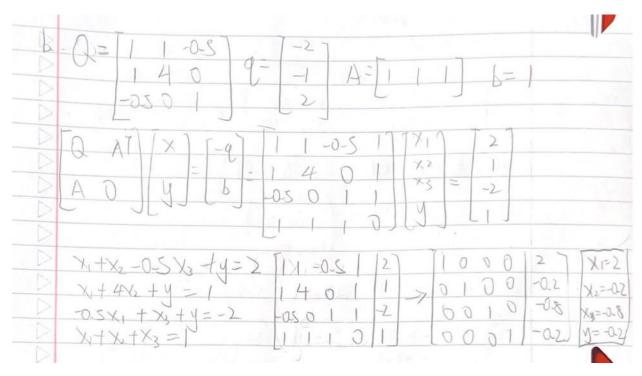
| 1. 02. | $A = \begin{pmatrix} 1 & -2 & e^2 \\ 1 & -1 & e^1 \\ 1 & 0 & e^0 \end{pmatrix} = \begin{pmatrix} -7.1474 \\ -3.3021 \\ 12274 \\ 6.1006 \end{pmatrix} = \begin{pmatrix} A^TAB = A^TB \\ 6.1006 \\ 11.0370 \end{pmatrix} = \begin{pmatrix} A^TAB = A^TB \\ A^TAB = A^TB \end{pmatrix}$   |
|--------|--|
|        | $B^* = (ATA)^-A^+B^- = (0.9975)$ $B = (ATA)^-A^+B^- = (0.2522)$  |
|        | $f(3) = -8.9435 + 5.0022 x + 0.2522e^{-x}$ $f(-3) = -8.9435 + f(0.5) = 3.6516 + f(2.5) = 13.5237$ $f(-3) = -8.9786 + f(0.5) = 3.6516 + f(2.5) = 13.5205$ $f(-3) = -6.054 + f(0.5) + f(0.5) = 0 + f(0.5) + f(0.5) + f(0.5) = 0 + f(0.5) $ |

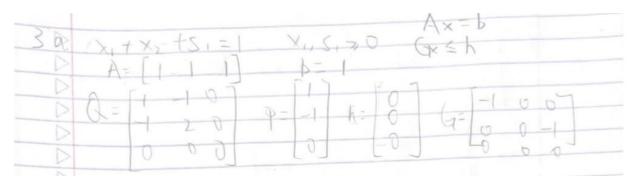
2a

| $2.9 + (x) = \frac{1}{2} (x_1 - \alpha_1)^2 + \frac{1}{2} (x_2 - \alpha_2)^2$   |
|---|
| = \frac{1}{\xi-2\x,\alpha_1+\alpha_1^2\frac{1}{2}(\xi^2-2\x,2\alpha_2+\alpha_2^2)}  |
| $= \frac{1}{2} \left( x_1^2 + x_2^2 \right) - \left( \alpha_1 x_1 + \alpha_2 x_2 \right) + \frac{1}{2} \left( \alpha_1^2 + \alpha_2^2 \right)$                        |
| $Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}  Q = \begin{bmatrix} -\alpha_1 \\ -\alpha_2 \end{bmatrix}  A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}  b = 1$ |
|   |
| $\begin{array}{c c} \begin{array}{c c} \hline \\ \hline $                             |
| LA OLUJE LO LA  |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$  |
| $\begin{array}{cccccccccccccccccccccccccccccccccccc$  |
| X, + X, ZX, XY = 02   |
| X12 X1 - Q2 Y2 Z2 Y2 Z2 Y2 Z2   |



2c

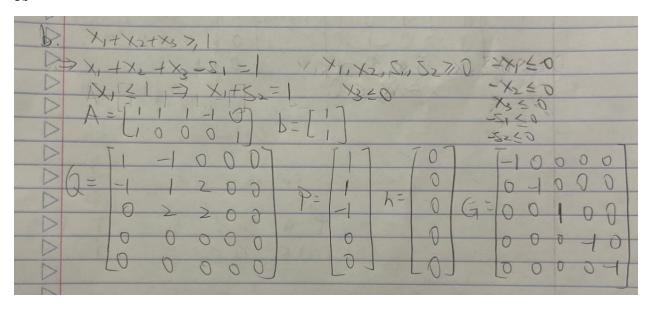
| 0 0 0 0 0  | $4 = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \end{bmatrix} b = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ |
|--|--|
| $\begin{array}{c c} \hline \begin{array}{c} \hline \end{array} \end{array} \begin{array}{c} \hline \end{array} \begin{array}{c} \hline \end{array} \begin{array}{c} \hline \end{array} \end{array} \begin{array}{c} \hline \end{array} \begin{array}{c} \hline \end{array} \end{array} \begin{array}{c} \hline \end{array} \begin{array}{c} \hline \end{array} \end{array} \begin{array}{c} \hline \end{array} \end{array} \begin{array}{c} \hline \end{array} \begin{array}{c} \hline \end{array} \end{array} \end{array} $ $ \end{array} $ | 11-0501   1   1   2   1   2   1   2   1   2   1   2   2  |
|  | 1 1 1 0 0 0 0 0 0  |
| D 11-0-5 0 1 1 2<br>D 14 0 0 1 1 11<br>D -0-5 0 1 0 1 -1 -2  | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |
| 000011-12  | $\begin{array}{c ccccccccccccccccccccccccccccccccccc$  |
| 0 0 0 0  | J LO 000 0 1 0.4022 Jyz=0.4022   |



```
pcost
                 dcost
                                            dres
                                     pres
                             gap
0: -2.2222e-01 -5.5556e-01
                             4e+00
                                     2e+00
                                            1e+00
 1: -1.5511e-01 -4.7625e-01
                                     2e-02
                                            1e-02
                              3e-01
 2: -2.3105e-01 -2.5436e-01
                              2e-02
                                     2e-04
                                           1e-04
3: -2.4975e-01 -2.5007e-01
                             3e-04
                                     2e-06
                                           1e-06
4: -2.5000e-01 -2.5000e-01
                                     2e-08
                                           1e-08
                             3e-06
5: -2.5000e-01 -2.5000e-01
                             3e-08
                                     2e-10 1e-10
Optimal solution found.
-0.2499999752186499
[ 4.96e-08]
[ 5.00e-01]
[ 5.00e-01]
```

The optimal solution is x1=4.96e-08, x2=5.0e-01, s1=5.0e-01. The optimal value is -0.250 which is minimized by the optimal solution. This is the minimum of the objective function.

