

# Homework 3

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Boltzmann Equation:

$$\begin{aligned}\frac{N_b}{N_a} &= \frac{g_b e^{-\frac{E_b}{kT}}}{g_a e^{-\frac{E_a}{kT}}} \\ &= \frac{g_b}{g_a} e^{-\frac{(E_b - E_a)}{kT}}\end{aligned}$$

Saha Equation:

$$\begin{aligned}P_e &= n_e kT \\ \frac{N_{i+1}}{N_i} &= \frac{2kT Z_{i+1}}{P_e Z_i} \left( \frac{2\pi m_e kT}{h^2} \right)^{\frac{3}{2}} e^{-\frac{x_i}{kT}}\end{aligned}$$

1.

$$\begin{aligned}5 &= \frac{1}{(2^2)} e^{-\frac{(0 - 10.2) * (1.6 * 10^{-19})}{(1.381 * 10^{-23})T}} \\ \ln(20) &= \frac{-(0 - 10.2) * (1.6 * 10^{-19})}{(1.381 * 10^{-23})T} \\ T &= \frac{-(0 - 10.2) * (1.6 * 10^{-19})}{(1.381 * 10^{-23})\ln(20)} \\ T &= 39450k\end{aligned}$$

2.

	5000k	12000k
$n = 2$	$5.4 * 10^{-11}$	$5.29 * 10^{-5}$
$n = 3$	$6.83 * 10^{-13}$	$8.53 * 10^{-6}$
$n = 4$	$1.48 * 10^{-13}$	$4.51 * 10^{-6}$
$n = 5$	$7.27 * 10^{-14}$	$3.35 * 10^{-6}$
$n = 6$	$4.94 * 10^{-14}$	$2.86 * 10^{-6}$

3. (a) When the temperature is doubled the ionization ratio changes by a factor of about  $3 * 10^5$
- (b) When the electron density is doubled, the ionization ratio changes by a factor of  $\frac{1}{2}$ .

(c) When the ionization potential is doubled, the ionization ratio is changed by a factor of  $1 * 10^{-10}$

4.  $T = 12000$

$$\frac{N_{II}}{N_I} = \frac{2(1.381*10^{-23})(12000)(2)}{(0.2)(1)} \left( \frac{2\pi(9.109*10^{-31})(1.381*10^{-23})(12000)}{(6.626*10^{-34})^2} \right)^{\frac{3}{2}} e^{\frac{-24.6(1.6*10^{-19})}{1.381*10^{-23}*12000}} = 0.510$$

$$\frac{N_{III}}{N_{II}} = \frac{2(1.381*10^{-23})(12000)(1)}{(0.2)(2)} \left( \frac{2\pi(9.109*10^{-31})(1.381*10^{-23})(12000)}{(6.626*10^{-34})^2} \right)^{\frac{3}{2}} e^{\frac{-54.4(1.6*10^{-19})}{1.381*10^{-23}*12000}} = 4.07 * 10^{-14}$$

$$\frac{N_{II}}{N_{total}} = \frac{0.510}{1+0.510} = 0.338$$