ST221 Introduction to Statistics

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Probability

Suppose you toss two 6-faced fair dice. Let X be the random variable representing the sum of the faces of the two dice.

a) What are the values of A, B, and C in the table below, that gives the probability mass function of X?

x	2	3	4	5	6	7	8	9	10	11	12
$P(X = x)$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	A	B	C	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

b) Calculate $P(X > 10)$.

c) Compute $E[X]$.

d) Compute $Var(X)$.

Bayes Theorem

In a particular hepatology clinic, about 30% of patients have been diagnosed with liver disease. Of the patients that have been diagnosed with liver disease, 80% of them consume alcohol excessively, while 10% of the patients that have not been diagnosed with liver disease drink excessively. For a patient selected at random

a) what is the probability that they drink excessively?

b) what is the probability that they have liver disease, given they drink excessively?

Poisson Random Variables

In a call centre, the number of calls an attendant answers follows a Poisson distribution with a mean of 5 calls per hour. The company has 8 attendants.

a) What is the probability that an attendant answers only one call in one hour?

b) What is the probability that an attendant answers at least one call in one hour?

The Normal Distribution

The weight, in kg, of fish in a lake follows a normal distribution with mean 20 and standard deviation 2.

a) What is the probability that a randomly selected fish weighs less than 21kg?

b) What is the probability that a randomly selected fish weighs more than 21kg?

c) What is the probability that a randomly selected fish weighs between 19kg and 21kg?

d) What is the probability that a randomly selected fish weighs exactly 20kg?

Hypothesis Testing

To study the efficacy of a diet, five people's weights were recorded before they started the diet and six months later. These are the results:

Before	80	70	65	95	100		
After	72	69	67	81	88	$\bar{x}$	s
Difference	8	1	−2	14	12	6.6	6.91

Assuming that the populations are normally distributed, test the hypothesis that the diet is effective and promotes weight loss.

a) State the null and alternative hypotheses. Explain the notation you use.

b) Calculate the test statistic.

c) Compute the p-value.

d) State the conclusions.