Practice Problems

seva6

November 2021

1 Chapter 4 Section 1

1.1 Easy Problems

1. $1, 5, 7, 11, 13, 17, 23$

2. Let R be a ring such that R is a field. That is to say that R is commutative and a division ring. Given that R is a division ring, it is true that R has a unit 1 and that for every $a \in R, a \neq 0$ there is a corresponding $a^{-1} \in R$ such that $a * a^{-1} = a^{-1} * a = 1$, the unit in R. This means that R has a multiplicative identity. Let $a, b \in R$ such that $a * b = 0$ where $a \neq 0$. This then implies, by the definition of a ring, that a^{-1} exists. Then we can say that $a^{-1}(ab) = a^{-1}(0) = 0 = (1)b = (a^{-1}a)b$, which implies that $b = 0$. Similarly, if $b \neq 0$ then $a = 0$. Therefore, this implies that whenever $ab = 0$, either $a = 0$ or $b = 0$ which means that R is an integral domain.

3. In order for an element $a \in \mathbb{Z}_n$ to have an inverse in $\mathbb{Z}_n$ it must be that $\gcd(a, n) = 1$. In order for $\mathbb{Z}_n$ be a field, each element $a \in \mathbb{Z}_n$ must have an inverse and it must be commutative. If n is not prime, we know that there exists $a < n$ such that $\gcd(a, n) \neq 1$. Therefore a would not have an inverse in $\mathbb{Z}_n$. Therefore, in order for each element to have an inverse in $\mathbb{Z}_n$ then n must be prime. Additionally, $\mathbb{Z}_n$ is commutative by virtue that $\mathbb{Z}_n$ is commutative by definition.

4. In example information.

5. In example information.

6. Just multiply 3 generic matrices.

7. Matrix multiplication.

8. a, d can be anything but $b = c = 0$.

9.

10.

1.2 Medium Problems

Not worth the busy work.

1.3 Hard Problems

Not worth the busy work.

2 Chapter 4 Section 2

1. Let R be a ring. That is to say that R is an abelian group under addition. Then for $n, m \in \mathbb{Z}$ and $a, b \in R$ then $(na)(mb) = (a + a + a + ...)(b + b + b + ...)$ n and m times respectively which is then $(ab + ab + ab + ...) + (ab + ab + ab...)$ where each parenthesis has n times and this is repeated m times. This then is to say that we have $ab + ab + ab...$ nm times in total. Therefore, $(na)(mb) = (nm)(ab)$.

2. Let R be an integral domain. That is to say that R is commutative and it is true that for $a, b \in R$ that $ab = 0$ then $a = 0$ or $b = 0$. Then consider for $a, b, c \in R$ such that $a \neq 0$ and $ab = ac$. Then we can say that $ab - ac = a(b - c) = 0$ since $a \neq 0$ we know that $b - c = 0$ which implies that $b = c$.

3. Homework 5 problem 5.

4.

5.

6.

3 Chapter 4 Section 3

3.1 Easy Problems

3.2 Medium Problems

3.3 Hard Problems

4 Chapter 4 Section 4

4.1 Easy Problems

4.2 Medium Problems

4.3 Hard Problems