1.



Then, we can know .

Similarly, we get .

Hence, the sequence of numbers is 3,6,4,3,6,4...

2.

Assume that  for any integer n, where  and 

So we have .

We add the first and the last equation, then we get .

Thus, .

We add the three equations together, then we get



Thus, , and 

And we plug in , we get . Then we plug in , we get .

Hence, the next seven numbers are 8,3,5,2,1,3,5.

3.

1. First we solve .

Since , we can know .

To solve , we multiply both sides by 55, then we get

.

Hence, the solution is .

1. First we solve .

Since , we can know .

To solve , we multiply both sides by 89, then we get

.

Hence, the solution is .

1. First we solve .

Since , we can know .

To solve , we multiply both sides by -5, then we get

.

Hence, the solution is .

4.

Since , .

Since , .

Since , .

Let , then 

Let , then .

Hence, 

5.

When , 

Suppose it is true when . Thus, .

When , we have



Thus, it is also true when .



6.

A.

Formula: 

|  |  |  |
| --- | --- | --- |
|  |  |  |
| 1 |  |  |
| 2 |  |  |
| 3 |  |  |

B.

Since when  is true, we assume that it is also true when .

Thus, .

When ,

Thus, it is also true when .

7.

When , .

Assume it is also true when . Thus, .

When ,



Since  and , we can know

.

Thus, it is true when .



8.

1. 

Thus,  are true.

Base case: when , n cents can be formed using 4-cent stamps and 7-cent stamps.

1. For any n where , we assume n cents can be formed using 4-cent stamps and 7-cent stamps.

C.  cents can also be formed using 4-cent stamps and 7-cent stamps.

D. When , we assume that k cents can be formed using  4-cent stamps and  7-cent stamps. Thus, .



If , it is obvious that we can use  4-cent stamps and  7-cent stamps to form  cents.

If , . According to our base case, , .



Thus, we can use and 3 7-cent stamps to form  cents.

Hence,  cents can also be formed using 4-cent stamps and 7-cent stamps.



E. When , we prove  by finding satisfying cases.

When , we assume  is true. And we can get  is true when  is true. Recursively, we can know for all ,  is true.

Hence, whenever ,  is true.

9.

When , . Thus, .

When , . Thus, .

Assume it is true when . Thus, we have .

When , .

Hence, it is true for all n.

10.

1. (2,3),(3,2),(4,6),(5,5),(6,4)
2. For the first 5 applications, it is obvious that according to A,  when .

Thus, we assume that it is true for the first  applications.

When it comes to  applications, either we do  or ,

since  and , we can always get

, which means

 and .

Thus,  is true when .

C. We currently have (0,0),(2,3),(3,2),(4,6),(5,5),(6,4) in our set, and these elements obviously satisfy the condition that  when .

So we assume that the first  elements all satisfy this condition.

We pick one element  out of  and do the application to make it the  element.

If we do , . Thus, .

If we do , . Thus, .

Thus, whatever application we do,  is always true.

Hence,  is true when .