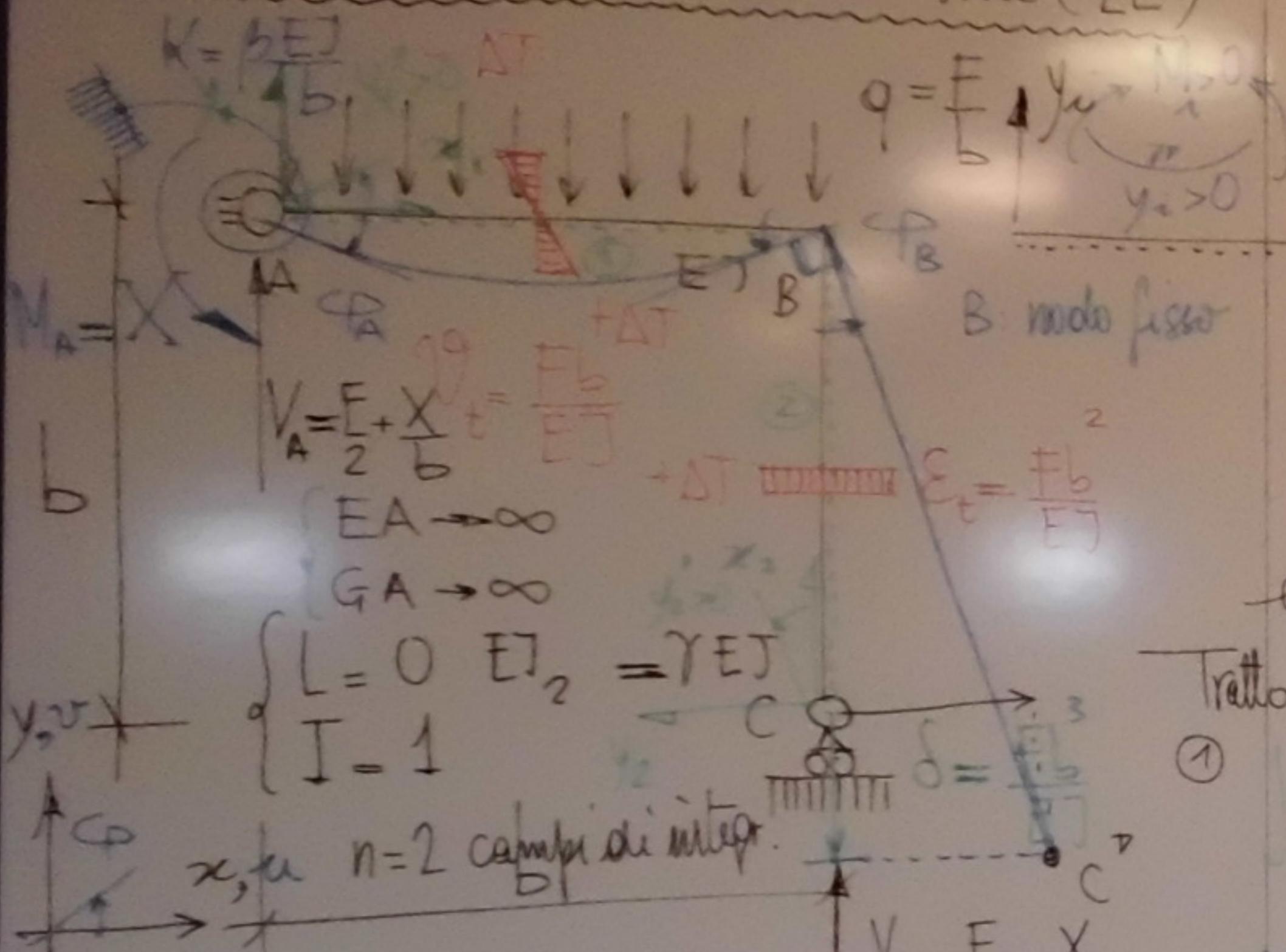


## Metodo della Linea Elastica (LE)



Generico tratto di trave:

$$x \approx y_i = y_{ie} + y_{it}$$

gradi di spost.

$$M_i(x_i) + EJ_i y_{ti}$$

$$EJ_i y_i = M_i(x_i) + EJ_i \vartheta_{ti}$$

$$y_i \approx y_i = M_i(x_i) + (EJ_i) \vartheta_{ti}$$

eq. ne differenziale della LE nel tratto i degli n tratti

Tutto:  $EJ y_1''(x_1) = -\frac{E}{b} \frac{x_1^2}{2} + \left(\frac{E}{2} + \frac{X}{b}\right) x_1 - X + F_b$

$$EJ y_1''(x_1) = -\frac{E}{b} \frac{x_1^5}{6} + \left(\frac{E}{2} + \frac{X}{b}\right) \frac{x_1^3}{2} + (F_b - X) x_1 + A_1$$

$$dr_1: EJ y_1''(x_1) = -\frac{E}{b} \frac{x_1^4}{24} + \left(\frac{E}{2} + \frac{X}{b}\right) \frac{x_1^3}{6} + (F_b - X) \frac{x_1^2}{2} + A_1 x_1 + A_2$$

