Homework 2

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Phys 662

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Problem 1 (Peskin 17.2)

(a) From chapter 8,

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} = \frac{\pi\alpha^2}{2s}\left(1 \pm \cos\theta\right)^2,$$

where the $+$ is for the $RL \to RL$ and $LR \to LR$ helicity states, and the minus is for the $LR \to RL$ and $RL \to LR$ states. The recall is *total*.

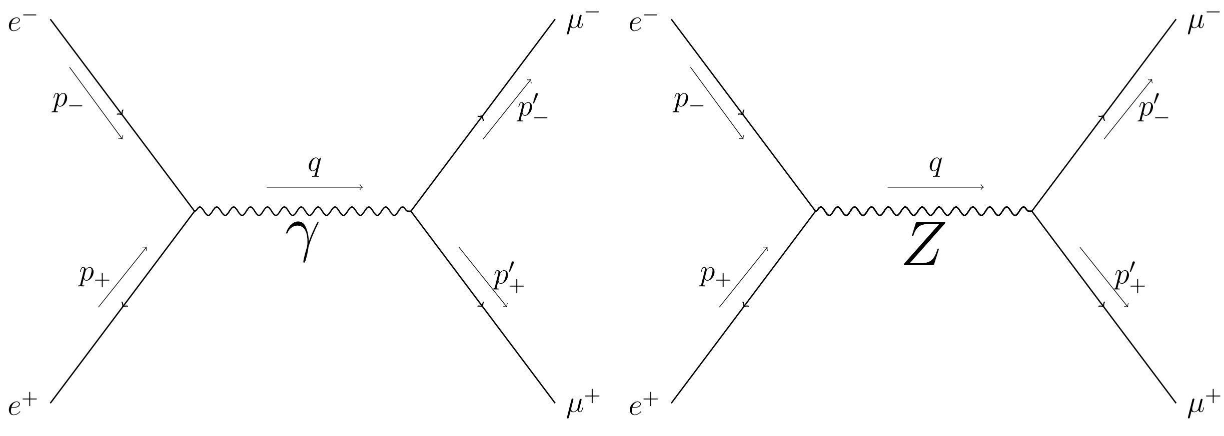

Figure 1: The Feynman diagram for $e^+e^- \to \mu^+\mu^-$

(b) Clearly, both diagrams are nearly identical. The only differences are the massiveness of the Z boson, and the couplings of the Z to e and μ. The virtual photon matrix element is

$$\mathcal{M}_\gamma(e_L^- e_R^+ \to \mu_L^- \mu_R^+) = e^2(1+\cos\theta) = g^2 s_w^2 (1+\cos\theta),$$

while the virtual Z matrix element is

$$\begin{aligned}\mathcal{M}_Z(e_L^- e_R^+ \to \mu_L^- \mu_R^+) &= \frac{g^2}{c_w^2}\frac{Q_{ZL}^2 8E^2}{s - m_Z^2 + i m_Z \Gamma_Z}\frac{1}{2}(1+\cos\theta) \\ &= \frac{g^2}{c_w^2}\frac{(\frac{1}{2} - s_w^2)^2 s}{s - m_Z^2 + i m_Z \Gamma_Z}(1+\cos\theta).\end{aligned}$$

The total matrix element is then

$$\begin{aligned}
\mathcal{M}(e_L^- e_R^+ \to \mu_L^- \mu_R^+) &= \mathcal{M}_\gamma + \mathcal{M}_\mathcal{Z} \\
&= g^2 s_w^2 (1+\cos\theta) + \frac{g^2}{c_w^2} \frac{(\frac{1}{2} - s_w^2)^2 s}{s - m_Z^2 + i m_Z^2} (1+\cos\theta) \\
&= g^2 s_w^2 (1+\cos\theta) \left(1 + \frac{1}{c_w^2 s_w^2} (\frac{1}{2} - s_w^2)^2 \frac{s}{s - m_Z^2 + i m_Z \Gamma_Z} \right)
\end{aligned}$$

The matrix element has the same θ dependence, so the cross section will be the same up to the term in large parenthesis above (squared).

(c) Plugging in the other charges / polarization product, we get

$$\begin{aligned}
\mathcal{M}(e_R^- e_L^+ \to \mu_R^- \mu_L^+) &= g^2 s_w^2 (1+\cos\theta) \left(1 + \frac{s_w^2}{c_w^2} \frac{s}{s - m_Z^2 + i m_Z \Gamma_Z^2} \right) \\
\mathcal{M}(e_L^- e_R^+ \to \mu_R^- \mu_L^+) &= g^2 s_w^2 (1-\cos\theta) \left(1 + \frac{1}{c_w^2} (-\frac{1}{2} + s_w^2) \frac{s}{s - m_Z^2 + i m_Z \Gamma_Z} \right) \\
\mathcal{M}(e_R^- e_L^+ \to \mu_L^- \mu_R^+) &= g^2 s_w^2 (1-\cos\theta) \left(1 + \frac{1}{c_w^2} (-\frac{1}{2} + s_w^2) \frac{s}{s - m_Z^2 + i m_Z \Gamma_Z} \right)
\end{aligned}$$

