1 Задача 2.5

$$u_t = u_{xx} \tag{1}$$

$U(t,x)$ - решение (1). $U_5(t,x) = (1+4ct)^{-1/2}\exp\{-\frac{cx^2}{1+4ct}\}\cdot U\left(\frac{x}{1+4ct}, \frac{t}{1+4ct}\right)$ - решение (1)?

$$\begin{aligned}
U_{5t} = &-\frac{1}{2}4c(1+4ct)^{-3/2}\exp\left\{-\frac{cx^2}{1+4ct}\right\}U\left(\frac{x}{1+4ct}, \frac{t}{1+4ct}\right) + \\
&+ (1+4ct)^{-1/2}\left(-\exp\left\{-\frac{cx^2}{1+4ct}\right\}\frac{x^2}{4}\left(-\frac{1}{\left(\frac{1}{4c}+t\right)^2}\right)\right)U\left(\frac{x}{1+4ct}, \frac{t}{1+4ct}\right) + \\
&+ (1+4ct)^{-1/2}\exp\left\{-\frac{cx^2}{1+4ct}\right\}\left(U_x\left(\frac{x}{1+4ct}, \frac{t}{1+4ct}\right)\left(-\frac{4cx}{(1+4ct)^2}\right) + \right. \\
&\left. + U_t\left(-\frac{x}{1+4ct}, \frac{t}{1+4ct}\right)\frac{1}{16c^2t^2}\right)
\end{aligned} \tag{2}$$

$$\left(\frac{x}{1+4ct}\right)_t = \frac{x}{4c}\left(-\frac{1}{\left(\frac{1}{4c}+t\right)^2}\right) = -\frac{4cx}{1+8ct+16ct^2}$$

$$\left(\frac{t}{1+4ct}\right)_t = \frac{1}{4c}\left(\frac{4ct}{1+4ct}\right)_t = \frac{1}{4c}\left(1-\frac{1}{4ct}\right)_t = \frac{1}{16c^2t^2}$$

$$\begin{aligned}
\left(\exp\left\{-\frac{cx^2}{1+4ct}\right\}\right)_{xx} &= \left(-\frac{2cx}{1+4ct}\exp\left\{-\frac{cx^2}{1+4ct}\right\}\right)_x = \\
&= -\left(\frac{2c}{1+4ct}\exp\left\{-\frac{cx^2}{1+4ct}\right\} - \frac{4c^2x^2}{(1+4ct)^2}\exp\left\{-\frac{cx^2}{1+4ct}\right\}\right) = \\
&= \exp\left\{-\frac{cx^2}{1+4ct}\right\}\left(\left(\frac{2cx}{1+4ct}\right)^2 - \frac{2cx}{1+4ct}\right)
\end{aligned}$$

$$\begin{aligned}
U_{5xx} = &(1+4ct)^{-1/2}(\exp\left\{-\frac{cx^2}{1+4ct}\right\}\left(\left(\frac{2cx}{1+4ct}\right)^2 - \frac{2cx}{1+4ct}\right)U\left(\frac{x}{1+4ct}, \frac{t}{1+4ct}\right) + \\
&+ 2\left(-\frac{2cx}{1+4ct}\exp\left\{-\frac{cx^2}{1+4ct}\right\}\frac{1}{1+4ct}U_x\left(\frac{x}{1+4ct}, \frac{t}{1+4ct}\right)\right) + \\
&+ \exp\left\{-\frac{cx^2}{1+4ct}\right\}U_{xx}\left(\frac{x}{1+4ct}\right)^2)
\end{aligned} \tag{3}$$

Сопоставляя коэффициенты при U, U_x, U_{xx} и учитывая, что $U_{xx} = U_t$ получаем, что U_5 является решением (1).

2 Задача 2.13

Если источник подключен к левому концу:

$$\begin{cases} u_x(0,t) = -\frac{Q}{kS}, \\ u_x(l,t) = 0. \end{cases} \tag{4}$$

К правому:

$$\begin{cases} u_x(0,t) = 0, \\ u_x(l,t) = \frac{Q}{kS}. \end{cases} \tag{5}$$

3 Задача 2.15

$$\begin{cases} u_t = a^2 u_{xx}, \\ u(0,t) = u(l,t) = 0, \\ u(x,0) = 0. \end{cases} \tag{6}$$