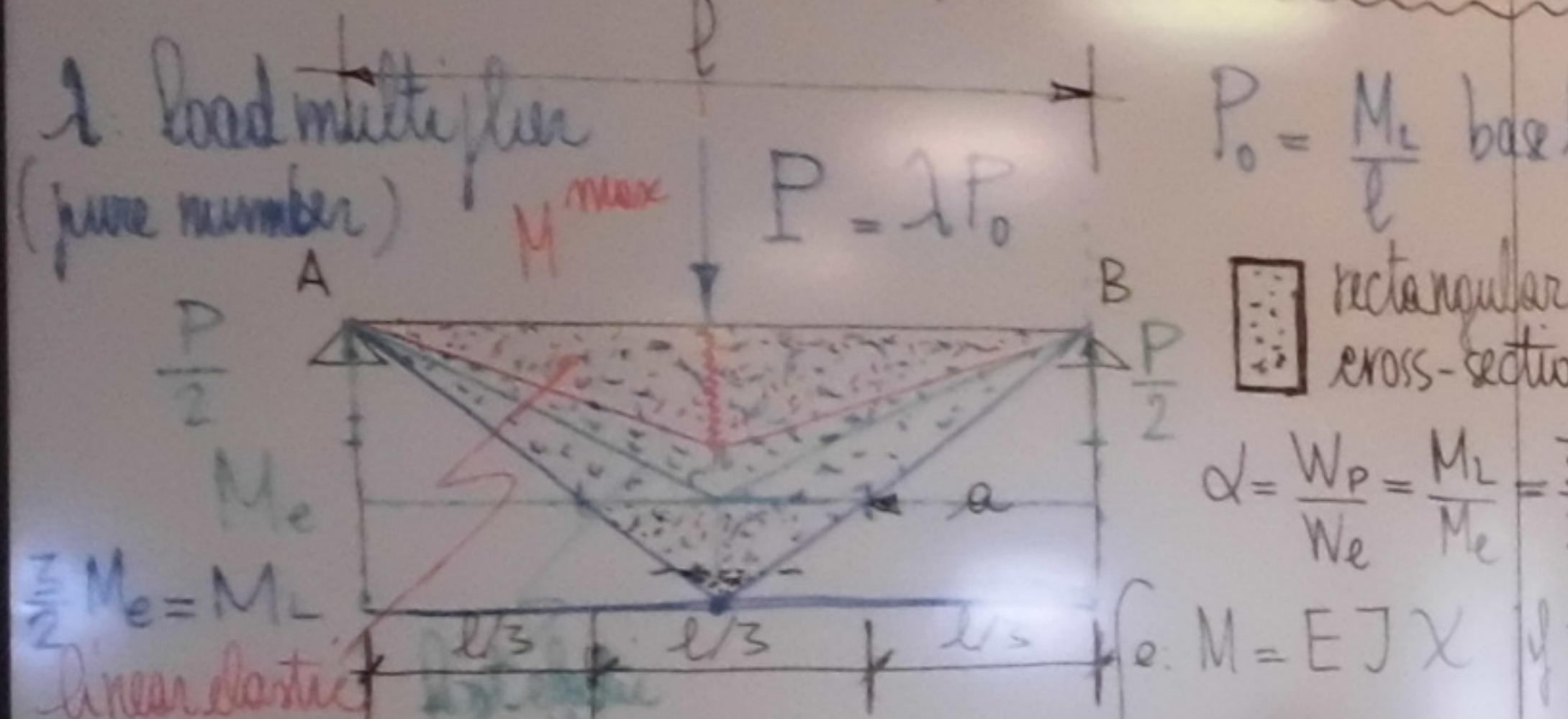


Plastic hinge concept and structural plastic collapse



- Let us refer to a statically-determined structure where the bending moment distribution is set just by equilibrium. Specifically, the maximum moment is obtained as:

$$M_{max} = \frac{P l}{2} = \frac{P l}{4} = \lambda \frac{M_e l}{4} = \lambda M_e$$

- On increasing the amount of $P = 2P_0$ leading to plastic collapse

$$\alpha = \frac{W_p}{W_e} = \frac{M_e}{M_c} = \frac{3}{2} \text{ shape factor}$$

$$M = EI X$$

$$X \leq X_e$$

$$M = M_e (1 - \frac{1}{3} \frac{(X_e)^2}{X})$$

$$= M_e (\frac{3}{2} - \frac{(X_e)^2}{X})$$

$$\frac{M}{M_e} = \frac{11}{12} M_e \approx 0.916 M_e$$

$$X \leq X_e$$

$$X \rightarrow \infty, M \rightarrow M_c$$

$$\sigma \propto \epsilon$$

$$M_e$$

$$X \leq X_e$$

- First yield is achieved when:

$$N_{max}^e = M_e \Rightarrow \frac{1}{4} M_e = M_e \Rightarrow \lambda = 4 \frac{M_e}{M_c} = \frac{8}{3}$$

- Then, linear elastic range applies to $0 \leq \lambda \leq \frac{8}{3}$

- Going beyond that, plasticification applies, to a central region that has a max extension of $\frac{l}{3}$ at $M_{max}^e = M_c \Rightarrow \frac{2}{4} M_c = M_c \Rightarrow \lambda = 4$

