Complexity of Jet Reconstruction Algorithms

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UO
PHYS 662

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  - Measuring Algorithmic Complexity
  - Analysis of FastJet, SISCone, and their predecessors

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  - Measure invariant masses of constituent particles
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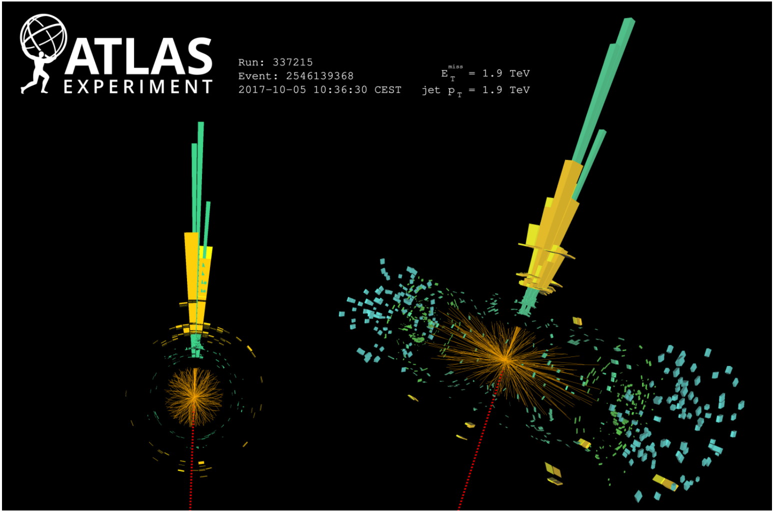
- Jets:
  - A collimated spray of stable particles arising from fragmentation and hadronization of a parton after a collision.

Review of Jet Algorithms (cont.)

Example Jet



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  - Necessary for faithful comparison of data to experiment

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  - Examples include K_t, $\bar{K}_t$, and Cambridge/Aachen

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  - Also, $o()$, $\Theta(x)$, $\Omega()$, $\omega()$ with related definitions

Complexity of K_t (pre-FastJet)

The Naïve Algorithm

Algorithm Naïve K_t

repeat
    for particle pair (i, j) **do**
        $d_{Bi} \leftarrow p_{ti}^2$ ▷ Calculated once for each i
        $R_{ij}^2 \leftarrow (\eta_i - \eta_j)^2 + (\phi_i - \phi_j)^2$
        $d_{ij} \leftarrow \min(p_{ti}^2, p_{tj}^2) R_{ij}$
    end for
    $d_{\min} \leftarrow \min(\{d_{ij}\} \cup \{d_{Bi}\})$
    if $d_{\min}$ is d_{Bi} **then**
        Add particle i to list of jets, remove from list of particles
    else
        Merge particles i, j
    end if
until no particles remain

Pre-FastJet K_t Complexity (cont.)

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- Also easy to see that the space complexity is $\mathcal{O}(N^2)$.

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- That is, particles i and j are geometric *nearest neighbors*
- Therefore, we don't need the $\mathcal{O}(N^2)$ table d_{ij}!
- We only need an n-element list of nearest neighbors $d_{i\mathcal{G}_i}$

Complexity of FastJet

The Smart Algorithm

Algorithm `FastJet` K_t

for particle i **do**
  $\mathcal{G}_i \leftarrow$ nearest neighbor of particle i
  $d_{i\mathcal{G}_i}, d_{Bi}$ calculated as previously
end for
repeat
  $d_{\min} \leftarrow \min(\{\mathcal{G}_i\} \cup \{d_{Bi}\})$
  Merge or Remove according to $d_{\min}$ as previously
  Update $\mathcal{G}_i$, $d_{i\mathcal{G}_i}$, d_{Bi}.
until no particles remain

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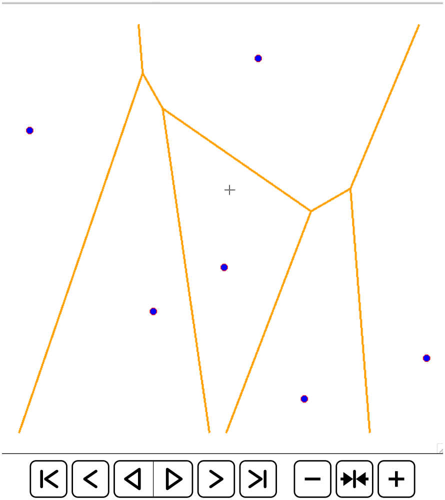
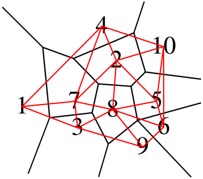
    - Updating $\mathcal{G}_i$ in $\mathcal{O}(N)$ is a nontrivial geometric affect
- However, we can do better!

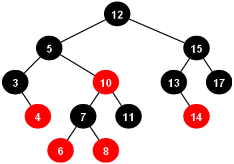
Complexity of FastJet (cont.)

