Midterm

Phys 610, Winter '24

February 17, 2024

Questions

Consider the line element

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{\sqrt{r^2+b^2}}\right)\mathrm{d}t^2 + \left(1 - \frac{2M}{\sqrt{r^2+b^2}}\right)^{-1}\mathrm{d}r^2 + (r^2+b^2)(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi)$$

where M and b are positive, real constants. Note that we are using geometrical units here where $G = c = 1$.

1. (15 points) Write down the metric and inverse metric in the t, r, θ, ϕ coordinate system.
2. (15 points) Write down the non-zero Christoffel symbols $\Gamma^{\alpha}_{\beta\gamma}$ for this metric. (Hint: of the 40 Christoffel symbols that are distinct by taking account of symmetry under $\beta \leftrightarrow \gamma$, only 9 are non-zero.)
3. (15 points) Write the geodesic equation in this metric for a massive particle *i.e.* write equations for $\ddot{t}, \ddot{r}, \ddot{\theta}, \ddot{\phi}$ where the double-dots are with respect to τ.
4. (15 points) Calculate the effective potential for timelike orbits for arbitrary M and b.
5. (10 points) Do bound orbital solutions exist for $M = 0$? Why or why not?
6. (10 points) Build a numerical solver for integrating the geodesic equation given an initial position, ε, and l. Here, ε corresponds to the script E (i.e. energy) that was used in class. Explore a bit; try different geometries and initial conditions. Can you find any interesting geometries and/or trajectories?
7. (10 points) Do bound orbital solutions exist for $b = 1$, $M = 1$, $l = 5$? If so, show a trajectory.
8. (10 points) What kind of trajectories are possible in $b = 3$, $M = 1$ geometry with $\varepsilon = -0.05$, $l = 2$?

WARNING: Don't use any off-the-shelf integrator as a black box. Think about the physics of the problem and how you can check for errors in the integration. Do you trust the trajectories you've computed?