Game Theory Optimal Play in the Game of Texas Hold 'em

Author:

Seán Paul Gill

Supervisors:

Prof. Vicenc Torra

Dr. Klara Stokes

An Interim Project Report submitted in partial fulfilment of the requirements of the Maynooth University for the degree of Bachelor of Engineering.

Submission Date

10th December 2019

Faculty of Engineering
Department of Electronic Engineering,
Maynooth University

Abstract

This project is based around solving the game of Texas Hold 'em through the implementation of various different technologies.

This report will outline the technologies used in the project also giving an insight to the important factors with decision making in Texas Hold 'em. After delving into the project formulation & background, the Literature Review will be carried out.

The Literature Review will provide a summary of three Literature Pieces that are in the same field as this project. The Literature Review will bring the reader up to date on the range of ideas and knowledge about Texas Hold 'em Solvers.

The report will then conclude with the workplan (including a Gannt Chart) of the project referencing the main objectives laid out at the beginning of the project.

Declaration

I declare that this report and the project it describes is my original work only. I have not plagiarized or excessively quoted the work of others, nor have I colluded with others to represent collaborative work as my own. I confirm that I have appropriately cited all information derived from the published and unpublished work of others.

Signed: _______________ *Student Number:* _______________ *Date:* _______________

Acknowledgements

I would like to acknowledge the Maynooth University Library for the access to multiple research documents.

I would like to acknowledge Vicenc Torra for supervision in the generation of this report.

I would like to acknowledge the consistent efforts of Electronic Engineering Department at Maynooth University to maintain the health and safety standards.

Table of Contents

Table of Figures

1 Introduction

Texas Hold 'em (also known as Hold 'em) is one of the most popular card-games of all-time. The objective of the game is very simple – win the value in the centre of the table, known as the pot. Although, the objective is simple, the game itself is incredibly advanced. The perfect solution for the game of Texas Hold 'em does not exist which is why it is considered "the only pure game left". [1]

Hold 'em is a game founded on probability, strategy, logic, game theory and psychology. Hold 'em is a card-game where an edge can be obtained. In 2019, resources for the game are not scarce by any means. The game is constantly evolving with the aid of new strategies and technologies. Due to this, any edge you can gain is crucial. One of the most popular ways to gain an edge would be to play Game Theory Optimally.

In Hold 'em, the term Game Theory Optimal, 'GTO', gets thrown around to signal a few different concepts. Game Theory Optimal refers to "thoughts about opponent modeling and thinking about situations in terms of ranges and probabilities, as opposed to being strictly results oriented". [2] A game theory optimal solution has precise mathematical definitions. There are numerous GTO solvers in the world today that try their best to guarantee positive expected value.

Having a positive expected value is not necessarily proportional to your winnings. With in the game of Hold 'em, there might be an instance where you are an 80% favorite to win the game. And of course, through the law of averages, you shall see profit in the long run but how often does this instance occur? This is one of the many problems with solving the game of Hold 'em. Therefore, focusing on making the right decision is far more important than focusing on results. So, this leads to many questions; How do we know what the right decision is?

The aim of this project is to gain an edge against other players in the game of Texas Hold 'em. This report will firstly focus on the project scope (problem statement & objective). Henceforth, delving into the background of the project and highlighting the current existing solutions (Literature Review). After investigating the current technologies & strategies to solve the problem statement, the Work Plan of the project will be presented.

2 Project Scope

The project scope consists of the project goals, deliverables, features, functions, tasks, deadlines, and ultimately costs. Essentially, the project scope highlights the work needed to deliver the project successfully.

The project brief was devised in Summer 2019 and submitted to the Department of Engineering, Maynooth University. After thorough investigation, the project was accepted as it covers various prerequisites of the Degree in Electronic & Computer Engineering. The brief consisted of a project description, the main objectives and the resources required.

2.1 Problem Statement [3]

Statement

Implementation of RFID cards & microcontrollers to ensure Game Theory Optimal play in the game of Texas Hold 'em.

Project Description

The game of Texas Hold 'em is arguably the most famous of all card games. In Hold 'em, the term Game Theory optimal, 'GTO', gets thrown around to signal a few different concepts. It refers to "thoughts about opponent modelling and thinking about situations in terms of ranges and probabilities, as opposed to being strictly results oriented."

A game theory optimal solution to a game has precise mathematical definitions. There are numerous GTO solvers for the game of Texas Hold 'em, most of which are offered at a high cost. The perfect solution for the game of Texas Hold 'em does not exist which is why it is considered "the only pure game left".

The live showing of big Hold 'em games supplemented with visual displays of current probabilities against each of the player's hands is a massive part of the game. Without some of the technology implemented (RFID/Computer Vision), the beautiful game of Hold 'em would not be exposed to the public, resulting in a decrease in popularity & revenue of the game.

The aim of this project is to setup a Texas Hold 'em playing environment where the cards are read using RFID technology and the edge is gained from a specific microcontroller / FPGA calculating the decision that delivers the highest expected value.

Figure 1 - Project Description captured from Project Brief [3]

2.2 Main Objectives

The project has been broken down into the following objectives:

- Perform a literature review of current systems that are relevant to this area
- Design, build and test a playing environment with RFID cards and RFID readers.
- Build and integrate the system to a PC/microcontroller/FPGA.
- Simulate & Run Tests (large sample sizes) against different types of AI and Human players. Defining ranges for different types of players
- Creatively inventing GTO methods that could be implemented using a microcontroller.
- Build and run tests of all the different GTO solving options.
- Enhancement in the GTO solver taking inspiration from some of the existing on the market such as "Pio Solver".
- Reduction of the system to test the solver against humans and have a fully functional playing environment with AI providing GTO solutions and leaving the option up to the Hero.

The main objectives above are not in a linear progression formula. The Work Plan section provides a deeper insight into the workflow and how the objectives will be achieved.

2.3 Resources Required

- a PC
- RFID Cards and Readers
- Microcontroller/FPGA
- Lab time for building/testing
- All **prerequisites** met.

Prerequisites: Programming, Circuit Design, Radio Frequency Management, Embedded Systems.

3 Background

As mentioned at the start of the report, the objective of Hold ‘em is simple but the gameplay itself is incredibly advanced. I, Seán Paul Gill, have 5 years of experience playing the game at different levels. I started playing Texas Hold ‘em when I was 16 years old with a group of friends. I observed the beauty and purity of the game from continuing to increase my skill level over the years. As of now, I have played in massive tournaments against professional players (Jamie Staples, Fintan Hand) and observed the solvability of the game. Over the years I have played, I have learned glossary terms, strategies, common mistakes and more. Based on the level of difficulty of the game, I feel an insight on how the game is played is necessary.

3.1 The Game of Texas Hold ‘em

Two cards, known as the hole-cards, are dealt face down to each player. Once the hole cards are dealt, the decision-making starts. There are four decision-making points, known as betting rounds. Between these betting rounds, community cards are dealt face up. Community cards are globally used by every player. Figure 1 illustrates the different betting rounds, and how the community cards are dealt throughout.

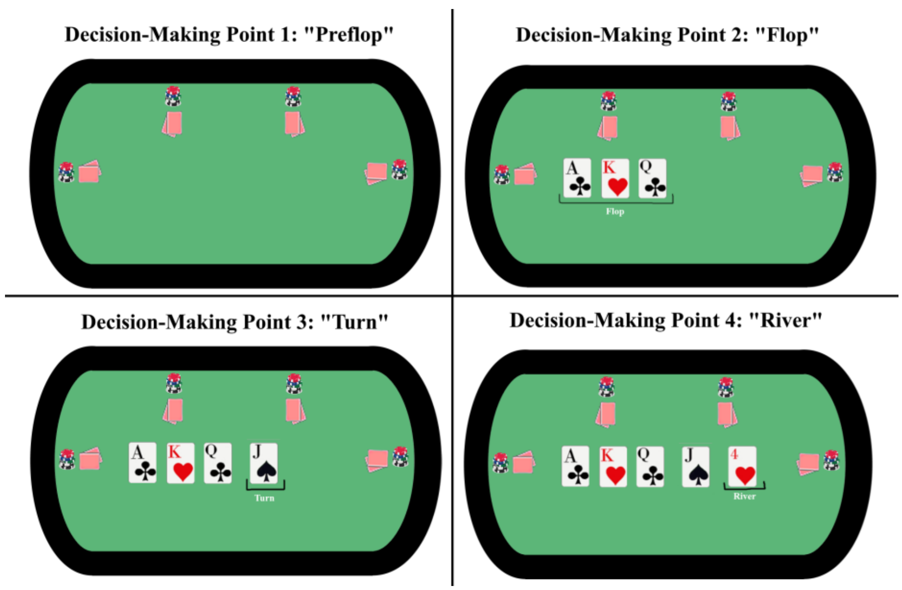


Figure 2 - Decision Making Points/Betting Rounds in Texas Hold ’em

Throughout the betting rounds, each player seeks their best five card poker hand. Each player has the choice of choosing their best hand using a combination of their hole cards and the five community cards. For every decision-making point, a player has the option of checking, calling, raising or folding. The player who has the best hand at the end (and has not folded) will win the pot (the money bet across the game).

