Exercise Set 2

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Exercise 1

First let's calculate the derivative

$$\partial_x \psi = \left(\pm \frac{i\eta}{\hbar} \sqrt{p(x)} - \frac{\eta p'(x)}{2(p(x))^{3/2}} \right) \exp\left[\pm \frac{i}{\hbar} \int^x p(x')\mathrm{d}x' \right]$$

Multiplying by ψ^* will cancel the exponential, so

$$\begin{aligned} j(x) &= \frac{\hbar}{m} \operatorname{Im} \left[\frac{\eta}{\sqrt{p(x)}} \left(\pm \frac{i\eta}{\hbar} \sqrt{p(x)} - \frac{\eta p'(x)}{2(p(x))^{3/2}} \right) \right] \\ &= \pm \frac{\eta^2}{m} \end{aligned}$$

Exercise 2

The turning points are given by

$$V(x_{1,2}) = E \implies m\omega^2 x_{1,2}^2 = E \implies x_{1,2} = \mp \sqrt{\frac{E}{m\omega^2}}$$

Now,

$$\begin{aligned} \oint p(x)\mathrm{d}x &= \int_{-\sqrt{\frac{E}{m\omega^2}}}^{\sqrt{\frac{E}{m\omega^2}}} \sqrt{2mE - 2m^2\omega^2 x^2}\mathrm{d}x \\ &= 4m\omega \int_{-\sqrt{\frac{E}{m\omega^2}}}^{\sqrt{\frac{E}{m\omega^2}}} \sqrt{\frac{E}{m\omega^2} - x^2} \\ &= 4m\omega \frac{E}{m\omega^2} \frac{\pi}{2} \\ &= \frac{2\pi E}{\omega} = \left(n + \frac{1}{2} \right) h \\ &\implies E = \left(n + \frac{1}{2} \right) \hbar\omega \end{aligned}$$