(d)

$$\begin{aligned}
\sigma(\cos\theta > 0) &= \int_0^1 \mathrm{d}\cos\theta \, (1+\cos\theta)^2 \\
&= \frac{1}{3} (1+\cos\theta)^3 \Big|_{\cos\theta=0}^{1} \\
&= \frac{7}{3} \\
\sigma(\cos\theta < 0) &= \int_{-1}^0 \mathrm{d}\cos\theta \, (1+\cos\theta)^2 \\
&= \frac{1}{3} (1+\cos\theta)^3 \Big|_{\cos\theta=-1}^{0} \\
&= \frac{1}{3}
\end{aligned}$$

$$\implies \frac{\sigma(\cos\theta>0) - \sigma(\cos\theta<0)}{\sigma(\cos\theta>0) + \sigma(\cos\theta<0)} = \frac{7-1}{7+1} = \frac{3}{4}$$

(e) In the limit $s \ll m_Z^2$, the breit-wigner factors go to zero (to leading order), reducing the cross sections to the ones calculated in chapter 8. The $LR \to LR$ and $RL \to RL$ states have $A_{FB} = 3/4$, while the other two states have $A_{FB} = -3/4$[1]. Clearly, then, the unpolarized process has vanishing forward-backward asymmetry in this limit.

[1] I did the integrals; they're practically identical to the ones in part (d), do I really need to type them out?

When $s = m_Z^2$, the Breit-wigner factors reduce to $-im_Z/\Gamma_Z$. Neglecting the photon contribution, the cross sections are

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\left(e_L^- e_R^+ \to \mu_L^- \mu_R^+\right) = \left[g^2 s_w^2(1+\cos\theta)\frac{1}{c_w^2 s_w^2}(\frac{1}{2}-s_w^2)^2\frac{m_Z}{\Gamma_Z}\right]^2$$
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\left(e_R^- e_L^+ \to \mu_R^- \mu_L^+\right) = \left[g^2 s_w^2(1+\cos\theta)\frac{s_w^2}{c_w^2}\frac{m_Z}{\Gamma_Z}\right]^2$$
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\left(e_L^- e_R^+ \to \mu_R^- \mu_L^+\right) = \left[g^2 s_w^2(1-\cos\theta)\frac{1}{c_w^2}(-\frac{1}{2}+s_w^2)\frac{m_Z}{\Gamma_Z}\right]^2$$
$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\left(e_R^- e_L^+ \to \mu_L^- \mu_R^+\right) = \left[g^2 s_w^2(1-\cos\theta)\frac{1}{c_w^2}(-\frac{1}{2}+s_w^2)\frac{m_Z}{\Gamma_Z}\right]^2$$

Let's preemptively drop common terms that will end up canceling. The total cross section is

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta} \propto (f_1+f_2)(1+\cos\theta)^2 + 2f_3(1-\cos\theta)^2$$

where

$$f_1 = \frac{\left(\frac{1}{2}-s_w^2\right)^4}{s_w^4}$$
$$f_2 = s_w^4$$
$$f_3 = \left(\frac{1}{2}-s_w\right)^2$$

The forward and backward integrals give

$$\sigma_{>0} = \frac{7}{3}(f_1+f_2) - \frac{2}{3}f_3$$
$$\sigma_{<0} = \frac{1}{3}(f_1+f_2) - \frac{14}{3}f_3$$

Hence,

$$A_{FB} = \frac{6(f_1+f_2)+12f_3}{8(f_1+f_2)-16f_3}$$

(f) In the ultra-high energy limit, the Breit-Wigner factor goes to 1, and we get

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\cos\theta}\left(e_L^- e_R^+ \to \mu_L^- \mu_R^+\right) = \left[g^2 s_w^2(1+\cos\theta)\left(1+\frac{1}{c_w^2 s_w^2}(\frac{1}{2}-s_w^2)^2\right)\right]^2$$

(g) The B diagram gives

$$(-\frac{1}{2}g')^2(1+\cos\theta)$$

The A diagram gives

$$(-\frac{1}{2}g)^2(1+\cos\theta)$$

I tried takeing all the s_w and c_ws in (f) and turning them into gs and g's, but I can't get them to look equal =(.

(h) For $RL \to RL$ The A diagram doesn't contribute, but the B diagram gives

$$(-g')^2(1+\cos\theta)$$

For $LR \to RL$ and $RL \to LR$ The A diagram also doesn't contribute, but B gives

$$(\frac{1}{2}g')^2(1-\cos\theta)$$