Homework 3

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Phys 663

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Problem 1

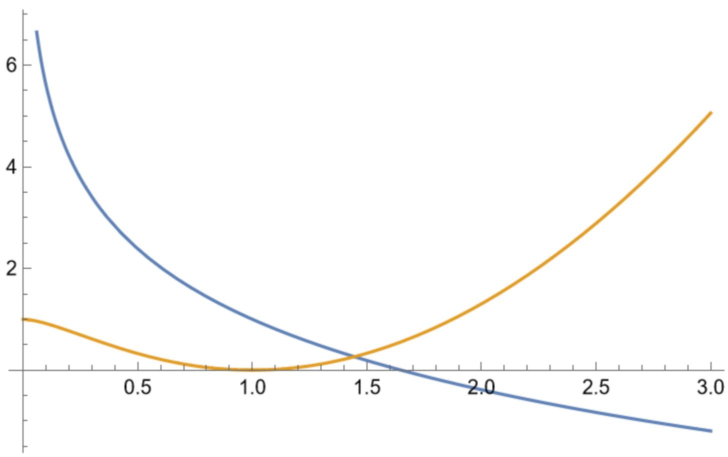


Figure 1: Example Plots of S and T.

(a) Let $x = m_u/m_d$. Then,

$$S = \frac{N_c}{6\pi}\left[1 - 2Y\ln x^2\right]$$
$$T = \frac{m_u^2 N_c}{16\pi\sin^2\theta_W\cos^2\theta_W m_Z^2}\left[1 + x^2 - \frac{2x}{x^2-1}\ln\left(x^2\right)\right]$$

Now,

$$S = 0 \implies x = e^{Y/4} = x_0.$$

s is monotonically deceasing in x, so S is positive when $x < x_0$ and negative when $x > x_0$. To show that T is non-negative, it suffices to show that its minimum is ≥ 0. The derivative of T with respect to x is (ignoring the constants out front)

$$\frac{\mathrm{d}T}{\mathrm{d}x} = 2x(3 - 4x^2 + x^4 + 4\ln(x))(x^2-1)^{-2}.$$

Clearly the derivative has a zero at $x = 0$. The value of the derivative at $x = 1$ is undefined, but it's limit is 0 (see Mathematica printout). Any other zeros will be given by

$$\begin{aligned} \frac{\mathrm{d}T}{\mathrm{d}x} &= 0 \\ \implies 2x(3 - 4x^2 + x^4 + 4\ln(x))(x^2-1)^{-2} &= 0 \\ \implies 3 - 4x^2 + x^4 + 4\ln(x) &= 0 \end{aligned}$$

For $x \geq \sqrt{2}$, the polynomial part is strictly increasing and the logarithm is positive, so there can be no other zeros. Hence, $x = 1$ is the global minimum. The value of T is undefined at $x = 1$, but its limit is 0. Therefore, T is always non-negative.

(b) Defining $m_u = m_d + \Delta$, we have that

$$\begin{aligned} S &= \frac{N_c}{6\pi} - \frac{2YN_c}{3\pi m_d}\Delta + O(\Delta^2) \\ T &= \frac{N_c}{12\pi \sin^2\theta_W \cos^2\theta_W m_Z^2}\Delta^2 + O(\Delta^3) \end{aligned}$$

(see Mathematica printout). Clearly, T is suppressed in the $\Delta = 0$ limit.

(c) In the degenerate mass limit, $N_c = 6\pi S$. Then,

$$S_{\text{new}} = -0.01 \pm 0.07 \implies N_c = -0.2 \pm 1.3$$

I'm not really sure what the hint for this part of the problem is getting at. Given that this value of S_{new} is 1 within uncertainty, it seems like this allows for a new extra heave generation of fermions with degenerate masses.

1 Problem 2

(a) Using the given values we find that $\theta_W = 0.50452$ and $\sin^2(\theta_W) = 0.23366$ (see Mathematica printout).

(b) We find that $m_W = 80.3573$ GeV. Given that the LHC average is 80.366 ± 0.017, our estimate is only 0.0087 GeV, or 0.5σ, from the central value.

