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# Meta-Analytic Criterion Profile Analysis

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***Abstract***

Intraindividual patterns or configurations are intuitive explanations for phenomena, and popular in both lay and research contexts. Criterion profile analysis (CPA; Davison & Davenport, 2002) is a well-established, regression-based pattern matching procedure that identifies a pattern of predictors that optimally relate to a criterion of interest and quantifies the strength of that association. Existing CPA methods require individual-level data, limiting opportunities for reanalysis of published work, including research synthesis via meta-analysis and associated corrections for psychometric artifacts. In this article, we develop methods for meta-analytic criterion profile analysis (MACPA), including new methods for estimating cross-validity and fungibility of criterion patterns. We also review key methodological considerations for applying MACPA, including homogeneity of studies in meta-analyses, corrections for statistical artifacts, and second-order sampling error. Finally, we present example applications of MACPA to published meta-analyses from organizational, educational, personality, and clinical psychological literatures. *R* code implementing these methods is provided in the *configural* package, available at https://cran.r-project.org/package=configural and at https://doi.org/10.17605/osf.io/aqmpc.

***Translational Abstract***

Patterns or configurations of predictors are popular ways for researchers and science consumers to understand phenomena. Criterion profile analysis (CPA; Davison & Davenport, 2002) is a regression-based pattern matching procedure that identifies patterns of predictors that are maximally related to a criterion of interest. This technique allows researchers to consider a new perspective on the relationship between a set of predictors and a criterion—that the criterion is associated most with a specific pattern or configuration of predictors. Existing CPA methods require individual-level data, limiting opportunities for research synthesis via meta-analysis or reanalysis of published research. In this article, we develop methods for meta-analytic criterion profile analysis (MACPA), including new methods for estimating cross-validation performance and sensitivity analyses. We also review methodological considerations and caveats for MACPA, including homogeneity of studies in meta-analyses, correction for statistical artifacts, and second-order sampling error. Finally, we present example applications of MACPA to published meta-analyses from organizational, educational, personality, and clinical psychological literatures. *R* code implementing these methods is provided. Application of MACPA in both new meta-analyses and reanalysis of existing correlational research can open new avenues of inquiry into the potential mechanisms driving important outcomes in psychological research.

*Keywords:* criterion profile analysis, meta-analysis, configural, pattern analysis, fungible regression weights

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Many researchers and practitioners are interested in determining if a specific configuration or pattern of characteristics relates to high scores on a criterion variable. Clinicians are interested in the specific pattern of symptoms that is most indicative of a disorder; human resources managers seek to know the configuration of competencies that is most predictive of job performance; and

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The methods described in this article have been implemented in the *configural* package for *R*, available at https://cran.r-project.org/package=configural and https://doi.org/10.17605/osf.io/aqmpc.

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community psychologists aim to find the particular combination of design factors that makes the most effective intervention (cf. Davison & Kuang, 2000; Kulas, 2013; Meehl, 1950). In all these applications, the question is not whether prominent patterns exist, but rather if a specific pattern of given predictors explains key outcomes and how much variation is accounted for by such a pattern.

To address these important questions, Davison and Davenport (2002) developed a procedure called *criterion profile analysis* (CPA; cf. Culpepper, 2008). CPA is a two-stage procedure built on multiple regression. In the first stage, a pattern of predictor scores is identified that optimally relates to a criterion variable. This pattern is called the *criterion pattern*. In the second stage, the strength of the association between the pattern and the criterion variable is quantified. In doing so, each person's profile of predictor scores is reexpressed in terms of (a) their overall profile elevation and (b) the similarity of their individual pattern to the optimal criterion pattern. Level and pattern scores are then used in a new regression model to estimate the amount of variation due to predictor configurations (*pattern effect*) and the overall profile level (*level effect*). By decomposing prediction from a set of variables into configural versus simple accumulation effects, CPA helps explicate their theoretical relationships to a criterion and informs assessment by indicating whether attention to or interpretation of predictor configurations is warranted (Davison, Davenport, Chang, Vue, & Su, 2015). CPA has been applied in work psychology (e.g., identifying patterns of traits and behaviors associated with career interests; Dilchert, 2007; Wiernik, 2016; Wiernik, Dilchert, & Ones, 2016; individual and team performance; Caughlin, 2011; Grand, Pearce, & Kozlowski, 2013; Kulas, 2013; Shen, 2011; and career success; Booth, Murray, Overduin, Mathews, & Furnham, 2016), educational psychology (e.g., patterns of abilities and experiences predictive of educational success; Chan, 2006; and college major choice; Culpepper, Davenport, & Davison, 2008; Davison, Jew, & Davenport, 2014), social psychology (e.g., patterns of social identity variables linked to well-being; Lyons, 2015; Perry, 2008; Swinburne Romine, 2011), and the wider sciences (e.g., tooth wear patterns that distinguish monkey taxa; Morse, Daegling, McGraw, & Pampush, 2013).

Despite promising applications of CPA, existing procedures are limited because they require primary individual-level data as input to compute individual pattern and level scores; that is, current procedures preclude using CPA with summary (i.e., secondary or meta-analytic) data. This limitation is considerable because it prevents reanalysis of published data to test intraindividual pattern effects. Likewise, CPA cannot be used in combination with meta-analysis. By pooling results across multiple samples, meta-analysis reduces biasing effects of sampling error and can address biases from other statistical artifacts (e.g., measurement error and selection effects, such as range restriction; Schmidt & Hunter, 2015; Wiernik & Dahlke, 2020; cf. Davison, Chang, & Davenport, 2014). Thus, this limitation constrains the ability of CPA to contribute to the accumulation of scientific knowledge via research synthesis. In response, the purpose of this article is to extend CPA procedures to summary data, and to show how, in combination with psychometric meta-analysis, *meta-analytic criterion profile analyses* (MACPA) can be used to advance research by examining pattern effects as part of integrative research synthesis.

## Objectives and Contributions of the Current Article

In this article, we describe procedures for meta-analytic criterion profile analysis. First, we distinguish between types of pattern relationships before reviewing the logic and procedures of CPA. Next, we show how the key statistics of CPA can be computed from summary data and present methods to limit overfitting and sensitivity analyses that aid in interpreting the shape of criterion patterns. Third, we describe methods to integrate CPA with psychometric meta-analysis, including correcting for biasing effects of statistical artifacts. Finally, we provide examples of MACPA using meta-analyses from organizational, educational, personality, and clinical psychological literatures. We close by discussing the unique considerations for combining CPA and meta-analysis and highlighting promising areas for future research. *R* code to implement all analyses is available in the *configural* package at https://cran.r-project.org/package=configural and at https://doi.org/10.17605/osf.io/aqmpc.

Taken together, this article makes several contributions. First, we show how CPA pattern and level effects can be estimated without access to individual-level data, opening avenues for using published data and interpreting meta-analyses from a configural pattern perspective. Second, we provide methods for constructing confidence intervals and statistical tests for parameters, including in meta-analysis, allowing for consideration of sampling uncertainty when interpreting CPA and MACPA results. Third, we present statistical methods to adjust the pattern effects to reduce overfitting and provide unbiased estimates of the population pattern effect or expected cross-validity in new samples. Fourth, we address challenges associated with integrating CPA with psychometric meta-analysis, including correcting for statistical artifacts, estimating uncertainty, and handling nonpositive definite matrices and effect size heterogeneity. Fifth, to assess the sensitivity of results, we present procedures for fungible profile analysis (Waller, 2008), which help researchers to determine whether results are robust to minor perturbations of a given input correlation matrix. We conclude by presenting several example applications of MACPA to address substantive and applied questions from the literature.

## Types of Pattern Relationships Among Variables

Psychological researchers are interested in a variety of types of pattern relationships (also called configural relationships) among variables (Cronbach & Gleser, 1953; Hoffman, 1960). A relationship between two variables, $X_1$ and $X_2$, is said to be configural "if the interpretation of one item of information [$X_1$] is contingent on a second [$X_2$]" (Hoffman, 1960, p. 122). As noted by Hoffman, configural relationships can take an infinite variety of mathematical forms, but he focused on two types: (a) *interactive*, which are patterns that have a multiplicative form (e.g., $X_1 \cdot X_2$); and (b) *contrastive*, which are represented mathematically by a difference between two or more variables (e.g., $X_1 - X_2$). Ipsative pattern relationships are a special case of the contrastive relationship in which there are $p$ variables ($j = 1, \ldots, p$) and the relationship has the form $X_1 - \frac{1}{p}\sum_j X_j$. For an ipsative relationship, a person's score on a variable, $X_1$, is interpreted relative to the mean of all variables for that person. Such relationships give rise to questions of whether there is within-person variability in scores across predictors and whether this variability relates to important outcomes.

Examining contrastive pattern relationships entails decomposing variance in scores on multiple measures into between-persons (i.e., differences in profile level or elevation) and within-persons (i.e., differences in relative patterns of higher and lower scores, irrespective of profile elevation) components, followed by examining these components' relationships to other variables (Davison, Kim, & Close, 2009). For example, Davison, Jew, and Davenport (2014) examined how between-person (level) and within-person (pattern) components of SAT scores predicted university student major choice. They found that between-person differences in scores predicted some majors (e.g., higher overall ability predicted choice of math/physics and engineering; lower overall ability predicted choice of education major). However, they found that within-person differences in score patterns had much stronger relationships with major choices (e.g., a within-person pattern of higher SAT Math vs. SAT Verbal predicted choice of business, math, or physical science majors; a within-person pattern of higher SAT Verbal vs. SAT Math predicted choice of humanities majors). These authors concluded that within-person configurations of SAT scores were the more important driver of major choice (cf. Wang, Eccles, & Kenny, 2013). Thus, *contrastive pattern relations* concern the predictive utility of within-person configurations of a set of predictor scores (or subscale scores).[1]

By contrast, *interactive pattern relationships* concern whether the relationship between two variables changes as a function of a third variable. For example, is a personality trait a stronger predictor of behavior when situational pressures are weak versus strong (Judge & Zapata, 2015), or is ability a stronger predictor of performance when motivation is high versus low (Van Iddekinge, Aguinis, Mackey, & DeOrtentiis, 2018)? Interactive pattern relationships are represented by multiplicative terms in polynomial regression and follow-up analyses (Cohen, Cohen, West, & Aiken, 2003; Shanock, Baran, Gentry, Pattison, & Heggestad, 2010). A key distinction between these two types of pattern relationships among variables is that contrastive pattern relationships concern decomposing variance in a set of predictors into between-person and within-person components, whereas interactive pattern relationships concern *expanding* the set of predictors to include multiplicative composites of two or more variables.

Contrastive and interactive represent two distinct types of pattern relationships. One, both, or neither, can be present when describing relationships of a set of predictors to a criterion variable. For example, Van Iddekinge, Aguinis, Mackey, and DeOrtentiis (2018) meta-analytically tested the hypothesis that ability and motivation have an interactive pattern relationship to work performance. They found that multiplicative ability–motivation composites had negligible incremental validity over linear ability and motivation terms, indicating little evidence for an interactive pattern relationship. However, if one reanalyzes their meta-analytic correlation matrix using the method detailed in this article, there is evidence of a contrastive pattern relationship.[2] In other words, there is a configural relationship in the Van Iddekinge et al. (2018) data, but it has a contrastive form, not an interactive form posited by the authors. As noted earlier, we extend methods for analyzing configural pattern relationships *of the contrastive pattern type* by showing how Davison and Davenport's (2002) CPA technique can be applied to meta-analytic data.

# Criterion Profile Analysis

## Logic and Procedures of Criterion Profile Analysis

Davison and Davenport's (2002) CPA is a two-stage procedure. First, ordinary least squares (OLS) regression[3] is used to identify the configural pattern of predictor scores that is optimally related to a criterion. Davison and Davenport (2002, Appendix A) proved this *criterion pattern* can be described as a vector of contrast coefficients $\boldsymbol{\beta}^* = \boldsymbol{\beta} - \mathbf{1}\bar{\beta} = [\beta_j - \bar{\beta}]'$, where $\boldsymbol{\beta}$ is a column vector of regression coefficients, $\mathbf{1}$ is a column vector of 1s, and $\bar{\beta}$ is the mean regression coefficient. Second, for each person, two new profile scores are computed: (a) their *profile level score*, which is the overall elevation of their predictor profile; it is computed as the within-person mean across the $p$ predictors in the model: $x_{lev_i} = \sum (x_{1i}, x_{2i}, x_{3i}, \ldots, x_{pi})/p$; and (b) their *criterion pattern similarity score*, which is the similarity of a their profile pattern to the criterion pattern; it is computed as the covariance between their predictor score vector and the criterion pattern: $Cov_{\beta_i^*} = \sum_j^p x_{ij}\beta_j^*/p$. Profile level and pattern scores, in turn, are used to predict the criterion in a new regression model that examines the relative explanatory power of the two effects. The predictive power of the criterion pattern for the criterion variable is captured by the correlation between the criterion and the profile similarity scores, $Cov_{\beta_i^*}$. Davison and Davenport (2002) showed that the regression model, $\hat{y}_i = \beta_{pat}Cov_{\beta_i^*} + \beta_{lev}x_{lev_i}$, has the same $R^2$ as the OLS regression model. Thus, the CPA regression model can be understood as the combined impact of the elevation of score profiles and the similarity of profile patterns to the optimal criterion pattern.

Although these CPA procedures are straightforward, they are limited by requiring individual-level data to compute key statistics. Accordingly, new methods are needed to estimate the CPA regression model using summary data. Sufficient statistics for the model are correlations of the criterion variable with (a) profile level scores, $r_{lev,y}$, and (b) criterion pattern similarity scores, $r_{pat,y}$, as well as (c) the correlation between profile level and criterion pattern similarity scores, $r_{lev,pat}$ (Cohen et al., 2003, p. 70). Below, we show how to compute these correlations from summary data.

## Procedures for Extending Criterion Profile Analysis to Summary Data

The theory of composites (Nunnally, 1978, pp. 166, 177; see also Ghiselli, Campbell, & Zedeck, 1981, p. 163) shows that

[1] A related question to contrastive pattern relationships is whether subscores of a measure have interpretative meaning or if their sole contribution is adding to a person's total score (Davison et al., 2015; Haberman, 2008; Ree, Earles, & Teachout, 1994). If subscores incrementally predict over a total score, this indicates a contrastive pattern relationship is present. For any specific total score, people with the same total score, but different within-person subscore patterns, will tend to have different expected criterion scores.

[2] Both the ability–motivation profile level ($r_{lev}$ = .50, $\beta_{lev}$ = .50, $\sqrt{\Delta R^2_{lev}}$ = .50) and profile pattern ($r_{pat}$ = .11, $\beta_{pat}$ = .11, $\sqrt{\Delta R^2_{lev}}$ = .11) offer unique prediction. This result can be interpreted as indicating that, although both constructs predict, other factors being equal, employees who are relatively more capable than they are motivated tend to perform better.

[3] CPA can also be applied with weighted least squares or generalized linear models (cf. Davison, Jew, & Davenport, 2014).

multiple regression can be conceptualized as a composite correlation weighted by the regression coefficients. Likewise, CPA level and pattern effects can be understood as alternative sets of weights for computing composite correlations. Thus, the correlation between predictor profile level and the criterion, $r_{lev,y}$, is estimable as a unit-weighted composite correlation:

$$r_{lev,y} = (\mathbf{1}'\boldsymbol{R}_{xx}\mathbf{1})^{-1/2}\mathbf{1}'\boldsymbol{r}_{xy} = \frac{\sum_j 1 \cdot r_{x_j y}}{\sqrt{\sum_{j,k} 1 \cdot 1 \cdot r_{x_j x_k}}} = \frac{\sum_j r_{x_j y}}{\sqrt{\sum_{j,k} r_{x_j x_k}}}, \tag{1}$$

where $\boldsymbol{r}_{xy}$ is the column vector of correlations between each predictor variable and the criterion, $\boldsymbol{R}_{xx}$ is the correlation matrix among the predictors, and $\mathbf{1}$ is a column vector of 1s.

