Simulation of Financial Transactions Using Monte Carlo Methods

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Presented is our simulation of a set of financial interactions using Monte Carlo methods. We analyze equilibrium wealth distributions under a variety of assumptions. These include savings criterion, taxation and preference for past transaction partners.

INTRODUCTION

In order to study the wealth distribution is society, we model financial transactions between financial agents. To simulate these transactions, we impose a Monte Carlo technique which generates a random number that must pass sampling rules corresponding to our physical system.

THEORY

From empirical studies[1], we know that the higher end of the wealth distribution exhibits the form:

$$w_m \propto m^{-1-\alpha}, \tag{1}$$

In our report we wish to recover this distribution.

Conservation of Money in Transactions

Consider two agents, i and j, with initial amounts of money m_i' and m_j' respectively. A transaction occurs between the two, leaving them with a final amount of money m_i and m_j. The total amount of money between the agents is conserved:

$$m_i + m_j = m_i' + m_j'. \tag{2}$$

We can model that the final amount of money held by agent i depends on a random factor, ϵ, sampled from a normal distribution:

$$m_i' = \epsilon(m_i + m_j),$$

As a result, the amount of money held by agent j is given by:

$$m_j' = (1-\epsilon)(m_i + m_j).$$

When equilibrium has been reached, a Gibb's distribution describes this conservation law:

$$w_m = \beta \exp\left(-\beta m\right), \tag{3}$$

where

$$\beta = \frac{1}{\langle m \rangle},$$

and $\langle m \rangle = \sum_i m_i/N = m_0$, the average money. From this we see that the majority of the money will be shared among a few agents whereas a large majority of agents will each hold a small amount of money.

SAVINGS AND TRANSACTIONS

If we impose that an agent has a savings criterion, λ, which is some percentage of an agent's total money that they wish to save, the final money of agent i can be described by.

$$m_i' = \lambda m_i + \epsilon(1-\lambda)(m_i + m_j),$$

and as a result of conservation of money:

$$m_j' = \lambda m_j + (1-\epsilon)(1-\lambda)(m_i + m_j).$$

These can be written as

$$m_i' = m_i + \delta m$$

and

$$m_j' = m_j - \delta m,$$

with

$$\delta m = (1-\lambda)(\epsilon m_j - (1-\epsilon)m_i).$$

Nearest Neighbors Interaction

The agents tend to have preferences for whom to interact with. This interaction of nearest neighbors can be described by a nonrandom probability given by:

$$p_{ij} \propto |m_i - m_j|^{-\alpha}, \tag{4}$$

for the interaction of agents i and j. It is thus most likely for agents to interact with those who are financially close to them.

Previous Transactions

In addition to an increased likelihood for transaction in the case of being financial neighbors, an increased likelihood of transaction comes about in the case of previous interactions occurred. This probability combined with the nearest neighbors likelihood can be written as:

$$p_{ij} \propto |m_i - m_j|^{-\alpha} \left(c_{ij} + 1\right)^{\gamma}, \tag{5}$$

where c_{ij} represents the number of previous interactions that have taken place between i and j. The factor 1 satisfies the condition that agents who have not been interacting with have the opportunity to interact with other agents in the future.

ALGORITHMS AND METHODS

Monte Carlo Methods

A Monte Carlo sampling technique can be applied to a physical system with a known probability distribution function (PDF). We can impose a sampling rule and generate random numbers that are tested against the sampling rule. Those that pass the sampling rule are counted for that corresponding point in the PDF. In our case, we calculated a PDF based on our simple model of financial transactions, and we generate a random number to test against the sampling rule

```
def transactions(agents,num,interact,lam, alpha,
    gamma):
    i = 0
    while i < num:
        #exchange quantifier and agent
            identification
        ep = epsilon()
        one = rand_agent()
        two = rand_agent()
        if one == two:
            two = rand_agent()
        #assign wealth values and the delM value
        Mone = agents[one]
        Mtwo = agents[two]
        Msum = Mone + Mtwo
        dM = np.round((1 - lam) * (ep * Mtwo - (1 -
            ep) * Mone),2)
        #determine if a transaction will occur
        nearestfriend = np.abs(Mone -
            Mtwo)**(-alpha) * (interact[one,two] +
            1)**gamma
        chance = np.random.uniform(0,1)
        if nearestfriend > 1:
            Moneprime = Mone + dM
            Mtwoprime= Mtwo - dM
            interact[one,two] += 1
            #update agent wealth
            agents[one] = Moneprime
            agents[two] = Mtwoprime
        elif chance <= nearestfriend:
            Moneprime = Mone + dM
            Mtwoprime= Mtwo - dM
            interact[one,two] +=1
            #update agent wealth
            agents[one] = Moneprime
            agents[two] = Mtwoprime
        else:
            pass
        i += 1
```

