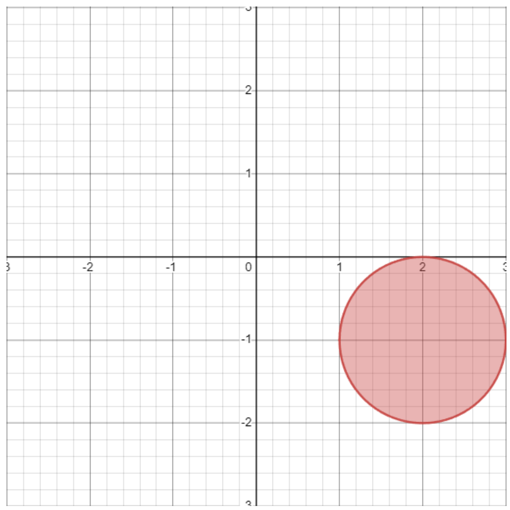
Homework 2

Sean Eva

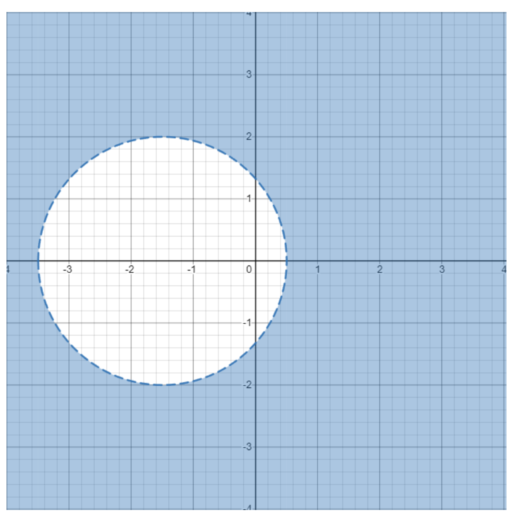
Math 4320

[1]