It is important to emphasize the fact that this is only a high-level description of the game and there are a lot of more situational rules that are only picked up through experience and a large rulebook. The website PokerListings.com provides the **full** rules to the game giving situational examples. [4]

3.1 History of Solving Texas Hold 'em

3.1.1 Popularity Growth of Hold 'em

The game is said to originate in the early 1900s in Robstown, Texas. [5] The game continued to grow with new rules changing the game massively. Texas Hold 'em was different to most card-games that were out there. "Draw poker, you bet only twice; hold 'em, you bet four times. That meant you could play strategically. This was more of a thinking man's game." [6] The following quote is from Crandell Addington in 1959 and signifies the beauty of the game. The fact that the game could be approached probabilistically & strategically for one player to gain an edge.

Throughout the years of the 1980s to the 2000s, the popularity of the game started to grow. The popularity of Hold 'em was due to a mix of exposure on the internet, television and popular literature. People started to realize that there was a lot more profitability in the game due to the skill-gap it naturally holds.

Someone who saw the profitability in the game of Texas Hold 'em early on was Chris Moneymaker. Chris Moneymaker, a 27-year old poker amateur and accountant from Tennessee, United States gained the $10,000 entry to the 2003 World Series of Poker through a $89 satellite tournament. [7] Coming from an accounting background, he was mathematically gifted which assisted him in learning a significant amount of the mathematics involved with Texas Hold 'em. Not only did he win the $10,000 entry, he went on to win the entire 2003 Main Event scooping $2,500,000. The story of how an amateur poker player beat some of the best poker players in the world inspired millions to play Texas Hold 'em and initiated the growth of the game. (also known as the Moneymaker Effect)

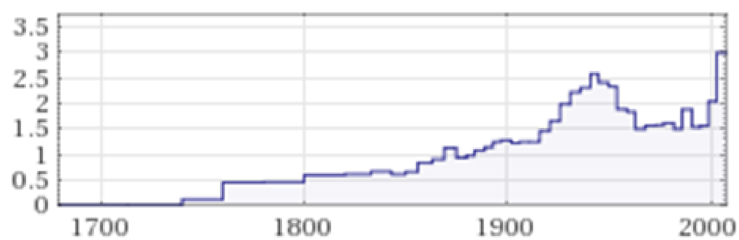


Figure 3: (from 1700 to 2008) (in occurrences per million word per year) [8]

Figure 3 illustrates the word frequency of 'poker' where the exponential growth can be observed from 2000 to this current day. There are other terms used to describe the growth other than the Moneymaker effect. The **poker boom** was another term used to describe the growth in the game of No-Limit Texas Hold 'em, which is the **type of game/poker that this project is focused on**. The seeds of the boom were planted in 1998 with the movie "Rounders" starring Matt Damon. [9]

Texas Hold 'em started to become widely televised. Classic movies like James Bond included a High Stakes Texas Hold 'em scene central to the plot of movie Casino Royale. There was a constant flow of Literature and movies coming out about the Hold 'em. Strategies, technologies continued to evolve resulting in the popularity of the game today.

The Massachusetts Institute of Technology (MIT) has mandatory poker modules that their students must enroll in. [10] MIT is rated as one of the best universities in the world. The fact that a university as prestigious as MIT are including mandatory poker modules signifies the amount that one can learn from the game. Anyone who has interest in the skills required for poker can start learning today due to the vast **free** resources that are available. There's much more than game-theory to poker, it teaches you a lot about life. Liv Boeree is a science communicator and games specialist. She is a professional poker player and covered a TED talk [11] titled "3 decision making lessons from a poker champion".

Although the growth of Texas Hold 'em was heavily revolved around the influence of the internet, Television and Literature, it's important to observe the technology advances that occurred. Without the evolution of technology, the game would not be as theoretical as it is today.

3.1.2 The Evolution of Technology

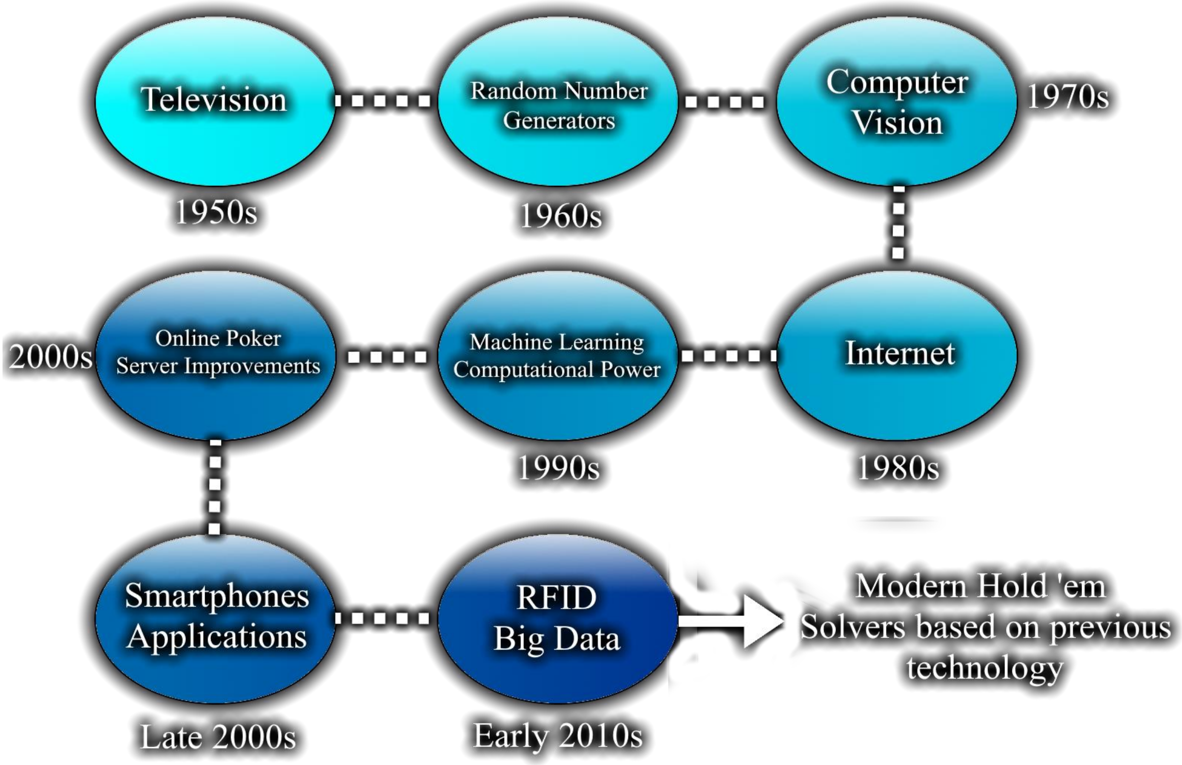


Figure 4: Evolution of technology resulting leading to modern-day Hold 'em

The evolution of technology over the years has increased the solvability of Hold 'em. Finding exact optimal solutions is quite expensive. (cost) Not only do you need to own a recognized Solver Software; you need to find the computational power. The computational power generally requires the purchase of a top-of-the-range PC or using Cloud Computing. The high power is necessary as the solver tries to find the best decision for **EVERY** possible combination in in that hand.

The evolution of technology has made this project plausible. RFID Technology and Microcontrollers are the two focal technologies in this project.

3.2 RFID Technology

Radio-frequency identification (**RFID**) is the "wireless non-contact use of radio frequency waves to transfer data." [12] One may store information onto an RFID tag. Electromagnetic fields are then used to identify the tags. The advantages of RFID tags compared to the likes of a Barcode is that the tag only has to be within a certain distance of the reader to transfer the held information.

The first application of RFID technology was in World War 2. They were used to detect whether a plane was a friend/foe. [13] Although this was the first sign of RFID technology, it was not practically made available until the 2010s as depicted in **Figure 4.** With the technology constantly improving, the cost decreases. This is one of the reasons why they are being implemented in this project.

There are three primary frequency ranges associated with the transmission of the radio waves – Ultra-High Frequency, High Frequency and Low Frequency. All these frequencies are illustrated in **Figure 5**.

