Università degli studi di Bergamo

Scuola di Ingegneria (Dalmine)

CCS Ingegneria Edile

LM-24 Ingegneria delle Costruzioni Edili

Complementi di Scienza delle Costruzioni

(ICAR/08 - SdC; 6 CFU)

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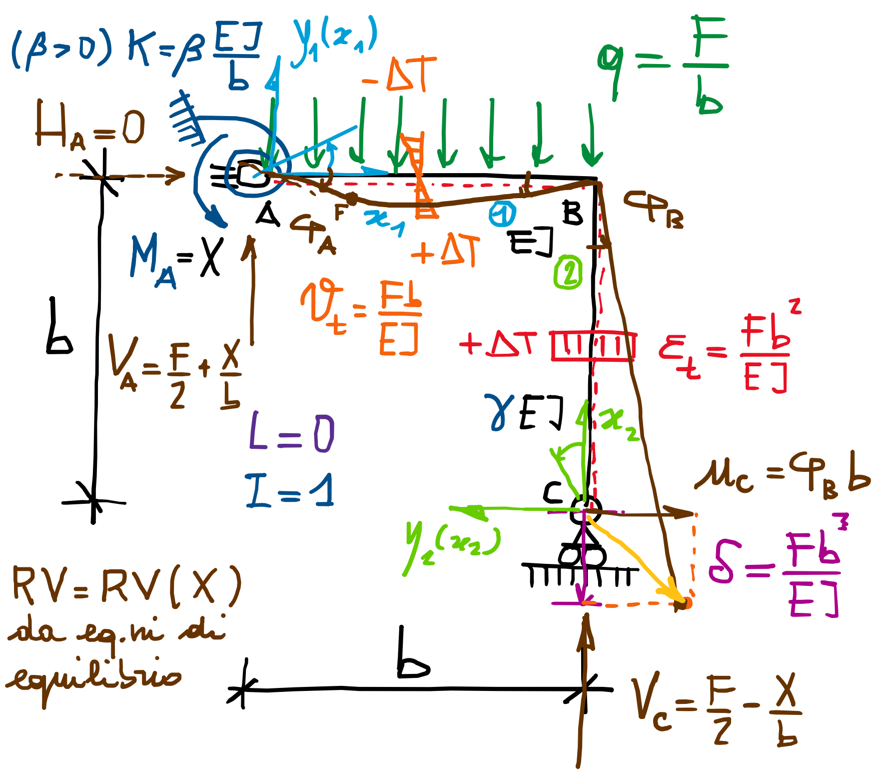
LEZIONE 09

Soluzione tramite Metodo della Linea Elastica (LE)



Curvatura totale del generico campo i-esimo: (n campi di integrazione)

$$y_i''(x_i) \underset{|y_i'| \ll 1}{\simeq} \chi_i(x_i) = \underbrace{\chi_{ie}(x_i)}_{\frac{M_i(x_i)}{EJ_i}} + \underbrace{\chi_{it}(x_i)}_{\vartheta_{it}(x_i)}$$

Legge di B-E-N — curvatura termica impressa

$$\overbrace{EJ_i(x_i)}^{\gamma_i(x_i)\,EJ}\, y_i''(x_i) = M_i(x_i) + EJ_i(x_i)\,\vartheta_{it}(x_i)$$

$$\boxed{EJ \cdot y_i''(x_i) = \frac{M_i(x_i)}{\gamma_i(x_i)} + EJ \cdot \vartheta_t(x_i)}$$

eq. differenziale del 2° ordine

Tratto 1:

$$EJ\, y_1''(x_1) = M_1(x_1) + \cancel{EJ}\,\frac{Fb}{\cancel{EJ}} = -\frac{F}{b}\frac{x_1^2}{2} + \overbrace{\left(\frac{F}{2}+\frac{X}{b}\right)}^{V_A} x_1 - X + Fb$$

$\int dx_i$

$$EJ\, y_1'(x_1) = -\frac{F}{b}\frac{x_1^3}{6} + \left(\frac{F}{2}+\frac{X}{b}\right)\frac{x_1^2}{2} + (Fb - X)\,x_1 + A_1$$

