Problem 1

In[1]:=
```
P = 1 mW; A = 1 mm^2;
```

In[2]:=
```
μ = e a_0; UnitConvert[μ, "Debyes"]
```

Out[2]= 2.541746473 D

In[3]:=
```
E_0 = Sqrt[2 P / (c ε_0 A)]; UnitConvert[E_0, "V/m"]
```

Out[3]= 868.021098 V/m

In[4]:=
```
Ω = μ E_0 / ħ; UnitConvert[Ω, "MHz"]
```

Out[4]= 69.7855727 MHz

Problem 2

In[5]:=
```
Clear[Ω];
c1 = {1, 0, 0, 0};
c2 = {0, 0, 0, 0};
```

In[8]:=
```
For[j = 2, j < 5, j++,
 c1[[j]] = Integrate[(-i/2) e^(i δ t) c2[[j - 1]], {t, 0, t}] // FullSimplify;
 c2[[j]] = Integrate[(-i/2) e^(-i δ t) c1[[j - 1]], {t, 0, t}] // FullSimplify;
]
```

In[9]:=
```
combine[order_, coeff_] = Sum[Indexed[coeff, j] Ω_0^(j-1), {j, 1, order + 1}];
sqrMag[z_] := Collect[FullSimplify[ComplexExpand[Abs[z]^2]], Ω_0];
showOrder[order_] := StringForm["Order ``: \n
 c_1 ~ `` ⇒ |c_1|^2 ~ `` \n
 c_2 ~ `` ⇒ |c_2|^2 ~ ``\n\n",
   order, combine[order, c1], sqrMag[combine[order, c1]],
   combine[order, c2], sqrMag[combine[order, c2]]];
```

In[12]:=
```
Column[Table[showOrder[i], {i, 0, 3}]]
```

Out[12]=

Order 0:

$$c_1 \sim 1 \Rightarrow |c_1|^2 \sim 1$$

$$c_2 \sim 0 \Rightarrow |c_2|^2 \sim 0$$

Order 1:

$$c_1 \sim 1 \Rightarrow |c_1|^2 \sim 1$$

$$c_2 \sim \frac{\left(-1+e^{-i\,t\,\delta}\right)\,\Omega_0}{2\,\delta} \Rightarrow |c_2|^2 \sim \frac{\text{Sin}\left[\frac{t\,\delta}{2}\right]^2\,\Omega_0^2}{\delta^2}$$

Order 2:

$$c_1 \sim 1 + \frac{\left(-1+e^{i\,t\,\delta}-i\,t\,\delta\right)\,\Omega_0^2}{4\,\delta^2} \Rightarrow |c_1|^2 \sim 1 + \frac{(-1+\text{Cos}[t\,\delta])\,\Omega_0^2}{2\,\delta^2} + \frac{\left(2+t^2\,\delta^2-2\,\text{Cos}[t\,\delta]-2\,t\,\delta\,\text{Sin}[t\,\delta]\right)\,\Omega_0^4}{16\,\delta^4}$$

$$c_2 \sim \frac{\left(-1+e^{-i\,t\,\delta}\right)\,\Omega_0}{2\,\delta} \Rightarrow |c_2|^2 \sim \frac{\text{Sin}\left[\frac{t\,\delta}{2}\right]^2\,\Omega_0^2}{\delta^2}$$

Order 3:

$$c_1 \sim 1 + \frac{\left(-1+e^{i\,t\,\delta}-i\,t\,\delta\right)\,\Omega_0^2}{4\,\delta^2} \Rightarrow |c_1|^2 \sim 1 + \frac{(-1+\text{Cos}[t\,\delta])\,\Omega_0^2}{2\,\delta^2} + \frac{\left(2+t^2\,\delta^2-2\,\text{Cos}[t\,\delta]-2\,t\,\delta\,\text{Sin}[t\,\delta]\right)\,\Omega_0^4}{16\,\delta^4}$$