Figure 5: Electromagnetic Spectrum with emphasis on radio-waves [122]

All of these three frequency ranges possess their own applications, cost and advantages/disadvantages. All of these properties are manifested in **Table 1.**

	Low Frequency	High-Frequency	Ultra-High Frequency	
			Active	Passive
General Frequency Range	30 – 300 kHz	3000 – 30000 kHz	300 - 3000 MHz	300 - 3000 MHz
Primary Frequency Range	125 – 134 kHz	13560 kHz	433 MHz	860-960 MHz
Read Range	0.10 m	0.3 m	10-100+ m	25 m
Cost	0.75 – 5 €	0.20 - 10.00 €	25 - 50 €	0.10 - 20 €
Applications	Animal Tracking, Access Control, Car Key-Fob	Library Books, Personal ID Cards, **Poker/Gaming Chips**, NFC Applications	Vehicle Tracking, Auto-Manufacturing, Mining..	Supply Chain Tracking, Manufacturing, Pharmaceuticals, Tolling, Race Timing..
Advantages	Works well near Liquids & Metals, Global Standards	NFC Global Protocols, Larger Memory Options, Global Standards	Very Long Read Range, Large Memory Capacity and High Data Transmission Rates	Long Read Range, Low Cost Per Tag, Global Standards, High Data Transmission Rates
Disadvantages	Short Range, Limited quantity of memory, low data transmission rate, high production cost	Short Read Range, Low Data Transmission Rate	High Per Tag Cost, Shipping Restrictions, Complex Software, High Interference from Metal & Liquids	High Equipment Costs, Moderate Memory Capacity, High Interference from Metal and Liquids

Table 1: Comparing RFID Technology

The information above was gathered from comparing prices across the web and notable sources. [14] Bolded in Table 1 is **Poker/Gaming Chips**. This is under the column for High-Frequency RFID. High-Frequency radio-waves are used for poker cards and chips as the reading range is ideal. Although, recently, somebody exploited the High-Frequency RFID technology used.

Mike Postle, an American “professional” poker player claimed himself to fame after his 2019 cheating scandal. As a previous employee for the Stones Casino in Sacramento, he had significant knowledge about the RFID technology that was in place at the poker tables. The RFID technology was in place in the casino in order to stream the games whilst showing each player’s cards and their current odds at every instance. At every instance, he was able to hack into the RFID system and find out his opponent’s hole-cards. [15]

Albeit, there are disadvantages with the RFID Technology, it is generally effective for its purpose. The RFID Technology involved in this project will be implemented in the following manner:

1. Cheap high-frequency tags will be purchased and attached to a standard deck of 52 playing cards with all tags holding their unique value.

2. There will be two high-frequency readers in place. One to identify the hole-cards of the hero and the other to identify the post-flop cards.
3. The information of each cards will then be passed to the microcontroller where the "solving" occurs.

3.3 Microcontrollers

Microcontrollers are essentially small computers on a Metal-Oxide-Semiconductor integrated chip. They are designed in a way that they take an input from the physical-world and control/compute necessary functions based on the input. There are various different types of Microcontrollers, but they have several things in common:

- **CPU (central processing unit):** Executes tasks/programs defined by the user.
- **Hard-Disk:** Where the CPU loads the tasks/programs from.
- **I/O Devices (input-output):** The CPU carries out programs on the input devices and produces an output on the output devices.

The microcontroller is the so-called "brains" of the AI for this project. It takes the input from the RFID reader and produces an output in the terms of the correct decision to make.

The Microcontroller technology will be implemented in the following manner:

1. Identifying the information from the RFID readers and assigning them to a **Base Class** (onto a program of some type) for the Hero.
2. Initially, the impacting factors and modeling of players discussed in 3.4.1Impacting Factors and Parameters to Model will be manually entered as an input to the microcontroller. By the end of the project, the objective would be that this process is automatic.
3. For every decision-making point, the microcontroller will produce the optimal decision. This decision will be based on a weighted decision-making function mentioned in 3.4.3 Decision Making Function.

3.4 Preflop Analysis

From the general overview given in **3.1 The Game of Texas Hold 'em**, the game was broken down into four decision making points; Preflop, Flop, Turn and River. Cards are dealt differently based on the decision-making point as illustrated in **Figure 2.** The author firmly believes that Preflop is the most important decision-making point. Bad Preflop play is the start of an inevitable path. If you are choosing to play a statistically poor hand before you start, your expected value in the long run will be poor. "Preflop Charts are the most efficient and absolute quickest way to improve your poker game." [16] This quote from Doug Polk indicates the importance of preflop charts. Preflop Charts are charts generated from a Nash Calculator.

A Nash Calculator generates the optimal decision based on input parameters. These input parameters are all situational and based on the current game-state. Another way of thinking about it is, a **decision-making function** is defined by the user. This function is weighted based on the current game-state's parameters & opponents' model. This section will lay down the schematic & creation of this decision-making function.

3.4.1 Impacting Factors

With the game of Texas Hold 'em, there are factors that a player can't dictate or assume. These factors do not change once a game has started. For example, the number of players at the table is a factor that a player cannot determine. These factors decree a certain amount of weight to the decision-making function. The factors to be considered preflop are:

Number of Players (N)

A standard Texas Hold 'em table consists of anywhere between 2 and 9 players. The more players there are, the narrower your range becomes. This is due to the fact there are more players that could potentially have a better hand than you. There are solvers such as simplepoker.com that offer different solutions based on the number of players. [17]

Number of Big Blinds (BB)

The number of big blinds denotes how many full circuits you can survive before being blinded out. You are said to be in a push-fold strategy when you have less than 15 Big Blinds. A push-fold strategy means that you have two decisions – All in (stake all of your chips) or fold (don't play that specific round).

Labels of Players

How do we label other players? Are they labelling us? Do we set try ourselves a label? Can we define their behavior through generic labels? Some of the standard/classic poker labels are:

- **Fish –** They play almost every round. Their sizes are not optimal. They are passive and poor players.

- **Maniac –** Similar to fish but instead of being passive they are hyper aggressive. They try bluff you constantly
- **Shark –** The Shark strategizes. One must avoid playing against the shark. They understand the game the most at the table.

There is an endless number of labels that are thrown around in the poker community. Labelling people quickly is a great skill in Hold 'em as you can get obtain your decisions a lot faster. A full list of 'classic' poker labels can be found at http://pokerinabox.com/player-categories-and-labels/.

Position

Position refers to the order in which the players are sitting around the table. Players who are first to act are in "early position"; players who act later are in "late position". There are lots of terms thrown around to denote the position of a player. I will standardize the position's names according to the following figure.

Figure 6: 9 Players Table Structure with Positions [18]

UTG: Under the gun. They are the first people to act pre-flop.	**D:** Dealer. The best place to be sitting. Always last to act on every single decision-making point.
MP: Middle-Position. They are situated in middle and have an advantage against the UTG.	**SB:** Small Blind, they must pay half the big blind and are always first to act post-flop.
CO: Cut-off. To the right of the dealer button. They have an advantage against every player except for the Dealer	**BB:** Big Blind, they must pay the full big blind.

The above figure depicts the position names for a table of 9 players. The same terminology is used for smaller tables but there might not be as many UTGs or MPs. For 4+ players, there is always a CO, D, SB and BB.

Every round the dealer button moves one position clockwise. This ensures that everyone gets to be in every position and can't avoid the blinds.

Effective Stack-Size

The effective stack-size is the size of the stack that is at risk in the given round. Take for example, two people are against each other, Bob and Seán. Bob has €100 but Seán only has €20. In the case, the effective stack-size is €20 because Seán only has €20 on the table. Therefore, all Bob can win/lose is €20.

A Balanced Range

Do I have a proper number of "bluffs" and "value" hands when I bet or raise? Bluffs are hands that are statistically poor, and you don't want your opponent to play against you. Value hands are statistically strong hands, and you want your opponent to play against you. It's important to keep a balanced range because otherwise your opponents can always assume that you have a good hand. Having a balanced range is also known as **polarization**.

3.4.2 Modelling Players

As previously mentioned in the **Labels of Players,** it is useful to be able to label an opponent as quickly as possible. Although, what happens if you label a player incorrectly? To construct a strong & reliable label, you need to be able to model a player based on their previous hand-history. The larger database you hold on a certain player, the more conclusions you can draw on their approach to the game. This section will delve into the ways of thinking about an opponent's hole-cards. Ultimately, what two cards are they holding? A background to some of the opponent modeling parameters will be also discussed with their connection to the decision-making function.

Hand Vs Hand, Hand Vs Range, Range Vs Range

There are different dimensions that people think about when playing poker. These are the three ways to deduce what hole-cards another player has.

