Homework 23

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Phys 662

February 7, 2024

Problem 1 (Peskin 10.1)

$$f_{\text{val}}(x) = A(1+ax)(1-x)^{\beta}$$

$$\int_0^1 \mathrm{d}x\ A(1+ax)(1-x)^{\beta} = A\frac{a+\beta+2}{\beta^2+3\beta+2} = n_q$$

(a) The equation above was solved in mathematica with $n_q = 2$ for u and $n_q = 1$ for d, with the results in the table below.

	u	d
$Q = 3.1$ GeV	$A = 9.23$	$A = 5.56$
$Q = 100$ GeV	$A = 11.5$	$A = 6.77$

(b) Average momentums fractions:

	u	d
$Q = 3.1$ GeV	$x = 0.351$	$x = 0.153$
$Q = 100$ GeV	$x = 0.295$	$x = 0.187$

(c) Total momentum fraction:

	Valence Momentum Fraction
$Q = 3.1$ GeV	$x_u + x_d = 0.505$
$Q = 100$ GeV	$x_u + x_d = 0.482$

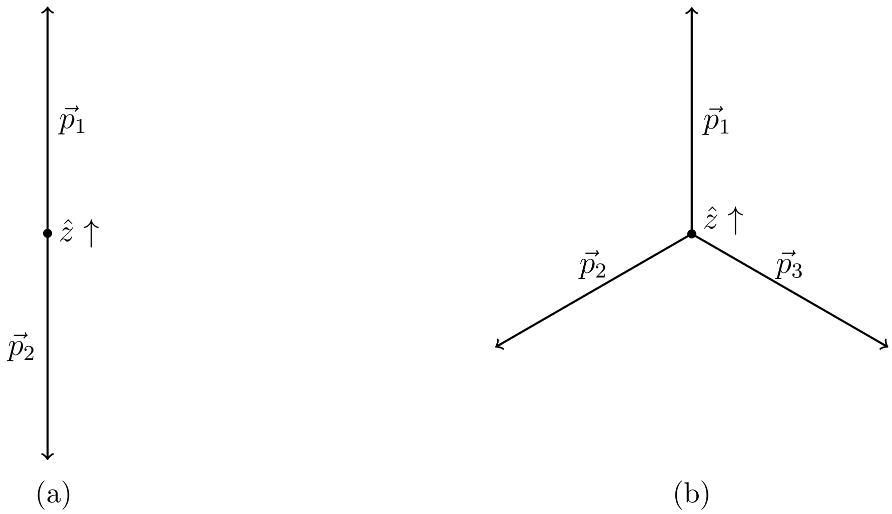

Figure 1: (a) A 2-jet event. (b) A 3-jet event.

Problem 2

For this problem I will refer to "directional thrust", which I'll define as

$$\tau(\hat{n}) \coloneqq \sum_i |\vec{p}_i \cdot \hat{n}|,$$

such that the actual thrust is given by

$$\tau = \frac{\max_{\hat{n}} \tau(\hat{n})}{\sum_i |\vec{p}_i|}.$$

(a) Choose a coordinate system such that $\hat{n}_0 = \hat{z}$, such as depicted in Figure 1a. Denote the common magnitude of the vectors by p. By the symmetry of the event, the directional thrust is maximized in the $\pm\hat{z}$ directions (and minimized in directions orthogonal to $\hat{z}$.) Therefore, the thrust is given by

$$\tau = \frac{|\vec{p}_1 \cdot \hat{n}| + |\vec{p}_2 \cdot \hat{n}|}{|\vec{p}_1| + |\vec{p}_2|} = \frac{|p| + |-p|}{2p} = 1$$

(b) Choose a coordinate system as in part (a), and again denote the magnitude of the vectors by p. Again, by the symmetry of the event, the directional thrust is maximized in the $\pm\hat{z}$ directions (or, equivalently, in the directions parallel to $\vec{p}_2$ or $\vec{p}_3$). Noting that

$$|\vec{p}_2 \cdot \hat{z}| = \left| |\vec{p}_2| \cos\frac{2\pi}{3} \right| = \left| -\frac{1}{2}|\vec{p}_2| \right| = \frac{p}{2},$$

and similarly for $\vec{p}_3$, we have that the thrust is

$$\tau = \frac{p + \frac{p}{2} + \frac{p}{2}}{3p} = \frac{2}{3}$$

(c) For a spherically symmetric event, we can evaluate the directional thrust in any direction to get the full thrust. Choosing the $\hat{z}$ direction,

$$\tau(\hat{z}) = \int \mathrm{d}\Omega|\vec{p}\cdot\hat{z}| = 2\pi \int_0^\pi \sin\theta \mathrm{d}\theta\ |\vec{p}||\cos\theta| = 2\pi p.$$

Therefore,

$$\tau = \frac{2\pi p}{\int \mathrm{d}\Omega|\vec{p}|} = \frac{2\pi p}{\int \mathrm{d}\Omega p} = \frac{2\pi p}{4\pi p} = \frac{1}{2}$$

Problem 3

(a)

$$\begin{aligned}\left[\Sigma_a t_R^a t_R^a, t_R^b\right] &= \Sigma_a \left(t_R^a\left[t_R^a, t_R^b\right] + \left[t_R^a, t_R^b\right]t_R^a\right)\\ &= \Sigma_{ac} f^{abc}\left(t_R^a t_R^c + t_R^c t_R^a\right)\\ &= 0\end{aligned}$$

The last equality comes from the fact that the structure constants are antisymmetric in the contracted indices (a and c), while the object it's contracted with (the anticommutator of t_R^a and t_R^c) is symmetric in those indices.

(b) (see attached pdf)

(c) (see attached pdf)

In[∘]:=
```
g1 = {{0, 1, 0}, {1, 0, 0}, {0, 0, 0}} / 2;
g2 = {{0, -ⅈ, 0}, {ⅈ, 0, 0}, {0, 0, 0}} / 2;
g3 = {{1, 0, 0}, {0, -1, 0}, {0, 0, 0}} / 2;
g4 = {{0, 0, 1}, {0, 0, 0}, {1, 0, 0}} / 2;
g5 = {{0, 0, -ⅈ}, {0, 0, 0}, {ⅈ, 0, 0}} / 2;
g6 = {{0, 0, 0}, {0, 0, 1}, {0, 1, 0}} / 2;
g7  = {{0, 0, 0}, {0, 0, -ⅈ}, {0, ⅈ, 0}} / 2;
g8 = {{1, 0, 0}, {0, 1, 0}, {0, 0, -2}} / Sqrt[12];
```

In[∘]:=
```
λ = {g1, g2, g3, g4, g5, g6, g7, g8};
```

In[∘]:=
```
(#.#) & /@ λ  // Total // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{4}{3} & 0 & 0 \\ 0 & \frac{4}{3} & 0 \\ 0 & 0 & \frac{4}{3} \end{pmatrix}$$

In[∘]:=
```
comm[x_, y_] := x.y - y.x;
```
$f[a\_, b\_, c\_] := \frac{1}{2\,ⅈ} \mathrm{Tr}[\mathrm{comm}[λ[\![a]\!], λ[\![b]\!]].λ[\![c]\!]];$

In[∘]:=
```
f[2, 5, 7]
```

Out[∘]= $\frac{1}{8}$

In[∘]:=
```
f[2, 7, 5]
```

Out[∘]= $-\frac{1}{8}$