

Università degli studi di Bergamo

Scuola di Ingegneria (Dolmine)

CCS Ingegneria Edile

LM-24 Ingegneria delle Costruzioni Edili

Complementi di Scienza delle Costruzioni

(ICAR/08 - SdC ; 6 CFU)

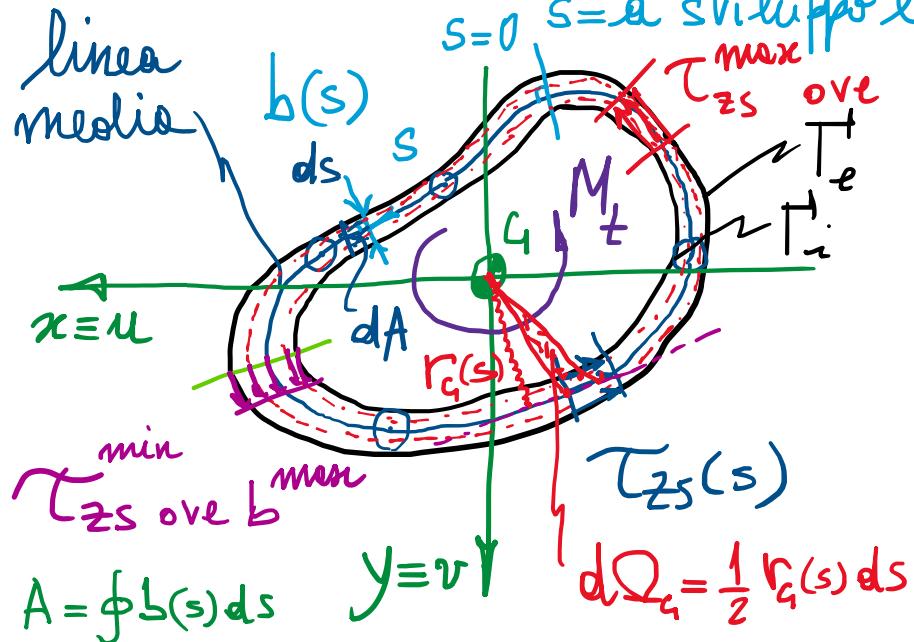
A.A. 2022/2023

prof. Egidio RIZZI

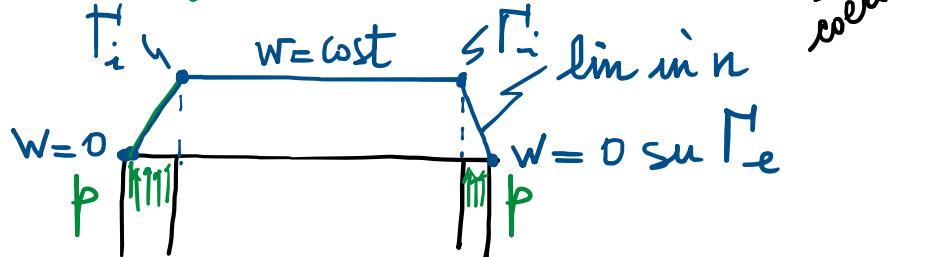
egidio.rizzi@unibg.it

LEZIONE 23

Torsione nei profili sottili chiusi (monocellulari)  biconnessi (2 tagli per separare) con continuità



- Analogie delle membrane:



$$\left\{ \begin{array}{l} \tau_{zs} = \frac{\partial \varphi}{\partial n} = \text{cost} \quad \text{tendenze cost sullo spessore} \\ \tau_{zn} = -\frac{\partial \varphi}{\partial s} \approx 0 \quad \text{tendenze nulle in } s \end{array} \right.$$

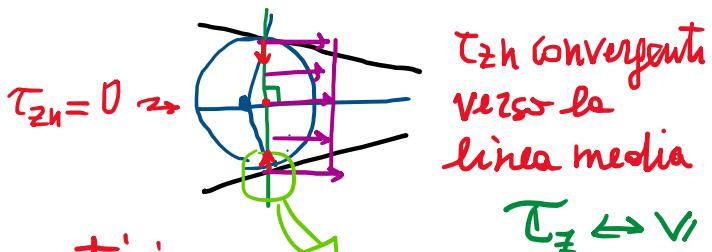
- Spessore $b(s)$ in generale variabile lungo lo sviluppo del profilo, rimanendo "piccolo": $b(s) \ll a$ $|b'(s)| \ll 1$ ($a > 10 b$)