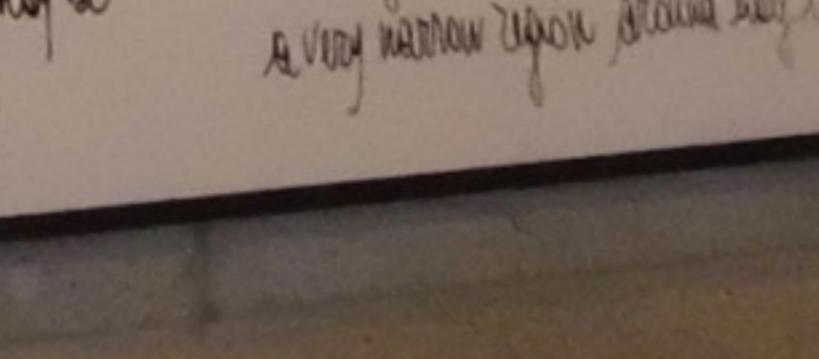
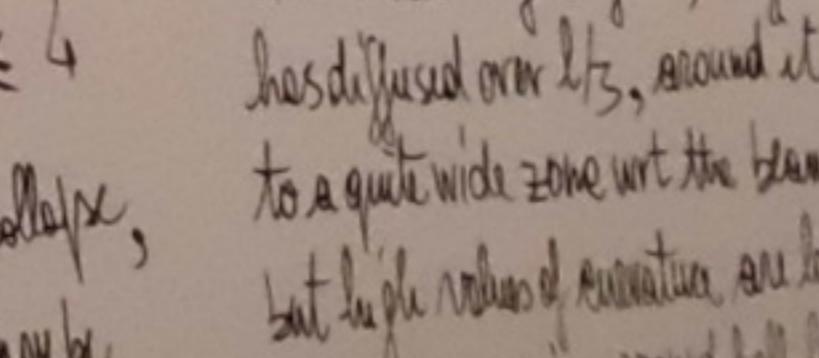
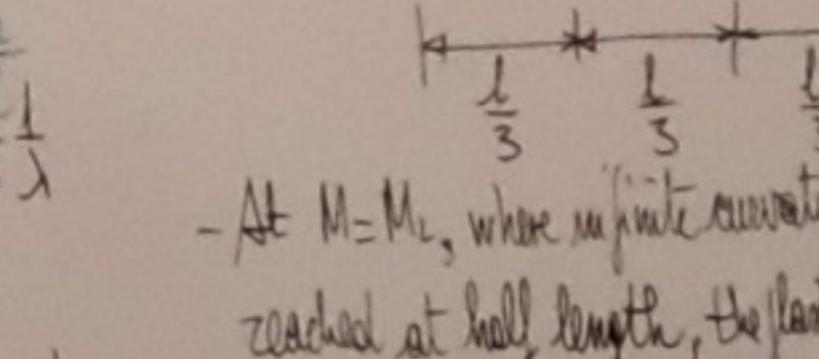
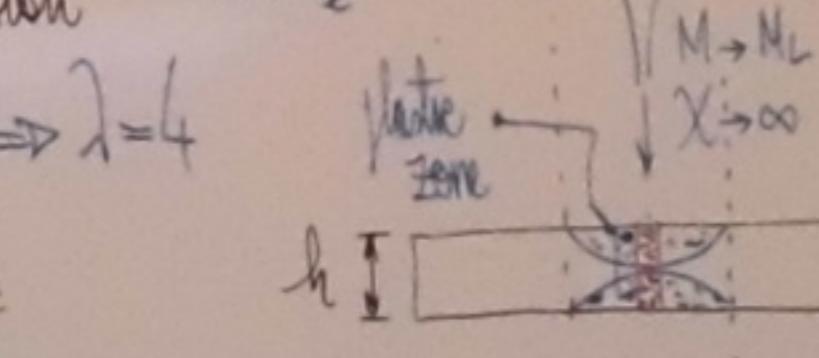
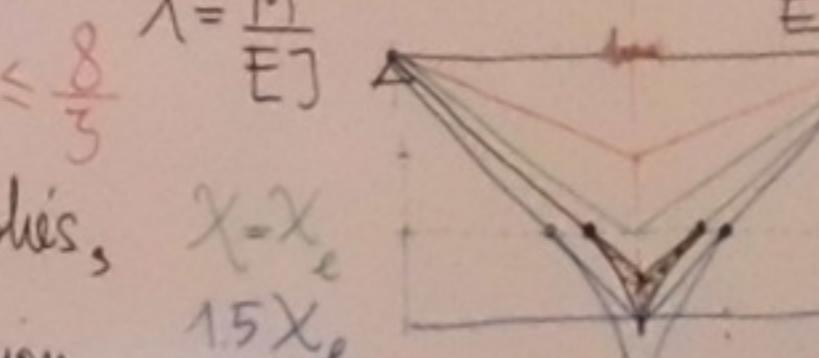
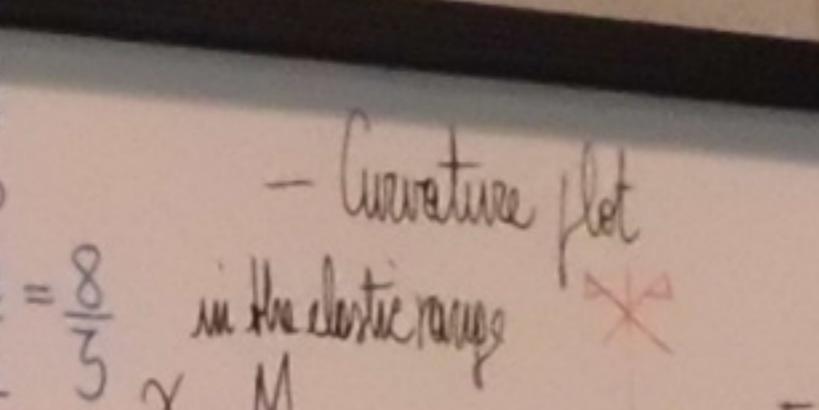
$$M_a = \frac{P}{2} a = M_e \Rightarrow a = \frac{2 M_e}{P} = \frac{2 M_e}{2 M_c} = \frac{M_e}{M_c}$$

$$\lambda = \frac{8}{3}; \frac{a}{l} = \frac{1}{2}$$

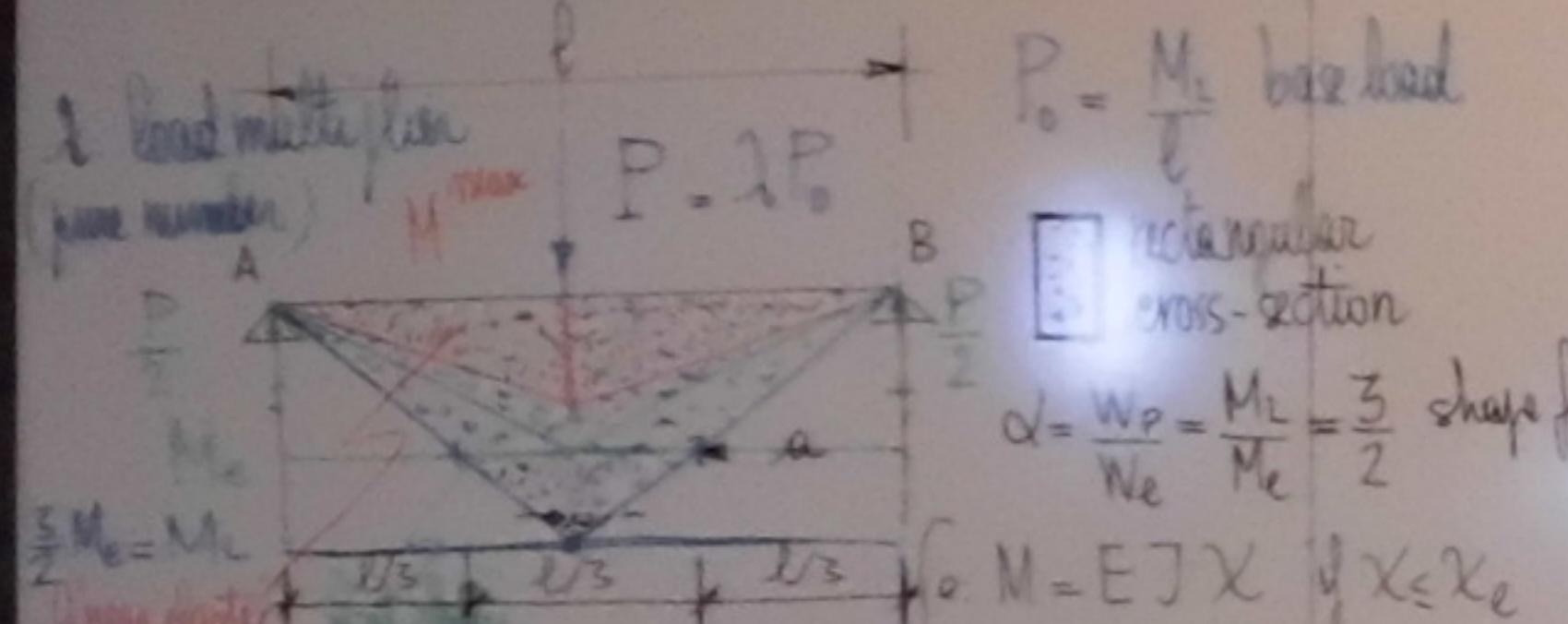
$$\lambda = 4; \frac{a}{l} = \frac{1}{3}$$

- Elasto-plastic range applies when $\frac{8}{3} \leq \lambda \leq 4$

and $\lambda = \lambda_c = 4$ obviously marks plastic collapse, in the sense that infinite local curvature may be reached in the cross-section at half length



Plastic hinge concept and structural plastic collapse



Let us refer to a statically-determined structure where the bending moment distribution is set just by equilibrium. Specifically, the maximum moment is obtained as:

$$M = \frac{P l}{2} = \frac{P l}{4}$$

$$\alpha = \frac{W_p}{W_e} = \frac{M_e}{M_c} = \frac{3}{2}$$

shape factor

- First yield is achieved when:

$$M_{\max}^{\text{yield}} = M_c \Rightarrow \frac{1}{4} M_c = M_c \Rightarrow \lambda = 4 \frac{M_c}{M_e} = \frac{8}{3}$$

Then, linear elastic range applies to $0 < \lambda < \frac{8}{3}$

- Going beyond that, plasticification applies to a central region that has a max extension of $\frac{l}{3}$ at $M_{\max}^{\text{yield}} = M_c \Rightarrow \frac{2}{4} M_c = M_c \Rightarrow \lambda = 4$

$$M_c - \frac{P}{2} \alpha = M_c \Rightarrow \alpha = \frac{2 M_c}{P} = \frac{2 M_c}{2 M_c} = 1$$

