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Value in t periods with simple interest:
$C_0 \times [1 + (\text{simple interest rate per period} \times t)] = C_0 + (C_0 \times \text{simple interest rate per period} \times t)$

$FV_t = C_0 \times (1+r)^t$
Financial calculator: In either BEGIN or END mode, FV is the future value in N periods from the reference point (time 0) of a cash flow equal to -PV at the reference point with an interest rate, return, etc. of I% per period

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$FV_t = C_k \times (1+r)^{(t-k)}$

$PV_0 = PV = C_t / (1 + r)^t$
Financial calculator: In either BEGIN or END mode, PV is the opposite of the present value as of the reference point (time 0) of a cash flow equal to FV that takes place in N periods from the reference point, with a discount rate of I% per period

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$PV_0 = PV = C_0 + [C_1/(1+r)^1] + [C_2/(1+r)^2] + \ldots + [C_{t-1}/(1+r)^{t-1}] + [C_t/(1+r)^t]$

PV for a fixed perpetuity $= [C/(1+r)] + [C/(1+r)^2] + [C/(1+r)^3] + \ldots = C / r$
Rate of return for a fixed perpetuity $= r = C / PV$
Cash flow for a fixed perpetuity $= C = PV \times r$

PV for a growing perpetuity $= C_1/(1+r) + [C_1(1+g)]/(1+r)^2 + [C_1(1+g)^2]/(1+r)^3 + \ldots = C_1 / (r - g)$
Rate of return for a growing perpetuity $= r = [C_1 / PV] + g$
First cash flow for a growing perpetuity $= C_1 = PV \times (r - g)$
Growth rate for a growing perpetuity $= g = r - [C_1 / PV]$
$C_k = C_1 \times (1 + g)^{k-1}$ which is the same as $C_t = C_1 \times (1 + g)^{(t-1)}$
Also, $C_b = C_a \times (1 + g)^{(b-a)}$ so $g = [(C_b / C_a)^{[1/(b-a)]}] - 1$

PV for an annuity $= [C/(1+r)] + [C/(1+r)^2] + \ldots + [C/(1+r)^t]$
$= C \times [\{1 - 1/(1+r)^t\} / r] = C \times [(1/r) - 1/\{r(1+r)^t\}] = (C/r) \times [1 - (1/\{(1+r)^t\})]$
Financial calculator: In END mode, PV is the opposite of the present value as of the reference point (time 0) of a series of N regular cash flows equal to PMT per period where the first regular cash flow takes place 1 period from the reference point, the last cash flow takes place N periods from the reference point, and the discount rate is I% per period

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PV for an annuity due $= (1+r) \times$ PV for an annuity $= C + [C/(1+r)] + [C/(1+r)^2] + [C/(1+r)^3] + \ldots + [C/(1+r)^{t-1}]$
$= (1+r) \times C \times [\{1 - 1/(1+r)^t\} / r] = (1+r) \times C \times [(1/r) - 1/\{r(1+r)^t\}] = C + (C/r) \times [1 - (1/\{(1+r)^{t-1}\})])$
Financial calculator: In BEGIN mode, PV is the opposite of the present value as of the reference point (time 0) of a series of N regular cash flows equal to PMT per period where the first regular cash flow takes place at the reference point, the last cash flow takes place N-1 periods from the reference point, and the discount rate is I% per period

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$PV_0 = PV = PV_k / (1+r)^k$

$FV_t = [C_0 \times (1+r)^t] + [C_1 \times (1+r)^{t-1}] + [C_2 \times (1+r)^{t-2}] + \ldots + [C_k \times (1+r)^{t-k}] + \ldots + [C_{t-1} \times (1 + r)^1] + [C_t]$

FV for an annuity $= [C_1 \times (1+r)^{t-1}] + [C_2 \times (1+r)^{t-2}] + \ldots + C_t$
$= (1+r)^t \times C \times [\{1 - 1/(1+r)^t\} / r] = C \times [\{(1+ r)^t - 1\} / r] = (1+r)^t \times C \times [(1/r) - 1/\{r(1+r)^t\}]$
Financial calculator: In END mode, FV is the future value in N periods from the reference point (time 0) of a series of N regular cash flows equal to -PMT per period where the first regular cash flow takes place 1 period from the reference point, the last cash flow takes place N periods from the reference point, and the interest rate, return, etc. is I% per period