$$EJ y_2''(x_2) = B_1$$

$$EJ y_2''(x_2) = B_2$$

$$dr_2: EJ y_2''(x_2) = B_1 x_2 + B_2$$

$$X, A_1, A_2; B_1, B_2$$

compatto di motori fissa legata al sistema di ref. locali

int. rigido

Scrivere delle condizioni al contorno ( $n_\alpha = I+2n$ )

$$y_1(0) = 0 \quad \text{condiz. in A}$$

$$y_1'(0) = -P_A = -\frac{X}{K} \quad \text{molla rilasciata in A}$$

$$y_1(b) = -S + \varepsilon_4 b = 0 \quad \text{ad inc. in C} + EA \rightarrow \infty$$

$$y_1'(b) = y_2(b) \quad \text{cont. alle zonate in B}$$

$$y_2(b) = 0 \quad \text{ezima in A} + EA \rightarrow \infty$$

Impozizione delle

$$y_1(0) = 0 \Rightarrow A_2 = 0$$

$$EJ y_1'(0) = -\frac{X}{b} EJ = -\frac{b}{K} X \Rightarrow A_1(b) = -\frac{b}{K} X$$

$$EJ y_1'(b) = 0 \Rightarrow -\frac{E}{b} \frac{b^4}{24} + \left(\frac{E}{2} + \frac{X}{b}\right) \frac{b^3}{6} + (F_b - X) \frac{b^2}{2} + A_2(b) = 0 \Rightarrow$$

$$EJ y_1'(b) = EJ y_2'(b) \Rightarrow -\frac{E}{b} \frac{b^3}{6} + \left(\frac{E}{2} + \frac{X}{b}\right) \frac{b^2}{2} + (F_b - X) b - A_2(b) = 0 \Rightarrow$$

$$EJ y_2'(b) = 0 \Rightarrow B_1 b + B_2 = 0 \Rightarrow B_2 = -B_1 b = -\frac{B_1}{3} F_b$$

Linee elastiche finali presenti:  $X, x$  non sfuggono

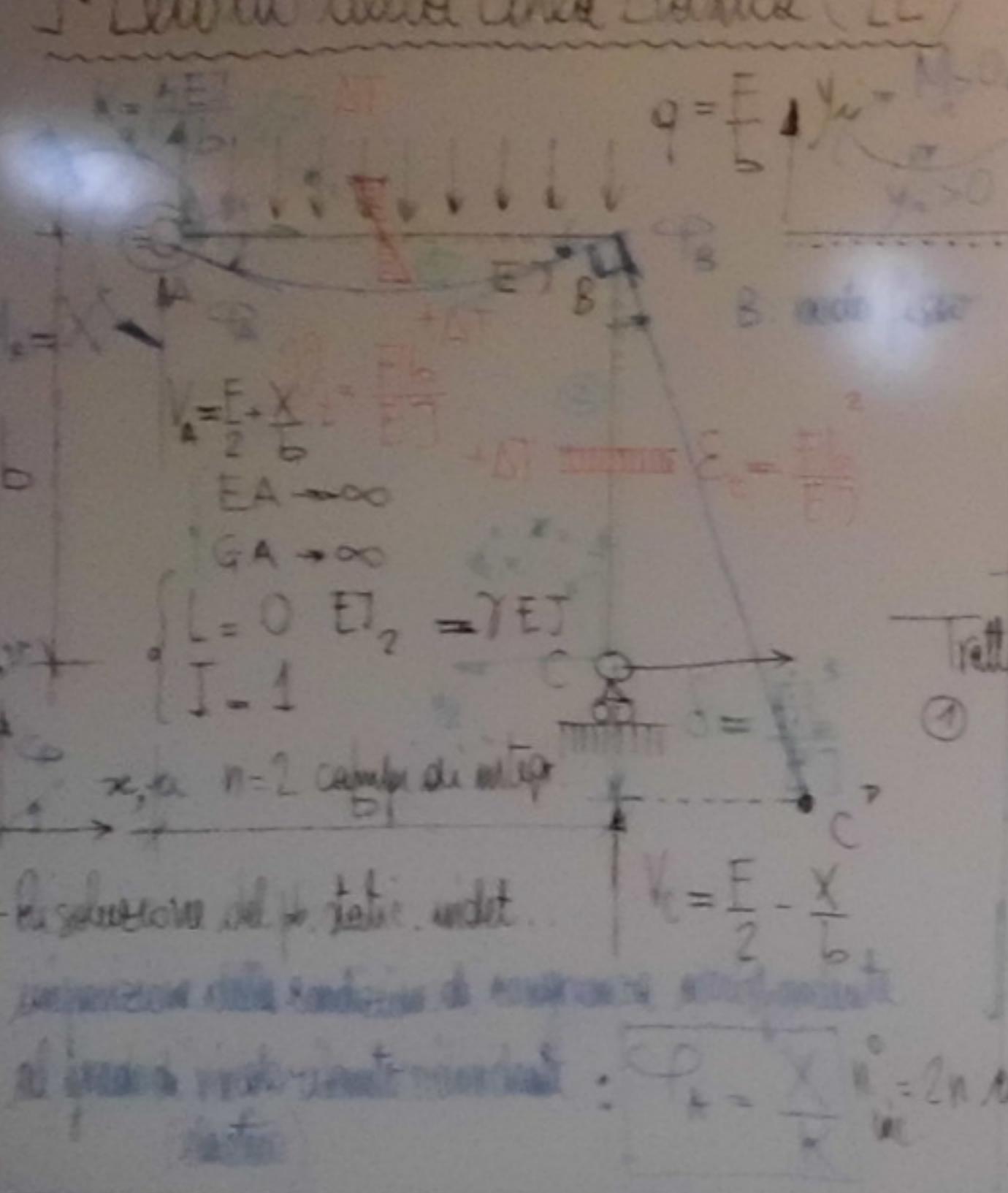
$X \rightarrow RV(X)$  tram. RV globale  $\rightarrow$  N.T. linea