Similarly, the correlation between criterion pattern similarity and the criterion, $r_{pat,y}$, is estimable as a composite correlation weighted by $\boldsymbol{\beta}^*$, the criterion pattern vector:

$$r_{pat,y} = (\boldsymbol{\beta}^{*\prime}\boldsymbol{R}_{xx}\boldsymbol{\beta}^*)^{-1/2}\boldsymbol{\beta}^{*\prime}\boldsymbol{r}_{xy} = \frac{\sum_j \beta_j^* r_{x_j y}}{\sqrt{\sum_{j,k} \beta_j^*\beta_k^* r_{x_j x_k}}} \tag{2}$$

Finally, the correlation between profile level and criterion pattern similarity, $r_{lev,pat}$, can be calculated using the formula for the correlation between two composites (Waller, 2008, p. 692):

$$r_{lev,pat} = \left(\boldsymbol{\beta}^{*\prime}\boldsymbol{R}_{xx}\boldsymbol{\beta}^*\mathbf{1}'\boldsymbol{R}_{xx}\mathbf{1}\right)^{-1/2}\boldsymbol{\beta}^{*\prime}\boldsymbol{R}_{xx}\mathbf{1}$$

$$= \frac{\sum_{j,k} 1 \cdot \beta_k^* \cdot r_{x_j x_k}}{\sqrt{\sum_{j,k} 1 \cdot 1 \cdot r_{x_j x_k}}\sqrt{\sum_{j,k} \beta_j^*\beta_k^* r_{x_j x_k}}}$$

$$= \frac{\sum_{j,k} \beta_k^* \cdot r_{x_j x_k}}{\sqrt{\sum_{j,k} r_{x_j x_k}}\sqrt{\sum_{j,k} \beta_j^*\beta_k^* r_{x_j x_k}}} \tag{3}$$

These three correlations can then be used to calculate the CPA regression coefficients and the full-model $R^2$ (Cohen et al., 2003, p. 70):

$$\boldsymbol{\beta}_{CPA} = (\beta_{pat} \quad \beta_{lev})' = \left(\frac{r_{lev,y}r_{lev,pat} - r_{pat,y}}{r_{lev,pat}^2 - 1} \quad \frac{r_{pat,y}r_{lev,pat} - r_{lev,y}}{r_{lev,pat}^2 - 1}\right)' \tag{4}$$

$$R^2 = \left(2 \cdot r_{pat,y}r_{lev,y}r_{lev,pat} - r_{pat,y}^2 - r_{lev,y}^2\right) / \left(r_{lev,pat}^2 - 1\right) \tag{5}$$

Appendix A presents proofs for these equations, and Appendix B provides methods for computing confidence intervals and statistical hypothesis tests for CPA parameters. Using Equations 1–5, researchers can compute all key CPA results (i.e., full-model $R^2$, zero-order correlations, regression coefficients, and $\Delta R^2$ for level and pattern effects) using only the correlation matrix.[4] This advance opens the possibility of applying CPA in secondary data analysis and meta-analysis.

## Meta-Analytic Criterion Profile Analysis

Meta-analysis is a technique for reducing biases in research findings by statistically pooling results from multiple studies and correcting for statistical artifacts, such as sampling error, measurement error, and selection biases (Schmidt, 1992; Schmidt & Hunter, 1977). These artifacts are present in all studies and bias observed correlations between variables away from true construct relationships. They can also distort CPA results, leading to incorrect conclusions about the shape and predictive validity of criterion-related predictor configurations. To avoid these distortions, these artifacts can be corrected by combining CPA with psychometric meta-analysis (Schmidt & Hunter, 2015), a procedure we call *meta-analytic criterion profile analyses* (MACPA). In the sections below, we discuss how statistical artifacts impact MACPA and describe procedures to correct their effects.

### Sampling Error

Random sampling error can distort observed correlations in a single sample from their true population values. As with multiple regression, sample size requirements for CPA can be great, especially if the number of predictors is large (Cohen et al., 2003). If sample sizes are small, the shape of the observed criterion pattern and the size of the level and pattern effects can vary widely across samples. By pooling results from many studies in a meta-analytic correlation matrix, MACPA reduces sampling error, allowing for increased power and more precise estimation of CPA parameters than would be possible in a single sample (Viswesvaran & Ones, 1995).

### Measurement Error

Classical measurement error variance (i.e., unreliability) systematically biases correlations toward zero, leading to conclusions that constructs have weaker relationships than they really do (Schmidt & Hunter, 1996; Viswesvaran, Ones, Schmidt, Le, & Oh, 2014). In CPA, measurement error has three effects. First, it flattens the amplitude of the criterion pattern, making the peaks and valleys of pattern features less pronounced and more difficult to detect. Second, it weakens the criterion correlations for both profile level and criterion pattern similarity, resulting in more pessimistic conclusions about the explanatory power of the predictor set. Third, it causes relatively more explanatory power to be attributed to the level effect rather than to the pattern effect, producing inaccurate inferences about their relative influence. Thus, measurement error systematically obscures configural pattern effects in empirical relationships.

Two methods are available to correct for measurement error in CPA. First, each correlation in the matrix can be individually corrected for unreliability. This is the approach taken for most applications of multivariate techniques to meta-analytic correlation matrices (Viswesvaran & Ones, 1995). When contributing meta-analyses do not report information to correct for measurement error (e.g., Example 2 below), estimates can be drawn from other meta-analyses in the literature. Second, the reliability of individuals' profile level and criterion pattern similarity scores can be

[4] We present formulas for conducting CPA using correlation matrices, because standardized coefficients are the output of most meta-analyses. In cases where variable scales are meaningful (e.g., econometric analyses), researchers may conduct CPA using covariance matrices (cf. Davison & Davenport, 2002). Associated formulas are presented in Appendix A.

estimated and used to directly correct $r_{lev,y}$, $r_{pat,y}$, and $r_{lev,pat}$. Bulut, Davison, and Rodriguez (2017) present methods for estimating the reliability of profile level scores (labeled $\hat{\rho}_{BB'}$) and pattern similarity scores (labeled $\hat{\rho}_{WW'}$) using either parallel forms or split-half reliability. Though both methods of correction are viable, they have different effects. When CPA is performed using a corrected correlation matrix, the amplitude of the criterion pattern and validity of profile level and pattern effects will all be correctly estimated without the attenuating bias of measurement error. By comparison, directly correcting $r_{lev,y}$, $r_{pat,y}$, and $r_{lev,pat}$ will yield unbiased estimates of profile level and pattern effects, whereas the shape of the criterion pattern will remain attenuated, making interpretations of results more difficult.[5]

## Selection Effects

Selection effects (i.e., range restriction, range enhancement, attrition effects, or collider bias; Elwert & Winship, 2014; Sackett & Yang, 2000) and the related phenomenon of base rates that do not adequately reflect the population of interest (McGrath & Meyer, 2006; R. P. Steel, Shane, & Griffeth, 1990) also induce artifacts needing correction. For example, relationships among personnel assessments differ among job applicants versus incumbent employees (Sackett, Lievens, Berry, & Landers, 2007), and correlations among psychopathology symptoms vary between general-population versus more-restricted clinical samples (Neale & Kendler, 1995; Westreich, 2012). Depending on selection mechanism, selection effects can attenuate, inflate, or even reverse signs of correlations (Dawes, 1975; Wiernik & Dahlke, 2020). In CPA, strong selection effects can seriously distort the shape of the criterion pattern and the relative influence of the level and pattern effects, particularly if selection causes correlation signs to flip. If all variables are subject to the same selection mechanism (e.g., job knowledge, interpersonal skill, and dependability all make a candidate more likely to be hired), then the general shape of the criterion pattern (i.e., which variables are high and low) will typically be the same for observed correlations versus correlations corrected for selection. However, the amplitude of the pattern, and the relative contributions of level and pattern effects can be quite different. More seriously, if the variables are under different selection pressures (e.g., people *high* on trait Openness but *low* on Emotional Stability tend to develop psychosis-related psychopathology), then the criterion patterns can have entirely different shapes if estimated from observed versus corrected correlation matrices.

Thus, a critical step for accurate MACPA results is specifying the appropriate population for which interferences are to be drawn. Then, each correlation in the meta-analytic matrix should be corrected to reflect the target population. Importantly, corrections applied to each correlation in the matrix may differ if the studies contributing to each meta-analysis are based on samples from different populations (e.g., if predictor intercorrelations come from general-population samples, but predictor–criterion correlations come from selected samples). Depending on available data, several methods exist to correct for selection effects and to estimate the correlations in the target population (Dahlke & Wiernik, 2019; Sackett & Yang, 2000). Such corrections can be applied to data at the individual-level, study-level, or even post hoc to uncorrected meta-analytic results. Correction formulas for bivariate indirect and direct selection are particularly flexible and can be applied in a wide variety of settings (Dahlke & Wiernik, 2019).

## Other Artifacts

Other artifacts can also bias MACPA results, including artificial polytomization (Alf & Abrahams, 1975; Hunter & Schmidt, 1990) and use of coarse measurement scales (Aguinis, Pierce, & Culpepper, 2009). If applicable, these can be corrected prior to MACPA to improve the accuracy of inferences.

# Correcting Pattern Effect $r_{pat}$, β, and $\Delta R^2$ for Overfitting

A limitation of multiple regression is its tendency to overfit coefficients to idiosyncratic features of small samples, leading to reduced cross-validity (or shrinkage) in new samples (Wainer, 1976). Correcting for overfitting is critical when evaluating model fit, and can be done using cross-validation or statistical estimators (Shieh, 2008). In the context of CPA, overfitting causes the pattern effect to be overestimated, resulting in a positive bias in $r_{pat,y}$. To address this, Davison and Davenport (2002) recommended a two-fold cross-validation procedure that involves estimating the criterion pattern in one subsample and then using it to compute pattern scores in the other subsample (and vice versa). However, this procedure requires individual-level data. Consequently, a statistical shrunken-$R^2$ estimator must be used for MACPA or when calculating CPA from summary statistics.[6]

Several shrunken-$R^2$ estimators are available to estimate the population or cross-validity $R^2$ for a regression model (see Shieh, 2008; and Cattin, 1980, for reviews) and can also be used to estimate the population or cross-validity $r_{pat,y}$. First, rearrange Equation 5 to express $r_{pat,y}$ in terms of $r_{lev,y}$, $r_{lev,pat}$, and full-model $R^2$ (a proof of this result is given in Appendix C):

$$r_{pat,y} = r_{lev,y} r_{lev,pat} + \sqrt{(1 - r^2_{lev,pat})(R^2 - r^2_{lev,y})} \tag{6}$$

The population or cross-validity correlation between criterion pattern similarity and the criterion can be estimated by substituting $R^2$ with an appropriate estimate of the population or cross-validity $R^2$:

$$\hat{r}_{pat,y} = r_{lev,y} r_{lev,pat} + \sqrt{\max\{(1 - r^2_{lev,pat})(\tilde{R}^2 - r^2_{lev,y}), 0\}} \tag{7}$$

[5] In addition to the classical disattenuation formula, other error-in-variables regression models might also be used to correct for measurement error, especially for more complex error structures (Fuller, 1987; Schennach, 2016).

[6] This approach corrects for overfitting by adjusting the estimated correlation between the OLS criterion pattern and the criterion variable downward. This adjusted $r_{pat,y}$ provides a more accurate estimate of the predictive validity of a person's similarity to the OLS criterion pattern in the population or new samples than the naïve unadjusted pattern effect correlation. An alternative approach to reduce overfitting is to estimate a criterion pattern from a non-OLS set of regression coefficients, such as ridge or LASSO regression coefficients (Hastie, Tibshirani, & Friedman, 2009). These regularization routines bias the coefficients for predictors toward a common value, directly limiting overfitting and overestimation of the pattern effect.

Although the most common estimator is the adjusted $r^2_{adj}$ statistic reported by most statistical packages, in a comprehensive simulation study, Shieh (2008) found that other estimators were less biased without substantially increasing computational complexity. Based on these results, the population correlation between criterion pattern similarity and the criterion is best estimated using one of two methods.

If the objective is to develop a model for how a profile of predictors predicts a criterion in the population (i.e., descriptive applications of CPA), estimate the population correlation by substituting $\tilde{R}^2$ with $\hat{\rho}^{2+}_{\mathrm{P}}$:

$$\hat{\rho}^{2+}_{\mathrm{P}} = \max\left\{1 - \left(\frac{N-3}{N-p-1}\right)(1-R^2)\left(1 + \frac{2(1-R^2)}{N-p-2.3}\right), 0\right\} \tag{8}$$

$$\hat{\rho}_{pat,y} = r_{lev,y} r_{lev,pat} + \sqrt{\max\{(1 - r^2_{lev,pat})(\hat{\rho}^{2+}_{\mathrm{P}} - r^2_{lev,y}),\ 0\}} \tag{9}$$

where $p$ is the number of predictors contributing to the profile pattern, and $N$ is the sample size.

If the objective is to estimate the performance of a criterion pattern for predicting outcomes in a specific sample (i.e., prescriptive applications of CPA; Davison, Chang, & Davenport, 2014), estimate the expected cross-validity correlation by substituting $\tilde{R}^2$ with $\hat{\rho}^{2+}_{\mathrm{C.BR\text{–}P}}$:

$$\hat{\rho}^{2+}_{\mathrm{C.BR\text{–}P}} = \max\left\{\frac{(N-p-3)\hat{\rho}^{4+}_{\mathrm{B}} + \hat{\rho}^{2+}_{\mathrm{P}}}{(N-2p-2)\hat{\rho}^{2+}_{\mathrm{P}} + p}, 0\right\}, \tag{10}$$

where $\hat{\rho}^{4+}_{\mathrm{B}} = \hat{\rho}^{4+}_{\mathrm{P}} - \dfrac{2p(1-\hat{\rho}^{2+}_{\mathrm{P}})^2}{(N-1)(N-p+1)}$

$$\hat{\rho}_{C_{pat,y}} = r_{lev,y} r_{lev,pat} + \sqrt{\max\{(1 - r^2_{lev,pat})(\hat{\rho}^{2+}_{\mathrm{C.BR\text{–}P}} - r^2_{lev,y}),\ \ 0\}} \tag{11}$$

## Assessing Criterion Pattern Sensitivity Using Fungible Weights Analysis

CPA equations show that the configuration of regression coefficients can be interpreted as an indicator of the optimal within-person configuration of predictors for the criterion. However, even if the CPA pattern effect is substantial, caution may still be warranted in interpreting the specific shape of the criterion pattern. Waller (2008) showed that the relative pattern of larger and smaller regression coefficients can sometimes change dramatically with only a small loss in model fit. That is, if one reduces model $R^2$ by a very small amount (e.g., reduce $R^2 = .005$, a value possibly within sampling error and unlikely to change substantive interpretations), some sets of possible regression coefficients yielding that reduced $R^2$ value may differ greatly from the configuration of OLS coefficients. The CPA criterion pattern depends on the relative pattern of regression coefficients, so, when applying CPA, researchers should first evaluate the sensitivity of the criterion pattern before making substantive interpretations about its shape.

