Homework 5

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Problem 1

On page 28 of the ATLAS paper, they quote a lower limit of 4.2 TeV for the mass of the W'. Using the collider reach site, I find the following predictions for the given colliders:

$\sqrt{s}$	$\mathcal{L}^{-1}$	$m_{W'}$ Limit
14 TeV	3000fb^{-1}	5.9 Tev
27 TeV	15000fb^{-1}	11.5 Tev
100 TeV	300000fb^{-1}	42.9 Tev

Problem 2

(a) The full lagrangian is given by

$$\mathscr{L} = \sum_f \bar{\Psi}_{fi} \left(i\not{D}_{ij} - m_f \delta_{ij} \right) \Psi_{fj} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a,$$

where

$$F_{\mu\nu} = 2\partial_{[\mu} A_{\nu]},$$
$$G^a_{\mu\nu} = 2\partial_{[\mu} B^a_{\nu]} + g f^{abc} B^a_\mu B^c_\nu,$$

and

$$\not{D}^i_j = \gamma^\mu \left(\partial_\mu \delta_{ij} - i g t^a_{ij} B^a_\mu - i Q_f e A^\mu \delta_{ij} \right).$$

With $m_f = 0$ for all flavors, the symmetries of the lagrangian are $SU(3)_{L\times R} \times U(1)_B$. I believe there should also be an $SU(2)$ for rotation between down and strange since they have the same charge (a sort of "strange isospin", if you will).

(b) Turning on the masses brakes the $SU(3)_{L\times R}$, though we still have the $U(1)_B$. If the masses are equal, we retain a $SU(3)_{L+R}$ symmetry, and if at least the down and strange masses are the same we retain that $SU(2)$.

Problem 3

(a) As a function of $u = \vec{\phi} \cdot \vec{\phi}$, the potential is given by

$$V(u) = \lambda(u - v^2)^2.$$

Setting the derivative to zero and solving for u, we find

$$\begin{aligned} 0 &= \frac{\mathrm{d}V}{\mathrm{d}u} \\ &= 2(x - v^2) \\ \implies \quad u &= v^2 \end{aligned}$$

(b) We have that

$$\begin{aligned} \delta \left\langle \vec{\phi} \right\rangle &= R(\alpha) \left\langle \vec{\phi} \right\rangle - \left\langle \vec{\phi} \right\rangle \\ &= \exp(-i\alpha_a T^a) \left\langle \vec{\phi} \right\rangle - \left\langle \vec{\phi} \right\rangle \\ &\approx (\mathbb{I} - \alpha_a \epsilon^a_{bc}) \left\langle \vec{\phi} \right\rangle - \left\langle \vec{\phi} \right\rangle \\ &= -\alpha_a \epsilon^a_{bc} \left\langle \vec{\phi}_c \right\rangle \end{aligned}$$

Plugging in 1,2, and 3 for a, we see that

$$\begin{aligned} \delta \left\langle \vec{\phi}_1 \right\rangle &= \alpha_1 \left(\phi_2 - \phi_3\right) = \alpha_1 v \\ \delta \left\langle \vec{\phi}_2 \right\rangle &= \alpha_1 \left(\phi_3 - \phi_1\right) = \alpha_2 v \\ \delta \left\langle \vec{\phi}_1 \right\rangle &= \alpha_1 \left(\phi_2 - \phi_1\right) = 0. \end{aligned}$$

So, generators 1 and 2 are broken.

(c) The mass squared matrix evaluates as

$$\begin{aligned} M^2_{ij} &= \frac{\partial^2 V}{\partial\tilde{\phi}_i \partial\tilde{\phi}_j}\Big|_{\tilde{\phi}=0} \\ &= \begin{pmatrix} 4\left(\tilde{\phi}_1^2 + \tilde{\phi}_2^2 + \tilde{\phi}_3^2 + 2v\tilde{\phi}_3\right) + 8\tilde{\phi}_1^2 & 8\tilde{\phi}_1\tilde{\phi}_2 & 8\tilde{\phi}_1\left(v + \tilde{\phi}_3\right) \\ 8\tilde{\phi}_1\tilde{\phi}_2 & 4\left(\tilde{\phi}_1^2 + \tilde{\phi}_2^2 + \tilde{\phi}_3^2 + 2v\tilde{\phi}_3\right) + 8\tilde{\phi}_2^2 & 8\tilde{\phi}_2\left(v + \tilde{\phi}_3\right) \\ 8\tilde{\phi}_1\left(v + \tilde{\phi}_3\right) & 8\tilde{\phi}_2(v + \tilde{\phi}_3) & 4\left(\tilde{\phi}_1^2 + \tilde{\phi}_2^2 + \tilde{\phi}_3^2 + 2v\tilde{\phi}_3\right) + 8\left(v + \tilde{\phi}_3\right)^2 \end{pmatrix} \end{aligned}$$

$$= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 8v^2 \end{pmatrix}$$

Since the mass squared matrix is diagonal, we can clearly see that the zero eigenvectors are 1 and 2, while the nonzero is 3, in agreement with (b).