Problem 2

In[1]:=
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<< Notation`
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In[2]:=
$$\text{Symbolize}[\pi^0];\ \text{Symbolize}[\pi^+];\ \text{Symbolize}[\pi^-];$$
$$\text{Symbolize}[K^0];\ \text{Symbolize}[K^+];\ \text{Symbolize}[K^-];\ \text{Symbolize}[\overline{K}^0];$$
$$\text{Symbolize}[\eta];\ \text{Symbolize}[f_\pi];\ \text{Symbolize}[\Sigma^+];$$
$$\text{Symbolize}[m_u];\ \text{Symbolize}[m_d];\ \text{Symbolize}[m_s];$$
$$\text{Symbolize}[Q_L];\ \text{Symbolize}[Q_R];\ \text{Symbolize}[L_2];$$

In[7]:=
$$\$\text{Assumptions} = \{m_u \in \mathbb{R},\ m_d \in \mathbb{R},\ m_s \in \mathbb{R},\ f_\pi \in \mathbb{R}\};$$

In[8]:=
$$\Pi = \left\{\left\{\pi^0 + \frac{1}{\sqrt{3}}\eta,\ \sqrt{2}\,\pi^+,\ \sqrt{2}\,K^+\right\},\right.$$
$$\left.\left\{\sqrt{2}\,\pi^-,\ -\pi^0 + \frac{1}{\sqrt{3}}\eta,\ \sqrt{2}\,K^0\right\},\ \left\{\sqrt{2}\,K^-,\ \sqrt{2}\,\overline{K}^0,\ \frac{-2}{\sqrt{3}}\eta\right\}\right\};$$
$$M = \text{DiagonalMatrix}[\{m_u,\ m_d,\ m_s\}];$$
$$\text{MatrixForm /@ }\{\Pi,\ M\}\text{ // Row}$$

Out[10]=
$$\begin{pmatrix} \frac{\eta}{\sqrt{3}} + \pi^0 & \sqrt{2}\,\pi^+ & \sqrt{2}\,K^+ \\ \sqrt{2}\,\pi^- & \frac{\eta}{\sqrt{3}} - \pi^0 & \sqrt{2}\,K^0 \\ \sqrt{2}\,K^- & \sqrt{2}\,\overline{K}^0 & -\frac{2\eta}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}$$

In[11]:=
$$n = 2;$$
$$\Sigma = \sum_{i=0}^{n} \frac{1}{i\,!}\left(\frac{i}{f_\pi}\right)^i \text{MatrixPower}[\Pi,\ i];$$
$$\Sigma^+ = \sum_{i=0}^{n} \frac{1}{i\,!}\left(\frac{-i}{f_\pi}\right)^i \text{MatrixPower}[\Pi,\ i];$$
$$\text{MatrixForm /@ }\{\Sigma,\ \Sigma^+\}\text{ // Row}$$

Out[14]=
$$\left(\begin{matrix}
1 + \frac{i\left(\frac{\eta}{\sqrt{3}}+\pi^0\right)}{f_\pi} - \frac{2K^- K^+ + \left(\frac{\eta}{\sqrt{3}}+\pi^0\right)^2 + 2\pi^-\pi^+}{2f_\pi^2} & \frac{i\sqrt{2}\,\pi^+}{f_\pi} - \frac{2\overline{K}^0 K^+ + \sqrt{2}\left(\frac{\eta}{\sqrt{3}}-\pi^0\right)\pi^+ + \sqrt{2}\left(\frac{\eta}{\sqrt{3}}+\pi^0\right)\pi^+}{2f_\pi^2} & \frac{i\sqrt{2}\,K^+}{f_\pi} - \frac{-2\sqrt{\frac{2}{3}}\,K^+\eta + \sqrt{\cdot}}{} \\
\frac{i\sqrt{2}\,\pi^-}{f_\pi} - \frac{2K^0 K^- + \sqrt{2}\left(\frac{\eta}{\sqrt{3}}-\pi^0\right)\pi^- + \sqrt{2}\left(\frac{\eta}{\sqrt{3}}+\pi^0\right)\pi^-}{2f_\pi^2} & 1 + \frac{i\left(\frac{\eta}{\sqrt{3}}-\pi^0\right)}{f_\pi} - \frac{2\overline{K}^0 K^0 + \left(\frac{\eta}{\sqrt{3}}-\pi^0\right)^2 + 2\pi^-\pi^+}{2f_\pi^2} & \frac{i\sqrt{2}\,K^0}{f_\pi} - \frac{-2\sqrt{\frac{2}{3}}\,K^0\eta + \sqrt{\cdot}}{} \\
\frac{i\sqrt{2}\,K^-}{f_\pi} - \frac{-2\sqrt{\frac{2}{3}}\,K^-\eta + \sqrt{2}\,K^-\left(\frac{\eta}{\sqrt{3}}+\pi^0\right) + 2\overline{K}^0\pi^-}{2f_\pi^2} & \frac{i\sqrt{2}\,\overline{K}^0}{f_\pi} - \frac{-2\sqrt{\frac{2}{3}}\,\overline{K}^0\eta + \sqrt{2}\,\overline{K}^0\left(\frac{\eta}{\sqrt{3}}-\pi^0\right) + 2K^-\pi^+}{2f_\pi^2} & 1 - \frac{2i\eta}{\sqrt{3}\,f_\pi} - \frac{2\overline{K}^0 \cdots}{}
\end{matrix}\right.$$

