Problem 2

In[11]:=
$$\{\hat{u}, \hat{v}, \hat{w}\} = \text{IdentityMatrix}[3];$$
$$\vec{R}_0 = -\hat{w};$$
$$\phi = \text{ArcTan}\left[\frac{\sqrt{\Omega^2 - \Omega_0{}^2}}{\Omega_0}\right];$$

In[14]:=
$$\left(\text{RotationMatrix}[-\phi, \hat{v}].\text{RotationMatrix}[\theta, \hat{u}].\text{RotationMatrix}[\phi, \hat{v}].\vec{R}_0 \text{ /. } \left\{\sqrt{\Omega^2 - \Omega_0{}^2} \to \delta\right\}\right) \text{ // FullSimplify // MatrixForm}$$

Out[14]//MatrixForm=
$$\begin{pmatrix} \frac{\delta\,(-1+\text{Cos}[\theta])\,\Omega_0}{\Omega^2} \\ \frac{\text{Sin}[\theta]\,\Omega_0}{\Omega} \\ -1 - \frac{(-1+\text{Cos}[\theta])\,\Omega_0^2}{\Omega^2} \end{pmatrix}$$

Problem 3

In[15]:=
$$\{\sigma_x, \sigma_y, \sigma_z\} = \text{Table}[\text{PauliMatrix}[i], \{i, 1, 3\}];$$
$$\{\sigma_+, \sigma_-\} = \frac{1}{2}(\sigma_x \mp i\,\sigma_y);\ \sigma_0 = \sigma_+.\sigma_-;$$
$$\text{rho} = \{\{\rho_{11}, \rho_{12}\}, \{\rho_{21}, \rho_{22}\}\};$$

In[18]:=
$$(-\gamma\,(\sigma_0.\text{rho} + \text{rho}.\sigma_0) + \gamma_2\,\sigma_-.\text{rho}.\sigma_+ + 2\,\Gamma\,\sigma_0.\text{rho}.\sigma_0 \text{ // FullSimplify}) \text{ /. } \{-\gamma + \Gamma \to -\gamma_2 / 2\} \text{ // MatrixForm}$$

Out[18]//MatrixForm=
$$\begin{pmatrix} \gamma_2\,\rho_{22} & -\gamma\,\rho_{12} \\ -\gamma\,\rho_{21} & -\gamma_2\,\rho_{22} \end{pmatrix}$$

Problem 4

In[19]:=
$$H_d = \frac{-\hbar}{2}\,\Omega_0\,\sigma_z + \hbar\,\dot{\theta}\,\sigma_y;$$

In[20]:=
$$\frac{1}{i\,\hbar}\,\text{comm}[H_d, \text{rho}] \text{ // FullSimplify // MatrixForm}$$

Out[20]//MatrixForm=
$$\begin{pmatrix} -\dot{\theta}\,(\rho_{12} + \rho_{21}) & \dot{\theta}\,(\rho_{11} - \rho_{22}) + i\,\rho_{12}\,\Omega_0 \\ \dot{\theta}\,(\rho_{11} - \rho_{22}) - i\,\rho_{21}\,\Omega_0 & \dot{\theta}\,(\rho_{12} + \rho_{21}) \end{pmatrix}$$