1. If  and  is a prime number, then  .

2.



(a) 

(b) 

(c) 

(d) 

3.

(a) Any of next 6 digits can be 0 and 1. Thus, there are  strings.

(b) Any of the 3 digits in the middle can be 0 and 1. Thus, there are  strings.

(c) According to , since the number of strings ending with 111 is obviously equal to that of strings beginning with 101. Thus, there are  strings.

4.

(a)

Note that the first digit cannot be 0. We can first ignore this condition and then subtract the number of numbers that begin with 0.

Four 7s: There rest 2 numbers can either be the same or not. Thus, there are



Five 7s: 

Six 7s: 

6-digit numbers in total: 

Thus, there are  numbers.

(b) There are  numbers that has at least four 7s. Thus, there are also 1309 numbers that has at least four 9s.

5.

(a) 

(b) 

(c) Followed by 1 letter or decimal digit: . Followed by 2 letters or decimal digits: . Thus, there are 34632 variable names.

6.

(a) Start with A: . End with D: . Both: .

Thus, there are  rearrangements.

(b) Total permutations: . AA appears: .

Thus, there are  rearrangements.

7.

(a) We can consider it as putting 20 balls into 5 boxes. And we only need to put 5-1=4 times.

Then we have  solutions.

(b) Since , we can put 3 balls in to .Then the equation becomes

. Thus, there are  solutions.

Since, we can consider , which is equivalent to .

Similarly, we get .

Thus, there  solutions for ,.

Hence, we have  solutions for ,.

8.



Thus, the coefficient is .

9.

(a) There are 8 letters, 3 As and 2 Rs. Thus, there are  different strings.

(b) We can combine As together as 1 letter. Thus, there  different strings.

10.

When , , . It is true.

Assume that it is true for all . Thus, we have . When ,



Thus, it is true for .

Hence, we have  for all .