Exercise Set 9

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Exercise 1

(a) The $\{|0\rangle, |1\rangle\}$ basis is already defined such that $|\psi_1\rangle$ and $|\psi_2\rangle$ are equally spaced about $|0\rangle$. In the case that $p_0 = p_1$, the Q matrix is proportional to σ_x, so it's eignevectors are $|0\rangle \pm |1\rangle$, which are *also* equally spaced about $|0\rangle$, so they must also be equally spaced about $|\psi_1\rangle$ and $|\psi_2\rangle$.

(b) Q is still Hermitian, so its distinct eigenvalues correspond to orthogonal eignevectors. To see that the Π_0 axis should be closer to $|\psi_0\rangle$ than Π_1 is to $|\psi_1\rangle$ (in the case that $p_0 > p_1$), consider the edge-case of a vanishingly small p_1. In this case, the problem of state "descrimination" is more of a problem of state "confirmation"; merely measure the state by projecting onto the expected state. If the measurment fails, it certianly was not in the expected state.

Exercise 2

The POVM element for the inconclusive result has the form $\mathcal{I} - |+\rangle\langle+| - |-\rangle\langle-|$. Since $|+\rangle$ and $|-\rangle$ are linearly independent pure states, subtracting them effectively subtracts all the 'purity' from the state, leaving the "I know nothing" state. Thus, no more information may be extracted by further measurement.