FIG. 1: Function that applies a Monte Carlo sampling technique to simulate which transactions occur in the Stock Market. Here we impose nearest neighbor and previous transaction considerations.

UNIT TESTS

In an effort to test the functionality of our code we perform unit tests. The first of these is done to ensure that equilibrium has been reached in the system, before we measure wealth distributions. To do this, the standard deviation of the wealth distribution is tested against a set tolerance. The tolerance is the highest value of standard deviation that we see fit for our sampling data. If the tolerance is not met, more sampling must be done to reach this tolerance.

For the next test it follows to test that money is actually conserved as money conservation[2] is a central theme for this simulation. We look for deviations in the final total money from the initial total money. These are indicative of an unexplainable loss or gain in money.

RESULTS AND DISCUSSIONS

Money Conservation in Transactions

Our simple model with only considerations of money conservation was implemented for a number of transactions much greater than those required to reach equilibrium. The resulting plot for 100000 transactions [fig2] shows the general exponential trend of the Gibbs distribution for wealth. This corresponds to a stock market where the majority of the agents individually hold small amounts of money. We can further identify the shape

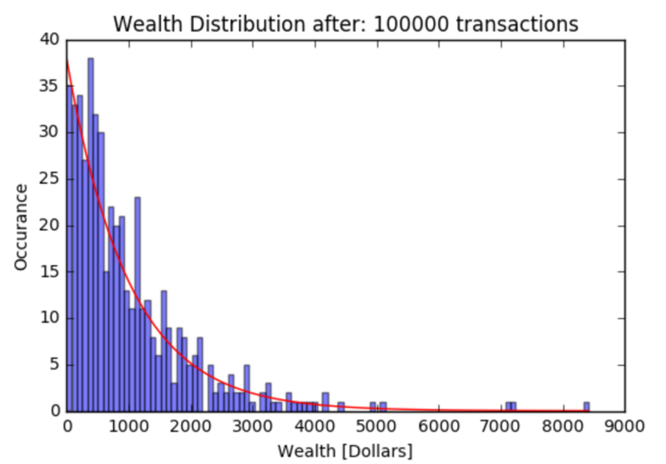

FIG. 2: The characteristic Gibbs distribution resulting from the simple money conservation model for 100000 transactions. The majority of agents hold the lowest amount of the wealth.

of this plot using the logarithm of the distribution vs wealth. The resulting plot will be linear with a slope corresponding to the inverse of the individual initial money held by each agent.

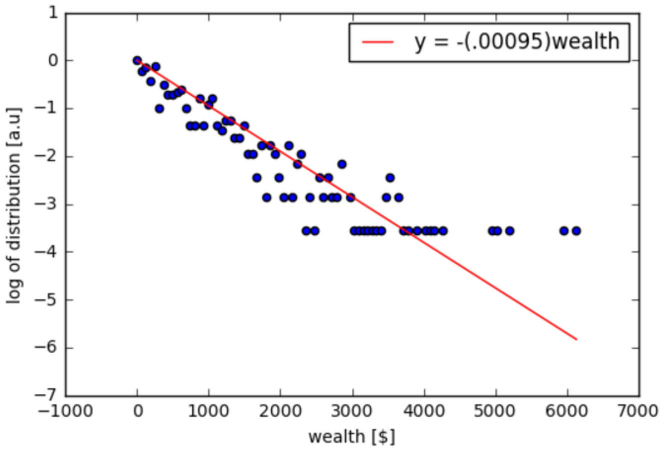

FIG. 3: $Log(Gibbs \cdot m_0)$ vs wealth for a starting individual wealth of \$1000. The slope displays the $1/m_0$ or approximately 1/1000 value we would expect. At a wealth of approximately \$3500 we see that our technique is no longer satisfactory to predict the behavior of the wealth distribution.

