Exercise Set 2

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Phys 633

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Monday

Exercise 1

$$\Delta_1 = S_0 V S_0 = P_0 V P_0$$

$$\begin{aligned}
\mathrm{Tr}[\Delta_1] &= \mathrm{Tr}[P_0 V P_0] \\
&= \mathrm{Tr}[P_0 V] \\
&= \langle \psi_0 | V | \psi_0 \rangle \\
&= V_{00}
\end{aligned}$$

$$\delta E_1 = \lambda \, \mathrm{Tr}[\Delta_1] = \lambda V_{00}$$

Exercise 2

$$\Delta_2 = P_0 V Q_0 G_0(E_0) Q_0 V P_0 = P_0 V \sum_{\alpha \neq 0} \frac{|\alpha\rangle\langle\alpha|}{E_0 \alpha} V P_0$$

$$\begin{aligned}
\mathrm{Tr}[\Delta_2] &= \mathrm{Tr}\left[P_0 V \sum_{\alpha \neq 0} \frac{|\alpha\rangle\langle\alpha|}{E_0 \alpha} V P_0 \right] \\
&= \mathrm{Tr}\left[P_0 \sum_{\alpha \neq 0} \frac{V_{0\alpha} V_{\alpha 0}}{E_{0\alpha}} \right] \\
&= \sum_{\alpha \neq 0} \frac{\left| V_{0\alpha} \right|^2}{E_{0\alpha}}
\end{aligned}$$

$$\delta E_2 = \lambda^2 \, \mathrm{Tr}[\Delta_2] = \lambda^2 \sum_{\alpha \neq 0} \frac{\left| V_{0\alpha} \right|^2}{E_{0\alpha}}$$

Exercise 3

$$
\begin{aligned}
\Delta_3 &= (0011\cdot) + (0101\cdot) + (1001\cdot) + (1010\cdot) + (1100\cdot) + (1001\cdot) + (0002\cdot) + (0020\cdot) + (0200\cdot) + (2000\cdot) \\
&\overset{\text{Tr}}{=} (0110\cdot) + (1001\cdot) + (0020\cdot) + (0200\cdot) \\
&\overset{\text{Tr}}{=} (0110\cdot) + (1\cdot 100) + (0\cdot 020) + (0\cdot 002) \\
&\overset{\text{Tr}}{=} (0110\cdot) + (200) - (020) - (002) \\
&\overset{\text{Tr}}{=} (0110\cdot) + (020) \\
&\overset{\text{Tr}}{=} P_0 V \sum_{\alpha\neq 0} \frac{|\alpha\rangle\langle\alpha|}{E_{0\alpha}} V \sum_{\beta\neq 0} \frac{|\beta\rangle\langle\beta|}{E_{0\beta}} V P_0 - P_0 V \sum_{\alpha\neq 0} \frac{|\alpha\rangle\langle\alpha|}{E_{0\alpha}^2} V P_0
\end{aligned}
$$

$$
\begin{aligned}
\text{Tr}[\Delta_3] &= \text{Tr}\left[P_0 V \sum_{\alpha\neq 0} \frac{|\alpha\rangle\langle\alpha|}{E_{0\alpha}} V \sum_{\beta\neq 0} \frac{|\beta\rangle\langle\beta|}{E_{0\beta}} V P_0\right] - \text{Tr}\left[P_0 V \sum_{\alpha\neq 0} \frac{|\alpha\rangle\langle\alpha|}{E_{0\alpha}^2} V P_0\right] \\
&= \text{Tr}\left[P_0 \sum_{\alpha,\beta\neq 0} \frac{V_{0\alpha}V_{\alpha\beta}V_{\beta 0}}{E_{0\alpha}E_{0\beta}}\right] - \text{Tr}\left[P_0 \sum_{\alpha\neq 0} \frac{V_{0\alpha}V_{\alpha 0}}{E_{0\alpha}}\right] \\
&= \sum_{\alpha,\beta\neq 0} \frac{V_{0\alpha}V_{\alpha\beta}V_{\beta 0}}{E_{0\alpha}E_{0\beta}} - \sum_{\alpha\neq 0} \frac{\left|V_{0\alpha}\right|^2}{E_{0\alpha}}
\end{aligned}
$$

$$
\delta E_3 = \lambda^3\,\text{Tr}[\Delta_3] = \lambda^3 \sum_{\alpha,\beta\neq 0} \frac{V_{0\alpha}V_{\alpha\beta}V_{\beta 0}}{E_{0\alpha}E_{0\beta}} - \lambda^3 \sum_{\alpha\neq 0} \frac{\left|V_{0\alpha}\right|^2}{E_{0\alpha}}
$$

Tuesday

Exercise 1

The full potential is the sum of the centrifugal barrier potential and the Coloumb potential:

$$
V(r) = V_{\text{CB}}(r) + V_{\text{C}}(r) = \frac{\hbar^2 l(l+1)}{2\mu r^2} - \frac{\hbar c\alpha}{r}
$$

No matter the coefficients, the r^{-2} term will dominate near $r = 0$, causing $V(r)$ to approach $+\infty$. Also independent of the coefficients is the fact that as $r \to \infty$, the $-r^{-1}$ term will dominate, causing $V(r)$ to approach 0 from below. These two facts imply that $V(r)$ must have a local minimum (i.e. a potential well) somwhere after it becomes negative.

Exercise 2

The 2P state has $l \neq 0$, so the position-amplitude goes to 0 at the origin. The perturbation potential is only non-zer *very* near the origin, so it's effect is negligable. Therefore the energy of the 2P state does not change, and the 1S-2P transition energy change is just the change in the 1S state's energy.