

Homework 2

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Problem 1 (Berman 2.9)

Problem 2 (Berman 2.17)

Problem 3

(a) Given that

$$H_0 = \hbar \begin{pmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{pmatrix},$$

we have that

$$\begin{aligned} \exp(iH_0t/\hbar) &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + it \begin{pmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{pmatrix} + \frac{(it)^2}{2} \begin{pmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{pmatrix}^2 + \frac{(it)^3}{3!} \begin{pmatrix} \omega_1 & 0 \\ 0 & \omega_2 \end{pmatrix}^3 + \dots \\ &= \begin{pmatrix} 1 + i\omega_1t + \frac{1}{2}(i\omega_1t)^2 + \frac{1}{3!}(i\omega_1t)^3 + \dots & 0 \\ 0 & 1 + i\omega_2t + \frac{1}{2}(i\omega_2t)^2 + \frac{1}{3!}(i\omega_2t)^3 + \dots \end{pmatrix} \\ &= \begin{pmatrix} \exp(i\omega_1t) & 0 \\ 0 & \exp(i\omega_2t) \end{pmatrix}. \end{aligned}$$

(b) In the interaction representation, we factor out this free phase evolution by writing

$$|\psi(t)\rangle_I = \bar{c}_1(t)e^{-i\omega_1t}|1\rangle + \bar{c}_2(t)e^{-i\omega_2t}|2\rangle,$$

that is, we make the (time-dependent) unitary transformation

$$|\psi(t)\rangle_S \rightarrow |\psi(t)\rangle_I = U(t) |\psi(t)\rangle_S,$$

where

$$U(t) = \begin{pmatrix} e^{-i\omega_1t} & 0 \\ 0 & e^{-i\omega_2t} \end{pmatrix}.$$

We get the effective interaction hamiltonian by making the inverse transformation on V :

$$V_I = U^\dagger V U = \hbar\Omega_0 \cos(\omega t) \begin{pmatrix} 0 & e^{-i\omega_0t} \\ e^{i\omega_0t} & 0 \end{pmatrix},$$

where the phase ϕ has been absorbed into the (complex) Rabi frequency.

Problem 4

(a)