SDM5008 ACR - HW2 SID: 12443028

1. Spatial Velocity:

(a). $^{o}\dot{C} = \frac{d}{dt}[C_x(t) \quad 0 \quad 0]^T = [v \quad 0 \quad 0]^T$

(b). $^{o}\dot{A} = \frac{d}{dt}[A_x(t) \quad 0 \quad 0]^T = [v \quad 0 \quad 0]^T$

(c). zero velocity

(d). $2V$

(e). Take the center as the reference point:

$^{o}\mathcal{V} = (^{o}w, {}^{o}v_r)$. where $^{o}w = [0 \quad -|\frac{v}{r}| \quad 0]^T$, $^{o}v_r = [v \quad 0 \quad 0]^T$

(f). $^{c}\mathcal{V} = (^{c}w, {}^{c}v_r)$, where $^{c}w = [0 \quad -|\frac{v}{r}| \quad 0]^T$, $^{c}v_r = [0 \quad 0 \quad 0]^T$

2. Modern Robotics Exercise - 3.21

(a.) Using Wei Zhang's Notation instead:

$T_{ab} \to {}^{a}T_b$, $p_a \to {}^{a}p$

$$\Rightarrow {}^{a}T_b = \begin{bmatrix} {}^{a}R_b & {}^{a}O_b \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & -100 \\ 1 & 0 & 0 & 300 \\ 0 & 0 & 1 & 500 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^{b}R_a = {}^{a}R_b^T = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \cancel{{}^{b}O_a} =$$

$$\underset{\sim}{^{b}r} = {}^{b}R_a {}^{a}p = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 800 \\ 0 \end{bmatrix} = \begin{bmatrix} 800 \\ 0 \\ 0 \end{bmatrix}$$

r_b in Modern Robotics' notation

(b). ${}^{b}T_{c} = {}^{b}T_{a}\,{}^{a}T_{c}$. we need to determine ${}^{b}T_{a}$, ${}^{a}T_{c}$ first:

$${}^{a}T_{c} = \begin{bmatrix} {}^{a}R_{c} & {}^{a}O_{c} \\ 0 & 1 \end{bmatrix}, \quad {}^{a}R_{c} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}, \quad {}^{a}O_{c} = \begin{bmatrix} 0 \\ 800 \\ 0 \end{bmatrix}$$

$$\Rightarrow {}^{a}T_{c} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 800 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

For ${}^{b}T_{a}$: ${}^{b}T_{a} = \begin{bmatrix} {}^{b}R_{a} & {}^{b}O_{a} \\ 0 & 1 \end{bmatrix}$, ${}^{b}R_{a}$ is known by (a), ${}^{b}O_{a} = \overrightarrow{O_bO_a} = \vec{r} - \vec{p}$

$$\Rightarrow {}^{b}O_{a} = \begin{bmatrix} 800 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 800 \\ 0 \end{bmatrix} = \begin{bmatrix} 800 \\ -800 \\ 0 \end{bmatrix}$$

$$\Rightarrow {}^{b}T_{a} = \begin{bmatrix} 0 & 1 & 0 & 800 \\ -1 & 0 & 0 & -800 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \text{Above all: } {}^{b}T_{c} = \begin{bmatrix} 0 & 1 & 0 & 800 \\ -1 & 0 & 0 & -800 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 800 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \frac{\sqrt{3}}{2} & -\frac{1}{2} & 1600 \\ -1 & 0 & 0 & -800 \\ 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3. Exercise - 3.28.

$${}^{b}\omega = {}^{b}R_{s}\,{}^{s}\omega = ({}^{s}R_{b})^{T}\,{}^{s}\omega = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 3 \\ -1 \\ -2 \end{bmatrix}$$

4. Exercise - 5.5

(a). $${}^{s}P_{(\theta)} = \begin{bmatrix} L + d\sin\theta \\ L - d\cos\theta \\ 0 \end{bmatrix}$$

(b). $${}^{s}\dot{p} = \begin{bmatrix} d\cos\theta\dot{\theta} \\ d\sin\theta\dot{\theta} \\ 0 \end{bmatrix} = \begin{bmatrix} d\cos\theta \\ d\sin\theta \\ 0 \end{bmatrix}$$

