Homework 9

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Phys 684

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Problem 1

Problem

An atom has to recoil when emitting a photon. Calculate the velocity of a Na atom after the emission of a photon (assume that the atom is initially at rest and the optical transition takes place at the D_2 line with $\lambda = 589$ nm). If we do not ignore the recoil energy, what will be the corresponding Doppler shift?

Solution

Using the calculations in the attached Mathematica notebook, we find that the resulting velocity is approximately 3 cm/s. This motion produces a red shift in the wavelength of the light of approximately 5.8×10^{-8} nm.

Problem 2

Problem

For the D_2 transition of the Na atom, what is the Doppler cooling limit (assume $\gamma_2/2\pi = 10$ MHz)? What is the temperature limit of single photon recoil for the D_2 transition?

Solution

Using the calculations in the attached Mathematica notebook, we find that the Doppler cooling limit is $T_D \approx 120\ \mu$K. The temperature associated with a single recoil event is $T_R \approx 2.4\ \mu$K.

Problem 3 (Berman 5.2)

Problem

Calculate the maximum force on an atom produced by a monochromatic, plane-wave field having Rabi frequency $\Omega_0/2\pi = 20$ MHz, given that $\gamma_2/2\pi = 10$ MHz and there are no collisions. Assume that $v_z = 200$ m/s, that the resonance wavelength is $\lambda_0 = 628$ nm, and that the field can be tuned within 1 GHz of resonance. Calculate the acceleration that this force produces for an atom having atomic mass 23.

Solution

Using the calculations in the attached Mathematica notebook, we find a maximum force of about 0.46 Attonewtons at a detuning of about 54.4 MHz. Associated with this force is an acceleration of about 1.2×10^7 m/s^2.

Problem 4 (Berman 5.3)

Problem

For a 5-mW standing-wave laser field having a waist area of 4 mm^2, calculate the well depth of the ground-state potential produced by the field in units of the recoil energy $(\hbar^2 k^2)/2M$ assuming a detuning of 3γ. Repeat the calculation for a FORT (Far Off-Resonance optical-dipole Trap), in which the laser field has a power of 100 mW and is focused to a spot diameter of 20 μm. The detuning is $20\gamma_2$. Take $\gamma_2/2\pi = 6$ MHz, $\lambda = 780$ nm, $M = {}^{85}$Rb mass, and $(\mu_x)_{21} = -0.57 e a_0$. Also calculate the frequency spacing at the bottom of the wells assuming that the potentials can be approximated as harmonic in that region. Can atoms cooled to the Doppler limit of laser cooling be trapped in these potentials? Explain.

Solution

Using the calculations in the attached Mathematica notebook, we find that the well depth for the 5-mW standing laser to be approximately 360 times the recoil energy. For the 100-mW case, we find that the well depth is approximately 4.6×10^7 the recoil energy. In both cases, the CoM energy associated with the Doppler cooling limit is several orders of magnitude smaller than the potential depth, so atoms cooled to the Doppler limit should indeed be trappable by these potentials.

The optical potential has the form

$$V(z) = V_{\text{opt}} \sin(kz).$$

Near a minimum, this is approximated by

$$V(z) \approx V_{\text{opt}} \left(\frac{(kz)^2}{2} - 1 \right).$$

Ignoring the offset, we have that

$$\begin{aligned} V &\sim \frac{1}{2} V_{\text{opt}} k^2 z^2 \\ &= \frac{1}{2} m \tilde{\omega}^2 z^2 \\ \implies \tilde{\omega} &= \sqrt{\frac{V_{\text{opt}} k^2}{m}} \end{aligned}$$

Using the the given values, we find that the potentials produce frequency spacings of $\tilde{\omega} \approx$ 0.1 MHz and 37 MHz, respectively.

Problem 1)

In[1]:=
```
M = UnitConvert[ sodium ELEMENT [ atomic mass ], "Kilograms"]; StringForm["M = ``", M]
λ = Quantity[589, "nanometers"];
k = 2π/λ; StringForm["k = ``", UnitConvert[k, "inverse nm"] // N]
p = ℏ k; StringForm["p = ``", UnitConvert[p, "Kg m/s"] // N]
v = p/M; StringForm["v = ``", UnitConvert[v, "cm/s"]]
```

Out[1]= M = $3.81754100 \times 10^{-26}$ kg

Out[3]= k = 0.0106675 /nm

Out[4]= p = 1.12497×10^{-27} kg m/s

Out[5]= v = 2.94684318 cm/s

In[6]:=
$$\lambda' = \frac{2\pi c}{\frac{2\pi c}{\lambda} - k v}$$
```
; StringForm["Doppler shifted frequency λ' = ``. So, Δλ = ``", λ', λ' - λ]
```

Out[6]= Doppler shifted frequency λ' = 589.0000000578964075 nm . So, Δλ = $5.78964075 \times 10^{-8}$ nm

