

Exercise Set 6

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Monday

Exercise 1

$$\begin{aligned}\int_{-\infty}^{\infty} dE \delta_t(E_f - E) \delta_t(E - E_i) &= \frac{1}{(2\pi\hbar)^2} \int_{-\infty}^{\infty} dE \int_{-t/2}^{t/2} dt_1 \int_{-t/2}^{t/2} dt_2 e^{i(E_f - E)t_1/\hbar} e^{i(E - E_f)t_2/\hbar} \\ &= \frac{1}{(2\pi\hbar)^2} \int_{-\infty}^{\infty} dE \int_{-t/2}^{t/2} dt_1 \int_{-t/2}^{t/2} dt_2 e^{iE(t_2 - t_1)/\hbar} e^{iE_f t_1/\hbar} e^{-iE_i t_2/\hbar} \\ &= \frac{1}{2\pi\hbar} \int_{-t/2}^{t/2} dt_1 \int_{-t/2}^{t/2} dt_2 \delta(t_1 - t_2) e^{iE_f t_1/\hbar} e^{-iE_i t_2/\hbar} \\ &= \frac{1}{2\pi\hbar} \int_{-t/2}^{t/2} dt' e^{i(E_f - E_i)t'/\hbar} \\ &= \delta_t(E_f - E_i)\end{aligned}$$

Tuesday

Exercise 1

We're just doing the substitutions $E_{\mathbf{f}} \rightarrow E_{\mathbf{f}} \pm \hbar\omega$

Exercise 2

$$\begin{aligned}\langle x | e^{\pm ikx} | p \rangle &= \int_{-\infty}^{\infty} dx' \langle x | e^{\pm ikx} | x' \rangle \langle x' | p \rangle \\ &= \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} dx' \delta(x - x') e^{\pm ikx} e^{ipx/\hbar} \\ &= \frac{1}{\sqrt{2\pi\hbar}} e^{i(p \pm \hbar k)x/\hbar} \\ &= \langle x | p \pm \hbar k \rangle\end{aligned}$$

Exercise 3

$$\begin{aligned}\lim_{E_{\mathfrak{F}} \rightarrow 0} e^{iE_{\mathfrak{F}}t/2\hbar} \delta_t(E_{\mathfrak{F}}) &= \lim_{E_{\mathfrak{F}} \rightarrow 0} e^{iE_{\mathfrak{F}}t/2\hbar} \frac{\sin(E_{\mathfrak{F}}t/2\hbar)}{\pi E_{\mathfrak{F}}} \\ &= \lim_{E_{\mathfrak{F}} \rightarrow 0} \frac{\sin(E_{\mathfrak{F}}t/2\hbar)}{\pi E_{\mathfrak{F}}} \\ &= \lim_{E_{\mathfrak{F}} \rightarrow 0} \frac{t}{2\pi\hbar} \cos(E_{\mathfrak{F}}t/2\hbar) \\ &= \frac{t}{2\pi\hbar}\end{aligned}$$