Waller and Jones (2009) presented an algorithm to evaluate the sensitivity of the pattern of regression coefficients. For a given loss in model fit ($\lambda R^2$), their algorithm identifies the set of alternative regression coefficients (called *fungible weights*) with this model fit that is maximally dissimilar from the OLS regression weights. This algorithm can be adapted for CPA to identify an alternative contrastive pattern with a specified $\lambda R^2$ that is maximally dissimilar from the criterion pattern. We call this procedure *fungible profile analysis*. First, select an amount of loss of model fit ($\lambda R^2$) for computing fungible weights. Values should be small enough that the loss of fit would not alter substantive conclusions.[7] We recommend $\lambda R^2 = -.005$ and $\lambda R^2 = -.010$ to examine sensitivity under relatively smaller and larger decrements in model fit. Next, for each value of $\lambda R^2$, use the Waller and Jones (2009) optimization procedure to identify the alternative regression coefficients that are maximally dissimilar from the OLS coefficients (labeled $\boldsymbol{a}$). Finally, mean-center $\boldsymbol{a}$ to compute a pattern vector, $\boldsymbol{a}^*$, and compare $\boldsymbol{a}^*$ to the criterion pattern, $\boldsymbol{\beta}^*$. If the shape of the maximally dissimilar pattern vector, $\boldsymbol{a}^*$, differs substantially from the criterion pattern, $\boldsymbol{\beta}^*$, with only a small loss in model fit, this indicates that diverse predictor profiles can produce the same criterion outcomes. Examples of such substantial differences could include a prominent peak in the criterion profile pattern disappearing in the maximally dissimilar fungible pattern (e.g., see Example 3, below) or the relative order of predictors in the criterion profile pattern switching places in the maximally dissimilar fungible pattern. In these cases, researchers should be cautious about interpreting the substantive meaning of the observed criterion pattern.

## Accuracy of Proposed Methods

MACPA combines three well-established analytic techniques—CPA, psychometric meta-analysis, and meta-analytic structural equation modeling (MASEM; Viswesvaran & Ones, 1995)—each of which has been the subject of extensive research and simulation work. We summarize key findings below and highlight the relevance and implications of these simulation studies for MACPA.

Davison, Davenport, Chang, Vue, and Su (2015) conducted simulations of CPA to test the incremental validity of profile pattern effects and level effects for conditions varying across five parameters: (a) sample size ($N = 100$; 300); (b) total model $R^2$ value ($R^2 = .20$, .40, .60); (c) predictor intercorrelations ($r = .00$, .35, .70); (d) relative sizes of pattern and level effects ($\beta_{lev} > 0$, $\beta_{pat} = 0$; $\beta_{lev} = 0$, $\beta_{pat} > 0$; $\beta_{lev} = 2 \times \beta_{pat}$; $\beta_{lev} = \beta_{pat}$); and (e) residual error distributions (Normal; $\chi^2$). When there was no

[7] We recommend focusing on effect size magnitude when choosing a value for $\lambda R^2$. The loss in model fit ($\lambda R^2$) should be chosen so that the reduced value would not lead to meaningfully different theoretical or practical conclusions. For example, a researcher would likely conclude that $R^2 = .09$ or .08 ($R = .30$ or .28; $\lambda R^2 = .01$) reflect similar levels of predictive validity (cf. Anvari & Lakens, 2019; Calin-Jageman & Cumming, 2019; Kruschke, 2018). What constitutes a meaningful theoretical or practical difference will of course depend on the specific research area or application in question. Another approach would be to adopt an equivalence testing framework (Tryon, 2001) and to choose $\lambda R^2$ to correspond to the lower bound of the confidence interval for $R^2$ (e.g., for $R^2 = .09$ [.04, .19], $\lambda R^2 = -.05$). This would reflect the largest loss in model fit that is statistically equivalent to (i.e., not significantly different from) the observed $R^2$. However, this approach will often result in $\lambda R^2$ values that reflect practically different levels of predictive validity (e.g., many researchers would interpret $R = .20$ [$R^2 = .04$] as substantively smaller than $R = .30$ [$R^2 = .09$]; Wiernik, Kostal, Wilmot, Dilchert, & Ones, 2017). Accordingly, this equivalence testing procedure will often identify $\lambda R^2$ values that are too large for use in fungible profile analysis.

pattern effect ($\beta_{pat} = 0$), CPA had nominal Type I error rates across conditions. When the pattern effect fully accounted for all prediction ($\beta_{lev} = 0$), CPA had power near 1.0 across conditions. When pattern and level effects were equal, power to detect a nonzero pattern effect remained $>.90$ for uncorrelated predictors. By comparison, power diminished as predictor intercorrelation increased, unless total model $R^2$ was also large ($R^2 = .40$; .60), and this loss was greater when the pattern effect was relatively smaller than the level effect ($\beta_{lev} = 2 \times \beta_{pat}$).[8] Taken together, the authors concluded CPA can reliably differentiate pattern versus level effects in a set of predictors when *one* of these effects account for most of the predictive variance. However, when *both* pattern and level effects are present, and predictors are substantially correlated, CPA is less able to detect pattern effects unless $R^2$ or $N$ is large (Davison et al., 2015). Therefore, by combining samples to increase $N$, MACPA can increase power to detect small, nonzero pattern effects, compared with traditional CPA.

For MACPA to yield accurate CPA parameter estimates, the correlation matrix submitted to MACPA must likewise contain accurate estimates. Several studies have evaluated the accuracy of methods for meta-analysis of correlations. Although various methods for weighting correlations or Fisher $z'$-transformed correlations yield similar results, sample size-weighted correlations typically yield the most accurate estimates of the mean and standard deviation of the population effect size distribution (Brannick, Potter, Benitez, & Morris, 2019; Hafdahl, 2010; Schulze, 2004; cf. Bakbergenuly, Hoaglin, & Kulinskaya, 2019).[9] Further, when measurement error, selection effects, or other statistical artifacts are present, making corrections for them yields more accurate estimates of population construct intercorrelations (Beatty, Barratt, Berry, & Sackett, 2014; Dahlke & Wiernik, 2019; Hedges & Olkin, 1985; Schmidt et al., 2009). Because contrastive configural patterns are strongly influenced by patterns of variable intercorrelations, it is particularly important to correct for selection effects (cf. Sackett et al., 2007). Dahlke and Wiernik (2019) reviewed available correction models for the common case of indirect (incidental) selection impacting both variables in a correlation. They found that their proposed procedures for individual-correction and artifact-distribution meta-analyses yielded highly accurate results, and that information to apply these corrections is more commonly available than most researchers realize.

Finally, considerable research has explored the accuracy of results of multivariate analyses (e.g., multiple regression, path analysis, structural equations modeling) when conducted using meta-analytic correlation matrices (Becker, 1992; Jak, 2015; Viswesvaran & Ones, 1995). MASEM has become widespread in psychology (Landis, 2013; Michel, Viswesvaran, & Thomas, 2011; Sheng, Kong, Cortina, & Hou, 2016). Simulation study results show MASEM yields accurate parameter estimates (Cheung & Cheung, 2016; Furlow & Beretvas, 2005; Hafdahl, 2008; Oort & Jak, 2016; Rosopa & Kim, 2017; Sheng et al., 2016), with increasing accuracy as total sample size and number of studies included in a meta-analysis increases.

Based on the preceding evidence, MACPA builds on a firm foundation: CPA can accurately detect contrastive pattern relationships, especially when the total sample size is large. Meta-analysis not only functions to increase total sample size $N$, but it also yields more accurate estimates of the mean and standard deviation of population effects, especially when statistical corrections are incorporated. Accordingly, MACPA represents a valid approach for examining contrastive pattern effects. We also note that our recommended methods for correcting for overfitting and sensitivity analyses via fungible profile analysis are based on results of extensive analytic and simulation work by Shieh (2008) and Waller (2008; Waller & Jones, 2009), respectively.

## Worked Examples of Meta-Analytic Criterion Profile Analysis

In this section, we apply MACPA to four published meta-analyses from multiple psychology subfields. Table 1 shows meta-analytic intercorrelation matrices, and Table 2 presents MACPA summary statistics. Figure 1 shows criterion pattern plots, and Figure 2 shows fungible pattern plots.

### Example 1: MACPA With Fully-Corrected Correlation Matrices—Work Characteristics and Job Performance Ratings

First, we apply MACPA to a meta-analytic correlation matrix that has been corrected for statistical artifacts. Humphrey, Nahrgang, and Morgeson (2007) meta-analyzed relations of 10 task design and social work characteristics with supervisor-ratings of employee job performance. Humphrey et al. (2007) corrected all variables for measurement error, but insufficient information was available to correct for range restriction. We chose this meta-analysis because it is plausible that a work design strategy that emphasizes some features (e.g., feedback), but deemphasizes others (e.g., increasing autonomy), might be more effective than alternatives for improving job performance. That is, a specific *configuration* of characteristics may have incremental validity over simply higher overall levels of positive work features.

We tested this hypothesis by applying MACPA to the meta-analytic correlation matrix. As Figure 1A displays, the criterion pattern showed that performance was highest for employees whose jobs had higher levels of feedback from others, higher complexity, and less variety of skill demands, compared with other work characteristics. Comparing predictive power for level and pattern effects, Table 2 shows both the level effect ($r_{lev} = .31$) and pattern effect ($r_{pat} = .37$) related strongly to performance. Further, supporting our hypothesis, the pattern effect showed substantial incremental validity over the level effect ($\sqrt{\Delta R^2} = .48$). Accordingly, results indicate that simply providing more work design features is not always optimal and that attention should be paid to providing an optimal configu-

[8] However, it should be noted that, in such cases, the true effect size for the population pattern effect will be small, so inferences that such effects are close to zero would not necessarily be inaccurate. Much larger sample sizes would be needed to yield narrow enough confidence intervals to consistently differentiate such small $\Delta R^2_{\text{pat}}$ from zero.

[9] Other weights that are also not dependent on sample-specific estimates of sampling error variance, such as inverse-variance weights calculated using the mean correlation as a fixed effect size, also perform well and better than simple sample-size weights in some situations (Bakbergenuly et al., 2019; Brannick et al., 2019).

Table 1
*Meta-Analytic Matrices Used for Criterion Profile Analyses*

| Variable | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Example 1. Work characteristics predicting job performance[a] | | | | | | | | | | |
| 1. Autonomy (AU) | | | | | | | | | | |
| 2. Skill variety (SV) | .64 | | | | | | | | | |
| 3. Task variety (TV) | .46 | .52 | | | | | | | | |
| 4. Task significance (TS) | .50 | .62 | .52 | | | | | | | |
| 5. Task identification (TI) | .55 | .37 | .39 | .39 | | | | | | |
| 6. Feedback from the job (FJ) | .53 | .50 | .40 | .56 | .49 | | | | | |
| 7. Job complexity (JC) | .43 | .51 | .62 | .31 | .22 | .21 | | | | |
| 8. Interdependence (IN) | .29 | .61 | .18 | .50 | .19 | .41 | .37 | | | |
| 9. Feedback from others (FO) | .48 | .37 | .10 | .36 | .31 | .57 | .01 | .33 | | |
| 10. Social support (SS) | .38 | .36 | .21 | .39 | .24 | .27 | .12 | .46 | .38 | |
| 11. Supervisor-rated job performance | .23 | .07 | .23 | .23 | .17 | .20 | .37 | .18 | .28 | .12 |
| Example 2. GRE subtests predicting graduate student GPA[b] | | | | | | | | | | |
| 1. GRE verbal (V) | | | | | | | | | | |
| 2. GRE quantitative (Q) | .56 | | | | | | | | | |
| 3. GRE analytical (A) | .77 | .73 | | | | | | | | |
| 4. Graduate school GPA | .34 | .32 | .36 | | | | | | | |
| Example 3. Big Five traits predicting trait mindfulness[c] | | | | | | | | | | |
| 1. Emotional Stability (ES) | | .31 | .33 | .27 | .09 | .57 | | | | |
| 2. Agreeableness (A) | .24 | | .41 | .20 | .19 | .32 | | | | |
| 3. Conscientiousness (C) | .27 | .32 | | .19 | .12 | .41 | | | | |
| 4. Extraversion (Ex) | .22 | .16 | .15 | | .33 | .20 | | | | |
| 5. Openness (O) | .07 | .15 | .09 | .26 | | .19 | | | | |
| 6. Trait mindfulness | .47 | .26 | .34 | .17 | .15 | | | | | |
| Example 4. Big Five traits predicting psychological disorders[d] | | | | | | | | | | |
| 1. Emotional Stability (ES) | | | | | | | | | | |
| 2. Agreeableness (A) | .31 | | | | | | | | | |
| 3. Conscientiousness (C) | .33 | .41 | | | | | | | | |
| 4. Extraversion (Ex) | .27 | .20 | .19 | | | | | | | |
| 5. Openness (O) | .09 | .19 | .12 | .33 | | | | | | |
| 6. Major depressive disorder (MDD) | −.47 | −.06 | −.36 | −.25 | −.08 | | | | | |
| 7. Social phobia (SP) | −.41 | .11 | −.34 | −.37 | −.15 | | | | | |
| 8. Substance use disorder (SUD) | −.36 | −.27 | −.44 | −.16 | −.07 | | | | | |

*Note*. GRE = graduate record examination; GPA = grade point average.
[a] Correlations corrected for measurement error from Humphrey, Nahrgang, and Morgeson (2007); harmonic mean $N$ = 3,444; work characteristic intercorrelation $k$s range 2–111; work characteristic–performance correlation $k$s range 2–42. [b] Correlations corrected for criterion measurement error and predictor range restriction from Kuncel, Hezlett, and Ones (2001); harmonic mean $N$ = 5,089; GRE intercorrelation $k$s range 2–7; GRE–GPA correlation $k$s range 2–103. [c] Observed correlations below the diagonal, correlations corrected for measurement error above the diagonal; Big Five–mindfulness correlations from Hanley and Garland (2017), Big Five intercorrelations from Davies, Connelly, Ones, and Birkland (2015); mindfulness reliability estimated as $\alpha$ = .85 (Giluk, 2009); harmonic mean $N$ = 17,060; Big Five intercorrelation $k$s range 148–211; Big Five–mindfulness correlation $k$s range 25–45. [d] Correlations corrected for measurement error, Big Five–disorder correlations from Kotov, Gamez, Schmidt, and Watson (2010), Big Five intercorrelations from Davies et al. (2015); harmonic mean $N$s = 49,409 (MDD), 25,283 (SP), 51,626 (SUD); Big Five intercorrelation $k$s range 148–211; Big Five–disorder correlation $k$s range 4–63.



ration of design features. These results support findings from work design research that so-called "enriched jobs" can sometimes overwhelm employees and reduce motivation and, therefore, job performance.

Because several job characteristics correlated strongly (e.g., autonomy and skill variety, $\rho$ = .64), we applied fungible profile analysis with $\lambda R^2$ values of −.005 and −.010 to examine the sensitivity of the shape of the criterion pattern. Results presented in Figure 2A show that feedback from others, job complexity, and (low) skill variety were consistently the most prominent peaks and valleys of the criterion profile pattern over minor losses of predictive power. Thus, fungible profile analyses support the conclusion that jobs providing challenging work and clarity from feedback, without overwhelming employees with diverse skill demands, are the most effective for motivating high performance levels. In this example, both the profile level and pattern effects are important.

