No.

密码学 hw1

2.1

(a) 7503 mod 81

7503 = 92×81+51, therefore 7503 mod 81 must = 51

(b) (−7503) mod 81

−7503 = −93×81+30, therefore (−7503) mod 81 must = 30

(c) 81 mod 7503

81 = 0×7503+81, therefore 81 mod 7503 must = 81

(d) (−81) mod 7503

−81 = −1×7503+7422, ~~thefe~~ therefore (−81) mod 7503 must = 7422

2.8 List all invertible elements in Z_m for m = 28, 33 and 35.

Z_{28}: 1, 3, 5, 9, 11, 13, 15, 17, 19, 23, ~~24~~ 25, 26, 27.

Z_{33}: 1, 2, 4, 5, 7, 8, 10, 13, 14, 16, 17, 19, 20, 23, 25, 26, 28, 29, 31, 32

Z_{35}: 1, 2, 3, 4, 6, 8, 9, 11, 12, 13, ~~14~~, 16, 17, 18, 19, 22, 23, 24, 26, 27, 29
31, 32, 33, 34.

判定依据 gcd(a, m) = 1，那么a有逆元.

2.9 For $1 \le a \le 28$, determine a^{-1} mod 29 by trail and error

($ab \equiv 1$ mod n，b是a在模n下的逆元 记作a^{-1})

$1^{-1}=30$ $2^{-1}=15$ $3^{-1}=10$ $4^{-1}=22$ $5^{-1}=6$ $6^{-1}=5$ $7^{-1}=25$

$8^{-1}=11$ $9^{-1}=13$ $10^{-1}=3$ $11^{-1}=8$ $12^{-1}=17$ $13^{-1}=9$, $14^{-1}=27$

$15^{-1}=2$ $16^{-1}=20$ $17^{-1}=12$ $18^{-1}=21$ $19^{-1}=26$ $20^{-1}=16$ $21^{-1}=18$

$22^{-1}=4$ $23^{-1}=24$ $24^{-1}=23$ $25^{-1}=7$ $26^{-1}=19$, $27^{-1}=14$, $28^{-1}=28$

hw2

2.7 Determine the number of keys in an Affine Cipher over Z_m for $m=30,100$ and 1225.

Answer:

(1) $30=2\times3\times5$, $\phi(30)=1\times2\times4=8$, The affine cipher over Z_{30} has $30\times8=240$ keys

(2) $100=2^2\times5^2$, $\phi(100)=(2^2-2)(5^2-5)=40$. The affine cipher over Z_{100} has $100\times40=4000$ keys

(3) $1225=5^2\times7^2$, $\phi(1225)=(5^2-5)(7^2-7)=840$. The affine cipher over Z_{1225} has $1225\times840=1029000$ keys.

2.10 Suppose that $K=(5,21)$ is a key in a Affine Cipher over Z_{29}

(a) Express the decryption function $d_k(y)$ in the form $d_k(y)=a'y+b'$, where $a',b'\in Z_{29}$

加密: $e_k(x)=(ax+b) \bmod 29$

解密: $d_k(y)=a'^{-1}(y-b') \bmod 29$

$a\cdot a'^{-1}\equiv1 \bmod 29$ $\quad 5\cdot a'\equiv1 \bmod 29 \quad a'=6$

$b'=-a'\cdot b \bmod 29 = -126 \bmod 29 = -5\times29+19 \quad \therefore b'=19$

$d_k = 6y+19 \bmod 29 \quad (d_k(y)=a'^{-1}(y+b') \bmod 29)$

(b) Prove that $d_k(e_k(x))=x$ for all $x\in Z_{29}$

$a=5 \quad b=21 \quad a'=6 \quad b'=19$

proof: $d_k(e_k(x)) = a'^{-1}(e_k(x)+b') \bmod 29$

$= 6(5x+21+19) \bmod 29$

$= 6(5x+11) \bmod 29$

$= (30x+66) \bmod 29$

$= (29x+x+66) \bmod 29$ $\quad\because$ ($29x$ 与29整除)

