Quantum Hadrodynamics

From Fundamental Fields to Nuclear Phenomena

Sean Ericson

UO
PHYS 663

June 13, 2024

A Brief History of Nuclear Physics

From Plumb Pudding to Nuclear Apocalypse

A Brief History of Nuclear Physics

From Plumb Pudding to Nuclear Apocalypse

- 1904: J.J. Thompson proposes plumb pudding model of atoms after discovering electrons in 1897.

A Brief History of Nuclear Physics

From Plumb Pudding to Nuclear Apocalypse

- 1904: J.J. Thompson proposes plumb pudding model of atoms after discovering electrons in 1897.
- 1911-1917: Rutherford discovers nucleus, proton.

A Brief History of Nuclear Physics

From Plumb Pudding to Nuclear Apocalypse

- 1904: J.J. Thompson proposes plumb pudding model of atoms after discovering electrons in 1897.
- 1911-1917: Rutherford discovers nucleus, proton.
- 1935: Chadwick discovers neutron.

A Brief History of Nuclear Physics

From Plumb Pudding to Nuclear Apocalypse

- 1904: J.J. Thompson proposes plumb pudding model of atoms after discovering electrons in 1897.
- 1911-1917: Rutherford discovers nucleus, proton.
- 1935: Chadwick discovers neutron.
- 1942: Fermi, first successful reactor (CP-1).

A Brief History of Nuclear Physics

From Plumb Pudding to Nuclear Apocalypse

- 1904: J.J. Thompson proposes plumb pudding model of atoms after discovering electrons in 1897.
- 1911-1917: Rutherford discovers nucleus, proton.
- 1935: Chadwick discovers neutron.
- 1942: Fermi, first successful reactor (CP-1).
- 1945: Trinity test

A Brief History of Nuclear Physics

From Plumb Pudding to Nuclear Apocalypse

- 1904: J.J. Thompson proposes plumb pudding model of atoms after discovering electrons in 1897.
- 1911-1917: Rutherford discovers nucleus, proton.
- 1935: Chadwick discovers neutron.
- 1942: Fermi, first successful reactor (CP-1).
- 1945: Trinity test
- 2023: BARBENHEIMER

A Brief Intro to Nuclear Physics

Iso-what now??

A Brief Intro to Nuclear Physics

Iso-what now??

- Nucleons

A Brief Intro to Nuclear Physics

Iso-what now??

- Nucleons
  - $m_p \approx 938.272$ MeV

A Brief Intro to Nuclear Physics

Iso-what now??

- Nucleons
  - $m_p \approx 938.272$ MeV
  - $m_n \approx 939.565$ MeV

A Brief Intro to Nuclear Physics

Iso-what now??

- Nucleons
  - $m_p \approx 938.272$ MeV
  - $m_n \approx 939.565$ MeV
  - $\Delta m/m_p \approx 0.14\%$

A Brief Intro to Nuclear Physics

Iso-what now??

- Nucleons
  - $m_p \approx 938.272$ MeV
  - $m_n \approx 939.565$ MeV
  - $\Delta m/m_p \approx 0.14\%$
- Nuclei

A Brief Intro to Nuclear Physics

Iso-what now??

- Nucleons
  - $m_p \approx 938.272$ MeV
  - $m_n \approx 939.565$ MeV
  - $\Delta m/m_p \approx 0.14\%$
- Nuclei
  - Z: # of protons ($Z_1 = Z_2 \implies$ "isotopes")

A Brief Intro to Nuclear Physics

Iso-what now??

- Nucleons
  - $m_p \approx 938.272$ MeV
  - $m_n \approx 939.565$ MeV
  - $\Delta m / m_p \approx 0.14\%$
- Nuclei
  - Z: # of protons ($Z_1 = Z_2 \implies$ "isotopes")
  - N: # of neutrons ($N_1 = N_2 \implies$ "isotones")

A Brief Intro to Nuclear Physics

Iso-what now??

- Nucleons
  - $m_p \approx 938.272$ MeV
  - $m_n \approx 939.565$ MeV
  - $\Delta m / m_p \approx 0.14\%$
- Nuclei
  - Z: # of protons ($Z_1 = Z_2 \implies$ "isotopes")
  - N: # of neutrons ($N_1 = N_2 \implies$ "isotones")
  - $A = Z + N$: Total # of nucleons ($A_1 = A_2 \implies$ "isobars")

A Brief Intro to Nuclear Physics

Iso-what now??

- Nucleons
  - $m_p \approx 938.272$ MeV
  - $m_n \approx 939.565$ MeV
  - $\Delta m/m_p \approx 0.14\%$
- Nuclei
  - Z: # of protons ($Z_1 = Z_2 \implies$ "isotopes")
  - N: # of neutrons ($N_1 = N_2 \implies$ "isotones")
  - $A = Z + N$: Total # of nucleons ($A_1 = A_2 \implies$ "isobars")
- Iso(baric)spin

A Brief Intro to Nuclear Physics

Iso-what now??

- Nucleons
  - $m_p \approx 938.272$ MeV
  - $m_n \approx 939.565$ MeV
  - $\Delta m/m_p \approx 0.14\%$
- Nuclei
  - Z: # of protons ($Z_1 = Z_2 \implies$ "isotopes")
  - N: # of neutrons ($N_1 = N_2 \implies$ "isotones")
  - $A = Z + N$: Total # of nucleons ($A_1 = A_2 \implies$ "isobars")
- Iso(baric)spin
  - p, n are just two different "isospin charge" states of the nucleon

A Brief Intro to Nuclear Physics

Iso-what now??