Dynamic Voronoi Diagram



Better Complexity Analysis





Better Complexity Analysis

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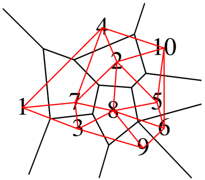

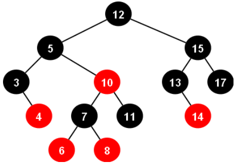

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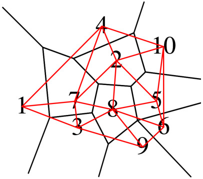

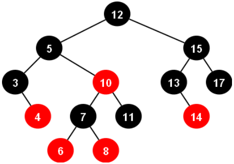

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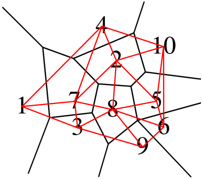

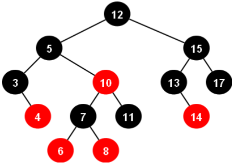

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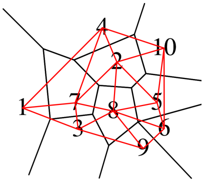

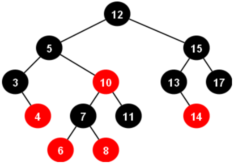

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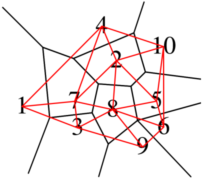
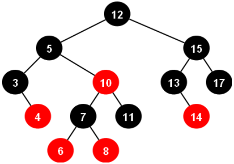
- Represent $d_{i\mathcal{G}_i}$ as a Red-Black tree





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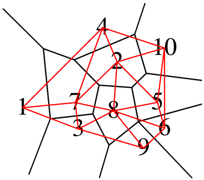

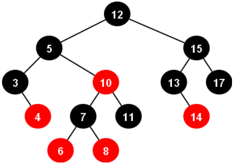

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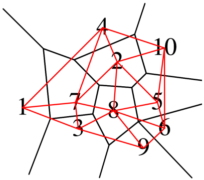

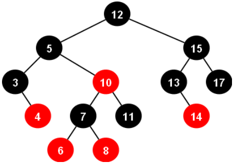

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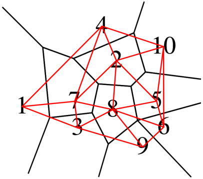

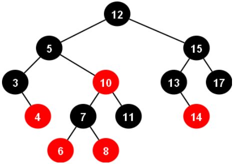

- A Voronoi diagram for NN calculations
  - Constructed in $\mathcal{O}(n \ln(N))$ time
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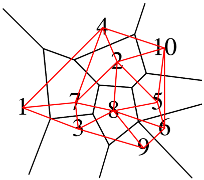

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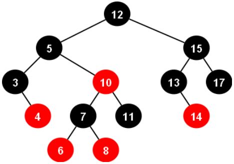

Complexity of IC-SM

A Naïve Cone-Finding Algorithm

Algorithm Iterative Cone with Split-Merge

repeat
    $p_t^* \leftarrow \max(\{p_{ti}\})$ ▷ seed axis
    repeat
        $p_j \leftarrow$ sum of momenta of particles within R of seed axis
        if $p_t^* == p_j$ **then**
            Label p_t^* a protojet, remove all particles
        else
            $p_t^* \leftarrow p_j$
        end if
    until The jet axis and seed axis coincide
until No seeds remain
Run Split-Merge on the protojets

Complexity of IC-SM

The Split-Merge Procedure

Algorithm Split-Merge

Remove all protojets with $p_t < p_{t,\text{cut}}$
repeat
    Determine hardest protojet i
    Determine hardest protojet j such that i and j share particles
    if no such j exists **then**
        i is a final jet; remove particles
    else
        if $p_{t,\text{shared}} < f \times p_{tj}$ **then**
            Split particles between protojets
        else
            Replace i and j with their merger
        end if
    end if
until no protojets left

Complexity of IC-SM

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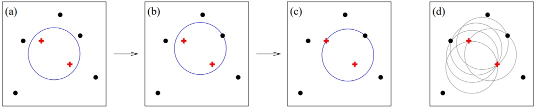
- Total complexity: $\mathcal{O}(N^2 n)$ (can be optimized to $\mathcal{O}(N^2)$ with 2D tree data structures)

Complexity of SISCone

Algorithm SISCone

```
for particle i do
    Find all particles j within 2R of i
    if no such j exists then
        i is made a protojet
    else
        Find circles of radius R with i j on their circumference
        for each circle do
            for each permutations of edge point containment do
                Label circle as cone
                Check cone stability w.r.t the edge points
            end for
        end for
    end if
end for
Explicitly check all cones not labeled unstable for stability
```

Improved Cone-Finding



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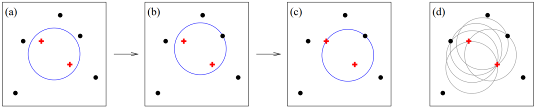

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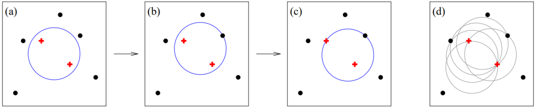

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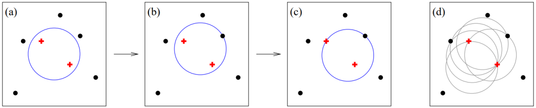

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- Seedlessness $\implies$ inherent IR safety

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- Have a good spring break!

References

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