

# Quantum Hadrodynamics

## From Fundamental Fields to Nuclear Phenomena

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UO  
PHYS 663

June 13, 2024



# A Brief History of Nuclear Physics

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- ▶  $p = |\frac{1}{2}, \frac{1}{2}\rangle, n = |\frac{1}{2}, -\frac{1}{2}\rangle; \pi^\pm = |1, \pm 1\rangle, \pi^0 = |1, 0\rangle$

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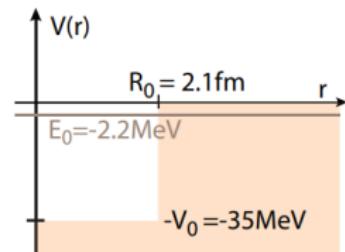
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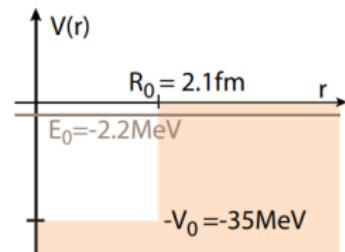
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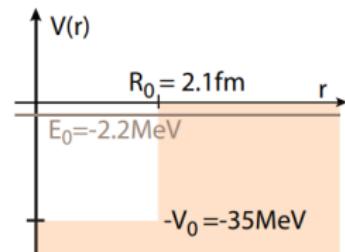
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- ▶ No bound excited states! (experimentally confirmed)



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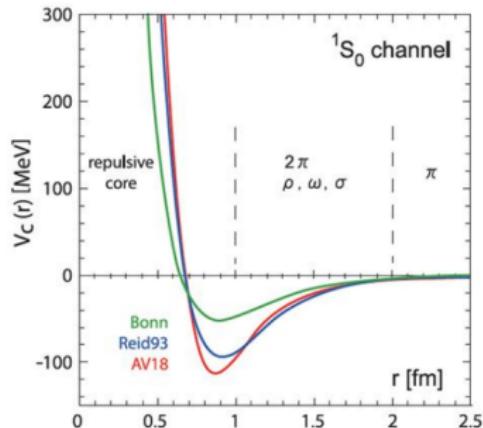
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- Thus  $|1, 0\rangle_t$  is an excited state (not bound), so charge-invariance of the  $N$ - $N$  force implies  $|1, 1\rangle_t$  (*pp*) and  $|1, -1\rangle_t$  (*nn*) don't exist as bound states.

# Understanding the $N$ - $N$ Interaction

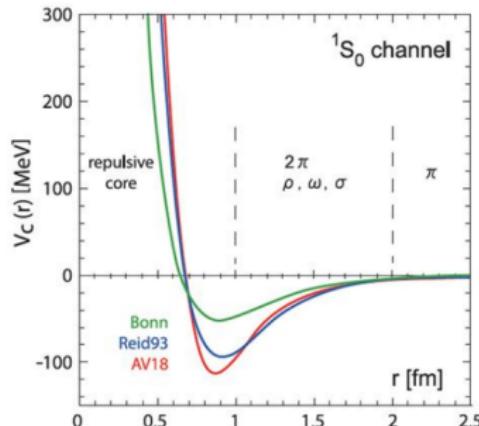
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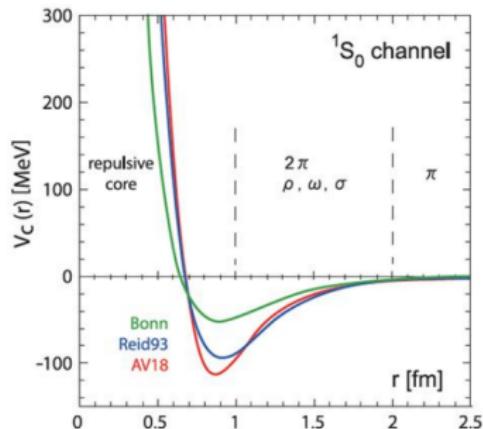
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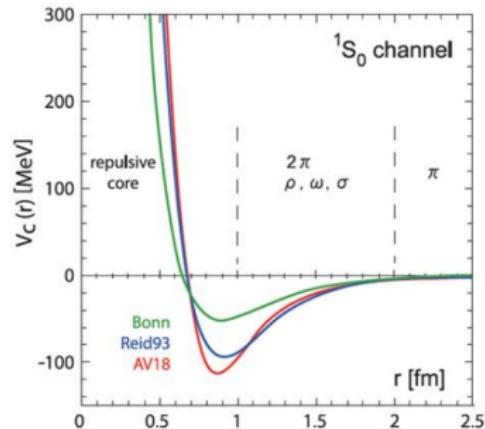