- Analogie idrodinamiche:

- traiettorie che abbracciano l'intero profilo $\sim \parallel$ alle linee medie.
- velocità \rightarrow tens. tangenziali $\sim \parallel$ alle l.m. $\rightarrow \tau_{zs}$
- distribuz. di velocità (τ_{zs}) cost. sullo spessore
- "flusso delle tensioni tangenziali": portata costante

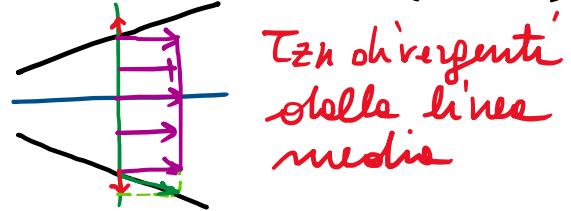
$$q(s) = \tau_{zs}(s) b(s) = q = \text{cost} \Leftrightarrow Q = VA = \text{cost}$$

$\rightarrow b \downarrow (b' < 0)$



antisimm. sullo spessore $\tau_{zn} \uparrow$ $\tau_{zs} \leftrightarrow V$ $\tau_{zs} \parallel \text{linee medie}$

$\rightarrow b \uparrow (b' > 0)$



$\tau_{zn} \ll \tau_{zs} \Rightarrow 0$

- Formula di Rudolf BREDT (~ 1896): [ragionamento "isostatico"] \rightarrow soluzione approssimata
- Equivalenza statica tra M_t e le $T_{zs}(s)$ come sopra discussi:

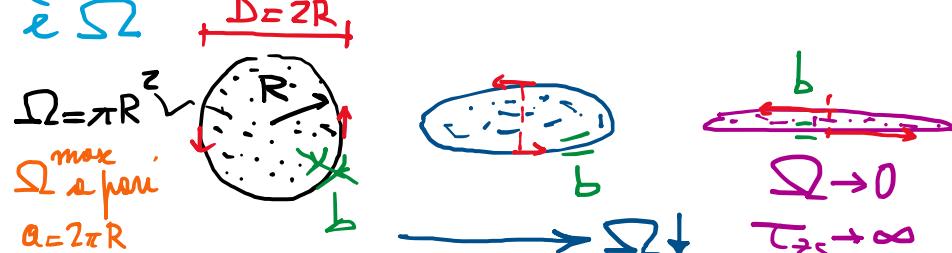
$$M_t = \oint \underbrace{T_{zs}(s) b(s)}_{\text{d}A} ds \cdot r_g(s)$$

$\Delta p: q(s) = q = \text{cost}$

$$= 2q \oint \underbrace{\frac{1}{2} r_g(s) ds}_{d\Omega_g} \quad d\Omega_g (\Omega_g(s)): \text{arie settoriale riferita a } G$$

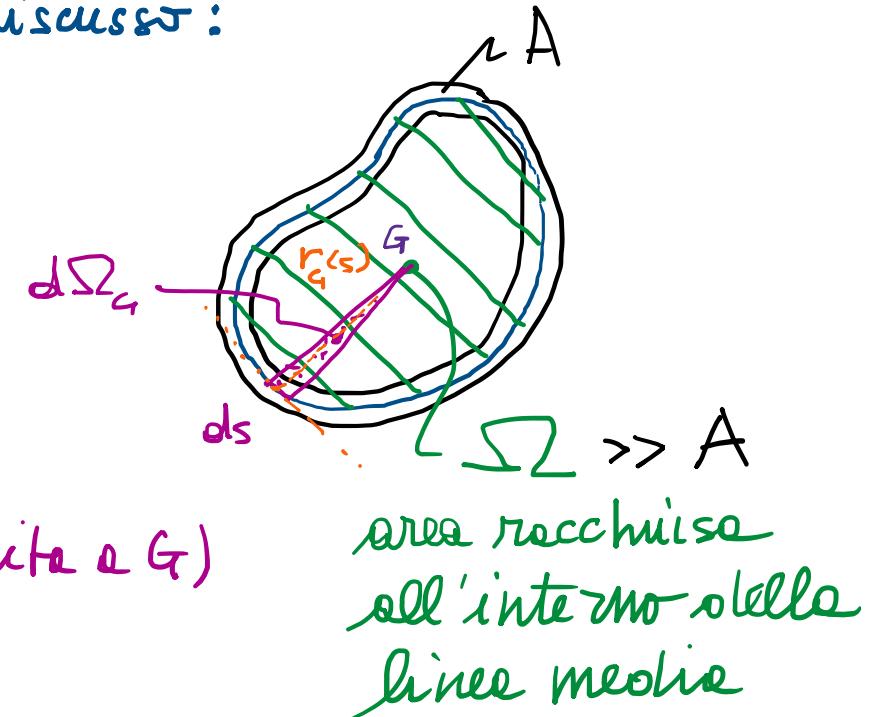
$$\left\{ M_t = 2q \Sigma \right\} \Rightarrow q = T_{zs}(s) b(s) = \frac{M_t}{2\Sigma}$$