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FV for an annuity due $= (1+r) \times$ FV for an annuity $= [C_0 \times (1+r)^t] + [C_1 \times (1+r)^{t-1}] + \ldots + [C_{t-1} \times (1+r)^1]$
$= (1+r)^{t+1} \times C \times [\{1 - 1/(1+r)^t\} / r] = (1+r) \times C \times [\{(1+r)^t - 1\} / r] = (1+r)^{t+1} \times C \times [(1/r) - 1/\{r(1+r)^t\}]$
Financial calculator: In BEGIN mode, FV is the future value in N periods from the reference point (time 0) of a series of N regular cash flows equal to -PMT per period where the first regular cash flow takes place at the reference point, the last cash flow takes place N-1 periods from the reference point, and the interest rate, return, etc. is I% per period

APR = annual percentage rate = # periods in a year × periodic rate = # periods in a year × $[(1 + \text{EAR})^{1/\#\text{ periods in a year}} – 1]$

EAR = Effective annual rate = $[(1 + \text{periodic rate})^{\#\text{ of periods in a year}}] – 1 = [1 + (\text{APR}/\#\text{ periods per year})]^{\#\text{ periods per year}} – 1$

Periodic rate = [APR / # periods per year] = $[(1 + \text{EAR})^{(1\ /\ \#\text{ of periods in a year})} – 1]$

EAR with continuous compounding = $(e^{\text{APR}}) – 1$

Bond value = $[\text{cpn}/(1+r)^1] + [\text{cpn}/(1+r)^2] +\ldots+ [\text{cpn}/(1+r)^t] + [\text{face}/(1+r)^t]$
$= \{\text{cpn} \times [\{1 – 1/(1+r)^t\} / r]\} + \{\text{face}/(1+r)^t\} = \{\text{cpn} \times [(1/r) – 1/\{r(1+r)^t\}]\} + \{\text{face}/(1+r)^t\}$
Financial calculator: Bond value equals -PV, where PV is the opposite of the present value as of the reference point (time 0) of N coupon payments equal to PMT per period where each coupon equals the coupon rate multiplied by the face value divided by the number of coupons per year, the first coupon is paid 1 period from the reference point (END mode), N is the number of coupons paid before maturity and equals number of coupons per year multiplied by the number of years to maturity, the discount rate is I% per period, where I% equals the bond's yield-to-maturity divided by the number of coupons per year, and FV is the face (or par) value of the bond

r for a bond = discount rate per period, where a period equals 1 year divided by the number of coupons per year

Coupon payment = (coupon rate × face value) / number of coupons per year
= total aggregate dollar amount of coupons per year / number of coupons per year

Total aggregate dollar amount of coupons per year = coupon rate × face value = coupon rate × par value
= coupon payment × number of coupons per year

Coupon rate = annual coupon rate = total aggregate dollar amount of coupons per year / face value

YTM = yield-to-maturity = expected annual return for a bond (as an APR)
= r × the number of coupon payments per year
= discount rate per period × the number of coupon payments per year

Current yield = total aggregate dollar amount of coupons per year / bond value
= (coupon rate × face value) / bond value

Total dollar return = cash flows from investment + capital gain
= initial value × percentage return = initial value × return

Capital gain = ending value – initial value

Percentage return = return
= total dollar return ÷ initial value
= (cash flows from investment + capital gain) ÷ initial value
= (cash flows from investment + ending value – initial value) ÷ initial value

Percentage return for a bond = return for a bond
= (coupons + capital gain) ÷ initial bond value
= (coupons + ending bond value – initial bond value) ÷ initial bond value

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Return for a stock = (dividends + capital gain) ÷ initial stock value
= (dividends + ending stock value – initial stock value) ÷ initial stock value
= $(D_1 + P_1 – P_0) / P_0$ when time 1 is today or earlier (so all relevant time periods have taken place)
= dividend yield + capital appreciation yield

Dividend yield = dividends / initial value

Capital appreciation yield = capital gain / initial value = (ending value – initial value) / initial value
Ending value = initial value × (1 + capital appreciation yield)

Expected total dollar return = expected cash flows from investment + expected capital gain
= expected initial value × expected percentage return = expected initial value × expected return

Expected capital gain = expected ending value – expected initial value

Expected percentage return = expected return
= expected total dollar return ÷ expected initial value
= (expected cash flows from investment + expected capital gain) ÷ expected initial value
= (expected CFs from investment + expected ending value – expected initial value) ÷ expected initial value

Expected return for a stock = (expected dividends + expected capital gain) ÷ expected initial stock value
= (expected dividends + expected ending stock value – expected initial stock value) ÷ expected initial stock value
= $(D_1 + P_1 – P_0) / P_0$ when time 0 is today or later (so not all relevant time periods have taken place)
= expected dividend yield + expected capital appreciation yield

Expected dividend yield = expected cash flows from investment / expected initial value
= expected dividends / expected initial value

