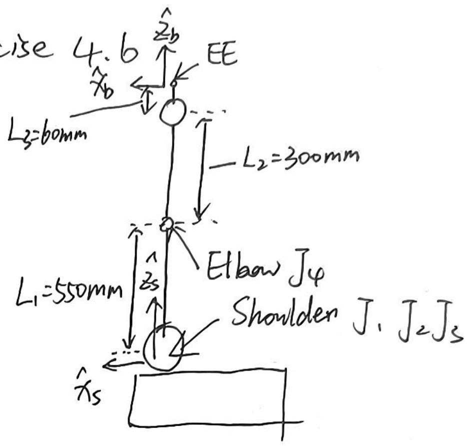
SDM5008 ACR - HW3 SID: 12443028

1. Mordern Robotics. Exercise 4.6

WAM 7R robot arm:



The space frame axes S_i for WAM robot:

$$\hat{S}_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \hat{S}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \hat{S}_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

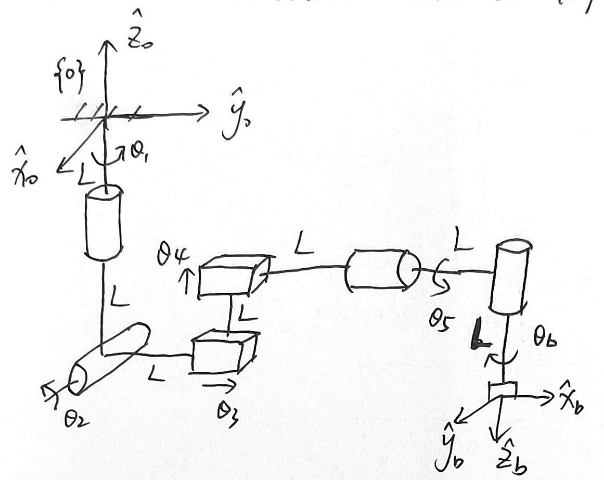
$$\hat{S}_4 = \begin{bmatrix} \hat{w}_4 \\ \hat{v}_4 \end{bmatrix}, \quad \hat{w}_4 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \hat{v}_4 = \text{"}h\hat{s} - w\times q\text{"} = -\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\times\begin{bmatrix} 0 \\ 0 \\ L_1 \end{bmatrix} = \begin{bmatrix} -L_1 \\ 0 \\ 0 \end{bmatrix}$$

$$\hat{S}_5 = \begin{bmatrix} \hat{w}_5 \\ \hat{v}_5 \end{bmatrix}, \quad \hat{w}_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \hat{v}_5 = \text{"}h\hat{s} - w\times q\text{"} = -\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\times\begin{bmatrix} 0 \\ 0 \\ L_1+L_2 \end{bmatrix} = 0$$

$$\hat{S}_6 = \begin{bmatrix} \hat{w}_6 \\ \hat{v}_6 \end{bmatrix}, \quad \hat{w}_6 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad \hat{v}_6 = -\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}\times\begin{bmatrix} 0 \\ 0 \\ L_1+L_2 \end{bmatrix} = \begin{bmatrix} -L_1-L_2 \\ 0 \\ 0 \end{bmatrix}$$

$$\hat{S}_7 = \begin{bmatrix} \hat{w}_7 \\ \hat{v}_7 \end{bmatrix}, \quad \hat{w}_7 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad \hat{v}_7 = -\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}\times\begin{bmatrix} 0 \\ 0 \\ L_1+L_2+L_3 \end{bmatrix} = 0$$

2. Mordern Robotics - Exercise 4.9



a). Zero Configuration M of EE:

$M = T(R, p)$, where $R = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$

b). $S_1 = [0\ 0\ 1 \,|\, 0\ 0\ 0]^T$, $S_2 = [1\ 0\ 0 \,|\, 0\ 2L\ 0]^T$, $S_3 = [0\ 0\ 0 \,|\, 0\ \dot{\theta}_3\ 0]^T$

$S_4 = [0\ 0\ 0 \,|\, 0\ 0\ \dot{\theta}_4]^T$, $S_5 = [0\ 1\ 0 \,|\, -2L\ 0\ 0]^T$, $S_6 = [0\ 0\ -1 \,|\, 3L\ 0\ 0]^T$

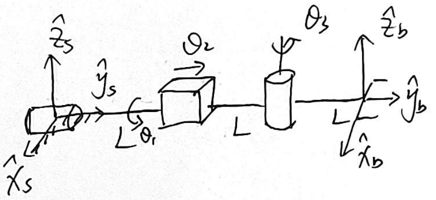
c). ${}^bS_1 = [0\ 0\ -1 \,|\, 0\ -3L\ 0]^T$, ${}^bS_2 = [0\ 1\ 0 \,|\, 0\ 0\ 3L]^T$, ${}^bS_3 = [0\ 0\ 0 \,|\, \dot{\theta}_3\ 0\ 0]^T$

${}^bS_4 = [0\ 0\ 0 \,|\, 0\ 0\ -\dot{\theta}_4]^T$, ${}^bS_5 = [1\ 0\ 0 \,|\, 0\ -L\ 0]^T$, ${}^bS_6 = [0\ 0\ 1 \,|\, 0\ 0\ 0]^T$

3. Modern Robotics - Exercise 5.8 (a)

Find ${}^sJ_{\xi}(\theta)$ for arbitrary configurations $\theta \in R^3$

RPR Robot:



$${}^s\bar{S}_1 = [0\ 1\ 0 \mid 0\ 0\ 0]^T, \quad {}^s\bar{S}_2 = [0\ 0\ 0 \mid 0\ \dot{\theta}_2\ 0]^T$$

$${}^s\bar{S}_3 = [0\ 0\ 1 \mid 2L\ 0\ 0]$$

$${}^sJ_1 = {}^sS_1(\theta) = {}^s\bar{S}_1 = [0\ 1\ 0 \mid 0\ 0\ 0]^T$$

$${}^sJ_2 = \cancel{\theta}\,{}^sS_2(\theta) = [Ad_{\hat{T}(\theta_1)}]\,{}^s\bar{S}_2 \text{, where } \hat{T}(\theta_1) = e^{[{}^s\bar{S}_1]\theta_1}$$

$${}^sJ_3 = {}^sS_3(\theta) = [Ad_{\hat{T}(\theta_1,\theta_2)}]\,{}^s\bar{S}_3 \text{. where } \hat{T}(\theta_1,\theta_2) = e^{[{}^s\bar{S}_1]\theta_1} e^{[{}^s\bar{S}_2]\theta_2}$$

$$\Rightarrow {}^sJ_{\xi}(\theta) = \cancel{[{}^sJ_1 \xi(\theta_1}$$

$$= [{}^sJ_1 \mid {}^sJ_2(\theta_1) \mid {}^sJ_3(\theta_1,\theta_2)]$$