Senior Fresh Computational Physics Laboratory Report

Laboratory 2: The Pendulum

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1 Introduction

Often in physics there are differential equations which are impossible / very difficult to solve analytically. A dynamic modelling problem which takes such a differential form is the motion of a simple pendulum. Without small angle approximation it is impossible find an analytical solution to the equations of motion of a simple pendulum. Two methods of resolving such a dynamic system is to approximate by trapezoid method or the Range-Kutta method.

The initial aim of these exercises is to plot the dynamic motion of a simple pendulum with linear, non-linear, damped non-linear, and damped-driven non-linear motion profiles. This involved varying the variables in the following equation of motion:

$$f(\theta, \omega, t) = \frac{-g}{l} \sin\theta - k\omega + \cos t\phi$$

Following from this; an additional aim is to compare the trapezoid method **(Fig 2.1)** and the Range-Kutta method of second order approximation **(Fig 2.2)**. The trapezoid method involves approximating the integration of a curve by finding the area of trapezoids which align roughly with the curve which in this case will provide a first order approximation of dynamic motion profile of the pendulum. The Range-Kutta method is similar to the trapezoid method but provides a second order approximation which better models a dynamic system.

2 Methodology

2.1 Exercise 1:

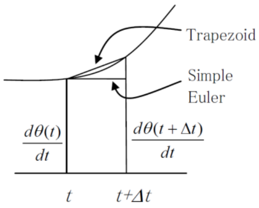

Figure 2.1: **The Trapezoid Method**

In exercise 1: the trapezoid method was used to model the dynamic motion of a simple linear pendulum (plot the pendulum angle and angular velocity) which is a simple pendulum with small angle approximation applied:

$$f(\theta) = \frac{-g}{l} theta$$

The trapezoid method gives a first order approximation of the curve through getting the area of a number of trapezoids which roughly align with it **(Fig 2.1)**.

2.2 Exercise 2:

In exercise 2: the the trapezoid method was used to model the dynamic motion of a simple non-linear pendulum which is a simple pendulum without an analytical solution to its motion which requires approximation to applied:

$$f(\theta) = \frac{-g}{l} \sin\theta$$

The non-linear pendulum is an area where such approximation is required as an analytical solution is not possible.

2.3 Exercise 3:

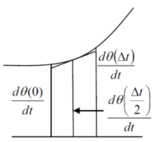

Figure 2.2: **The Range-Kutta Method**

In exercise 3: The pendulum angle per unit time was modelled for the simple non-linear pendulum using both the trapezoid method and the Range-Kutta method and they were compared on a single graph.

The Range-Kutta approximation is similar to the trapazoid method but provides a second order approxmation in contrast to the less accurate first order approximation of the trapezoid method. The differences between these approximations for an exagerated example were recorded.

2.4 Exercise 4:

In exercise 4: The dynamic motion was modeled for a simple damped non-linear pendulum using the superior Range-Kutta method. The damped non-linear pendulum obeys the following equation of motion:

$$f(\theta, \omega) = \frac{-g}{l}\sin\theta - k\omega$$

2.5 Exercise 5:

In exercise 5: The dynamic motion was modeled using phase portraits (Pendulum Angle / Angular Velocity at each instant) for a simple, driven, damped, non-linear pendulum which obeys the following equation of motion:

$$f(\theta, \omega, t) = \frac{-g}{l}\sin\theta - k\omega + \cos t\phi$$

Instead of a plot of the pendulum angle and angular velocity per unit time of each of these, a phase portrait of these quantities against each other was plotted whilst ignoring the transient motion which precedes the bulk of the dynamics. This was repeated for different amplitudes of the driving force while keeping all else constant.

3 Results

3.1 Exercise 1:

In exercise 1: The dynamic motion of the simple linear pendulum was modelled for differing initial pendulum angles and initial angular velocities.