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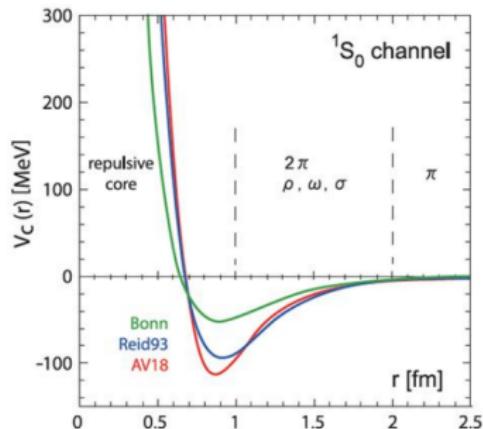


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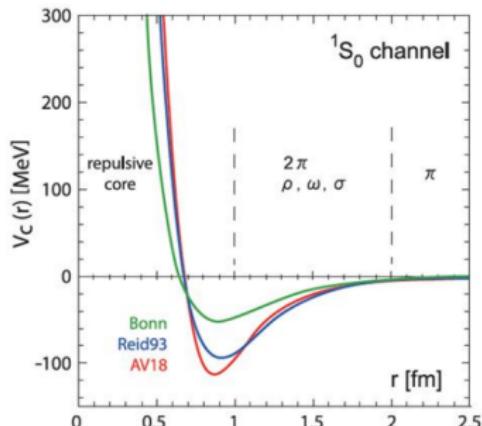
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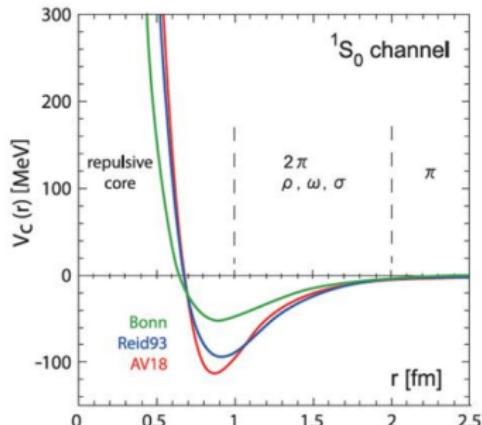
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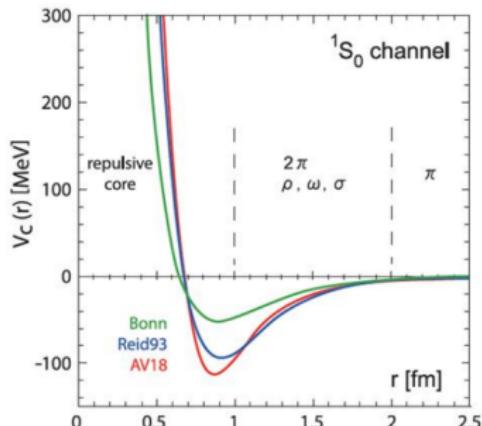
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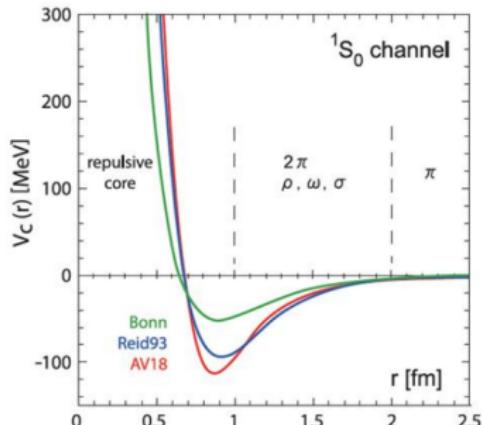
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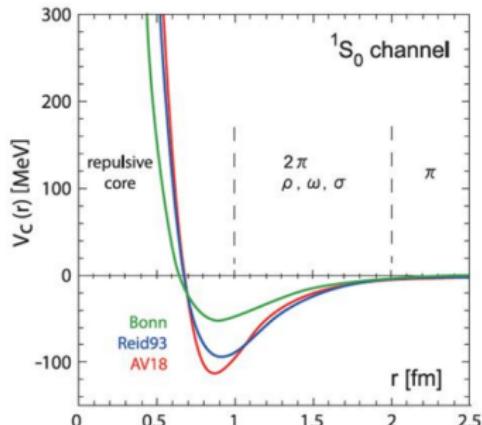
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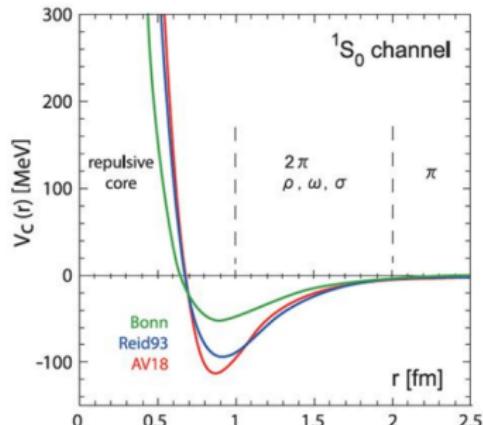
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$$P_r \psi(\vec{r}_1, \vec{r}_2) = \psi(\vec{r}_2, \vec{r}_1), \quad P_\sigma = \frac{1}{2} (1 + \vec{\sigma}_1 \cdot \vec{\sigma}_2)$$