a)

In[15]:=
```
L = c fπ^3 Tr[M.Σ + M†.Σ†] // FullSimplify
```

Out[15]=

$$\frac{1}{3} c f_\pi \left(-6 \overline{K}^0 K^0 (m_d + m_s) - 6 K^- K^+ (m_s + m_u) + 6 f_\pi^2 (m_d + m_s + m_u) - (m_d + 4 m_s + m_u) \eta^2 + 2 \sqrt{3} (m_d - m_u) \eta \pi^0 - 3 (m_d + m_u) \left(\pi^0\right)^2 - 6 (m_d + m_u) \pi^- \pi^+\right)$$

In[16]:=
```
-Coefficient[L, K+ K-]
```

Out[16]=

$$2 c f_\pi (m_s + m_u)$$

In[17]:=
```
-Coefficient[L, K0 K̄0]
```

Out[17]=

$$2 c f_\pi (m_d + m_s)$$

In[18]:=
```
-2 Coefficient[L, η^2] // Simplify
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Out[18]=

$$\frac{2}{3} c f_\pi (m_d + 4 m_s + m_u)$$

b)

In[19]:=
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QL = DiagonalMatrix[{2/3, -1/3, -1/3}];
QR = QL;
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In[21]:=
```
L2 = d fπ^3 Tr[QL.Σ.QR.Σ† + Σ.QR.Σ†.QL] // FullSimplify
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Out[21]=

$$\frac{1}{54 f_\pi} d \Big(7 \eta^4 + 4 \sqrt{3} \eta^3 \pi^0 + 6 \eta^2 \left(4 \overline{K}^0 K^0 + 4 K^- K^+ + 5 \left(\pi^0\right)^2 - 2 \pi^- \pi^+\right) + 12 \sqrt{3} \eta \left(\left(\pi^0\right)^3 - \sqrt{2} \left(\overline{K}^0 K^+ \pi^- + K^0 K^- \pi^+\right) + 2 \pi^0 (2 K^- K^+ + \pi^- \pi^+)\right) + 3 \Big(24 f_\pi^4 + 8 \left(\overline{K}^0\right)^2 \left(K^0\right)^2 + 5 \left(\pi^0\right)^4 - 12 \sqrt{2} \pi^0 \left(\overline{K}^0 K^+ \pi^- + K^0 K^- \pi^+\right) - 72 f_\pi^2 (K^- K^+ + \pi^- \pi^+) + 4 \left(\pi^0\right)^2 \left(2 \overline{K}^0 K^0 + 2 K^- K^+ + 5 \pi^- \pi^+\right) + 4 (K^- K^+ + \pi^- \pi^+) \left(-2 \overline{K}^0 K^0 + 5 K^- K^+ + 5 \pi^- \pi^+\right)\Big)\Big)$$

In[22]:=
```
-Coefficient[L2, K+ K-] /. {π0 → 0, π+ → 0, π- → 0, K0 → 0, K̄0 → 0, η → 0}
```

Out[22]=

$$4 d f_\pi$$

In[23]:=
```
-Coefficient[L2, π+ π-] /. {π0 → 0, K+ → 0, K- → 0, K0 → 0, K̄0 → 0, η → 0}
```

Out[23]=

$$4 d f_\pi$$

```
In[24]:= -Coefficient[L₂, K⁰ K̄⁰] /. {π⁰ → 0, π⁺ → 0, π⁻ → 0, K⁺ → 0, K⁻ → 0, η → 0}
Out[24]=
0

In[25]:= -Coefficient[L₂, π⁰ π⁰] /. {π⁺ → 0, π⁻ → 0, K⁺ → 0, K⁻ → 0, K⁰ → 0, K̄⁰ → 0, η → 0}
Out[25]=
0

In[26]:= -2 Coefficient[L₂, η²] /. {π⁰ → 0, π⁺ → 0, π⁻ → 0, K⁺ → 0, K⁻ → 0, K⁰ → 0, K̄⁰ → 0}
Out[26]=
0
```