Exercise Set 1

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Exercise 1

First let's right out all the probabilites we'll need:

$$\begin{aligned}
P(sick) &= x \\
P(+|sick) &= 1 - x \\
P(-|sick) &= x \\
P(+|\neg sick) &= x \\
P(+) &= P(+|sick)P(sick) + P(+|\neg sick)P(\neg sick) = 2x(1-x) \\
x &= 0.001
\end{aligned}$$

Where the symbols + and - represent positive and negative test results (respectively), while *sick* and $\neg sick$ represent being sick and healthy (respectively). Now the posterior probability of actually having Bayes' syndrom, given that you have tested positive, is simply

$$P(sick|+) = \frac{P(+|sick)P(sick)}{P(+)} = \frac{x(1-x)}{2x(1-x)} = \frac{1}{2}.$$

In this case, the prior is the probability of any random person having Bayes' syndrome ($x = 0.001$), the likelihood is the probability of a person with Bayes' syndrome testing positive for it ($1 - x = 0.999$), and the renormalization factor is the probability of *any* test for Bayes' syndrome being positive.

Exercise 2

The relevent probabilites are

$$\begin{aligned}
P(sick) &= 0.5 \\
P(+|sick) &= 0.6 \\
P(-|sick) &= 0.4 \\
P(-|\neg sick) &= 1 \\
P(-) &= P(-|sick)P(sick) + P(-|\neg sick)P(\neg sick) = 0.5(0.4 + 1) = 0.7
\end{aligned}$$

The probability of being sick despite a negative test is therefore

$$P(sick|-) = \frac{P(-|sick)P(sick)}{P(-)} = \frac{0.4 * 0.5}{0.7} = \frac{2}{7}$$

In the case that the test's sensitivity is 90%, the likelihood and renormalization factor change to

$$\begin{aligned} P(-|sick) &= 0.1 \\ P(sick) &= 0.5(0.1+1) = .55 \end{aligned}$$

and the posterior probability becomes

$$P(sick|-) = \frac{0.1 * 0.5}{0.55} = \frac{1}{11}$$

Exercise 3

(a)

$$B = A + (B - A) \implies \frac{1}{B} B \frac{1}{A} = \frac{1}{B} A \frac{1}{A} + \frac{1}{B}(B - A)\frac{1}{A} \implies \frac{1}{A} = \frac{1}{B} + \frac{1}{B}(B - A)\frac{1}{A}$$

(b) Let $A = z - H_0 - \lambda V$, $B = z - H_0$. Then, $B - A = \lambda V$ and

$$\frac{1}{z - H_0 - \lambda V} = \frac{1}{z - H_0} + \frac{1}{z - H_0}(\lambda V)\frac{1}{z - H_0 - \lambda V}.$$

Because $G(z) = 1/A$ and $G_0(z) = 1/B$, this shows that

$$G(z) = G_0(z) + \lambda G_0(z) V G(z).$$