```
In[1]:= Symbolize[ω0]; Symbolize[ω0']; Symbolize[ω']; Symbolize[δ']; Symbolize[Ω0]; Sy
        Symbolize[c1]; Symbolize[c2]; Symbolize[c3];

In[3]:= $Assumptions = {t > 0, ω ∈ ℝ, ω' ∈ ℝ, ω0 ∈ ℝ, ω0' ∈ ℝ, Ω0 ∈ ℝ, Ω0' ∈ ℝ, γ2,1 > 0, γ3,1 > 0, δ ∈

In[4]:= comm[a_, b_] := a.b - b.a;
```

Problem 1

```
In[6]:= c = Sqrt[Abs[Ω0]^2 + Abs[Ω0']^2];

In[9]:= U = 1/c {{Ω0', 0, Ω0}, {0, 1, 0}, {Ω0*, 0, -Ω0'*}}; U // MatrixForm // TraditionalForm
        H = ℏ {{-2 δ, Ω0*/2, 0}, {Ω0/2, 0, Ω0'/2}, {0, Ω0'*/2, -2 δ'}}; H // MatrixForm // TraditionalFo
```

Out[9]//TraditionalForm=

$$\begin{pmatrix} \frac{\Omega_0'}{\sqrt{|\Omega_0'|^2+|\Omega_0|^2}} & 0 & \frac{\Omega_0}{\sqrt{|\Omega_0'|^2+|\Omega_0|^2}} \\ 0 & \frac{1}{\sqrt{|\Omega_0'|^2+|\Omega_0|^2}} & 0 \\ \frac{(\Omega_0)^*}{\sqrt{|\Omega_0'|^2+|\Omega_0|^2}} & 0 & -\frac{(\Omega_0')^*}{\sqrt{|\Omega_0'|^2+|\Omega_0|^2}} \end{pmatrix}$$

Out[10]//TraditionalForm=

$$\begin{pmatrix} -2\,\delta\,\hbar & \frac{1}{2}\,\hbar\,(\Omega_0)^* & 0 \\ \frac{\Omega_0\,\hbar}{2} & 0 & \frac{\hbar\,\Omega_0'}{2} \\ 0 & \frac{1}{2}\,\hbar\,(\Omega_0')^* & -2\,\hbar\,\delta' \end{pmatrix}$$

```
In[11]:= U.H.U† /. {δ' → δ} // FullSimplify // MatrixForm // TraditionalForm
```

Out[11]//TraditionalForm=

$$\begin{pmatrix} -2\,\delta\,\hbar & \frac{\Omega_0\,\hbar\,\Omega_0'}{(\Omega_0')^2+\Omega_0^2} & 0 \\ \frac{\Omega_0\,\hbar\,\Omega_0'}{(\Omega_0')^2+\Omega_0^2} & 0 & \frac{\Omega_0^2\,\hbar}{(\Omega_0')^2+\Omega_0^2}-\frac{\hbar}{2} \\ 0 & \frac{\Omega_0^2\,\hbar}{(\Omega_0')^2+\Omega_0^2}-\frac{\hbar}{2} & -2\,\delta\,\hbar \end{pmatrix}$$

Problem 2

In[13]:=
```
rho = Table[Subscript[ρ, i, j], {i, 3}, {j, 3}]; rho // MatrixForm
```

Out[13]//MatrixForm=

$$\begin{pmatrix} \rho_{1,1} & \rho_{1,2} & \rho_{1,3} \\ \rho_{2,1} & \rho_{2,2} & \rho_{2,3} \\ \rho_{3,1} & \rho_{3,2} & \rho_{3,3} \end{pmatrix}$$

In[14]:=
$$\texttt{rhoDot} = \frac{-i}{\hbar}\ \texttt{comm[H, rho] // FullSimplify; rhoDot // MatrixForm}$$