Trasformazione della linea elastica

Calcolo dei campi di spost. da nodi fissi alla linea

## Metodo della Linea Elastica (LE)

Generico tratto di trave:



Scrivere delle condizioni al contorno ( $n_{cc} = I + 2n$ )

- $y_1(0) = 0$  (zero in A)  $= 5$
- $y_1''(0) = -\varphi_A = -\frac{X}{K}$  (molla rotile in A)
- $y_1(b) = -S + \varepsilon_t b = 0$  (condiz. di congruenza)
- $y_1''(b) = y_2(b)$  (lung. fermaria su BC)
- $y_2(b) = 0$  (cont. alla rotile in B)
- $y_2''(b) = 0$  (ezernita in A + EA  $\rightarrow \infty$ )

eq. di differenziale delle LE nel tratto i degli n tratti

$$\text{Tratto: } EJy_1''(x_1) = -\frac{E}{b^2} x_1^2 + \left(\frac{F}{2} + \frac{X}{b}\right)x_1 - X + Fb$$

$$\text{Tratto: } EJy_2''(x_2) = 0 + 0$$

$$EJy_1''(x_1) = -\frac{E}{b^2} x_1^2 + \left(\frac{F}{2} + \frac{X}{b}\right)x_1 + A_1$$

$$EJy_2''(x_2) = B_1$$

$$EJy_1''(x_1) = -\frac{E}{b^2} x_1^2 + \left(\frac{F}{2} + \frac{X}{b}\right)x_1^2 + (Fb - X)x_1 + A_1$$

$$EJy_2''(x_2) = B_1 x_2 + B_2$$

$$EJy_1''(x_1) = -\frac{E}{b^2} x_1^4 + \left(\frac{F}{2} + \frac{X}{b}\right)x_1^3 + (Fb - X)x_1^2 + A_1 x_1 + A_2$$

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$$EJy_1''(x_1) = -\frac{E}{b^2} x_1^4 + \left(\frac{F}{2} + \frac{X}{b}\right)x_1^3 + (F$$

$$\varphi_B = \frac{B_1}{EJ} = \frac{A_1}{EJ}$$

$$\varphi_A = \frac{X}{K} = \frac{b}{\beta EJ} \frac{13}{8} \frac{1}{3+\beta} Fb$$

$$B_1 = \frac{13}{48} \frac{6+\beta}{3+\beta} \frac{Fb}{EJ}$$

$$\varphi_B^{BS} = -\varphi_A = -\frac{13}{8} \frac{1}{3+\beta} \frac{Fb^2}{EJ}$$

Impostazione delle c.e.

$$y_1(0) = 0 \Rightarrow A_2 = 0$$

$$EJ y_1''(0) = -\frac{X}{EJ} = -\frac{b}{\beta} X \Rightarrow A_1(X) = -\frac{b}{\beta} X$$

$$EJ y_1(b) = 0 \Rightarrow -\frac{E}{b} \frac{b}{24} + \left(\frac{E}{2} + \frac{X}{b}\right) \frac{b}{6} + \left(Fb - X\right) \frac{b^2}{2} + \frac{b}{\beta} X b + A_2 = 0 \Rightarrow X = \frac{13}{8} \frac{\beta}{3+\beta} Fb$$

$$EJ y_1'(b) = EJ y_2'(b) \Rightarrow -\frac{E}{b} \frac{b}{6} + \left(\frac{E}{2} + \frac{X}{b}\right) \frac{b}{2} + \left(Fb - X\right) b + \frac{b}{\beta} X = B_1 \Rightarrow B_1(X) = \frac{13}{12} Fb - \frac{1+\beta}{2\beta} X b$$

$$EJ y_2(b) = 0 \Rightarrow B_1 b + B_2 = 0 \Rightarrow B_2 = -B_1 b = -\frac{13}{48} \frac{6+\beta}{3+\beta} Fb^3$$

- Linee elastiche finali (pr sost. X e post. di integr.)