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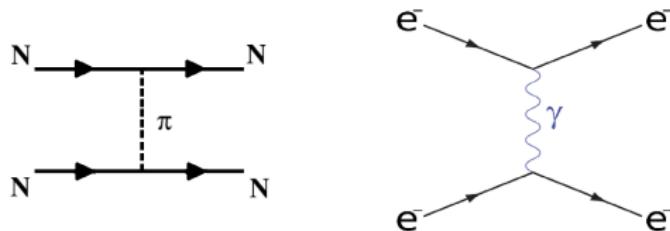
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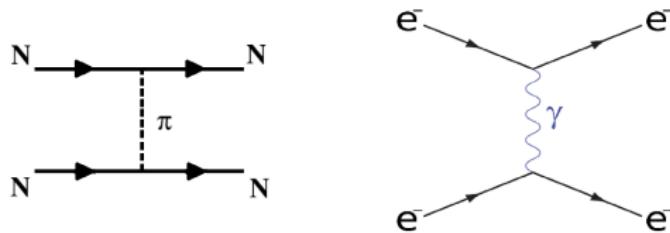
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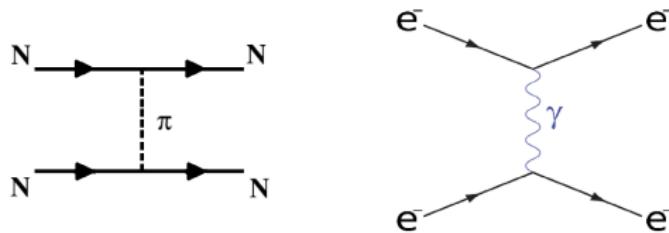


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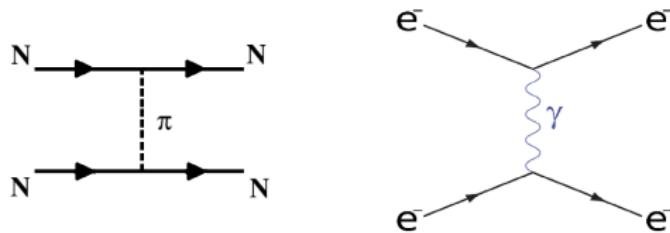


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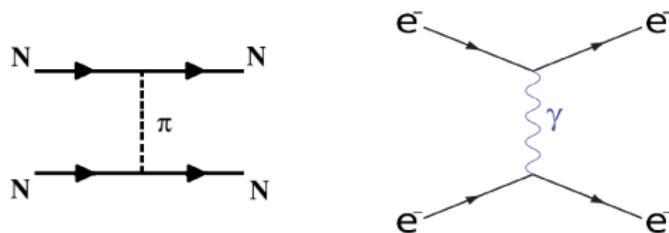


