Midterm

Phys 614

Due: 10:30 AM, Thursday, May 4, 2023

Problem 1

For a two-dimensional system of non-interacting Bosons in a box, calculate the low temperature behavior of $\mu(T)$.

Problem 2

For arbitrary spatial dimensions $d > 2$, show that, for non-interacting Bosons in a box,

$$\mu(T) \propto |T - T_c|^{X(d)} \tag{2.1}$$

as $T \to T_c$ from above, where T_c is the Bose-Einstein condensation temperature, and find an explicit expression for $X(d)$.

Problem 3

For the system in problem 2, show that the specific heat $C_V(T)$ for $T \gtrapprox T_c$, obeys

$$C_V(T) - C_V(T_c) \propto |T - T_c|^{-\alpha(d)} \tag{3.1}$$

and find an explicit expression for $\alpha(d)$.

Problem 4

Find the behavior of the specific heat $C_V(T)$ as $T \to 0$ in this problem.

Problem 5

N non-interacting Bosons move in a 1-dimensional box of length L. There is an attractive potential in the box that creates a single bound state of energy $\epsilon_0 < 0$.

(a) Assuming that $k_B T_c \ll |\epsilon_0|$, show that the system Bose condenses at a temperature T_c, and calculate T_c. For what values of $\rho \equiv \frac{N}{L}$ is the $k_B T_c$ you found actually $\ll |\epsilon_0|$?

(b) Assuming $k_B T_c \gg |\epsilon_0|$, show that the system again Bose condenses, and calculate T_c. For what values of ρ is $k_B T_c$ actually $\gg |\epsilon_0|$?

(c) Are your results for (a) and (b) consistent with each other?