Homework 1

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Problem 1

(a) Using $Q = 1$, $R = 4 \times 10^7$m, $B = 10$ Tesla, we find

$$\begin{aligned} E &= Q\left(\frac{R}{\text{meter}}\right)\left(\frac{B}{\text{Tesla}}\right) 0.3 \text{ GeV} \\ &= (4 \times 10^7)(10)(0.3) \text{ GeV} \\ &= 1.2 \times 10^7 \text{ GeV} \end{aligned}$$

(b) Power loss due to synchrotron radiation is given by

$$P = \frac{0.3\gamma^4}{R/\text{meter}} \text{ eV/s}.$$

The time to complete one loop is given by $2\pi R$. Then,

$$E = \frac{0.3\gamma^4}{R/\text{meter}} 2\pi R \text{ eV} = 0.6\pi\left(\frac{E}{m_e}\right)^4 \text{ eV}$$

$$\implies E \approx 77 \text{ MeV}$$

(c) We have that

$$N_{\text{turns}} = \frac{\gamma\tau c}{2\pi R} = \frac{\tau c E}{2\pi R m_\mu}.$$

With $E = 10$TeV, and plugging in the mass and lifetime of the muon, this gives

$$N_{\text{turns}} \approx 2300$$

(d) The event rate is given by

$$\frac{\mathrm{d}N}{\mathrm{d}t} = \sigma\mathscr{L}.$$

Given a cross section of $\sigma = 100$ nb and an instantaneous luminosity of $10\text{nb}^{-1}\text{s}^{-1}$ (from google), this gives a total event rate of

$$\frac{\mathrm{d}N}{\mathrm{d}t} = 1000 \text{ Hz},$$

which seems rather low... If you can only record events at 100 Hz, then your trigger needs a suppression factor of 10.

(e) Using the power formula stated in part (b), with $E = 200$ GeV and $2\pi R = 2.7 \times 10^4$m, we find that the power per particle is

$$P = 4.1 \times 10^{-2} \text{ W}.$$

With 10^{12} particles in the ring, that's a total power of

$$P_{\text{total}} = 4.1 \times 10^10 \text{ W} = 41 \text{ gigawatts}$$

Problem 2

(a)

(b) The Feynman diagram for the process is shown in Figure 1.

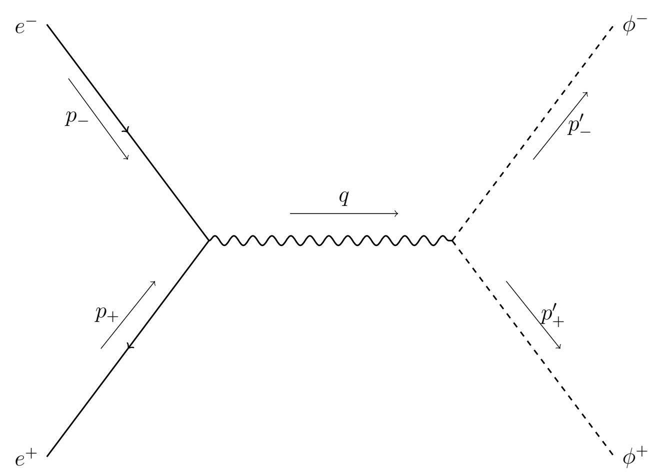

Figure 1: The Feynman diagram for $e^+e^- \to \phi^+\phi^-$

2a)

$d\cos\theta = \sin\theta\, d\theta$

b) $\langle \phi^- \phi^+ | j^\mu(x) | 0 \rangle = (p_- - p_+)^\mu e^{i(p_- + p_+)\cdot x}$; $\langle 0 | j^\mu(x) | e^-_R e^+_L \rangle = 2\sqrt{2}\, E\, \varepsilon_+^\mu$; $\langle 0 | j^\mu(x) | e^-_L e^+_R \rangle = -2\sqrt{2} E \varepsilon_-^\mu$; $\varepsilon_\pm^\mu = (0, 1, \pm i, 0)$

$$M_{RL} = \frac{2\sqrt{2}E}{q^2} e^{i(p_+' + p_-')\cdot x} e^2 (p_-' - p_+')^\mu (\varepsilon_+)_\mu \; ; \; M_{LR} = -\frac{2\sqrt{2}E}{q^2} e^{+i(p_+' + p_-')\cdot x} e^2 (p_-' - p_+')^\mu (\varepsilon_-)_\mu$$

$p_- = E(1,0,0,1)$ $p_+ = E(1,0,0,-1)$ $p_-' = E(1, s_\theta, 0, c_\theta)$ $p_+' = E(1, -s_\theta, 0, -c_\theta)$; $(p_-' - p_+') = 2E(0, \sin\theta, 0, \cos\theta)$; $(p_+' + p_-') = 2E(1,0,0,0)$

$$M_{RL} = 4\sqrt{2}\, e^{i2Et} e^2 \sin\theta \; ; \; M_{LR} = +4\sqrt{2}\, e^2 e^{2iEt} \sin\theta$$

$$\sigma = \frac{1}{2E_{cm}^2} \int d\Pi_2 |M|^2 = \frac{1}{2E_{cm}^2} \frac{1}{8\pi} \int \frac{d\cos\theta}{2} |M|^2 =$$

Spin-avg $\sigma = \frac{1}{4} \cdot \frac{2 \cdot 16 \cdot 2 \cdot e^4}{2E_{cm}^2\, 8\pi} \cdot \int \frac{1}{2} \sin^2\theta\, d\cos\theta = \frac{64}{E_{cm}^2\, 4 \cdot 16\pi} \int_0^\pi \sin^3\theta\, d\theta = \frac{64 \cdot 3 e^4}{16\pi} = \frac{3 \cdot 16}{\pi E_{cm}^2} e^4 =$

$$\frac{d\sigma}{d\cos\theta} = \frac{64}{4 \cdot 16} (1 - \cos^2\theta)$$