Expected capital appreciation yield = expected capital gain / expected initial value
= (expected ending value – expected initial value) / expected initial value
Expected ending value = expected initial value × (1 + expected capital appreciation yield)

Stock value
$P_0 = [(D_1 + P_1)/(1 + R)]$
$= [D_1/(1 + R)] + [(D_2 + P_2)/(1 + R)^2] = [D_1/(1 + R)] + [D_2/(1 + R)^2] + [P_2/(1 + R)^2]$
$= [D_1/(1 + R)] + [D_2/(1 + R)^2] +...+ [(D_N + P_N)/(1 + R)^N] = [D_1/(1+R)] + [D_2/(1+R)^2] +...+ [D_N/(1+R)^N] + [P_N/(1+R)^N]$
$= [D_1/(1 + R)] + [D_2/(1 + R)^2] + ...$

R for a stock is the annual expected return for the stock divided by the number of possible dividends per year

Expected stock value
$P_t = [(D_{t+1} + P_{t+1}) / (1 + R)]$
$= [D_{t+1}/(1 + R)] + [(D_{t+2} + P_{t+2})/(1 + R)^2] = [D_{t+1}/(1 + R)] + [D_{t+2}/(1 + R)^2] + [P_{t+2}/(1 + R)^2]$
$= [D_{t+1}/(1 + R)] + [D_{t+2}/(1 + R)^2] + ... + [(D_{t+N} + P_{t+N})/(1 + R)^N]$
$= [D_{t+1}/(1 + R)] + [D_{t+2}/(1 + R)^2] + ...$

Constant dividend (no-growth) model
$P_0 = D / R$
$R = D / P_0$ and $D = R \times P_0$

Constant dividend (no-growth) model
$P_t = D / R$
$R = D / P_t$ and $D = R \times P_t$

Constant dividend growth model
$P_0 = D_1 / (R – g)$
$D_k = D_1 \times (1 + g)^{k – 1}$ which is the same as $D_t = D_1 \times (1 + g)^{t – 1}$
Also, $D_b = D_a \times (1 + g)^{b-a}$ so $g = [(D_b / D_a)^{1/(b-a)}] – 1$
$R = (D_1 / P_0) + g$ and $D_1 = P_0 \times (R – g)$ and $g = R – (D_1 / P_0)$

Constant dividend growth model
$P_t = D_{t+1} / (R – g)$
$R = (D_{t+1} / P_t) + g$ and $D_{t+1} = P_t \times (R – g)$ and $g = R – (D_{t+1} / P_t)$

Non-constant dividend growth model
$P_0 = [D_1/(1 + R)] + [D_2/(1 + R)^2] + ... + [(D_N + P_N)/(1 + R)^N]$ where $P_N = D_{N+1} / (R – g)$

Net present value = NPV = $C_0 + [C_1/(1+r)^1] + [C_2/(1+r)^2] + \ldots + [C_t/(1+r)^t]$ *Financial calculator*: npv(discount rate, C_0, {C_1, C_2, …, last non-zero expected cash flow}) produces NPV
Internal rate of return = IRR = discount rate such that the present value of a project's expected cash flows is zero $0 = C_0 + [C_1/(1+IRR)^1] + [C_2/(1+IRR)^2] + \ldots + [C_t/(1+IRR)^t]$ *Financial calculator*: irr(C_0, {C_1, C_2, …, last non-zero expected cash flow}) produces IRR
The payback period is the length of time that it takes for the cumulative expected cash flows produced by a project to equal the initial investment
If payback period is between t and t+1 years, then the portion of year t+1 needed to produce the cash for payback = expected CF needed for payback after t years / expected CF in year t+1 = [investment – cumulative expected CFs produced through time t] / expected CF in year t+1 = [investment – ($C_1 + C_2 + \ldots + C_t$)] / C_{t+1}
The discounted payback period is the length of time that it takes for the cumulative discounted expected cash flows produced by a project to equal the initial investment
If discounted payback period is between t and t+1 years, then the portion of year t+1 needed to produce the discounted cash for discounted payback = expected DCF needed for discounted payback after t years / expected DCF in year t+1 = [investment – cumulative expected DCFs produced through time t] / expected DCF in year t+1 = [investment – {PV(C_1) + PV(C_2) + ... + PV(C_t)}] / PV(C_{t+1})
Relevant cash flow for a project = incremental expected cash flow = expected cash flow with project – expected cash flow without project
Relevant cash flow for a project = operating cash flow + cash flow effects from changes in net working capital + cash flow from capital spending + terminal value
Operating cash flow = OCF = net income + depreciation
Project net income = EBIT – taxes
Project EBIT = project earnings before interest and taxes = project taxable income = revenue – costs – depreciation
Project taxes = taxable income × tax rate = EBIT × tax rate
Revenue = sales = number of units sold × average price per unit
Total costs = costs = total expenses = expenses = fixed costs + variable costs
Variable costs = number of units sold × average variable cost per unit
Annual straight-line depreciation = (investment – amount item is depreciated to) / depreciable life = (investment – amount item is depreciated to) / useful life Note: depreciation expense can only be taken during useful life
Annual MACRS depreciation = investment × relevant rate
Net working capital = NWC = current assets – current liabilities = CA – CL
For project analysis, CA = cash + securities + receivables + inventories and CL = payables
Change in NWC = ΔNWC = NWC at end of period – NWC at start of period, except for the initial change in NWC at time 0, which equals NWC at time 0 $\Delta NWC_t = NWC_t - NWC_{t-1}$ and $\Delta NWC_{t+1} = NWC_{t+1} - NWC_t$
Cash flow effect from change in NWC = opposite of the change in NWC = -ΔNWC
Cash flow from asset sale = sale price of asset – taxes paid on sale of asset
Taxes paid on sale of asset = (sale price of asset – book value of asset) × tax rate = taxable gain on asset × tax rate
Book value of asset = initial investment – accumulated depreciation
Accumulated depreciation = cumulative sum of all depreciation taken for an asset