## Example 2: MACPA Showing a Negligible Pattern Effect—GRE Scores and Graduate School GPA

Second, we apply MACPA to another fully corrected meta-analytic correlation matrix. Kuncel, Hezlett, and Ones (2001) examined the validity of graduate record examination (GRE) subtest scores for predicting graduate school success criteria, including grade point average (GPA). Kuncel et al. (2001)

Table 2
*Results of Meta-Analytic Criterion Profile Analyses*

| | Total | | Level effect | | | | Pattern effect | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Input matrix and variables | $R$ | $R^2$ | $r$ | $r^2$ | $\sqrt{\Delta R^2}$ | β | $r$ | $r^2$ | $\sqrt{\Delta R^2}$ | β | $r_{lev,pat}$ |
| 1. Job characteristics and performance | .57 | .33 | .31 | .10 | .43 | .45 | .37 | .14 | .48 | .50 | −.29 |
| | *.42, .73* | *.15, .50* | *.26, .36* | *.07, .12* | *.24, .63* | | *.26, .48* | *.08, .20* | *.26, .70* | | *−.43, −.15* |
| 2. GRE subtests predicting graduate GPA | .38 | .15 | .38 | .15 | .15 | .38 | .02 | .0004 | .00 | .00 | .05 |
| | *.34, .43* | *.11, .18* | *.34, .42* | *.11, .18* | *.11, .18* | | *.00, .34* | *.00, .11* | *.00, .22* | | *−1.0, 1.0* |
| 3. Big Five traits and mindfulness | | | | | | | | | | | |
| Observed | .54 | .29 | .47 | .22 | .46 | .46 | .27 | .07 | .26 | .26 | .02 |
| | *.50, .57* | *.25, .33* | *.43, .50* | *.18, .25* | *.41, .51* | | *.22, .32* | *.05, .10* | *.19, .34* | | *−.007, .04* |
| Corrected | .63 | .40 | .54 | .29 | .54 | .54 | .33 | .11 | .32 | .32 | .01 |
| | *.58, .67* | *.34, .45* | *.50, .58* | *.24, .34* | *.47, .60* | | *.27, .39* | *.07, .15* | *.24, .41* | | *−.02, .04* |
| 4. Big Five traits and psychological disorders | | | | | | | | | | | |
| Major depressive disorder | .56 | .31 | −.39 | .15 | −.39 | −.39 | .40 | .16 | .40 | .40 | −.002 |
| | *.50, .62* | *.24, .38* | *−.44, −.34* | *.11, .19* | *−.47, −.30* | | *.32, .48* | *.09, .23* | *.31, .50* | | *−.03, .03* |
| Social phobia | .65 | .42 | −.37 | .14 | −.38 | −.38 | .52 | .27 | .53 | .53 | .02 |
| | *.51, .79* | *.24, .60* | *−.45, −.29* | *.08, .20* | *−.61, −.15* | | *.34, .71* | *.08, .46* | *.28, .78* | | *−.01, .05* |
| Substance use disorder | .50 | .25 | −.41 | .17 | −.40 | −.40 | .30 | .09 | .28 | .28 | −.05 |
| | *.42, .58* | *.17, .33* | *−.47, −.35* | *.12, .22* | *−.50, −.30* | | *.21, .39* | *.04, .14* | *.14, .41* | | *−.09, −.01* |

*Note.* GRE = graduate record examination; GPA = grade point average. Results based on estimated population $R^2$ (Equation 9), adjusted using the harmonic mean sample size of contributing meta-analyses; $R$ = total regression model multiple correlation; $r$ = zero-order correlation between effect and criterion; $\sqrt{\Delta R^2}$ = signed square root of incremental $R^2$ (i.e., semipartial correlation) for effect beyond the other effect; β = standardized regression coefficient for model including both level and pattern effects; $r_{lev,pat}$ = correlation between level and pattern effects; values in italics are 95% delta method confidence intervals (Appendix D).

corrected criterion variables for measurement error and the GRE subscales for range restriction relative to the applicant pool; however, they did not correct GRE scores for measurement error, as admissions decisions are made based on observed test scores. We chose this meta-analysis because decision makers often wonder whether there is value in attending to specific subtests or if only the GRE total composite is relevant. We examined this question for the three primary GRE subtests (verbal, quantitative, and analytical).

As Figure 1B shows, the criterion pattern was flat, with no pronounced high or low predictors. Likewise, Table 2 shows that all the predictive power of the GRE subscales for graduate GPA are attributable to the profile level ($r_{lev}$ = .38), not to the within-person profile pattern ($r_{pat}$ = .02, $\sqrt{\Delta R^2}$ = .00). These findings were consistent under fungible profile analysis (see Figure 2B). Thus, results suggest that evaluation of the GRE for predicting graduate GPA should focus on the total score, not the scores of its subtests. In this example, only the profile level effect, not the pattern effect, is important for predicting the criterion.

## Example 3: Combining Results from Multiple Meta-Analyses and Applying Post-Hoc Reliability Corrections—Big Five Personality Traits and Trait Mindfulness

Third, we combined meta-analytic correlations from multiple published meta-analyses and corrected for artifacts, before applying MACPA. Hanley and Garland (2017) meta-analyzed relationships between the Big Five personality traits and trait mindfulness. We combined their results with a meta-analytic correlation matrix *among* the Big Five (Davies, Connelly, Ones, & Birkland, 2015). To correct for measurement error, we used internal consistencies for the Big Five and trait mindfulness reported by Davies, Connelly, Ones, and Birkland (2015) and Giluk (2009), respectfully. Both meta-analyses were based on general population samples, so we did not correct for range restriction. Some scholars argue that compound personality traits, such as mindfulness, should be conceptualized as a configuration of high and low levels across multiple traits (Judge & Erez, 2007; Ones & Viswesvaran, 2001; Ones, Wiernik, Wilmot, & Kostal, 2016; Stanek & Ones, 2018). Thus, we chose this meta-analysis to evaluate this hypothesis for trait mindfulness.

As Figure 1C shows, the criterion pattern was marked by elevated Emotional Stability, as well as low levels of Extraversion and Openness. Low Agreeableness initially appeared to be an additional marker, but it was unstable in the fungible profile analysis (see Figure 2C). Illustrating the importance of measurement error correction, the criterion pattern was more pronounced and accounted for more variance when MACPA was conducted using corrected correlations.

Comparing level and pattern effects, Table 2 shows trait mindfulness relates substantially to both an individual's overall level of traits relevant to adaptation ($r_{lev}$ = .54) and the degree to which their trait configuration matches the prototypical mindfulness criterion pattern ($r_{pat}$ = .33, $\sqrt{\Delta R^2}$ = .32). Thus, though each of the Big Five relates positively to trait mindfulness, mindfulness tendencies are strongest for those whose personality configurations are marked by higher emotional adjustment and relatively lower needs for cognitive exploration and social stimulation, compared

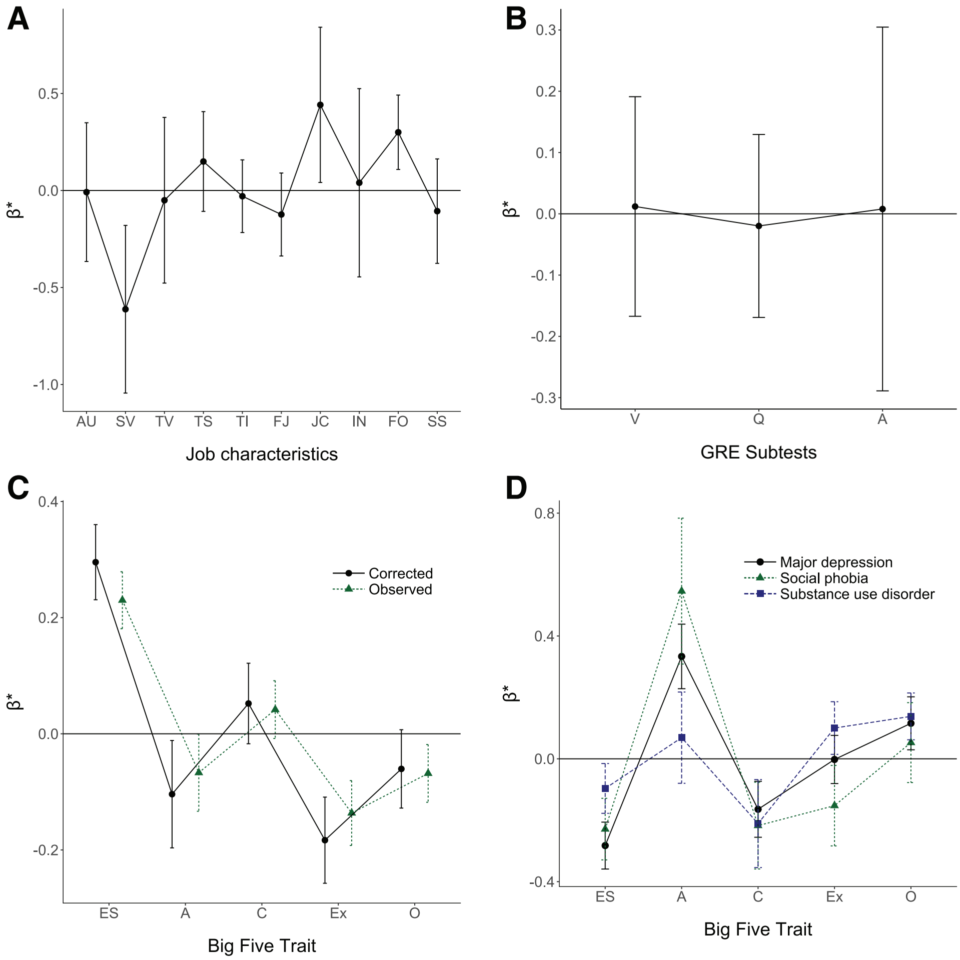


*Figure 1.* Criterion patterns for example methods for meta-analytic criterion profile analysis (MACPA) analyses. Error bars are 95% delta method confidence intervals. GRE = graduate record examination; GPA = grade point average. See the online article for the color version of this figure.

with their other traits. Thus, in this example, once again, both the profile level and pattern effects are important.

## Example 4: Comparing MAPCA Profiles for Multiple Criteria—Personality Traits and Psychological Disorders

Fourth, we compared MACPA criterion patterns across related criterion variables. Kotov, Gamez, Schmidt, and Watson (2010) meta-analyzed relationships between personality traits and diagnoses for several psychological disorders. Kotov et al. (2010) corrected correlations for measurement error. We combined results with the corrected Big Five matrix described in Example 3. We chose this meta-analysis to examine whether trait configurations are risk factors for specific disorders or if disorders reflect overall low levels of adaptive traits (cf. psychopathology "P factor"; Markon, Krueger, & Watson, 2005), as well as whether different disorders are associated with similar or distinct trait patterns.

Results showed that all diagnoses related significantly to both profile level ($r_{lev}$ ranged −.38 to −.40) and profile pattern ($r_{pat}$ ranged .30 to .52, $\sqrt{\Delta R^2}$ ranged .28 to .53). However, each diagnosis had its own distinct criterion pattern (see Figure 1D). Major depressive disorder was marked by high Agreeableness and low Emotional Stability and Conscientiousness. Social phobia had a similar configuration, with the addition of low Extraversion. Substance use disorder was marked with a configuration of low Conscientiousness, as well as high levels of Extraversion and Openness. Fungible profile analyses (Figures 2D–2F) showed that the shapes of these criterion patterns were quite stable. Thus, individuals are most at risk for a specific disorder when their trait profile combines a high overall level of maladaptive traits along with a key configuration of high and low tendencies. For example, depression is associated with a trait configuration where a person's needs for interdependence and empathy (high Agreeableness relative to other traits) are out of step with their low capacity for emotional or impulse control (low Emotional Stability and Conscientiousness). In contrast, substance use is associated with a trait configuration marked by personal weaknesses for impulse control (low Conscientiousness), as well as relatively elevated needs for stimulation and novelty (high Extraversion and Openness). In this

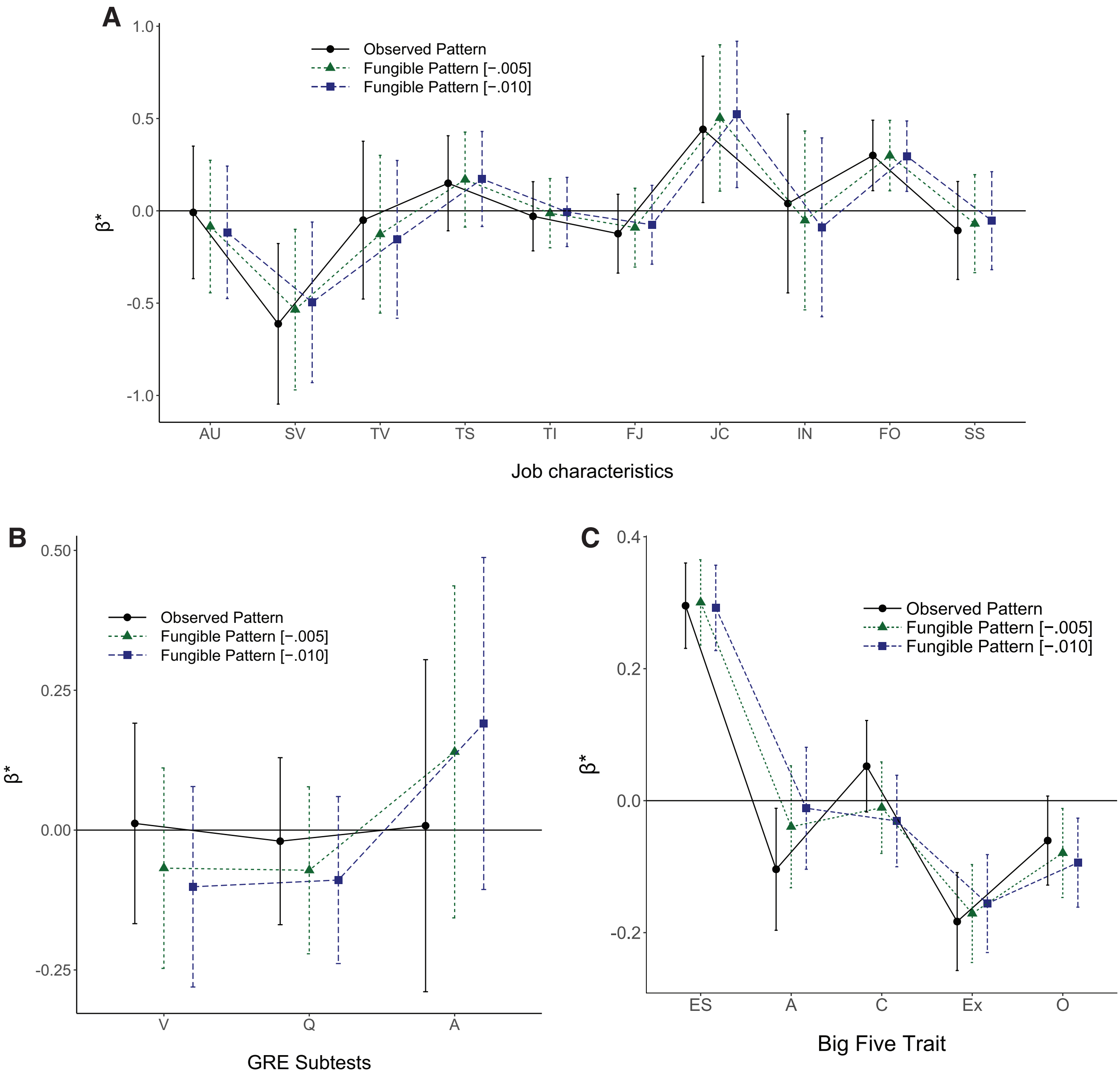


*Figure 2.* Fungible patterns for example methods for meta-analytic criterion profile analysis (MACPA) analyses. Error bars are 95% delta method confidence intervals for the observed criterion pattern elements. GRE = graduate record examination; GPA = grade point average. See the online article for the color version of this figure. (*Figure continues on next page.*)



example, both profile level and pattern effects are important for predicting the criterion, with the shape of the optimal pattern varying across disorders.