$\int dx_i$

$$EJ\, y_1(x_1) = -\frac{F}{b}\frac{x_1^4}{24} + \left(\frac{F}{2}+\frac{X}{b}\right)\frac{x_1^3}{6} + (Fb - X)\frac{x_1^2}{2} + \underline{A_1 x_1 + A_2}$$

Tratto 2:

$$EJ\, y_2''(x_2) = \frac{0}{\gamma} + EJ\;0 = 0$$

$$EJ\, y_2'(x_2) = B_1$$

$$EJ\, y_2(x_2) = \underline{B_1 x_2 + B_2}$$

comp. moto rigido rototraslatorio

Incognite $2n + I = 5$

A_1, A_2, B_1, B_2; X

costanti di integrazione; inc. iperst.

Scrittura e imposizione delle condizioni al contorno (c.c.) [in numero pari a $n_{cc} = 2n + I = 5$]

- $y_1(0) = 0$ — cerniera in A
- $y_1'(0) = -\frac{X}{K}$ — molla elastica rotazionale in A
- $y_1(b) = -\delta + \varepsilon_t b = 0$ — carrello in C con cedimento δ + $\left(\Delta \ell_t^{BC} = \varepsilon_t b\right)$
- $y_1'(b) = y_2'(b)$ — continuità della rotazione in B $\;\varphi_B$
- $y_2(b) = 0$ — cerniera in A + inest. assiale di AB ($EA_{AB} \to \infty$)

$M_A = X$ $\quad K \quad$ $\varphi = \frac{X}{K}$ opposta a $M_A = X$ sull'asta

spost. assoluti

B è nodo fisso ($u_B = v_B = 0$)

$\circlearrowleft \; \varphi_A = \frac{A_1}{EJ}$

$EJ\, y_1(0) = 0 \Rightarrow \boxed{A_2 = 0}$

$EJ\, y_1'(0) = A_1 = -\frac{X}{K} EJ = -\frac{X}{\beta \frac{EJ}{b}} \cancel{EJ} = -\frac{b}{\beta} X \Rightarrow \boxed{A_1 = A_1(X) = -\frac{b}{\beta} X} \rightarrow A_1 = -\frac{b}{\cancel{\beta}} \frac{13}{8} \frac{\cancel{\beta}}{3+\beta} Fb = \boxed{-\frac{13}{8} \frac{1}{3+\beta} Fb^2 = A_1}$

$EJ\, y_1(b) = -\frac{F}{b}\frac{b^4}{24} + \left(\frac{F}{2} + \frac{X}{b}\right)\frac{b^3}{6} + (Fb - X)\frac{b^2}{2} + A_1 b + A_2 = 0 \Rightarrow \left(\frac{1}{6} - \frac{1}{2} - \frac{1}{\beta}\right) X b^2 = \left(\frac{1}{24} - \frac{1}{12} - \frac{1}{2}\right) Fb^3$

$\left(-\frac{1}{3} - \frac{1}{\beta}\right) X = \frac{1-2-12}{24} Fb = -\frac{13}{24} Fb$

$\frac{3+\beta}{3\beta} X = \frac{13}{8} Fb$

$\boxed{X = \frac{13}{8} \frac{\beta}{3+\beta} Fb}$

$\beta \to 0,\; X \to 0$

$\beta \to \infty,\; X \to \frac{13}{24} Fb$

$EJ\, y_1'(b) = -\frac{F}{b}\frac{b^3}{6} + \left(\frac{F}{2} + \frac{X}{b}\right)\frac{b^2}{2} + (Fb - X) b + A_1 = EJ\, y_2'(b) = B_1 = \frac{13}{12} Fb^2 - \frac{2+\beta}{2\beta} X b$

$EJ\, y_2(b) = B_1 b + B_2 = 0 \Rightarrow B_2 = -B_1 b \rightarrow u_C = \varphi_B b \quad (v_C = -\delta)$

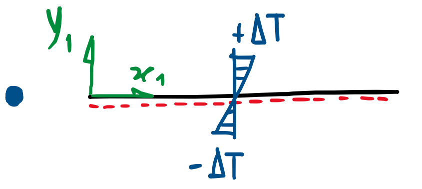
$\boxed{B_2 = -\frac{13}{48} \frac{6+\beta}{3+\beta} Fb^3} \qquad \boxed{B_1 = \frac{13}{48} \frac{6+\beta}{3+\beta} Fb^2}$