- Nucleons
  - $m_p \approx 938.272$ MeV
  - $m_n \approx 939.565$ MeV
  - $\Delta m/m_p \approx 0.14\%$
- Nuclei
  - Z: # of protons ($Z_1 = Z_2 \implies$ "isotopes")
  - N: # of neutrons ($N_1 = N_2 \implies$ "isotones")
  - $A = Z + N$: Total # of nucleons ($A_1 = A_2 \implies$ "isobars")
- Iso(baric)spin
  - p, n are just two different "isospin charge" states of the nucleon
  - $p = \left|\frac{1}{2}, \frac{1}{2}\right\rangle$, $n = \left|\frac{1}{2}, -\frac{1}{2}\right\rangle$; $\pi^{\pm} = |1, \pm 1\rangle$, $\pi^0 = |1, 0\rangle$

^{2}D: The Simplest Nucleus

Well, besides ^{1}H

^2D: The Simplest Nucleus

Well, besides ^1H

- Properties of the deuteron

^2D: The Simplest Nucleus

Well, besides ^1H

- Properties of the deuteron
  - Mean-square radius 2.1 fm (0.8 fm for p)

^2D: The Simplest Nucleus

Well, besides ^1H

- Properties of the deuteron
  - Mean-square radius 2.1 fm (0.8 fm for p)
  - Binding energy $E_B \approx 2.25$ MeV

^2D: The Simplest Nucleus

Well, besides ^1H

- Properties of the deuteron
  - Mean-square radius 2.1 fm (0.8 fm for p)
  - Binding energy $E_B \approx 2.25$ MeV
- Square Well Approximation

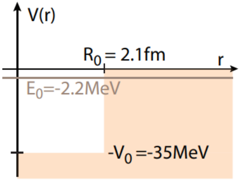
^2D: The Simplest Nucleus

Well, besides ^1H

- Properties of the deuteron
  - Mean-square radius 2.1 fm (0.8 fm for p)
  - Binding energy $E_B \approx 2.25$ MeV
- Square Well Approximation
  - Reduce p-n two-body problem to the equivalent one-body problem.

^2D: The Simplest Nucleus

Well, besides ^1H

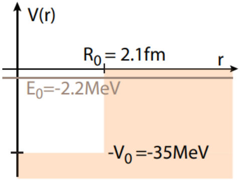


- Properties of the deuteron
  - Mean-square radius 2.1 fm (0.8 fm for p)
  - Binding energy $E_B \approx 2.25$ MeV
- Square Well Approximation
  - Reduce p-n two-body problem to the equivalent one-body problem.
  - Assume a 3D square well potential of depth V_0

^2D: The Simplest Nucleus

Well, besides ^1H

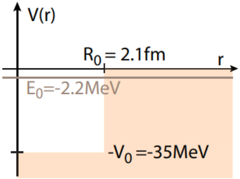
- Properties of the deuteron
  - Mean-square radius 2.1 fm (0.8 fm for p)
  - Binding energy $E_B \approx 2.25$ MeV



- Square Well Approximation
  - Reduce p-n two-body problem to the equivalent one-body problem.
  - Assume a 3D square well potential of depth V_0
  - $R_0 = 2.1$ fm, $E_0 = -2.2$ MeV $\implies$ $V_0 = -35$ MeV

^2D: The Simplest Nucleus

Well, besides ^1H



- Properties of the deuteron
  - Mean-square radius 2.1 fm (0.8 fm for p)
  - Binding energy $E_B \approx 2.25$ MeV
- Square Well Approximation
  - Reduce p-n two-body problem to the equivalent one-body problem.
  - Assume a 3D square well potential of depth V_0
  - $R_0 = 2.1$ fm, $E_0 = -2.2$ MeV $\implies$ $V_0 = -35$ MeV
  - No bound excited states! (experimentally confirmed)

Other Dinucleoic Nuclei?

pp and *nn*

Other Dinucleoic Nuclei?

pp and *nn*

- The general form of the wavefunction of the nucleus is

$$\Psi = \psi_{\text{space}} \chi_s^{m_s} \phi_t^{m_t}$$

Other Dinucleoic Nuclei?

pp and *nn*

- The general form of the wavefunction of the nucleus is

$$\Psi = \psi_{\text{space}} \chi_s^{m_s} \phi_t^{m_t}$$

- Deuteron composed of two distinguishable fermions $\Longrightarrow$ antisymmetric wavefunction [1].

Other Dinucleoic Nuclei?

pp and *nn*

- The general form of the wavefunction of the nucleus is

$$\Psi = \psi_{\text{space}} \chi_s^{m_s} \phi_t^{m_t}$$

- Deuteron composed of two distinguishable fermions $\implies$ antisymmetric wavefunction [1].
- Experimentally, $J = S + L = 1$. $L > 0$ is incompatible with a bound state, so $S = 1$.

Other Dinucleoic Nuclei?

pp and *nn*

- The general form of the wavefunction of the nucleus is

$$\Psi = \psi_{\text{space}} \chi_s^{m_s} \phi_t^{m_t}$$

- Deuteron composed of two distinguishable fermions $\implies$ antisymmetric wavefunction [1].
- Experimentally, $J = S + L = 1$. $L > 0$ is incompatible with a bound state, so $S = 1$.
- Then, ψ_{space}, $\chi_s^{m_s}$ symmetric $\implies$ $\phi_t^{m_t} = |0, 0\rangle$ (antisymm)

Other Dinucleoic Nuclei?

pp and *nn*

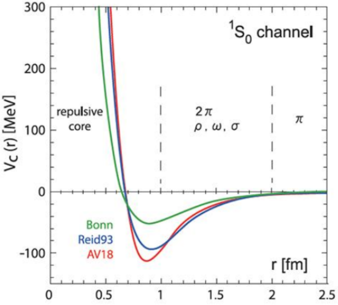
- The general form of the wavefunction of the nucleus is

$$\Psi = \psi_{\text{space}} \chi_s^{m_s} \phi_t^{m_t}$$

- Deuteron composed of two distinguishable fermions $\implies$ antisymmetric wavefunction [1].
- Experimentally, $J = S + L = 1$. $L > 0$ is incompatible with a bound state, so $S = 1$.
- Then, ψ_{space}, $\chi_s^{m_s}$ symmetric $\implies$ $\phi_t^{m_t} = |0, 0\rangle$ (antisymm)
- Thus $|1, 0\rangle_t$ is an excited state (not bound), so charge-invariance of the N-N force implies $|1, 1\rangle_t$ (*pp*) and $|1, -1\rangle_t$ (*nn*) don't exist as bound states.

Understanding the N-N Interaction

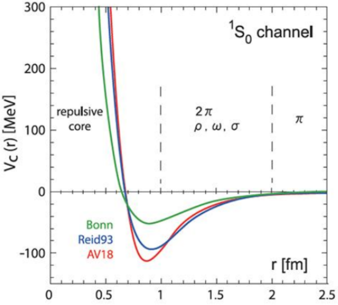
A potential for confusion...



Understanding the N-N Interaction

A potential for confusion...