- X → RV(X) trovo RV finali → N, T, M finali

- Tracciamento della linea elastica (deformata)

- Calcolo di comp. di spost. di nodi caratteristici

$$M_c = \varphi_B b =$$

$$= -\frac{B_e}{EJ} = \frac{13}{48} \frac{6+\beta}{3+\beta} \frac{Fb^3}{EJ}$$

$$\varphi_A^{BS} = -\varphi_A = -\frac{13}{8} \frac{1}{3+\beta} \frac{Fb^2}{EJ}$$

Commenti sulla scrittura delle c.c.

$$EJ_i \ddot{y}_i = +M_i - EJ_i \dot{v}_i$$

$$v_{2(b)} = -\psi_2(b)$$

$$\psi_2(b) = \psi_2(b)$$

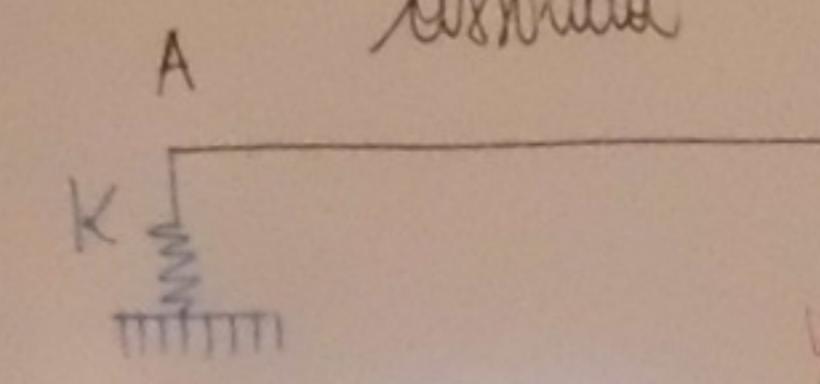
$$= -\psi_2^*(b)$$

$$\psi_2(b) = \psi_2(b)$$

$$= -\psi_2^*(b)$$

Molle elastiche:

assolute

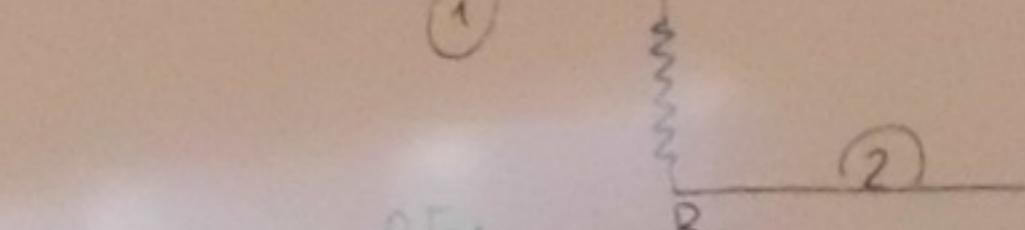


$$v_A = -\psi_1(b)$$

$$v_A = \frac{1}{K} F_A$$

$$\psi_1(b) = -\frac{F_A}{K}$$

relativa



$$v_{B1} = v_B$$

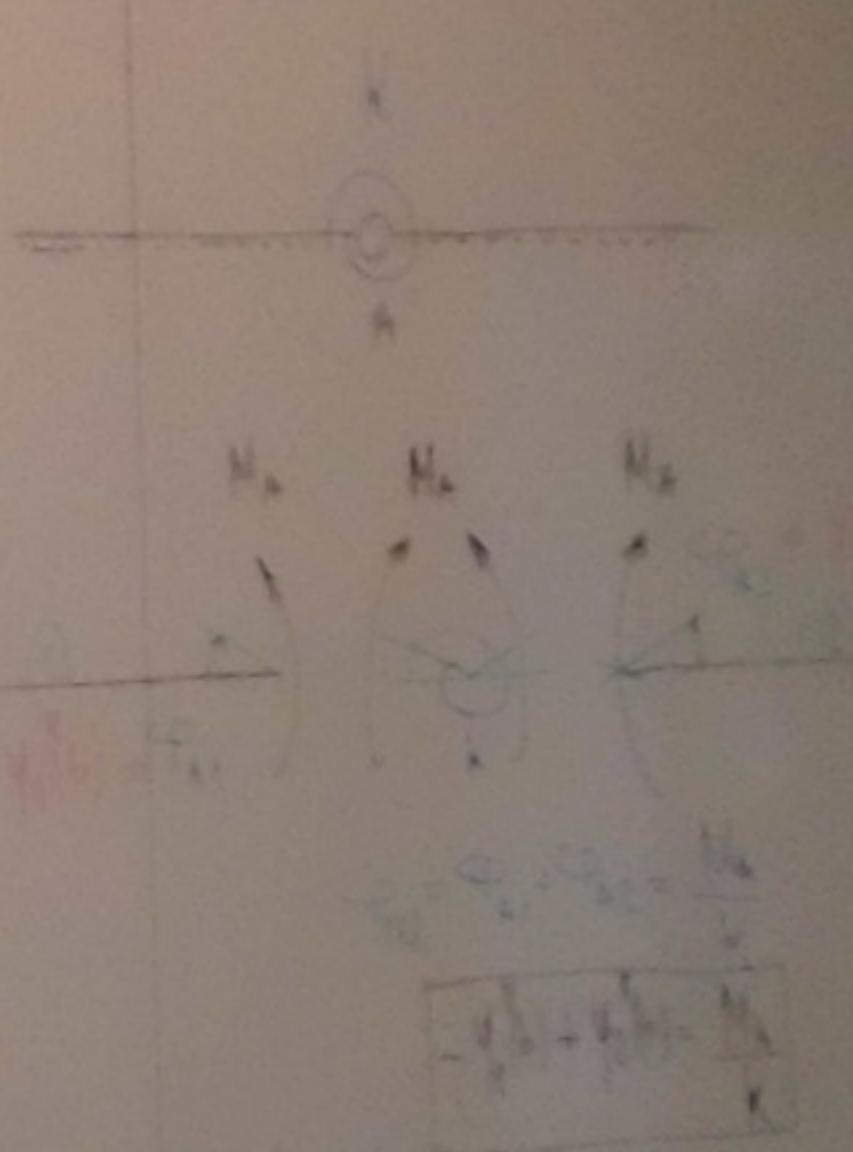
$$v_{B2} = v_B$$

$$-v_{B1} - v_{B2} = \frac{v_B}{K} \rightarrow \psi_1(b) + \psi_2(b) = -\frac{v_B}{K}$$

