

# MATH 4802

## Presentation 3, Group 4

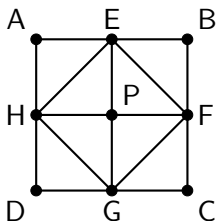
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## Problem: 2009 A1

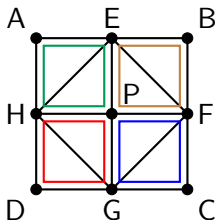
Let  $f$  be a real-valued function on the plane such that for every square  $ABCD$  in the plane,  $f(A) + f(B) + f(C) + f(D) = 0$ . Does it follow that  $f(P) = 0$  for all points  $P$  in the plane?

# Solution



Let  $P$  be any point in the plane. Define an arbitrary square  $ABCD$  in the plane with center  $P$  and corners labeled  $A, B, C, D$ . Let the midpoints of  $AB, BC, CD, DA$  be  $E, F, G, H$  (respectively).

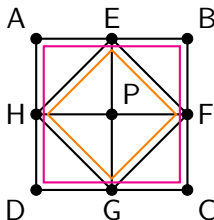
# Solution



We note  $AEPH$ ,  $EBFP$ ,  $PFCG$ ,  $HPGD$  are all squares, so

$$\begin{array}{rcll}
 f(A) & & + f(E) & + f(H) + f(P) = 0 \\
 f(B) & & + f(E) + f(F) & + f(P) = 0 \\
 f(C) & & + f(F) + f(G) & + f(P) = 0 \\
 + & f(D) & + f(G) + f(H) + f(P) & = 0 \\
 \hline
 (f(A) + f(B) + f(C) + f(D)) & & + 2(f(E) + f(F) + f(G) + f(H)) & + 4f(P) = 0
 \end{array}$$

# Solution



$$\begin{aligned} & (f(A)+f(B)+f(C)+f(D)) \\ & \quad + 2(f(E)+f(F)+f(G)+f(H)) \\ & \quad \quad \quad + 4f(P) = 0 \end{aligned}$$

We know that  $ABCD$  and  $EFGH$  are both squares, so the pink and orange sums are both 0. This leaves us with  $4f(P) = 0 \implies f(P) = 0$ . As  $P$  was chosen arbitrarily we conclude  $f(P) = 0$  for all points  $P$  in the plane.