In[5]:=
```
Symbolize[ δ₀ ]; Symbolize[ Ω₀ ]; Symbolize[ T₀ ]; Symbolize[ H_I ];
Symbolize[ ψ_I ]; Symbolize[ c₁ ]; Symbolize[ c₂ ];
Symbolize[ s_θ ]; Symbolize[ c_θ ];
$Assumptions = {T > 0};
```

# Problem 1

In[9]:=
```
Ω₀[t_] = A e^-(t/T)^2;
R[t_] = √(δ^2 + (Ω₀[t])^2) // FullSimplify;
```

In[11]:=
```
Integrate[Ω₀[t], {t, -∞, t}]
```

Out[11]=

$$\frac{1}{2} A \sqrt{\pi}\, T \left(1 + \text{Erf}\left[\frac{t}{T}\right]\right)$$

In[12]:=
```
c_θ[t_] = √(1/2 (1 + δ/R[t])) // FullSimplify;
s_θ[t_] = √(1/2 (1 - δ/R[t])) // FullSimplify;
```

In[15]:=
```
s_θ[-∞] // FullSimplify
```

Out[15]=

$$\frac{\sqrt{1 - \frac{\delta}{\sqrt{\delta^2}}}}{\sqrt{2}}$$

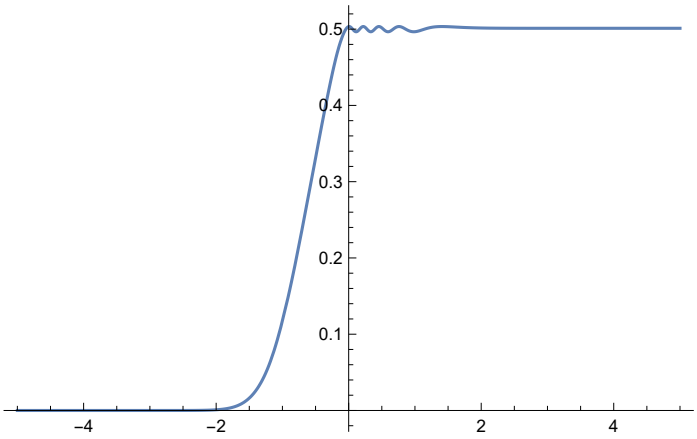
# Problem 2

In[ ]:=
```
δ[t_] = δ₀ (1 - e^(t/T))^3 HeavisideTheta[-t];
H_I[t_] = 1/2 {{0, Ω₀[t] e^-i δ[t]}, {Ω₀[t] e^i δ[t], 0}};
ψ_I[t_] = {{c₁[t]}, {c₂[t]}};
```

In[•]:=
```
T₀ = 5;
T = 1;
δ₀ = 30;
A = 30;
soln1 =
  NDSolve[{i D[ψI[t], t] == HI[t].ψI[t], ψI[-T₀] == {{1}, {0}}}, c₁, {t, -T₀, T₀}];
soln2 = NDSolve[{i D[ψI[t], t] == HI[t].ψI[t], ψI[-T₀] == {{1}, {0}}}, c₂, {t, -T₀, T₀}];
soln12 = Union[soln1, soln2];
```

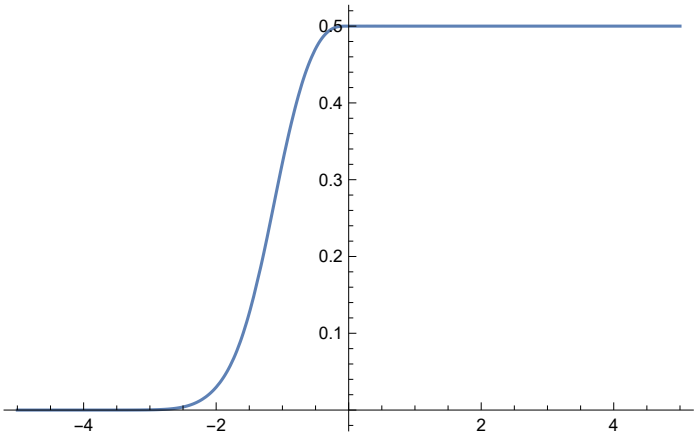
In[•]:=
```
Plot[Evaluate[Abs[c₂[x]]^2 /. soln2], {x, -T₀, T₀}, PlotRange → All]
```

Out[•]=




In[•]:=
```
Plot[Evaluate[Evaluate[Abs[c₁[x] Conjugate[c₂[x]]] /. soln1] /. soln2],
 {x, -T₀, T₀}, PlotRange → All]
```

Out[•]=




# Problem 3

In[•]:=
```
a
```

Out[•]=

a

# Problem 4

In[•]:= a

Out[•]=

a

# Problem 5

In[•]:= "$\Omega = \sqrt{\delta^2 + A^2}$;"

$c_1 = e^{-i\delta t/2}\left(\text{Cos}\left[\frac{\Omega t}{2}\right] + \frac{i\delta}{\Omega}\text{Sin}\left[\frac{\Omega t}{2}\right]\right)$;

$c_2 = -\frac{iA}{\Omega}e^{i\delta t/2}\text{Sin}\left[\frac{\Omega t}{2}\right]$;

Out[•]=

$\Omega = \sqrt{\delta^2 + A^2}$;

In[•]:= $c_1$ // TrigToExp // Simplify

Out[•]=

$$\frac{e^{-\frac{1}{2}it\delta}e^{-\frac{1}{2}it\Omega}\left(\left(-1+e^{it\Omega}\right)\delta+\left(1+e^{it\Omega}\right)\Omega\right)}{2\Omega}$$

In[•]:= $c_2$ // TrigToExp // Simplify

Out[•]=

$$-\frac{Ae^{\frac{1}{2}it(\delta-\Omega)}\left(-1+e^{it\Omega}\right)}{2\Omega}$$

In[•]:= $$\frac{\left(\left(-1+e^{it\Omega}\right)\delta+\left(1+e^{it\Omega}\right)\Omega\right)}{2\Omega}$$ // FullSimplify

Out[•]=

$$\frac{-\delta+\Omega+e^{it\Omega}(\delta+\Omega)}{2\Omega}$$