

Non-Degenerate Perturbation Theory

Sean Ericson

March 22, 2024

Notation

$$V_{00} = \langle \psi_0 | V | \psi_0 \rangle$$

$$E_{0\alpha} = E_0 - E_\alpha$$

$$\Sigma_{x_1 x_2 \cdots x_n} = \sum_{\alpha_1, \alpha_2, \cdots \alpha_n \neq 0} \frac{V_{0\alpha_1} V_{\alpha_1 \alpha_2} \cdots V_{\alpha_n 0}}{E_{0\alpha_1}^{x_1} E_{0\alpha_2}^{x_2} \cdots E_{0\alpha_n}^{x_n}}$$

Examples

$$\Sigma_1 = \sum_{\alpha \neq 0} \frac{|V_{0\alpha}|^2}{E_{0\alpha}}$$

$$\Sigma_{12} = \sum_{\alpha_1, \alpha_2 \neq 0} \frac{V_{0\alpha_1} V_{\alpha_1 \alpha_2} V_{\alpha_2 0}}{E_{0\alpha_1} E_{0\alpha_2}^2}$$

Energy Corrections

$$\text{Tr}[\Delta_1] = V_{00}$$

$$\text{Tr}[\Delta_2] = \Sigma_1$$

$$\text{Tr}[\Delta_3] = \Sigma_{1,1} - V_{00}\Sigma_2$$

$$\begin{aligned}\text{Tr}[\Delta_4] = & -\Sigma_1\Sigma_2 + \Sigma_{1,1,1} \\ & - V_{00}(\Sigma_{1,2} + \Sigma_{2,1}) \\ & + V_{00}^2\Sigma_3\end{aligned}$$

$$\begin{aligned}\text{Tr}[\Delta_5] = & \Sigma_1\Sigma_{1,2} + \Sigma_1\Sigma_{2,1} + \Sigma_{1,1}\Sigma_2 - \Sigma_{1,1,1,1} \\ & - V_{00}(-2\Sigma_1\Sigma_3 + \Sigma_{1,1,2} + \Sigma_{1,2,1} - \Sigma_2^2 + \Sigma_{2,1,1}) \\ & + V_{00}^2(\Sigma_{1,3} + \Sigma_{2,2} + \Sigma_{3,1}) \\ & - V_{00}^3\Sigma_4\end{aligned}$$

$$\begin{aligned}\text{Tr}[\Delta_6] = & -\Sigma_1^2\Sigma_3 + \Sigma_1\Sigma_{1,1,2} + \Sigma_1\Sigma_{1,2,1} - \Sigma_1\Sigma_2^2 + \Sigma_1\Sigma_{2,1,1} + \Sigma_{1,1}\Sigma_{1,2} + \Sigma_{1,1}\Sigma_{2,1} + \Sigma_{1,1,1}\Sigma_2 - \Sigma_{1,1,1,1,1} \\ & + V_{00}(2\Sigma_1\Sigma_{1,3} + 2\Sigma_1\Sigma_{2,2} + 2\Sigma_1\Sigma_{3,1} + 2\Sigma_{1,1}\Sigma_3 - \Sigma_{1,1,1,2} - \Sigma_{1,1,2,1} + 2\Sigma_{1,2}\Sigma_2 - \Sigma_{1,2,1,1} + 2\Sigma_2\Sigma_{2,1} - \Sigma_{2,1,1,1}) \\ & + V_{00}^2(-3\Sigma_1\Sigma_4 + \Sigma_{1,1,3} + \Sigma_{1,2,2} + \Sigma_{1,3,1} - 3\Sigma_2\Sigma_3 + \Sigma_{2,1,2} + \Sigma_{2,2,1} + \Sigma_{3,1,1}) \\ & - V_{00}^3(\Sigma_{1,4} + \Sigma_{2,3} + \Sigma_{3,2} + \Sigma_{4,1}) \\ & + V_{00}^4\Sigma_5\end{aligned}$$