- **Hand Vs Hand:** This is by far the worst way of deducing your opponent's cards in poker. You are playing your two cards against their two cards. This thought-process is extremely exploitable. The thought-process is perfect exhibited in this video: https://www.youtube.com/watch?v=St4Q55amkO4. Jennifer Tilly in this video checks an incredibly strong hand on the "river" because she thought her opponent had the one possible combination ([King, King]) that could beat her. She didn't think of the other 300+ combinations her opponent **could have had**. The other players at the table were in disbelief.
- **Hand Vs Range:** This is an improvement to the first method but still not fully optimal. The player is thinking about the cards that their opponent **could** have (their range). They then play their exact hand-strength based on their opponents' range.

- **Range Vs Range:** This is the most optimal thought-process in poker. The player is constructing a range for their opponent but also for themselves. They are thinking what are the two cards that I could potentially possess. This means that they can follow a balanced strategy and not only be playing when they have value hands.

Ranges in Hold 'em

In order to think in Hold 'em through ranges, it's important to know the standard range chart.

AA	AKs	AQs	AJs	ATs	A9s	A8s	A7s	A6s	A5s	A4s	A3s	A2s
AKo	KK	KQs	KJs	KTs	K9s	K8s	K7s	K6s	K5s	K4s	K3s	K2s
AQo	KQo	QQ	QJs	QTs	Q9s	Q8s	Q7s	Q6s	Q5s	Q4s	Q3s	Q2s
AJo	KJo	QJo	JJ	JTs	J9s	J8s	J7s	J6s	J5s	J4s	J3s	J2s
ATo	KTo	QTo	JTo	TT	T9s	T8s	T7s	T6s	T5s	T4s	T3s	T2s
A9o	K9o	Q9o	J9o	T9o	99	98s	97s	96s	95s	94s	93s	92s
A8o	K8o	Q8o	J8o	T8o	98o	88	87s	86s	85s	84s	83s	82s
A7o	K7o	Q7o	J7o	T7o	97o	87o	77	76s	75s	74s	73s	72s
A6o	K6o	Q6o	J6o	T6o	96o	86o	76o	66	65s	64s	63s	62s
A5o	K5o	Q5o	J5o	T5o	95o	85o	75o	65o	55	54s	53s	52s
A4o	K4o	Q4o	J4o	T4o	94o	84o	74o	64o	54o	44	43s	42s
A3o	K3o	Q3o	J3o	T3o	93o	83o	73o	63o	53o	43o	33	32s
A2o	K2o	Q2o	J2o	T2o	92o	82o	72o	62o	52o	42o	32o	22

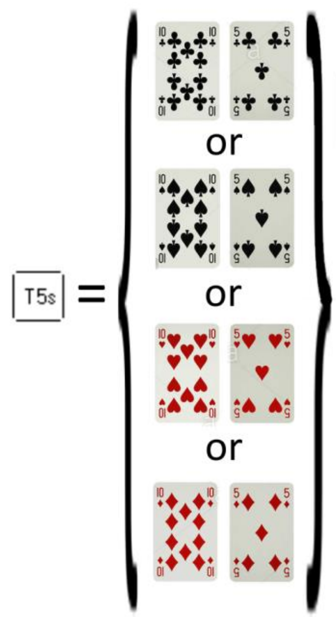


Figure 7: Standard Empty Range Chart with Key

This standard range chart consists of all the possible starting hands in poker. It is a 13x13 Matrix with the pairs running across the left-right diagonal. The **'s'/'o'** beside the two cards dictate if they are suited (same suit) or off-suit respectively. The Key beside the empty range chart shows an example of T5s which is all 4 combinations of where [10, 5] are the same suit.

After labeling a certain player, it's then important to construct a range for that type of player. Once all of the factors and opponent's parameters are created, one can construct a range for that player.

The following figure shows a typical Shark Vs Fish's range when both players are under the gun. The shark only plays the premium hands and fish plays every possible hand. The range of the Shark is generated through Nash Equilibrium using the factors previously discussed and the parameters of his opponents' model.

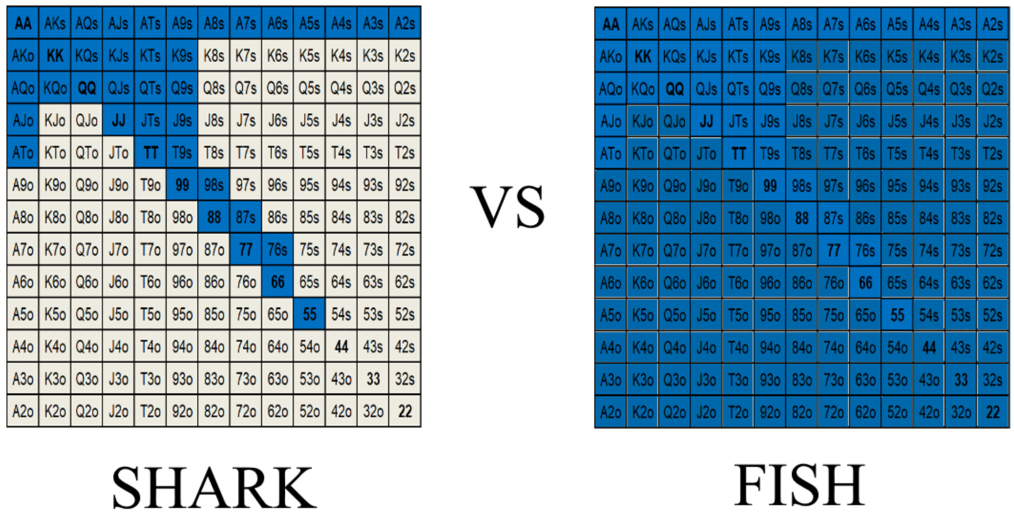


Figure 8: Shark VS Fish Range UTG

So, what are the parameters used to model our opponents?

Parameters to Model

When it comes to modelling an opponent in poker, there are endless parameters that can be chosen to observe. The opponent modelling process of this project consists of the following parameters: [19]

- **VPIP:** Voluntarily put in pot. What percentage of hands does your opponent play?
- **FF:** Fold Frequency. What percent of the time does your opponent fold?
- **PFR:** Preflop Raise Percentage. How often are they raising?
- **AF:** Aggression Factor. How aggressive is a player is being?
- **3BET:** Re-raising and raise. How often are they re-raising a raise?
- **F3BET:** Folding to re-raise of a raise. How often are they folding when a player re-raises their raise.
- **N_Samples:** The number of data-points collected. The higher, the more reliable the data.
- **ATS:** Attempt to steal blind. How often are they trying to stealing the small and big blind?
- **FTS:** Folding to blind steal. How often are they folding when a player tries to steal the blind?

Through observing these parameters alone, strong conclusions can be drawn about a player's preflop tendencies. These parameters are captured and transferred to the decision-making function.

3.4.3 Decision Making Function

The most important function in this project is this decision-making function. As previously stated, this function is weighted based on the **3.4.1 Impacting** Factors and **Parameters** to Model **of Opponent's Model.** The decision-making function is defined as:

$$f(x) = \{\, fold, check, call, raise \,\}$$

Where

$$x = \sum_{n=0}^{15} a_n \begin{cases} N & - \ Number\ of\ Players \\ BB & - \ Big\ Blinds \\ L & - \ Label \\ P & - \ Position \\ E & - \ Effective\ Stacksize \\ B & - \ Balanced\ Range \\ VPIP & - \ Voluntarily\ put\ in\ pot \\ FF & - \ Fold\ Frequency \\ PFR & - \ Preflop\ Raise\ Percentage \\ AF & - \ Aggression\ Factor \\ 3BET & - \ Three\ Bet \\ F3BET & - \ Fold\ to\ Three\ Bet \\ N_{Samples} & - \ Number\ of\ Datapoints \\ ATS & - \ Attempt\ to\ Steal\ Blinds \\ FTS & - \ Folding\ to\ Blind\ Steal \end{cases}$$

Equation 1: Decision Making Function

Equation 1, x, is the input to the decision-making function. $\mathbf{a_n}$ – is the weight of the particular parameter. The discovery of this variable will occur in second semester when we are finding the optimal decision-making function.

Based on the input of x, the decision-making function will produce the optimal solution in terms of folding, checking, calling or raising the optimal amount.

4 Literature Review

The idea of solving Texas Hold 'em has been around for a long time. People have claimed to solve variations of the game Texas Hold 'em such as 2 player Pot Limit Hold 'em. Pot Limit Hold 'em is where a player can-not bet more than what is in the pot. This means that an AI can call if they have the direct odds. For example, if the AI knows that it has a 40% of hitting a club and winning the hand, then the AI won't bet more than 40% of what is currently in the pot. This is the general concept behind an AI who can solve Pot-Limit Hold 'em.