$= (x+66) \bmod 29$

$= (x+29\times2+8) \bmod 29$

$= (x+8) \bmod 29 = x \bmod 29 + 8 \bmod 29$

$= x \bmod 29.$

2.15 Determine the inverse of the following matrices over Z_{26}

(a) $\begin{pmatrix} 2 & 5 \\ 9 & 5 \end{pmatrix}$

$A^{-1} = \frac{1}{\det(A)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$\det(A) = 2\times5 - 5\times9 = -35$

-35在模26下的逆元 $-35\times6 \not\equiv 1 \bmod 26$

$\Rightarrow -35\times23 = -26\times31+1$

$-805 = -806+1$

$\therefore -35$在模26下的逆元为23

$A^{-1} = 23\times\begin{pmatrix} 5 & -5 \\ -9 & 2 \end{pmatrix} \bmod 26$

$= \begin{pmatrix} 11 & 15 \\ 1 & 20 \end{pmatrix}$ $\begin{cases} 115 = 26\times4+11 \\ -115 = -5\times26+15 \\ -207 = -8\times26+1 \\ 46 = 26\times1+20 \end{cases}$

2.16 (a) permutation of $\{1, \cdots, 8\}$:

x	1	2	3	4	5	6	7	8
$\pi(x)$	4	1	6	2	7	3	8	5

~~π^{-1}: 2 4 6 1 8 3 5 7~~

π^{-1}: [2,4,6,1,8,3,5,7]

(b) m=8 分为4块每块8个字符.

原始 TGEEMNEL | NNTDROEO | ~~EOAAHDOE | TCSHAE~~

AAHDOETC | SHAEIRLM

用π^{-1}解密后: ETNGEELM | DNONETOR | DAEATHCO | ESRHLAMI

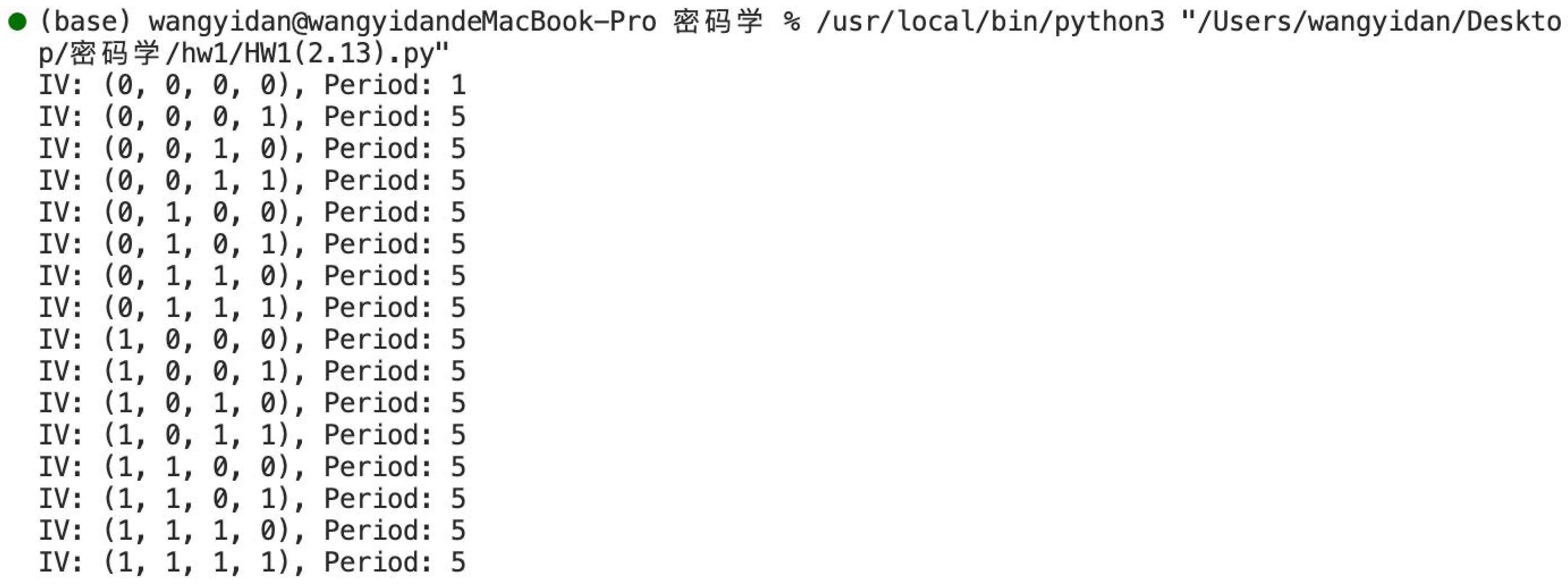
2.18 Consider the following linear recurrence over $\mathbb{Z}_2$ of degree four:

