
Problem 1

```
In[1]:= P = 1 mW ; A = 1 mm2 ;
```

```
In[2]:= μ = e a0 ; UnitConvert[μ, "Debyes"]
```

```
Out[2]= 2.541746473 D
```

```
In[3]:= E0 =  $\sqrt{\frac{2 P}{c \epsilon_0 A}}$  ; UnitConvert[E0, "V/m"]
```

```
Out[3]= 868.021098 V/m
```

```
In[4]:= Ω =  $\frac{\mu E_0}{\hbar}$  ; UnitConvert[Ω, "MHz"]
```

```
Out[4]= 69.7855727 MHz
```

Problem 2

```
In[5]:= Clear[Ω];
```

```
c1 = {1, 0, 0, 0};
```

```
c2 = {0, 0, 0, 0};
```

```
In[8]:= For[j = 2, j < 5, j++,
```

```
    c1[[j]] = Integrate[ $\frac{-i}{2} e^{i \delta t} c2[[j-1]]$ , {t, 0, t}] // FullSimplify;
```

```
    c2[[j]] = Integrate[ $\frac{-i}{2} e^{-i \delta t} c1[[j-1]]$ , {t, 0, t}] // FullSimplify;
```

```
]
```

```
In[9]:= combine[order_, coeff_] = Sum[Index[coeff, j] Ωj-1, {j, 1, order+1}];
```

```
sqrMag[z_] := FullSimplify[ComplexExpand[Abs[z]2]];
```

```
showOrder[order_] := StringForm["Order ``: \n
```

```
c1 ~ `` ⇒ |c1|2 ~ `` \n
```

```
c2 ~ `` ⇒ |c2|2 ~ `` \n\n",
```

```
order, combine[order, c1], sqrMag[combine[order, c1]],
```

```
combine[order, c2], sqrMag[combine[order, c2]]];
```

In[12]:= Column[Table[showOrder[i], {i, 0, 3}]]

Order 0:

$$c_1 \sim 1 \Rightarrow |c_1|^2 \sim 1$$

$$c_2 \sim 0 \Rightarrow |c_2|^2 \sim 0$$

Order 1:

$$c_1 \sim 1 \Rightarrow |c_1|^2 \sim 1$$

$$c_2 \sim \frac{(-1+e^{-i t \delta}) \Omega_0}{2 \delta} \Rightarrow |c_2|^2 \sim \frac{\sin\left[\frac{t \delta}{2}\right]^2 \Omega_0^2}{\delta^2}$$

Out[12]= Order 2:

$$c_1 \sim 1 + \frac{(-1+e^{i t \delta}-i t \delta) \Omega_0^2}{4 \delta^2} \Rightarrow |c_1|^2 \sim 1 + \frac{8 \delta^2 (-1+\cos[t \delta]) \Omega_0^2 + (2+t^2 \delta^2 - 2 \cos[t \delta] - 2 t \delta \sin[t \delta]) \Omega_0^4}{16 \delta^4}$$

$$c_2 \sim \frac{(-1+e^{-i t \delta}) \Omega_0}{2 \delta} \Rightarrow |c_2|^2 \sim \frac{\sin\left[\frac{t \delta}{2}\right]^2 \Omega_0^2}{\delta^2}$$

Order 3:

$$c_1 \sim 1 + \frac{(-1+e^{i t \delta}-i t \delta) \Omega_0^2}{4 \delta^2} \Rightarrow |c_1|^2 \sim 1 + \frac{8 \delta^2 (-1+\cos[t \delta]) \Omega_0^2 + (2+t^2 \delta^2 - 2 \cos[t \delta] - 2 t \delta \sin[t \delta]) \Omega_0^4}{16 \delta^4}$$

$$c_2 \sim \frac{(-1+e^{-i t \delta}) \Omega_0}{2 \delta} + \frac{(2-i t \delta+e^{-i t \delta} (-2-i t \delta)) \Omega_0^3}{8 \delta^3} \Rightarrow |c_2|^2 \sim \frac{\left(t \delta \cos\left[\frac{t \delta}{2}\right] \Omega_0^3 - 2 \sin\left[\frac{t \delta}{2}\right] \Omega_0 (-2 \delta^2 + \Omega_0^2)\right)^2}{16 \delta^6}$$

Problem 3

In[13]:= $\sigma_x = \text{PauliMatrix}[1]; \sigma_z = \text{PauliMatrix}[3];$

In[14]:= $H_0 = -\frac{\hbar \omega_0}{2} \sigma_z;$

$V = \hbar \Omega_0 \cos[\omega t - \phi] \sigma_x;$

$U = \text{MatrixExp}[-i H_0 t / \hbar];$

```
In[17]:= V_I = U†.V.U // ComplexExpand // TrigToExp // FullSimplify;
          V_I // MatrixForm
```

```
Out[18]//MatrixForm=
```

$$\begin{pmatrix} 0 & e^{-i t \omega_0} \cos[\phi - t \omega] \hbar \Omega_0 \\ e^{i t \omega_0} \cos[\phi - t \omega] \hbar \Omega_0 & 0 \end{pmatrix}$$

Problem 4

```
In[19]:= M = -1/2 {{γ1, i Ω0}, {i Ω0, γ2}};
```

```
ψ0 = {{1}, {0}};
```

```
ψ[t_] = {a1[t], a2[t]};
```

```
In[22]:= soln = MatrixExp[M t].ψ0 // FullSimplify;
```

```
soln = (Numerator[soln] /. {Sqrt[(γ1 - γ2)^2 - 4 Ω0^2] -> χ}) /
        (Denominator[soln] /. {Sqrt[(γ1 - γ2)^2 - 4 Ω0^2] -> χ});
```

```
StringForm["c1 = `` \nc2 = `` \n|c2|^2 = ``", soln[[1]][1] // FullSimplify,
          soln[[2]][1] // FullSimplify, SqrMag[soln[[2]][1]] // FullSimplify]
```

```
Out[24]= c1 =
```

$$\frac{e^{-\frac{1}{4} t (\chi + \gamma_1 + \gamma_2)} \left(\chi + \gamma_1 - \gamma_2 + e^{\frac{t \chi}{2}} (\chi - \gamma_1 + \gamma_2) \right)}{2 \chi}$$

$$c_2 = -\frac{i e^{-\frac{1}{4} t (\chi + \gamma_1 + \gamma_2)} \left(-1 + e^{\frac{t \chi}{2}} \right) \Omega_0}{\chi}$$

$$|c_2|^2 = \frac{\left(1 + e^{t \chi} - 2 \sqrt{e^{t \chi}} \right) \Omega_0^2}{\sqrt{e^{t (\chi + \gamma_1 + \gamma_2)}} \chi^2}$$

```
In[25]:= soln = soln /. {γ1 -> γ, γ2 -> γ, χ -> 2 i Ω0};
```

```
StringForm["c1 = `` \nc2 = `` \n|c2|^2 = ``", soln[[1]][1] // FullSimplify,
          soln[[2]][1] // FullSimplify, SqrMag[soln[[2]][1]] // FullSimplify]
```

```
Out[26]= c1 =
```

$$e^{-\frac{t \gamma}{2}} \cos\left[\frac{t \Omega_0}{2}\right]$$

$$c_2 = -i e^{-\frac{t \gamma}{2}} \sin\left[\frac{t \Omega_0}{2}\right]$$

$$|c_2|^2 = \frac{\sin^2\left[\frac{t \Omega_0}{2}\right]}{\sqrt{e^{2 t \gamma}}}$$