Savings and Transactions

We imposed a savings criterion on transactions. As the savings criterion increases, the Gibbs distribution becomes less prominent. As the criterion reaches 1, the distribution mirrors a normal distribution about the initial amount of wealth held by an individual. This reflects a stock market where agents save all of their money instead of going through with transactions.

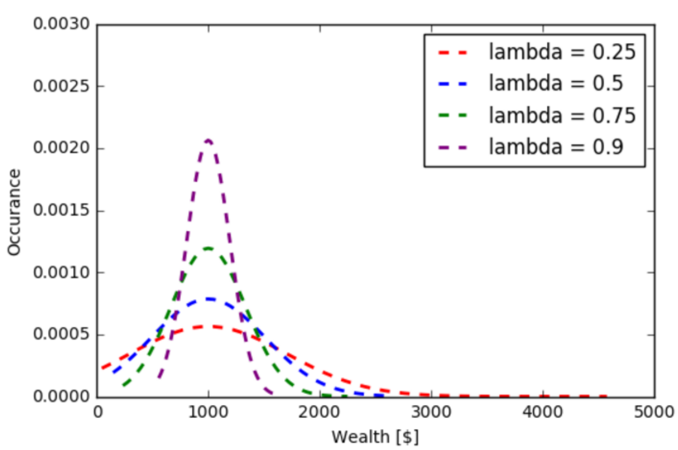

FIG. 4: Wealth distribution for various values of savings criterion, λ. At low savings criterion, the wealth is distributed with a relatively large standard deviation about the beginning wealth of each agent, but at high savings criterion, the standard deviation becomes relatively small. This is the result of a low number of agents making transactions.

Nearest Neighbors Interaction

The addition of agent preference in transactions is modeled for various values of α and for $N = 500$ and 1000. As α increases, we see that the agents becomes more distributed amongst amount of wealth. The high end of the wealth distribution follows the distribution in [eq 1]. This is obvious on a log-log plot as the slope is of the form $-(1+\alpha)$. So we would expect the slope should be decreasing as α increases. This is clear for 500 transactions, [fig5], however for 1000 transactions, [fig6], we see a deviation from this.

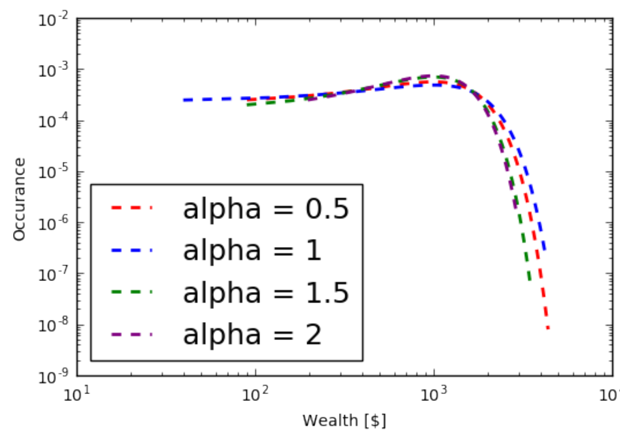

FIG. 5: Wealth distribution for various values of α, the strength of the nearest neighbors interaction, for 500 financial transactions. The savings criterion in this case is 0.5.