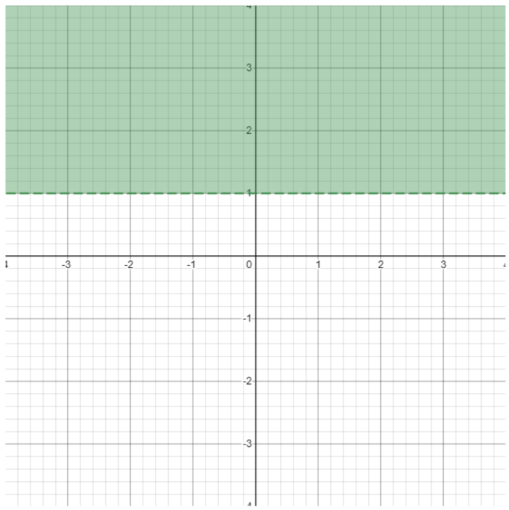
(a) Not a domain



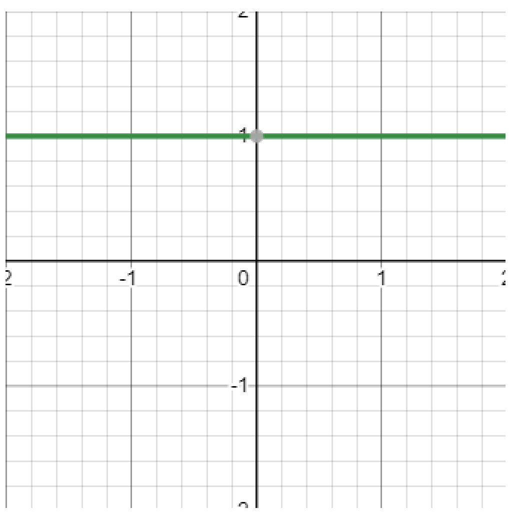
(b) A domain



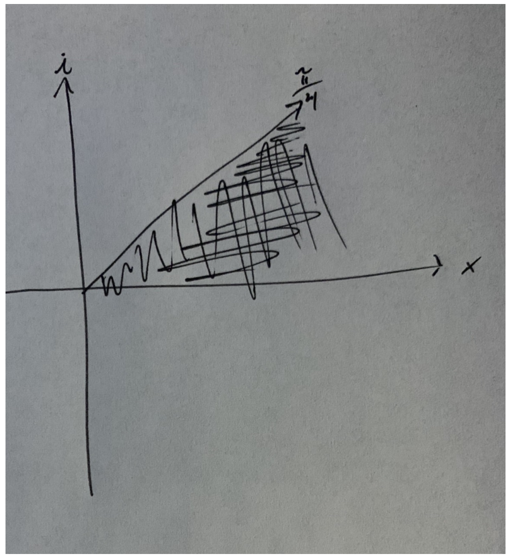
(c) A domain

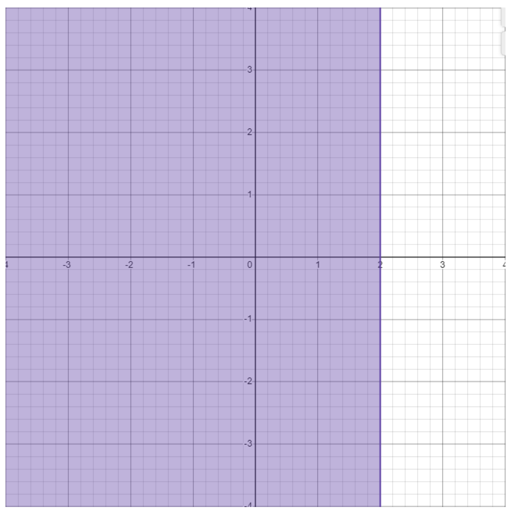


(d) Not a domain



(e) Not a domain





(f) Not a domain

[2]

The only set from Exercise 1 that is neither open nor closed is part e $0 \leq arg(z) \leq \pi/4 (z \neq 0)$ because there is no $\epsilon > 0$ that the area would be encompassed in for all values of r.

[5]

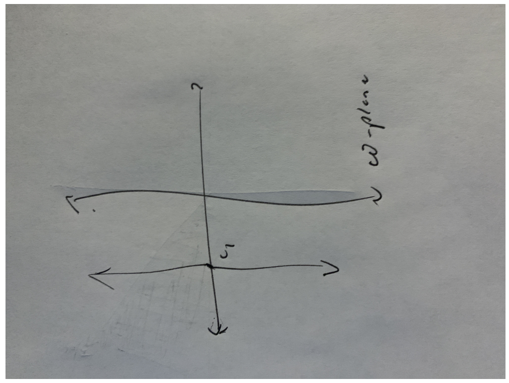
Proof. Let the two sets be as they are in the problem statement. We can consider these two sets as open balls where one is centered at $x = 0$ and $x = 2$ with radius 1. Let us then say that these two sets are jointed, where we could define z_0 as being within both sets. Then we have $x = |z_0 - (z_0 - 2)| \leq |z_0| + |z_0 - 2| < 2$ which is a contradiction since it states that $2 < 2$ which is false. Therefore, we know that these two sets are disjointed. Since these two sets are open and disjointed, it follows then that they are disconnected. $\square$

[1]

$f(z) = (\frac{z+\bar{z}}{2})^2 - (\frac{z-\bar{z}}{2i})^2 - 2(\frac{z-\bar{z}}{2i}) + i(2(\frac{z+\bar{z}}{2}) - 2(\frac{z+\bar{z}}{2})(\frac{z-\bar{z}}{2i})) = \frac{z^2+\bar{z}^2+2z\bar{z}}{4} - \frac{z^2+\bar{z}^2-2z\bar{z}}{-4} - \frac{z-\bar{z}}{i} + i(z + \bar{z}) - \frac{z^2-\bar{z}^2}{2} = \bar{z}^2 + 2iz$. Therefore, we have that $f(z) = \bar{z}^2 + 2iz$ as desired.

[6]

Let us take the given transformation of $\omega = z^2 = (x+iy)^2 = (x^2 - y^2) + 2ixy$ thus making $u = x^2 - y^2$ and $v = 2xy$. Then for the first hyperbola, we know that $c_1 = (x^2 - y^2)$ so then we know that $\omega = c_1 + iv$. So it is a straight line as:



For the second hyperbola, we have that $c_2 = 2xy$ and with the same transofmration wer have that $\omega = u + ic_2$ which would again make the image a straight like as: