Homework 15

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1. (a) Density: $D = \frac{m}{v}$, Volume of a sphere: $V = \frac{4}{3}\pi r^3$.
   Density of a neutron: $D = \frac{1.7*10^{-27}}{\frac{4}{3}\pi(1.8*10^{-15})^3} = 6.959 * 10^{17}\frac{\text{kg}}{\text{m}^3}$.
   Density of a neutron star: $D = \frac{1.4*(1.989*10^{30})}{\frac{4}{3}\pi(10*1000)^3} = 6.648 * 10^{17}\frac{\text{kg}}{\text{m}^3}$.
   The density of a neutron star is ten times that of just a neutron.

   (b) It appears that since the density of a neutron star is ten times that of just a neutron that the neutrons within the star are overlapping.

2. The mass of the Earth $= 5.972 * 10^{24}$kg. Then,

$$6.648 * 10^{17} = \frac{5.972 * 10^{24}}{\frac{4}{3}\pi(r)^3}$$
$$r = \sqrt[3]{\frac{5.972 * 10^{24}}{\frac{4}{3}\pi(6.648 * 10^{17})}}$$
$$r = 128.958\text{m} = 0.129\text{km}$$

3. Conservation of angular momentum: $I_0\omega_0 = I_1\omega_1$. Then,

$$M(6.96 * 10^8)^2\frac{1}{30 * 24 * 3600} = M(12 * 1000)^2\omega$$
$$(6.96 * 10^8)^2\frac{1}{30 * 24 * 3600} = (12000)^2\omega$$
$$\omega = 1297.840$$
$$P = \frac{1}{1297.840}$$
$$P = 7.71 * 10^{-4}\text{s}$$

4. (a) Maximum wavelength of emission: $\lambda_{max} = \frac{0.0029}{T}$. Then,

$$\lambda_{max} = \frac{0.0029}{4.4 * 10^7}$$
$$\lambda_{max} = 6.591 * 10^{-11}\text{m}.$$

   This wavelength is in the X rays to Gamma rays part of the spectrum.

(b) Stefan-Boltzmann: $\frac{L_1}{L_1} = (\frac{R_1}{R_2})^2(\frac{T_1}{T_2})^4$. Then,

$$L = (\frac{10000}{6.96 * 10^8})^2(\frac{4.4 * 10^7}{5800})^4$$
$$L = 6.84 * 10^5.$$

Therefore, the neutron star is about $6.84 * 10^5 L_{\text{Sun}}$ or approximately $2.64 * 10^{32}$

5. Kepler's 3rd Law: $M * P^2 = a^3$. Then,

$$MP^2 = a^3$$
$$M(\frac{7.25}{24 * 365.25})^2 = (1.163 * 10^{-2})^3$$
$$M = 2.300 M_\odot.$$

6. Schwarzschild Radius: $R_S = \frac{2GM}{c^2}$

(a)

$$\begin{aligned} R_S &= \frac{2GM}{c^2} \\ &= \frac{2(6.67 * 10^{-11})(5.972 * 10^{24})}{(3 * 10^8)^2} \\ &= 8.85 * 10^{-3} \text{ m} \end{aligned}$$

(b)

$$\begin{aligned} R_S &= \frac{2GM}{c^2} \\ &= \frac{2(6.67 * 10^{-11})(5.683 * 10^{26})}{(3 * 10^8)^2} \\ &= 8.42 * 10^{-1} \text{ m} \end{aligned}$$

(c)

$$\begin{aligned} R_S &= \frac{2GM}{c^2} \\ &= \frac{2(6.67 * 10^{-11})(1.989 * 10^{30})}{(3 * 10^8)^2} \\ &= 2.948 * 10^3 \text{ m} \end{aligned}$$

7. In order to find the density we need to find the radius of the event horizon it would be contained in. Schwarzschild Radius: $\frac{2GM}{c^2}$. Then,

$$\begin{aligned} R &= \frac{2GM}{c^2} \\ &= \frac{2(6.67 * 10^{-11})(20 * (1.989 * 10^{30}))}{(3 * 10^8)^2} \\ &= 5.896 * 10^4 \text{ m.} \end{aligned}$$

Using this radius we can calculate the density: $D = \frac{M}{\frac{4}{3}\pi r^3}$. Then,

$$\begin{aligned}
D &= \frac{M}{\frac{4}{3}\pi r^3} \\
&= \frac{20 * (1.989 * 10^{30}}{\frac{4}{3}\pi(5.896)^3} \\
&= 4.633 * 10^{16}\,\frac{\mathrm{kg}}{\mathrm{m}^3}.
\end{aligned}$$

This density is less than the density of a neutron star by about 2 orders of magnitude.