

Exercise Set 7

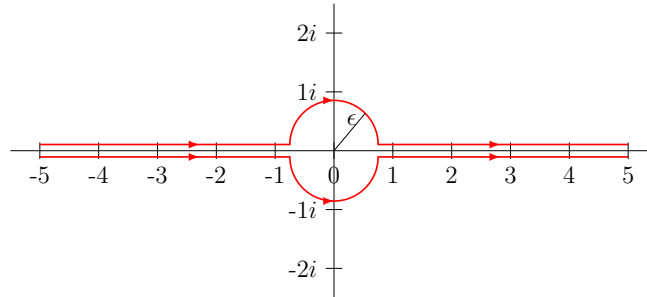
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Phys 633

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Monday

Exercise 1



For any function $f(z)$ that is analytic through an ϵ -disk centered on the origin, as $\epsilon \rightarrow 0$ we can approximate the value of f for points on the circular portions of the above contour integrals by its value at the origin $f(0)$. In the case of an integrand of the form $f(z)/z$, the circular portion on the upper integral gives

$$-f(0) \int_{\pi}^0 \frac{1}{\epsilon e^{i\theta}} \epsilon d\theta = f(0) \int_0^{\pi} e^{-i\theta} d\theta = -2if(0),$$

while the circular portion on the lower integral gives

$$f(0) \int_{\pi}^{2\pi} \frac{1}{\epsilon e^{i\theta}} \epsilon d\theta = f(0) \int_{\pi}^{2\pi} e^{-i\theta} d\theta = 2if(0),$$

The circular portions of both integrals therefore cancel, and we're left with just two integrals over the real line that skip the origin, verifying the equivalence of the two definitions.

Tuesday

Echoes and meeps!