2 Problem 3 (Peskin 18.2)

(a) Going off of (14.45), it seems like maybe

$$\langle 0 | \bar{s}\gamma^5 d | K^0 \rangle = \frac{f_\pi m_{K^0}^2}{(m_d + m_s)\Delta'}$$

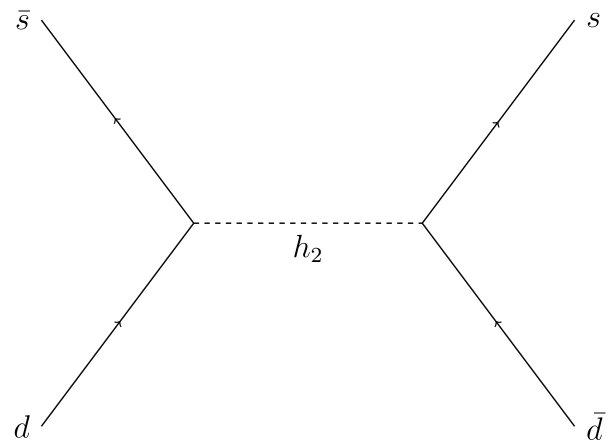


Figure 2: s-Channel exchange of h_2.

(b) The value of the above diagram is given by

$$i\left(\frac{iy_2}{\sqrt{2}}\right)^2 \bar{v}(\vec{p}_{\bar{s}})\gamma^5 u(\vec{p}_d)\frac{i}{p^2 - m_h^2 + i\epsilon}\bar{u}(\vec{p}_s)\gamma^5 v(\vec{p}_{\bar{d}})$$

(c) Using $y_s = \sqrt{2}m_s/v$, we get a Yukawa coupling of 5.5×10^{-4}.

Very disappointed with this last problem. I should have came back to your office a second time to talk more about it.

HW 3 Problem 1

In[4]:=
```
Symbolize[ N_c ]; Symbolize[ m_u ]; Symbolize[ m_d ];
Symbolize[ m_Z ]; Symbolize[ θ_W ];
$Assumptions =
  {N_c > 0, m_u ≥ 0, m_d ≥ 0, m_Z > 0, θ_W > 0, Y ∈ Reals, Δ ∈ Reals, x ∈ Reals};
```

a)

In[6]:=
$$S = \frac{N_c}{6\pi}\left(1 - 2\,Y\,\mathrm{Log}\left[\frac{m_u{}^2}{m_d{}^2}\right]\right);$$

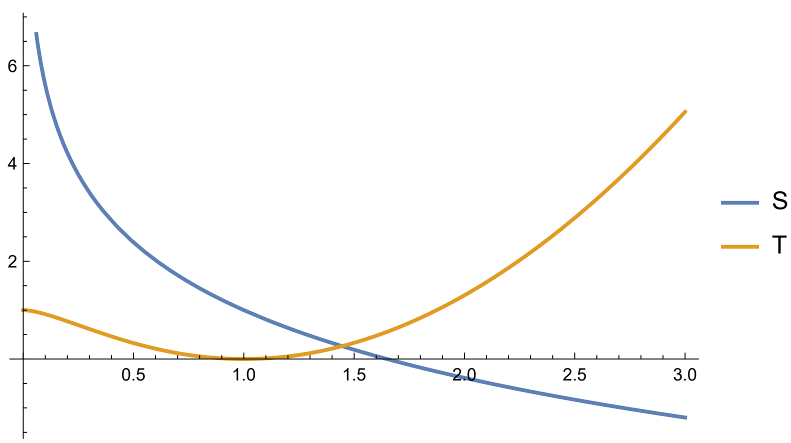
$$T = \frac{N_c}{16\pi\,(\mathrm{Sin}[\theta_W]\,\mathrm{Cos}[\theta_W]\,m_Z)^2}\left(m_u{}^2 + m_d{}^2 - \frac{2\,m_u{}^2\,m_d{}^2}{m_u{}^2 - m_d{}^2}\,\mathrm{Log}\left[\frac{m_u{}^2}{m_d{}^2}\right]\right);$$

In[8]:=
$$s = 1 - \mathrm{Log}[x^2];$$

$$t = 1 + x^2 - \frac{2\,x^2}{x^2 - 1}\,\mathrm{Log}[x^2];$$

In[10]:=
```
Plot[{s, t}, {x, 0, 3}, PlotLegends → {"S", "T"}]
```

Out[10]=



In[11]:=
```
tDeriv = D[t, x] // FullSimplify
```

Out[11]=
$$\frac{2\,x\,(3 - 4\,x^2 + x^4 + \mathrm{Log}[x^4])}{(-1 + x^2)^2}$$

In[22]:=
```
Limit[tDeriv, x → 1] // Quiet
```
Out[22]=
```
0
```

In[23]:=
```
Limit[t, x → 1] // Quiet
```
Out[23]=
```
0
```

B)