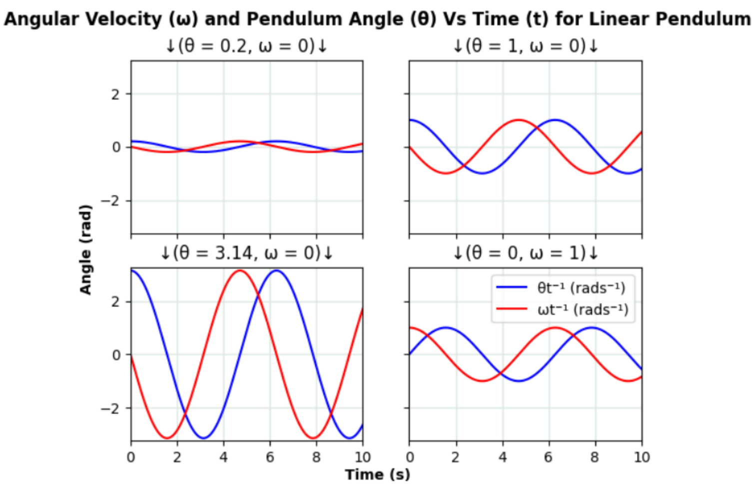

Figure 3.1: **Linear Motion Model**

3.2 Exercise 2:

In exercise 2: The dynamic motion of the simple non-linear pendulum was modelled for differing initial pendulum angles and initial angular velocities.

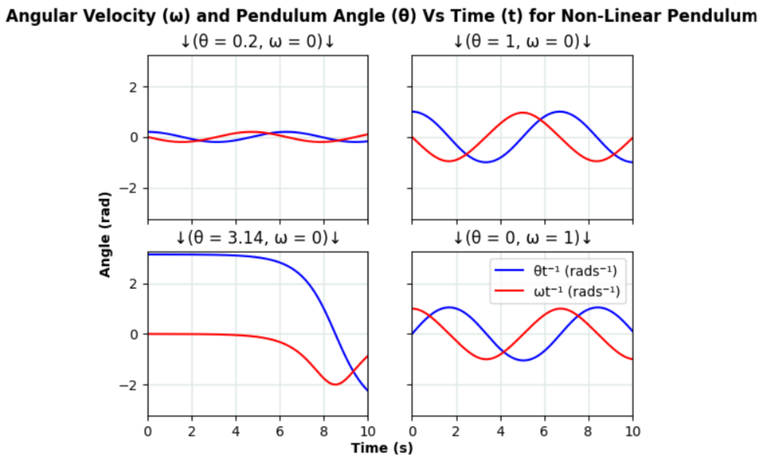

Figure 3.2: **Non-Linear Motion Model**

3.3 Linear Vs Non-Linear Pendulum:

The Linear and Non-Linear pendulum are similar in three of these examples but for the extreme example where the initial angle of the pendulum is 3.14 radians (almost vertical) The simple linear pendulum model provides a sinusoidal pattern while the non-linear pendulum model does not for the pendulum angle and angular velocity per unit time graphs which shows a clear difference between the two being that the linear pendulum does not deviate enough under extreme conditions.

3.4 Exercise 3:

In exercise 3: The pendulum angle per unit time was plotted using both the trapezoid method and the Range-Kutta for the exaggerated value of 3.14 radians. These values were plotted on the same graph in order to view the difference between the first order trapezoid method the second order Range-Kutta Method.