## Supplemental Examples

The above examples show the utility of MACPA for exploring previously unexamined configural effects in meta-analytic data. These examples focused on effects at the individual level. Although space prohibits more examples here, it is important to note that MACPA can also be used to examine configural effects at the group level and as part of meta-regression. Examples of these applications from organizational psychology, human resource management, and social psychology are included in the online supplemental materials.

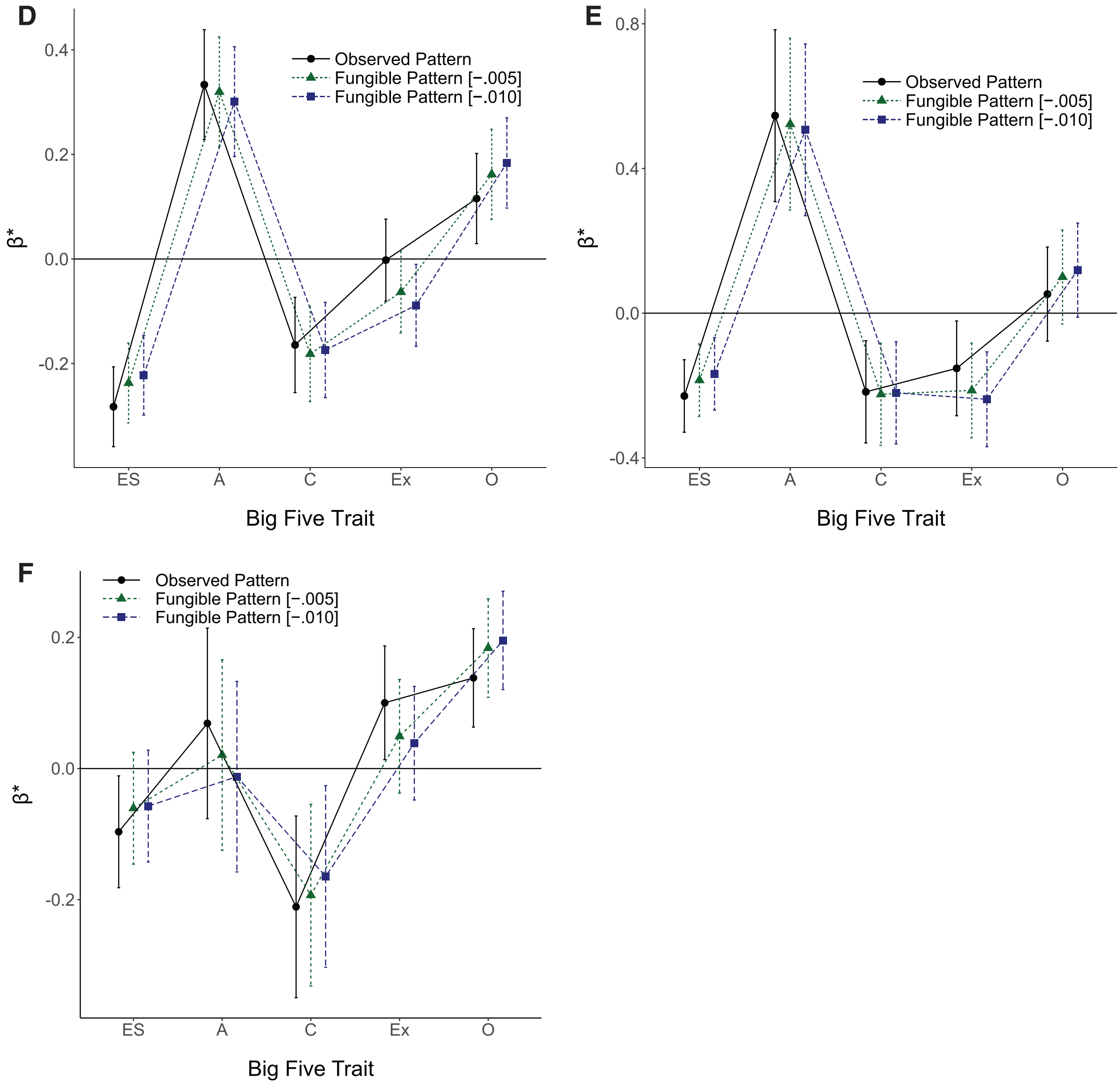


*Figure 2.* (*continued*)



## Additional Considerations and Caveats

A key potential application of MACPA is to examine profile level and pattern effects for sets of predictors that may have never been examined in any single study. Accordingly, researchers can apply MACPA to a correlation matrix compiled using results from many individual bivariate meta-analyses. However, conducting MACPA with such synthetic correlation matrices presents several challenges that researchers must address to ensure accurate and meaningful results.

### Comparability of Included Correlations

The most critical consideration when conducting MACPA with synthetic correlation matrices is to ensure that all correlations are comparable in terms of the populations they represent. Researchers should avoid constructing a “franken-matrix”, which contains correlations from wildly different populations or contexts. Accordingly, researchers should verify that all contributing meta-analyses had similar inclusion criteria for factors relevant for the research questions at hand, such as age and type of participants (e.g., children, clinical patients, etc.) and study design features (e.g., cross-sectional vs. longitudinal). For example, if the objective is to determine whether configurations of cognitive abilities predict performance for different university majors, it would be improper to use correlations from meta-analyses with elementary school samples. Similarly, all correlations should be comparable in terms of corrections for statistical artifacts (i.e., correcting for measurement error[10] and selection effects to reflect the same target population).

## Improper Matrices

When correlations in a meta-analytic matrix are based on different samples, the resulting matrix can sometimes be improper (i.e., nonpositive definite). This means that the specific correlation matrix used could not be observed with real data in a single sample. Improper matrices are a concern because they prohibit multivariate analyses using maximum likelihood estimation and can prevent computation of standard errors in generalized least squares methods (Yuan, 2016). Counterintuitively, improper matrices are more likely to occur when corrections for statistical artifacts are applied. This does not mean that meta-analytic correlation matrices or artifact corrections are invalid, but rather that a value from the credibility interval other than the mean must be used for one or more correlations.[11] In these cases, MACPA can be performed by "smoothing" the matrix to obtain a positive-definite correlation matrix that is as close as possible to the original.[12]

## Choice of Sample Size

Individual cells of meta-analytic correlation matrices are often based on different total sample sizes. This raises the question as to what sample size(s) is appropriate to use for MACPA. Sample sizes are used in two ways in multivariate analyses: (a) to test overall structural equations model fit and to compute fit indices, and (b) to estimate uncertainty for model parameters. Because MACPA, like OLS regression, uses a fully saturated structural equation model, the overall model fit test is perfect by constraint ($\chi^2 = 0$, $df = 0$). Consequently, MACPA sample sizes are only needed to estimate confidence intervals for MACPA parameters. A common solution is to use the arithmetic or harmonic mean sample size across the contributing meta-analyses (Landis, 2013). Viswesvaran and Ones (1995) recommended the harmonic mean (i.e., the reciprocal of the mean reciprocal sample size), which helps to conservatively balance uncertainty associated with larger and smaller sample sizes. An alternative, more accurate approach is to estimate uncertainty using the delta method (cf. Becker, 1992; Olkin & Finn, 1995). Here, variances of MACPA parameters are estimated as a weighted sum of the variances of each element of the meta-analytic correlation matrix. This method incorporates the uncertainty associated with each contributing meta-analysis when determining the overall uncertainty in the MACPA results. Delta method standard errors for each MACPA statistic are given in Appendix D, and *R* code implementing these methods in the *configural package*, available at https://cran.r-project.org/package=configural and at https://doi.org/10.17605/osf.io/aqmpc.

## Second-Order Sampling Error

Second-order sampling error is also a concern in applying MACPA. If a contributing meta-analysis is based on a small number of samples ($k$), substantial uncertainty will remain in the distribution, even if it is based on a large total $N$. The reason is because small-$k$ meta-analyses have little power to estimate effect size heterogeneity between-studies (i.e., $\tau$ or $SD_\rho$). Thus, if any contributing meta-analysis has small $k$, MACPA conclusions should be considered tentative. Using delta method estimators (see Appendix D) to construct confidence intervals is a solution that can better capture the impact of second-order sampling error, particularly if it is combined with a Bayesian estimate of $SD_\rho$ (P. D. G. Steel, Kammeyer-Mueller, & Paterson, 2015).

## Heterogeneity

Most meta-analyses of psychological constructs report nonnegligible heterogeneity in effect sizes, which suggests the presence of unaccounted for moderators or artifacts. As a result, caution is urged when interpreting MACPA results in the presence of substantial effect size heterogeneity. In these cases, results may only apply to certain subpopulations or in limited contexts. Consequently, when analyzing heterogeneous matrices, whenever possible, moderator analyses should be conducted first to identify subpopulations or contexts where effects are less variable. Afterward, MACPA can be conducted on more homogeneous matrices. If substantial heterogeneity remains, sensitivity analyses should be run to examine how results change if values from various points in meta-analyses' credibility intervals are substituted for mean values. Yu, Downes, Carter, and O'Boyle (2016) present bootstrap methods for constructing credibility intervals around model parameters in meta-analytic structural equations models (see also Hedges, 2016). These methods can also be used to construct credibility intervals for MACPA parameters. If credibility intervals for MACPA parameters are wide, strong inferences about observed criterion patterns should be avoided.

# Discussion

Criterion profile analysis (CPA) provides a unique alternative perspective on the meaning of predictor–criterion relations. It shows that OLS regression models can be interpreted as the joint influence of profile elevation and within-person configurations of scores on the criterion (Davison & Davenport, 2002). CPA iden-

[10] If pattern and level scores will be used to inform decision making (e.g., to select employees or diagnose disorders), their operational validity should be estimated by correcting for measurement error in the criterion variable only, as decisions in practice must be made using observed scores, rather than error-free true scores (Schmidt & Hunter, 2015). Further, the intercorrelations among the predictors used in the MACPA should also be values not corrected for measurement error.

[11] Alternative possibilities are (a) second-order sampling error perturbed one or more correlations from their true values, (b) correlations in the matrix do not all represent the same population, or (c) some artefact values were poor estimates.

[12] Kracht and Waller (2018a, 2018b) compared the accuracy of smoothing algorithms and found that the method by Higham (2002; implemented in *R* as the `Matrix::nearPD()` function) yields the most accurate results.

tifies the configuration of predictor scores that optimally relates to the criterion variable, but current methods are limited to primary data. The methods detailed in this article expand the applicability of CPA to a wider range of research contexts, including reanalysis of published studies from a pattern perspective and integrating CPA into the research synthesis process through meta-analytic criterion profile analysis (MACPA).

## Contributions

This article makes four methodological contributions. First, we showed how CPA pattern and level effects can be estimated without access to individual-level data, which opens the possibility of reanalyzing published data and interpreting meta-analyses from a pattern perspective. We presented methods for constructing confidence intervals and statistical tests for parameters, including in meta-analyses, allowing for consideration of sampling uncertainty when interpreting results. Second, we presented statistical methods to adjust the pattern effect ($r_{pat,y}$) to reduce overfitting and provide unbiased estimates of the population pattern effect, or expected cross-validity, in new samples. Procedures allow for overfitting corrections in MACPA applications, but can also be more practical and accurate when applied to primary data than cross-validation (cf. Kromrey & Hines, 1996). Third, we addressed critical challenges associated with integrating CPA with psychometric meta-analysis, including correcting for statistical artifacts, estimating uncertainty, and handling nonpositive definite matrices and effect size heterogeneity. Fourth, to assess the sensitivity of CPA results, we presented procedures for fungible profile analysis (Waller, 2008), which can help researchers to determine whether results are robust to minor perturbations of the correlation matrix. We recommend that researchers routinely apply these fungible profile analyses before drawing conclusions about the shape and criterion-related validity of criterion patterns.

## Implications

The pattern perspective enabled by MACPA has applications for both psychological theory and assessment practice.

**Implications for psychological research and theory.** MACPA can be used *descriptively* to characterize the nature of relationships between predictor variables and a criterion. For example, a strong pattern effect might suggest that configurations of risk factors uniquely predispose individuals to psychological disorders, rather than simple cumulative exposure (cf. Dohrenwend, 2006). MACPA could also be used to compare criterion patterns across related constructs (e.g., different disorders, compound personality traits) as a method for evaluating construct similarity or distinctiveness. Further, MACPA can illuminate within-person processes through which predictors influence outcomes. For example, Davison, Jew, and Davenport (2014) found that motivation to pursue a STEM major was highest among individuals who had stronger mathematics versus verbal ability, regardless of their overall general ability level. Similarly, Dilchert (2007) found that individuals who emphasized Extraversion, but deemphasized Agreeableness, at work tended to be most interested and effective in leadership roles. Finally, the intraindividual and configural processes suggested by MACPA are fruitful avenues for longitudinal study.

**Implications for psychological assessment practice.** For assessment practice, MACPA can be used *prescriptively* to facilitate effective interpretation and application of assessment information. Although optimal data-driven selection decisions can be made by selecting top-down using regression-based composite scores, in applied practice, most decisions are made using some form of holistic or subjective judgments. This can be problematic because human judgment is notoriously poor at reliably integrating multiple pieces of data to make predictions (Kuncel, Klieger, Connelly, & Ones, 2013). A major reason for the enduring popularity of subjective judgments is judges' interest in incorporating configural effects into decisions (cf. Kulas, 2013). Accordingly, MACPA can provide an avenue to improve the validity of subjective judgments by steering judges to focus on empirically defensible, criterion-valid configural effects, rather than spurious or irrelevant predictor patterns. MACPA can be used to identify the criterion pattern for a desired outcome, and then judges can be trained to identify candidates that match this prototypical pattern (cf. Asendorpf, 2006). This can simplify subjective judgments involving many predictors by reducing them to only *two* pieces of information: (a) an overall impression of quality (i.e., profile level); and (b) similarity to a prototype profile (i.e., criterion pattern similarity; cf. Davenport & Davison, 2012). Judgments based on these two pieces of information are likely to be more reliable than those based on the original larger predictor set; thus, MACPA can help to improve holistic decision making in education, personnel selection, clinical diagnosis, and other contexts.

Training judges to attend to criterion-relevant profile features identified through MACPA can also enhance the accessibility and acceptability of quantitative assessment data to decision-makers. A likely reason for the persistence of categorical type-based personality assessments in lay and applied practice (e.g., Myers-Briggs Type Indicator), despite contrary evidence (McCrae & Costa, 1989; Wilmot, Haslam, Tian, & Ones, 2018), is that typological systems are easier for untrained users to understand and interpret than multi-indicator score profiles—particularly when presented as percentiles or other norm-referenced scores. By describing the predictive validity of an assessment battery in terms of a characteristic successful "type," decision-makers may be more willing to rely on information derived from construct- and criterion-valid, multidimensional, quantitative measures versus easier-to-understand, but less valid, categorical, or qualitative assessments.

