Exercise Set 8

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Monday

Exercise 1

$$\begin{aligned}
\langle \tilde{x}(t_1)\tilde{y}(t_2)\rangle &= \mathrm{tr}\left[\tilde{x}(t_1)\tilde{y}(t_2)\rho_0\right] \\
&= \mathrm{tr}\left[\tilde{x}(t_1)e^{iH_0t_2/\hbar}\tilde{y}(0)e^{-iH_0t_2/\hbar}\rho_0\right] \\
&= \mathrm{tr}\left[e^{-iH_0t_2/\hbar}\tilde{x}(t_1)e^{iH_0t_2/\hbar}\tilde{y}(0)\rho_0\right] \\
&= \mathrm{tr}\left[\tilde{x}(t_1-t_2)\tilde{y}(0)\right] \\
&= \langle \tilde{x}(t_1-t_2)\tilde{y}(0)\rangle
\end{aligned}$$

Tuesday

Exercise 1

(a) Given that

$$\omega_{eg} = \frac{3\alpha^2 m_e c^2}{8}, \quad \left|\vec{r}_{eg}\right|^2 = \left|z_{ge}\right|^2 = \frac{2^{15}a_0}{3^{10}}, \quad \alpha = \frac{1}{4\pi\epsilon_0}\frac{e^2}{\hbar c}$$

We have

$$\begin{aligned}
\Gamma &= \frac{\omega_{eg}^3 e^2 \left|\vec{r}_e g\right|^2}{3\pi\epsilon_0\hbar c^3} \\
&= \frac{3^3}{2^9}\frac{\alpha^6 m_e^3 c^6}{\hbar^3}\frac{e^2}{3\pi\epsilon_0\hbar c^3}\frac{2^{15}}{3^{10}}\frac{\hbar^2}{m_e^2 c^2\alpha^2} \\
&= \frac{2^6 m_e c e^2\alpha^4}{3^8\hbar^2\pi\epsilon_0} \\
&= \left(\frac{2}{3}\right)^8 \frac{m_e c^2\alpha^5}{\hbar}
\end{aligned}$$

(b) Plugging the above expression into WolframAlpha gives

$$6.268315 \times 10^8 \text{Hz}$$

which is pretty good!