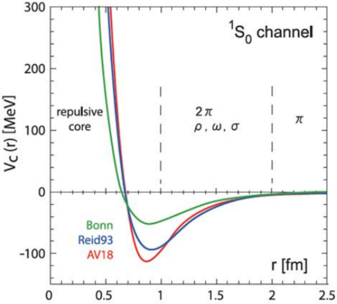
- Potential characterized by three parts:



Understanding the N-N Interaction

A potential for confusion...

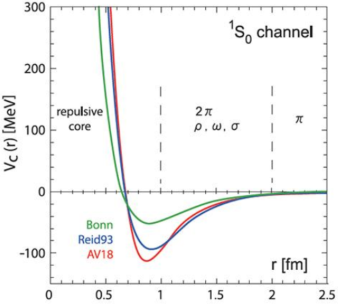
- Potential characterized by three parts:
  - $r \gtrsim 2$ fm: weakly attractive



Understanding the N-N Interaction

A potential for confusion...

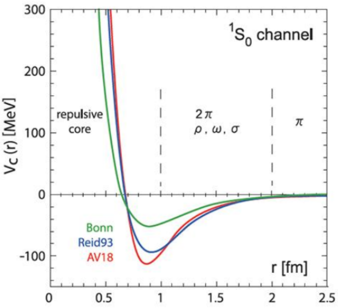
- Potential characterized by three parts:
  - $r \gtrsim 2$ fm: weakly attractive
  - $1 \lesssim r \lesssim 2$ fm: strongly attractive



Understanding the N-N Interaction

A potential for confusion...

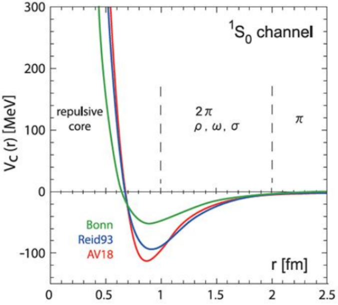
- Potential characterized by three parts:
  - $r \gtrsim 2$ fm: weakly attractive
  - $1 \lesssim r \lesssim 2$ fm: strongly attractive
  - $r \lesssim 1$ fm: very strongly repulsive



Understanding the N-N Interaction

A potential for confusion...

- Potential characterized by three parts:
  - $r \gtrsim 2$ fm: weakly attractive
  - $1 \lesssim r \lesssim 2$ fm: strongly attractive
  - $r \lesssim 1$ fm: very strongly repulsive

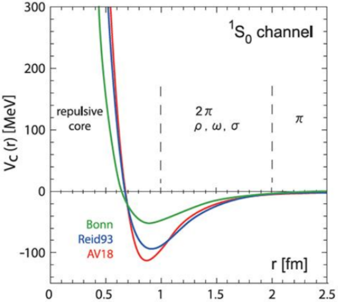


- Most general potential [1]:

Understanding the N-N Interaction

A potential for confusion...

- Potential characterized by three parts:
  - $r \gtrsim 2$ fm: weakly attractive
  - $1 \lesssim r \lesssim 2$ fm: strongly attractive
  - $r \lesssim 1$ fm: very strongly repulsive

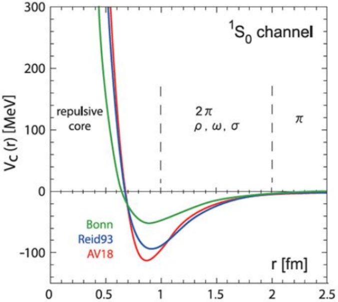


- Most general potential [1]:

$$V = V_c + V_s(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T S_{12}(\vec{r}) + V_T' S_{12}(\vec{p}) + V_{LS}(\vec{L} \cdot \vec{S})$$

Understanding the N-N Interaction

A potential for confusion...



- Potential characterized by three parts:
  - $r \gtrsim 2$ fm: weakly attractive
  - $1 \lesssim r \lesssim 2$ fm: strongly attractive
  - $r \lesssim 1$ fm: very strongly repulsive

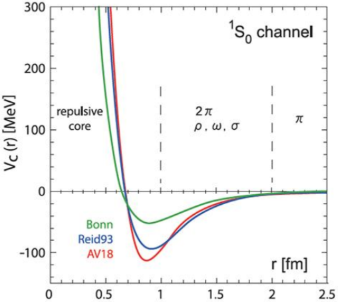
- Most general potential [1]:

$$V = V_c + V_s(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T S_{12}(\vec{r}) + V_T' S_{12}(\vec{p}) + V_{LS}(\vec{L} \cdot \vec{S})$$

where

Understanding the N-N Interaction

A potential for confusion...



- Potential characterized by three parts:
  - $r \gtrsim 2$ fm: weakly attractive
  - $1 \lesssim r \lesssim 2$ fm: strongly attractive
  - $r \lesssim 1$ fm: very strongly repulsive

- Most general potential [1]:

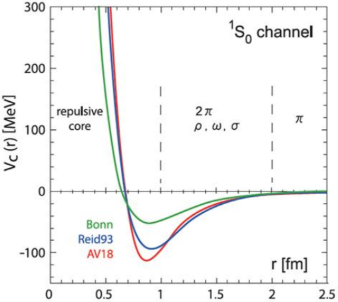
$$V = V_c + V_s(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T S_{12}(\vec{r}) + V_T' S_{12}(\vec{p}) + V_{LS}(\vec{L} \cdot \vec{S})$$

where

$$S_{12}(\vec{r}) = 3\,(\vec{\sigma}_1 \cdot \hat{r})\,(\vec{\sigma}_2 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$

Understanding the N-N Interaction

A potential for confusion...



- Potential characterized by three parts:
  - $r \gtrsim 2$ fm: weakly attractive
  - $1 \lesssim r \lesssim 2$ fm: strongly attractive
  - $r \lesssim 1$ fm: very strongly repulsive

- Most general potential [1]:

$$V = V_c + V_s(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T S_{12}(\vec{r}) + V'_T S_{12}(\vec{p}) + V_{LS}(\vec{L} \cdot \vec{S})$$

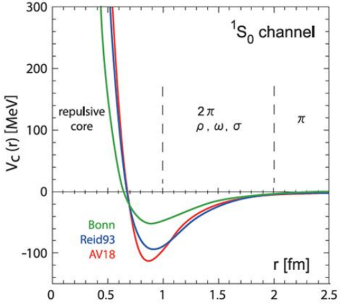
where

$$S_{12}(\vec{r}) = 3\,(\vec{\sigma}_1 \cdot \hat{r})\,(\vec{\sigma}_2 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$
$$V_c = V_w(r) + V_r(r)P_r + V_\sigma(r)P_\sigma + V_t(r)P_t$$

Understanding the N-N Interaction

A potential for confusion...

