Full Name: ______________________________

EEL 4750 / EEE 5502 (Fall 2021) – HW #6 **Due: 4:00 PM ET, Oct. 4, 2021**

Concept Questions 6

Question #1: Complete the Canvas questions here: `https://ufl.instructure.com/courses/437179/assignments/4812579/`

Question #2: **Study for the exam next week.** You can find old / practice exams here: `http://smartdata.ece.ufl.edu/eee5502/lecture.html?lecture=18`.

Theory Questions 6

Question #1:

Consider three signals $x_1(t)$ and $x_2(t)$ and $x_3(t)$ such that their Fourier transforms satisfy

$$\begin{aligned} X_1(\Omega) &= 0 \quad , \quad 30 \le |\Omega| \\ X_2(\Omega) &= 0 \quad , \quad |\Omega| \le 15, |\Omega| \ge 20 \\ X_3(\Omega) &= 10^{-\Omega}, \quad |\Omega| < \infty \end{aligned}$$

Determine the minimum frequency Ω_s at which we must sample the following signals in order to prevent aliasing. If aliasing is unavoidable, write that instead of a frequency.

(a) $x(t) = x_1(t) + x_2(t)$

(b) $x(t) = x_1(t) * x_3(t)$

(c) $x(t) = x_1(t)x_2(t)$

(d) $x(t) = \cos(3.6\pi t + 9.23)$

(e) $x(t) = u(t-1) - u(t-4)$

(f) $x(t) = \delta(t)$

Implementation Questions 6

In this assignment, we want to observe the effects of sampling and applying reconstruction filters. For this, it will be helpful to have an inverse continuous-time Fourier transform. Therefore, we are providing a new function `x = ICTFT(X,t,w)` that computes the ICTFT $X(\Omega)$ at desired angular frequencies `w` from $x(t)$ (`x`) at times `t`. The function approximates the ICTFT by:

$$x[n] = \int_{-\infty}^{\infty} X(\Omega)e^{+j\omega t}\, d\Omega \approx \Delta W \sum_{n=-\infty}^{\infty} X(\Omega)e^{+jn(\Omega\Delta W)} \; ,$$

where ΔW is the difference between each consecutive value in the `w` vector.

Question #1: Use our CTFT function to plot the approximate continuous-time Fourier transform of the following "continuous-time" signals after sampling with the given cyclic sampling rate f_s. Then apply an **ideal** low pass reconstruction filter (i.e., do your filtering in the frequency domain) [1] and compute the inverse CTFT. Use a time range for `t` from -10 to 10 and use a frequency range of `w=-4*pi:pi/20:(4*pi-pi/20)`.

(a) $x(t) = \cos((3\pi/4)t) \quad , \quad f_s = 1/2$

(b) $x(t) = \cos((3\pi/4)t) \quad , \quad f_s = 1$

(c) $x(t) = \cos((3\pi/4)t) \quad , \quad f_s = 4$

(d) $x(t) = u(t+3) - u(t-3) \quad , \quad f_s = 1/2$

(e) $x(t) = u(t+3) - u(t-3) \quad , \quad f_s = 1$

(f) $x(t) = u(t+3) - u(t-3) \quad , \quad f_s = 4$

Question #2: **(Optional – Not Graded)** In this class, we will use a linear frequency-modulated "chirp" signal as a common example signal, a signal whose instantaneous frequency $\omega[n]$ changes linearly with time. Chirp signals are commonly used in RADAR processing to extract time-varying time-shifts (distance from a target) and frequency/Doppler-shifts (velocity of a target). They are defined by

$$x[n] = \cos\left(2\pi(f_1/(2T))t^2\right) \; .$$

(a) Create a function `x = chirp(t,f1,T)` that computes the chirp signal for final frequency `f1` and signal length `T` (in seconds).

(b) Plot the time-domain and the CTFT magnitude response for a chirp signal for final frequency `f1 = 2000`, length `T = 0.1` s. Choose your sampling rate so that $x[n]$ is over-sampled.

(c) Plot the time-domain and the CTFT magnitude response for a chirp signal for final frequency `f1 = 2000`, length `T = 0.1` s. Choose your sampling rate so that $x[n]$ is critically-sampled.

(d) Plot the time-domain and the CTFT magnitude response for a chirp signal for final frequency `f1 = 2000`, length `T = 0.1` s. Choose your sampling rate so that $x[n]$ is under-sampled.

[1] Note: Due to how this is setup (i.e., we are not zero-ing values but instead removing values), the gain for this filter will be 1, not T_s, as discussed in the lecture