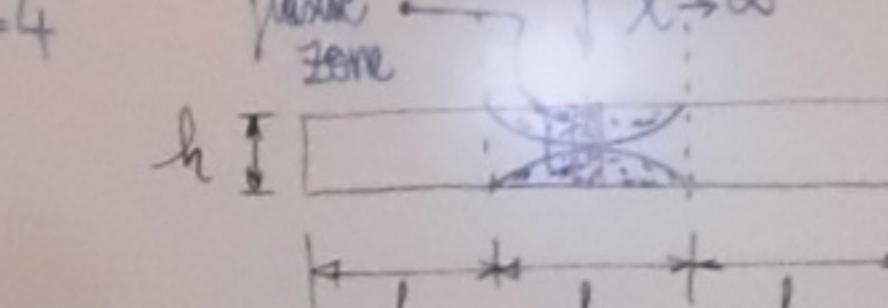
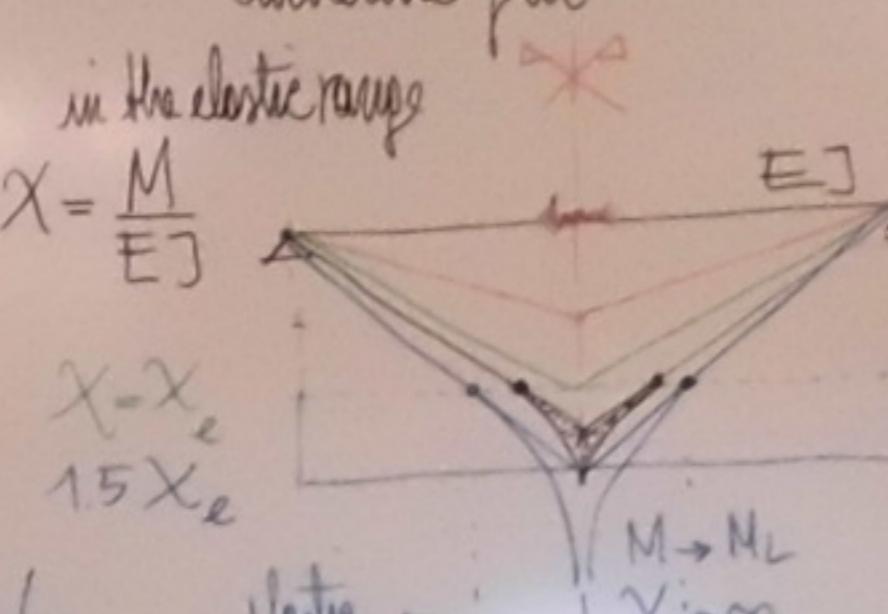
$$\lambda = 8; \frac{\alpha}{l} = \frac{1}{2} \quad \frac{\alpha}{l} = \frac{2}{3} \frac{M_c}{M_e} = \frac{4}{3} \lambda$$

$$\lambda = 4; \frac{\alpha}{l} = \frac{1}{3}$$

- Elasto-plastic range applies when $\frac{8}{3} \leq \lambda \leq 4$

and $\lambda = \lambda_c = 4$ obviously marks plastic collapse, in the sense that infinite local curvature may be reached in the cross-section at half length

- Curvature plot



- This leads to the concept of plastic hinge, where a localized rotation may occur in the plastic range at constant $M = M_c$.

$$\int X dx = \int \frac{dP}{dx} dx = \Delta \phi$$

high curve

finite rotation

plastic hinge mechanism

assumption of structural behaviour

$\Delta \phi \cdot L_c = 2 \frac{M_c \Delta \phi}{l} = 2 M_c \Delta \phi = 0$

arbitrary

internal force dissipation

$M_c \Delta \phi = D \Rightarrow \lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

$D = 2 M_c \Delta \phi$

constant internal rotation in the plastic hinge

$\Delta \phi = 2 \pi / 3$

collapse multiplier

$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

collapse multiplier

$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

collapse multiplier

$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

collapse multiplier

$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

collapse multiplier

$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

collapse multiplier

$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

collapse multiplier

$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

collapse multiplier

$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

collapse multiplier

$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

collapse multiplier

$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

collapse multiplier

$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

collapse multiplier

$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

collapse multiplier

$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

collapse multiplier

$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

collapse multiplier

$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

collapse multiplier

$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

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$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

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work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

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constant internal rotation in the plastic hinge

$\lambda_c = 4$

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constant internal rotation in the plastic hinge

$\lambda_c = 4$

collapse multiplier

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work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

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work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

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constant internal rotation in the plastic hinge

$\lambda_c = 4$

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$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

collapse multiplier

$\lambda_c = 4$

work produced in the plastic hinge at constant $M = M_c$

constant internal rotation in the plastic hinge

$\lambda_c = 4$

collapse multiplier

$\lambda_c = 4$ </

Plastic hinge concept and structural static collapse

$$P = \frac{M_L}{l}$$

critical load

cross-section

$$\alpha = \frac{M_y}{M_c} = \frac{3}{2}$$

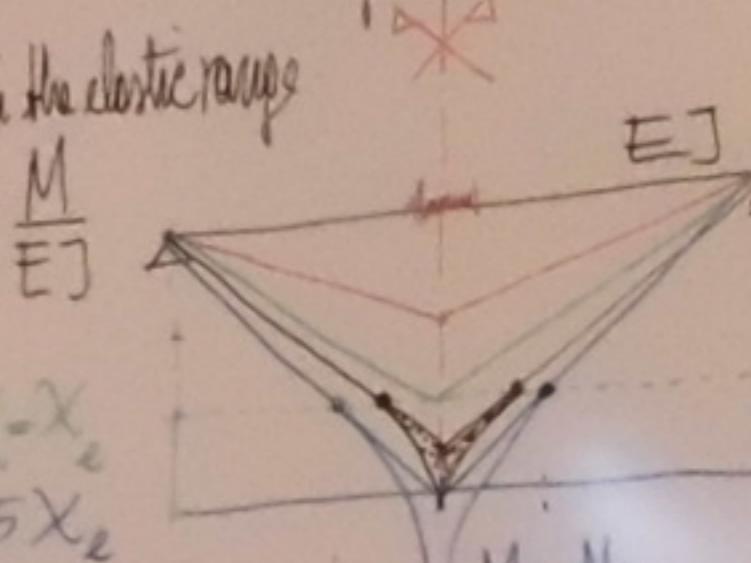
$$\text{First yield is achieved when } \frac{M}{M_c} = \frac{1}{4} \Rightarrow \frac{1}{4} M_L = M_c \Rightarrow l = 4 \frac{M_L}{M_c} = \frac{8}{3}$$

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in the elastic range

$$X = \frac{M}{EI}$$

- Curvature plot



Then, three elastic range applies to $0 \leq \lambda \leq \frac{8}{3}$

Going beyond that, plasticification applies, to a central region that has a max extension of $\frac{l}{3}$ at $M = M_L \Rightarrow \frac{2}{3} M_L = M_c \Rightarrow \lambda = 4$

