Homework 1

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Problem 1

We can get the electric field amplitude from the intensity, as

$$I = \frac{P}{A} = \frac{1}{2} c \epsilon_0 E_0^2 \implies E_0 = \sqrt{\frac{2P}{c\epsilon_0 A}} \approx 868 \text{ V/m}.$$

A rough but simple estimate for the dipole moment is just $ea_0 \approx 2.5$ Debye. The Rabi frequency is then

$$\Omega_0 = \frac{\mu E_0}{\hbar} \approx 70 \text{ MHz}$$

See the end of the document for a printout of the Mathematica notebook used for these calculations.

Problem 2

Under the rotating wave approximation, we neglect the counter-rotating term and get as our differential equation (neglecting bars on the 'c's)

$$\begin{aligned} \dot{c}_1 &= -\frac{1}{2} i \Omega_0 e^{i\delta t} c_2 \\ \dot{c}_2 &= -\frac{1}{2} i \Omega_0 e^{-i\delta t} c_1. \end{aligned}$$

The Rabi frequency is directly proportional to the applied electric field. In the weak-field limit, we can perturbatively expand the amplitudes as

$$c_i = c_i^{(0)} + \Omega_0 c_i^{(1)} + \Omega_0^2 c_i^{(2)} + \cdots .$$

To zero-th order, the amplitudes are given by the initial conditions $c_1^{(0)} = c_1(0) = 1$, and $c_2^{(0)} = c_2(0) = 0$. Now we go to first order and plug into the differential equation:

$$\begin{aligned} & \frac{\mathrm{d}}{\mathrm{d}t}\left(c_1^{(0)} + \Omega_0 c_1^{(1)}\right) = -\frac{i}{2}\Omega_0 e^{i\delta t}\left(c_2^{0} + \Omega_0 c_2^{(1)}\right) \\ \implies \quad & \Omega_0 \dot{c}_1^{(1)} = -\frac{i}{2}\Omega_0^2 e^{i\delta t} c_2^{(1)} \end{aligned}$$

Matching terms proportional to Ω_0 gives

$$\begin{aligned} \dot{c}_1^{(1)} &= 0 \\ \implies \quad c_1^{(1)} &= c_1^{(1)}(0) = 0. \end{aligned}$$

For c_2, we find

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\left(c_2^{(0)} + \Omega_0 c_2^{(1)}\right) &= -\frac{i}{2}\Omega_0 e^{-i\delta t}\left(c_1^{(0)} + \Omega_0 c_1^{(1)}\right) \\ \implies \qquad \Omega_0 \dot{c}_2^{(1)} &= -\frac{i}{2}\Omega_0 e^{-i\delta t} + O(\Omega^2) \\ \implies \qquad c_2^{(1)} &= -\frac{i}{2}\int_0^t \mathrm{d}t' e^{-i\delta t'} \\ &= \frac{1}{2\delta}\left(e^{-i\delta t} - 1\right). \end{aligned}$$

So, to first order we have that

$$c_1 = 1 \tag{1}$$

$$c_2 = \frac{\Omega_0}{2\delta}\left(e^{-i\delta t} - 1\right) \tag{2}$$

Repeating the process for second order,

Problem 3

(a)

(b)

(c)

Problem 4

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In[1]:= P = 1 mW ; A = 1 mm^2 ;

        μ = e a_0 ; UnitConvert[μ, "Debyes"]

Out[2]= 2.541746473 D

In[3]:= E_0 = Sqrt[2 P / (c ε_0 A)] ; UnitConvert[E_0, "V/m"]

Out[3]= 868.021098 V/m

In[4]:= Ω_0 = μ E_0 / ℏ ; UnitConvert[Ω_0, "MHz"]

Out[4]= 69.7855727 MHz
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$$P = 1\,\mathrm{mW};\quad A = 1\,\mathrm{mm}^2;\quad \mu = e\,a_0;\quad E_0 = \sqrt{\frac{2P}{c\,\varepsilon_0\,A}};\quad \Omega_0 = \frac{\mu E_0}{\hbar}$$