$$z_{i+4} = (z_i + z_{i+1} + z_{i+2} + z_{i+3}) \bmod 2,$$

$i \geq 0$. For each of the 16 possible initialization vectors $(z_0, z_1, z_2, z_3) \in (\mathbb{Z}_2)^4$, determine the period of the resulting keystream.

Answer:

```
from itertools import product

# 定义 LFSR 函数
def lfsr(initial_state):
state = initial_state[:] # 复制初始状态，避免修改原始输入
seen_states = {tuple(state)} # 使用集合存储已见状态
count = 0

while True:
# 应用线性递归关系
new_bit = (state[0] + state[1] + state[2] + state[3]) % 2
state = state[1:] + [new_bit] # 移位并添加新状态
count += 1
# 如果新状态已经见过，结束循环
if tuple(state) in seen_states:
break
seen_states.add(tuple(state))
return count

# 遍历所有可能的初始向量（IVs），因为 Z2 的范围是 [0, 1]
ivs = list(product(range(2), repeat=4))

# 计算每个 IV 的周期
periods = {iv: lfsr(list(iv)) for iv in ivs}

# 打印周期
for iv, period in periods.items():
print(f"IV: {iv}, Period: {period}")
```



```
● (base) wangyidan@wangyidandeMacBook-Pro 密码学 % /usr/local/bin/python3 "/Users/wangyidan/Deskto
  p/密码学/hw1/HW1(2.13).py"
  IV: (0, 0, 0, 0), Period: 1
  IV: (0, 0, 0, 1), Period: 5
  IV: (0, 0, 1, 0), Period: 5
  IV: (0, 0, 1, 1), Period: 5
  IV: (0, 1, 0, 0), Period: 5
  IV: (0, 1, 0, 1), Period: 5
  IV: (0, 1, 1, 0), Period: 5
  IV: (0, 1, 1, 1), Period: 5
  IV: (1, 0, 0, 0), Period: 5
  IV: (1, 0, 0, 1), Period: 5
  IV: (1, 0, 1, 0), Period: 5
  IV: (1, 0, 1, 1), Period: 5
  IV: (1, 1, 0, 0), Period: 5
  IV: (1, 1, 0, 1), Period: 5
  IV: (1, 1, 1, 0), Period: 5
  IV: (1, 1, 1, 1), Period: 5
```

2.23 Suppose we are told that the plaintext

`breathtaking`

yields the ciphertext

`RUPOTENTOIFV`

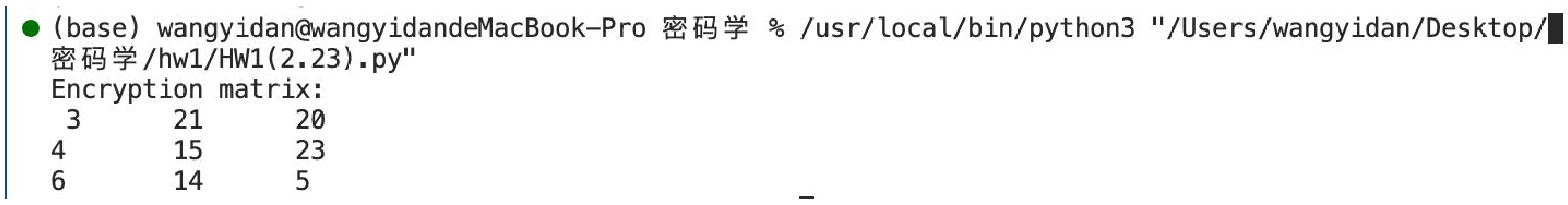
where the *Hill Cipher* is used (but m is not specified). Determine the encryption matrix.

