Non-Degenerate Perturbation Theory

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Notation

$$V_{00} = \langle \psi_0 | V | \psi_0 \rangle$$

$$E_{0\alpha} = E_0 - E_\alpha$$

$$\Sigma_{x_1 x_2 \cdots x_n} = \sum_{\alpha_1, \alpha_2, \cdots \alpha_n \neq 0} \frac{V_{0\alpha_1} V_{\alpha_1 \alpha_2} \cdots V_{\alpha_n 0}}{E_{0\alpha_1}^{x_1} E_{0\alpha_2}^{x_2} \cdots E_{0\alpha_n}^{x_n}}$$

Examples

$$\Sigma_1 = \sum_{\alpha \neq 0} \frac{\left| V_{0\alpha} \right|^2}{E_{0\alpha}}$$

$$\Sigma_{12} = \sum_{\alpha_1, \alpha_2 \neq 0} \frac{V_{0\alpha_1} V_{\alpha_1 \alpha_2} V_{\alpha_2 0}}{E_{0\alpha_1} E_{0\alpha_2}^2}$$

Energy Corrections

$$\begin{aligned}
\mathrm{Tr}[\Delta_1] &= V_{00} \\
\mathrm{Tr}[\Delta_2] &= \Sigma_1 \\
\mathrm{Tr}[\Delta_3] &= \Sigma_{1,1} \\
&\quad - V_{00}\Sigma_2
\end{aligned}$$