

```

In[180]:=
$Assumptions = { $\theta \in \mathbb{R}$ ,  $r > 0$ ,  $\phi \in \mathbb{R}$ ,  $\tau \in \mathbb{R}$ };
n = 2;
coord = { $\theta$ ,  $\phi$ };
vel = {Dt[ $\theta$ ,  $\tau$ ], Dt[ $\phi$ ,  $\tau$ ]};

In[184]:=
affine :=
  affine = FullSimplify[Table[ $\frac{1}{2} * \text{Sum}[(\text{inversemetric}[[i, s]] * (D[\text{metric}[[s, j]], \text{coord}[[k]] +$ 
    D[metric[[s, k], coord[[j]]] - D[metric[[j, k], coord[[s]]]),
    {s, 1, n}], {i, 1, n}, {j, 1, n}, {k, 1, n}]]];

In[185]:=
riemann :=
  riemann = Simplify[Table[D[affine[[i, j, l]], coord[[k]] - D[affine[[i, j, k]], coord[[l]] +
    Sum[affine[[s, j, l]]  $\times$  affine[[i, k, s]] - affine[[s, j, k]]  $\times$  affine[[i, l, s]], {s, 1, n}],
    {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}]]];

In[186]:=
ricci :=
  ricci = Simplify[Table[Sum[riemann[[i, j, i, l]], {i, 1, n}], {j, 1, n}, {l, 1, n}]]];

In[187]:=
Rscalar = Simplify[Sum[inversemetric[[i, j]]  $\times$  ricci[[i, j]], {i, 1, n}, {j, 1, n}]]];

In[188]:=
listaffine :=
  Table[If[UnsameQ[affine[[i, j, k]], 0], {ToString[R[coord[[i]], coord[[j]], coord[[k]]],
    affine[[i, j, k]]}], {i, 1, n}, {j, 1, n}, {k, 1, j}];

In[189]:=
listriemann := Table[If[UnsameQ[riemann[[i, j, k, l]], 0],
  {ToString[R[coord[[i]], coord[[j]], coord[[k]], coord[[l]]], riemann[[i, j, k, l]]},
  {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, k-1}];

In[190]:=
listricci := Table[If[UnsameQ[ricci[[j, l]], 0],
  {ToString[R[coord[[j]], coord[[l]]], ricci[[j, l]]}, {j, 1, n}, {l, 1, j}];

In[191]:=
metric = DiagonalMatrix[{ $r^2$ ,  $r^2 \sin[\theta]^2$ ]}];
inversemetric = Simplify[Inverse[metric]];

In[193]:=
metric // MatrixForm
inversemetric // MatrixForm

Out[193]//MatrixForm=

$$\begin{pmatrix} r^2 & 0 \\ 0 & r^2 \sin[\theta]^2 \end{pmatrix}$$


Out[194]//MatrixForm=

$$\begin{pmatrix} \frac{1}{r^2} & 0 \\ 0 & \frac{\csc[\theta]^2}{r^2} \end{pmatrix}$$


```

In[195]:=

```
TableForm[Partition[DeleteCases[Flatten[listaffine], Null], 2], TableSpacing -> {2, 2}]
```

Out[195]//TableForm=

```
 $\Gamma[\theta, \phi, \phi] \quad -\cos[\theta] \sin[\theta]$   

 $\Gamma[\phi, \phi, \theta] \quad \cot[\theta]$ 
```

In[196]:=

```
TableForm[Partition[DeleteCases[Flatten[listriemann], Null], 2], TableSpacing -> {2, 2}]
```

Out[196]//TableForm=

```
 $R[\theta, \phi, \phi, \theta] \quad -\sin[\theta]^2$   

 $R[\phi, \theta, \phi, \theta] \quad 1$ 
```

In[197]:=

```
TableForm[Partition[DeleteCases[Flatten[listricci], Null], 2], TableSpacing -> {2, 2}]
```

Out[197]//TableForm=

```
 $R[\theta, \theta] \quad 1$   

 $R[\phi, \phi] \quad \sin[\theta]^2$ 
```

In[198]:=

```
Rscalar
```

Out[198]=

$$\frac{2}{r^2}$$

In[199]:=

```
Integrate[ $\sqrt{\text{Det}[\text{metric}]}$  Rscalar, {\theta, 0, \pi}, {\phi, 0, 2\pi}]
```

Out[199]=

$$8\pi$$