Average annual return = arithmetic average return = arithmetic average annual return = arithmetic mean return
= arithmetic mean annual return = arithmetic return = arithmetic annual return = mean return = mean annual return
= (return in year 1 + return in year 2 + … + return in year n) / n
= (1/n)(return in year 1) + (1/n)(return in year 2) + … + (1/n)(return in year n)

Compound return = compound annual return = geometric average return = geometric average annual return
= geometric mean return = geometric mean annual return = geometric return = geometric annual return
= [(1 + return in year 1) × (1 + return in year 2) × … × (1 + return in year n)]$^{(1/n)}$ – 1
= [(ending value / starting value)]$^{(1/n)}$ – 1 with no interim CFs or with reinvestment of any interim CFs

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(1 + compound annual return)^{n}
= (1 + return in year 1) × (1 + return in year 2) × … × (1 + return in year n)
= (ending value / starting value) with no interim CFs or with reinvestment of any interim CFs

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(1 + real rate) = (1 + nominal rate) ÷ (1 + inflation rate)
Real rate = [(1+nominal rate) ÷ (1+inflation rate)] – 1

(1 + nominal rate) = (1 + real rate) × (1 + inflation rate)
Nominal rate = [(1+real rate) × (1+inflation rate)] – 1
= the "regular" or "normal" return, expected return, discount rate, etc. used throughout the course

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(1+inflation rate) = (1+nominal rate) ÷ (1+real rate)
Inflation rate = [(1+nominal rate) ÷ (1+real rate)] – 1

Variance of R based on past returns = sample variance
= {[1/(n-1)] × [R_1 – mean(R)]^2} + {[1/(n-1)] × [R_2 – mean(R)]^2} + … + {[1/(n-1)] × [R_t – mean(R)]^2} + … + {[1/(n-1)] × [R_n – mean(R)]^2}

Standard deviation of R based on past returns = sample standard deviation
= √variance of R based on past returns = √sample variance

Expected return based on possible future outcomes = E(R) = [p(1) × R(1)] + [p(2) × R(2)] + … + [p(S) × R(S)]

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p(s) is the probability of state (or outcome) s occurring, where the sum of all probabilities equals 1, which is 100%
R(s) is the return in state (or outcome) s

Variance of returns based on future outcomes
= {p(1) × [R(1) – E(R)]^2} + {p(2) × [R(2) – E(R)]^2} + … + {p(S) × [R(S) – E(R)]^2}

Standard deviation of returns based on future outcomes = √variance of returns based on future outcomes

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Portfolio return = R_p = [x_1 × R_1] + [x_2 × R_2] + ... + [x_n × R_n]

Expected portfolio return = E(R_p) = [x_1 × E(R_1)] + [x_2 × E(R_2)] + ... + [x_n × E(R_n)]

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x_i = (value of holdings of asset i in the portfolio) / (total value of the portfolio) where the sum of all weights equals 1, which is 100%, and the total value of the portfolio is the sum of the holdings of all the assets in the portfolio

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Expected return = return on risk-free asset + risk premium = risk-free rate + risk premium
= required return for financial asset like a stock or bond

Risk premium = expected return – return on risk-free asset = expected return – risk-free rate

