Homework 3

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Problem 1

Problem 2

Problem 3 (Berman 3.8)

Problem 4 (Berman 3.10)

Problem 5 (Berman 3.7)

In[4]:=
```
Symbolize[ Ω₀ ];
Symbolize[ σₓ ]; Symbolize[ σᵧ ]; Symbolize[ σz ];
Symbolize[ σ₊ ]; Symbolize[ σ₋ ]; Symbolize[ σ₀ ];
```

In[7]:=
```
$Assumptions = {ω ∈ ℝ, t ∈ ℝ, θ ∈ ℝ, ϕ ∈ ℝ};
σₓ = PauliMatrix[1]; σᵧ = PauliMatrix[2]; σz = PauliMatrix[3];
σ₊ = 1/2 (σₓ - i σᵧ); σ₋ = 1/2 (σₓ + i σᵧ); σ₀ = σ₊.σ₋;
Comm[A_, B_] := A.B - B.A;
CleanUp[x_] := TraditionalForm[MatrixForm[FullSimplify[x]]];
```

Problem 1

In[12]:=
```
H_FI[t_] = ħ/2 (-δ[t] σz + Re[Ω₀[t]] σₓ + Im[Ω₀[t]] σᵧ);
rho = {{ρ11, ρ12}, {ρ21, ρ22}};
ρ̇ = 1/(i ħ) Comm[H_FI[t], rho];
ρ̇ // CleanUp
```

Out[15]//TraditionalForm=

$$\begin{pmatrix} -\frac{1}{2} i (\rho_{21} \Omega_0(t)^* - \rho_{12} \Omega_0(t)) & \frac{1}{2} i ((\rho_{11} - \rho_{22}) \Omega_0(t)^* + 2 \rho_{12} \delta(t)) \\ -\frac{1}{2} i (2 \rho_{21} \delta(t) + (\rho_{11} - \rho_{22}) \Omega_0(t)) & \frac{1}{2} i (\rho_{21} \Omega_0(t)^* - \rho_{12} \Omega_0(t)) \end{pmatrix}$$

Problem 2

In[16]:=
```
{{Tr[ρ̇.σₓ], Tr[ρ̇.σᵧ], Tr[ρ̇.σz]}}ᵀ // CleanUp
```

Out[16]//TraditionalForm=

$$\begin{pmatrix} (\rho_{11} - \rho_{22}) \operatorname{Im}(\Omega_0(t)) + i (\rho_{12} - \rho_{21}) \delta(t) \\ (\rho_{22} - \rho_{11}) \operatorname{Re}(\Omega_0(t)) - (\rho_{12} + \rho_{21}) \delta(t) \\ -i (\rho_{21} \Omega_0(t)^* - \rho_{12} \Omega_0(t)) \end{pmatrix}$$

Problem 3

In[17]:=
```
rho[t_] = {{a[t], b[t]}, {c[t], d[t]}};
H[t_] = ħ/2 (-ω₀ σz + Ω₀ e^(i ω t) σ₊ + (Ω₀ e^(i ω t) σ₊)†);
rhoDot[t_] =
  1/(i ħ) (Comm[H[t], rho[t]] - i ħ γ (σ₀.rho[t] + rho[t].σ₀) + 2 i ħ γ σ₋.rho[t].σ₊);
rhoDot[t] // CleanUp
```

In[21]:=
```
DSolve[{a'[t] == (rhoDot[t][[1, 1]] /. {d[t] → 1 - a[t], c[t] → b[t]*}),
    b'[t] == (rhoDot[t][[1, 2]] /. {d[t] → 1 - a[t], c[t] → b[t]*}),
    a[0] == 1, c[0] == 0} // FullSimplify, {a, b}, t] // CleanUp
```

Problem 4

In[23]:=
```
ψ = {{Cos[θ / 2]}, {e^(i ϕ) Sin[θ / 2]}};
((ψ†.#.ψ)[[1]]) & /@ {σx, σy, σz} // CleanUp
```

Out[24]//TraditionalForm=

$$\begin{pmatrix} \sin(\theta)\cos(\phi) \\ \sin(\theta)\sin(\phi) \\ \cos(\theta) \end{pmatrix}$$

Problem 5

In[25]:=