Full Name: ____

EEL 4750 / EEE 5502 (Fall 2021) – HW #2 **Due: 4:00 PM ET, Sept. 8, 2021**

Concept Questions 2

Question #1: Complete the Canvas questions here: `https://ufl.instructure.com/courses/437179/assignments/4812573`

Theory Questions 2

Question #1: *(Convolution Bounds)* Consider the expression for convolution given two signals

$$c[n] = x[n] * y[n] = \sum_{m=-\infty}^{\infty} x[m]y[n-m]$$

$$x[n] = 0 \quad \text{for} \quad n < L_x \quad \text{and} \quad n > U_x$$

$$y[n] = 0 \quad \text{for} \quad n < L_y \quad \text{and} \quad n > U_y$$

The signals has length $U_x - L_x + 1$ and $U_y - L_y + 1$, respectively. Show that $x[n] * y[n]$ satisfies

$$x[n] * y[n] = 0 \quad \text{for} \quad n < L_x + L_y \quad \text{and} \quad n > U_x + U_y \ .$$

That is, the result has length $U_x + U_y - (L_x + L_y) + 1$. Illustrations and correct intuition are acceptable. *[Side note: Versions of this is a common DSP interview question.]*

Question #2: *(Inner Product)* The inner product is an important operation in signal processing, statistics, and machine learning. The inner product between length-N signals $x[n]$ and $y[n]$ is

$$c = \sum_{n=-\infty}^{\infty} x[n]y[n]$$

(a) Show that when $y[n] = x[n]$, the inner product is the energy of $x[n]$.

(b) Let $y[n]$ be $y[m-n]$ in the inner product equation between length-N signals. Show that this is equivalent the convolution operation, i.e.,

$$c[m] = \sum_{n=-\infty}^{\infty} x[n]y[m-n] = x[m] * y[m]$$

(c) Let $y[n]$ be $y[n+m]$ in the inner product equation between length-N signals. Show that this is equivalent the correlation operation, i.e.,

$$a[m] = \sum_{n=-\infty}^{\infty} x[n]y[n+m] = x[-m] * y[m]$$

(d) Let $y[n] = x[n-M]$. Show the $a[m]$ is maximum when $m = M$. [1]

(e) How is this last property valuable in pattern recognition? (*Note: This is used particularly everywhere and is the fundamental for a lot of machine learning, computer vision, radar, sonar, statistical signal processing, etc.*)

[1] Hint: Use the Cauchy-Schwarz Inequality: $\left|\sum_{n=0}^{N-1} x[n]y[n]\right|^2 \leq \sum_{n=0}^{N-1} |x[n]|^2 \sum_{n=0}^{N-1} |y[n]|^2$

Implementation Questions 2

Question #1: (*Impulse responses*) Construct a vector representation of the impulse response for each system below. Each impulse response should be in the range of `n=-5:5`, resulting in vectors of length 11.

(a) $y_1[n] = \frac{1}{2}x[n-1]$

(b) $y_2[n] = \frac{1}{2}x[n+1] + x[n-3]$

(c) $y_3[n] = \sum_{m=-M}^{M} m(-1)^m x[n-m]$, with M = 5. *Hint: Can be done in one short line of code.*

Question #2: (*Convolution*) Construct the input signal

$$x[n] = u[n] - u[n-4]$$

for input `n=-5:5`. Construct a vector representation of the impulse response for each of the following systems, as in Question 1. Then use the MATLAB convolution function `conv` to compute each system's output. Also, construct a output vector `n` that can be used to plot the outputs using `stem` while properly aligning the x-axis (needed because the output of `conv` will be a different length than `x`) [2].

(a) $y_4[n] = x[n]$

(b) $y_5[n] = x[n-2] - 2x[n+5]$

(c) $y_6[n] = x[n-1] + x[n+1]$

(d) $y_7[n] = \sum_{m=-M}^{M}(-1)^m x[n-2m]$, with M = 2.

Question #3: (*Finding a hidden signal*) Oftentimes we want to find small signals (e.g., a face or a radar reflection) buried in larger signals (e.g., a picture or a stream of measurements). The correlation operation gives us the tools to accomplish this. Use the `load` command to load the provided MAT file, which contains a noisy signal called `noisy_message` and a smaller signal called `code`. The signal `code` can be found somewhere within `noisy_message`, but is hidden by noise. Use correlation to find the location of `code` in `noisy_message` (i.e., the index at which `code` starts in `noisy_message`). [3]

A third signal is provided that may be helpful: `test_message` contains `code` surrounded by silence, and can be used to easily test your method. Note that `code` has different locations in `noisy_message` and `test_message`, and this question only requires you to locate `code` in `noisy_message`.

Hint: Recall that correlation is identical to convolution, but with time reversal involved. Correlation can be achieved using some combination of the functions `conv` *and* `fliplr` *(or* `flipud`*, depending on how you define you variables).*

[2] Use Theory Question #1 to help you define n

[3] See Theory Question #2 for how this works