## Homework 8

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## Problem 1

$$\Gamma = cT^n; \quad N_{\rm int} = \int_t^\infty \mathrm{d}t' \Gamma(t')$$

For a radiation dominated universe

$$a(t) = \sqrt{\frac{t}{t_0}} \implies H = \frac{1}{2t}, \quad H_0 = \frac{1}{2t_0}$$

$$T \propto \frac{1}{a}$$

$$\implies \Gamma = c \left(\frac{t}{t_0}\right)^{n/2}$$

$$\implies N_{\text{int}} = c \int_t^{\infty} dt' \left(\frac{t}{t_0}\right)^{n/2}$$

$$= \frac{2c}{n-2} \frac{t^{-(n-2)/2}}{(2H_0)^{n/2}} \quad (n > 2)$$

Now,

$$N_{\text{int}}(t_d) = 1$$

$$\Rightarrow \frac{2c}{n-2} \frac{t_d^{-(n-2)/2}}{(2H_0)^{n/2}} = 1$$

$$\Rightarrow t_d = \frac{1}{2} \left(\frac{n-2}{c}\right)^{\frac{-2}{n-2}} H_0^{\frac{-n}{n-2}}.$$

Finally,

$$\frac{\Gamma(t)}{H(t)} = c \frac{t^{(2-n)/2}}{t_0^{-n/2}}$$

$$\Rightarrow \frac{\Gamma(t_d)}{H(t_d)} = \frac{n-2}{2},$$

which is greater than 1 for n > 4

## Problem 2

i) The entropy density is given by

$$s_0 = \frac{2\pi^2}{45} g_{*s} T_{\gamma}^3$$

$$= \frac{2\pi^2}{45} \left( 2 + \frac{7}{8} \times 3 \times 2 \left( \frac{T_{\nu}}{T_{\gamma}} \right)^3 \right) T_{\gamma}^3$$

$$= \frac{2\pi^2}{45} \left( 2 + \frac{21}{4} \frac{4}{11} \right) (2.73 \text{ K})^3$$

$$= 39.4 \text{ K}^3$$

$$\approx 4 \times 10^{-38} \text{ GeV}^3$$
7.7

The critical density is

$$\rho_c = \frac{3H_0^2}{8\pi G} = 5 \times 10^{-6} \text{ GeV cm}^{-3}$$

the dark matter density is then

$$\rho_{\rm DM} = \Omega_{\rm DM} \rho_c \approx 1.3 \times 10^{-6} \rm GeV cm^{-3}$$

The number density,  $n_{\rm DM}$  is given by

Using

I get

ii)

 $n_{\rm DM}=Y_{\rm DM}s_0.$  See /cell  $Y_{\rm DM}\sim 0.2 \frac{g}{g_{*}}\approx 0.007$  Such to the Such that  $S \approx 3\times 10^{-40}~{\rm GeV^3}$ 

 $n_{\rm DM} \approx 3 \times 10^{-10} \; {\rm GeV}^{\circ}$   $\Omega_{\rm DM} \approx 0.25 = \frac{\rho_{\rm DM}}{\rho_c} = \frac{m_{\rm DM} n_{\rm DM}}{\rho_c}$   $\implies m_{\rm DM} = \frac{\Omega_{\rm DM} \rho_c}{n_{\rm DM}} \approx 37 \; {\rm eV}$ 

This is wayyyy below the weak scale. But I'm sure my yield calculation was nonsense =P.

## Problem 3

Given that

$$\rho_{\rm DM} = 0.3 \text{ GeV cm}^{-3},$$
 $R = 20 \text{ kpc},$ 
 $\langle \sigma v \rangle = 3 \times 10^{-26} \text{ cm}^3 \text{s}^{-1},$ 
 $m_{\rm DM} = 100 \text{ GeV}$ 

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we have that

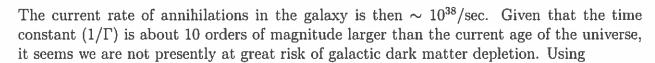
$$\Gamma \approx n \langle \sigma v \rangle$$

$$= \frac{\rho_{\rm DM}}{m_{\rm DM}} \langle \sigma v \rangle$$

$$\approx 1 \times 10^{-28} \, {\rm s}^{-1}$$

The total number of dark matter particles within the given radius is

$$N = \frac{4}{3}\pi R^3 \frac{\rho_{\rm DM}}{m_{\rm DM}} \approx 10^{66}$$



$$\rho_{\rm DM} = \Omega_{\rm DM} \rho_c = \frac{3\Omega_{\rm DM} H_0^2}{8\pi G} \approx 1.2 \times 10^{-6} \; {\rm GeV cm}^{-3}$$

as the universal dark matter density, we find a universal dark matter annihilation rate of

$$\Gamma \approx 10^{-34} \text{ s}^{-1}$$