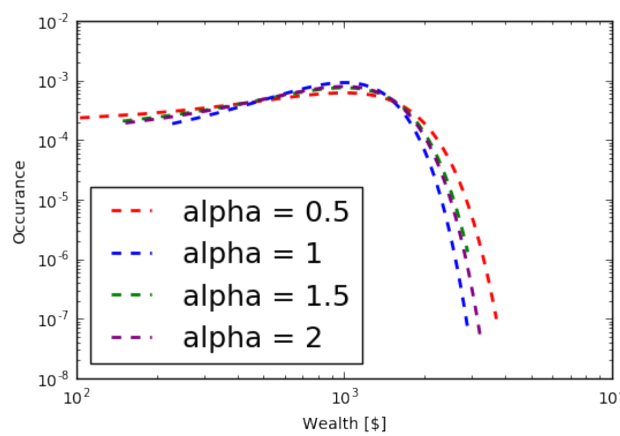

FIG. 6: Wealth distribution for various values of α, the strength of the nearest neighbors interaction, for 1000 financial transactions. The savings criterion in this case is 0.5.

Nearest Neighbors and Previous Transactions

We considered the previous transactions interaction at various values of gamma for 1000 financial transactions and for nearest neighbors interaction strength $\alpha = 1$[fig7] and $\alpha = 1$[fig8]. These plot gives us a different perspective on the nearest neighbors interaction. With $\alpha = 1$ we see that agents have a larger variety of wealth than for $\alpha = 2$. It is also obvious that below \$1000, the distribution of wealth is more uniform for $\alpha = 1$ than for $\alpha = 2$. The high end of the wealth distribution follows the distribution in [eq 1]. This is obvious on a log-log plot as the slope is of the form $-(1 + \alpha)$. So we would expect the slope in [fig7] to less than the slope in [fig8], as it is.

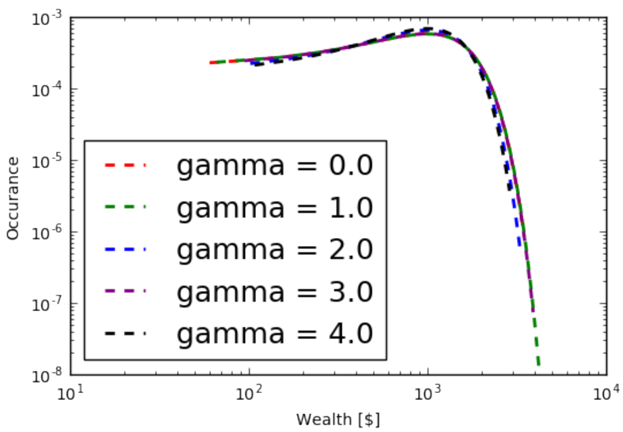

FIG. 7: Wealth distribution for various values of γ, the strength of the previous transactions interaction, for $\alpha = 1$ and 1000 financial transactions.

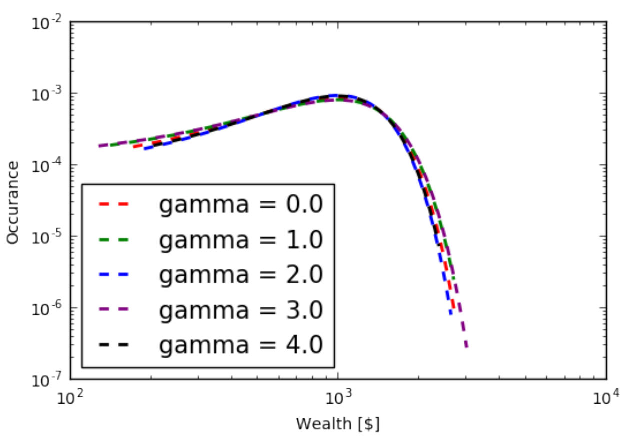

FIG. 8: Wealth distribution for various values of γ, the strength of the previous transactions interaction, for $\alpha = 2$ and 1000 financial transactions.

CONCLUSIONS

We analyzed the wealth distributions in a stock market for considerations of various market factors including money conservation, savings criterion and agent preference for transactions. We recovered a Gibbs distribution that describes the case of money conservation only. For nearest neighbors considerations, the tail end of the wealth distribution generally followed the predicted distribution in [eq1].

REFERENCES

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