1 Задача 4(а)

Вычислить интеграл:

$$\int_1^3 \sqrt{\frac{x-1}{3-x}} \frac{dx}{(x+2)^2}$$

$$\int_1^3 \sqrt{\frac{x-1}{3-x}} \frac{dx}{(x+2)^2} = \int_1^3 (x-1)^{\frac{1}{2}} (3-x)^{-\frac{1}{2}} (x+2)^{-2} dx = \left|_{t=\frac{x-1}{2}} \int_0^1 2t^{\frac{1}{2}} (2-2t)^{-\frac{1}{2}} (3+2t)^{-2} dt = \right.$$

$$= \frac{1}{4} \int_0^1 t^{\frac{1}{2}} (1-t)^{-\frac{1}{2}} \left(\frac{3}{2}+t\right)^{-2} dt$$

Положим $t = \frac{x-1}{3-x} = -1 + \frac{2}{3-x}$. Тогда:

$$3 - x = \frac{2}{t+1} \Rightarrow x = 3 - \frac{2}{t+1} = \frac{3t+1}{t+1}$$

$$dx = -2d\frac{1}{t+1} = \frac{2dt}{(t+1)^2}$$

$$\frac{1}{(x+2)^2} = \frac{1}{\left(5 - \frac{2}{t+1}\right)^2} = \frac{(t+1)^2}{(5t+3)^2}$$