Answer:

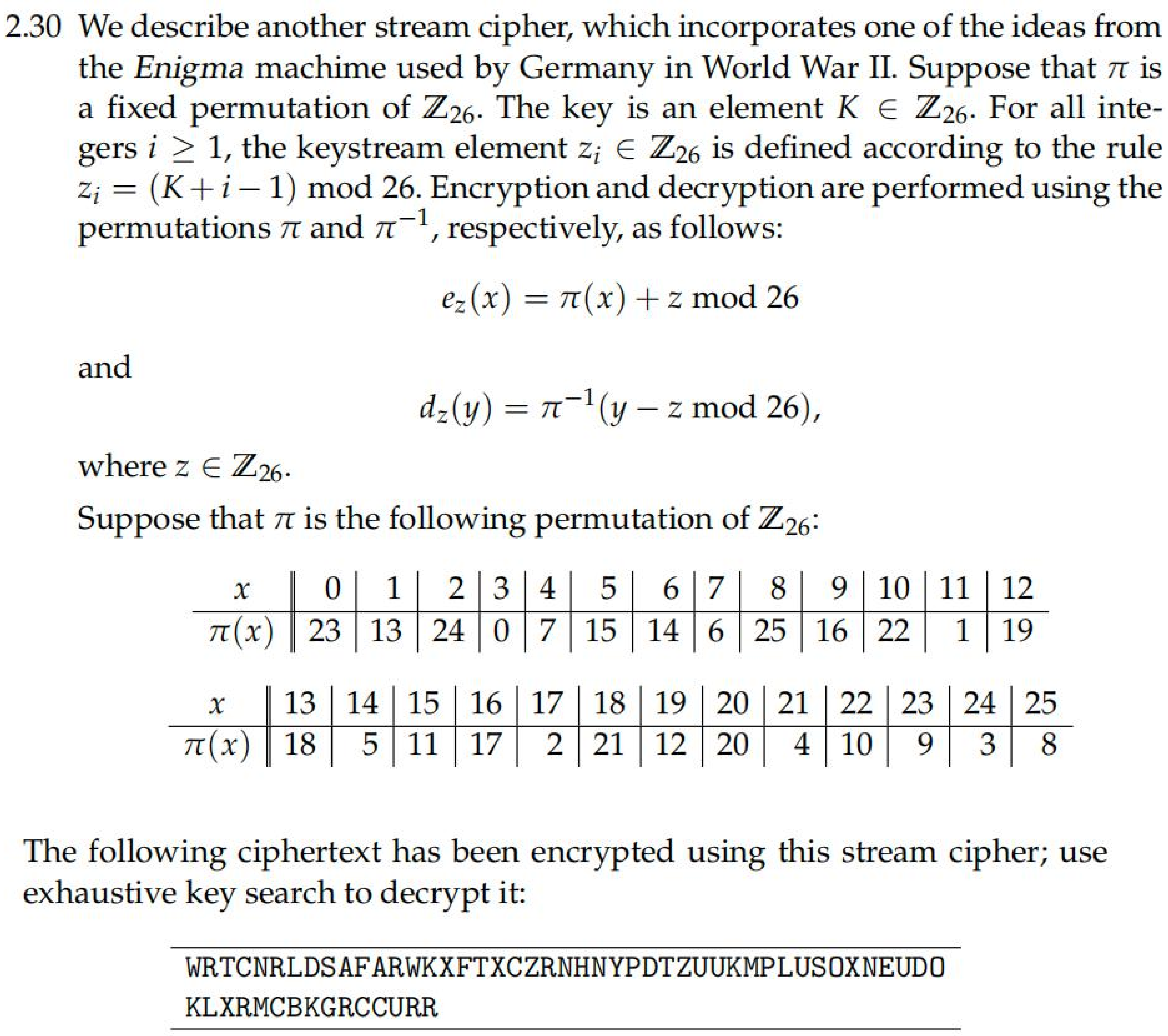
```
from sympy import Matrix, mod_inverse

# 将字母转换为数字
def letters_to_numbers(letters):
return [ord(letter) - ord('A') for letter in letters.upper()]

# 将文本转换为数字，假设它已经是大写且没有空格或非字母字符
def text_to_numeric(text):
return [letters_to_numbers(text[i:i+3]) for i in range(0,
len(text), 3)]

# 使用明文和密文求解加密矩阵
def solve_hill_cipher(plaintext, ciphertext):
# 将明文和密文分割成大小为3的块，并转换为数字
plaintext_blocks = text_to_numeric(plaintext)
ciphertext_blocks = text_to_numeric(ciphertext)

# 使用第一个块来确定加密矩阵
P = Matrix(plaintext_blocks[0:3])
C = Matrix(ciphertext_blocks[0:3])

# 求加密矩阵A，使P * A = C mod 26
# P求模26的倒数解A
try:
P_inv = P.inv_mod(26)
except ValueError as e:
return str(e), None # If the inverse doesn't exist, return
the error message

A = P_inv * C % 26
return None, A

# 给定明文和密文
plaintext = "breathtaking"
ciphertext = "RUPOTENTOIFV"
```

```
# 假设 n = 3，求解加密矩阵
error, encryption_matrix = solve_hill_cipher(plaintext,
ciphertext)

# 检查矩阵是否找到并打印出来
if encryption_matrix:
# Format and print the matrix
matrix_as_list = encryption_matrix.tolist()
formatted_matrix = '\n'.join(['\t'.join(map(str, row)) for
row in matrix_as_list])
print("Encryption matrix:\n", formatted_matrix)
else:
# If there was an error, print it
print("Error:", error)
```