In[14]:=
```
Series[S /. {mu → md + Δ}, {Δ, 0, 1}] // TraditionalForm
```
Out[14]//TraditionalForm=

$$\frac{N_c}{6\,\pi} - \frac{2\,\Delta\,(Y\,N_c)}{3\,(\pi\,m_d)} + O(\Delta^2)$$

In[15]:=
```
Series[T /. {mu → md + Δ}, {Δ, 0, 2}] // TraditionalForm
```
Out[15]//TraditionalForm=

$$\frac{\Delta^2\,N_c\,\csc^2(\theta_W)\,\sec^2(\theta_W)}{12\,\pi\,m_Z^2} + O(\Delta^3)$$

c)

In[16]:=
```
degenerateT = Limit[T, mu → md] // Quiet
```
Out[16]=
```
0
```

In[19]:=
```
degenerateS = Limit[S, mu → md] // Quiet
```
Out[19]=

$$\frac{N_c}{6\,\pi}$$

In[20]:=
```
Snew = Around[-0.01, 0.07]
```
Out[20]=

-0.01 ± 0.07

In[21]:=
```
6 π Snew
```
Out[21]=

-0.2 ± 1.3

HW 3 Problem 2

In[7]:=
```
Symbolize[ m_t ]; Symbolize[ m_b ];
Symbolize[ m_Z ]; Symbolize[ θ_W ];
Symbolize[ m_W ];
Symbolize[ G_f ];
Symbolize[ α_{m_Z} ];
m_t = 172.8 GeV // UnitConvert;
m_b = 4.18 GeV // UnitConvert;
m_Z = 91.188 GeV // UnitConvert;
G_f = 0.00001166 /GeV^2 // UnitConvert;
α_{m_Z} = 1 / 127.951 // UnitConvert;
```

a)

In[13]:=
```
soln = NSolve[(Sin[2 θ_W] == Sqrt[(4 π α_{m_Z}) / (Sqrt[2] G_f m_Z^2)]), θ_W] // Quiet;
StringForm["θ_W = ``", θ_W /. soln[[1]]]
StringForm["`` = ``", Sin[θ_W]^2 // TraditionalForm, Sin[θ_W]^2 /. soln[[1]] ]
θ_W = θ_W /. soln[[1]];
```

$$\sin(2\theta_W) = \sqrt{\frac{4\pi\alpha_{m_Z}}{\sqrt{2}\,G_f\,m_Z^2}}$$

Out[14]=

θ_W = 0.5045265403000181`

Out[15]=

$\sin^2(\theta_W)$ = 0.23366881784571636`

B)

In[17]:=

```
T = 3/(16 π (Sin[θW] Cos[θW] mZ)^2) (mt^2 + mb^2 - (2 mt^2 mb^2)/(mt^2 - mb^2) Log[mt^2/mb^2]);
mW = mZ Sqrt[Cos[θW]^2 + (αmz Cos[θW]^2)/(Cos[θW]^2 - Sin[θW]^2) (Cos[θW]^2 T)];
StringForm["mW = ``", UnitConvert[mW, "Gigaelectronvolts"]]
```

$$T = \frac{3}{16\pi\,(\mathrm{Sin}[\theta_W]\,\mathrm{Cos}[\theta_W]\,m_Z)^2}\left(m_t{}^2 + m_b{}^2 - \frac{2\,m_t{}^2\,m_b{}^2}{m_t{}^2 - m_b{}^2}\,\mathrm{Log}\left[\frac{m_t{}^2}{m_b{}^2}\right]\right);$$

$$m_W = m_Z\sqrt{\mathrm{Cos}[\theta_W]^2 + \frac{\alpha_{m_z}\,\mathrm{Cos}[\theta_W]^2}{\mathrm{Cos}[\theta_W]^2 - \mathrm{Sin}[\theta_W]^2}\left(\mathrm{Cos}[\theta_W]^2\,T\right)};$$

StringForm["m_W = ``", UnitConvert[m_W, "Gigaelectronvolts"]]

Out[19]=

m_W = 80.3573 GeV