Homework 3

Sean Ericson
Phys 610

October 18, 2022

1.2.3

(a)

	()	(12)	(23)	(13)	(123)	(132)
()	()	(12)	(23)	(13)	(123)	(132)
(12)	(12)	()	(132)	(123)	(13)	(23)
(23)	(23)	(123)	()	(132)	(12)	(13)
(13)	(13)	(132)	(123)	()	(23)	(12)
(123)	(123)	(23)	(13)	(12)	(132)	()
(132)	(132)	(13)	(12)	(23)	()	(123)

Note: in the table above the row headers left-multiply the column headers. S_3 is *not* abelian, as it's multiplication table is not symmetric.

(b) The subgroups of S_3 are

$$\{()\}, \quad \{(), (12)\}, \quad \{(), (13)\}, \quad \{(), (23)\}, \quad \{(), (123), (132)\}$$

and all are abelian.

1.2.4

Let $H \leq G$. Then

$$b \in H \implies b^{-1} \in H \implies a \vee b^{-1} \in H \quad \checkmark$$

Now, let $a, b \in H \implies a \vee b^{-1} \in H$. Then,

$$a = b \implies a \vee a^{-1} \in H$$

so H has a neutral element. Also,

$$a = e \implies e \vee b^{-1} = b \in H$$

so H has inverses. Next,

$$a \vee \left(b^{-1}\right)^{-1} = a \vee b \in H$$

so H is closed. Finally, since G is a group, the operation is associative, so we have that

$$G \leq H$$

1.3.1

(a) Let $a, b, c, d \in \mathbb{Z}$. Then,

$$\frac{a}{b}, \frac{c}{d} \in \mathbb{Q} \implies \frac{a}{b} + \frac{c}{d} = \frac{ad+bc}{bd} \in \mathbb{Q},$$

which demonstrates closure. The opperation is obviously associative *and* commutative. The neutral element is 0. Finally, all elements have an inverse:

$$\frac{a}{b} \in \mathbb{Q} \implies -\frac{a}{b} \in \mathbb{Q}$$

(b) Let's make the addition table:

	θ	e
θ	θ	e
e	e	θ

From this table we an see that $(F, +)$ forms an additve group with neutral element θ. Excluding the neutral element, the group $(F, \cdot)$ as defined is the trivial group. Therefore, $(F, +, \cdot)$ is a field.

1.4.1

Let $V = C$, and define addition as

$$(f+g)(x) = f(x) + g(x)$$

Now let $\alpha, \beta \in \mathbb{R}$. Then define scalar multiplication as

$$(\alpha\beta f)(x) = ((\alpha\beta))(f(x)) = \alpha(\beta f(x))$$