Homework 8

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Problem 1

$$\Gamma = cT^n; \quad N_{\text{int}} = \int_t^\infty \mathrm{d}t'\Gamma(t')$$

For a radiation dominated universe

$$a(t) = \sqrt{\frac{t}{t_0}} \implies H = \frac{1}{2t}, \quad H_0 = \frac{1}{2t_0}$$

$$\begin{aligned}
T &\propto \frac{1}{a} \\
\implies \qquad \Gamma &= c\left(\frac{t}{t_0}\right)^{n/2} \\
\implies \qquad N_{\text{int}} &= c\int_t^\infty \mathrm{d}t'\left(\frac{'}{t_0}\right)^{n/2} \\
&= \frac{2c}{n-2}\frac{t^{-(n-2)/2}}{(2H_0)^{n/2}} \quad (n > 2)
\end{aligned}$$

Now,

$$\begin{aligned}
N_{\text{int}}(t_d) &= 1 \\
\implies \qquad \frac{2c}{n-2}\frac{t_d^{-(n-2)/2}}{(2H_0)^{n/2}} &= 1 \\
\implies \qquad t_d &= \frac{1}{2}\left(\frac{n-2}{c}\right)^{\frac{-2}{n-2}} H_0^{\frac{-n}{n-2}}.
\end{aligned}$$

Finally,

$$\begin{aligned}
\frac{\Gamma(t)}{H(t)} &= c\frac{t^{(2-n)/2}}{t_0^{-n/2}} \\
\implies \qquad \frac{\Gamma(t_d)}{H(t_d)} &= \frac{n-2}{2},
\end{aligned}$$

which is greater than 1 for $n > 4$

Problem 2

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Problem 3

Given that

$$\begin{aligned}
\rho_{\text{DM}} &= 0.3 \text{ GeVcm}^{-3}, \\
R &= 20 \text{ kpc}, \\
\langle \sigma v \rangle &= 3 \times 10^{-26} \text{ cm}^3\text{s}^{-1}, \\
m_{\text{DM}} &= 100 \text{ GeV}
\end{aligned}$$

we have that

$$\begin{aligned}
\Gamma &\approx n \langle \sigma v \rangle \\
&= \frac{\rho_{\text{DM}}}{m_{\text{DM}}} \langle \sigma v \rangle \\
&\approx 1 \times 10^{-28} \text{ s}^{-1}
\end{aligned}$$

The total number of dark matter particles within the given radius is

$$N = \frac{4}{3}\pi R^3 \frac{\rho_{\text{DM}}}{m_{\text{DM}}}.$$

The time for all of that to decay is

$$t_{\text{decay}} = \frac{4\pi R^3}{3 \langle \sigma v \rangle} \approx 3 \times 10^{94} \text{ s},$$

so there is certainly no risk of depletion *any* time soon. The universal dark matter density is