

Homework 8

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Problem 1

$$\Gamma = cT^n; \quad N_{\text{int}} = \int_t^\infty dt' \Gamma(t') \quad \checkmark$$

For a radiation dominated universe

$$a(t) = \sqrt{\frac{t}{t_0}} \implies H = \frac{1}{2t}, \quad H_0 = \frac{1}{2t_0}$$

$$T \propto \frac{1}{a}$$

$$\implies \Gamma = c \left(\frac{t}{t_0} \right)^{n/2}$$

$$\begin{aligned} \implies N_{\text{int}} &= c \int_t^\infty dt' \left(\frac{t'}{t_0} \right)^{n/2} \\ &= \frac{2c}{n-2} \frac{t^{-(n-2)/2}}{(2H_0)^{n/2}} \quad (n > 2) \quad \checkmark \end{aligned}$$

Now,

$$\begin{aligned} N_{\text{int}}(t_d) &= 1 \\ \implies \frac{2c}{n-2} \frac{t_d^{-(n-2)/2}}{(2H_0)^{n/2}} &= 1 \quad \checkmark \end{aligned}$$

$$\implies t_d = \frac{1}{2} \left(\frac{n-2}{c} \right)^{\frac{-2}{n-2}} H_0^{\frac{-n}{n-2}}.$$

Finally,

$$\begin{aligned} \frac{\Gamma(t)}{H(t)} &= c \frac{t^{(2-n)/2}}{t_0^{-n/2}} \\ \implies \frac{\Gamma(t_d)}{H(t_d)} &= \frac{n-2}{2}, \quad \checkmark \end{aligned}$$

which is greater than 1 for $n > 4$

But is it correct 1 for n

Problem 2

i) The entropy density is given by

$$\begin{aligned}
 s_0 &= \frac{2\pi^2}{45} g_{*s} T_\gamma^3 \\
 &= \frac{2\pi^2}{45} \left(2 + \frac{7}{8} \times 3 \times 2 \left(\frac{T_\nu}{T_\gamma} \right)^3 \right) T_\gamma^3 \\
 &= \frac{2\pi^2}{45} \left(2 + \frac{21}{4} \frac{4}{11} \right) (2.73 \text{ K})^3 \\
 &= 39.4 \text{ K}^3 \\
 &\approx 4 \times 10^{-38} \text{ GeV}^3 \quad ???
 \end{aligned}$$

The critical density is

$$\rho_c = \frac{3H_0^2}{8\pi G} = 5 \times 10^{-6} \text{ GeVcm}^{-3} \quad \checkmark$$

the dark matter density is then

$$\rho_{\text{DM}} = \Omega_{\text{DM}} \rho_c \approx 1.3 \times 10^{-6} \text{ GeVcm}^{-3} \quad \checkmark$$

The number density, n_{DM} is given by

$$n_{\text{DM}} = Y_{\text{DM}} s_0 \quad \checkmark$$

Using

$$Y_{\text{DM}} \sim 0.2 \frac{g}{g_{*s}} \approx 0.007$$

I get

$$n_{\text{DM}} \approx 3 \times 10^{-40} \text{ GeV}^3$$

See Sect 5.2
and turn
section 5.2

ii)

$$\Omega_{\text{DM}} \approx 0.25 = \frac{\rho_{\text{DM}}}{\rho_c} = \frac{m_{\text{DM}} n_{\text{DM}}}{\rho_c}$$

$$\Rightarrow m_{\text{DM}} = \frac{\Omega_{\text{DM}} \rho_c}{n_{\text{DM}}} \approx 37 \text{ eV}$$

This is wayyyy below the weak scale. But I'm sure my yield calculation was nonsense =P.

Problem 3

Given that

$$\begin{aligned}
 \rho_{\text{DM}} &= 0.3 \text{ GeVcm}^{-3}, \\
 R &= 20 \text{ kpc}, \\
 \langle \sigma v \rangle &= 3 \times 10^{-26} \text{ cm}^3 \text{s}^{-1}, \\
 m_{\text{DM}} &= 100 \text{ GeV}
 \end{aligned}$$

20
40

we have that

$$\begin{aligned}\Gamma &\approx n \langle \sigma v \rangle \\ &= \frac{\rho_{\text{DM}}}{m_{\text{DM}}} \langle \sigma v \rangle \\ &\approx 1 \times 10^{-28} \text{ s}^{-1}\end{aligned}$$

✓

The total number of dark matter particles within the given radius is

$$N = \frac{4}{3} \pi R^3 \frac{\rho_{\text{DM}}}{m_{\text{DM}}} \approx 10^{66}$$

✓

The current rate of annihilations in the galaxy is then $\sim 10^{38}/\text{sec}$. Given that the time constant ($1/\Gamma$) is about 10 orders of magnitude larger than the current age of the universe, it seems we are not presently at great risk of galactic dark matter depletion. Using

$$\rho_{\text{DM}} = \Omega_{\text{DM}} \rho_c = \frac{3\Omega_{\text{DM}} H_0^2}{8\pi G} \approx 1.2 \times 10^{-6} \text{ GeV cm}^{-3}$$

as the universal dark matter density, we find a universal dark matter annihilation rate of

$$\Gamma \approx 10^{-34} \text{ s}^{-1}$$

✓

$\frac{3.0}{3.0}$