$$\begin{aligned}
\text{Tr}[\Delta_7] = & -\Sigma_1^2\Sigma_{1,3} - \Sigma_1^2\Sigma_{2,2} - \Sigma_1^2\Sigma_{3,1} - 2\Sigma_1\Sigma_{1,1}\Sigma_3 + \Sigma_1\Sigma_{1,1,1,2} + \Sigma_1\Sigma_{1,1,2,1} - 2\Sigma_1\Sigma_{1,2}\Sigma_2 + \Sigma_1\Sigma_{1,2,1,1} - 2\Sigma_1\Sigma_2\Sigma_{2,1} \\
& + \Sigma_1\Sigma_{2,1,1,1} + \Sigma_{1,1}\Sigma_{1,1,2} + \Sigma_{1,1}\Sigma_{1,2,1} - \Sigma_{1,1}\Sigma_2^2 + \Sigma_{1,1}\Sigma_{2,1,1} + \Sigma_{1,1,1}\Sigma_{1,2} + \Sigma_{1,1,1}\Sigma_{2,1} + \Sigma_{1,1,1,1}\Sigma_2 - \Sigma_{1,1,1,1,1,1} \\
& + V_{00}(-3\Sigma_1^2\Sigma_4 + 2\Sigma_1\Sigma_{1,1,3} + 2\Sigma_1\Sigma_{1,2,2} + 2\Sigma_1\Sigma_{1,3,1} - 6\Sigma_1\Sigma_2\Sigma_3 + 2\Sigma_1\Sigma_{2,1,2} + 2\Sigma_1\Sigma_{2,2,1} + 2\Sigma_1\Sigma_{3,1,1} + 2\Sigma_{1,1}\Sigma_{1,3} \\
& \quad + 2\Sigma_{1,1}\Sigma_{2,2} + 2\Sigma_{1,1}\Sigma_{3,1} + 2\Sigma_{1,1,1}\Sigma_3 - \Sigma_{1,1,1,1,2} - \Sigma_{1,1,1,2,1} + 2\Sigma_{1,1,2}\Sigma_2 - \Sigma_{1,1,2,1,1} + \Sigma_{1,2}^2 + 2\Sigma_{1,2}\Sigma_{2,1} + 2\Sigma_{1,2,1}\Sigma_2 \\
& \quad - \Sigma_{1,2,1,1,1} - \Sigma_2^3 + 2\Sigma_2\Sigma_{2,1,1} + \Sigma_{2,1}^2 - \Sigma_{2,1,1,1,1}) \\
& + V_{00}^2(-3\Sigma_1\Sigma_{1,4} - 3\Sigma_1\Sigma_{2,3} - 3\Sigma_1\Sigma_{3,2} - 3\Sigma_1\Sigma_{4,1} - 3\Sigma_{1,1}\Sigma_4 + \Sigma_{1,1,1,3} + \Sigma_{1,1,2,2} + \Sigma_{1,1,3,1} - 3\Sigma_{1,2}\Sigma_3 + \Sigma_{1,2,1,2} + \Sigma_{1,2,2,1} \\
& \quad - 3\Sigma_{1,3}\Sigma_2 + \Sigma_{1,3,1,1} - 3\Sigma_2\Sigma_{2,2} - 3\Sigma_2\Sigma_{3,1} - 3\Sigma_{2,1}\Sigma_3 + \Sigma_{2,1,1,2} + \Sigma_{2,1,2,1} + \Sigma_{2,2,1,1} + \Sigma_{3,1,1,1}) \\
& - V_{00}^3(-4\Sigma_1\Sigma_5 + \Sigma_{1,1,4} + \Sigma_{1,2,3} + \Sigma_{1,3,2} + \Sigma_{1,4,1} - 4\Sigma_2\Sigma_4 + \Sigma_{2,1,3} + \Sigma_{2,2,2} + \Sigma_{2,3,1} - 2\Sigma_3^2 + \Sigma_{3,1,2} + \Sigma_{3,2,1} + \Sigma_{4,1,1}) \\
& + V_{00}^4(\Sigma_{1,5} + \Sigma_{2,4} + \Sigma_{3,3} + \Sigma_{4,2} + \Sigma_{5,1}) \\
& - V_{00}^5\Sigma_6
\end{aligned}$$