- Potential characterized by three parts:
  - $r \gtrsim 2$ fm: weakly attractive
  - $1 \lesssim r \lesssim 2$ fm: strongly attractive
  - $r \lesssim 1$ fm: very strongly repulsive



- Most general potential [1]:

$$V = V_c + V_s(\vec{\sigma}_1 \cdot \vec{\sigma}_2) + V_T S_{12}(\vec{r}) + V_T' S_{12}(\vec{p}) + V_{LS}(\vec{L} \cdot \vec{S})$$

where

$$S_{12}(\vec{r}) = 3\,(\vec{\sigma}_1 \cdot \hat{r})\,(\vec{\sigma}_2 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2$$
$$V_c = V_w(r) + V_r(r)P_r + V_\sigma(r)P_\sigma + V_t(r)P_t$$
$$P_r\psi(\vec{r}_1, \vec{r}_2) = \psi(\vec{r}_2, \vec{r}_1),\ P_\sigma = \frac{1}{2}(1 + \vec{\sigma}_1 \cdot \vec{\sigma}_2)$$

One Boson Exchange Potentials (OBEPs)

Oh baby, OBEPs!

One Boson Exchange Potentials (OBEPs)

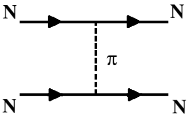
Oh baby, OBEPs!

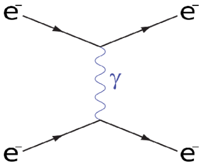
- Yukawa [5]: Short range of nuclear force could be explained by exchange of massive meson

One Boson Exchange Potentials (OBEPs)

Oh baby, OBEPs!

- Yukawa [5]: Short range of nuclear force could be explained by exchange of massive meson

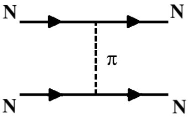


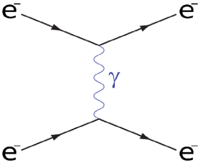


One Boson Exchange Potentials (OBEPs)

Oh baby, OBEPs!

- Yukawa [5]: Short range of nuclear force could be explained by exchange of massive meson



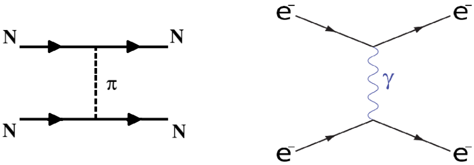


$$0 = \left(\partial_\mu \partial^\mu + m_\pi^2\right) \phi^{(\pi)}$$

One Boson Exchange Potentials (OBEPs)

Oh baby, OBEPs!

- Yukawa [5]: Short range of nuclear force could be explained by exchange of massive meson

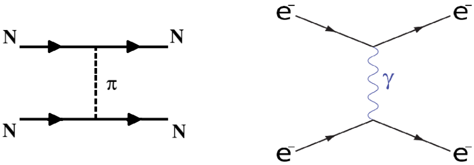


$$
\begin{aligned}
0 &= \left(\partial_\mu \partial^\mu + m_\pi^2\right) \phi^{(\pi)} \\
\implies \quad \partial_t^2 \phi^{(\pi)} &= \left(\nabla^2 - m_\pi^2\right) \phi^{(\pi)}
\end{aligned}
$$

One Boson Exchange Potentials (OBEPs)

Oh baby, OBEPs!

- Yukawa [5]: Short range of nuclear force could be explained by exchange of massive meson

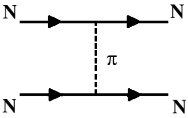


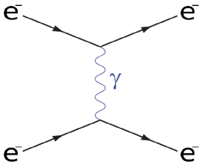
$$
\begin{aligned}
0 &= \left(\partial_\mu \partial^\mu + m_\pi^2\right)\phi^{(\pi)} \\
\implies \quad \partial_t^2 \phi^{(\pi)} &= \left(\nabla^2 - m_\pi^2\right)\phi^{(\pi)} \\
\implies \quad \phi^{(\pi)} &\sim e^{-m_\pi r}/r
\end{aligned}
$$

One Boson Exchange Potentials (OBEPs)

Oh baby, OBEPs!

- Yukawa [5]: Short range of nuclear force could be explained by exchange of massive meson



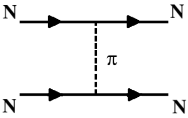


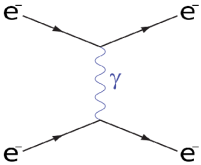
$$
\begin{aligned}
0 &= \left(\partial_\mu \partial^\mu + m_\pi^2\right) \phi^{(\pi)} \\
\Longrightarrow \quad \partial_t^2 \phi^{(\pi)} &= \left(\nabla^2 - m_\pi^2\right) \phi^{(\pi)} \\
\Longrightarrow \quad \phi^{(\pi)} &\sim e^{-m_\pi r}/r \\
\Longrightarrow \quad V &= g^2 e^{-m_\pi r}/r
\end{aligned}
$$

One Boson Exchange Potentials (OBEPs)

Oh baby, OBEPs!

- Yukawa [5]: Short range of nuclear force could be explained by exchange of massive meson





$$
\begin{aligned}
0 &= \left(\partial_\mu \partial^\mu + m_\pi^2\right)\phi^{(\pi)} \\
\implies \quad \partial_t^2 \phi^{(\pi)} &= \left(\nabla^2 - m_\pi^2\right)\phi^{(\pi)} \\
\implies \quad \phi^{(\pi)} &\sim e^{-m_\pi r}/r \\
\implies \quad V &= g^2 e^{-m_\pi r}/r
\end{aligned}
$$

- This is the famous *Yukawa potential*

OBEPs Continued

For a better EP, try OPEP!

OBEPs Continued

For a better EP, try OPEP!

- Yukawa potential lacks necessary spin dependence, tensor component

OBEPs Continued

For a better EP, try OPEP!

- Yukawa potential lacks necessary spin dependence, tensor component
    - ${}^2D$ is actually a addmixture of $\sim 96\%$ ${}^3S_1$ and $\sim 4\%$ ${}^3D_1$ states

OBEPs Continued

For a better EP, try OPEP!

- Yukawa potential lacks necessary spin dependence, tensor component
  - ^2D is actually a addmixture of $\sim 96\%$ $^3S_1$ and $\sim 4\%$ $^3D_1$ states
- A pseudoscalar isovector field $\phi^{(\pi)}$, however, gives

OBEPs Continued

For a better EP, try OPEP!