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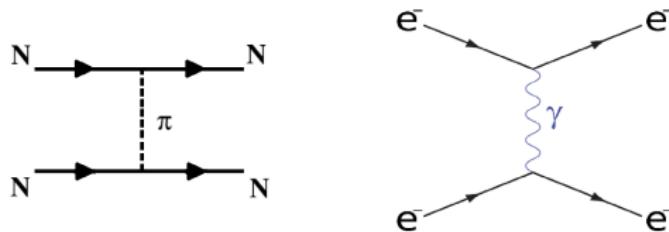


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$$\begin{aligned} V_{\text{OPEP}} = & \frac{f_\pi^2}{12\pi} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left[ \vec{\sigma}_1 \cdot \vec{\sigma}_2 \left( \frac{e^{-m_\pi r}}{r} - \frac{4\pi}{m_\pi^2} \delta^3(\vec{r}) \right) \right. \\ & \left. + S_{12}(\vec{r}) \left( 1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2} \right) \frac{e^{-m_\pi r}}{r} \right] \end{aligned}$$

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- A pseudoscalar isovector field  $\phi^{(\pi)}$ , however, gives

$$V_{\text{OPEP}} = \frac{f_\pi^2}{12\pi} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \left[ \vec{\sigma}_1 \cdot \vec{\sigma}_2 \left( \frac{e^{-m_\pi r}}{r} - \frac{4\pi}{m_\pi^2} \delta^3(\vec{r}) \right) + S_{12}(\vec{r}) \left( 1 + \frac{3}{m_\pi r} + \frac{3}{m_\pi^2 r^2} \right) \frac{e^{-m_\pi r}}{r} \right]$$

- This One Pion Exchange Potential captures the qualitative aspects of the long range portion of the  $N$ - $N$  interaction

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- Each spin-isospin channel contributes a  $V_c + V_T S_{12} + V_{LS} \vec{L} \cdot \vec{S}$

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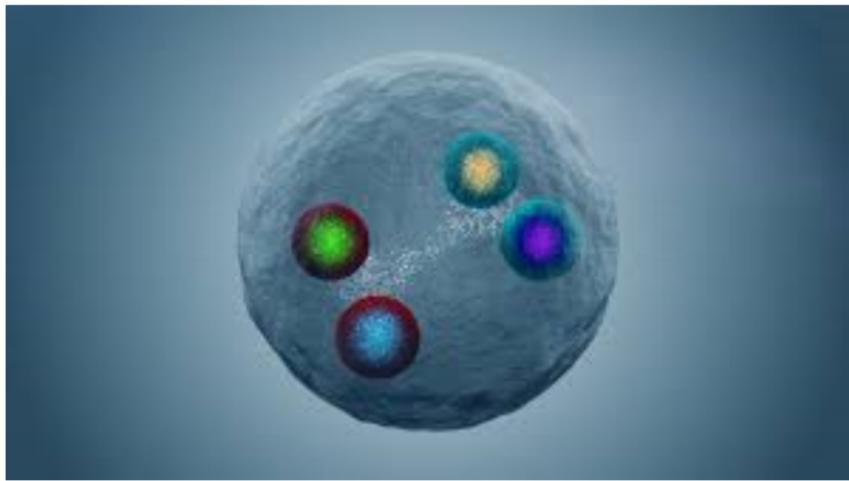
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- Why??
  - ▶ Large width  $\rightarrow$  difficult to differentiate from background
  - ▶ Abnormal substructure: almost certainly a tetraquark state with subdominant but non-negligible  $q\bar{q}$  component.