(c). $${}^{s}T_{b} = \begin{bmatrix} {}^{s}R_{b} & {}^{s}O_{b} \\ 0 & 1 \end{bmatrix}, \quad {}^{s}R_{b} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad {}^{s}O_{b} = \begin{bmatrix} L \\ L \\ 0 \end{bmatrix}$$

$$\Rightarrow {}^{s}T_{b} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & L \\ \sin\theta & \cos\theta & 0 & L \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(d). ${}^{b}\mathcal{V} = [\omega, V_r]^T$, $|\omega| = \dot{\theta}$, $V_r = 0$

$$\Rightarrow {}^{b}\mathcal{V} = [0 \;\; 0 \;\; 1 \;\vdots\; 0 \;\; 0 \;\; 0]^T$$

(e). ${}^s\mathcal{V} = [{}^s\omega, {}^sv_r]^T$, $|{}^s\omega| = \dot\theta$, ${}^bv_r = {}^s\dot p = \begin{bmatrix} d\cos\theta \\ d\sin\theta \\ 0 \end{bmatrix}$

$$\Rightarrow {}^s\mathcal{V} = [0 \quad 0 \quad \dot\theta \ \vdots\ d\cos\theta \quad d\sin\theta \quad 0]^T$$

(f). ${}^s\mathcal{V}$ and ${}^b\mathcal{V}$ is connected by a transformation matrix. ${}^bX_s$ (or ${}^sX_b$)

$${}^sX_b = \begin{bmatrix} {}^sR_b & 0 \\ [{}^sO_b]\,{}^sR_b & {}^sR_b \end{bmatrix}, \text{ where } {}^sR_b = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$${}^sO_b = [L + d\sin\theta, L - d\overset{\cos\theta}{\cancel{\sin\theta}}, 0]^T$$

$\cancel{\Rightarrow {}^sX_b = [}$ R and $[{}^sO_b]\,{}^sR_b = \begin{bmatrix} 0 & 0 & L - d\overset{\cos\theta}{\cancel{\sin\theta}} \\ 0 & 0 & -L - d\sin\theta \\ L(\sin\theta - \cos\theta) - d\cos 2\theta & L(\sin\theta + \cos\theta) & 0 \end{bmatrix}$

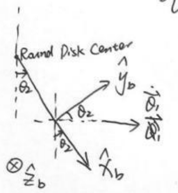
(g) twist of (d) has nothing to do with $\dot P$ from (b)

5. Exercise - 5.6

(a) We see from the disk (2D view)

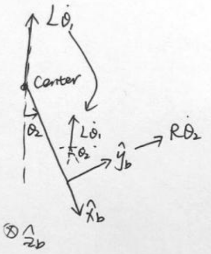
At $t = 0$: $\hat z_b$ is poing outward of the paper

For ${}^b\omega$:



$${}^b\omega = \begin{bmatrix} \sin\theta_2\,\dot\theta_1 \\ \cos\theta_2\,\dot\theta_1 \\ \dot\theta_2 \end{bmatrix} = \begin{bmatrix} \sin t \\ \cos t \\ 1 \end{bmatrix}$$

For ${}^bv$:



$${}^bv = \begin{bmatrix} -L\dot\theta_1\cos\theta_2 \\ L\dot\theta_1\sin\theta_2 + R\dot\theta_2 \\ \cancel{-R\dot\theta_2}\ 0 \end{bmatrix} = \begin{bmatrix} -20\cos t \\ 20\sin t + 10 \\ 0\ \cancel{10\cos t} \end{bmatrix}$$

(b) $\dot P(t) = {}^sv_{(t)} = {}^sR_{b(t)}\,{}^bv_{(t)}$

$${}^sR_{b(t)} = \begin{bmatrix} \cos t & 0 & \sin t \\ 0 & 1 & 0 \\ -\sin t & 0 & \cos t \end{bmatrix} \begin{bmatrix} \cos t & -\sin t & 0 \\ \sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cancel{10}\ -20\cos t \\ 20\sin t + 10 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -20\cos t - 10\sin t\cos t \\ 10\cos t \\ 20\sin t + 10\sin^2 t \end{bmatrix}$$