Problem Set #2

Phys 614

Due: Who the hell knows when

Problem 1

N non-interacting Bosons of mass m move in a spherically symmetric potential (in three dimensions)

$$U(\vec{r}) = \frac{1}{2} m\omega r^2 \qquad (r \equiv |\vec{r}|) \tag{1}$$

A) Calculate the Bose-Einstein condensation temerature T_c of this system.

B) Calculate the number of atoms in the ground state as a function of temperature $T < T_c$. (Note: this is a reasonable model for the famous Bose-Einstein condensation experiments of Cornell and Weinmann, who created the potential with a "magnetic trap")

Problem 2

Using the result of (1), plus your knowledge of the ground state wavefunction for a harmonic oscillator, calculate the density of atoms at the origin for $T < T_c$, and show that it is larger than the corresponding density for $T \gtrapprox T_c$ (e.g. $T = 2T_c$) by a factor of order $\sqrt{N}$, which, of course, is huge. Comment on the implications of this result for the applicability of the non-interacting approximation in this experiment.

Problem 3

Returning to Bose-Einstein condensation of particles in a box (rather than a harmonic potential), show that there is no Bose-Einstein condensation in $d = 2$ dimensions of space. (Hint: calculate $\rho(T, \mu)$ by replacing sums by integrals in the usual way, and show that ρ is unbounded (i.e. diverges) as $\mu \to 0^-$ from below.)