$\varphi_B \geq |\varphi_A| \qquad \beta \geq 0$

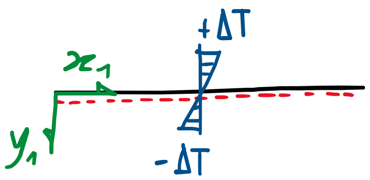
$\varphi_B = \frac{B_1}{EJ} = \frac{13}{8} \frac{1+\beta/6}{3+\beta} \frac{Fb^2}{EJ}$

– LE finali, sost. A_1, A_2, B_1, B_2 e X e tracciamento della deformata qualitativa.

– Attenzione ai segni su LE e c.c.!



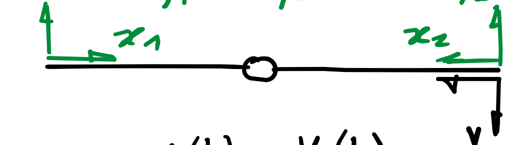
$$EJ\,y_1'' = +M - EJ\chi_t$$



$$EJ\,y_1'' = -M + EJ\chi_t$$

curvature elastiche e termiche

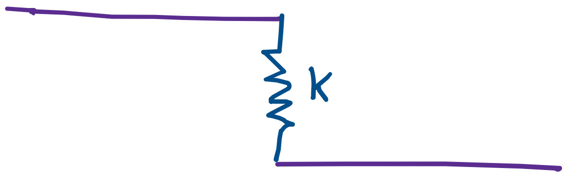
$y_1(b) = y_2(b)$



$y_1(b) = -y_2(b)$

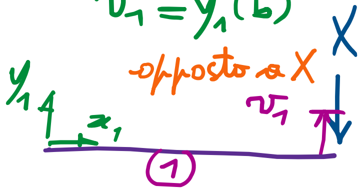
molla relativa

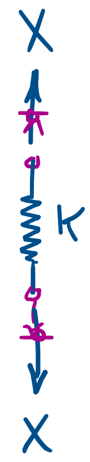
elongazionale



letto nel riferim. locale

$v_1 = y_1(b)$

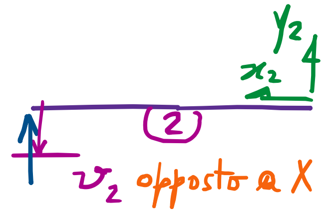




spost. relativo

$\Delta l = v_{rel} = v_1 + v_2$

$= \dfrac{X}{K}$ legge di Hooke

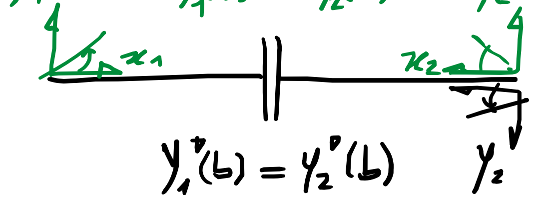


v_2 opposto a X

X $\quad v_2 = -y_2(b)$

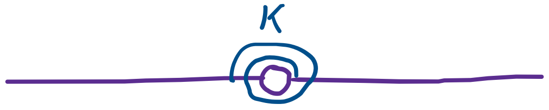
$$\boxed{y_1(b) - y_2(b) = \frac{X}{K}}$$

$y_1'(b) = -y_2'(b)$

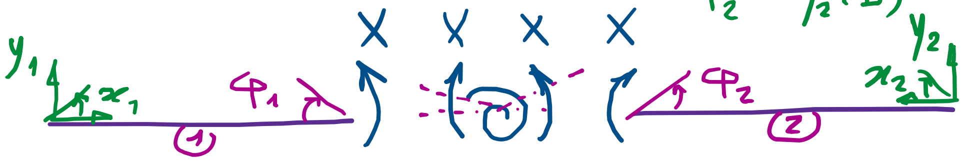


$y_1'(b) = y_2'(b)$

rotazionale



$\varphi_2 = -y_2'(b)$



$\varphi_1 = -y_1'(b)$

$$\boxed{-y_1'(b) - y_2'(b) = \frac{X}{K}}$$

$\Delta\varphi = \varphi_{rel} = \varphi_1 + \varphi_2$

$= \dfrac{X}{K}$