Many theories have been proposed to solve the game of Texas Hold 'em. There have been endless amounts of Literature written around the game of Texas Hold 'em or more generally poker. This Literature Review will investigate and summarize Literature pieces that are similar to what this Final Year Project is attempting to achieve. The common themes of the literature are; Qualitive Analysis, Evolutionary Solving Algorithms and Artificial Intelligence.

4.1 AI in Heads-Up No-Limit Hold 'em [20]

DeepStack, the expert level AI that has supposedly solved the game of Heads Up No Limit Hold 'em. This particular game was just as "difficult to solve as Go, but with the addition of both players not knowing the entire game-state." DeepStack focuses on minimizing the computation needed whilst providing a solution that is still optimal. The concepts used in DeepStack are:

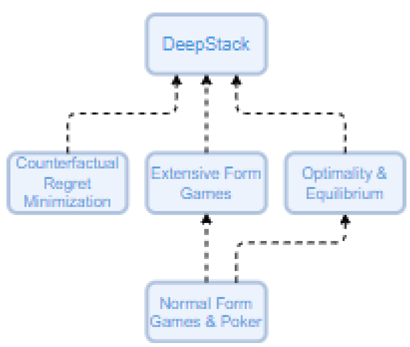


Figure 9: Concepts used in DeepStack

The Normal Form Block is a block that consists of the Game Theory associated with Hold 'em. It includes various strategies for different spots that have already been extensively solved. Storing these strategies means that there will be less computational power as a particular spot will not have to be re-calculated.

The Optimality & Equilibrium block denotes the reasoning with games. Can the game be optimally solved? This block deals with the Nash Calculator that was previously mentioned in the background section. Along with Nash, the report delved into Pareto Optimal strategy and how every game must have it. Pareto optimality is a state of allocation of resources from which it is impossible to reallocate so as to make any one individual or preference criterion better off without making at least one individual or preference criterion worse off.

The Extensive Form Games block explains what happens when both players don't act at the same time. With games like poker where people make their move in an order, a lot more information can be drawn from an opponent.

The final block, **counterfactual regret minimization**, **CFR**, is an algorithm that is known to solve large imperfect-information games. "It converges to an equilibrium by iteratively traversing the game tree." [21]

4.1.1 Results:

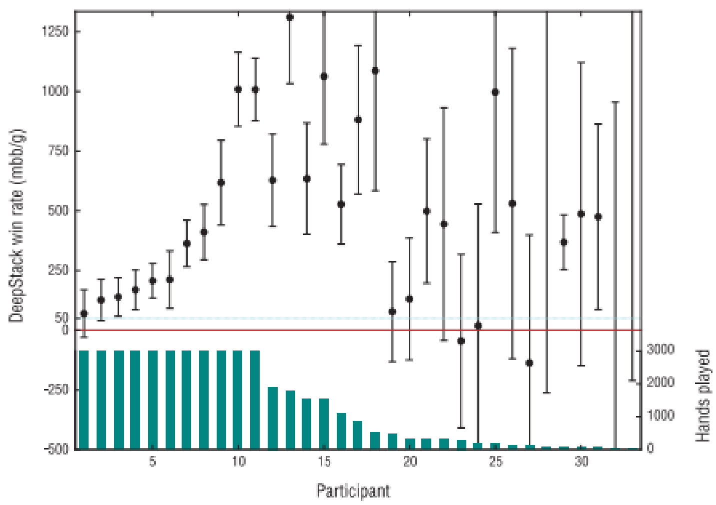


Figure 10: Performance of professional poker players against DeepStack

Over all games played, DeepStack won 492 mbb/g. The solid green bars at the bottom of the figure depict the amount of games completed by the professionals. The black circles indicate the win-rate against that certain player and the black lines is the standard error.

4.1.2 Conclusion:

This report shows that the game of Heads-Up No Limit Hold 'em does not need to be solved using Deep Learning methods. Using counterfactual regret minimization shows positive expected value even when playing against experts of the game.

4.2 Qualitative Analysis of Gamblers' Perceptions in Hold 'em [22]

A common stereotype associated with Hold 'em / Poker is that it is a form of gambling. The authors of this report set out to examine the perceptions "regarding the excessive behaviors and the nature of the skill involved. The qualitative thematic analysis and a comparative analysis on problem and social gamblers were performed" [22] The definition of gambling is taking a risky action in the hope of a desired result. This report set out to decipher the question of: Do the skills involved with Hold 'em support that it deserves to be set apart from the term gambling?

4.2.1 Results

Throughout the Literature, the author consistently brings up the involvement of emotions and the effect that they have on our decision making. The following figure demonstrates the cognitive functions involved with decision making in Hold 'em. The chart was collected from quantitative analysis on "16 regular HE gamblers"

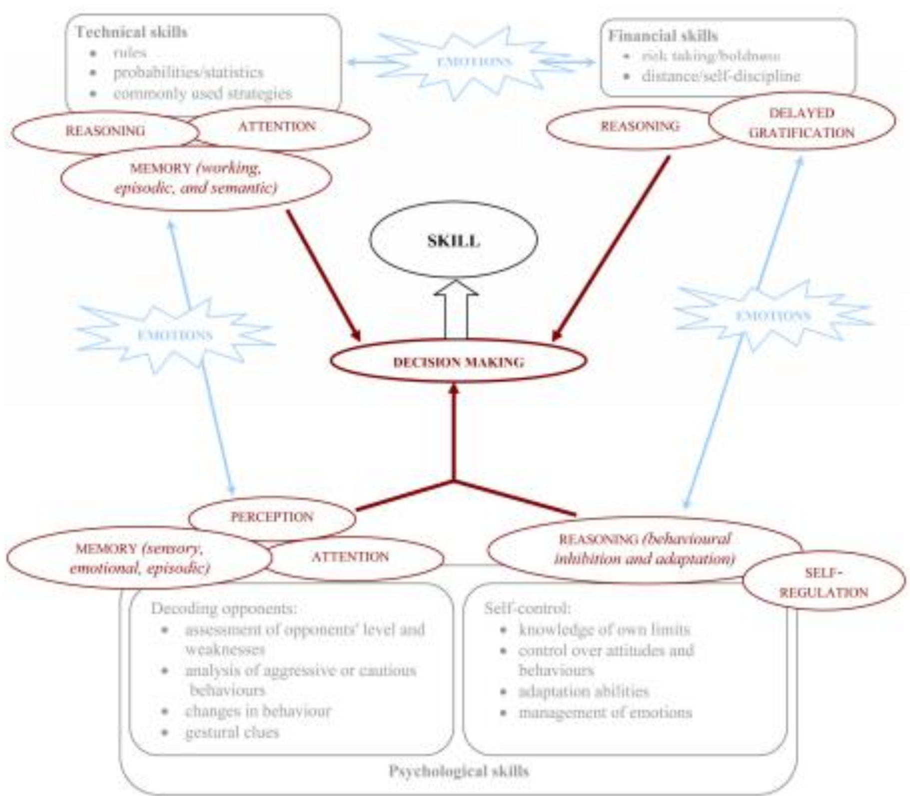


Figure 11: "Interactions between the three components of skills and the cognitive functions involved"[22]

From the quantitative analysis, three main skills were drawn. Technical Skills (game-theory optimal and playing strategically), psychological skills (self-regulation & non-"tilted" decisions) and financial skills (not staking more than they should, maintaining a bank-roll).

4.2.3 Conclusion

The report concluded that it is unfair to classify hold ‘em within the field of gambling. Through ensuring that if all the skills are being followed correctly, the game of Hold ’em is not a form of gambling. This report fits perfectly when considering the case of “Game Theory Optimal Play in Texas Hold ‘em”. This piece of literature illustrates the huge problem with the human playing Hold ‘em. Humans must maintain self-control of their emotions to continue playing game-theory optimally whereas an AI does not.

4.3 Superhuman AI for Multiplayer Poker [23]

In 2017, as mentioned in **4.1** AI in Heads-Up No-Limit Hold ‘em [, No Limit Heads Up Hold ‘em is solved. This was a massive leap in the right direction for solving poker. The next step was, how do we solve when there are more players? This report lays out the solution it came up with for a game of 6 player No Limit Hold ‘em.

The main questions at the beginning of this revolutionary research project were:

- How do we deal with incomplete information?
- How do we achieve a Nash equilibrium with 6 players?
- How do we teach the AI skills like bluffing?

Pluribus unlike many other AI does not rely on deep/reinforcement learning. “Pluribus’s main strategy was computed through playing against copies of itself.” It starts playing randomly and gradually improves as it determines which actions are better against its previous self. This algorithm is similar to genetic machine learning algorithms.

To reduce the computational power involved, the AI makes smart abstractions. The first type of abstraction would be the AI limiting its bet sizes. It won’t bet €200 and then change to €201 as that would be a waste of computational power. The other form of abstraction would be on information. For instance, a straight up to a queen will be considered the same as a straight up to a king.

