

Problem 1 (Peskin 14.2)

In[1]:= << Notation`

In[2]:= Symbolize[α_s]; Symbolize[α_{s0}]; Symbolize[Q_0]; Symbolize[b_0]; Symbolize[n_f];
 Symbolize[m_0]; Symbolize[m_f]; Symbolize[m_u]; Symbolize[m_d];
 Symbolize[m_s]; Symbolize[m_c]; Symbolize[m_t]; Symbolize[m_b];

In[4]:= \$Assumptions = {Q > 0};

In[5]:= $m_u = 0.0022$;
 $m_d = 0.0047$;
 $m_s = 0.096$;
 $m_c = 1.28$;
 $m_b = 4.18$;
 $m_t = 164$;

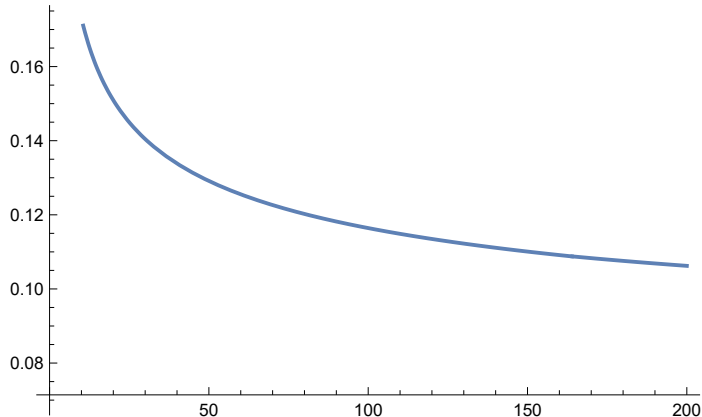
In[11]:= $b_0 = 11 - \frac{2}{3} n_f$;

$f[q_, q0_, \alpha0_] := \frac{\alpha0}{1 + \left(\frac{b_0 \alpha0}{2\pi}\right) \text{Log}[q / q0]}$;

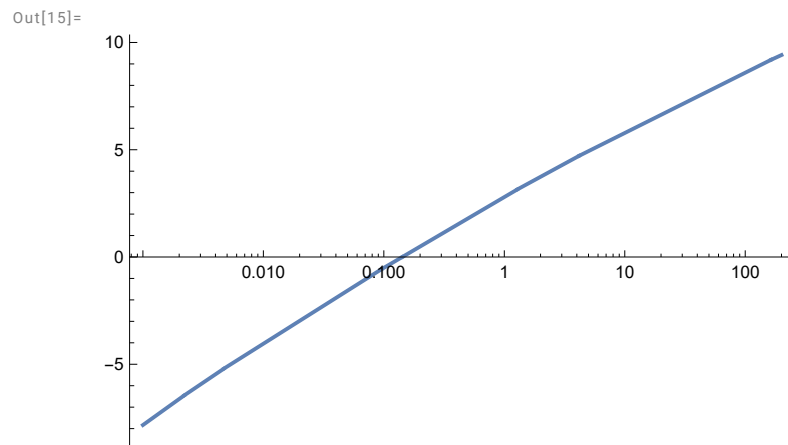
$\alpha_s[Q_] := \text{Piecewise}[\{\{f[Q, m_t - 1, \alpha_s[m_t - 1]] /. n_f \rightarrow 6, m_t < Q\},$
 $\{f[Q, 91, 0.118] /. n_f \rightarrow 5, m_b \leq Q < m_t\},$
 $\{f[Q, m_b, \alpha_s[m_b]] /. n_f \rightarrow 4, m_c \leq Q < m_b\}, \{f[Q, m_c, \alpha_s[m_c]] /. n_f \rightarrow 3,$
 $m_s \leq Q < m_c\}, \{f[Q, m_s, \alpha_s[m_s]] /. n_f \rightarrow 2, m_d \leq Q < m_s\},$
 $\{f[Q, m_d, \alpha_s[m_d]] /. n_f \rightarrow 1, m_u \leq Q < m_d\}, \{f[Q, m_u, \alpha_s[m_u]] /. n_f \rightarrow 0, Q < m_u\}\}\};$

In[14]:= Plot[$\alpha_s[Q]$, {Q, 0.001, 200}]

Out[14]=



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In[15]:= LogLinearPlot[ $\frac{1}{\alpha_s[Q]}$ , {Q, 0.001, 200}]
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a)

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In[16]:=  $\alpha_s[m_b]$ 
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Out[16]=
0.212056

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In[17]:=  $\alpha_s[2]$ 
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Out[17]=
0.26752

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In[18]:=  $\alpha_s[m_c]$ 
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Out[18]=
0.317851

b)

$$m[Q_, m0_, Q0_] := m0 \left(\frac{\alpha_s[Q]}{\alpha_s[Q0]} \right)^{\frac{4}{b_0}}$$

c)

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In[32]:=
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m[2, m_c, m_c] /. n_f -> 4
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Out[32]=
1.17835

d)

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In[38]:= n_f = 5;  
         m[m_b, m_u, 2]  
         m[m_b, m_d, 2]  
         m[m_b, m_s, 2]  
         m[m_b, m_c, 1.28]
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Out[39]=  
0.00197001
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Out[40]=  
0.00420866
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Out[41]=  
0.0859641
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Out[42]=  
1.05772
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