Homework 1

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Phys 610

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1.1.1

(a) If the set contains itself, then by definition it doesn *not* contain itself, a contradiction. Alternatively, if the set does not contain itself, then by definition it *does* contain itself, also a contradiction. Formerly,

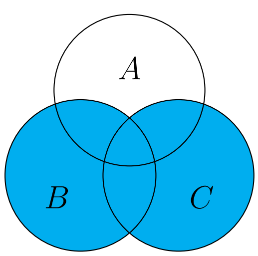
$$M \in M \implies M \notin M$$

$$M \notin M \implies M \in M$$

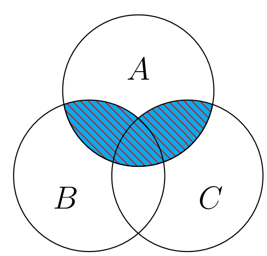
(b) If the barber shaves himself, then he is by definition *not* the barber. If he is not the barber, then he must not shave himself.

(c) The barber may now be a woman, robot, or dog, therefore avoiding the paradox.

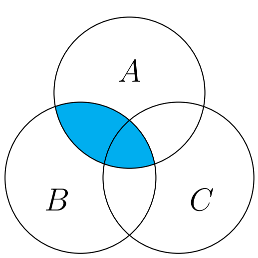
1.1.2



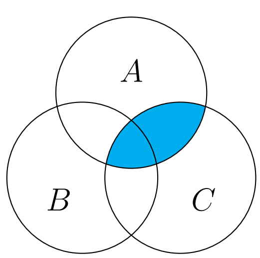
$B \cup C$



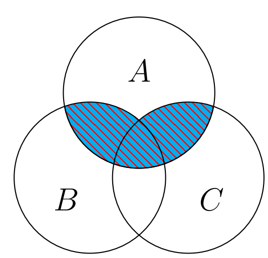
$A \cap (B \cup C)$



$A \cap B$



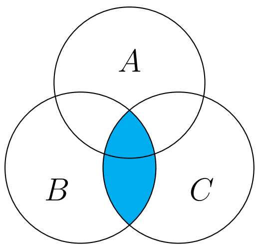
$A \cap C$



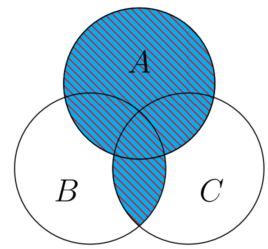
$(A \cap B) \cup (A \cap C)$

Thus,

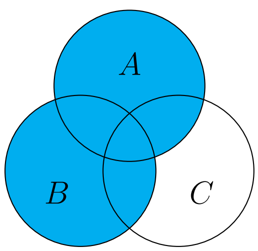
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$



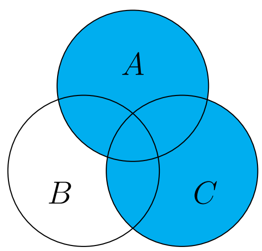
$B \cap C$



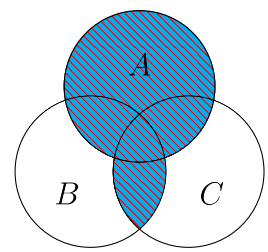
$A \cup (B \cap C)$



$A \cup B$



$A \cup C$



$(A \cup B) \cap (A \cup C)$

Thus,

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

1.1.3

(a) $f(m) = m^2 + 1$ is a true mapping, but it is neither surjective nor injective:

$$f(X) = \{1, 2, 3, \cdots\} \neq \mathbb{Z} \implies \text{Not Surjective}$$

$$\forall a \in \mathbb{Z} \quad f(a) = f(-a) \implies \text{Not Injective}$$

(b) $f(n) = n + 1$ is a true mapping and it is injective, but it is not surjective:

$$f(X) = \{2, 3, 4, \cdots\} \neq \mathbb{N} \implies \text{Not Surjective}$$

(c) $f(x) = \log x$ is not a true mapping from $\mathbb{Z}$ to $\mathbb{R}$, as $\log x$ is only defined for positive values.

(d) $f(x) = e^x$ is a true mapping and it is injective, but not surjective:

$$f(X) = \{x; x \in \mathbb{R}, x > 0\} \neq \mathbb{R} \implies \text{Not Surjective}$$

1.1.4

(a) f is surjective for all tripples $(0, 1, c)$, as any line with slope 1 and integer y-intercept will hit all other integers.

(b) f is injective for all tripples $(0, b, c)$, as any non-zero value for a will result in a 'parabola'.