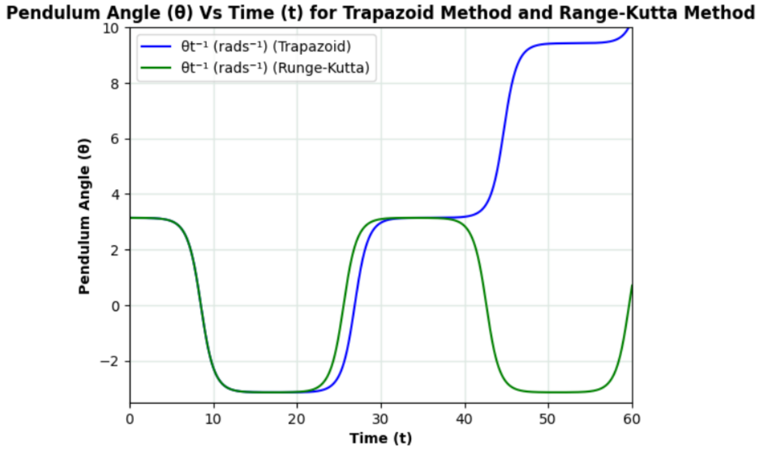

Figure 3.3: **Trapezoid Vs Range-Kutta Method**

The Trapezoid method exploded giving values which increased in an inclined sinusoidal pattern which goes against the conservation of energy while the Range Kutta method provided a more accurate demonstration of the motion under these starting conditions.

3.5 Exercise 4:

In exercise 4: The pendulum angle and angular velocity per unit time was plotted using the Range-Kutta method. These values were plotted on separate graphs in order to analyse the motion of simple damped non-linear pendulum. These results show that without a driving force the motion of the dampened pendulum quickly ceases.

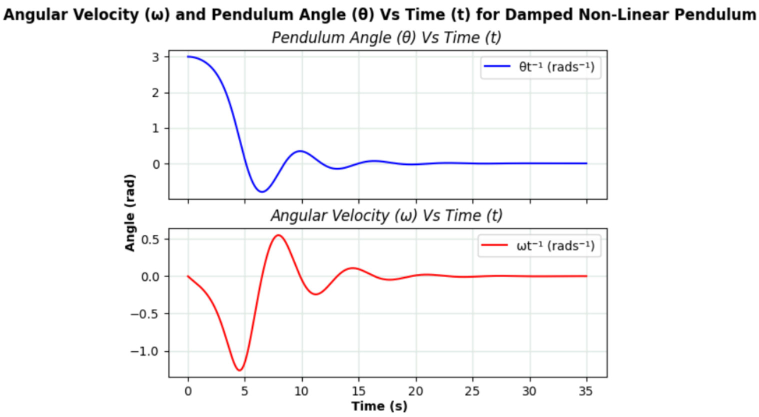

Figure 3.4: **Non-Linear Damped Motion Model**

3.6 Exercise 5:

In exercise 5: the phase portraits for different driving force amplitudes were plotted whilst ignoring the initial transient motion. This resulted in periodic and aperiodic motions. As the the driving for amplitudes approached 1.5 the motion became more and more chaotic contrasting the uniformly periodic motion in while the driving force amplitude was 0.9 in **Figure 3.5**

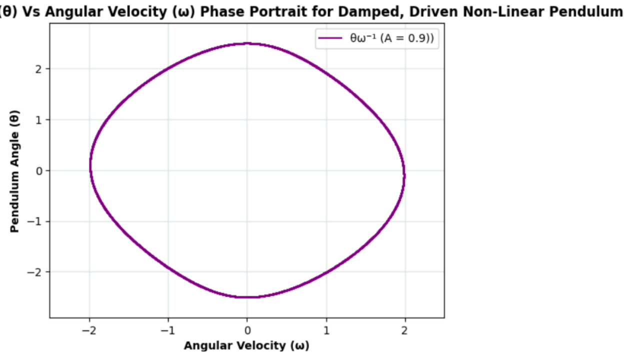

Figure 3.5: **Phase Portrait for Driving Force Amplitude of 0.9**

4 Conclusions

The aim of these exercises was to model the dynamic motion of various stages of complexity of the simple pendulum and compare the first order trapezoid method of approximation to that of the Range-Kutta method. The results above indicate that the accuracy of each method indicate the Range-Kutta method is more accurate than the trapezoid method which aligns with the understanding that first order approximations are less accurate than second order ones. Overall the goals of each exercise were met to sufficient levels of successs.