因为明文有 12 个字符所以加密矩阵的类型可以是 2*2,3*3,4*4 和 6*6 在代码中我尝试使用 3*3 的加密矩阵并输出了结果。



```
● (base) wangyidan@wangyidandeMacBook-Pro 密码学 % /usr/local/bin/python3 "/Users/wangyidan/Desktop/
  密码学/hw1/HW1(2.23).py"
  Encryption matrix:
   3      21      20
  4       15      23
  6       14      5
```



2.30 We describe another stream cipher, which incorporates one of the ideas from the *Enigma* machine used by Germany in World War II. Suppose that π is a fixed permutation of $\mathbb{Z}_{26}$. The key is an element $K \in \mathbb{Z}_{26}$. For all integers $i \geq 1$, the keystream element $z_i \in \mathbb{Z}_{26}$ is defined according to the rule $z_i = (K + i - 1) \bmod 26$. Encryption and decryption are performed using the permutations π and π^{-1}, respectively, as follows:

$$e_z(x) = \pi(x) + z \bmod 26$$

and

$$d_z(y) = \pi^{-1}(y - z \bmod 26),$$

where $z \in \mathbb{Z}_{26}$.

Suppose that π is the following permutation of $\mathbb{Z}_{26}$:

x	0	1	2	3	4	5	6	7	8	9	10	11	12
$\pi(x)$	23	13	24	0	7	15	14	6	25	16	22	1	19

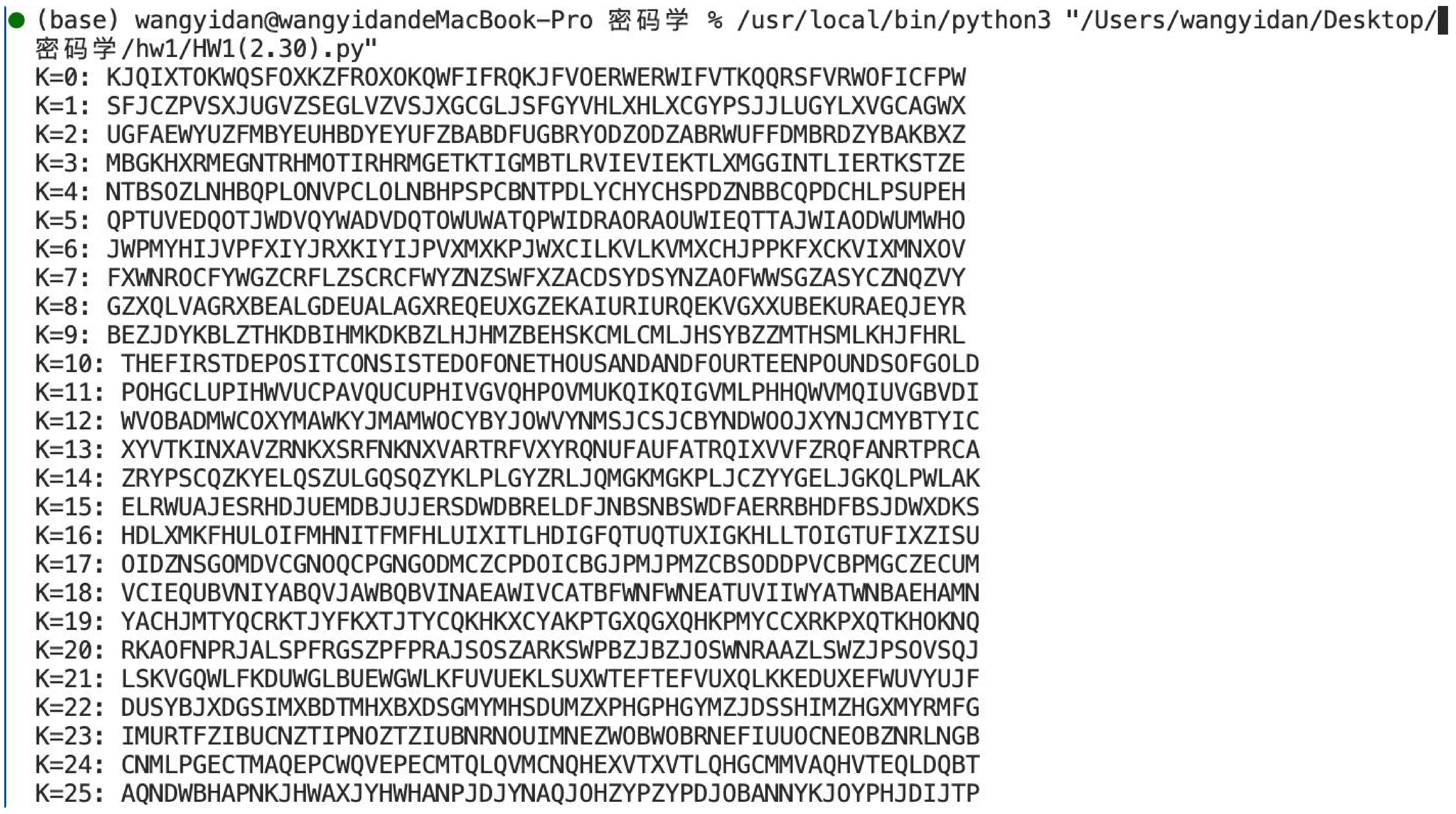
x	13	14	15	16	17	18	19	20	21	22	23	24	25
$\pi(x)$	18	5	11	17	2	21	12	20	4	10	9	3	8

The following ciphertext has been encrypted using this stream cipher; use exhaustive key search to decrypt it:

```
WRTCNRLDSAFARWKXFTXCZRNHNYPDTZUUKMPLUSOXNEUDO
KLXRMCBKGRCCURR
```