## Future Directions

Several avenues exist for future methodological research to expand and extend MACPA. One is to incorporate level and pattern effects for a set of predictors into a broader structural model for a criterion. For example, researchers interested in personality–job performance relationships may wonder whether the effects of optimal trait configuration are mediated by specific goal pursuit strategies (cf. Barrick, Mount, & Li, 2013). This question can be addressed by estimating mediation models incorporating one or more latent personality criterion pattern variables (cf. Davison, Chang, & Davenport, 2014). A second future direction is to further decompose contrastive pattern effects into effects attributable to *profile shape* (i.e., which predictors are elevated or depressed in the profile) and *profile scatter* (i.e., amplitude of profile peaks and valleys). Traditional methods for separating these effects (e.g., Cronbach & Gleser, 1953) may not fully account for the relationships of these profile features with external criteria. Future developments in this area may yield more accurate profile-based predictions and enhance configuration-based judgments in theory and practice. Future studies might also explicitly examine random-effects heterogeneity in CPA effects by meta-analyzing the results of CPA conducted in primary studies (cf. Becker & Wu, 2007). Finally, the most important and promising area for future research is applying the MACPA methods described here to existing literatures. Many areas of psychological research can benefit from considering whether observed predictor–criterion relationships represent pattern or level effects. We expect applications of MACPA to reanalysis of existing correlational research and new meta-analyses alike, to open new and exciting avenues of inquiry into potential mechanisms driving important criteria and outcomes.

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(*Appendices follow*)

## Appendix A

### Derivations of Equations for Correlations With Profile Level and Criterion Pattern From Summary Data

This appendix provides proofs for Equations 1–3.

### Level Effect

For each person $i$, let $X_i = (x_{1i}, x_{2i}, x_{3i}, \ldots, x_{pi})'$ be a vector of $p$ predictor scores and $y$ be a score on a criterion variable. The most common form of data for meta-analytic criterion profile analysis is a matrix of meta-analytic mean correlations. Such data treat $y_i$ and each $x_{ji}$ as standard scores with mean = 0 and $SD$ = 1. Formulas for unstandardized variables are presented below. Let $x_{lev_i} = \sum X_i/p$ be a person's profile level, their average score across the $p$ predictors. Following from the theory of composites (Ghiselli et al., 1981, p. 163), the correlation between $\boldsymbol{x}_{lev}$ and $\boldsymbol{y}$ is:

$$r_{lev,y} = \frac{\sum y_i \frac{1}{p}(x_{1i} + x_{2i} + x_{3i} + \ldots + x_{pi})}{N \cdot SD_y \cdot SD_{x_{lev}}} \tag{A1-a}$$

$$= \frac{\sum y_i \frac{1}{p}(x_{1i} + x_{2i} + x_{3i} + \ldots + x_{pi})}{N \cdot 1 \cdot \sqrt{\sum_{j,k} \frac{1}{p} \cdot \frac{1}{p} \cdot r_{x_j x_k}}} \tag{A1-b}$$

$$= \frac{\sum y_i(x_{1i} + x_{2i} + x_{3i} + \ldots + x_{pi})}{N \cdot \sqrt{\sum_{j,k} r_{x_j x_k}}} \tag{A1-c}$$

$$= \frac{\sum \left(\frac{y_i x_{1i}}{N} + \frac{y_i x_{2i}}{N} + \frac{y_i x_{3i}}{N} + \ldots + \frac{y_i x_{pi}}{N}\right)}{\sqrt{\sum_{j,k} r_{x_j x_k}}} \tag{A1-d}$$

Which yields the result in Equation (1):

$$r_{lev,y} = \frac{r_{y_i x_{1i}} + r_{y_i x_{2i}} + r_{y_i x_{3i}} + \ldots + r_{y_i x_{pi}}}{\sqrt{\sum_{j,k} r_{x_j x_k}}} = \frac{\sum_j r_{x_j y}}{\sqrt{\sum_{j,k} r_{x_j x_k}}}$$

The analogous formula for the unstandardized profile level effect is:

$$^{(\text{unstd})}r_{lev,y} = \frac{\boldsymbol{c}'_{\boldsymbol{xy}}\mathbf{1}}{SD_y\sqrt{\mathbf{1}'\boldsymbol{C}_{\boldsymbol{xx}}\mathbf{1}}}$$

$$= \frac{\sum_j 1 \cdot c_{x_j y}}{SD_y\sqrt{\sum_{j,k} 1 \cdot 1 \cdot c_{x_j x_k}}}$$

$$= \frac{\sum_j c_{x_j y}}{SD_y\sqrt{\sum_{j,k} c_{x_j x_k}}}, \tag{A2}$$

where $\boldsymbol{c}_{\boldsymbol{xy}}$ is the column vector of covariances between the predictor variables and the criterion, $\boldsymbol{C}_{\boldsymbol{xx}}$ is the variance-covariance matrix among the $x$ variables, $\mathbf{1}$ is a column vector of 1s, and $SD_y$ is the standard deviation of the criterion variable.

### Pattern Effect

Continuing from above, let $X_i^* = (x_{1i} - x_{lev_i}, x_{2i} - x_{lev_i}, x_{3i} - x_{lev_i}, \ldots, x_{pi} - x_{lev_i})'$ be each person's profile pattern (i.e., the vector of deviations of each of the $p$ predictor scores from the person's mean score). From Davison and Davenport (2002, Equation 2), $\boldsymbol{\beta}^* = [\beta_j - \bar{\beta}]'$ is the criterion pattern. Let $Cov_{\boldsymbol{\beta}_i^*} = \frac{1}{p}\sum_j x_{ji}^*\beta_j^*$ be each person's criterion pattern similarity score, which is the covariance between their profile pattern and the criterion pattern. $Cov_{\boldsymbol{\beta}_i^*}$ is a composite score of $X_i^*$ with weights $\beta_j^*/p$:

$$Cov_{\boldsymbol{\beta}_i^*} = \frac{1}{p}\sum_j x_{ji}^*\beta_j^* \tag{A3-a}$$

$$= \frac{1}{p}\sum_j (x_{ji} - x_{lev_i})\beta_j^* \tag{A3-b}$$

$$= \frac{1}{p}\sum_j (x_{ji}\beta_j^* - x_{lev_i}\beta_j^*) \tag{A3-c}$$

$$= \frac{1}{p}\left[\sum_j x_{ji}\beta_j^* - \sum_j x_{lev_i}\beta_j^*\right] \tag{A3-d}$$

$$= \frac{1}{p}\left[\sum_j x_{ji}\beta_j^* - x_{lev_i}\sum_j \beta_j^*\right] \tag{A3-e}$$

$$= \frac{1}{p}\left[\sum_j x_{ji}\beta_j^* - x_{lev_i} \cdot 0\right] \tag{A3-f}$$

$$Cov_{\boldsymbol{\beta}_i^*} = \frac{1}{p}\sum_j x_{ji}\beta_j^* \tag{A3-g}$$

Following the same logic as for the level effect, the correlation between $\boldsymbol{Cov}_{\boldsymbol{\beta}^*}$ and $\boldsymbol{y}$ is:

$$r_{pat,y} = \frac{\sum y_i \frac{1}{p}(x_{1i}\beta_1^* + x_{2i}\beta_2^* + x_{3i}\beta_3^* + \ldots + x_{pi}\beta_p^*)}{N \cdot SD_y \cdot SD_{Cov_{\boldsymbol{\beta}^*}}} \tag{A4-a}$$

$$= \frac{\sum y_i \frac{1}{p}(x_{1i}\beta_1^* + x_{2i}\beta_2^* + x_{3i}\beta_3^* + \ldots + x_{pi}\beta_p^*)}{N \cdot 1 \cdot \sqrt{\sum_{j,k} \frac{\beta_j^*}{p} \cdot \frac{\beta_k^*}{p} \cdot r_{x_j x_k}}} \tag{A4-b}$$

$$= \frac{\sum y_i(x_{1i}\beta_1^* + x_{2i}\beta_2^* + x_{3i}\beta_3^* + \ldots + x_{pi}\beta_p^*)}{N \cdot \sqrt{\sum_{j,k} \beta_j^*\beta_k^* r_{x_j x_k}}} \tag{A4-c}$$

$$= \frac{\sum \left(\beta_1^*\frac{y_i x_{1i}}{N} + \beta_2^*\frac{y_i x_{2i}}{N} + \beta_3^*\frac{y_i x_{3i}}{N} + \ldots + \beta_p^*\frac{y_i x_{pi}}{N}\right)}{\sqrt{\sum_{j,k} \beta_j^*\beta_k^* r_{x_j x_k}}} \tag{A4-d}$$

(*Appendices continue*)

Which yields the result in Equation (2):

$$r_{pat,y} = \frac{\beta_1^* r_{y_i x_{1i}} + \beta_2^* r_{y_i x_{2i}} + \beta_3^* r_{y_i x_{3i}} + \ldots + \beta_p^* r_{y_i x_{pi}}}{\sqrt{\sum_{j,k} \beta_j^* \beta_k^* r_{x_j x_k}}} = \frac{\sum_j \beta_j^* r_{x_j y}}{\sqrt{\sum_{j,k} \beta_j^* \beta_k^* r_{x_j x_k}}}$$

The analogous formula for the unstandardized profile pattern effect is:

$$^{(\text{unstd})}r_{pat,y} = \frac{\boldsymbol{c}'_{xy}\boldsymbol{b}^*}{SD_y\sqrt{\boldsymbol{b}^{*\prime}\,\boldsymbol{C}_{xx}\boldsymbol{b}^*}} = \frac{\sum_j b_j^* c_{x_j y}}{SD_y\sqrt{\sum_{j,k} b_j^* b_k^* c_{x_j x_k}}} \tag{A5}$$

where $\boldsymbol{b}^* = \boldsymbol{b} - \overline{b}$ is the unstandardized criterion pattern computed by ipsatizing the unstandardized regression coefficients around their mean, and other variables are defined as in Equation A2.

## Level–Pattern Correlation

The above equations show that CPA profile and level effects are two alternative composites of the predictor variables with different weighting schemes. Accordingly, following Ghiselli et al. (1981, p. 174), the correlation between these two composites is given by Equation (3):

$$r_{lev,pat} = \frac{\boldsymbol{\beta}^{*\prime}\boldsymbol{R}_{xx}\mathbf{1}}{\sqrt{\boldsymbol{\beta}^{*\prime}\boldsymbol{R}_{xx}\boldsymbol{\beta}^*}\sqrt{\mathbf{1}'\boldsymbol{R}_{xx}\mathbf{1}}} = \frac{\sum_{j,k}\beta_k^* \cdot r_{x_j x_k}}{\sqrt{\sum_{j,k} r_{x_j x_k}}\sqrt{\sum_{j,k}\beta_j^*\beta_k^* r_{x_j x_k}}}$$

The analogous formula for the correlation between unstandardized profile level and criterion pattern similarity is:

$$r_{lev,pat} = \frac{\boldsymbol{b}^{*\prime}\boldsymbol{C}_{xx}\mathbf{1}}{\sqrt{\boldsymbol{b}^{*\prime}\boldsymbol{C}_{xx}\boldsymbol{b}^*}\sqrt{\mathbf{1}'\boldsymbol{C}_{xx}\mathbf{1}}} = \frac{\sum_{j,k} b_k^* \cdot c_{x_j x_k}}{\sqrt{\sum_{j,k} c_{x_j x_k}}\sqrt{\sum_{j,k} b_j^* b_k^* c_{x_j x_k}}} \tag{A6}$$

# Appendix B

## Single-Sample CPA Confidence Intervals and Sampling Distributions

For the full multiple regression model,

$$F_{full} = \left(\frac{R^2}{1-R^2}\right)\cdot\frac{N-p-1}{p}, \tag{B1}$$

where $R^2$ is the full-model $R^2$, $N$ is the sample size, and $p$ is the number of predictors. $F_{full}$ follows the noncentral $F$ distribution (Shieh, 2006):

$$F_{full}\,|\,\Delta_{full} \sim F(p,\ N-p-1, \Delta_{full}) \text{ and } \Delta_{full} \sim \left(\frac{R^2}{1-R^2}\right)\cdot\chi^2(N-1). \tag{B2}$$

Here, the test statistic, $F_{full}$, follows an $F$ distribution with $p$ and $N - p - 1$ degrees of freedom and noncentrality parameter, $\Delta_{full}$. The noncentrality parameter captures the degree to which the shape of the $F$ distribution is altered when $R^2 > 0$. When $R^2 = 0$, $\Delta_{full}$ also equals 0, and the distribution is reduced to the ordinary central $F$ distribution. The noncentrality parameter, $\Delta_{full}$, follows a $\chi^2$ distribution with $N - 1$ degrees of freedom, multiplied by the quantity $R^2/[1 - R^2]$.

The sampling error variance for $R^2$ is well-approximated (Cohen et al., 2003) by:

$$SE^2_{R^2} = \frac{4R^2(1-R^2)(N-p-1)^2}{(N^2-1)(N+3)}. \tag{B3}$$

An approximate confidence interval can be constructed for $R^2$ as $R^2 \pm z_{\alpha/2}\,SE_{R^2}$, where $z_{\alpha/2}$ is the 100(1 − α/2)th percentile of the standard normal distribution. An approximate confidence interval for $R$ can be constructed as $\sqrt{R^2 \pm z_{\alpha/2}\,SE_{R^2}}$. Alternative approximate and exact interval methods are also available (Helland, 1987; Mendoza & Stafford, 2001; Shieh, 2006, 2009).

The CPA level and pattern effects are a decomposition of the full-model $R^2$, so they have similar sampling distributions, variances, and confidence interval methods.

## Level Effect

For the correlation between profile level and the criterion, $r_{lev,y}$,

$$F_{lev} = \left(\frac{r^2_{lev,y}}{1-r^2_{lev,y}}\right)\cdot(N-2), \tag{B4}$$

with $F_{lev}$ following the noncentral $F$ distribution:

$$F_{lev}\,|\,\Delta_{lev} \sim F(1,\ N-2, \Delta_{lev}) \text{ and } \Delta_{lev} \sim \left(\frac{r^2_{lev,y}}{1-r^2_{lev,y}}\right)\cdot\chi^2(N-1). \tag{B5}$$

Here, the test statistic, $F_{lev}$, follows an $F$ distribution with 1 and $N - 2$ degrees of freedom and noncentrality parameter, $\Delta_{lev}$. The noncentrality parameter, $\Delta_{lev}$, follows a $\chi^2$ distribution with $N - 1$ degrees of freedom, multiplied by the quantity $r^2_{lev,y}/[1 - r^2_{lev,y}]$.

(*Appendices continue*)

As $r_{lev,y}$ is a unit-weighted composite correlation, its sampling error variance is well-known as approximately (Schmidt & Hunter, 2015):

$$SE^2_{r_{lev,y}} = \frac{(1 - r^2_{lev,y})^2}{N - 1} \quad \text{(B6)}$$

$$SE^2_{r^2_{lev,y}} = \frac{4r^2_{lev,y}(1 - r^2_{lev,y})^2}{N - 1}. \quad \text{(B7)}$$

Approximate confidence intervals for $r_{lev,y}$ can be computed using Fisher's (1921) $r$-to-$z'$ transformation.[13] The hypothesis that the level effect fully accounts for the variables' predictive power can be tested using $R^2 - r^2_{lev,y} \sim F(p - 1, N - p - 1)$.