A true *color* image of a tetraquark captured by the ACME collaboration



# Beyond $^2D$ : The Nuclear Many Body Problem

Two's company, three's a crowd

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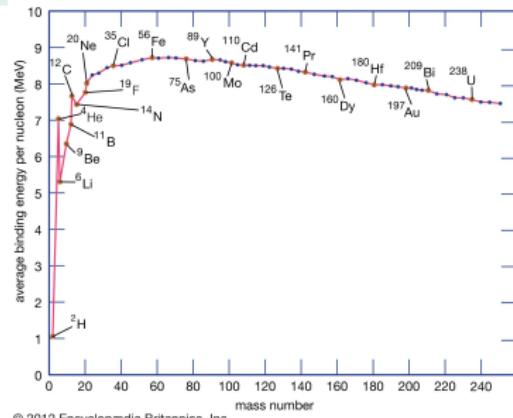
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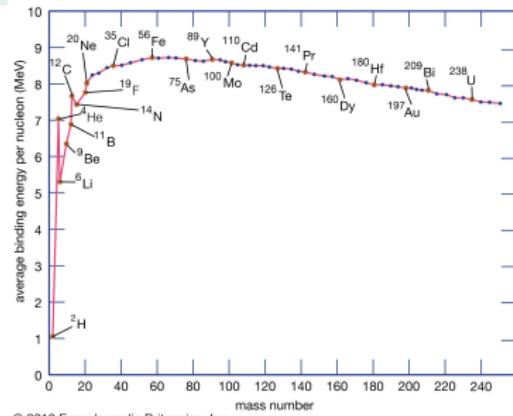
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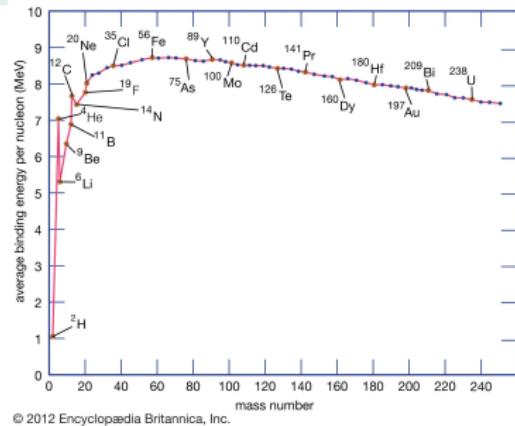
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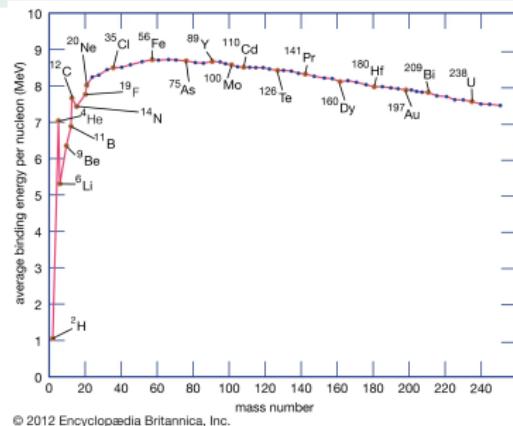
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$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i>j} V_{ij}(\vec{r}_i, \vec{r}_j) \rightarrow \sum_i \left( \frac{p_i^2}{2m} + U(\vec{r}_i) \right) + H_{\text{res}}$$

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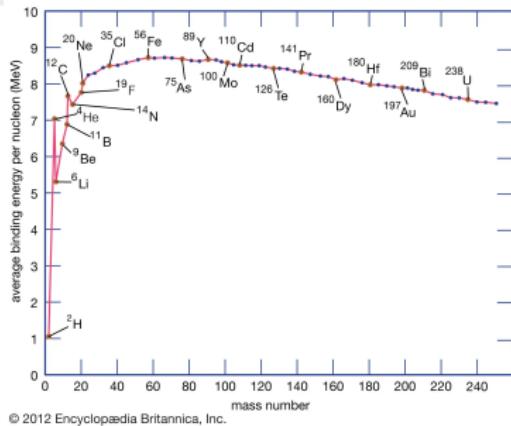
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  - ▶ The rest of the force,  $H_{\text{res}}$  is treated *perturbatively*

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Who doesn't love a good Lagrangian

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- The nucleon mass is shifted by the scalar:

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- Additionally, the energy is shifted by the vector:

$$E^\pm(\vec{k}) = g_\omega \langle \omega^0 \rangle \pm \sqrt{\left( \vec{k} - g_\omega \vec{\omega} \right)^2 + M^{*2}}$$

# References

- [1] C. Bertulani. *Nuclear Physics in a Nutshell*. In a Nutshell. Princeton University Press, 2007.
- [2] R. Machleidt. High-precision, charge-dependent bonn nucleon-nucleon potential. *Physical Review C*, 63(2), Jan. 2001.
- [3] J. R. Peláez. From controversy to precision on the sigma meson: A review on the status of the non-ordinary f0(500) resonance. *Physics Reports*, 658:1–111, Nov. 2016.
- [4] B. D. Serot and J. D. Walecka. *Relativistic Nuclear Many-Body Theory*, pages 49–92. Springer US, Boston, MA, 1992.
- [5] H. Yukawa. *On the Interaction of Elementary Particles*. Number pt. 4 in On the Interaction of Elementary Particles. Tokto Imperial University, 1935.