Problem 2)

In[7]:=
```
γ2 = 2π Quantity[10, "MHz"];
```
$$T = \frac{\hbar \gamma_2}{4 k}$$
```
; StringForm["T_D = ``", UnitConvert[T, "μK"] // N]
```
$$\text{StringForm}\left[\text{"}T_R = \text{``"}, \text{UnitConvert}\left[\frac{p^2}{M k}, \text{"μK"}\right]\right]$$

Out[8]= T_D = 119.981 μK

Out[9]= T_R = 2.40112338 μK

Problem 3)

In[10]:=
$$\Omega_0 = 2\pi\,\text{Quantity}[20, \text{"MHz"}];$$
$$v = \text{Quantity}[200, \text{"m/s"}];$$
$$\lambda = \text{Quantity}[628, \text{"nm"}];$$
$$\delta' = \text{Quantity}[1, \text{"GHz"}];$$
$$M = \text{Quantity}[23, \text{"amu"}];$$
$$\gamma' = \frac{\gamma_2}{2}\sqrt{1 + 2\frac{\Omega_0{}^2}{\gamma_2{}^2}};$$
$$\beta = \hbar\, k^2\, \Omega_0{}^2 \frac{\gamma_2\,\delta}{2\left(\delta^2 + (\gamma')^2\right)^2};$$
$$F = v\,\beta;$$

In[18]:=
```
{maxForce, detuning} = Maximize[{F, {-δ' < δ < δ'}}, δ];
```

In[19]:=
```
StringForm["A maximum force of `` is achieved by δ = ``",
 UnitConvert[maxForce, "aN"] // N,
 UnitConvert[δ /. detuning, "MHz"] // N]
StringForm["This force produces an acceleration of ``",
 maxForce / M // UnitConvert]
```

Out[19]= A maximum force of 0.461906 aN is achieved by δ = 54.414 MHz

Out[20]= This force produces an acceleration of 1.209418168×10^7 m/s^2

Problem 4)

In[21]:=
$$P = \text{Quantity}[5, \text{"mW"}];$$
$$a = \text{Quantity}[4, \text{"mm\^{}2"}];$$
$$\gamma_2 = 2\pi\,\text{Quantity}[6, \text{"MHz"}];$$
$$\lambda = \text{Quantity}[780, \text{"nm"}];$$
$$\mu = -0.57\ e\ a_0;$$
$$M = \text{UnitConvert}\left[\{\boxed{\text{rubidium-85 ISOTOPE}}\}[\boxed{\text{atomic mass}}], \text{"Kg"}\right];$$
$$k = \frac{2\pi}{\lambda};\ I = \frac{P}{a};\ \delta = \frac{3}{2}\gamma_2;$$
$$\Omega_0 = \sqrt{\frac{2\,\text{Abs}[\mu]^2\, I}{\hbar^2\,\varepsilon_0\,c}};$$
$$E_r = \text{UnitConvert}\left[\frac{\hbar^2 k^2}{2M}, \text{"J"}\right][[1]];$$

In[30]:=
```
V = UnitConvert[ħ Ω0^2/(4 δ), "J"];
StringForm["In units of the recoil energy, the well has a depth of ``", V/Er // N]
Ed = ħ γ2/4;
StringForm["Doppler cooling can achieve atomic CoM energies (in units of the well depth) of
ω̃ = Sqrt[V k^2/M][[1]];
StringForm["The frequency spacing near the bottom of the well is ``", UnitConvert[ω̃/(2 π), "MHz
```

Out[31]= In units of the recoil energy, the well has a depth of 360.33715442066836`

Out[33]= Doppler cooling can achieve atomic CoM energies (in units of the well depth) of 1.07786030

Out[35]= The frequency spacing near the bottom of the well is 0.103679 MHz

In[36]:=
```
P = Quantity[100, "mW"];
a = π Quantity[10, "μm"]^2;
I = P/a; δ = 3 γ2;
Ω0 = Sqrt[2 Abs[μ]^2 I/(ħ^2 ε0 c)];
```

In[40]:=
```
V = UnitConvert[ħ Ω0^2/(4 δ), "J"];
StringForm["In units of the recoil energy, the well has a depth of ``", V/Er]
```

Out[41]= In units of the recoil energy, the well has a depth of 4.587955144457358`*^7

In[42]:=
```
Ed = ħ γ2/4;
StringForm["Doppler cooling can achieve atomic CoM energies (in units of the well depth) of
```

Out[43]= Doppler cooling can achieve atomic CoM energies (in units of the well depth) of 8.46549504

In[44]:= $\tilde{\omega} = \sqrt{\frac{V\,k^2}{M}}[[1]];$

StringForm[“The frequency spacing near the bottom of the well is ``”, UnitConvert[$\frac{\tilde{\omega}}{2\,\pi}$, “MH

Out[45]= The frequency spacing near the bottom of the well is 36.9951 MHz