$$M_c - \frac{P}{2} x = M_c \Rightarrow x = \frac{2M_c}{P} = \frac{2M_L}{l}$$

$$x = \frac{2}{3} M_L = \frac{4}{3} l$$

$$= \frac{2}{3} \cdot \frac{4}{3} l = \frac{8}{9} l$$

$$= \frac{2}{3} \cdot \frac{2}{3} M_L = \frac{4}{9} M_L$$

$$= \frac{4}{9} M_L \cdot \frac{4}{3} l = \frac{16}{27} M_L l$$

$$= \frac{16}{27} M_L l \cdot \frac{1}{2} = \frac{8}{27} M_L l$$

$$= \frac{8}{27} M_L l \cdot \frac{1}{2} = \frac{4}{27} M_L l$$

$$= \frac{4}{27} M_L l \cdot \frac{1}{2} = \frac{2}{27} M_L l$$

$$= \frac{2}{27} M_L l \cdot \frac{1}{2} = \frac{1}{27} M_L l$$

$$= \frac{1}{27} M_L l \cdot \frac{1}{2} = \frac{1}{54} M_L l$$

$$= \frac{1}{54} M_L l \cdot \frac{1}{2} = \frac{1}{108} M_L l$$

$$= \frac{1}{108} M_L l \cdot \frac{1}{2} = \frac{1}{216} M_L l$$

$$= \frac{1}{216} M_L l \cdot \frac{1}{2} = \frac{1}{432} M_L l$$

$$= \frac{1}{432} M_L l \cdot \frac{1}{2} = \frac{1}{864} M_L l$$

$$= \frac{1}{864} M_L l \cdot \frac{1}{2} = \frac{1}{1728} M_L l$$

$$= \frac{1}{1728} M_L l \cdot \frac{1}{2} = \frac{1}{3456} M_L l$$

$$= \frac{1}{3456} M_L l \cdot \frac{1}{2} = \frac{1}{6912} M_L l$$

$$= \frac{1}{6912} M_L l \cdot \frac{1}{2} = \frac{1}{13824} M_L l$$

$$= \frac{1}{13824} M_L l \cdot \frac{1}{2} = \frac{1}{27648} M_L l$$

$$= \frac{1}{27648} M_L l \cdot \frac{1}{2} = \frac{1}{55296} M_L l$$

$$= \frac{1}{55296} M_L l \cdot \frac{1}{2} = \frac{1}{110592} M_L l$$

$$= \frac{1}{110592} M_L l \cdot \frac{1}{2} = \frac{1}{221184} M_L l$$

$$= \frac{1}{221184} M_L l \cdot \frac{1}{2} = \frac{1}{442368} M_L l$$

$$= \frac{1}{442368} M_L l \cdot \frac{1}{2} = \frac{1}{884736} M_L l$$

$$= \frac{1}{884736} M_L l \cdot \frac{1}{2} = \frac{1}{1769472} M_L l$$

$$= \frac{1}{1769472} M_L l \cdot \frac{1}{2} = \frac{1}{3538944} M_L l$$

$$= \frac{1}{3538944} M_L l \cdot \frac{1}{2} = \frac{1}{7077888} M_L l$$

$$= \frac{1}{7077888} M_L l \cdot \frac{1}{2} = \frac{1}{1415576} M_L l$$

$$= \frac{1}{1415576} M_L l \cdot \frac{1}{2} = \frac{1}{2831152} M_L l$$

$$= \frac{1}{2831152} M_L l \cdot \frac{1}{2} = \frac{1}{5662304} M_L l$$

$$= \frac{1}{5662304} M_L l \cdot \frac{1}{2} = \frac{1}{11324608} M_L l$$

$$= \frac{1}{11324608} M_L l \cdot \frac{1}{2} = \frac{1}{22649216} M_L l$$

$$= \frac{1}{22649216} M_L l \cdot \frac{1}{2} = \frac{1}{45298432} M_L l$$

$$= \frac{1}{45298432} M_L l \cdot \frac{1}{2} = \frac{1}{90596864} M_L l$$

$$= \frac{1}{90596864} M_L l \cdot \frac{1}{2} = \frac{1}{181193728} M_L l$$

$$= \frac{1}{181193728} M_L l \cdot \frac{1}{2} = \frac{1}{362387456} M_L l$$

$$= \frac{1}{362387456} M_L l \cdot \frac{1}{2} = \frac{1}{724774912} M_L l$$

$$= \frac{1}{724774912} M_L l \cdot \frac{1}{2} = \frac{1}{1449549824} M_L l$$

$$= \frac{1}{1449549824} M_L l \cdot \frac{1}{2} = \frac{1}{2899099648} M_L l$$

$$= \frac{1}{2899099648} M_L l \cdot \frac{1}{2} = \frac{1}{5798199296} M_L l$$

$$= \frac{1}{5798199296} M_L l \cdot \frac{1}{2} = \frac{1}{11596398592} M_L l$$

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$$= \frac{1}{23192797184} M_L l \cdot \frac{1}{2} = \frac{1}{46385594368} M_L l$$

$$= \frac{1}{46385594368} M_L l \cdot \frac{1}{2} = \frac{1}{92771188736} M_L l$$

$$= \frac{1}{92771188736} M_L l \cdot \frac{1}{2} = \frac{1}{185542377472} M_L l$$

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$$= \frac{1}{371084754944} M_L l \cdot \frac{1}{2} = \frac{1}{742169509888} M_L l$$

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$$= \frac{1}{1484339019776} M_L l \cdot \frac{1}{2} = \frac{1}{2968678039552} M_L l$$

$$= \frac{1}{2968678039552} M_L l \cdot \frac{1}{2} = \frac{1}{5937356079104} M_L l$$

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$$= \frac{1}{11874712158208} M_L l \cdot \frac{1}{2} = \frac{1}{23749424316416} M_L l$$

$$= \frac{1}{23749424316416} M_L l \cdot \frac{1}{2} = \frac{1}{47498848632832} M_L l$$

$$= \frac{1}{47498848632832} M_L l \cdot \frac{1}{2} = \frac{1}{94997697265664} M_L l$$

$$= \frac{1}{94997697265664} M_L l \cdot \frac{1}{2} = \frac{1}{189995394531328} M_L l$$

$$= \frac{1}{189995394531328} M_L l \cdot \frac{1}{2} = \frac{1}{379990789062656} M_L l$$

$$= \frac{1}{379990789062656} M_L l \cdot \frac{1}{2} = \frac{1}{759981578125312} M_L l$$

$$= \frac{1}{759981578125312} M_L l \cdot \frac{1}{2} = \frac{1}{1519963156250624} M_L l$$

$$= \frac{1}{1519963156250624} M_L l \cdot \frac{1}{2} = \frac{1}{3039926312501248} M_L l$$

$$= \frac{1}{3039926312501248} M_L l \cdot \frac{1}{2} = \frac{1}{6079852625002496} M_L l$$

$$= \frac{1}{6079852625002496} M_L l \cdot \frac{1}{2} = \frac{1}{12159705250004992} M_L l$$

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$$= \frac{1}{389110568000159744} M_L l \cdot \frac{1}{2} = \frac{1}{778221136000319488} M_L l$$

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$$= \frac{1}{1556442272000638976} M_L l \cdot \frac{1}{2} = \frac{1}{3112884544001277952} M_L l$$

$$= \frac{1}{3112884544001277952} M_L l \cdot \frac{1}{2} = \frac{1}{6225769088002555904} M_L l$$

$$= \frac{1}{6225769088002555904} M_L l \cdot \frac{1}{2} = \frac{1}{12451538176005111808} M_L l$$

$$= \frac{1}{12451538176005111808} M_L l \cdot \frac{1}{2} = \frac{1}{24903076352010223616} M_L l$$

$$= \frac{1}{24903076352010223616} M_L l \cdot \frac{1}{2} = \frac{1}{49806152704020447232} M_L l$$

$$= \frac{1}{49806152704020447232} M_L l \cdot \frac{1}{2} = \frac{1}{99612305408040894464} M_L l$$

$$= \frac{1}{99612305408040894464} M_L l \cdot \frac{1}{2} = \frac{1}{199224610816081788928} M_L l$$

