Full Name: ______________________________

EEL 4750 / EEE 5502 (Fall 2021) – HW #8 **Due: 4:00 PM ET, Nov. 02, 2021**

Concept Questions 8

Question #1: Complete the Canvas questions here: `https://ufl.instructure.com/courses/437179/assignments/4812584`

Theory Questions 8

Question #1: Consider the analog biquad filter, an analog filter design that is very common because it is flexible (can be low-pass, high-pass, band-pass, or stop-band) and easy to manufacture. A biquad filter is most generally defined by:

$$H(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0} = \frac{K\left(s^2 + 2(\Omega_z/Q_z)s + \Omega_z^2\right)}{s^2 + 2(\Omega_p/Q_p)s + \Omega_p^2}$$

(a) Show that the system

$$\text{poles are equal to: } s = \Omega_p\left(-1/Q_p \pm \sqrt{1/Q_p^2 - 1}\right)$$

$$\text{zeros are equal to: } s = \Omega_z\left(-1/Q_z \pm \sqrt{1/Q_z^2 - 1}\right)$$

Note that with the Laplace transform, $s = j\omega + \sigma$ whereas the z-transform $z = e^{j\omega+\sigma}$. In both cases, the ω values correspond to angular frequency and the σ values correspond to the impulse response decay rate.

(b) When $\Omega_z = 0$, $Q_p = \infty$, and $\Omega_p > 0$, at what frequencies (as a function of Ω_p) are approximately in the analog filter's passband (in radians/s)?

(c) When $\Omega_z = 0$, $0 \leq Q_p \leq 1$, and $\Omega_p > 0$, at what frequencies (as a function of Ω_p) are approximately in the analog filter's passband (in radians/s)?

(d) When $\Omega_z = 0$, $Q_p = \sqrt{2}$, and $\Omega_p > 0$, at what frequencies (as a function of Ω_p) are approximately in the analog filter's passband (in radians/s)?

(e) Use the difference approximation method with a sampling rate of $f_s = 1$ to design a digital IIR filter based on the analog biquad filter. Under this approximation, show that

$$\text{poles are equal to: } z = \frac{1}{1 + \Omega_p\left(1/Q_p \pm \sqrt{1/Q_p^2 - 1}\right)}$$

$$\text{zeros are equal to: } z = \frac{1}{1 + \Omega_z\left(1/Q_z \pm \sqrt{1/Q_z^2 - 1}\right)}$$

(f) When $\Omega_z = 0$, $Q_p = \infty$, and Ω_p approaches infinity (i.e., $\Omega_p \gg 1$ Footnote:[1]), frequencies (as a function of Ω_p) are approximately in the digital filter's passband (in radians/s)?

(g) When $\Omega_z = 0$, $0 \leq Q_p \leq 1$, and Ω_p approaches infinity (i.e., $\Omega_p \gg 1$), frequencies (as a function of Ω_p) are approximately in the digital filter's passband (in radians/s)?

(h) When $\Omega_z = 0$ and Ω_p approaches zero (i.e., $\Omega_p \ll 1$ Footnote: [2]), what frequencies (as a function of Ω_p) are approximately in the analog filter's passband (in radians/s)?

(i) Based on these results, what type(s) of filter(s) do you the think the first difference method here would be most effective at approximating? Why?

[1]Hence, $1 + \Omega_p \approx \Omega_p$

[2]Hence, $1 + \Omega_p \approx 1$

Implementation Questions 8

Question #1: Consider a **negative** ideal low-pass, continuous-time filter defined by

$$H(\Omega) = u\left(\Omega + \Omega_c\right) - u\left(\Omega - \Omega_c\right)$$

and its DTFT equivalent frequency response

$$H(\omega) = u\left(\omega + \Omega_c/f_s\right) - u\left(\omega - \Omega_c/f_s\right) \qquad \text{(this repeats for every } 2\pi \text{ shift)}$$

For all of the components below, use a sampling rate of $f_s = 1$ Hz and a cut-off frequency of $\Omega_c = \pi/4$ rad/s. For every designed filter, ensure that the DC gain (i.e., at $\omega = 0$) is equal to 1.

(a) Use the windowing method with a Hann window [3] to design an FIR, linear phase low-pass filter of length $N = 50$ from $H(\Omega)$.[4] Plot the pole-zero plot for the filter and plot the magnitude and phase response.

(b) Use the frequency sampling method to design an FIR, linear phase low-pass filter of length $N = 50$ from $H(\Omega)$. Plot the pole-zero plot for the filter and plot the magnitude and phase response.

(c) Use the difference approximation method to design an IIR filter with $K = 1$, $\Omega_z = 20$, and $Q_p = 5$, $Q_z = 2$, and $\Omega_p = 0.8$ for the bi-quad filter from theory questions. Plot the pole-zero plot for the filter and plot the magnitude and phase response.

(d) Using filters in (a), (b) and (c) above, filter the following square waves defined in the template:

```
sqwaves{1} = square(2*pi*0.4*t);
sqwaves{2} = square(2*pi*1*t);
```

[3] A Hann window is defined by $w[n] = 0.5(1 - \cos(2\pi n/(N-1)))$ for $0 \le n \le N-1$, where N is the window length

[4] Note: You may need to use a `sinc` function. I am providing a `sinc_func` function in the template since MATLAB's `sinc` function uses a different definition, as discussed in class.