

Exercise Set 3

Sean Ericson
Phys 633

May 15, 2022

Monday

Exercise 1

$$[p^2, z] = 2i\hbar p_z = -2\hbar^2 \partial_z$$

Exercise 2

$$\begin{aligned} [p^2, r]f &= -\hbar^2 [\nabla^2, r]f \\ &= -\hbar^2 (\nabla^2(rf) - r(\nabla^2 f)) \\ &= -\hbar^2 ((\nabla^2 r)f + 2(\nabla r) \cdot (\nabla f) + r\nabla^2 f - r\nabla^2 f) \\ &= -2\hbar^2 \left(\frac{1}{r} - (\nabla r) \cdot \nabla \right) f \\ &= -2\hbar^2 \left(\frac{1}{r} - \hat{r} \cdot \nabla \right) f \\ \implies [p^2, r] &= -2\hbar^2 \left(\frac{1}{r} - \hat{r} \cdot \nabla \right) \end{aligned}$$

Tuesday

Exercise 1

$$\begin{aligned} \mathcal{U}_1 &= S_1 V P_0 = G_Q V P_0 \\ \mathcal{U}_2 &= S_1 V S_1 V P_0 + S_2 V S_0 V P_0 = G_Q V G_Q V P_0 - G_Q^2 V P_0 V P_0 \end{aligned}$$