Answer:

```
def decrypt(ciphertext, K, pi):
# 反置换字典
pi_inverse = {v: k for k, v in pi.items()}
# 解密函数
def dz(y, z):
return pi_inverse[(y - z) % 26]
plaintext = ''
for i, c in enumerate(ciphertext):
z = (K + i) % 26
y = ord(c) - ord('A')
x = dz(y, z)
plaintext += chr(x + ord('A'))
return plaintext
# 置换π
pi = {0: 23, 1: 13, 2: 24, 3: 0, 4: 7, 5: 15, 6: 14, 7: 6,
8: 25, 9: 16, 10: 22, 11: 1, 12: 19,
13: 18, 14: 5, 15: 11, 16: 17, 17: 2, 18: 21, 19: 12, 20: 20,
21: 4, 22: 10, 23: 9, 24: 3, 25: 8}
ciphertext =
"WRTCNRLDSAFARWKXFTXCZRNHNYPDTZUUKMPLUSOXNEUDOKLXRMCBKGRC
CURR"
# 尝试每个可能的密钥 K
for K in range(26):
plaintext = decrypt(ciphertext, K, pi)
print(f"K={K}: {plaintext}")
```



```
● (base) wangyidan@wangyidandeMacBook-Pro 密码学 % /usr/local/bin/python3 "/Users/wangyidan/Desktop/
  密码学/hw1/HW1(2.30).py"
  K=0: KJQIXTOKWQSFOXKZFROXOKQWFIFRQKJFVOERWERWIFVTKQQRSFVRWOFICFPW
  K=1: SFJCZPVSXJUGVZSEGLVZVSJXGCGLJSFGYVHLXHLXCGYPSJJLUGYLXVGCAGWX
  K=2: UGFAEWYUZFMBYEUHBDYEYUFZBABDFUGBRYODZODZABRWUFFDMBRDZYBAKBXZ
  K=3: MBGKHXRMEGNTRHMOTIRHRMGETKTIGMBTLRVIEVIEKTLXMGGINTLIERTKSTZE
  K=4: NTBSOZLNHBQPLONVPCLOLNBHPSPCBNTPDLYCHYCHSPDZNBBCQPDCHLPSUPEH
  K=5: QPTUVEDQOTJWDVQYWADVDQTOWUWATQPWIDRAORAOUWIEQTTAJWIAODWUMWHO
  K=6: JWPMYHIJVPFXIYJRXKIYIJPVXMXKPJWXCILKVLKVMXCHJPPKFXCKVIXMNXOV
  K=7: FXWNROCFYWGZCRFLZSCRCFWYZNZSWFXZACDSYDSYNZAOFWWSGZASYCZNQZVY
  K=8: GZXQLVAGRXBEALGDEUALAGXREQEUXGZEKAIURIURQEKVGXXUBEKURAEQJEYR
  K=9: BEZJDYKBLZTHKDBIHMKDKBZLHJHMZBEHSKCMLCMLJHSYBZZMTHSMLKHJFHRL
  K=10: THEFIRSTDEPOSITCONSISTEDOFONETHOUSANDANDFOURTEENPOUNDSOFGOLD
  K=11: POHGCLUPIHWVUCPAVQUCUPHIVGVQHPOVMUKQIKQIGVMLPHHQWVMQIUVGBVDI
  K=12: WVOBADMWCOXYMAWKYJMAMWOCYBYJOWVYNMSJCSJCBYNDWOOJXYNJCMYBTYIC
  K=13: XYVTKINXAVZRNKXSRFNKNXVARTRFVXYRQNUFAUFATRQIXVVFZRQFANRTPRCA
  K=14: ZRYPSCQZKYELQSZULGQSQZYKLPLGYZRLJQMGKMGKPLJCZYYGELJGKQLPWLAK
  K=15: ELRWUAJESRHDJUEMDBJUJERSDWDBRELDFJNBSNBSWDFAERRBHDFBSJDWXDKS
  K=16: HDLXMKFHULOIFMHNITFMFHLUIXITLHDIGFQTUQTUXIGKHLLTOIGTUFIXZISU
  K=17: OIDZNSGOMDVCGNOQCPGNGODMCZCPDOICBGJPMJPMZCBSODDPVCBPMGCZECUM
  K=18: VCIEQUBVNIYABQVJAWBQBVINAEAWIVCATBFWNFWNEATUVIIWYATWNBAEHAMN
  K=19: YACHJMTYQCRKTJYFKXTJTYCQKHKXCYAKPTGXQGXQHKPMYCCXRKPXQTKHOKNQ
  K=20: RKAOFNPRJALSPFRGSZPFPRAJSOSZARKSWPBZJBZJOSWNRAAZLSWZJPSOVSQJ
  K=21: LSKVGQWLFKDUWGLBUEWGWLKFUVUEKLSUXWTEFTEFVUXQLKKEDUXEFWUVYUJF
  K=22: DUSYBJXDGSIMXBDTMHXBXDSGMYMHSDUMZXPHGPHGYMZJDSSHIMZHGXMYRMFG
  K=23: IMURTFZIBUCNZTIPNOZTZIUBNRNOUIMNEZWOBWOBRNEFIUUOCNEOBZNRLNGB
  K=24: CNMLPGECTMAQEPCWQVEPECMTQLQVMCNQHEXVTXVTLQHGCMMVAQHVTEQLDQBT
  K=25: AQNDWBHAPNKJHWAXJYHWHANPJDJYNAQJOHZYPZYPDJOBANNYKJOYPHJDIJTP
```