## Pattern Effect

For the correlation between criterion pattern similarity and the criterion, $r_{pat,y}$,

$$F_{pat} = \left(\frac{r^2_{pat,y}}{1 - r^2_{pat,y}}\right) \cdot \frac{N - p}{p - 1}, \quad \text{(B8)}$$

with $F_{pat}$ following the noncentral $F$ distribution:

$$F_{pat} \mid \Delta_{pat} \sim F(p - 1,\ N - p, \Delta_{pat})$$

$$\text{and } \Delta_{lev} \sim \left(\frac{r^2_{pat,y}}{1 - r^2_{pat,y}}\right) \cdot \chi^2(N - 1). \quad \text{(B9)}$$

Here, the test statistic, $F_{pat}$, follows an $F$ distribution with $p - 1$ and $N - p$ degrees of freedom and noncentrality parameter, $\Delta_{pat}$. The noncentrality parameter, $\Delta_{pat}$, follows a $\chi^2$ distribution with $N - 1$ degrees of freedom, multiplied by the quantity $r^2_{pat,y}/[1 - r^2_{pat,y}]$.

The sampling error variance of $r^2_{pat,y}$ is well-approximated[14] using a formula like that for $R^2$:

$$SE^2_{r^2_{pat,y}} = \frac{4r^2_{pat,y}(1 - r^2_{pat,y})(N - p - 1)^2}{(N^2 - 1)(N + 3)} \quad \text{(B10)}$$

An approximate confidence interval can be constructed for $r^2_{pat,y}$ as $r^2_{pat,y} \pm z_{1-\alpha/2} SE_{r^2_{pat,y}}$, where $z_{1-\alpha/2}$ is the $100(1 - \alpha/2)$th percentile of the standard normal distribution. An approximate confidence interval for $r_{pat,y}$ can be constructed as $\sqrt{r^2_{pat,y} \pm z_{\alpha/2} SE_{r^2_{pat,y}}}$. Exact intervals can be constructed by adapting methods for $R^2$ (Shieh, 2006, 2009). The hypothesis that the pattern effect fully accounts for variables' predictive power can be tested using $R^2 - r^2_{pat,y} \sim F(1, N - p - 1)$.

## Criterion Pattern Elements

Davison and Davenport (2002, Appendix B) suggested an approximate hypothesis test for the elements of the criterion pattern vector, $\boldsymbol{\beta}^*$,[15] by assuming that the sampling error for $\boldsymbol{\beta}^*$ is the same as for the OLS regression coefficient vector, $\boldsymbol{\beta}$. However, this procedure yields confidence intervals for $\boldsymbol{\beta}^*$ that are too wide; it does not account for the sampling error variance the elements of $\boldsymbol{\beta}^*$ share because the same estimate of the mean regression coefficient, $\bar{\beta}$, is subtracted from each element of $\boldsymbol{\beta}$. More accurate confidence intervals account for the shared variance of $\bar{\beta}$ using the formula:

$$\boldsymbol{\Sigma}_{\boldsymbol{\beta}^*} = \boldsymbol{J}_{\boldsymbol{\beta}^*}\boldsymbol{\Sigma}_{\boldsymbol{r}}\boldsymbol{J}'_{\boldsymbol{\beta}^*} = \left[\frac{\partial \boldsymbol{\beta}^*}{\partial \boldsymbol{r}'_{\boldsymbol{xx}}} \quad \frac{\partial \boldsymbol{\beta}^*}{\partial \boldsymbol{r}'_{\boldsymbol{xy}}}\right]\boldsymbol{\Sigma}_{\boldsymbol{r}}\left[\frac{\partial \boldsymbol{\beta}^*}{\partial \boldsymbol{r}'_{\boldsymbol{xx}}} \quad \frac{\partial \boldsymbol{\beta}^*}{\partial \boldsymbol{r}'_{\boldsymbol{xy}}}\right]', \quad \text{(B11)}$$

where $\boldsymbol{\Sigma}_{\boldsymbol{\beta}^*}$ is the sampling error variance-covariance matrix for the elements of $\boldsymbol{\beta}^*$, $\boldsymbol{J}_{\boldsymbol{\beta}^*}$ is the Jacobian matrix of partial derivatives of $\boldsymbol{\beta}^*$ with respect to each unique element of the variable correlation matrix, $\boldsymbol{R}$, $\boldsymbol{\Sigma}_{\boldsymbol{r}}$ is the sampling covariance matrix among the unique elements of $\boldsymbol{R}$, $\boldsymbol{r}_{\boldsymbol{xy}}$ is the column vector of correlations between each predictor and the criterion, and $\boldsymbol{r}_{\boldsymbol{xx}}$ is the column vector of predictor intercorrelations (i.e., vecp($\boldsymbol{R}_{\boldsymbol{xx}}$); Nel, 1985). Formulas for $\boldsymbol{\Sigma}_{\boldsymbol{r}}$ are given by Olkin and Finn (1995) and Nel (1985).

The formula for $\boldsymbol{\beta}^*$ can be expressed in matrix algebra as:

$$\boldsymbol{\beta}^* = \boldsymbol{\beta} - \mathbf{1}\boldsymbol{p}^{-1\prime}\boldsymbol{\beta} = (\mathbf{I} - \mathbf{1}\boldsymbol{p}^{-1\prime})\boldsymbol{\beta} = \boldsymbol{Q}\boldsymbol{\beta}, \quad \text{(B12)}$$

where $\mathbf{1}$ is a column vector 1s of length $p$ (the number of predictors), $\boldsymbol{p}^{-1}$ is a column vector of length $p$ with each element equal to $1/p$, and $\mathbf{I}$ is a $p \times p$ identity matrix. Following the chain rule for matrix differentiation (Nel, 1980), $\boldsymbol{J}_{\boldsymbol{\beta}^*} = \boldsymbol{Q}\boldsymbol{J}_{\boldsymbol{\beta}}$, indicating that $\boldsymbol{J}_{\boldsymbol{\beta}^*}$ contains shrunken values of $\boldsymbol{J}_{\boldsymbol{\beta}}$.

Jones and Waller (2013, 2015) showed that the standard method for calculating the standard errors of $\boldsymbol{\beta}$ (Cohen et al., 2003) is negatively biased because it fails to account for the uncertainty in the sample standard deviations used to standardize the regression coefficients. Jones and Waller (2013) presented a more accurate delta method estimator of the sampling variance-covariance matrix of $\boldsymbol{\beta}$:

$$\boldsymbol{\Sigma}_{\boldsymbol{\beta}} = \boldsymbol{J}_{\boldsymbol{\beta}}\boldsymbol{\Sigma}_{\boldsymbol{r}}\boldsymbol{J}'_{\boldsymbol{\beta}} = \left[\frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{r}_{\boldsymbol{xx}}} \quad \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{r}_{\boldsymbol{xy}}}\right]\boldsymbol{\Sigma}_{\boldsymbol{r}}\left[\frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{r}_{\boldsymbol{xx}}} \quad \frac{\partial \boldsymbol{\beta}}{\partial \boldsymbol{r}_{\boldsymbol{xy}}}\right]', \quad \text{(B13)}$$

$$\boldsymbol{J}_{\boldsymbol{\beta}} = \left[-2\left[\boldsymbol{r}'_{\boldsymbol{xy}}\boldsymbol{R}^{-1}_{\boldsymbol{xx}} \otimes \boldsymbol{R}^{-1}_{\boldsymbol{xx}}\right]\boldsymbol{K}'_c \quad \boldsymbol{R}^{-1}_{\boldsymbol{xx}}\right] \quad \text{(B14)}$$

where $\boldsymbol{J}_{\boldsymbol{\beta}}$ is the Jacobian matrix of $\boldsymbol{\beta}$, $\otimes$ is the Kronecker product (Abadir & Magnus, 2005, p. 274), and $\boldsymbol{K}_c$ is a transition matrix (Nel, 1985, p. 143) that extracts the lower triangular elements from $\boldsymbol{R}_{\boldsymbol{xx}}$.

[13] Exact confidence intervals for $r_{lev,y}$ can be computed using $t_{lev} = \sqrt{N-2} \cdot r_{lev,y}/\sqrt{1-r^2_{lev,y}}$ with $t_{lev} \mid \delta_{lev} \sim t(N - 2, \delta_{lev})$ and $\delta_{lev} \sim \sqrt{\chi^2(N-1)} \cdot r_{lev,y}/\sqrt{1-r^2_{lev,y}}$.

[14] Simulation results supporting this approximation are available from the first author.

[15] Unless stated otherwise, all vectors are column vectors. The prime symbol indicates the transpose operation.

(*Appendices continue*)

Substituting the results of Equation B14 into Equation B11 yields:

$$\boldsymbol{\Sigma}_{\beta^*} = \boldsymbol{Q}\boldsymbol{J}_\beta\boldsymbol{\Sigma}_r\boldsymbol{J}_\beta'\boldsymbol{Q}, \tag{B15}$$

Each element of $\boldsymbol{\beta}^*$ then has a standard error of $SE_{\beta_j^*} = \sqrt{\mathrm{diag}(\boldsymbol{\Sigma}^2_{\beta^*})_j}$. The $100(1 - \alpha)\%$ confidence interval for the $j$th element of $\boldsymbol{\beta}^*$ can be estimated as $\beta_j^* \pm t_{1-\alpha/2,df}\, SE_{\beta_j^*}$, where $t_{1-\alpha/2,df}$ is the $100(1 - \alpha/2)$th percentile of the $t(N - p - 1)$ distribution.

The sampling error for the mean regression coefficient, $\bar{\beta}$, is computed as

$$SE^2_{\bar{\beta}} = \boldsymbol{J}_{\bar{\beta}}\boldsymbol{\Sigma}_r\boldsymbol{J}_{\bar{\beta}}', \tag{B16}$$

with

$$\boldsymbol{J}_{\bar{\beta}} = \left[-2\boldsymbol{p}^{-1\prime}\left[\boldsymbol{r}_{xy}'\boldsymbol{R}_{xx}^{-1} \otimes \boldsymbol{R}_{xx}^{-1}\right]\boldsymbol{K}_c \quad \boldsymbol{p}^{-1\prime}\boldsymbol{R}_{xx}^{-1}\right]. \tag{B17}$$

## Appendix C

## Derivation of Shrinkage-Adjusted and Cross-Validity Pattern Effects

The correlation between criterion pattern similarity and the criterion, $r_{pat,y}$, can be expressed in terms of (a) the correlation between profile level and the criterion, $r_{lev,y}$, (b) the correlation between profile level and criterion pattern similarity, $r_{lev,pat}$, and (c) the total regression model $R^2$.

$$R^2 = \boldsymbol{r}_{xy}'\boldsymbol{R}^{-1}\boldsymbol{r}_{xy} = \begin{bmatrix} r_{pat,y} & r_{lev,y} \end{bmatrix}\begin{bmatrix} 1 & r_{lev,pat} \\ r_{lev,pat} & 1 \end{bmatrix}^{-1}\begin{bmatrix} r_{pat,y} \\ r_{lev,y} \end{bmatrix} \tag{C1-a}$$

$$R^2 = \left(\frac{1}{r^2_{lev,pat} - 1}\right)\begin{bmatrix} r_{pat,y} & r_{lev,y} \end{bmatrix}\begin{bmatrix} -1 & r_{lev,pat} \\ r_{lev,pat} & -1 \end{bmatrix}\begin{bmatrix} r_{pat,y} \\ r_{lev,y} \end{bmatrix} \tag{C1-b}$$

$$R^2 = \left(\frac{1}{r^2_{lev,pat} - 1}\right)\left(2r_{pat,y}r_{lev,y}r_{lev,pat} - r^2_{pat,y} - r^2_{lev,y}\right) \tag{C1-c}$$

$$R^2 \cdot \left(r^2_{lev,pat} - 1\right) = 2r_{pat,y}r_{lev,y}r_{lev,pat} - r^2_{pat,y} - r^2_{lev,y} \tag{C1-d}$$

$$r^2_{pat,y} - 2r_{pat,y}r_{lev,y}r_{lev,pat} = -R^2 \cdot \left(r^2_{lev,pat} - 1\right) - r^2_{lev,y} \tag{C1-e}$$

$$r^2_{lev,y}r^2_{lev,pat} + r^2_{pat,y} - 2r_{pat,y}r_{lev,y}r_{lev,pat} = r^2_{lev,y}r^2_{lev,pat} - R^2 \cdot \left(r^2_{lev,pat} - 1\right) - r^2_{lev,y} \tag{C1-f}$$

$$(r_{lev,y}r_{lev,pat} - r_{pat,y})^2 = r^2_{lev,y} \cdot \left(r^2_{lev,pat} - 1\right) - R^2 \cdot \left(r^2_{lev,pat} - 1\right) \tag{C1-g}$$

$$(r_{lev,y}r_{lev,pat} - r_{pat,y})^2 = \left(r^2_{lev,pat} - 1\right)\left(r^2_{lev,y} - R^2\right) \tag{C1-h}$$

$$(r_{lev,y}r_{lev,pat} - r_{pat,y})^2 = \left(1 - r^2_{lev,pat}\right)\left(R^2 - r^2_{lev,y}\right) \tag{C1-i}$$

$$r_{lev,y}r_{lev,pat} - r_{pat,y} = \pm\sqrt{\left(1 - r^2_{lev,pat}\right)\left(R^2 - r^2_{lev,y}\right)} \tag{C1-j}$$

$$-r_{pat,y} = -r_{lev,y}r_{lev,pat} \pm \sqrt{\left(1 - r^2_{lev,pat}\right)\left(R^2 - r^2_{lev,y}\right)} \tag{C1-k}$$

$$r_{pat,y} = r_{lev,y}r_{lev,pat} \pm \sqrt{\left(1 - r^2_{lev,pat}\right)\left(R^2 - r^2_{lev,y}\right)} \tag{C1-l}$$

It can be assumed that the pattern effect is positively correlated with the criterion, yielding the result in Equation (6):

$$r_{pat,y} = r_{lev,y}r_{lev,pat} + \sqrt{\left(1 - r^2_{lev,pat}\right)\left(R^2 - r^2_{lev,y}\right)}$$

(*Appendices continue*)

# Appendix D

## Delta Method Confidence Intervals for MACPA Parameters

Approximate confidence intervals for MACPA parameters can be computed using the following sampling error formulas and critical values from a $t$ or standard normal distribution (cf. Alf & Graf, 1999; Cohen et al., 2003, p. 88). $R$ code implementing these methods is available in the *configural* package at https://cran.r-project.org/package=configural or at https://doi.org/10.17605/osf.io/aqmpc.

Each sampling error formula takes the general form:

$$SE_f^2 = \boldsymbol{J}_f \boldsymbol{\Sigma}_{\bar{r}} \boldsymbol{J}_f' = \left[\frac{\partial f}{\partial \bar{\boldsymbol{r}}_{xx}'} \quad \frac{\partial f}{\partial \bar{\boldsymbol{r}}_{xy}'}\right] \boldsymbol{\Sigma}_{\bar{r}} \left[\frac{\partial f}{\partial \bar{\boldsymbol{r}}_{xx}'} \quad \frac{\partial f}{\partial \bar{\boldsymbol{r}}_{xy}'}\right]', \tag{D1}$$

where $\boldsymbol{J}_f$ is the Jacobian matrix of partial derivatives of the formula for the specified MACPA parameter with respect to each element of the vector of meta-analytic mean correlations, $\bar{\boldsymbol{r}}$ (including the meta-analytic mean intercorrelations among the predictors, $\bar{\boldsymbol{r}}_{xx}$, and the meta-analytic mean criterion correlation for each predictor, $\bar{\boldsymbol{r}}_{xy}$) and $\boldsymbol{\Sigma}_{\bar{r}}$ is the sampling covariance matrix among the elements of $\bar{\boldsymbol{r}}$ (Becker, 1992).