During the self-play training, a version of the iterative Monte Carl CFR (MCCFR) algorithm is used. The outputs are superb, and this implementation is known as the blueprint strategy. This issue is that method does not run in real time and can only be used for statistical analysis after hands have completed.

4.3.1 Results

The pluribus was tested against group of players who have collectively won over $1,000,000 from tournaments such as the World Series of Poker.

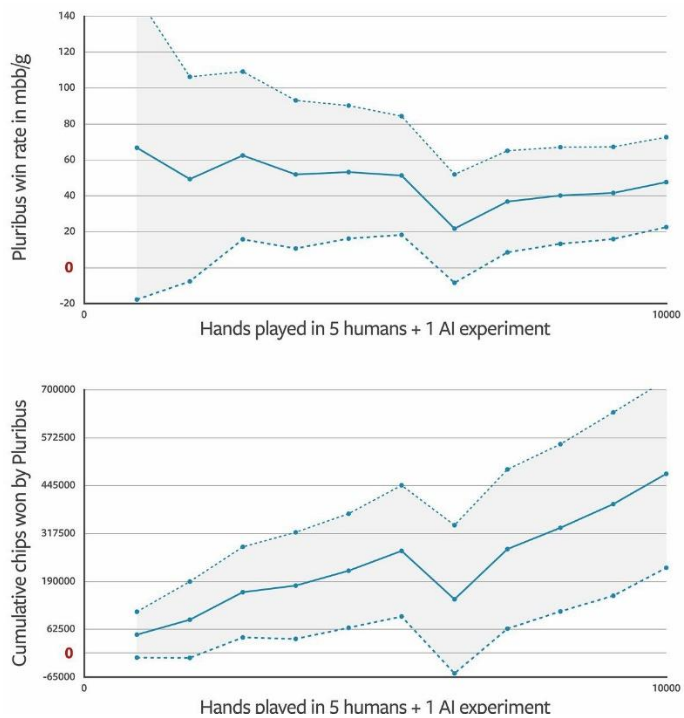


Figure 12: Results of Pluribus against professional poker players

The figure on top indicates the win-rate of pluribus, the bottom indications the number of chips won over the course of the games. The broken line on both plots indicates the standard error.

4.3.2 Conclusion

The results signify the success of the AI against the 5 other players but how is the data collected? 10,000 data-points against players chosen by the creators doesn't seem to be significant enough evidence that this revolutionary AI, Pluribus has solved 6 player No Limit Hold 'em. This report is fascinating in the methods that were implemented although the results don't exhibit a lot. With larger a dataset, the Pluribus could without a doubt conclude that they have solved the game of 6-player No Limit Hold 'em but until I see the day, I shall remain skeptical.

5 Work Plan and Gantt Chart

5.1 Semester 1

The overall work plan for this project can be broken down into objectives discussed in 2.2 Main Objectives. Breaking down the overall main objectives into a Gantt chart is beneficial. Through creating a Gantt Chart, the progress of a project is obvious is the Gantt chart is followed. The requirement of the Gantt Chart is knowing what tasks are ahead of you and given estimations on how long they should take.

The first step of the project was to design a project brief. Texas Hold ‘em and the edge that can be gained through playing optimally has always interested me. Based on this, a project brief was designed which was discussed in Section 2. After designing the project brief, confirmation was sought of the project from Andrew Meehan. Andrew Meehan accepted the project and informed me to find a supervisor. After looking into all the Lecturers that the Electronic Engineering department have, Dr. Klara Stokes seemed like the best option. As this project is heavily based around mathematics, I proceeded to ask Dr. Klara Stokes if she would supervise my project. Dr. Klara Stokes accepted to supervise.

After obtaining acceptance of the project and a supervisor, research for the project began. The research behind the resources required was initially focused on. Once that research had finished, I moved onto researching some existing solutions for Solving the game of Texas Hold ‘em. Within this time, my supervisor, Dr. Klara Stokes had referred me to Prof Vicenc Torra. Vicenc Torra is a professor in the Computer Science department at Maynooth University. Vicenc Torra is an expert in many fields in computer science and applied mathematics.

Weekly meetings were then organized between Vicenc and I. The primary focus of semester one was laying down all the research required for the project. Research into some Literature pieces began as a Literature Review needed to be performed for the Interim Report due on the 20th of December. Three Literature Pieces were chosen; DeepStack AI, Qualitative Analysis of Gambler’s Perceptions in Hold ‘em and the Superhuman AI solves 6-player hold ‘em. All of these Literature pieces were interconnected to my project and it served great purpose reading them and understanding the logic behind them.

After reading the Literature Pieces, the Problem Scope & Domain was redefined. The game of Texas Hold ‘em would be getting solved for the first decision making point only, preflop. Creating an AI to solve the entire game of Hold ‘em is PHD worthy and would require far more Future Work.

The interim report draft was created and sent to Andrew Meehan. Andrew Meehan gave further comments regarding the report which were taken on-board. The Interim Report was then modified and would be submitted on the 20th of December before 12 am and that would be the end of semester 1.

5.2 Semester 2

The work for semester 2 would begin on the 25th of January through ordering the necessary parts for the second semester. Whilst waiting for all the parts to arrive, designing of the RFID system would be carried out. After carrying out the design and the parts arrive, the build-test of the RFID playing environment would begin. This would take a few weeks to have up and running.

After implementing the RFID playing environment, the microcontroller would be implemented with the circuit. Once integrated, the RFID playing environment would then feed as an input to the microcontroller. Once the tests were passed, the game of Texas Hold 'em was built onto the microprocessor. Multiple games would be simulated without any function being performed on the input.

Once the electronic circuits would be implemented, the parameters for the decision-making function be built onto the microprocessor as a Base Class. Once all the factors & parameters discussed in this report were inbuilt, it would be time to start messing with the decision-making function. The decision-making function would have to built in a way that it can output the decision – fold, check, call, raise and the amount associated. The output of the decision would be outputted to a simple inbuilt screen on the microcontroller.

After setting up the decision-making function and testing out its functionality, optimization of the weights of the different parameters would begin. Through tuning and implementing different combination of weights for different factors and parameters, an optimal decision-making function will be found. The decision-making function will be accuracy-tested and attempt to construct Nash Equilibrium Charts and export them in a visual manner.

After solving the game of preflop based on the decision-making function, different GTO solutions would be explored. The implementation of the software PIOSOLVER would be integrated with the current microcontroller setup.

Finally, reducing the system and presenting it in the centre of the Electronic Engineering Department in Maynooth University. Results regarding the performance of the solver would be collected at this presentation. After collecting the results, the typing of the Final Year Report would commence. Ensuring that a draft is completed 15 days before the actual submission date to ensure that there is plenty of time to fix mistakes. A final year project poster would be created and presented in the centre of Electronic Engineering Department in Maynooth University.

5.3 Gantt Chart

Section 5.1 & 5.2 break down the goals to be achieved and the steps that are going to be taken to achieve them. The Gantt chart below contains a detailed overview of the estimated dates that have been laid out:

Game Theory Optimal in Texas Hold 'em

Seán Paul Gill

Project Start Date	10/14/2019 (Monday)
Display Week	1
Project Lead	Seán Paul Gill

WBS	TASK	START	END	DAYS	% DONE	WORK DAYS
1	**Semester 1**		-			-
1.1	Design Project Brief	Mon 10/14/19	Fri 10/18/19	5	100%	5
1.2	Research Resources Required	Sat 10/19/19	Tue 10/22/19	4	100%	2
1.3	Research Solving Methods	Tue 10/22/19	Thu 10/31/19	10	100%	8
1.4	Research Literature	Fri 11/01/19	Thu 12/12/19	42	100%	30
1.4.1	DeepStack AI	Fri 11/01/19	Thu 11/14/19	14	100%	10
1.4.2	Qualitative Analysis of Gamblers' Perceptions in Hold 'em	Fri 11/15/19	Thu 11/28/19	14	100%	10
1.4.3	Superhuman AI solves 6-player hold 'em	Fri 11/29/19	Thu 12/12/19	14	100%	10
1.5	Defining the Problem Scope	Sun 11/24/19	Sat 11/30/19	7	100%	5
1.6	Write Interim Report	Wed 12/04/19	Tue 12/10/19	7	100%	5
1.7	Re-type Interim from comments from draft	Tue 12/10/19	Thu 12/19/19	10	100%	8
1.8	Submit Interim Report	Fri 12/20/19	Fri 12/20/19	1	0%	1

