Exercise Set 1

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Exercise 1

(a) We want to approximate

$$n! = \int_0^\infty \mathrm{d}t\, e^{-t+n\log t}.$$

First we expand the function in the exponential about it's maximum. The maximum occurs at

$$\frac{\mathrm{d}}{\mathrm{d}t}(-t+n\log t) = 0 \implies \frac{n}{t} - 1 = 0 \implies t = n.$$

Expanding about that point gives

$$-(t-n) + n\log(t-n) \approx -n - n\log n - \frac{(t-n)^2}{2n}$$

(b) Plugging this back into the exponential we see that

$$\begin{aligned} n! &\approx e^{-n-n\log n} \int_0^\infty \mathrm{d}t\, e^{-\frac{(t-n)^2}{2n}} \\ &\approx \left(\frac{n}{e}\right)^n \sqrt{2\pi n} \end{aligned}$$

Exercise 2

(a) Given that $x = \frac{1}{2}\frac{F}{m}t^2$, we can see that $F \propto \frac{mx}{t^2}$ thus

$$\Delta F \propto \Delta x \frac{m}{\tau^2} = \sqrt{\frac{\hbar\tau}{m}}\frac{m}{\tau^2} = \sqrt{\frac{\hbar m}{\tau^3}}$$

(b) Multiplying the expression for ΔF by a quantity with dimensions of time (τ) gives

$$\Delta p \propto \tau\sqrt{\frac{\hbar m}{\tau^3}} = \sqrt{\frac{\hbar m}{\tau}}$$