PHYS 674 Condensed Matter
Final Exam

Due: Thursday, December 7, 12:00pm

Problem 1

Consider an $O(n)$ model with a "cubic symmetry breaking term" g, by which I mean the following model:

$$H = \int \mathrm{d}^d r \left[\frac{t}{2} \left| \vec{M} \right|^2 + u \left| \vec{M} \right|^4 + g \sum_{\alpha=1}^{n} M_\alpha^4 + \frac{c}{2} |\nabla M|^2 \right] \tag{1.1}$$

(a) Derive the RG recursion relations for this model to one loop order.

(b) Find the fixed points

(c) Identify the fixed point that controls the transition.

(d) Calculate the critical exponents α, β, and ν to $O(\epsilon = 4 - d)$

Hint: Your answers to parts (b), (c), and (d) could be qualitatively different for different values of n.

Problem 3

Suppose we apply a magnetic field to an X-Y model, this amounts to adding a symmetry breaking term

$$\Delta H = -\vec{h} \cdot \sum_i \vec{S}_i \tag{3.1}$$

to the standard X-Y Hamiltonian.

(a) Write the continuum version of this model, ignoring irrelevant terms.

(b) Show that the model includes a term

$$\Delta H = - \int \mathrm{d}^d r \cos(\theta(\vec{r})) \tag{3.2}$$

(c) Expand the cosine to all orders in θ, and represent each term by a (schematic) Feynman graph.

(d) Derive RG recursion relations for a_n, the coefficient of θ^n in the above expansion to linear order in the a_n. Show that, for the correct choice of χ_θ, the rescaling exponent for θ, the entire series can be resummed to give a $-h(l)\cos\left(\theta'(\vec{r'})\right)$, with a renormalized $h(l)$; and thereby derive the recursion relation for $h(l)$ to this order.

(e) Derive the recursion relation for $K(l)$ to this order, using the above choice of χ_θ.

(f) Show that, for $d > 2$, $h(l \to \infty) \to \infty$ (at least, as far as this perturbative calculation can tell). What does this imply about the $|\vec{q}| \ll \Lambda$ limit of the fluctuations $\left\langle |\theta(q)|^2 \right\rangle$?

(g) Show that, in $d = 2$, there is a phase transition at some critical temperature T_c in this model, in the sense that qualitatively different scaling behaviors apply for $T < T_c$ and $T > T_c$. Calculate T_c in terms of K.

(h) Calculate $\langle \cos\theta(r) \rangle$ in a $d = 2$ $L \times L$ square system, taking $h = 0$.