WBS	TASK	START	END	DAYS	% DONE	WORK DAYS
2	**Semester 2**		-			-
2.1	Order Parts	Sat 1/25/20	Sat 2/01/20	8	0%	5
2.2	Design RFID Card Setup	Sun 1/26/20	Sat 2/01/20	7	0%	5
2.3	Build and Test RFID playing environment	Sun 2/02/20	Thu 2/06/20	5	0%	4
2.4	Integrate Microcontroller/PC	Thu 2/06/20	Sun 2/09/20	4	0%	2
2.5	Program Texas Hold 'em on microcontroller	Mon 2/10/20	Fri 2/14/20	5	0%	5
2.6	Input Decision Making Parameters	Sat 2/15/20	Wed 2/19/20	5	0%	3
2.7	Tune Decision Making Function Parameters	Mon 2/17/20	Mon 2/24/20	8	0%	6
2.8	Optimize Decision Making Function	Sun 2/23/20	Thu 2/27/20	5	0%	4
2.9	Attempt Nash Calculator	Fri 2/28/20	Sun 3/01/20	3	0%	1
2.10	Incorparate Decision Making function with microcontroller	Sun 3/01/20	Sat 3/07/20	7	0%	5
2.11	Run Accuracy Tests	Fri 3/06/20	Wed 3/11/20	6	0%	4
2.12	Try new GTO solutions (range vs range)	Wed 3/11/20	Sat 3/14/20	4	0%	3
2.13	Try get hold of PIOSOLVER	Sat 2/15/20	Sun 3/15/20	30	0%	20
2.14	Reduction of entire system	Sun 3/15/20	Mon 3/23/20	9	0%	6
2.15	Test Against Humans	Tue 3/24/20	Wed 4/01/20	9	0%	7
2.16	Write Draft Report	Wed 4/01/20	Mon 4/20/20	20	0%	14
2.17	Create Presentation	Thu 4/02/20	Wed 4/22/20	21	0%	15
2.18	Create Poster	Fri 4/03/20	Fri 4/24/20	22	0%	16
2.19	Write Final Report	Thu 4/23/20	Sat 5/09/20	17	0%	12

Figure 13: Overview of the project layout (semester 1 - left, semester 2 - right)

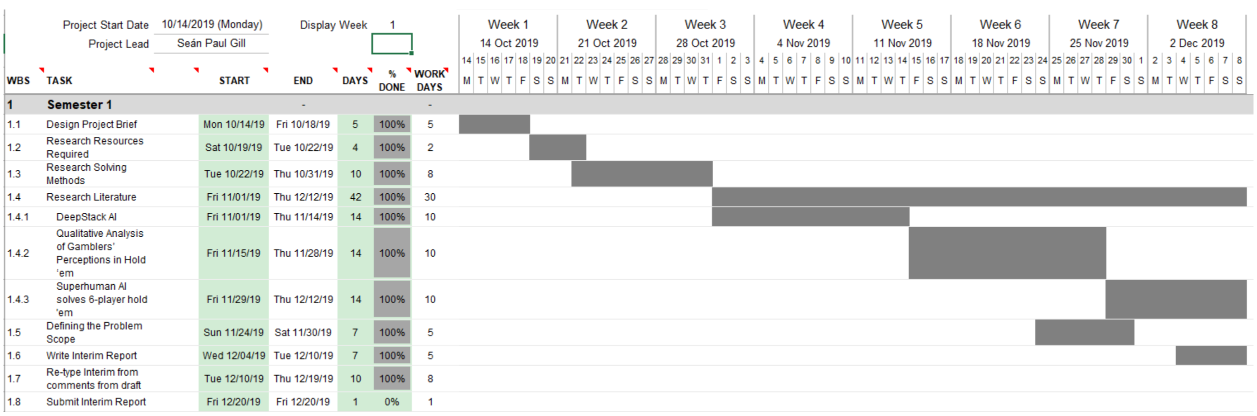
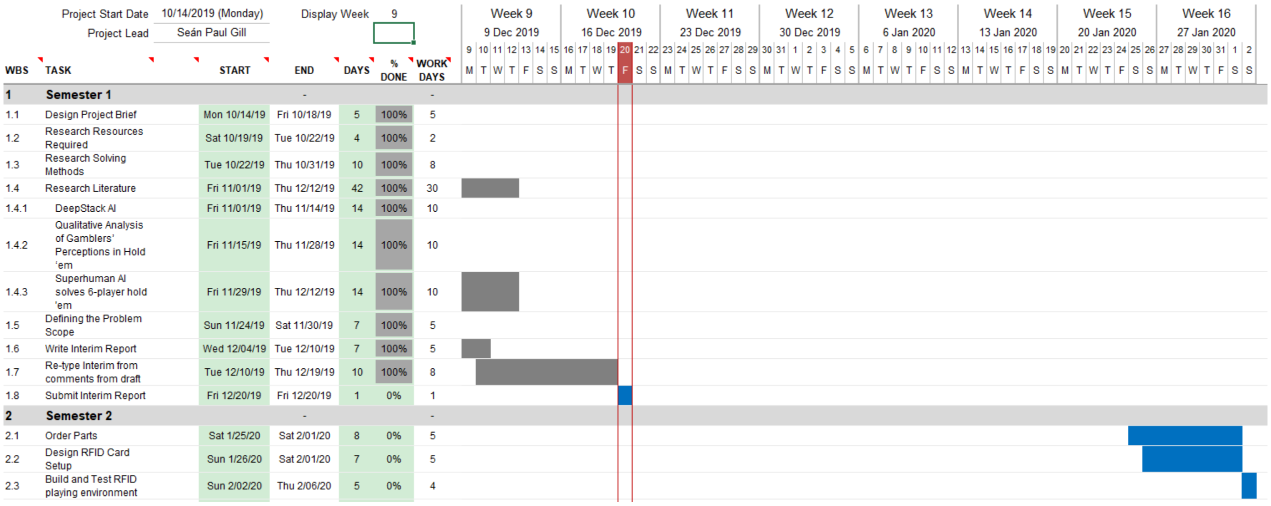
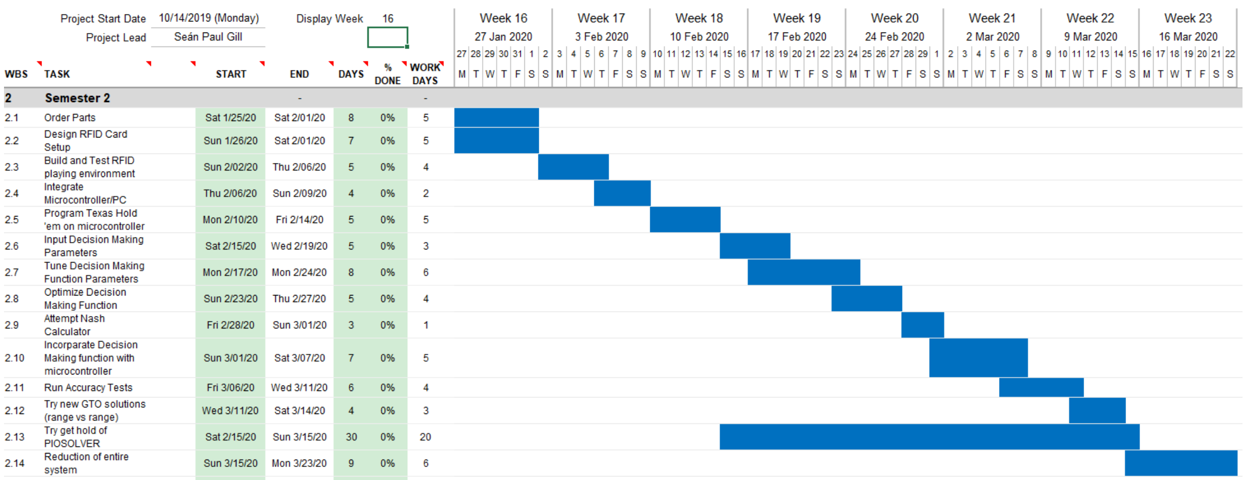


Figure 14: Gantt Chart (Week 1 - Week 8)



