HW 3 Problem 1

In[4]:=
```
Symbolize[N_c]; Symbolize[m_u]; Symbolize[m_d];
Symbolize[m_Z]; Symbolize[θ_W];
$Assumptions =
  {N_c > 0, m_u ≥ 0, m_d ≥ 0, m_Z > 0, θ_W > 0, Y ∈ Reals, Δ ∈ Reals, x ∈ Reals};
```

a)

In[6]:=
$$S = \frac{N_c}{6\pi}\left(1 - 2\,Y\,\mathrm{Log}\left[\frac{m_u{}^2}{m_d{}^2}\right]\right);$$

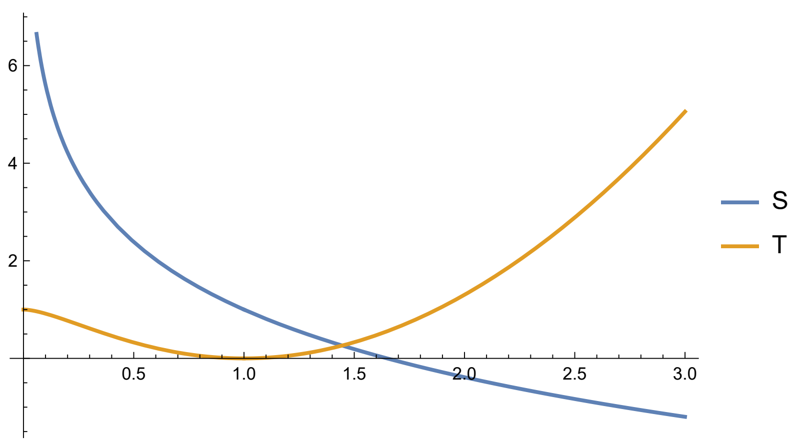
$$T = \frac{N_c}{16\pi\,(\mathrm{Sin}[\theta_W]\,\mathrm{Cos}[\theta_W]\,m_Z)^2}\left(m_u{}^2 + m_d{}^2 - \frac{2\,m_u{}^2\,m_d{}^2}{m_u{}^2 - m_d{}^2}\,\mathrm{Log}\left[\frac{m_u{}^2}{m_d{}^2}\right]\right);$$

In[8]:=
$$s = 1 - \mathrm{Log}\left[x^2\right];$$

$$t = 1 + x^2 - \frac{2\,x^2}{x^2 - 1}\,\mathrm{Log}\left[x^2\right];$$

In[10]:=
```
Plot[{s, t}, {x, 0, 3}, PlotLegends → {"S", "T"}]
```

Out[10]=



In[11]:=
```
tDeriv = D[t, x] // FullSimplify
```

Out[11]=
$$\frac{2\,x\,\left(3 - 4\,x^2 + x^4 + \mathrm{Log}\left[x^4\right]\right)}{\left(-1 + x^2\right)^2}$$

In[22]:=
```
Limit[tDeriv, x → 1] // Quiet
```
Out[22]=
```
0
```

In[23]:=
```
Limit[t, x → 1] // Quiet
```
Out[23]=
```
0
```

B)

In[14]:=
```
Series[S /. {mu → md + Δ}, {Δ, 0, 1}] // TraditionalForm
```
Out[14]//TraditionalForm=

$$\frac{N_c}{6\pi} - \frac{2\Delta(Y N_c)}{3(\pi m_d)} + O(\Delta^2)$$

In[15]:=
```
Series[T /. {mu → md + Δ}, {Δ, 0, 2}] // TraditionalForm
```
Out[15]//TraditionalForm=

$$\frac{\Delta^2 N_c \csc^2(\theta_W)\sec^2(\theta_W)}{12\pi m_Z^2} + O(\Delta^3)$$

c)

In[16]:=
```
degenerateT = Limit[T, mu → md] // Quiet
```
Out[16]=
```
0
```

In[19]:=
```
degenerateS = Limit[S, mu → md] // Quiet
```
Out[19]=

$$\frac{N_c}{6\pi}$$

In[20]:=
```
Snew = Around[-0.01, 0.07]
```
Out[20]=
```
-0.01 ± 0.07
```

In[21]:=
```
6 π Snew
```
Out[21]=
```
-0.2 ± 1.3
```