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Actual return = E(R) + U = expected return + unexpected return
= E(R) + systematic portion of unexpected return + unsystematic portion of unexpected return

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Total risk = systematic risk + unsystematic risk
= (risk from macroeconomic surprises + risk from individual surprises) in FNAN 303

β for a portfolio = $\beta_p = x_1\beta_1 + x_2\beta_2 + \ldots + x_n\beta_n$

CAPM: E(R_i) = R_f + (β_i × [E(R_M) – R_f]) for asset i and E(R_p) = R_f + (β_p × [E(R_M) – R_f]) for portfolio p
E(R_i) = R_f + (β_i × market premium) for asset i and E(R_p) = R_f + (β_p × market premium) for portfolio p

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Market risk premium = market premium = expected market return – risk-free rate = E(R_M) – R_f

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After-tax expected cost of debt = pre-tax cost of debt × (1 – Tc) = R_D × (1 – Tc) = YTM × (1 – Tc) for a bond

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With 3 capital structure items (common stock, preferred stock, and debt):
Weighted average cost of capital = WACC = R_A = [(E/V) × R_E] + [(P/V) × R_P] + [(D/V) × R_D × (1 – Tc)]
where V = E + P + D = value of firm's assets = value of firms' capital structure
E, P, and D = value of all of firm's common equity, preferred equity, and debt respectively
Where E, P, and D each = (number of that particular type of shares or bonds × price per that type of share or bond) ÷ V

For exam problems: Round values less than \$10,000 to nearest penny and values equal to or greater than \$10,000 to nearest dollar Round rates (and figures based on rates) to nearest hundredth of a percent Round variance and (weight for variance multiplied by the squared deviation) terms to 6 decimal places Round (asset weight multiplied by asset β) terms to 3 decimal places Round portfolio and asset β figures to 2 decimal places
Assume a rate (such as interest or return) is a compound rate unless told otherwise.
Assume a given rate is for a year unless told otherwise. More specifically, assume rate is an APR unless told otherwise.
Note that a quarter is 3 months, a semi-annual period (or half year) is 6 months, and a year is 12 months
Assume "in x periods" is equivalent to "in x periods from today" unless told otherwise.
Assume there is a "flat yield curve." Therefore, the rate (interest rate, discount rate, expected return, etc.) associated with any set of cash flows is not influenced by the timing of the cash flows.
The terms "value" and "market value" refer to present value unless it is explicitly indicated that some other value (like future value, book value, or face value) is relevant and being referred to.
For a given source of cash flows, assume that the discount rate is constant and the same for each individual time period.
Assume markets for financial investments and securities like stocks and bonds are well-functioning and that markets for projects and business activities are not well-functioning. However, assume that markets for assets such as buildings, plants, stores, etc. are well-functioning in questions that involve comparisons of risk and/or value.
Assume that the rate of return is greater than the growth rate with a growing perpetuity or a stock with a constantly growing dividend.
If a question asks for a "payment" or a "contribution" then the payment or contribution equals the magnitude of the cash flow and is a positive number.
Assume the term bond refers to a "typical" bond that pays regular, fixed coupon payments and all principal at maturity; par (or face) value is \$1,000; and amount initially borrowed is par (or face) value unless explicitly noted otherwise.
The terms "return" and "rate of return" refer to percentage return, not total dollar return.
When valuing a bond or stock at the same time that a coupon or dividend is paid, assume that the coupon or dividend is paid just before the bond or stock is valued. Also, if a bond or stock is sold at the same time that a coupon or dividend is paid, assume the security is sold just after the coupon or dividend is paid
Assume that the expected return for stocks, bonds, and other financial investments remains constant over time. It is the same for each individual time period.
Assume that a project has conventional cash flows if its expected cash flows are consistent with conventional cash flows. In other words, if the cash flows look conventional, assume they are conventional.
Assume any cash flows from terminal values are after-tax values
Assume expected returns, discount rates, and costs of capital are positive, unless explicitly noted as negative or given information that indicates rate is or may be negative.
Unless explicitly noted or told otherwise, use the arithmetic average return when computing an average (or a mean, which is the same thing). The terms "average" and "mean" refer to the arithmetic average.
Unless noted otherwise, assume the returns of all the assets in a portfolio (with more than 1 asset) do not move perfectly together in the same direction by the same relative amount.
Unless noted otherwise, assume that any information that is "announced" is information that was previously private and made public by the announcement.
Assume the expected return of the market portfolio is greater than the risk-free return and that both are greater than 0.
Assume that the expected cash flows and level of risk associated with a particular project are the same for all firms.