Homework 2

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Problem 1 (Berman 2.9)

Problem 2 (Berman 2.17)

Problem 3

(a) Given that

$$H_0 = \hbar \begin{pmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{pmatrix},$$

we have that

$$\begin{aligned} \exp(iH_0 t/\hbar) &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + it \begin{pmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{pmatrix} + \frac{(it)^2}{2} \begin{pmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{pmatrix}^2 + \frac{(it)^3}{3!} \begin{pmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{pmatrix}^3 + \cdots \\ &= \begin{pmatrix} 1 + i\omega_1 t + \frac{1}{2}(i\omega_1 t)^2 + \frac{1}{3!}(i\omega_1 t)^3 + \cdots & 0 \\ 0 & 1 + i\omega_2 t + \frac{1}{2}(i\omega_2 t)^2 + \frac{1}{3!}(i\omega_2 t)^3 + \cdots \end{pmatrix} \\ &= \begin{pmatrix} \exp(i\omega_1 t) & 0 \\ 0 & \exp(i\omega_2 t) \end{pmatrix}. \end{aligned}$$

(b) In the interaction representation, we factor out this free phase evolution by writing

$$|\psi(t)\rangle_{\mathrm{I}} = \bar{c}_1(t) e^{-i\omega_1 t} |1\rangle + \bar{c}_2(t) e^{-i\omega_2 t} |2\rangle\,,$$

that is, we make the (time-dependent) unitary transformation

$$|\psi(t)\rangle_{\mathrm{S}} \to |\psi(t)\rangle_{\mathrm{I}} = U(t)\, |\psi(t)\rangle_{\mathrm{S}}\,,$$

where

$$U(t) = \begin{pmatrix} e^{-i\omega_1 t} & 0 \\ 0 & e^{-i\omega_2 t} \end{pmatrix}.$$

We get the effective interaction hamiltonian by making the inverse transformation on V:

$$V_{\mathrm{I}} = U^\dagger V U = \hbar \Omega_0 \cos(\omega t) \begin{pmatrix} 0 & e^{-i\omega_0 t} \\ e^{i\omega_0 t} & 0 \end{pmatrix},$$

where the phase ϕ has been absorbed into the (complex) Rabi frequency.

Problem 4

(a)