$$v_{B1} = -\psi_1(b)$$

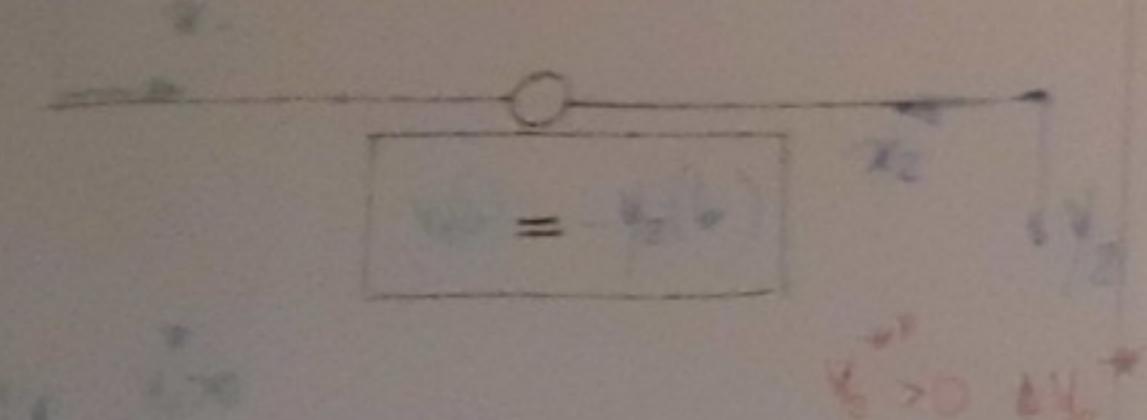
$$v_{B2} = -\psi_2(b)$$

caso per molle interne



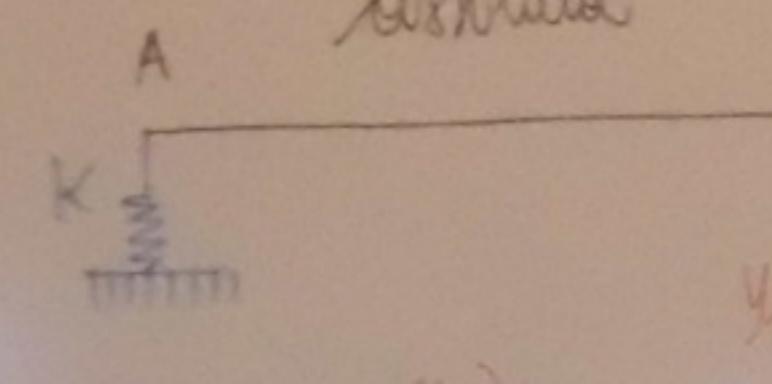
Commenti sulla scrittura della cc

$$v_x = -y_1(b) \quad \text{Eq. } y_1 = M_x - EJ_z v_x$$



$$\ddot{y}_1 = -y_1''(b)$$

Molle elastiche  
assolute

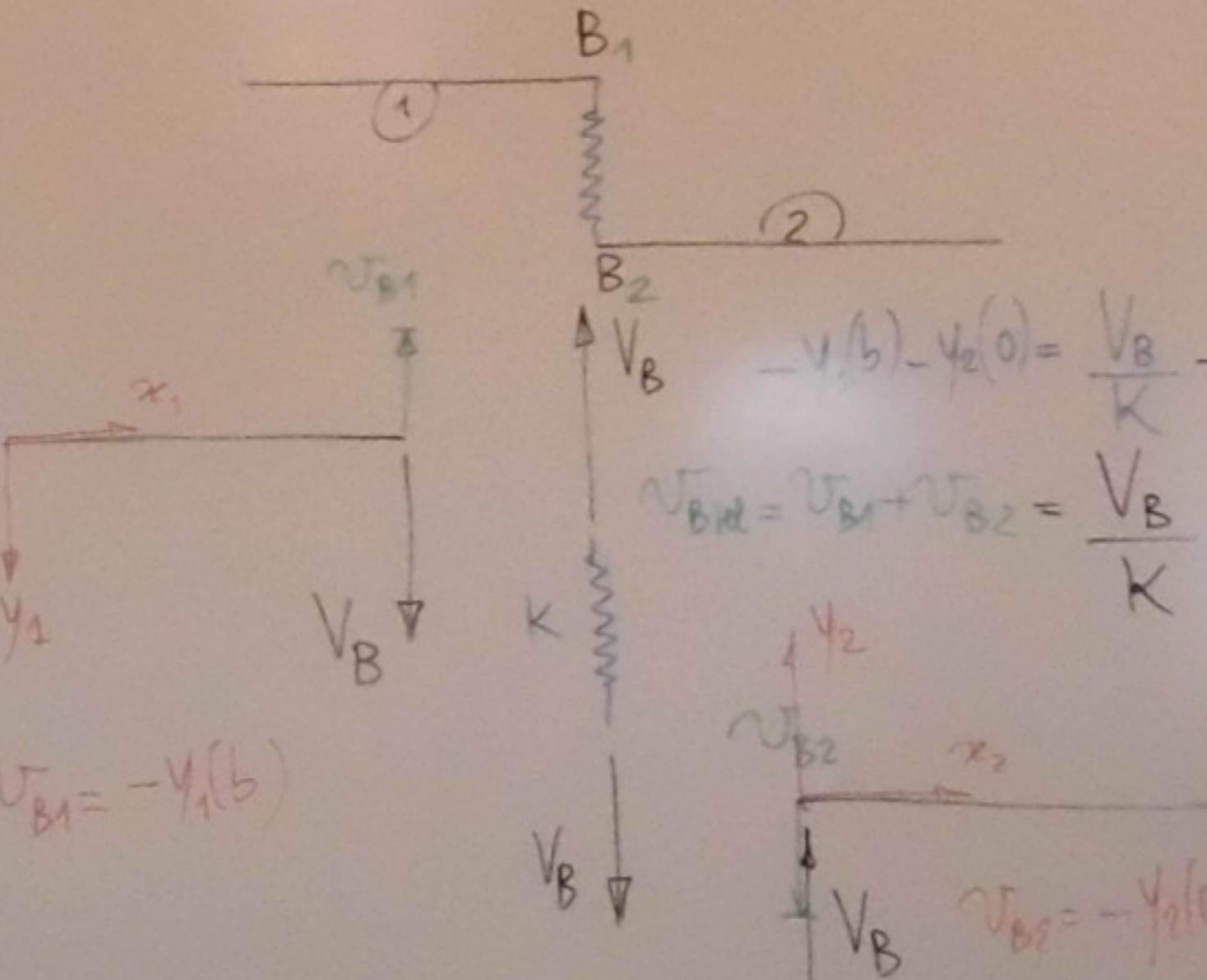