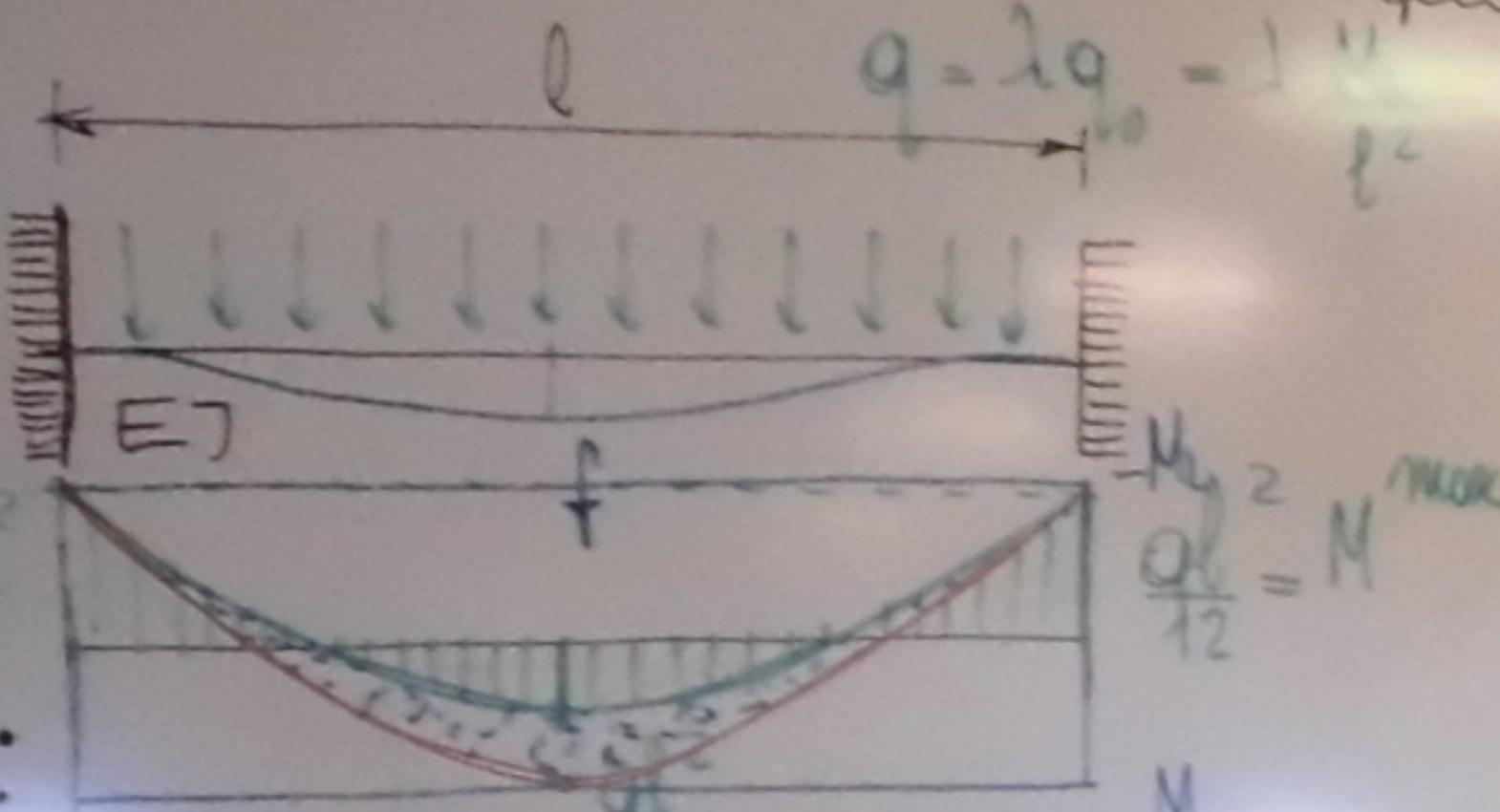
$$= \frac{1}{199224610816081788928} M_L l \cdot \frac{1}{2} = \frac{1}{398449221632163577856} M_L l$$

$$= \frac{1}{398449221632163577856} M_L l \cdot \frac{1}{2} = \frac{1}{796898443264327155712} M_L l$$

$$= \frac{1}{$$

Elasto-plastic evolutive analysis (in the plastic hinge concept)

electric solution:



$$f = \frac{1}{384} \frac{q l^4}{E J} = \frac{1}{96 \cdot 4} \frac{M_L l^2}{E J} \Rightarrow f = \frac{M_L l^2}{E J} = \frac{\lambda}{96 \cdot 4}$$

$$M_{\max} = \frac{q l^2}{8} = \frac{1}{8} \frac{M_L l^2}{E J} = \frac{1}{8} M_L$$

statically-undetermined structure (hypostatic).

- First yield: electric range: $0 < \lambda < \lambda_E = 8$

$$M_{\max} = \frac{q l^2}{12} = M_e \Rightarrow \frac{\lambda}{12} M_L = M_e \Rightarrow \lambda_e = \frac{12 M_e}{M_L} = 12 \frac{2}{3} = 8$$

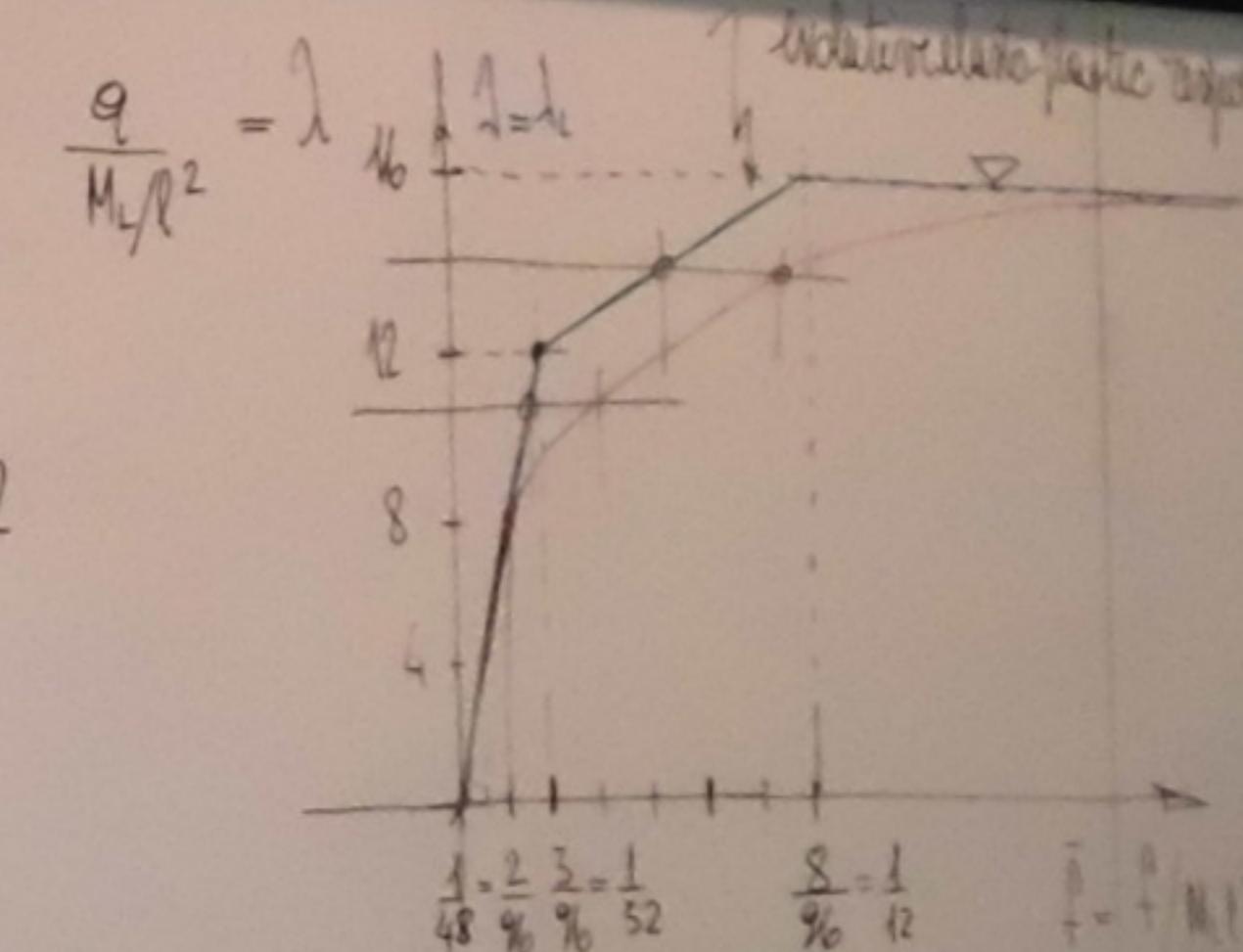
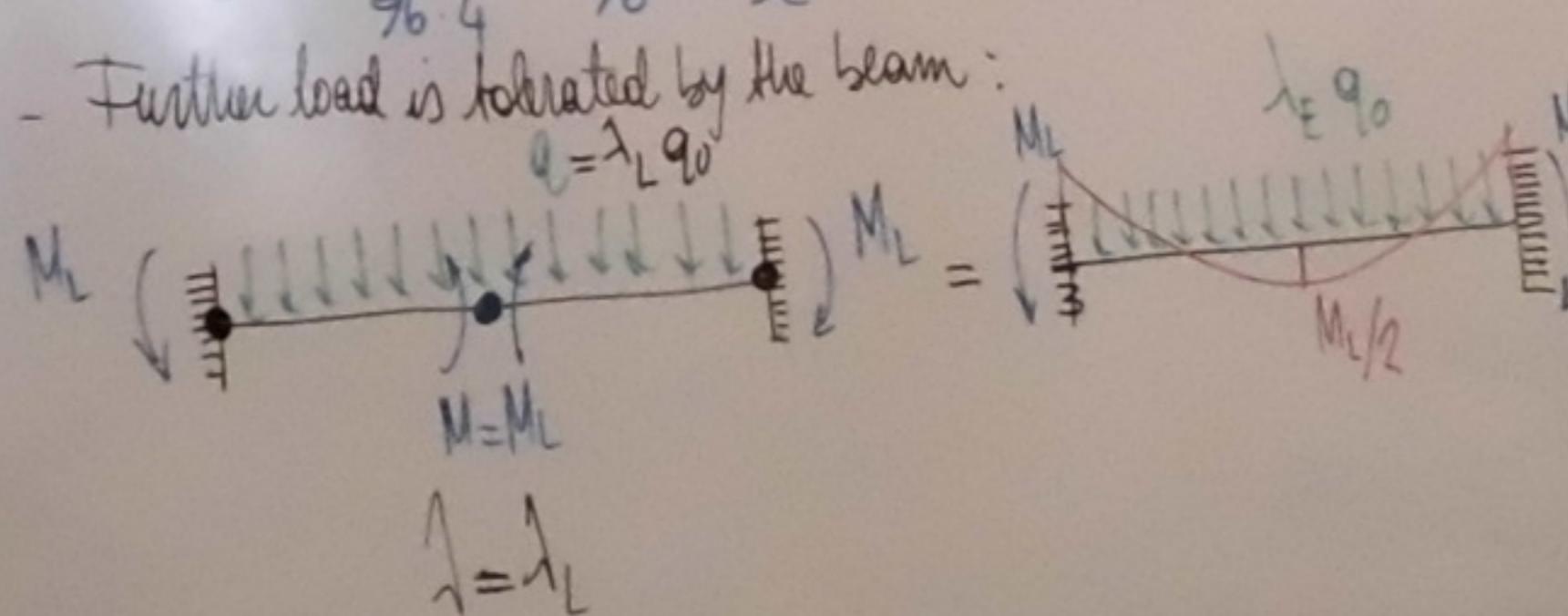
$$\bar{f} = \frac{8}{96 \cdot 4} = \frac{2}{96}$$

