Problem Set #3

Phys 614

Due: 10:30 Noon, Thursday, May 16, 2023

Problem 1

The Sun is a sphere of radius 6.96×10^5 km with a surface temperature of 6000K. The Earth is 1.496×10^8 km from the Sun. Assuming that

1. both the Earth's surface and the Sun's radiate as black bodies;
2. there is no heat exchange between different points on the Earth's surface,
3. that each point on the Earth's surface equilibriates at a temperature that remains constant all day, and is such that it radiates exactly as much heat in the course of a day as it absorbes from the sun during the day, and
4. The Earth is a perfect sphere;

calculate the temperature $T(L)$ on the earth's surface at lattitude L on March 21, the day that the Earth's axis is normal to the plane of the earth's orbit. Evaluate your answer for Eugene ($L = 44°$) and your home town. How well does the answer agree with your experience?
Note: lattitude is define as illustrated in Fig. 1.

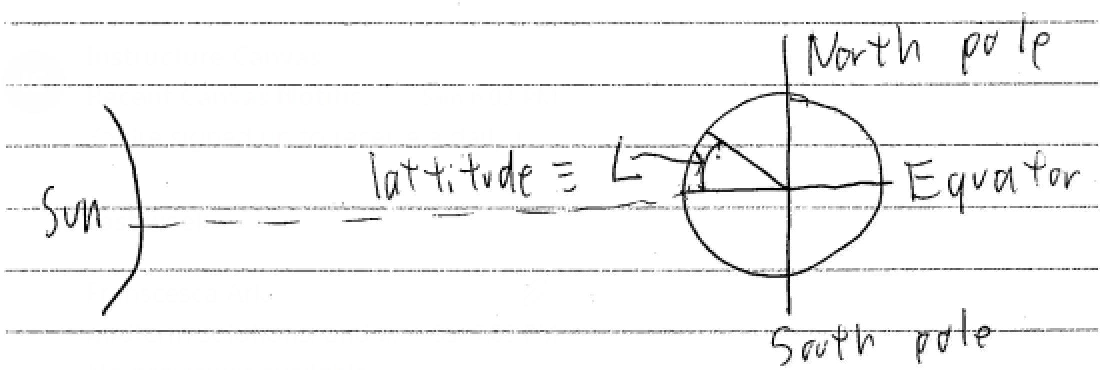

Figure 1: The definition of lattitude.

Problem 2

Hawking has argued that "black holes ain't black", rather, that they radiate due to the black holes' "swallowing up" one member of virtual particle-antiparticle pairs created near the schwarzschild radius, leaving the other to escape to infinity (see the discussion in Hawking's otherwise worthless "A Brief History of Time"). The radiation thus emmitted has a black body spectrum (what else, for a black hole?). Assuming that the peak in this black body spectrum occurs at a wavelength comparable ot the schwarzschild radius r_s of the black hole (defined as the radius at which the classical escape velocity equals the speed of light), calculate:

(a) The "temperature" of the black hole, as a function of its mass M.

(b) The total rate at which it emmits energy, assuming that the surface from which the emmision is occuring is a sphere of radius r_s. Express your answer solely in terms of $\hbar, M$, the gravitational constant G, and the speed of light c.

(c) Assuming all this emmitted energy is created by removing mass from the black hole, using $E = mc^2$, calculate the lifetime of a black hole as a function of its mass M. How does your answer compare with Hawking's statement that a black hole with the mass of the sun (2×10^{30} kg) "would take about 10^{66} years to evaporate completely".

Problem 3

Suppose the Earth was completely covered with a "greenhouse gas" which reflected back to the ground any radiation at wavelengths longer than visible light (i.e. $\lambda > 7 \times 10^{-}7\text{m}$). What temperature T_s would the earth's surface have to be to radiate away all of the heat recieved form the sun, assuming the surface was of uniform temperature and radiated as a black body?
Hint: calculate the integrated emmission from all frequencies $\omega > \omega_{\min} \equiv \frac{2\pi c}{\lambda_{\max}}$, assuming, to simplify the integral, that $\hbar\omega_{\min} \gg k_B T_s$. You may also assume that *all* of the sun's radiation reaches the earth's surface.