HW1

1.

$$\frac{V^2}{2} + \frac{\gamma}{\gamma-1}\frac{p}{\rho} = \frac{\gamma}{\gamma-1}\frac{p_0}{\rho_0}$$

Since $p = \rho RT$ (perfect gas law)

$$\frac{V^2}{2} + \frac{\gamma}{\gamma-1}RT = \frac{\gamma}{\gamma-1}RT_0$$

$$\frac{V^2}{2} = \frac{\gamma}{\gamma-1}R(T_0 - T) = \frac{\gamma}{\gamma-1}RT\left(\frac{T_0}{T} - 1\right)$$

$$\frac{\gamma-1}{\gamma}\frac{1}{RT}\frac{V^2}{2} = \frac{T_0}{T} - 1$$

$$\frac{T_0}{T} = 1 + \frac{\gamma-1}{2}\frac{V^2}{\gamma RT} = 1 + \frac{\gamma-1}{2}M^2 \quad (\text{since } a = \sqrt{\gamma RT})$$

Using the isentropic relation

$$\frac{p_0}{p} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}} = \left(1 + \frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$$

#

2.

$$P_0 - P = \frac{1}{2}\rho V^2 = \frac{1}{2}\frac{\gamma RT}{\gamma RT}\rho V^2 = \frac{1}{2}\gamma P \frac{V^2}{\gamma RT} = \underline{\underline{\frac{1}{2}\gamma P M^2}}$$ #

3.

Binomial expansion

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \cdots$$

for $|x|<1$

$$P_0 - P = P\left[\left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}} - 1\right]$$

$$\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{1}{\gamma-1}\right)$$

$$= P\left[\cancel{1} + \frac{\gamma}{\cancel{\gamma-1}}\frac{\cancel{\gamma-1}}{2}M^2 + \frac{1}{2}\underline{\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{\gamma}{\gamma-1}-1\right)}\left(\frac{\gamma-1}{2}\right)^2 M^4\right.$$

$$+ \frac{1}{3\times 2}\underline{\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{\gamma}{\gamma-1}-1\right)\left(\frac{\gamma}{\gamma-1}-2\right)}\left(\frac{\gamma-1}{2}\right)^3 M^6$$

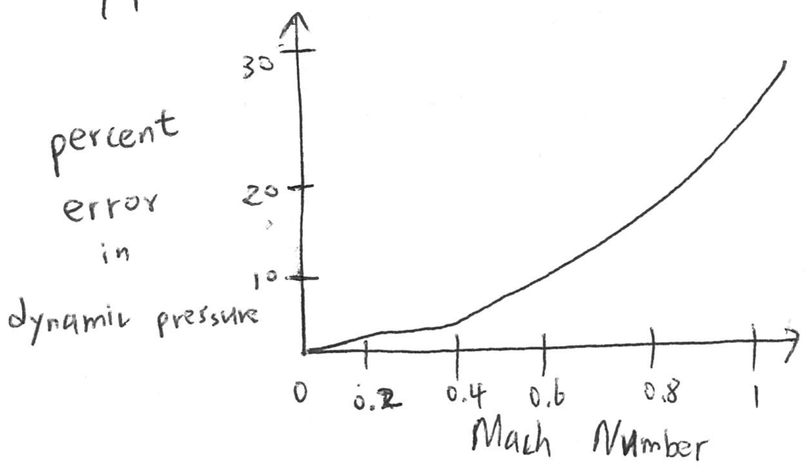
$$\left. + \cdots \quad - \cancel{1}\right]$$

$$\left(\frac{\gamma}{\gamma-1}\right)\left(\frac{1}{\gamma-1}\right)\left(\frac{\gamma}{\gamma}\right.$$

$$= P\left(\frac{\gamma}{2}M^2 + \frac{1}{8}\gamma M^4 + \frac{M^6}{48}(-\gamma^2+2\gamma) + \cdots\right)$$

$$= \underline{\underline{\frac{\gamma P}{2}M^2\left(1 + \frac{1}{4}M^2 + \frac{1}{24}M^4(-\gamma+2) + \cdots\right)}}$$ #

4.



$$\frac{P_0 - P}{\frac{1}{2}\gamma P M^2} - 1 = \left(\frac{1}{4} M^2 + \frac{1}{24} M^4 (-\gamma + 2) + \cdots \right)$$