$$c_2 \sim \frac{\left(-1+e^{-i\,t\,\delta}\right)\,\Omega_0}{2\,\delta} + \frac{\left(2-i\,t\,\delta+e^{-i\,t\,\delta}\,(-2-i\,t\,\delta)\right)\,\Omega_0^3}{8\,\delta^3} \Rightarrow |c_2|^2 \sim$$
$$\frac{\text{Sin}\left[\frac{t\,\delta}{2}\right]^2\,\Omega_0^2}{\delta^2} + \frac{\left(8\,t\,\delta^3\,\text{Cos}\left[\frac{t\,\delta}{2}\right]\,\text{Sin}\left[\frac{t\,\delta}{2}\right]-16\,\delta^2\,\text{Sin}\left[\frac{t\,\delta}{2}\right]^2\right)\,\Omega_0^4}{16\,\delta^6} + \frac{\left(t^2\,\delta^2\,\text{Cos}\left[\frac{t\,\delta}{2}\right]^2-4\,t\,\delta\,\text{Cos}\left[\frac{t\,\delta}{2}\right]\,\text{Sin}\left[\frac{t\,\delta}{2}\right]+4\,\text{Sin}\left[\frac{t\,\delta}{2}\right]^2\right)\,\Omega_0^6}{16\,\delta^6}$$

Problem 3

In[13]:=
```
σx = PauliMatrix[1]; σz = PauliMatrix[3];
```

In[14]:=
```
H0 = -(ħ ω0 / 2) σz;
V = ħΩ0 Cos[ω t - ϕ] σx;
U = MatrixExp[-i H0 t / ħ];
```

In[17]:=
```
V_I = U†.V.U // ComplexExpand // TrigToExp // FullSimplify;
V_I // MatrixForm
```

Out[18]//MatrixForm=

$$\begin{pmatrix} 0 & e^{-i t \omega_0} \cos[\phi - t\,\omega]\ \hbar\Omega_0 \\ e^{i t \omega_0} \cos[\phi - t\,\omega]\ \hbar\Omega_0 & 0 \end{pmatrix}$$

Problem 4

In[19]:=
```
M = -1/2 {{γ1, i Ω0}, {i Ω0, γ2}};
ψ0 = {{1}, {0}};
ψ[t_] = {a1[t], a2[t]};
```

In[22]:=
```
soln = MatrixExp[M t].ψ0 // FullSimplify;
subs1 = {Sqrt[(γ1 - γ2)^2 - 4 Ω0^2] → χ};
subs2 = {γ1 → 1/2 (γ^+ + Δγ), γ2 → 1/2 (γ^+ - Δγ)};
soln = (Numerator[soln] /. subs1) / (Denominator[soln] /. subs1);
soln =
  FullSimplify[Numerator[soln] /. subs2] / FullSimplify[Denominator[soln] /. subs2];
StringForm["c1 = `` \nc2 = `` \n|c2|^2 = ``", soln[[1]][[1]] // FullSimplify,
 soln[[2]][[1]] // FullSimplify, sqrMag[soln[[2]][[1]]] // FullSimplify]
```

Out[27]=

$$c_1 = \frac{e^{-\frac{1}{4} t (\chi+\gamma^+)} \left(\Delta\gamma + \chi + e^{\frac{t\chi}{2}} (-\Delta\gamma + \chi)\right)}{2\chi}$$

$$c_2 = -\frac{i\, e^{-\frac{1}{4} t (\chi+\gamma^+)} \left(-1 + e^{\frac{t\chi}{2}}\right) \Omega_0}{\chi}$$

$$|c_2|^2 = \frac{\left(1 + e^{t\chi} - 2\sqrt{e^{t\chi}}\right) \Omega_0^2}{\sqrt{e^{t(\chi+\gamma^+)}}\ \chi^2}$$

In[28]:=
```
soln = soln /. {Δγ → 0, γ^+ → 2 γ, χ → 2 i Ω0};
StringForm["c1 = `` \nc2 = `` \n|c2|^2 = ``", soln[[1]][[1]] // FullSimplify,
 soln[[2]][[1]] // FullSimplify, sqrMag[soln[[2]][[1]]] // FullSimplify]
```

Out[29]=

$$c_1 = e^{-\frac{t\gamma}{2}} \cos\left[\frac{t\,\Omega_0}{2}\right]$$

$$c_2 = -i\, e^{-\frac{t\gamma}{2}} \sin\left[\frac{t\,\Omega_0}{2}\right]$$

$$|c_2|^2 = \frac{\sin\left[\frac{t\,\Omega_0}{2}\right]^2}{\sqrt{e^{2t\gamma}}}$$