$$\begin{aligned}
\text{Tr}[\Delta_8] = & -\Sigma_1^3 \Sigma_4 + \Sigma_1^2 \Sigma_{1,1,3} + \Sigma_1^2 \Sigma_{1,2,2} + \Sigma_1^2 \Sigma_{1,3,1} - 3\Sigma_1^2 \Sigma_2 \Sigma_3 + \Sigma_1^2 \Sigma_{2,1,2} + \Sigma_1^2 \Sigma_{2,2,1} + \Sigma_1^2 \Sigma_{3,1,1} + 2\Sigma_1 \Sigma_{1,1} \Sigma_{1,3} + 2\Sigma_1 \Sigma_{1,1} \Sigma_{2,2} + 2\Sigma_1 \Sigma_{1,1} \Sigma_{3,1} \\
& + 2\Sigma_1 \Sigma_{1,1,1} \Sigma_3 - \Sigma_1 \Sigma_{1,1,1,1,2} - \Sigma_1 \Sigma_{1,1,1,2,1} + 2\Sigma_1 \Sigma_{1,1,2} \Sigma_2 - \Sigma_1 \Sigma_{1,1,2,1,1} + \Sigma_1 \Sigma_{1,2}^2 + 2\Sigma_1 \Sigma_{1,2} \Sigma_{2,1} + 2\Sigma_1 \Sigma_{1,2,1} \Sigma_2 - \Sigma_1 \Sigma_{1,2,1,1,1} \\
& - \Sigma_1 \Sigma_2^3 + 2\Sigma_1 \Sigma_2 \Sigma_{2,1,1} + \Sigma_1 \Sigma_{2,1}^2 - \Sigma_1 \Sigma_{2,1,1,1,1} + \Sigma_{1,1}^2 \Sigma_3 - \Sigma_{1,1} \Sigma_{1,1,1,2} - \Sigma_{1,1} \Sigma_{1,1,2,1} + 2\Sigma_{1,1} \Sigma_{1,2} \Sigma_2 - \Sigma_{1,1} \Sigma_{1,2,1,1} + 2\Sigma_{1,1} \Sigma_2 \Sigma_{2,1} \\
& - \Sigma_{1,1} \Sigma_{2,1,1,1} - \Sigma_{1,1,1} \Sigma_{1,1,2} - \Sigma_{1,1,1} \Sigma_{1,2,1} + \Sigma_{1,1,1} \Sigma_2^2 - \Sigma_{1,1,1} \Sigma_{2,1,1} - \Sigma_{1,1,1,1} \Sigma_{1,2} - \Sigma_{1,1,1,1} \Sigma_{2,1} - \Sigma_{1,1,1,1,1} \Sigma_2 + \Sigma_{1,1,1,1,1,1,1} \\
& + V_{00}(-3\Sigma_1^2 \Sigma_{1,4} - 3\Sigma_1^2 \Sigma_{2,3} - 3\Sigma_1^2 \Sigma_{3,2} - 3\Sigma_1^2 \Sigma_{4,1} - 6\Sigma_1 \Sigma_{1,1} \Sigma_4 + 2\Sigma_1 \Sigma_{1,1,1,3} + 2\Sigma_1 \Sigma_{1,1,2,2} + 2\Sigma_1 \Sigma_{1,1,3,1} - 6\Sigma_1 \Sigma_{1,2} \Sigma_3 \\
& + 2\Sigma_1 \Sigma_{1,2,1,2} + 2\Sigma_1 \Sigma_{1,2,2,1} - 6\Sigma_1 \Sigma_{1,3} \Sigma_2 + 2\Sigma_1 \Sigma_{1,3,1,1} - 6\Sigma_1 \Sigma_2 \Sigma_{2,2} - 6\Sigma_1 \Sigma_2 \Sigma_{3,1} - 6\Sigma_1 \Sigma_{2,1} \Sigma_3 + 2\Sigma_1 \Sigma_{2,1,1,2} + 2\Sigma_1 \Sigma_{2,1,2,1} \\