- In the plastic hinge hyp., first plastic hinges appear at: $0 < \lambda < \lambda_E = 12$

$$M_{\max} = \frac{q l^2}{12} = 1 \Rightarrow \frac{\lambda}{12} M_L = M_L \Rightarrow \lambda_E = 12$$

$$\bar{f}_E = \frac{12}{96 \cdot 4} = \frac{3}{96} = \frac{1}{32}$$

Further load is tolerated by the beam:

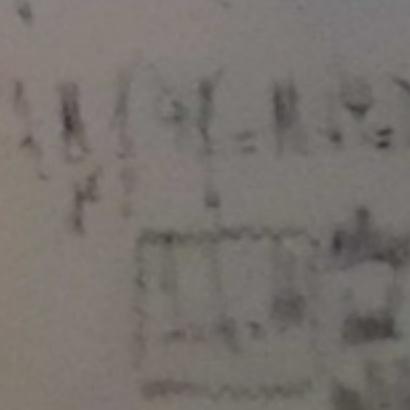


$$\Delta q = \Delta \lambda q_0$$

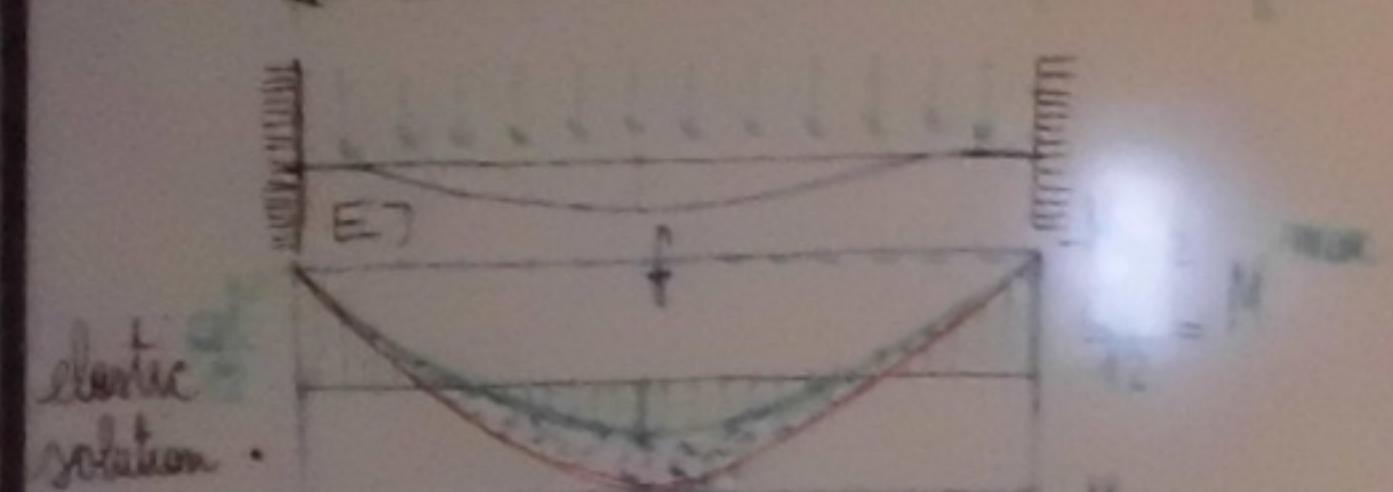
$$\Delta \lambda = \frac{5}{384} \frac{q_0 l^3}{E J} = \frac{5}{384} \frac{M_L l^2}{E J} = \frac{5}{384} \frac{M_L l^2}{8 E J} = \frac{5}{3072} \frac{M_L l^2}{E J} \approx \frac{8}{96}$$

post-yield load deflection
load-angle incrementing
additively, neglect the frame
rigidity and multiply

collapse mechanism



Elasto-plastic iterative analysis (on the plastic hinge concept)



$$\bullet f = \frac{1}{48} \frac{q l^3}{E J} - \frac{1}{96} \frac{M_L l}{E J} = \frac{\lambda}{E J}$$

statically indeterminate structure (hypostatic).

$$\bullet M = \frac{q l^2}{12} = M_L \quad \lambda = \frac{M}{M_L}$$

- First yield: elastic range: $0 \leq \lambda \leq \lambda_e = 8$

$$M_{\text{max}} = \frac{q l^2}{12} = M_e \Rightarrow \frac{\lambda}{12} M_L = M_e \Rightarrow \lambda_e = \frac{12 M_e}{M_L} = 12 \cdot \frac{2}{3} = 8$$