Out[14]//MatrixForm=

$$\begin{pmatrix} \frac{1}{2}\, i\, \Omega_0\, (\rho_{1,2} - \rho_{2,1}) & \frac{1}{2}\, i\, (4\, \delta\, \rho_{1,2} + \Omega_0\, (\rho_{1,1} - \rho_{2,2}) + \rho_{1,3}\, \Omega_0{}') & \frac{1}{2}\, i\, (-\Omega_0\, \rho_{2,3} + \\ -\frac{1}{2}\, i\, (4\, \delta\, \rho_{2,1} + \Omega_0\, (\rho_{1,1} - \rho_{2,2}) + \rho_{3,1}\, \Omega_0{}') & \frac{1}{2}\, i\, (\Omega_0\, (-\rho_{1,2} + \rho_{2,1}) + (\rho_{2,3} - \rho_{3,2})\, \Omega_0{}') & -\frac{1}{2}\, i\, (\Omega_0\, \rho_{1,3} + 4 \\ \frac{1}{2}\, i\, (\Omega_0\, \rho_{3,2} + 4\, \rho_{3,1}\, (-\delta + \delta') - \rho_{2,1}\, \Omega_0{}') & \frac{1}{2}\, i\, (\Omega_0\, \rho_{3,1} + 4\, \rho_{3,2}\, \delta' + (-\rho_{2,2} + \rho_{3,3})\, \Omega_0{}') & -\frac{1}{2}\, i \end{pmatrix}$$

In[18]:=
```
rhoDot[[1, 2]] // TraditionalForm
rhoDot[[1, 3]] // TraditionalForm
rhoDot[[2, 3]] // TraditionalForm
```

Out[18]//TraditionalForm=

$$\frac{1}{2}\, i\, (4\, \delta\, \rho_{1,2} + \rho_{1,3}\, \Omega_0{}' + \Omega_0\, (\rho_{1,1} - \rho_{2,2}))$$

Out[19]//TraditionalForm=

$$\frac{1}{2}\, i\, (4\, \rho_{1,3}\, (\delta - \delta') + \rho_{1,2}\, \Omega_0{}' - \Omega_0\, \rho_{2,3})$$

Out[20]//TraditionalForm=

$$-\frac{1}{2}\, i\, (4\, \rho_{2,3}\, \delta' + (\rho_{3,3} - \rho_{2,2})\, \Omega_0{}' + \Omega_0\, \rho_{1,3})$$

Problem 3

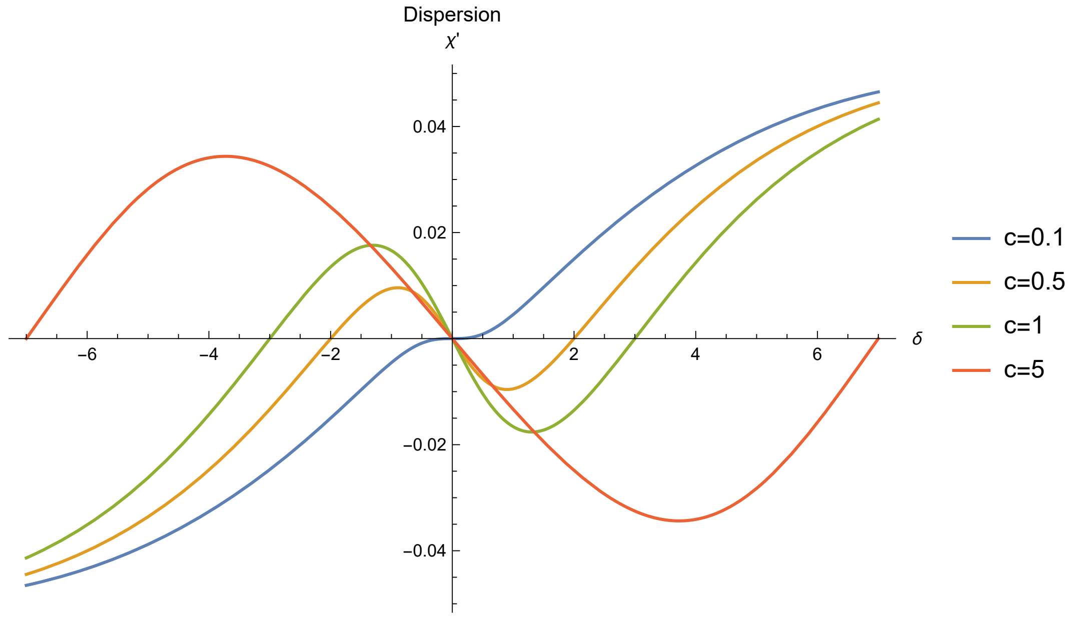
In[21]:=
```
Clear[c];
```

In[22]:=
$$\gamma_{3,1} = 1;$$
$$\gamma_{2,1} = 10\, \gamma_{3,1};$$
$$\Delta = \delta;$$
$$\Omega_0{}' = \sqrt{4\, c\, \gamma_{2,1}\, \gamma_{3,1}};$$
$$\chi = \frac{i}{\gamma_{2,1} + i\, \delta + \frac{(\Omega_0{}')^2}{4\, (\gamma_{3,1} + i\, \Delta)}};$$