- Yukawa potential lacks necessary spin dependence, tensor component
  - ^2D is actually a addmixture of $\sim 96\%\ ^3S_1$ and $\sim 4\%\ ^3D_1$ states
- A pseudoscalar isovector field $\phi^{(\pi)}$, however, gives

$$V_{\text{OPEP}} = \frac{f_\pi^2}{12\pi}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \Bigg[\vec{\sigma}_1 \cdot \vec{\sigma}_2 \left(\frac{e^{-m_\pi r}}{r} - \frac{4\pi}{m_\pi^2}\delta^3(\vec{r})\right) + S_{12}(\vec{r})\left(1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2}\right)\frac{e^{-m_\pi r}}{r}\Bigg]$$

OBEPs Continued

For a better EP, try OPEP!

- Yukawa potential lacks necessary spin dependence, tensor component
  - ^2D is actually a addmixture of $\sim 96\%$ $^3S_1$ and $\sim 4\%$ $^3D_1$ states
- A pseudoscalar isovector field $\phi^{(\pi)}$, however, gives

$$V_{\mathrm{OPEP}} = \frac{f_\pi^2}{12\pi}\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left[\vec{\sigma}_1 \cdot \vec{\sigma}_2 \left(\frac{e^{-m_\pi r}}{r} - \frac{4\pi}{m_\pi^2}\delta^3(\vec{r})\right) + S_{12}(\vec{r}) \left(1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2}\right) \frac{e^{-m_\pi r}}{r}\right]$$

- This One Pion Exchange Potential captures the qualitative aspects of the long range portion of the N-N interaction

Quantum Hadrodynamics: The Bonn Potential

Bonn appoténtial!

Quantum Hadrodynamics: The Bonn Potential

Bonn appoténtial!

- Including ρ, σ, and ω mesons allows description of the strongly attractive middle and strongly repulsive core

Quantum Hadrodynamics: The Bonn Potential

Bonn appoténtial!

- Including ρ, σ, and ω mesons allows description of the strongly attractive middle and strongly repulsive core
- The couplings are given by [2]

Quantum Hadrodynamics: The Bonn Potential

Bonn appoténtial!

- Including ρ, σ, and ω mesons allows description of the strongly attractive middle and strongly repulsive core
- The couplings are given by [2]

$$\mathcal{L}_{\pi^0 N} = -g_{\pi^0}\bar{\psi} i\gamma^5 \tau_3 \psi \phi^{(\pi^0)}$$

Quantum Hadrodynamics: The Bonn Potential

Bonn appoténtial!

- Including ρ, σ, and ω mesons allows description of the strongly attractive middle and strongly repulsive core
- The couplings are given by [2]

$$\mathcal{L}_{\pi^0 N} = -g_{\pi^0}\bar{\psi} i\gamma^5 \tau_3 \psi \phi^{(\pi^0)}$$
$$\mathcal{L}_{\pi^\pm N} = -\sqrt{2} g_{\pi^\pm}\bar{\psi} i\gamma^5 \tau_\pm \psi \phi^{(\pi^\pm)}$$

Quantum Hadrodynamics: The Bonn Potential

Bonn appoténtial!

- Including ρ, σ, and ω mesons allows description of the strongly attractive middle and strongly repulsive core
- The couplings are given by [2]

$$\mathcal{L}_{\pi^0 N} = -g_{\pi^0} \bar{\psi} i \gamma^5 \tau_3 \psi \phi^{(\pi^0)}$$
$$\mathcal{L}_{\pi^\pm N} = -\sqrt{2} g_{\pi^\pm} \bar{\psi} i \gamma^5 \tau_\pm \psi \phi^{(\pi^\pm)}$$
$$\mathcal{L}_{\sigma N} = -g_\sigma \bar{\psi} \psi \phi^{(\sigma)}$$

Quantum Hadrodynamics: The Bonn Potential

Bonn appoténtial!

- Including ρ, σ, and ω mesons allows description of the strongly attractive middle and strongly repulsive core
- The couplings are given by [2]

$$
\begin{aligned}
\mathcal{L}_{\pi^0 N} &= -g_{\pi^0}\bar{\psi} i\gamma^5 \tau_3 \psi \phi^{(\pi^0)} \\
\mathcal{L}_{\pi^\pm N} &= -\sqrt{2} g_{\pi^\pm}\bar{\psi} i\gamma^5 \tau_\pm \psi \phi^{(\pi^\pm)} \\
\mathcal{L}_{\sigma N} &= -g_{\sigma}\bar{\psi} \psi \phi^{(\sigma)} \\
\mathcal{L}_{\omega N} &= -g_{\omega}\bar{\psi} \gamma^\mu \psi \phi^{(\omega)}
\end{aligned}
$$

Quantum Hadrodynamics: The Bonn Potential

Bonn appoténtial!

- Including ρ, σ, and ω mesons allows description of the strongly attractive middle and strongly repulsive core
- The couplings are given by [2]

$$\begin{aligned}
\mathcal{L}_{\pi^0 N} &= -g_{\pi^0}\bar{\psi} i\gamma^5 \tau_3 \psi \phi^{(\pi^0)} \\
\mathcal{L}_{\pi^\pm N} &= -\sqrt{2} g_{\pi^\pm}\bar{\psi} i\gamma^5 \tau_\pm \psi \phi^{(\pi^\pm)} \\
\mathcal{L}_{\sigma N} &= -g_\sigma \bar{\psi}\psi\phi^{(\sigma)} \\
\mathcal{L}_{\omega N} &= -g_\omega \bar{\psi}\gamma^\mu \psi\phi^{(\omega)} \\
\mathcal{L}_{\rho N} &= -g_\rho \bar{\psi}\gamma^\mu \boldsymbol{\tau}\psi \cdot \boldsymbol{\phi}_\mu^{(\rho)} - \frac{f_\rho}{4M}\bar{\psi}\sigma^{\mu\nu}\boldsymbol{\tau}\psi \cdot \boldsymbol{F}_{\mu\nu}^{(\rho)}
\end{aligned}$$

Quantum Hadrodynamics: The Bonn Potential

Bonn appoténtial!

