Homework 4

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Problem 1

(a) Letting

$$\vec{R} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}; \quad \vec{\Omega} = \begin{pmatrix} \Omega' \\ -\Omega'' \\ \delta \end{pmatrix},$$

the evolution of the Bloch vector in the BLANK is given by

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{R} = \vec{\Omega} \times \vec{R} \implies \begin{cases} \dot{u} = -\delta v - \Omega_0'' w \\ \dot{v} = \delta u - \Omega_0' w \\ \dot{w} = \Omega_0' v + \Omega_0'' u. \end{cases}$$

Given that $\Omega_0'' = 0$, this simplifies to

$$\begin{aligned} \dot{u} &= -\delta v \\ \dot{v} &= \delta u - \Omega_0' w \\ \dot{w} &= \Omega_0' v. \end{aligned}$$

Then,

$$\begin{aligned} \ddot{v} &= \delta \dot{u} - \Omega_0' \dot{w} \\ &= -\left(\delta^2 + \Omega_0'^2\right) v \\ &= -\Omega^2 v \\ \implies \quad v(t) &= A\cos(\Omega t) + B\sin(\Omega t). \end{aligned}$$

Next, the initial condition $\vec{R}(0) = -\hat{w}$ implies $A = 0$, so

$$v(t) = B\sin(\Omega t).$$

Now solving for u,

$$\begin{aligned} \dot{u} &= -\delta v \\ \implies \quad u(t) &= u(t) - \delta \int_0^t \mathrm{d}t' v(t') \\ &= \frac{\delta B}{\Omega} \cos(\Omega t')\Big|_0^t \\ &= \frac{\delta B}{\Omega} \left(\cos(\Omega t) - 1\right) \end{aligned}$$

Problem 2

(a)

Problem 3

(a)

Problem 4

(a)