Riguardo l'entità delle T_{zs} , a parità di spessore, il param. geometrico cruciale è Σ



$$\left\{ T_{zs}(s) = \frac{M_t}{2\Sigma b(s)} \right\}$$

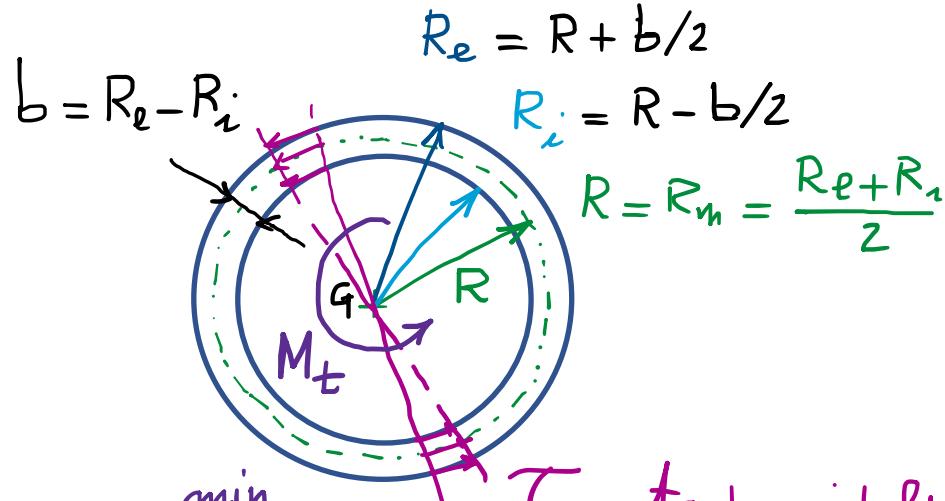
$$(idem per J) \quad T_{zs} \sim \Sigma^{-1}$$



Le $T_{zs}(s)$ risultano:

- dirett. prop. a M_t
 - invers. prop. a Σ
 - invers. prop. a $b(s)$
- $T_{zs}^{\text{max}} \text{ ore } b^{\text{min}} ; T_{zs}^{\text{min}} \text{ ore } b^{\text{max}}$

- Sezione circolare cava



$$\begin{aligned} r = R_i &: \tau^{\min} \\ r = R_m &: \tau^m \\ r = R_e &: \tau^{\max} \end{aligned}$$

τ_{zt} trapezoidali
nullo spessore
 $R_i \leq r \leq R_e$

$$\tau^m = \frac{M_E}{2\pi R^2 b \left[1 + \frac{1}{4} \left(\frac{b}{R} \right)^2 \right]}$$

$b \sim \frac{1}{10} R$

perete sottile

se $\frac{b}{R} \rightarrow 0$

torsione nei profili rotti chiusi

Bredt

$$\tau \sim \frac{M_t}{2\pi R^2 b}$$

Ω area racchiusa all'interno della linea media

$$\tau_{zt} = \frac{M_t}{J_G} r$$

$$J_G = \frac{\pi}{2} (R_e^4 - R_i^4)$$

$$= \frac{\pi}{2} (R_e^2 + R_i^2)(R_e^2 - R_i^2)$$

$$= \frac{\pi}{2} \left(\left(R + \frac{b}{2} \right)^2 + \left(R - \frac{b}{2} \right)^2 \right) \left(\left(R + \frac{b}{2} \right)^2 - \left(R - \frac{b}{2} \right)^2 \right)$$