- Including ρ, σ, and ω mesons allows description of the strongly attractive middle and strongly repulsive core
- The couplings are given by [2]

$$
\begin{aligned}
\mathcal{L}_{\pi^0 N} &= -g_{\pi^0}\bar{\psi} i\gamma^5 \tau_3 \psi \phi^{(\pi^0)} \\
\mathcal{L}_{\pi^\pm N} &= -\sqrt{2} g_{\pi^\pm}\bar{\psi} i\gamma^5 \tau_\pm \psi \phi^{(\pi^\pm)} \\
\mathcal{L}_{\sigma N} &= -g_\sigma \bar{\psi}\psi\phi^{(\sigma)} \\
\mathcal{L}_{\omega N} &= -g_\omega \bar{\psi}\gamma^\mu\psi\phi^{(\omega)} \\
\mathcal{L}_{\rho N} &= -g_\rho \bar{\psi}\gamma^\mu \boldsymbol{\tau}\psi \cdot \boldsymbol{\phi}_\mu^{(\rho)} - \frac{f_\rho}{4M}\bar{\psi}\sigma^{\mu\nu}\boldsymbol{\tau}\psi \cdot \boldsymbol{F}_{\mu\nu}^{(\rho)}
\end{aligned}
$$

- Each spin-isospin channel contributes a $V_c + V_T S_{12} + V_{LS}\vec{L}\cdot\vec{S}$

Interlude: The Mystery of the σ

- The σ meson has a somewhat bizarre history

Interlude: The Mystery of the σ

- The σ meson has a somewhat bizarre history
- Review of Particle Properties [3]:

Interlude: The Mystery of the σ

- The σ meson has a somewhat bizarre history
- Review of Particle Properties [3]:
  - 1964: several references list $\sim$ 390 MeV, width 50-150 MeV

Interlude: The Mystery of the σ

- The σ meson has a somewhat bizarre history
- Review of Particle Properties [3]:
    - 1964: several references list $\sim$ 390 MeV, width 50-150 MeV
    - 1967: σ or other light mesons listed, but considered not well established

Interlude: The Mystery of the σ

- The σ meson has a somewhat bizarre history
- Review of Particle Properties [3]:
  - 1964: several references list $\sim$ 390 MeV, width 50-150 MeV
  - 1967: σ or other light mesons listed, but considered not well established
  - 1976: All entries *vanish*

Interlude: The Mystery of the σ

- The σ meson has a somewhat bizarre history
- Review of Particle Properties [3]:
  - 1964: several references list $\sim$ 390 MeV, width 50-150 MeV
  - 1967: σ or other light mesons listed, but considered not well established
  - 1976: All entries *vanish*
  - 1996: σ returns as $f_0(400 - 1200)$ with width 500-1000 MeV

Interlude: The Mystery of the σ

- The σ meson has a somewhat bizarre history
- Review of Particle Properties [3]:
  - 1964: several references list ~ 390 MeV, width 50-150 MeV
  - 1967: σ or other light mesons listed, but considered not well established
  - 1976: All entries *vanish*
  - 1996: σ returns as $f_0(400 - 1200)$ with width 500-1000 MeV
  - 2012: Revision to $f_0(500)$, with mass 400-550 MeV, width 200-350 MeV

Interlude: The Mystery of the σ

- The σ meson has a somewhat bizarre history
- Review of Particle Properties [3]:
  - 1964: several references list $\sim$ 390 MeV, width 50-150 MeV
  - 1967: σ or other light mesons listed, but considered not well established
  - 1976: All entries *vanish*
  - 1996: σ returns as $f_0(400 - 1200)$ with width 500-1000 MeV
  - 2012: Revision to $f_0(500)$, with mass 400-550 MeV, width 200-350 MeV
- Why??

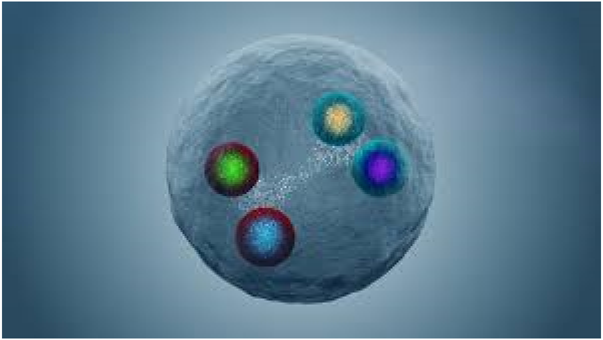
Interlude: The Mystery of the σ

- The σ meson has a somewhat bizarre history
- Review of Particle Properties [3]:
  - 1964: several references list $\sim$ 390 MeV, width 50-150 MeV
  - 1967: σ or other light mesons listed, but considered not well established
  - 1976: All entries *vanish*
  - 1996: σ returns as $f_0(400 - 1200)$ with width 500-1000 MeV
  - 2012: Revision to $f_0(500)$, with mass 400-550 MeV, width 200-350 MeV
- Why??
  - Large width $\rightarrow$ difficult to differentiate from background

Interlude: The Mystery of the σ

- The σ meson has a somewhat bizarre history
- Review of Particle Properties [3]:
    - 1964: several references list ~ 390 MeV, width 50-150 MeV
    - 1967: σ or other light mesons listed, but considered not well established
    - 1976: All entries *vanish*
    - 1996: σ returns as $f_0(400 - 1200)$ with width 500-1000 MeV
    - 2012: Revision to $f_0(500)$, with mass 400-550 MeV, width 200-350 MeV
- Why??
    - Large width $\rightarrow$ difficult to differentiate from background
    - Abnormal substructure: almost certainly a tetraquark state with subdominant but non-negligible $q\bar{q}$ component.