& + 2\Sigma_1 \Sigma_{2,2,1,1} + 2\Sigma_1 \Sigma_{3,1,1,1} + 2\Sigma_{1,1} \Sigma_{1,1,3} + 2\Sigma_{1,1} \Sigma_{1,2,2} + 2\Sigma_{1,1} \Sigma_{1,3,1} - 6\Sigma_{1,1} \Sigma_2 \Sigma_3 + 2\Sigma_{1,1} \Sigma_{2,1,2} + 2\Sigma_{1,1} \Sigma_{2,2,1} + 2\Sigma_{1,1} \Sigma_{3,1,1} \\
& + 2\Sigma_{1,1,1} \Sigma_{1,3} + 2\Sigma_{1,1,1} \Sigma_{2,2} + 2\Sigma_{1,1,1} \Sigma_{3,1} + 2\Sigma_{1,1,1,1} \Sigma_3 - \Sigma_{1,1,1,1,1,2} - \Sigma_{1,1,1,1,2,1} + 2\Sigma_{1,1,1,2} \Sigma_2 - \Sigma_{1,1,1,2,1,1} + 2\Sigma_{1,1,2} \Sigma_{1,2} \\
& + 2\Sigma_{1,1,2} \Sigma_{2,1} + 2\Sigma_{1,1,2,1} \Sigma_2 - \Sigma_{1,1,2,1,1,1} + 2\Sigma_{1,2} \Sigma_{1,2,1} - 3\Sigma_{1,2} \Sigma_2^2 + 2\Sigma_{1,2} \Sigma_{2,1,1} + 2\Sigma_{1,2,1} \Sigma_{2,1} + 2\Sigma_{1,2,1,1} \Sigma_2 - \Sigma_{1,2,1,1,1,1} \\
& - 3\Sigma_2^2 \Sigma_{2,1} + 2\Sigma_2 \Sigma_{2,1,1,1} + 2\Sigma_{2,1} \Sigma_{2,1,1} - \Sigma_{2,1,1,1,1,1}) \\
& - V_{00}^2(-6\Sigma_1^2 \Sigma_5 + 3\Sigma_1 \Sigma_{1,1,4} + 3\Sigma_1 \Sigma_{1,2,3} + 3\Sigma_1 \Sigma_{1,3,2} + 3\Sigma_1 \Sigma_{1,4,1} - 12\Sigma_1 \Sigma_2 \Sigma_4 + 3\Sigma_1 \Sigma_{2,1,3} + 3\Sigma_1 \Sigma_{2,2,2} + 3\Sigma_1 \Sigma_{2,3,1} - 6\Sigma_1 \Sigma_3^2 \\
& + 3\Sigma_1 \Sigma_{3,1,2} + 3\Sigma_1 \Sigma_{3,2,1} + 3\Sigma_1 \Sigma_{4,1,1} + 3\Sigma_{1,1} \Sigma_{1,4} + 3\Sigma_{1,1} \Sigma_{2,3} + 3\Sigma_{1,1} \Sigma_{3,2} + 3\Sigma_{1,1} \Sigma_{4,1} + 3\Sigma_{1,1,1} \Sigma_4 - \Sigma_{1,1,1,1,3} - \Sigma_{1,1,1,2,2} \\
& - \Sigma_{1,1,1,3,1} + 3\Sigma_{1,1,2} \Sigma_3 - \Sigma_{1,1,2,1,2} - \Sigma_{1,1,2,2,1} + 3\Sigma_{1,1,3} \Sigma_2 - \Sigma_{1,1,3,1,1} + 3\Sigma_{1,2} \Sigma_{1,3} + 3\Sigma_{1,2} \Sigma_{2,2} + 3\Sigma_{1,2} \Sigma_{3,1} + 3\Sigma_{1,2,1} \Sigma_3 \\