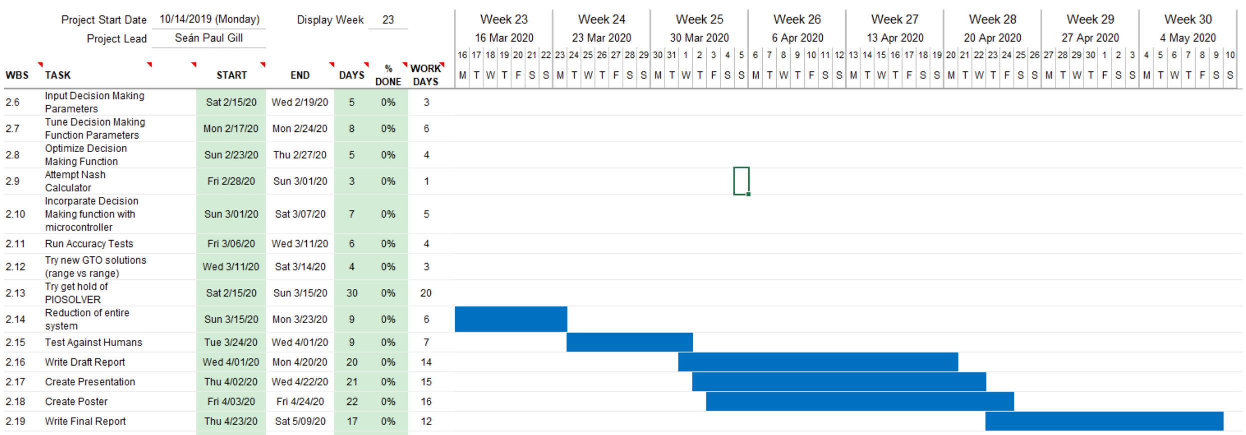
WBS	TASK	START	END	DAYS	% DONE	WORK DAYS
1	**Semester 1**		-			-
1.1	Design Project Brief	Mon 10/14/19	Fri 10/18/19	5	100%	5
1.2	Research Resources Required	Sat 10/19/19	Tue 10/22/19	4	100%	2
1.3	Research Solving Methods	Tue 10/22/19	Thu 10/31/19	10	100%	8
1.4	Research Literature	Fri 11/01/19	Thu 12/12/19	42	100%	30
1.4.1	DeepStack AI	Fri 11/01/19	Thu 11/14/19	14	100%	10
1.4.2	Qualitative Analysis of Gamblers' Perceptions in Hold 'em	Fri 11/15/19	Thu 11/28/19	14	100%	10
1.4.3	Superhuman AI solves 6-player hold 'em	Fri 11/29/19	Thu 12/12/19	14	100%	10
1.5	Defining the Problem Scope	Sun 11/24/19	Sat 11/30/19	7	100%	5
1.6	Write Interim Report	Wed 12/04/19	Tue 12/10/19	7	100%	5
1.7	Re-type Interim from comments from draft	Tue 12/10/19	Thu 12/19/19	10	100%	8
1.8	Submit Interim Report	Fri 12/20/19	Fri 12/20/19	1	0%	1
2	**Semester 2**		-			-
2.1	Order Parts	Sat 1/25/20	Sat 2/01/20	8	0%	5
2.2	Design RFID Card Setup	Sun 1/26/20	Sat 2/01/20	7	0%	5
2.3	Build and Test RFID playing environment	Sun 2/02/20	Thu 2/06/20	5	0%	4

Figure 15: Gantt Chart (Week 9 to Week 16)



WBS	TASK	START	END	DAYS	% DONE	WORK DAYS
2	**Semester 2**		-			-
2.1	Order Parts	Sat 1/25/20	Sat 2/01/20	8	0%	5
2.2	Design RFID Card Setup	Sun 1/26/20	Sat 2/01/20	7	0%	5
2.3	Build and Test RFID playing environment	Sun 2/02/20	Thu 2/06/20	5	0%	4
2.4	Integrate Microcontroller/PC	Thu 2/06/20	Sun 2/09/20	4	0%	2
2.5	Program Texas Hold 'em on microcontroller	Mon 2/10/20	Fri 2/14/20	5	0%	5
2.6	Input Decision Making Parameters	Sat 2/15/20	Wed 2/19/20	5	0%	3
2.7	Tune Decision Making Function Parameters	Mon 2/17/20	Mon 2/24/20	8	0%	6
2.8	Optimize Decision Making Function	Sun 2/23/20	Thu 2/27/20	5	0%	4
2.9	Attempt Nash Calculator	Fri 2/28/20	Sun 3/01/20	3	0%	1
2.10	Incorparate Decision Making function with microcontroller	Sun 3/01/20	Sat 3/07/20	7	0%	5
2.11	Run Accuracy Tests	Fri 3/06/20	Wed 3/11/20	6	0%	4
2.12	Try new GTO solutions (range vs range)	Wed 3/11/20	Sat 3/14/20	4	0%	3
2.13	Try get hold of PIOSOLVER	Sat 2/15/20	Sun 3/15/20	30	0%	20
2.14	Reduction of entire system	Sun 3/15/20	Mon 3/23/20	9	0%	6

Figure 16: Gantt Chart (Week 16 - Week 23)



WBS	TASK	START	END	DAYS	% DONE	WORK DAYS
2.6	Input Decision Making Parameters	Sat 2/15/20	Wed 2/19/20	5	0%	3
2.7	Tune Decision Making Function Parameters	Mon 2/17/20	Mon 2/24/20	8	0%	6
2.8	Optimize Decision Making Function	Sun 2/23/20	Thu 2/27/20	5	0%	4
2.9	Attempt Nash Calculator	Fri 2/28/20	Sun 3/01/20	3	0%	1
2.10	Incorparate Decision Making function with microcontroller	Sun 3/01/20	Sat 3/07/20	7	0%	5
2.11	Run Accuracy Tests	Fri 3/06/20	Wed 3/11/20	6	0%	4
2.12	Try new GTO solutions (range vs range)	Wed 3/11/20	Sat 3/14/20	4	0%	3
2.13	Try get hold of PIOSOLVER	Sat 2/15/20	Sun 3/15/20	30	0%	20
2.14	Reduction of entire system	Sun 3/15/20	Mon 3/23/20	9	0%	6
2.15	Test Against Humans	Tue 3/24/20	Wed 4/01/20	9	0%	7
2.16	Write Draft Report	Wed 4/01/20	Mon 4/20/20	20	0%	14
2.17	Create Presentation	Thu 4/02/20	Wed 4/22/20	21	0%	15
2.18	Create Poster	Fri 4/03/20	Fri 4/24/20	22	0%	16
2.19	Write Final Report	Thu 4/23/20	Sat 5/09/20	17	0%	12

Figure 17: Gantt Chart (Week 23 - Week 30)

References

[1] Doyle Brunson

[2] https://www.pokernews.com/strategy/game-theory-optimal-solutions-and-poker-afew-thoughts22654.htm

[3] Seán Paul Gill, Project Brief, Department of Electronic Engineering, Maynooth University

[4] Rules Texas Holdem, https://www.pokerlistings.com/poker-rules-texas-holdem

[5] Texas State Legislature – House (May 11, 2007). "80(R) HCR 109". House Resolution.

[6] Ghosts at the Table by Des Wilson – Page 119–122

[7] "Moneymaker method can show the way to a fortune". The Times.

[8] https://www.wolframalpha.com/input/?i=word+frequency+of+%22poker%22

[9] https://en.wikipedia.org/wiki/Rounders_(film)

[10] https://ocw.mit.edu/courses/sloan-school-of-management/15-s50-poker-theory-and-analytics-january-iap-2015/

[11] https://www.ted.com/talks/liv_boeree_3_lessons_on_decision_making_from_a_poker_champion

[12] https://www.atlasrfidstore.com/rfid-beginners-guide/

[13] https://www.rfidjournal.com/articles/view?1338

[14] "Frequently Asked Questions - RFID Journal". www.rfidjournal.com | Weis, Stephen A. (2007), RFID (Radio Frequency Identification): Principles and Applications, MIT CSAIL

[15] https://www.pokernews.com/news/2019/10/graphics-company-mike-postle-cheating-allegations-35609.htm

[16] Doug Polk, https://upswingpoker.com/preflop-poker-mistakes-avoid-at-all-cost/

[17] https://simplepoker.com/en/

[18] https://pokerglobal.info/knowledge-base/poker-table-position-positions-in-poker-post-2346/

[19] 19 https://www.blackrain79.com/2017/10/what-are-the-best-poker-hud-stats.html

[20] DeepStack: Expert-level artificial intelligence in heads-up no-limit poker - Matej Moravčík,1,2* Martin Schmid,1,2* Neil Burch,1 Viliam Lisý,1,3 Dustin Morrill,1 Nolan Bard,1 Trevor Davis,1 Kevin Waugh,1 Michael Johanson,1 Michael Bowling

[21] https://en.wikipedia.org/wiki/Rounders_(film)

[22] Deep Counterfactual Regret Minimization – Noam Brown, Adam Lerer

[23] Texas hold'em poker: a qualitative analysis of gamblers' perceptions - Gae¨lle Bouju,1,2 Marie Grall-Bronnec,1,2 Virginie Quistrebert-Davanne,1 Jean-Benoit Hardouin,2,3 & Jean-Luc Ve´nisse1,2

[24] Superhuman AI for multiplayer poker - Noam Brown, Tuomas Sandholm