A true *color* image of a tetraquark captured by the ACME collaboration



Beyond ^2D: The Nuclear Many Body Problem

Two's company, three's a crowd

Beyond ^2D: The Nuclear Many Body Problem

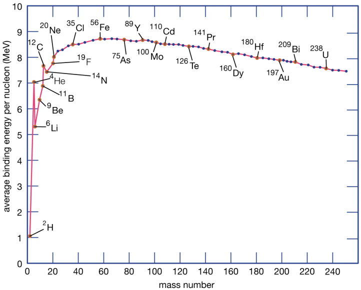
Two's company, three's a crowd

- More nucleons, more problems

Beyond ^2D: The Nuclear Many Body Problem

Two's company, three's a crowd

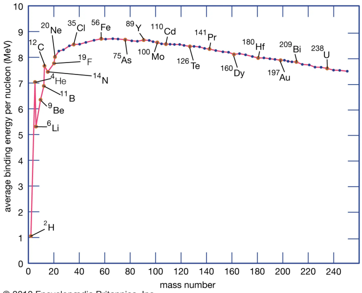
- More nucleons, more problems
- Observation: E_B/A approx. constant for $A \gtrsim 40$



Beyond ^2D: The Nuclear Many Body Problem

Two's company, three's a crowd

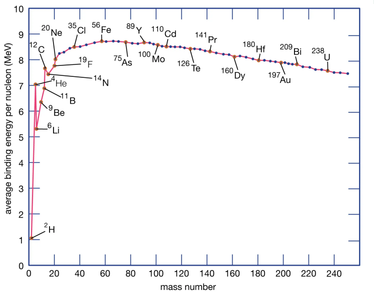
- More nucleons, more problems
- Observation: E_B/A approx. constant for $A \gtrsim 40$
- Enter: The Shell Model



Beyond ^2D: The Nuclear Many Body Problem

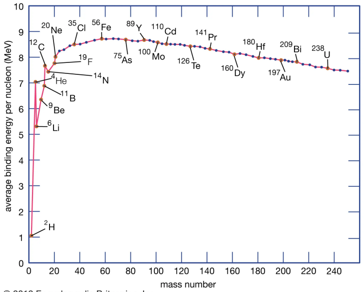
Two's company, three's a crowd

- More nucleons, more problems
- Observation: E_B/A approx. constant for $A \gtrsim 40$
- Enter: The Shell Model



$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i>j} V_{ij}(\vec{r}_i, \vec{r}_j) \to \sum_i \left(\frac{p_i^2}{2m} + U(\vec{r}_i) \right) + H_{\text{res}}$$

Beyond ^2D: The Nuclear Many Body Problem



Two's company, three's a crowd

- More nucleons, more problems
- Observation: E_B/A approx. constant for $A \gtrsim 40$
- Enter: The Shell Model

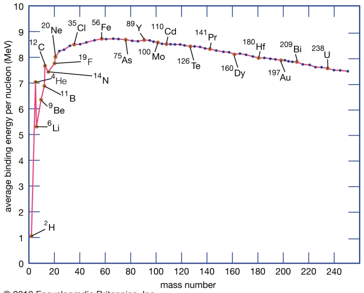
$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i>j} V_{ij}(\vec{r}_i, \vec{r}_j) \to \sum_i \left(\frac{p_i^2}{2m} + U(\vec{r}_i) \right) + H_{\text{res}}$$

- The average affect of the nuclear force is treated as a central potential

Beyond ^2D: The Nuclear Many Body Problem

Two's company, three's a crowd

- More nucleons, more problems
- Observation: E_B/A approx. constant for $A \gtrsim 40$
- Enter: The Shell Model



$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i>j} V_{ij}(\vec{r}_i, \vec{r}_j) \to \sum_i \left(\frac{p_i^2}{2m} + U(\vec{r}_i) \right) + H_{\text{res}}$$

- The average affect of the nuclear force is treated as a central potential
  - The rest of the force, H_{res} is treated *perturbatively*

Back to the Fields: QHD-I and QHD-II

Back to the Fields: QHD-I and QHD-II

Who doesn't love a good Lagrangian

Back to the Fields: QHD-I and QHD-II

Who doesn't love a good Lagrangian

$$\mathcal{L}_{\text{QHD-II}} = \bar{\psi}\left(i\not{D} - m\right)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu}$$
$$+ \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\boldsymbol{R}_{\mu\nu}\boldsymbol{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

Back to the Fields: QHD-I and QHD-II

Who doesn't love a good Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\text{QHD-II}} &= \bar{\psi}\left(i\not{D} - m\right)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} \\
&+ \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\boldsymbol{R}_{\mu\nu}\boldsymbol{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\
\Omega_{\mu\nu} &= \partial_\mu\omega_\nu - \partial_\nu\omega_\mu
\end{aligned}
$$

Back to the Fields: QHD-I and QHD-II

Who doesn't love a good Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\text{QHD-II}} = \bar{\psi}\left(i\not{D} - m\right)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} \\
+ \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\boldsymbol{R}_{\mu\nu}\boldsymbol{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}
\end{aligned}
$$

$$
\Omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu
$$

$$
\boldsymbol{R}_{\mu\nu} = \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu - g_\rho\left(\boldsymbol{\rho}_\mu \times \boldsymbol{\rho}_\nu\right)
$$

Back to the Fields: QHD-I and QHD-II

Who doesn't love a good Lagrangian

$$\mathcal{L}_{\text{QHD-II}} = \bar{\psi}\left(i\not{D} - m\right)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu}$$
$$+ \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\boldsymbol{R}_{\mu\nu}\boldsymbol{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$\Omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$$
$$\boldsymbol{R}_{\mu\nu} = \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu - g_\rho\left(\boldsymbol{\rho}_\mu \times \boldsymbol{\rho}_\nu\right)$$
$$U(\sigma) = \frac{\kappa}{3!}\sigma^3 + \frac{\lambda}{4!}\sigma^4$$

Back to the Fields: QHD-I and QHD-II

Who doesn't love a good Lagrangian

$$\mathcal{L}_{\text{QHD-II}} = \bar{\psi}\left(i\not{D} - m\right)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu}$$
$$+ \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\boldsymbol{R}_{\mu\nu}\boldsymbol{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$\Omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$$
$$\boldsymbol{R}_{\mu\nu} = \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu - g_\rho\left(\boldsymbol{\rho}_\mu \times \boldsymbol{\rho}_\nu\right)$$
$$U(\sigma) = \frac{\kappa}{3!}\sigma^3 + \frac{\lambda}{4!}\sigma^4$$

- QHD-I considers only σ and ω, while QHD-II uses the full Lagrangian above

Back to the Fields: QHD-I and QHD-II

Who doesn't love a good Lagrangian

$$\mathcal{L}_{\text{QHD-II}} = \bar{\psi}\left(i\not{D} - m\right)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu}$$
$$+ \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\boldsymbol{R}_{\mu\nu}\boldsymbol{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$\Omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$$
$$\boldsymbol{R}_{\mu\nu} = \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu - g_\rho\left(\boldsymbol{\rho}_\mu \times \boldsymbol{\rho}_\nu\right)$$
$$U(\sigma) = \frac{\kappa}{3!}\sigma^3 + \frac{\lambda}{4!}\sigma^4$$