& - \Sigma_{1,2,1,1,2} - \Sigma_{1,2,1,2,1} + 3\Sigma_{1,2,2} \Sigma_2 - \Sigma_{1,2,2,1,1} + 3\Sigma_{1,3} \Sigma_{2,1} + 3\Sigma_{1,3,1} \Sigma_2 - \Sigma_{1,3,1,1,1} - 6\Sigma_2^2 \Sigma_3 + 3\Sigma_2 \Sigma_{2,1,2} + 3\Sigma_2 \Sigma_{2,2,1} \\
& + 3\Sigma_2 \Sigma_{3,1,1} + 3\Sigma_{2,1} \Sigma_{2,2} + 3\Sigma_{2,1} \Sigma_{3,1} + 3\Sigma_{2,1,1} \Sigma_3 - \Sigma_{2,1,1,1,2} - \Sigma_{2,1,1,2,1} - \Sigma_{2,1,2,1,1} - \Sigma_{2,2,1,1,1} - \Sigma_{3,1,1,1,1}) \\
& - V_{00}^3(-4\Sigma_1 \Sigma_{1,5} - 4\Sigma_1 \Sigma_{2,4} - 4\Sigma_1 \Sigma_{3,3} - 4\Sigma_1 \Sigma_{4,2} - 4\Sigma_1 \Sigma_{5,1} - 4\Sigma_{1,1} \Sigma_5 + \Sigma_{1,1,1,4} + \Sigma_{1,1,2,3} + \Sigma_{1,1,3,2} + \Sigma_{1,1,4,1} - 4\Sigma_{1,2} \Sigma_4 \\
& + \Sigma_{1,2,1,3} + \Sigma_{1,2,2,2} + \Sigma_{1,2,3,1} - 4\Sigma_{1,3} \Sigma_3 + \Sigma_{1,3,1,2} + \Sigma_{1,3,2,1} - 4\Sigma_{1,4} \Sigma_2 + \Sigma_{1,4,1,1} - 4\Sigma_2 \Sigma_{2,3} - 4\Sigma_2 \Sigma_{3,2} - 4\Sigma_2 \Sigma_{4,1} - 4\Sigma_{2,1} \Sigma_4 \\
& + \Sigma_{2,1,1,3} + \Sigma_{2,1,2,2} + \Sigma_{2,1,3,1} - 4\Sigma_{2,2} \Sigma_3 + \Sigma_{2,2,1,2} + \Sigma_{2,2,2,1} + \Sigma_{2,3,1,1} - 4\Sigma_3 \Sigma_{3,1} + \Sigma_{3,1,1,2} + \Sigma_{3,1,2,1} + \Sigma_{3,2,1,1} + \Sigma_{4,1,1,1}) \\
& + V_{00}^4(-5\Sigma_1 \Sigma_6 + \Sigma_{1,1,5} + \Sigma_{1,2,4} + \Sigma_{1,3,3} + \Sigma_{1,4,2} + \Sigma_{1,5,1} - 5\Sigma_2 \Sigma_5 + \Sigma_{2,1,4} + \Sigma_{2,2,3} + \Sigma_{2,3,2} + \Sigma_{2,4,1} - 5\Sigma_3 \Sigma_4 + \Sigma_{3,1,3} \\
& + \Sigma_{3,2,2} + \Sigma_{3,3,1} + \Sigma_{4,1,2} + \Sigma_{4,2,1} + \Sigma_{5,1,1}) \\
& - V_{00}^5(\Sigma_{1,6} + \Sigma_{2,5} + \Sigma_{3,4} + \Sigma_{4,3} + \Sigma_{5,2} + \Sigma_{6,1}) \\
& + V_{00}^6 \Sigma_7
\end{aligned}$$