$$y_1 = -y_1(b) \quad \ddot{y}_1 = -y_1''(b)$$

$$y_1(b) = -\frac{F_A}{K}$$

$$\ddot{y}_1(b) = -\frac{F_A}{K}$$

relative



$$w_B1 - w_B(0) = \frac{V_B}{K} \rightarrow w_B(b) + w_B(0) = -\frac{V_B}{K}$$

$$V_{Brel} = V_{B1} + V_{B2} = \frac{V_B}{K}$$

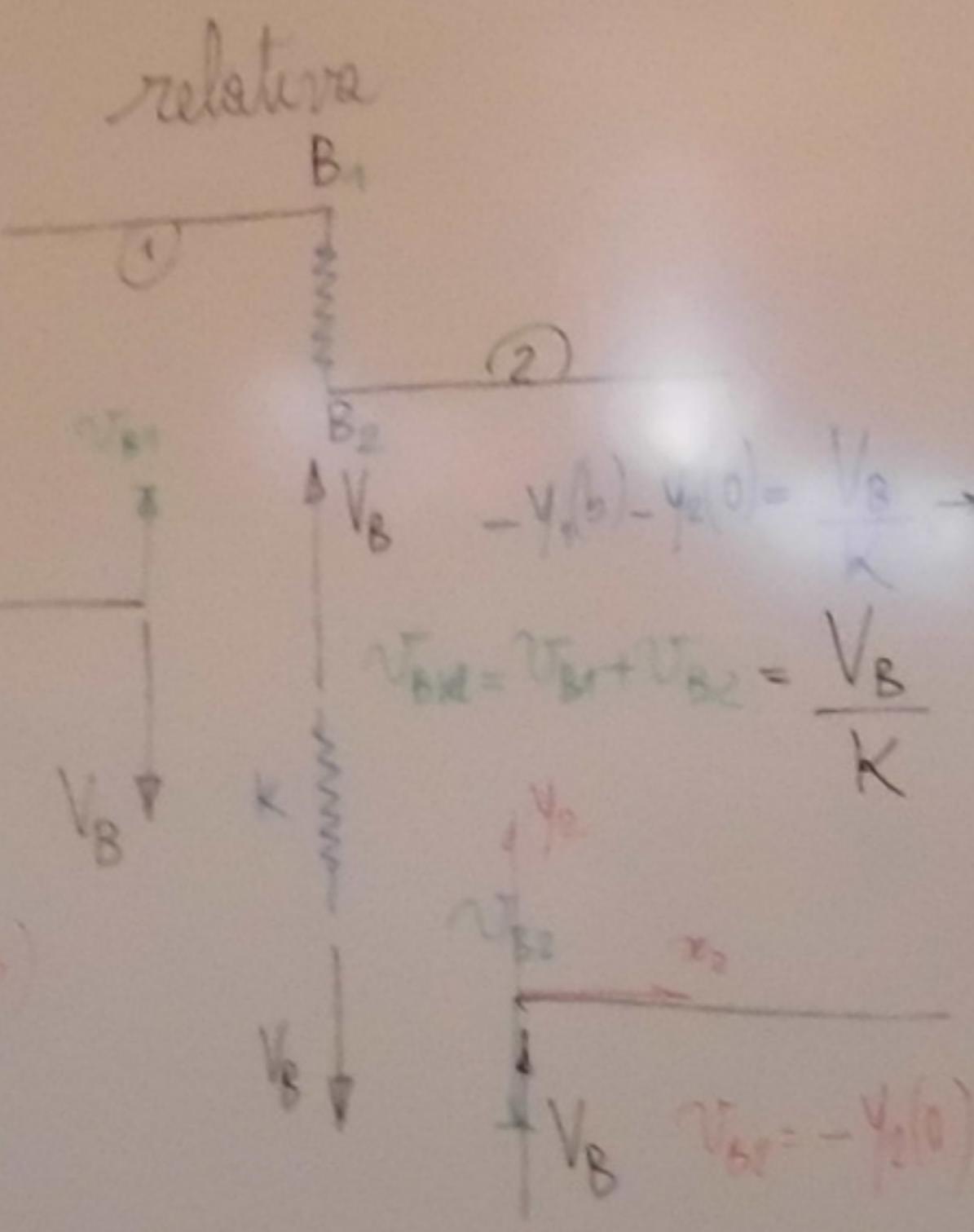
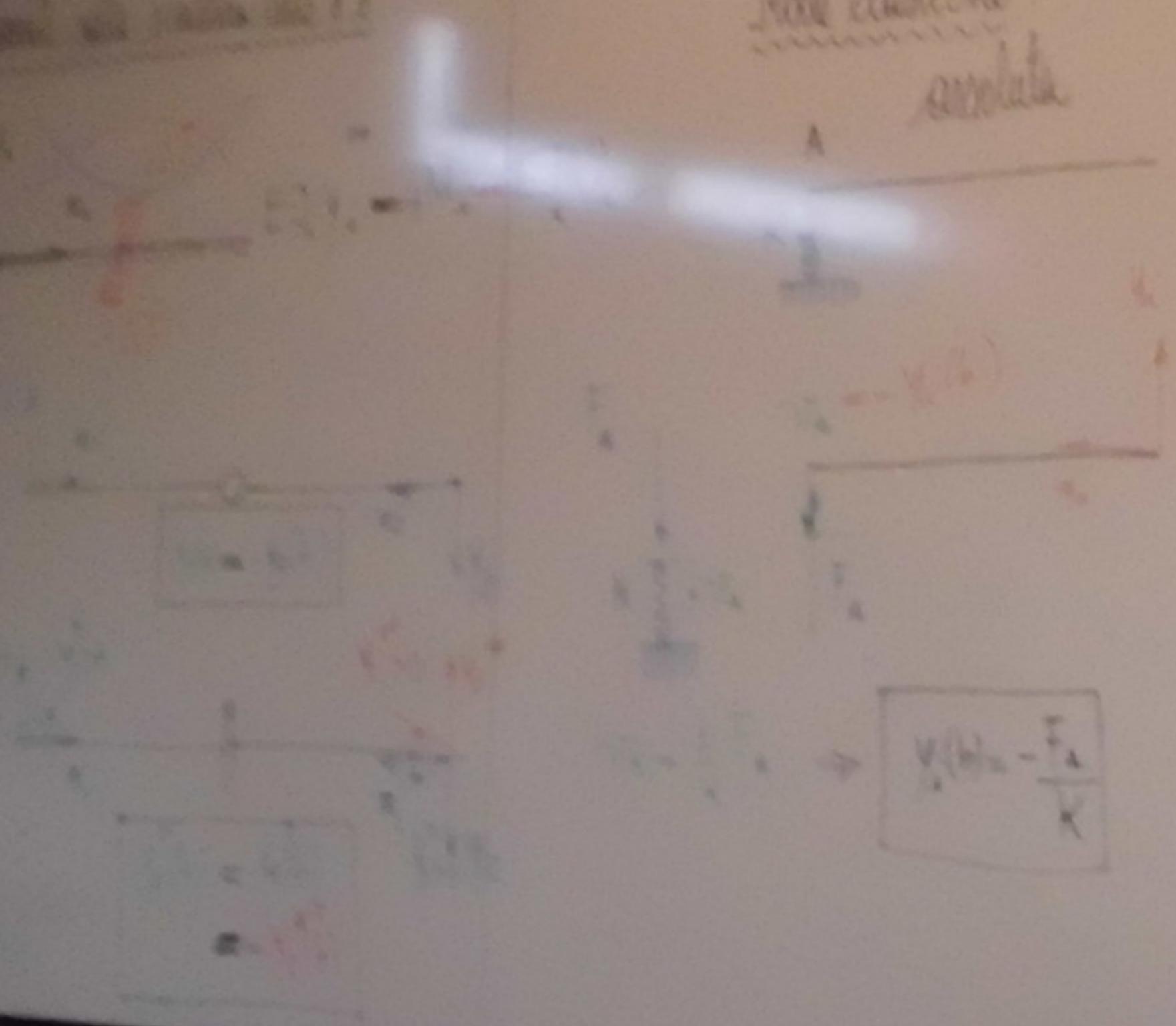
$$w_B2 - w_B(0) = -\frac{V_B}{K}$$

idem per molle rotazionali

$$\begin{aligned} & \text{Diagram showing a beam element with nodes A and B, fixed at A and free at B, with a clockwise moment } M_A \text{ at A.} \\ & \text{Horizontal displacement } x_1 \text{ at A and } x_2 \text{ at B.} \\ & \text{Angular displacements } \theta_1, \theta_2 \text{ at A and B.} \\ & \text{Equation: } -V_1(b) + V_2(b) = \frac{M_A}{K} \end{aligned}$$

