Exercise Set 4

Sean Ericson
Phys 632

January 31, 2022

Exercise 1

$$\begin{aligned}
[L_\alpha, p_\delta] &= [\epsilon_{\alpha\beta\gamma} r_\beta p_\gamma, p_\delta] \\
&= \epsilon_{\alpha\beta\gamma}[r_\beta, p_\delta]p_\gamma \\
&= i\hbar\epsilon_{\alpha\beta\gamma}\delta_{\beta\gamma}p_\gamma \\
&= i\hbar\epsilon_{\alpha\beta\gamma}p_\gamma
\end{aligned}$$

Exercise 2

$$\begin{aligned}
[L_x, L_y] &= [L_x, zp_x - xp_z] \\
&= [L_x, zp_x] - [L_x, xp_z] \\
&= [L_x, z]p_x - x[L_x, p_z] \\
&= i\hbar\left(yp_x - xp_y\right) \\
&= L_z
\end{aligned}$$

By cyclic permutation of the indicies,

$$[L_\alpha, L_\beta] = i\hbar\epsilon_{\alpha\beta\gamma}L_\gamma$$

Exercise 3

(a) 0; Violation of the triangle condition

(b) 0; m-quantum numbers don't add

(c) 1; One particle has no angular momentum, so the combined momentum must be that of the other.

(d) 0; By the above argument.

(e) 1/3; The $|00\rangle$ state has components $|1-1;11\rangle$, $|1,1;1,-1\rangle$, and $|1,0;1,0\rangle$.