- QHD-I considers only σ and ω, while QHD-II uses the full Lagrangian above
- Mesons/Nucleons described have intrinsic structure due to implied meson and baryon-antibaryon loops

Back to the Fields: QHD-I and QHD-II

Who doesn't love a good Lagrangian

$$\mathcal{L}_{\text{QHD-II}} = \bar{\psi}\left(i\not{D} - m\right)\psi + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}m_\sigma^2\sigma^2 - U(\sigma) - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu}$$
$$+ \frac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \frac{1}{4}\boldsymbol{R}_{\mu\nu}\boldsymbol{R}^{\mu\nu} + \frac{1}{2}m_\rho^2\boldsymbol{\rho}_\mu\boldsymbol{\rho}^\mu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$
$$\Omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$$
$$\boldsymbol{R}_{\mu\nu} = \partial_\mu\boldsymbol{\rho}_\nu - \partial_\nu\boldsymbol{\rho}_\mu - g_\rho\left(\boldsymbol{\rho}_\mu \times \boldsymbol{\rho}_\nu\right)$$
$$U(\sigma) = \frac{\kappa}{3!}\sigma^3 + \frac{\lambda}{4!}\sigma^4$$

- QHD-I considers only σ and ω, while QHD-II uses the full Lagrangian above
- Mesons/Nucleons described have intrinsic structure due to implied meson and baryon-antibaryon loops
- OPEP component averages out of bulk nuclear matter

Getting Practical Results: Mean Field Theory

When in doubt, integrate out

Getting Practical Results: Mean Field Theory

When in doubt, integrate out

- We solve the Euler-Lagrange equations for the meson fields, finding nucleon density source terms, e.g.

Getting Practical Results: Mean Field Theory

When in doubt, integrate out

- We solve the Euler-Lagrange equations for the meson fields, finding nucleon density source terms, e.g.

$$\partial_\nu \Omega^{\nu\mu} + m_\omega^2 \omega^\mu = g_\omega \bar{\psi} \gamma^\mu \psi$$

Getting Practical Results: Mean Field Theory

When in doubt, integrate out

- We solve the Euler-Lagrange equations for the meson fields, finding nucleon density source terms, e.g.
$$\partial_\nu \Omega^{\nu\mu} + m_\omega^2 \omega^\mu = g_\omega \bar{\psi} \gamma^\mu \psi$$
- At high nucleon densities, the source terms become large and we can treat the mesons as classical fields ($\phi \to \langle \phi \rangle$)

Getting Practical Results: Mean Field Theory

When in doubt, integrate out

- We solve the Euler-Lagrange equations for the meson fields, finding nucleon density source terms, e.g.
$$\partial_\nu \Omega^{\nu\mu} + m_\omega^2 \omega^\mu = g_\omega \bar{\psi} \gamma^\mu \psi$$
- At high nucleon densities, the source terms become large and we can treat the mesons as classical fields ($\phi \to \langle \phi \rangle$)
- For QHD-I (neglecting the σ potential), the Lagrangian becomes [4]

Getting Practical Results: Mean Field Theory

When in doubt, integrate out

- We solve the Euler-Lagrange equations for the meson fields, finding nucleon density source terms, e.g.
$$\partial_\nu \Omega^{\nu\mu} + m_\omega^2 \omega^\mu = g_\omega \bar{\psi} \gamma^\mu \psi$$
- At high nucleon densities, the source terms become large and we can treat the mesons as classical fields ($\phi \to \langle \phi \rangle$)
- For QHD-I (neglecting the σ potential), the Lagrangian becomes [4]
$$\mathcal{L}_{\text{MFT-I}} = \bar{\psi} \left(\not{D} - m \right) \psi - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu$$

Getting Practical Results: Mean Field Theory

When in doubt, integrate out

- We solve the Euler-Lagrange equations for the meson fields, finding nucleon density source terms, e.g.
$$\partial_\nu \Omega^{\nu\mu} + m_\omega^2 \omega^\mu = g_\omega \bar{\psi} \gamma^\mu \psi$$
- At high nucleon densities, the source terms become large and we can treat the mesons as classical fields ($\phi \to \langle \phi \rangle$)
- For QHD-I (neglecting the σ potential), the Lagrangian becomes [4]
$$\mathcal{L}_{\text{MFT-I}} = \bar{\psi} \left(\not{D} - m \right) \psi - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu$$
- The nucleon mass is shifted by the scalar: $M \to M^* = m - g_\sigma \langle \sigma \rangle$

Getting Practical Results: Mean Field Theory

When in doubt, integrate out

- We solve the Euler-Lagrange equations for the meson fields, finding nucleon density source terms, e.g.
$$\partial_\nu \Omega^{\nu\mu} + m_\omega^2 \omega^\mu = g_\omega \bar{\psi} \gamma^\mu \psi$$
- At high nucleon densities, the source terms become large and we can treat the mesons as classical fields ($\phi \to \langle \phi \rangle$)
- For QHD-I (neglecting the σ potential), the Lagrangian becomes [4]
$$\mathcal{L}_{\text{MFT-I}} = \bar{\psi} \left(\not{D} - m \right) \psi - \frac{1}{2} m_\sigma^2 \sigma^2 + \frac{1}{2} m_\omega^2 \omega_\mu \omega^\mu$$
- The nucleon mass is shifted by the scalar: $M \to M^* = m - g_\sigma \langle \sigma \rangle$
- Additionally, the energy is shifted by the vector: $E^\pm(\vec{k}) = g_\omega \langle \omega^0 \rangle \pm \sqrt{\left(\vec{k} - g_\omega \vec{\omega} \right)^2 + M^{*2}}$

References

[1] C. Bertulani. *Nuclear Physics in a Nutshell*. In a Nutshell. Princeton University Press, 2007.

[2] R. Machleidt. High-precision, charge-dependent bonn nucleon-nucleon potential. *Physical Review C*, 63(2), Jan. 2001.

[3] J. R. Peláez. From controversy to precision on the sigma meson: A review on the status of the non-ordinary f0(500) resonance. *Physics Reports*, 658:1–111, Nov. 2016.

[4] B. D. Serot and J. D. Walecka. *Relativistic Nuclear Many-Body Theory*, pages 49–92. Springer US, Boston, MA, 1992.

[5] H. Yukawa. *On the Interaction of Elementary Particles*. Number pt. 4 in On the Interaction of Elementary Particles. Tokto Imperial University, 1935.