$$= \frac{\pi}{2} \left(R^2 + \frac{b^2}{4} + Rb + R^2 + \frac{b^2}{4} - Rb \right).$$

$$\left(R^2 + \frac{b^2}{4} + Rb - R^2 - \frac{b^2}{4} + Rb \right)$$

$$= \frac{\pi}{2} 2 \left(R^2 + \frac{b^2}{4} \right) 2 Rb = 2\pi Rb \left(R^2 + \frac{b^2}{4} \right)$$

$$= 2\pi R^3 b \left[1 + \frac{1}{4} \left(\frac{b}{R} \right)^2 \right] = J_G$$

- Momento d'inerzia torsionale: valutabile, a volte del calcolo delle tang, via PLV

$$\frac{dL_e}{dz} = M_t \beta = \oint T_{zs} \frac{\cancel{T}_{zs}}{G} \overbrace{b(s) ds}^{\frac{dA}{ds}} = \frac{dL_i}{dz}$$

$$\beta = \frac{M_t}{GJ}; \quad J = \frac{M_t}{G\beta}$$

$$G = \frac{E}{2(1+\nu)}$$

$$J \sim \Omega^2$$

$$\cancel{M_t} \frac{M_t}{GJ} = \oint \frac{T_{zs}(s)}{G} b(s) ds$$

$$\frac{1}{J} = \oint \frac{M_t}{(2\Omega b)^2} b(s) ds = \frac{1}{4\Omega^2} \oint \frac{ds}{b(s)} \Rightarrow$$

$$J = \frac{4\Omega^2}{\oint \frac{ds}{b(s)}}$$

$$= \frac{4\Omega^2 b}{a_{\text{cost}}}$$

$$a = \oint ds$$

$$\frac{b}{a} \sim \frac{1}{10}$$

- Ingobbamento (riferito alla linea media) \rightarrow Centro di torsione

$$d\Psi_g = \frac{T_{zs}(s)}{G\beta} ds - 2d\Omega_g$$

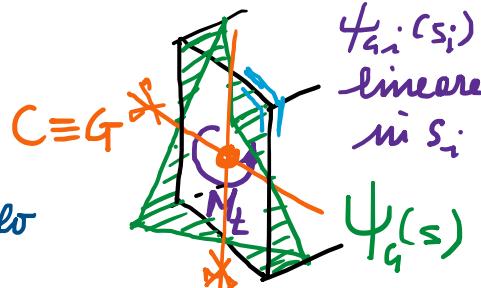
$$= \frac{J}{M_t} \frac{M_t}{2\Omega b(s)} ds - 2d\Omega_g$$

$$= \frac{2\Omega^2}{\oint \frac{ds}{b(s)}} \frac{1}{2\Omega} \frac{ds}{b(s)} - 2d\Omega_g$$

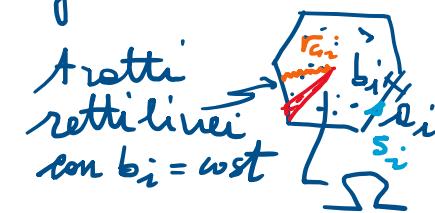
$$d\Psi_g = \frac{2\Omega}{\oint \frac{ds}{b(s)}} \frac{ds}{b(s)} - 2d\Omega_g \rightarrow \Psi_g(s) = \frac{2\Omega}{\oint \frac{ds}{b(s)}} \int_0^s \frac{ds}{b(s)} - 2\Omega_g(s) + \Psi_g^{\text{cost}}$$

ingobb. medio nullo

$$\bar{\Psi}_g = 0$$



Profili scatolari:



$$= \frac{4\Omega}{\sum_i b_i} = \frac{4\Omega}{\sum_i b_i}$$

$$a = \sum_i s_i$$

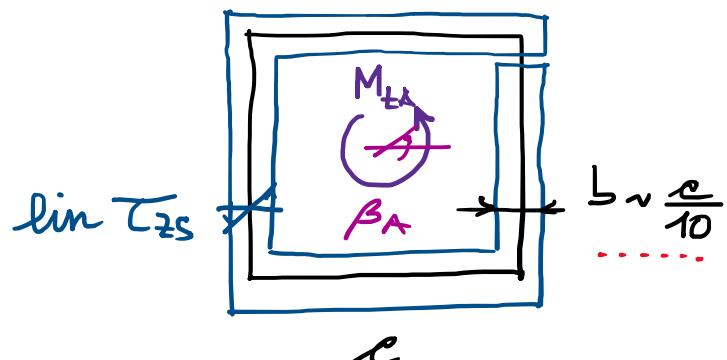
$$\left\{ \begin{array}{l} x_c = -\frac{1}{J_x} \oint \Psi_g(s) y(s) b(s) ds \\ y_c = \frac{1}{J_y} \oint \Psi_g(s) x(s) b(s) ds \end{array} \right.$$

coordinate del C. di To.
(se Esse disim, C è stesa esse)

Confronto a torsione tra profilo sottile aperto e profilo sottile chiuso

$$\beta = \frac{M_t}{GJ}$$

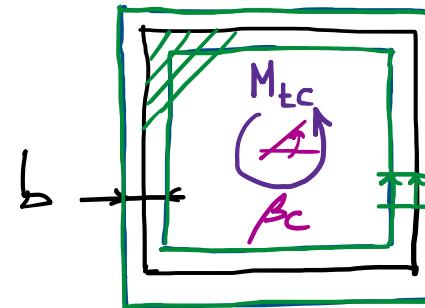
APERTO



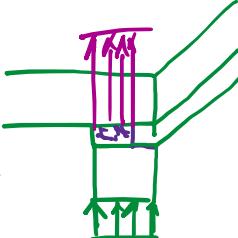
$\text{lin } \tau_{zs}$

The winner is...
profilo chiuso!

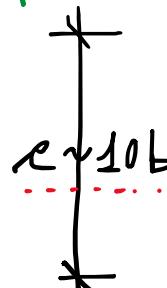
CHIUSO



soldature difettose:
innescio di fratture



$\tau \cdot b = \text{cost}$



$$\frac{c}{b} \sim 10$$

$$b > 1$$

... inerzia torsionale

$\therefore J_c = \frac{4 \Omega^2 b}{\sum_i \alpha_i} = \frac{4 (c^2)^2 b}{4 c} = c^3 b$

$$\tau_A = \frac{M_{tA}}{J_A} b = \frac{3}{4} \frac{M_{tA}}{c b^2} \quad \dots \text{tensione tangenziale}$$

(angolo unitario di)

$$\beta_A = \frac{M_{tA}}{G J_A} = \frac{3}{4} \frac{M_{tA}}{G c b^3} \quad \dots \text{"torsione"}$$

Ese:

- a parità di M_t : $M_{tA} = M_{tc}$

$$- \frac{\tau_A}{\tau_c} = \frac{3}{2} \frac{c}{b} \sim 15 \cdot 10$$

$$- \frac{\beta_A}{\beta_c} = \frac{3}{4} \left(\frac{c}{b} \right)^2 \sim 0.75 \cdot 100$$

- a parità di β : $\beta_A = \beta_c$

$$- \frac{M_{tc}}{M_{tA}} = \frac{3}{4} \left(\frac{c}{b} \right)^2 \sim 0.75 \cdot 100$$

Rapporti:

$$\frac{J_c}{J_A} = \frac{c^3 b}{4/3 c b^3} = \frac{3}{4} \left(\frac{c}{b} \right)^2 \ggg 1 \sim 0.75 \cdot 100 = 75$$

due ordini
di grandezza
 $\sim \left(\frac{c}{b} \right)^2$

$$\frac{\tau_A}{\tau_c} = \frac{3}{2} \frac{M_{tA}}{M_{tc}} \cancel{\frac{c^2 b}{b^2}} = \frac{M_{tA}}{M_{tc}} \frac{3}{2} \left(\frac{c}{b} \right)^1 \sim 1.5 \cdot 10 = 15$$

$\sim \left(\frac{c}{b} \right)$

$$\frac{\beta_A}{\beta_c} = \frac{M_{tA}}{M_{tc}} \frac{J_c}{J_A} = \frac{M_{tA}}{M_{tc}} \frac{3}{4} \left(\frac{c}{b} \right)^2$$

un ordine
di grandezza