### Criterion Pattern Vector

As shown in Appendix B, the elements of the criterion pattern vector, $\boldsymbol{\beta}^*$ have standard errors that are shrunken values of the standard errors of the standardized OLS regression coefficients, $\boldsymbol{\beta}$. The sampling error variance-covariance matrix for $\boldsymbol{\beta}^*$ in MACPA is:

$$\boldsymbol{\Sigma}_{\boldsymbol{\beta}^*} = \boldsymbol{Q}\boldsymbol{J}_{\boldsymbol{\beta}}\boldsymbol{\Sigma}_{\bar{r}}\boldsymbol{J}_{\boldsymbol{\beta}}'\boldsymbol{Q}, \tag{D2}$$

$$\boldsymbol{J}_{\boldsymbol{\beta}} = \left[-2\left[\bar{\boldsymbol{r}}_{xy}'\bar{\boldsymbol{R}}_{xx}^{-1} \otimes \bar{\boldsymbol{R}}_{xx}^{-1}\right]\boldsymbol{K}_c' \quad \bar{\boldsymbol{R}}_{xx}^{-1}\right] \tag{D3}$$

where $\boldsymbol{Q} = (\mathbf{I} - \mathbf{1}'\boldsymbol{p}^{-1})$, $\mathbf{1}'$ is a row vector of 1s of length $p$ (the number of predictors), $\boldsymbol{p}^{-1}$ is a column vector of length $p$ with each element equal to $1/p$, $\mathbf{I}$ is $p \times p$ identity matrix, $\bar{\boldsymbol{R}}_{xx}$ is the meta-analytic mean intercorrelation matrix among predictors, $\otimes$ is the Kronecker product (Abadir & Magnus, 2005, p. 274), and $\boldsymbol{K}_c$ is a transition matrix (Nel, 1985, p. 143) that extracts the lower triangular elements from $\bar{\boldsymbol{R}}_{xx}$. The standard error for each element of $\boldsymbol{\beta}^*$ is $SE_{\beta_j^*} = \sqrt{\text{diag}(\boldsymbol{\Sigma}_{\boldsymbol{\beta}^*}^2)_j}$.

### Total Model $\mathbf{R^2}$ and R

The total model $R^2$ in an OLS multiple regression is $R^2 = \boldsymbol{r}_{xy}'\boldsymbol{\beta}$. Applying principles of matrix differentiation (Laue, Mitterreiter, & Giesen, 2018; Nel, 1980), the Jacobian matrix for $R^2$, $\boldsymbol{J}_{R^2}$ is:

$$\boldsymbol{J}_{R^2} = [-2[\boldsymbol{\beta}' \otimes \boldsymbol{\beta}']\boldsymbol{K}_c' \quad 2\boldsymbol{\beta}']. \tag{D4}$$

The Jacobian matrix for total multiple correlation, $\boldsymbol{J}_R$, is:

$$\boldsymbol{J}_R = \left[-\frac{1}{R}[\boldsymbol{\beta}' \otimes \boldsymbol{\beta}']\boldsymbol{K}_c' \quad \frac{1}{R}\boldsymbol{\beta}'\right]. \tag{D5}$$

### Level Effect, Pattern Effect, and Correlation Between Profile Level and Criterion Pattern Similarity

Jacobian matrices for the level effect, pattern effect, and correlation between profile level and criterion pattern similarity can be obtained by differentiating Equations 1–3.

The Jacobian matrix for the level effect, $\boldsymbol{J}_{r_{lev,y}}$, is:

$$\boldsymbol{J}_{r_{lev,y}} = \left[-\frac{\bar{\boldsymbol{r}}_{xy}'\mathbf{1}}{(\mathbf{1}'\bar{\boldsymbol{R}}_{xx}\mathbf{1})^{3/2}}(\mathbf{1}' \otimes \mathbf{1}')\boldsymbol{K}_c' \quad \frac{1}{(\mathbf{1}'\bar{\boldsymbol{R}}_{xx}\mathbf{1})^{1/2}}\mathbf{1}'\right]. \tag{D6}$$

The Jacobian matrix for the squared level effect, $\boldsymbol{J}_{r_{lev,y}^2}$, is:

$$\boldsymbol{J}_{r_{lev,y}^2} = \left[-2\left(\frac{\bar{\boldsymbol{r}}_{xy}'\mathbf{1}}{\mathbf{1}'\bar{\boldsymbol{R}}_{xx}\mathbf{1}}\right)^2(\mathbf{1}' \otimes \mathbf{1}')\boldsymbol{K}_c' \quad 2\left(\frac{\bar{\boldsymbol{r}}_{xy}'\mathbf{1}}{\mathbf{1}'\bar{\boldsymbol{R}}_{xx}\mathbf{1}}\right)\mathbf{1}'\right]. \tag{D7}$$

The Jacobian matrix for the pattern effect, $\boldsymbol{J}_{r_{pat,y}}$, s:

$$\boldsymbol{J}_{r_{pat,y}} = \left[\frac{\partial \boldsymbol{r}_{pat,y}}{\partial \bar{\boldsymbol{r}}_{xx}'} \quad \frac{\partial \boldsymbol{r}_{pat,y}}{\partial \bar{\boldsymbol{r}}_{xy}'}\right], \quad \text{where} \tag{D8}$$

$$\begin{aligned}\frac{\partial r_{pat,y}}{\partial \bar{\boldsymbol{r}}_{xx}'} = \left(\frac{1}{\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{xx}\boldsymbol{\beta}^*}\right)^{1/2}\Bigg[&2(\boldsymbol{\beta}' \otimes \boldsymbol{r}'\boldsymbol{Q}\bar{\boldsymbol{R}}_{xx}^{-1}) \\ &+ \frac{\bar{\boldsymbol{r}}_{xy}'\boldsymbol{\beta}^*}{\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{xx}\boldsymbol{\beta}^*}\big((\boldsymbol{\beta}^{*\prime} \otimes \boldsymbol{\beta}^{*\prime}) - (\boldsymbol{\beta}' \otimes \boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{xx}\boldsymbol{Q}\bar{\boldsymbol{R}}_{xx}^{-1}) \\ &- (\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{xx}\boldsymbol{Q}\bar{\boldsymbol{R}}_{xx}^{-1} \otimes \boldsymbol{\beta}')\big)\Bigg]\boldsymbol{K}_c',\end{aligned} \tag{D9}$$

$$\frac{\partial r_{pat,y}}{\partial \bar{\boldsymbol{r}}_{xy}'} = \left(\frac{1}{\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{xx}\boldsymbol{\beta}^*}\right)^{1/2}\left[\bar{\boldsymbol{r}}_{xy}'\boldsymbol{Q}\bar{\boldsymbol{R}}_{xx}^{-1} + \boldsymbol{\beta}^{*\prime} - \left(\frac{\bar{\boldsymbol{r}}_{xy}'\boldsymbol{\beta}^*\boldsymbol{\beta}^{*\prime}\boldsymbol{R}\boldsymbol{Q}\bar{\boldsymbol{R}}_{xx}^{-1}}{\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{xx}\boldsymbol{\beta}^*}\right)\right]. \tag{D10}$$

The Jacobian matrix for the squared pattern effect, $\boldsymbol{J}_{r_{pat,Y}^2}$, is:

$$\boldsymbol{J}_{r_{pat,y}^2} = \left[\frac{\partial r_{pat,y}^2}{\partial \bar{\boldsymbol{r}}_{xx}'} \quad \frac{\partial r_{pat,y}^2}{\partial \bar{\boldsymbol{r}}_{xy}'}\right], \text{ where} \tag{D11}$$

$$\begin{aligned}\frac{\partial r_{pat,y}^2}{\partial \bar{\boldsymbol{r}}_{xx}'} = -2\frac{\bar{\boldsymbol{r}}_{xy}'\boldsymbol{\beta}^*}{\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{xx}\boldsymbol{\beta}^*}\Bigg[&2\big(\boldsymbol{\beta}' \otimes \bar{\boldsymbol{r}}_{xy}'\boldsymbol{Q}\bar{\boldsymbol{R}}_{xx}^{-1}\big) \\ &+ \frac{\bar{\boldsymbol{r}}_{xy}'\boldsymbol{\beta}^*}{\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{xx}\boldsymbol{\beta}^*}\big((\boldsymbol{\beta}^{*\prime} \otimes \boldsymbol{\beta}^{*\prime}) - \big(\boldsymbol{\beta}' \otimes \boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{xx}\boldsymbol{Q}\bar{\boldsymbol{R}}_{xx}^{-1}\big) \\ &- \big(\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{xx}\boldsymbol{Q}\bar{\boldsymbol{R}}_{xx}^{-1} \otimes \boldsymbol{\beta}'\big)\big)\Bigg]\boldsymbol{K}_c',\end{aligned} \tag{D12}$$

$$\frac{\partial r_{pat,y}^2}{\partial \bar{\boldsymbol{r}}_{xy}'} = \frac{2\bar{\boldsymbol{r}}_{xy}'\boldsymbol{\beta}^*}{\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{xx}\boldsymbol{\beta}^*}\left[\bar{\boldsymbol{r}}_{xy}'\boldsymbol{Q}\bar{\boldsymbol{R}}_{xx}^{-1} + \boldsymbol{\beta}^{*\prime} - \left(\frac{\bar{\boldsymbol{r}}_{xy}'\boldsymbol{\beta}^*\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{xx}\boldsymbol{Q}\bar{\boldsymbol{R}}_{xx}^{-1}}{\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{xx}\boldsymbol{\beta}^*}\right)\right]. \tag{D13}$$

(*Appendices continue*)

If the pattern effect is very small, Equation D8 may overestimate the variance of $r_{pat,y}$. In this case, constructing a confidence interval for $r^2_{pat,y}$ and taking the square roots of the bounds will be more accurate.

The Jacobian matrix for the profile level–criterion pattern similarity correlation, $\boldsymbol{J}_{r_{lev,pat}}$, is:

$$\boldsymbol{J}_{r_{lev,pat}} = \left[\frac{\partial r_{lev,pat}}{\partial \bar{\boldsymbol{r}}'_{\boldsymbol{xx}}} \quad \frac{\partial r_{lev,pat}}{\partial \bar{\boldsymbol{r}}'_{\boldsymbol{xy}}}\right], \text{ where} \tag{D14}$$

$$\begin{aligned}\frac{\partial r_{lev,pat}}{\partial \bar{\boldsymbol{r}}'_{\boldsymbol{xx}}} = &\left(\frac{\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{\boldsymbol{xx}}\boldsymbol{1}}{(\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{\boldsymbol{xx}}\boldsymbol{\beta}^{*}\boldsymbol{1}'\bar{\boldsymbol{R}}_{\boldsymbol{xx}}\boldsymbol{1})^{3/2}}(\boldsymbol{1}'\bar{\boldsymbol{R}}_{\boldsymbol{xx}}\boldsymbol{1}((\boldsymbol{\beta}'\otimes\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{\boldsymbol{xx}}\boldsymbol{Q}\bar{\boldsymbol{R}}_{\boldsymbol{xx}}^{-1})\right.\\ &+(\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{\boldsymbol{xx}}\boldsymbol{Q}\bar{\boldsymbol{R}}_{\boldsymbol{xx}}^{-1}\otimes\boldsymbol{\beta}')-(\boldsymbol{\beta}^{*\prime}\otimes\boldsymbol{\beta}^{*\prime}))-\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{\boldsymbol{xx}}\boldsymbol{\beta}^{*}(\boldsymbol{1}'\otimes\boldsymbol{1}'))\\ &\left.+\frac{2}{(\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{\boldsymbol{xx}}\boldsymbol{\beta}^{*}\boldsymbol{1}'\bar{\boldsymbol{R}}_{\boldsymbol{xx}}\boldsymbol{1})^{1/2}}((\boldsymbol{1}'\otimes\boldsymbol{\beta}^{*\prime})-(\boldsymbol{1}'\bar{\boldsymbol{R}}_{\boldsymbol{xx}}\boldsymbol{Q}\bar{\boldsymbol{R}}_{\boldsymbol{xx}}^{-1}\otimes\boldsymbol{\beta}'))\right)\boldsymbol{K}'_c,\end{aligned} \tag{D15}$$

$$\begin{aligned}\frac{\partial r_{lev,pat}}{\partial \bar{\boldsymbol{r}}'_{\boldsymbol{xy}}} = &\left(\frac{1}{\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{\boldsymbol{xx}}\boldsymbol{\beta}^{*}\boldsymbol{1}'\bar{\boldsymbol{R}}_{\boldsymbol{xx}}\boldsymbol{1}}\right)^{1/2}\left[\boldsymbol{1}'\bar{\boldsymbol{R}}_{\boldsymbol{xx}}\boldsymbol{Q}\bar{\boldsymbol{R}}_{\boldsymbol{xx}}^{-1}\right.\\ &\left.-\frac{\boldsymbol{1}'\bar{\boldsymbol{R}}_{\boldsymbol{xx}}\boldsymbol{\beta}^{*}\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{\boldsymbol{xx}}\boldsymbol{Q}\bar{\boldsymbol{R}}_{\boldsymbol{xx}}^{-1}}{\boldsymbol{\beta}^{*\prime}\bar{\boldsymbol{R}}_{\boldsymbol{xx}}\boldsymbol{\beta}^{*}}\right].\end{aligned} \tag{D16}$$

## Incremental Validity ($\Delta R^2$ and $\sqrt{\Delta R^2}$) of Level and Pattern Effects

Following Alf and Graf (1999), the sampling error variances for the incremental $R^2$ of the level effect over the pattern effect, $SE^2_{\Delta R^2_{lev}}$, and the incremental $R^2$ of the pattern effect over the level effect, $SE^2_{\Delta R^2_{pat}}$, are:

$$\begin{aligned}SE^2_{\Delta R^2_{lev}} \approx\ & 4R^2SE^2_R + 4r^2_{lev,y}SE^2_{r_{lev,y}}\\ &-8\left[\frac{r_{lev,y}(R^6+R^4(r^2_{lev,y}-3)+R^2(2-r^2_{lev,y})-r^2_{lev,y})}{2R^3(1-R^2)(1-r^2_{lev,y})}\right]\\ &\times SE_R SE_{r_{lev,y}} Rr_{lev,y}\end{aligned} \tag{D17}$$

$$\begin{aligned}SE^2_{\Delta R^2_{pat}} \approx\ & 4R^2SE^2_R + 4r^2_{pat,y}SE^2_{r_{pat,y}}\\ &-8\left[\frac{r_{pat,y}(R^6+R^4(r^2_{pat,y}-3)+R^2(2-r^2_{pat,y})-r^2_{pat,y})}{2R^3(1-R^2)(1-r^2_{pat,y})}\right]\\ &\times SE_R SE_{r_{pat,y}} Rr_{pat,y}\end{aligned} \tag{D18}$$

The sampling error variances for the square roots of the these values ($\sqrt{\Delta R^2}$; i.e., the semipartial correlation of profile or criterion pattern similarity with the criterion), $SE^2_{\sqrt{\Delta R^2_{lev}}}$, and the incremental $R^2$ of the pattern effect over the level effect, $SE^2_{\sqrt{\Delta R^2_{pat}}}$, are:

$$SE^2_{\sqrt{\Delta R^2_{lev}}} \approx SE^2_{\Delta R^2_{lev}} / (4\Delta R^2_{lev}) \tag{D19}$$

$$SE^2_{\sqrt{\Delta R^2_{pat}}} \approx SE^2_{\Delta R^2_{pat}} / (4\Delta R^2_{pat}) \tag{D20}$$

If the pattern effect is very small, Equation D20 may overestimate the variance of $\sqrt{\Delta R^2_{pat}}$. In this case, constructing a confidence interval for $\Delta r^2_{pat}$ and taking the square roots of the bounds will be more accurate.

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