$$\bar{f} = \frac{8}{96} = \frac{1}{12}$$

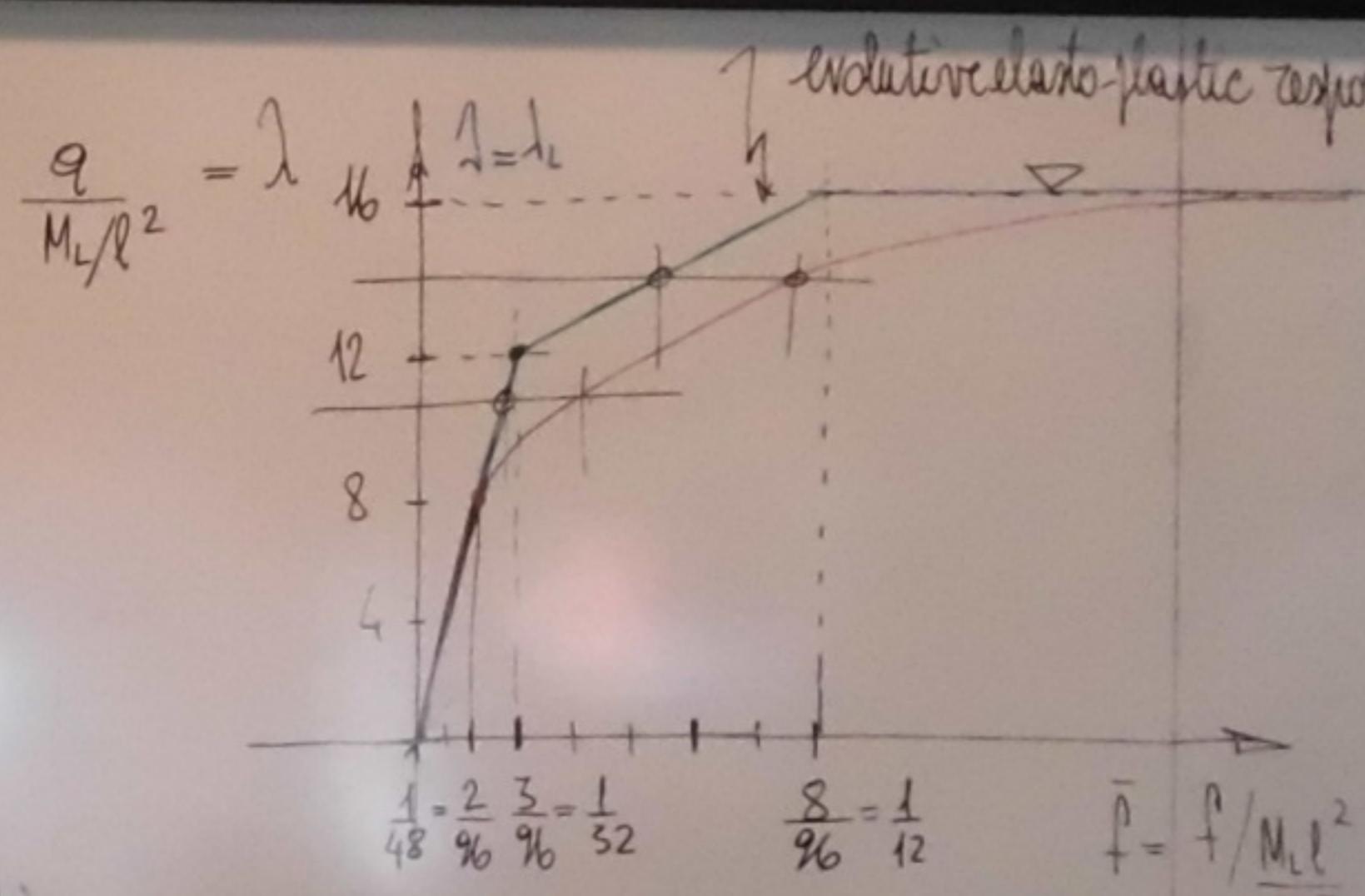
- In the plastic hinge hyp., first plastic hinges appear at: $0 \leq \lambda \leq \lambda_E = 12$

$$M_{\text{max}} = \frac{q l^2}{12} = M_L \Rightarrow \frac{\lambda}{12} M_L \leq \lambda_E = 12$$

$$\bar{f}_E = \frac{12}{96} = \frac{1}{8} = \frac{1}{52} M_L$$

- Further load is tolerated by the beam:

$$M_L (\lambda = \lambda_E) = M_L \quad \lambda = \lambda_E = \frac{M_L}{M_L} = \frac{1}{1} = \lambda_0$$

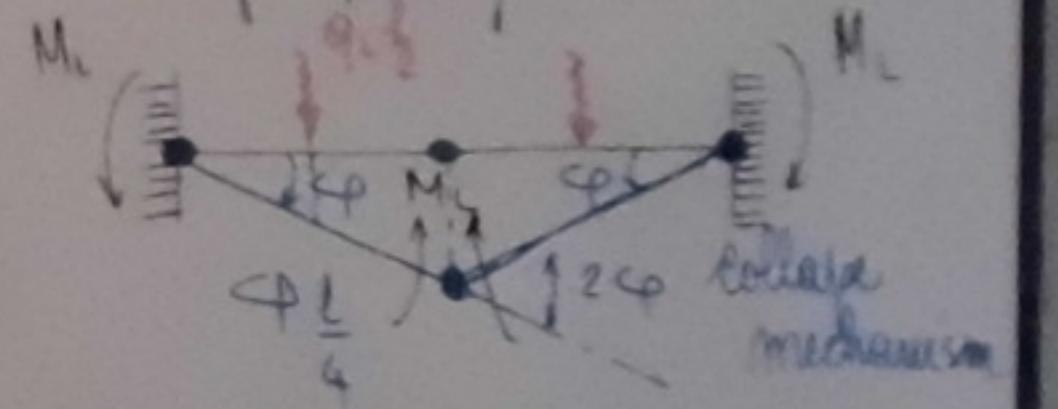


$$\Delta f = \frac{5}{384} \frac{\lambda q l^4}{E J}$$

$$\Delta \lambda = \frac{M_L^2}{8} = \frac{M_L^2}{2} \rightarrow \Delta \lambda = 4 = \frac{5}{96} \frac{M_L^2}{E J} = \frac{5}{96} \frac{M_L^2}{E J} \leftrightarrow \frac{8}{96}$$

evolution elasto-plastic response with sequence of plastic hinge activations
piece-wise linear load deflection curve
that, though underestimating
the deflections, targets the final
collapse load multiplier