$$\begin{aligned} \text{Tr}[\Delta_9] = & -\Sigma_1^3 \Sigma_{1,4} - \Sigma_1^3 \Sigma_{2,3} - \Sigma_1^3 \Sigma_{3,2} - \Sigma_1^3 \Sigma_{4,1} - 3\Sigma_1^2 \Sigma_{1,1} \Sigma_4 + \Sigma_1^2 \Sigma_{1,1,1,3} + \Sigma_1^2 \Sigma_{1,1,2,2} + \Sigma_1^2 \Sigma_{1,1,3,1} - 3\Sigma_1^2 \Sigma_{1,2} \Sigma_3 + \Sigma_1^2 \Sigma_{1,2,1,2} \\ & + \Sigma_1^2 \Sigma_{1,2,2,1} - 3\Sigma_1^2 \Sigma_{1,3} \Sigma_2 + \Sigma_1^2 \Sigma_{1,3,1,1} - 3\Sigma_1^2 \Sigma_{2,2} \Sigma_{2,2} - 3\Sigma_1^2 \Sigma_{2,3} \Sigma_{3,1} - 3\Sigma_1^2 \Sigma_{2,1} \Sigma_3 + \Sigma_1^2 \Sigma_{2,1,1,2} + \Sigma_1^2 \Sigma_{2,1,2,1} + \Sigma_1^2 \Sigma_{2,2,1,1} + \Sigma_1^2 \Sigma_{3,1,1,1} \\ & + 2\Sigma_1 \Sigma_{1,1} \Sigma_{1,1,3} + 2\Sigma_1 \Sigma_{1,1} \Sigma_{1,2,2} + 2\Sigma_1 \Sigma_{1,1} \Sigma_{1,3,1} - 6\Sigma_1 \Sigma_{1,1} \Sigma_2 \Sigma_3 + 2\Sigma_1 \Sigma_{1,1} \Sigma_{2,1,2} + 2\Sigma_1 \Sigma_{1,1} \Sigma_{2,2,1} + 2\Sigma_1 \Sigma_{1,1} \Sigma_{3,1,1} + 2\Sigma_1 \Sigma_{1,1,1} \Sigma_{1,3} \\ & + 2\Sigma_1 \Sigma_{1,1,1} \Sigma_{2,2} + 2\Sigma_1 \Sigma_{1,1,1} \Sigma_{3,1} + 2\Sigma_1 \Sigma_{1,1,1,1} \Sigma_3 - \Sigma_1 \Sigma_{1,1,1,1,1,2} - \Sigma_1 \Sigma_{1,1,1,1,2,1} + 2\Sigma_1 \Sigma_{1,1,1,2} \Sigma_2 - \Sigma_1 \Sigma_{1,1,1,2,1,1} + 2\Sigma_1 \Sigma_{1,1,2} \Sigma_{1,2} \\ & + 2\Sigma_1 \Sigma_{1,1,2} \Sigma_{2,1} + 2\Sigma_1 \Sigma_{1,1,2,1} \Sigma_2 - \Sigma_1 \Sigma_{1,1,2,1,1,1} + 2\Sigma_1 \Sigma_{1,2} \Sigma_{1,2,1} - 3\Sigma_1 \Sigma_{1,2} \Sigma_2^2 + 2\Sigma_1 \Sigma_{1,2} \Sigma_{2,1,1} + 2\Sigma_1 \Sigma_{1,2,1} \Sigma_{2,1} + 2\Sigma_1 \Sigma_{1,2,1,1} \Sigma_2 \\ & - \Sigma_1 \Sigma_{1,2,1,1,1,1} - 3\Sigma_1 \Sigma_2^2 \Sigma_{2,1} + 2\Sigma_1 \Sigma_2 \Sigma_{2,1,1,1} + 2\Sigma_1 \Sigma_{2,1} \Sigma_{2,1,1} - \Sigma_1 \Sigma_{2,1,1,1,1,1} + \Sigma_{1,1}^2 \Sigma_{1,3} + \Sigma_{1,1}^2 \Sigma_{2,2} + \Sigma_{1,1}^2 \Sigma_{3,1} + 2\Sigma_{1,1} \Sigma_{1,1,1} \Sigma_3 \\ & - \Sigma_{1,1} \Sigma_{1,1,1,1,2} - \Sigma_{1,1} \Sigma_{1,1,1,2,1} + 2\Sigma_{1,1} \Sigma_{1,1,2} \Sigma_2 - \Sigma_{1,1} \Sigma_{1,1,2,1,1} + \Sigma_{1,1} \Sigma_{1,2}^2 + 2\Sigma_{1,1} \Sigma_{1,2} \Sigma_{2,1} + 2\Sigma_{1,1} \Sigma_{1,2,1} \Sigma_2 - \Sigma_{1,1} \Sigma_{1,2,1,1,1} \\ & - \Sigma_{1,1} \Sigma_2^3 + 2\Sigma_{1,1} \Sigma_2 \Sigma_{2,1,1} + \Sigma_{1,1} \Sigma_{2,1}^2 - \Sigma_{1,1} \Sigma_{2,1,1,1,1} - \Sigma_{1,1,1} \Sigma_{1,1,1,2} - \Sigma_{1,1,1} \Sigma_{1,1,2,1} + 2\Sigma_{1,1,1} \Sigma_{1,2} \Sigma_2 - \Sigma_{1,1,1} \Sigma_{1,2,1,1} + 2\Sigma_{1,1,1} \Sigma_2 \Sigma_{2,1} \\ & - \Sigma_{1,1,1} \Sigma_{2,1,1,1} - \Sigma_{1,1,1,1} \Sigma_{1,1,2} - \Sigma_{1,1,1,1} \Sigma_{1,2,1} + \Sigma_{1,1,1,1} \Sigma_2^2 - \Sigma_{1,1,1,1} \Sigma_{2,1,1} - \Sigma_{1,1,1,1,1} \Sigma_{1,2} - \Sigma_{1,1,1,1,1} \Sigma_{2,1} - \Sigma_{1,1,1,1,1,1} \Sigma_2 + \Sigma_{1,1,1,1,1,1,1} \\ & + V_{00}(4\Sigma_1^3 \Sigma_5 - 3\Sigma_1^2 \Sigma_{1,1,4} - 3\Sigma_1^2 \Sigma_{1,2,3} - 3\Sigma_1^2 \Sigma_{1,3,2} - 3\Sigma_1^2 \Sigma_{1,4,1} + 12\Sigma_1^2 \Sigma_2 \Sigma_4 - 3\Sigma_1^2 \Sigma_{2,1,3} - 3\Sigma_1^2 \Sigma_{2,2,2} - 3\Sigma_1^2 \Sigma_{2,3,1} + 6\Sigma_1^2 \Sigma_3^2 - 3\Sigma_1^2 \Sigma_{3,1,2} \\ & - 3\Sigma_1^2 \Sigma_{3,2,1} - 3\Sigma_1^2 \Sigma_{4,1,1} - 6\Sigma_1 \Sigma_{1,1} \Sigma_{1,4} - 6\Sigma_1 \Sigma_{1,1} \Sigma_{2,3} - 6\Sigma_1 \Sigma_{1,1} \Sigma_{3,2} - 6\Sigma_1 \Sigma_{1,1} \Sigma_{4,1} - 6\Sigma_1 \Sigma_{1,1,1} \Sigma_4 + 2\Sigma_1 \Sigma_{1,1,1,1,3} + 2\Sigma_1 \Sigma_{1,1,1,2,2} \\ & + 2\Sigma_1 \Sigma_{1,1,1,3,1} - 6\Sigma_1 \Sigma_{1,1,2} \Sigma_3 + 2\Sigma_1 \Sigma_{1,1,2,1,2} + 2\Sigma_1 \Sigma_{1,1,2,2,1} - 6\Sigma_1 \Sigma_{1,1,3} \Sigma_2 + 2\Sigma_1 \Sigma_{1,1,3,1,1} - 6\Sigma_1 \Sigma_{1,2} \Sigma_{1,3} - 6\Sigma_1 \Sigma_{1,2} \Sigma_{2,2} \\ & - 6\Sigma_1 \Sigma_{1,2} \Sigma_{3,1} - 6\Sigma_1 \Sigma_{1,2,1} \Sigma_3 + 2\Sigma_1 \Sigma_{1,2,1,1,2} + 2\Sigma_1 \Sigma_{1,2,1,2,1} - 6\Sigma_1 \Sigma_{1,2,2} \Sigma_2 + 2\Sigma_1 \Sigma_{1,2,2,1,1} - 6\Sigma_1 \Sigma_{1,3} \Sigma_{2,1} - 6\Sigma_1 \Sigma_{1,3,1} \Sigma_2 \\ & + 2\Sigma_1 \Sigma_{1,3,1,1,1} + 12\Sigma_1 \Sigma_2^2 \Sigma_3 - 6\Sigma_1 \Sigma_2 \Sigma_{2,1,2} - 6\Sigma_1 \Sigma_2 \Sigma_{2,2,1} - 6\Sigma_1 \Sigma_2 \Sigma_{3,1,1} - 6\Sigma_1 \Sigma_{2,1} \Sigma_{2,2} - 6\Sigma_1 \Sigma_{2,1} \Sigma_{3,1} - 6\Sigma_1 \Sigma_{2,1,1} \Sigma_3 + 2\Sigma_1 \Sigma_{2,1,1,1,2} \\ & + 2\Sigma_1 \Sigma_{2,1,1,2,1} + 2\Sigma_1 \Sigma_{2,1,2,1,1} + 2\Sigma_1 \Sigma_{2,2,1,1,1} + 2\Sigma_1 \Sigma_{3,1,1,1,1} - 3\Sigma_{1,1}^2 \Sigma_4 + 2\Sigma_{1,1} \Sigma_{1,1,1,3} + 