5 Additional Information

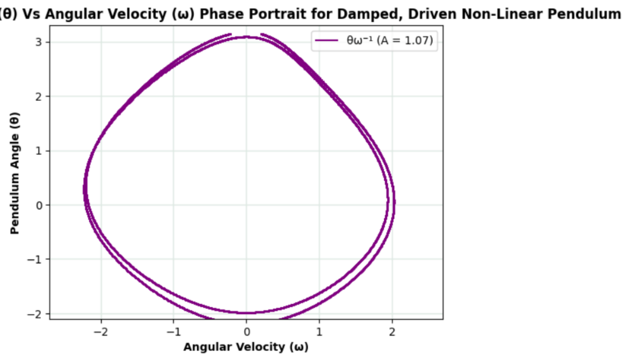

Figure 5.1: **Phase Portrait for Driving Force Amplitude of 1.07**

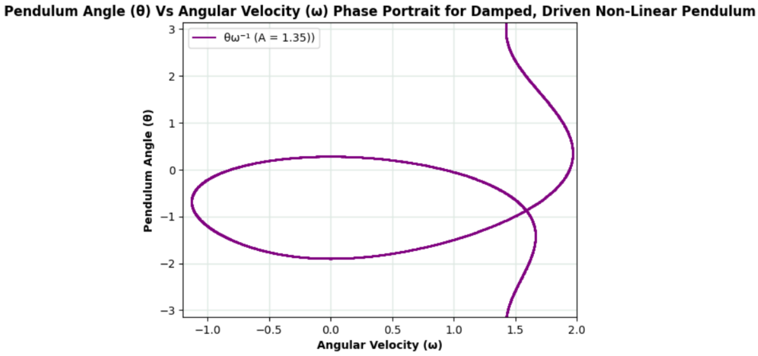

Figure 5.2: **Phase Portrait for Driving Force Amplitude of 1.35**

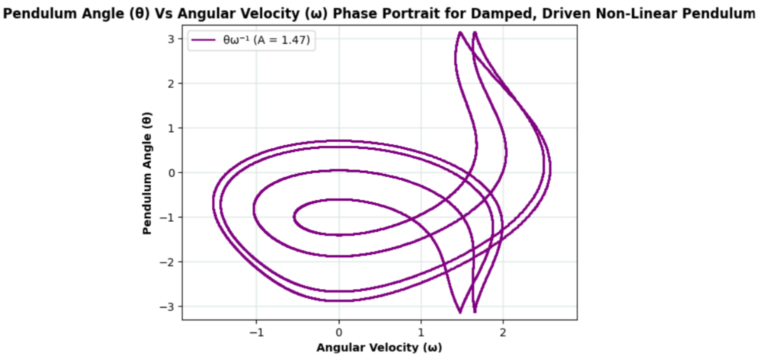

Figure 5.3: **Phase Portrait for Driving Force Amplitude of 1.47**

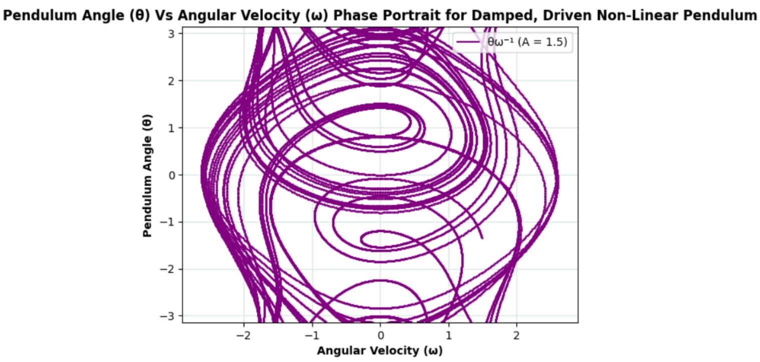

Figure 5.4: **Phase Portrait for Driving Force Amplitude of 1.5**