Molle elastiche  
angolari

angolari

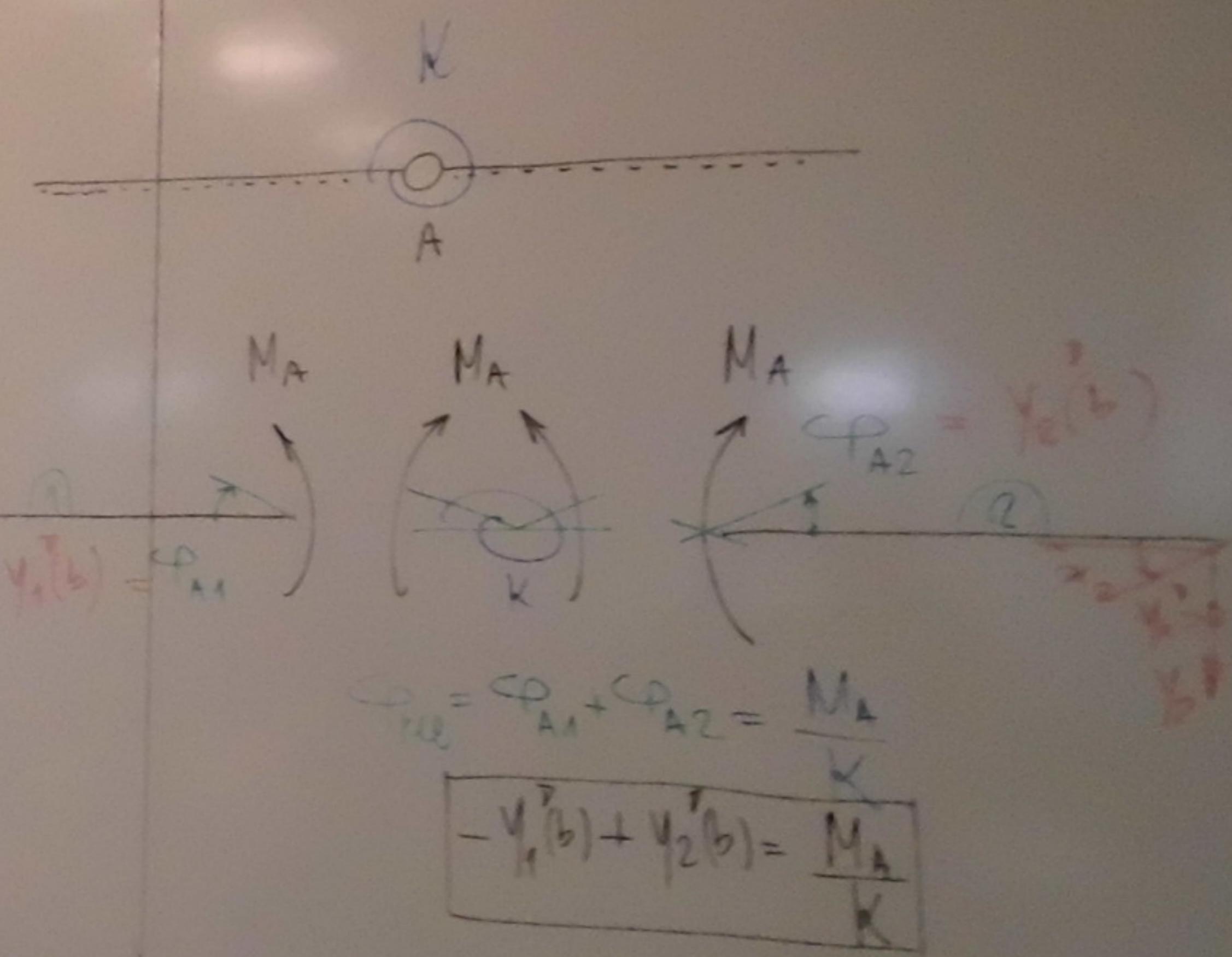


$$-y_1(b) - y_2(0) = \frac{V_B}{K} \rightarrow y_1(b) + y_2(0) = -\frac{V_B}{K}$$

$$V_{B12} = V_{B1} + V_{B2} = \frac{V_B}{K}$$

$$y_2(0) = -V_{B12}$$

lem per molle rotazionali



$$\phi_{rel} = \phi_{A1} + \phi_{A2} = \frac{M_A}{K}$$

$$-\dot{\phi}_{A1}(b) + \dot{\phi}_{A2}(b) = \frac{\ddot{M}_A}{K}$$