Full Name: ____

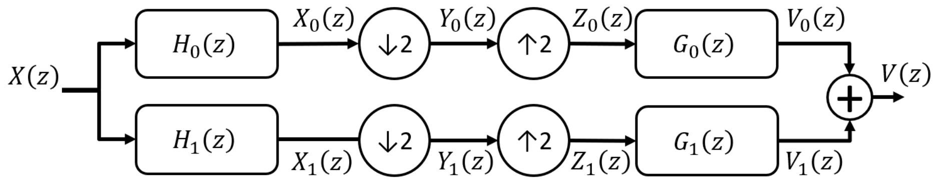
EEL 4750 / EEE 5502 (Fall 2021) – HW #10 **Due: 4:00 PM ET, Nov. 16, 2021**

Concept Questions 10

Question #1: Complete the Canvas questions here: `https://ufl.instructure.com/courses/437179/assignments/4812586`

Theory Questions 10

Question #1: Consider a 2-channel filter bank shown below.

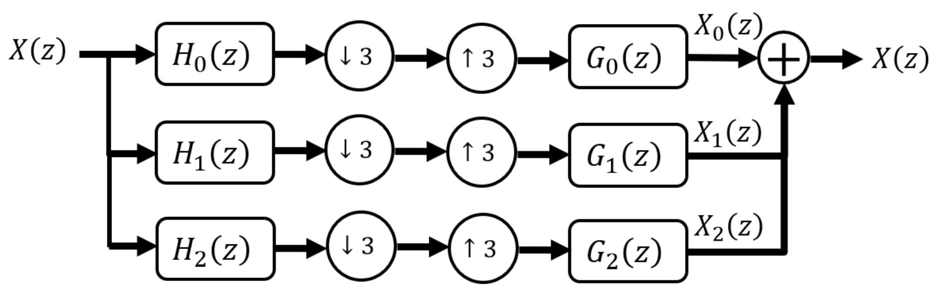


$$H_0(z) = G_0(z^{-1}) = (1/\sqrt{2})(1 - z^{-n})$$
$$H_1(z) = G_1(z^{-1}) = (1/\sqrt{2})(1 + z^{-n})$$

(a) Identify all natural numbers n so that the above 2-channel filter bank satisfies the alias canceling perfect reconstruction condition. Prove your claim.

(b) For all such natural numbers n for which the alias canceling perfect reconstruction condition is not satisfied, which of the two conditions is violated.

Question #2: Consider the three-channel filter bank below.



Let the three filters (which satisfy the orthogonal filter bank conditions) be defined by

$$G_0(z) = \frac{1}{\sqrt{6}}\left(\sqrt{2} + z^{-1} + \sqrt{3}z^{-2}\right) \quad , \quad G_1(z) = \frac{1}{\sqrt{6}}\left(-\sqrt{2} + 2z^{-1}\right)$$
$$G_2(z) = \frac{1}{\sqrt{6}}\left(\sqrt{2} + z^{-1} - \sqrt{3}z^{-2}\right)$$

(a) Rather than prove every condition, show that one filter satisfies

$$\sum_{k=0}^{M-1} G_m\left(e^{j(2\pi/M)k}z\right) G_m\left(e^{-j(2\pi/M)k}z^{-1}\right) = M \qquad \text{for all } \; m$$

and one pair of filters satisfies

$$\sum_{k=0}^{M-1} G_m\left(e^{j(2\pi/M)k}z\right) G_v\left(e^{-j(2\pi/M)k}z^{-1}\right) = 0 \qquad \text{for all } \; m \neq v$$

.

(b) Plot the frequency response of each transfer function $G_i(z)$. What do you observe?

Implementation Questions 10

Question #1: *(STFT with Overlap)* You may have noticed choppiness in the reconstructed time signal from HW 9. This is because there is no smooth transition from one frame to another.

(a) To remove the choppiness, create two new functions `stft_woverlap` and `istft_woverlap`. In these functions:

- In `stft_woverlap`, each time you compute the FFT, shift the frame by $W/2$ instead of W. Hence, each frame will have a 50% overlap with an adjacent frame.
- In `istft_woverlap`, after each IFFT, multiply the length-W signal with a Hann window

$$w[n] = \frac{1}{2}\left[1 - \cos\left(\frac{2\pi n}{W-1}\right)\right] .$$

  This creates a smooth transition between frames.
- In `istft_woverlap`, after computing each IFFT and multiplying each frame by a Hann window, sum the overlapping components of the adjacent frames. This process is known as overlap-add.

(b) Load `urquan.wav`, containing audio from the Ur-Quan Masters game. Use your `stft_woverlap` to compute the STFT of the audio with $W = 100$. Then use the `istft_woverlap` function to compute the inverse STFT and reconstruct the original signal.

(c) The template provides code that compute the STFT with and without an overlap, keeps only the largest values (a simple compression scheme), and then computes the respective ISTFT. How and why does the audio change due to your overlapping and adding?

Question #2: *(Filter Bank Implementation)* Use filter bank coefficients defined by a length-$2N$ modified discrete cosine transform:

$$g_k[n] = \frac{1}{\sqrt{N}} \cos\left(\frac{\pi}{N}\left(n + \frac{N+1}{2}\right)\left(k + \frac{1}{2}\right)\right) \quad , \quad h_k[n] = g_k[2N - n - 1]$$

The variable k corresponds to the k-th filter of frequency $\omega_k = (\pi/N)k$. [1]

For this question, load `urquan.wav` (audio from Ur-Quan Masters again) to get `x`.

(a) Implement a 100-channel analysis filter bank function (named `fb_analysis`) to transform a signal $x[n]$ into the output of 100 different filter coefficient streams $v_m[n]$ (note that each filter will have a length of 200 samples in this case). Note: use the `conv` function to implement each filter.

(b) Implement a 100-channel synthesis filter bank function (named `fb_synthesis`) to transform the filter coefficients $v_m[n]$ into the output signal $y[n]$. Note: use the `conv` function to implement each filter.

(c) The reconstruction should be perfect with the exception of a delay. Identify this delay [in samples].

(d) Why does this delay occur?

(e) Are there similarities or differences between the STFT with overlap and the filter bank?

[1] The modified discrete cosine filter bank is widely used in audio compression schemes (mp4, vorbis, etc.).