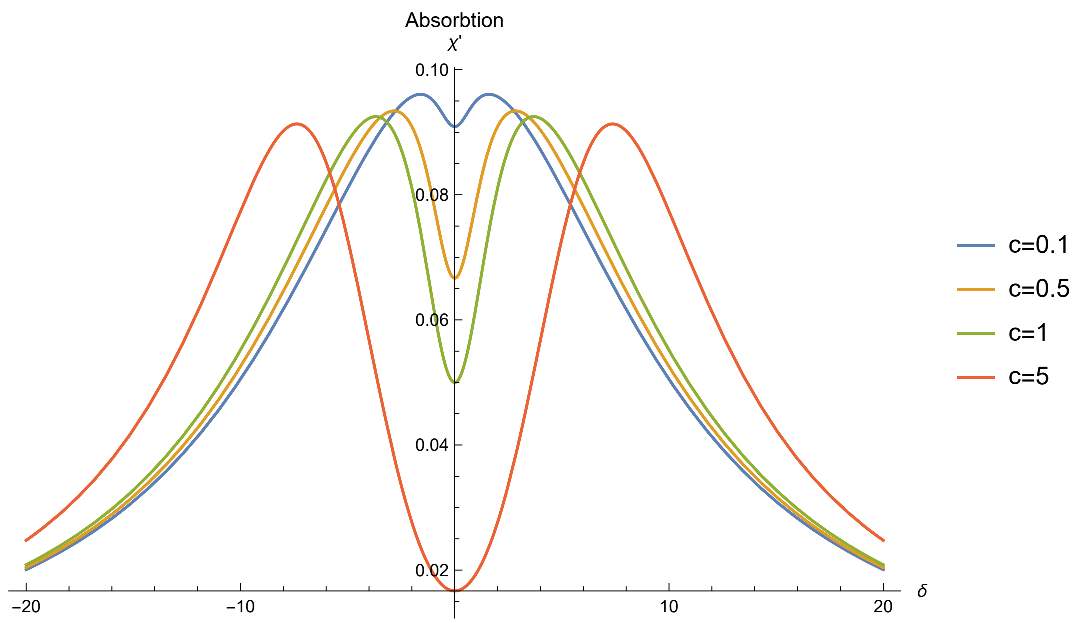
```
Cs = {0.1, 0.5, 1, 5};
legends = Table[StringForm["c=``", x], {x, Cs}];
```

In[29]:=
```
Plot[Evaluate@Table[Re[χ] /. c → x, {x, Cs}], {δ, -7, 7}, PlotLegends → legends, PlotLabel
Plot[Evaluate@Table[Im[χ] /. c → x, {x, Cs}], {δ, -20, 20}, PlotLegends → legends, PlotLab
```

Out[29]=



Out[30]=



Problem 4

Problem 5 (Berman 9.12)

In[ ]:=
```
inf = 90;
coefficients = {9.5, 10, 11};
delays = {0.75, 0, -0.75};
equations = {c1'[t] == -ⅈ a ⅇ^(-t^2) c2[t], c2'[t] == -ⅈ a (c1[t] ⅇ^(-t^2) - c3[t] ⅇ^(-(t - τ)^2)), c3'[t] == -ⅈ
initialConditions = {c1[-inf] == 1, c2[-inf] == 0, c3[-inf] == 0};
```

In[ ]:=
```
soln = ParametricNDSolveValue[Join[equations, initialConditions], c3, {t, -inf, inf}, {a,
```

In[ ]:=
```
m = Table[Evaluate[Abs[soln[coeff, delay][inf]]], {coeff, coefficients}, {delay, delays}];
TableOfValues1 = Prepend[m, Table[StringForm["a0 = ``", x], {x, coefficients}]];
TableOfValues2 = MapThread[Prepend, {TableOfValues1, Join[{""}, Table[StringForm["τ0 = ``",
Grid[TableOfValues2, Frame → All]
```

Out[ ]=

	a_0 = 9.5`	a_0 = 10	a_0 = 11
τ_0 = 0.75`	16.8383	16.8383	16.8383
τ_0 = 0	17.7245	17.7245	17.7245
τ_0 = -0.75	19.497	19.497	19.497