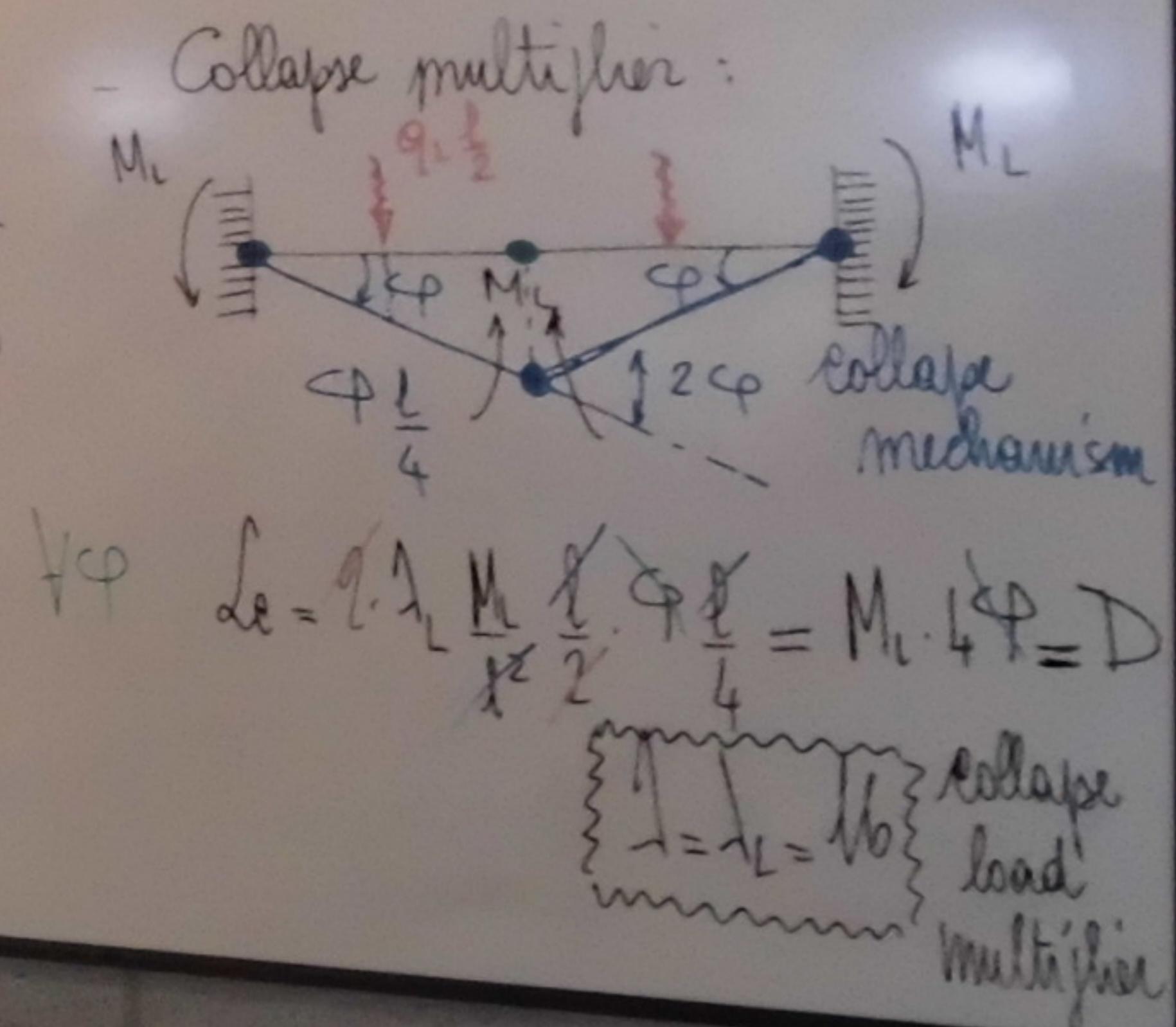
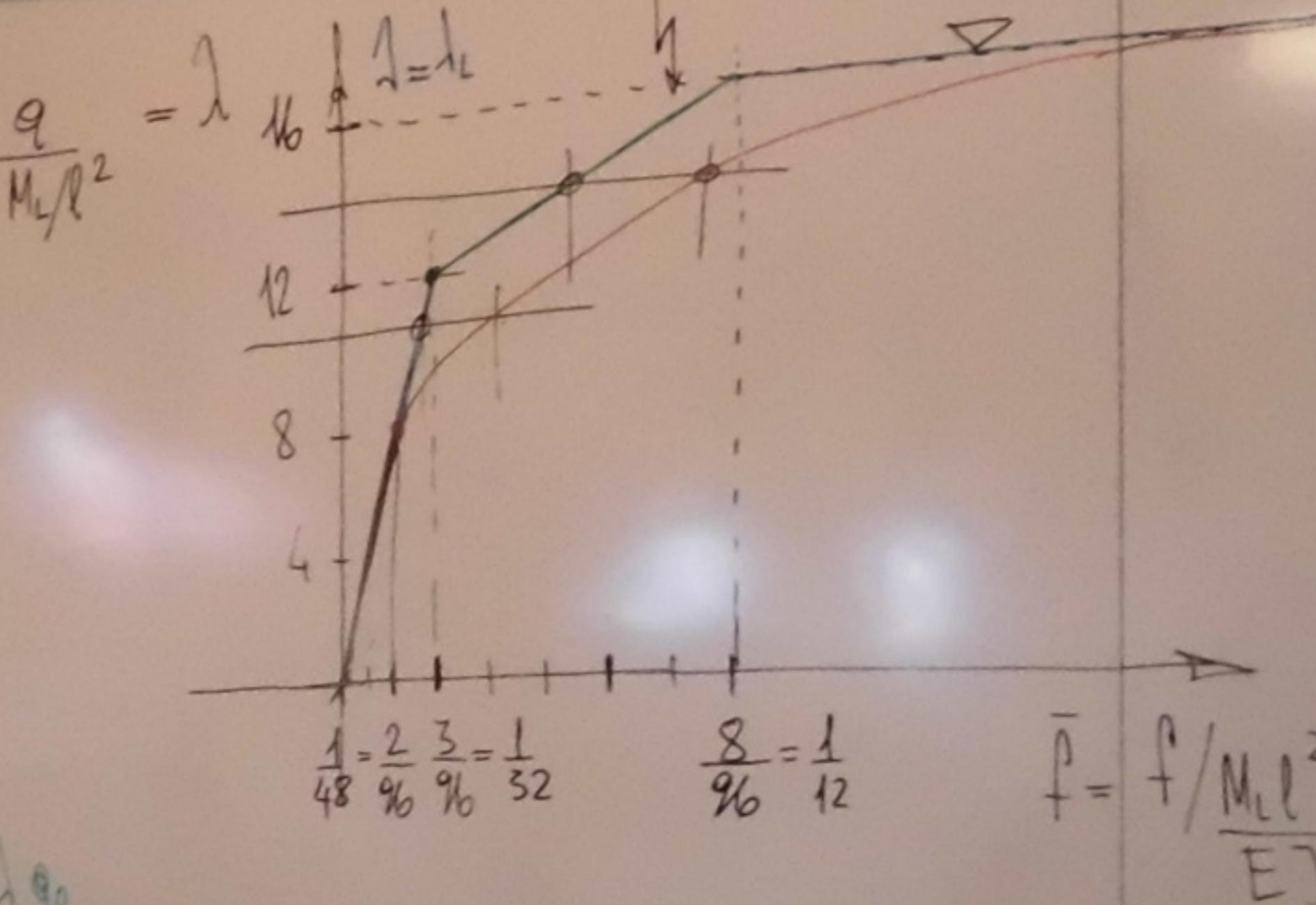
- Collapse multiplier:



$$\Delta \lambda = 12 \frac{M_L}{E J} \frac{q l^2}{4} = M_L \cdot 12 = D$$

collapse load multiplier

evolutive elasto-plastic response with sequence of plastic hinge activations
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- First yield: elastic range: $0 \leq \lambda \leq \lambda_E = 8$

$$\frac{M}{\frac{ql^2}{12}} = M_c \Rightarrow \frac{1}{12} M_c = M_c \Rightarrow \lambda_E = \frac{12 M_c}{M} = \frac{12 \cdot 2}{3} = 8$$

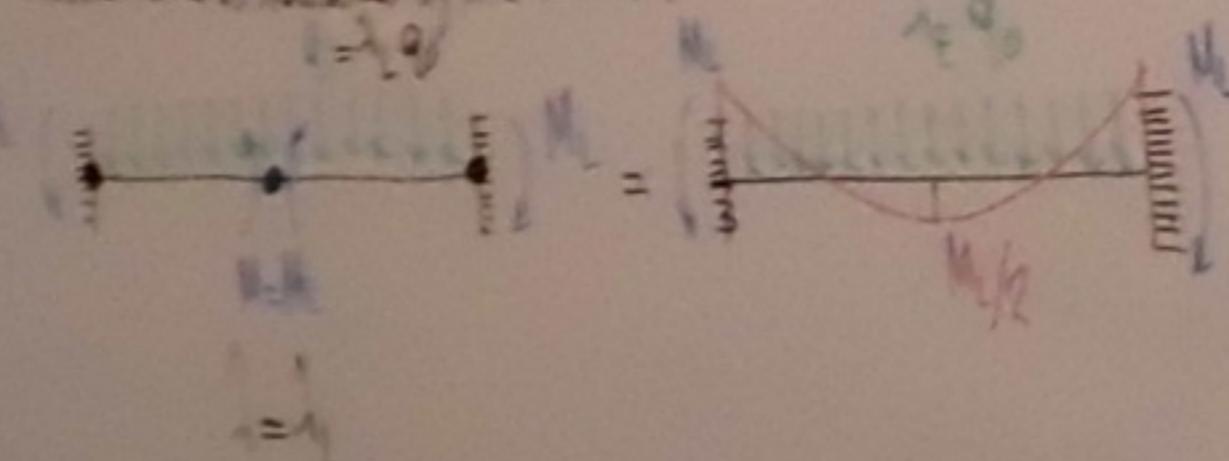
$$\bar{f} = \frac{8}{96} = \frac{1}{12}$$

- In the plastic hinge hyp., first plastic hinges appear at: $0 \leq \lambda \leq \lambda_E = 12$

$$\frac{M}{\frac{ql^2}{12}} = M_c \Rightarrow \frac{1}{12} M_c = M_c \Rightarrow \lambda_E = 12$$

$$\frac{M}{\frac{ql^2}{12}} = \frac{3}{52} = \frac{1}{52} M_c \quad \leftarrow M_c$$

Further load is resisted by the beam:



$$\Delta f = \frac{5}{384} \frac{M_c l^4}{EJ}$$

$$\frac{M^2}{l^2} = M_c \Rightarrow M = M_c \sqrt{\frac{l^2}{2}}$$

$$\frac{M^2}{l^2} = \frac{M_c^2}{2} \Rightarrow \Delta f = \frac{5}{96} \frac{M_c^2 l^2}{EJ} = \frac{5}{96} \frac{M_c l^2}{EJ} \Leftrightarrow \frac{8}{96}$$