2\Sigma_{1,1} \Sigma_{1,1,2,2} + 2\Sigma_{1,1} \Sigma_{1,1,3,1} - 6\Sigma_{1,1} \Sigma_{1,2} \Sigma_3 \\ & + 2\Sigma_{1,1} \Sigma_{1,2,1,2} + 2\Sigma_{1,1} \Sigma_{1,2,2,1} - 6\Sigma_{1,1} \Sigma_{1,3} \Sigma_2 + 2\Sigma_{1,1} \Sigma_{1,3,1,1} - 6\Sigma_{1,1} \Sigma_2 \Sigma_{2,2} - 6\Sigma_{1,1} \Sigma_2 \Sigma_{3,1} - 6\Sigma_{1,1} \Sigma_{2,1} \Sigma_3 + 2\Sigma_{1,1} \Sigma_{2,1,1,2} \\ & + 2\Sigma_{1,1} \Sigma_{2,1,2,1} + 2\Sigma_{1,1} \Sigma_{2,2,1,1} + 2\Sigma_{1,1} \Sigma_{3,1,1,1} + 2\Sigma_{1,1,1} \Sigma_{1,1,3} + 2\Sigma_{1,1,1} \Sigma_{1,2,2} + 2\Sigma_{1,1,1} \Sigma_{1,3,1} - 6\Sigma_{1,1,1} \Sigma_2 \Sigma_3 + 2\Sigma_{1,1,1} \Sigma_{2,1,2} + 2\Sigma_{1,1,1} \Sigma_{2,2,1} \\ & + 2\Sigma_{1,1,1} \Sigma_{3,1,1} + 2\Sigma_{1,1,1,1} \Sigma_{1,3} + 2\Sigma_{1,1,1,1} \Sigma_{2,2} + 2\Sigma_{1,1,1,1} \Sigma_{3,1} + 2\Sigma_{1,1,1,1,1} \Sigma_3 - \Sigma_{1,1,1,1,1,1,2} - \Sigma_{1,1,1,1,1,2,1} + 2\Sigma_{1,1,1,1,2} \Sigma_2 - \Sigma_{1,1,1,1,2,1,1} \\ & + 2\Sigma_{1,1,1,2} \Sigma_{1,2} + 2\Sigma_{1,1,1,2} \Sigma_{2,1} + 2\Sigma_{1,1,1,2,1} \Sigma_2 - \Sigma_{1,1,1,2,1,1,1} + \Sigma_{1,1,2}^2 + 2\Sigma_{1,1,2} \Sigma_{1,2,1} - 3\Sigma_{1,1,2} \Sigma_2^2 + 2\Sigma_{1,1,2} \Sigma_{2,1,1} + 2\Sigma_{1,1,2,1} \Sigma_{1,2} \\ & + 2\Sigma_{1,1,2,1} \Sigma_{2,1} + 2\Sigma_{1,1,2,1,1} \Sigma_2 - \Sigma_{1,1,2,1,1,1,1} - 3\Sigma_{1,2}^2 \Sigma_2 + 2\Sigma_{1,2} \Sigma_{1,2,1,1} - 6\Sigma_{1,2} \Sigma_2 \Sigma_{2,1} + 2\Sigma_{1,2} \Sigma_{2,1,1,1} + \Sigma_{1,2,1}^2 - 3\Sigma_{1,2,1} \Sigma_2^2 + 2\Sigma_{1,2,1} \Sigma_{2,1,1} \\ & + 2\Sigma_{1,2,1,1} \Sigma_{2,1} + 2\Sigma_{1,2,1,1,1} \Sigma_2 - \Sigma_{1,2,1,1,1,1,1} + \Sigma_2^4 - 3\Sigma_2^2 \Sigma_{2,1,1} - 3\Sigma_2 \Sigma_{2,1}^2 + 2\Sigma_2 \Sigma_{2,1,1,1,1} + 2\Sigma_{2,1} \Sigma_{2,1,1,1} + \Sigma_{2,1,1}^2 - \Sigma_{2,1,1,1,1,1,1}) \\ & - V_{00}^2(-6\Sigma_1^2 \Sigma_{1,5} - 6\Sigma_1^2 \Sigma_{2,4} - 6\Sigma_1^2 \Sigma_{3,3} - 6\Sigma_1^2 \Sigma_{4,2} - 6\Sigma_1^2 \Sigma_{5,1} - 12\Sigma_1 \Sigma_{1,1} \Sigma_5 + 3\Sigma_1 \Sigma_{1,1,1,4} + 3\Sigma_1 \Sigma_{1,1,2,3} + 3\Sigma_1 \Sigma_{1,1,3,2} + 3\Sigma_1 \Sigma_{1,1,4,1} \\ & - 12\Sigma_1 \Sigma_{1,2} \Sigma_4 + 3\Sigma_1 \Sigma_{1,2,1,3} + 3\Sigma_1 \Sigma_{1,2,2,2} + 3\Sigma_1 \Sigma_{1,2,3,$$