In[10]:=
```
Symbolize[ δ₀ ]; Symbolize[ Ω₀ ]; Symbolize[ T₀ ]; Symbolize[ H_I ];
Symbolize[ ψ_I ]; Symbolize[ c₁ ]; Symbolize[ c₂ ];
Symbolize[ s_θ ]; Symbolize[ c_θ ]; Symbolize[ ω₁ ]; Symbolize[ ω₂ ];
$Assumptions = {T > 0 , t ∈ ℝ, ω₁ > 0, ω₂ > 0, ω > 0};
```

Problem 1

In[•]:=
```
Ω₀[t_] = A e^-(t/T)^2;
R[t_] = √(δ^2 + (Ω₀[t])^2) // FullSimplify;
```

In[•]:=
```
Integrate[Ω₀[t], {t, -∞, t}]
```

Out[•]=

$$\frac{1}{2} A \sqrt{\pi}\, T \left(1 + \text{Erf}\left[\frac{t}{T}\right]\right)$$

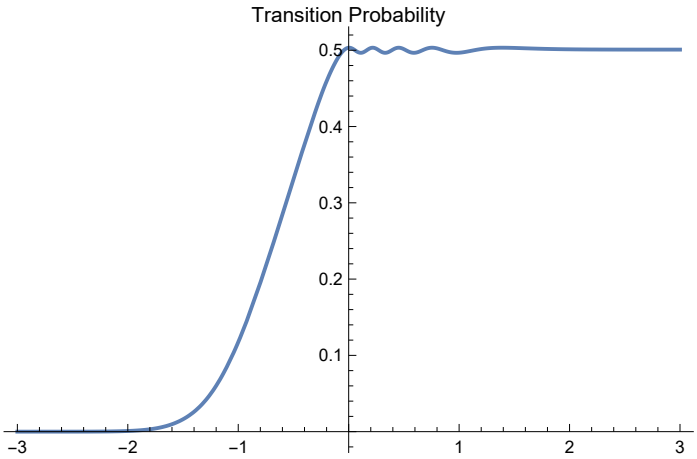
Problem 2

In[•]:=
```
δ[t_] = δ₀ (1 - e^(t/T))^3 HeavisideTheta[-t];
H_I[t_] = 1/2 {{0, Ω₀[t] e^-i δ[t]}, {Ω₀[t] e^i δ[t], 0}};
ψ_I[t_] = {{c₁[t]}, {c₂[t]}};

T₀ = 3;
T = 1;
δ₀ = 30;
A = 30;
soln1 =
  NDSolve[{i D[ψ_I[t], t] == H_I[t].ψ_I[t], ψ_I[-T₀] == {{1}, {0}}}, c₁, {t, -T₀, T₀}];
soln2 = NDSolve[{i D[ψ_I[t], t] == H_I[t].ψ_I[t], ψ_I[-T₀] == {{1}, {0}}}, c₂, {t, -T₀, T₀}];
```

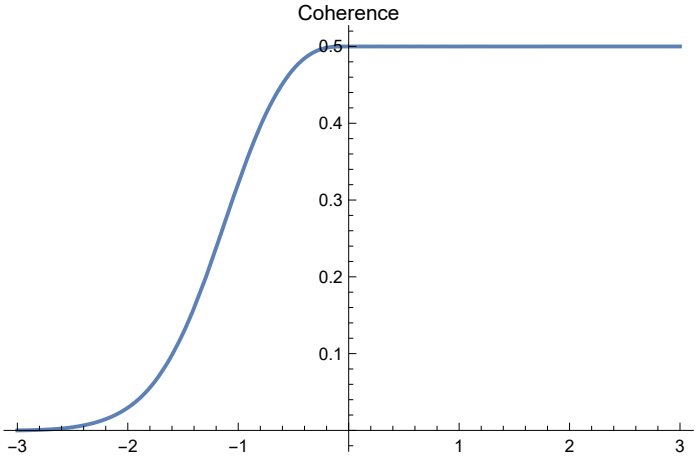
In[•]:=
```
Plot[Evaluate[Abs[c2[x]]^2 /. soln2], {x, -T0, T0},
 PlotRange → All, PlotLabel → "Transition Probability"]
```

Out[•]=



In[•]:=
```
Plot[Evaluate[Evaluate[Abs[c1[x] Conjugate[c2[x]]] /. soln1] /. soln2],
 {x, -T0, T0}, PlotRange → All, PlotLabel → "Coherence"]
```

Out[•]=



Problem 3

In[5]:=
```
U = {{ℯ^(-ⅈ ω1 t), 0}, {0, ℯ^(-ⅈ ω2 t)}};
V = {{0, Omega Cos[ω t]}, {Omega Cos[ω t], 0}};
```

In[7]:=
```
U†.V.U // FullSimplify // MatrixForm
```

Out[7]//MatrixForm=

$$\begin{pmatrix} 0 & e^{i t (\omega_1-\omega_2)}\ \text{Omega}\ \text{Cos}[t\,\omega] \\ e^{i t (-\omega_1+\omega_2)}\ \text{Omega}\ \text{Cos}[t\,\omega] & 0 \end{pmatrix}$$

In[16]:=
```
U = {{e^(-i ω t/2), 0}, {0, e^(i ω t/2)}};
U†.V.U // FullSimplify // MatrixForm
```

Out[17]//MatrixForm=

$$\begin{pmatrix} 0 & e^{i\,t\,\omega}\,\text{Omega}\,\text{Cos}[t\,\omega] \\ e^{-i\,t\,\omega}\,\text{Omega}\,\text{Cos}[t\,\omega] & 0 \end{pmatrix}$$

Problem 4

In[•]:=
```
R = √(δ^2 + Ω0^2);
(-2 Ω0/(R+δ))/((Ω0/(R+δ))^2 - 1) // FullSimplify
```

Out[•]=

$$\frac{\Omega_0}{\delta}$$

Problem 5

In[•]:=
```
"Ω = √(δ^2 + A^2);"
c1 = e^(-i δ t/2) (Cos[Ω t/2] + (i δ)/Ω Sin[Ω t/2]);
```

Out[•]=

$$\Omega = \sqrt{\delta^2 + A^2};$$

In[•]:=
```
c1 // TrigToExp // Simplify
```

Out[•]=

$$\frac{1}{2\,\Omega}e^{-\frac{1}{2}i\,t\,\delta}\,e^{-\frac{1}{2}i\,t\,\Omega}\left(\left(-1+e^{i\,t\,\Omega}\right)\delta+\left(1+e^{i\,t\,\Omega}\right)\Omega\right)$$

In[•]:=
```
((-1 + e^(i t Ω)) δ + (1 + e^(i t Ω)) Ω)/(2 Ω) // FullSimplify
```

Out[•]=

$$\frac{-\delta+\Omega+e^{i\,t\,\Omega}\,(\delta+\Omega)}{2\,\Omega}$$

In[•]:=
```
c2 = e^(i δ t/2) (Cos[Ω t/2] - i δ/Ω Sin[Ω t/2]);
```

In[•]:=
```
c2 // TrigToExp // FullSimplify
```

Out[•]=

$$\frac{e^{\frac{1}{2}i\,t\,(\delta-\Omega)}\left(\delta+\Omega+e^{i\,t\,\Omega}\,(-\delta+\Omega)\right)}{2\,\Omega}$$