3b



```
[0.0, 0.0, 0.0, 0.0, 0.0]]
p= matrix([1.0, 1.0, -1.0, 0.0, 0.0])
G= matrix([[-1.0, 0.0, 0.0, 0.0, 0.0],
         [0.0, -1.0, 0.0, 0.0, 0.0],
         [0.0, 0.0, 1.0, 0.0, 0.0],
         [0.0, 0.0, 0.0, -1.0, 0.0],
         [0.0, 0.0, 0.0, 0.0, -1.0]
h=matrix([0.0, 0.0, 0.0, 0.0, 0.0])
A=matrix([1.0, 1.0, 1.0, -1.0, 0.0,
        1.0, 0.0, 0.0, 0.0, 1.0, (2, 5)
b=matrix([1.0, 1.0])
sol = solvers.qp(Q, p, G, h, A, b)
print(sol["primal objective"])
print(sol["x"])
                      dcost
                                                       dres
       pcost
                                              pres
                                      gap
       1.0425e+00 -6.2500e-02
  0:
                                    1e+01
                                              3e+00
                                                       2e+00
  1:
       1.1152e+00 2.1995e-01
                                     9e-01
                                                       2e-02
                                              3e-02
  2:
       1.0288e+00 9.3043e-01 1e-01
                                              2e-03
                                                       2e-03
 Terminated (singular KKT matrix).
 1.028796956467899
 [ 3.66e-01]
 [ 6.34e-01]
 [-2.38e-01]
 [ 1.28e+00]
 [ 1.51e+00]
```

The optimal solution was found at x1=3.66e-01, x2=6.34e-01, x3=-2.38e-01, x3=-2.38e-01

| 4 or. | $R_1 = F[R_1] = 1.078$ $R_2 = 1.1524$ $R_3 = 1.15945$<br>$R_4 = 1.16545$ $R_5 = 1.0997$  |
|-------|--|
|       | Z= R-R 5= ZTZ  |
|       | mm f(x)= - > R; X; + X, TSX  |
|       | $\lambda \times 15 \times = \frac{1}{2} \times 10 \times \lambda = \frac{1}{2} 0  Q = 2 \lambda S$ $S1.  Z_{3-1}^{2} \times 1 = 1$ |
|       | X770, J-1-S  |

| Q=21S                                  |           |  |
|--|-----------|--|
| P= -R2 A-[1                            | 11117 6=1 |  |
| -R4<br>ER5                             | 007 107   |  |
| 9=0-10                                 | 0 0 h = 0 |  |
| 10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 | < 0       |  |

Using R-studio to calculate the centered data matrix and sample covariance matrix:

```
· ```{r}
 PortSelectData <- read.csv("PortSelectData.csv")</pre>
 mu_1 <- mean(PortSelectData[, 2], na.rm=T)</pre>
 mu_2 <- mean(PortSelectData[, 3], na.rm=T)</pre>
 mu_3 <- mean(PortSelectData[, 4], na.rm=T)</pre>
 mu_4 <- mean(PortSelectData[, 5], na.rm=T)</pre>
 mu_5 <- mean(PortSelectData[, 6], na.rm=T)</pre>
 paste("E[R_1]: ", mu_1)
paste("E[R_2]: ", mu_2)
 paste("E[R_2]: ", mu_2)
paste("E[R_3]: ", mu_3)
paste("E[R_4]: ", mu_4)
paste("E[R_5]: ", mu_5)
  [1] "E[R_1]: 1.078"
  [1] "E[R_2]: 1.1524"
  [1] "E[R_3]: 1.15945"
[1] "E[R_4]: 1.16545"
  [1] "E[R_5]: 1.0997"
 ```{r}
clean_port <- PortSelectData[-c(21, 22), -c(1, 7, 8, 9)]
R_j < c(mu_1, mu_2, mu_3, mu_4, mu_5)
for (i in 1:ncol(clean_port)){
  clean_port[, i]=clean_port[, i]-R_j[i]
X <- as.matrix(clean_port)</pre>
S \leftarrow 1/(nrow(clean\_port)-1) * t(X) %*% X
colnames(S) <- NULL</pre>
rownames(S) <- NULL
                    [,1]
                                   [,2]
   [,3]
   [,4]
  [1,] 0.0010230526 0.000195000 0.0001472105 -0.0002766842 0.0003207895
 [2,] 0.0001950000 0.018573621 0.0189497579 0.0195837053 0.0052181263 [3,] 0.0001472105 0.018949758 0.0201629974 0.0225924711 0.0045108789 [4,] -0.0002766842 0.019583705 0.0225924711 0.0326845763 0.0043572474
  [5,] 0.0003207895 0.005218126 0.0045108789 0.0043572474 0.0064332737
```

```
#4 lambda = 1
from cvxopt import matrix, solvers
lamb = 1
Q= 2* lamb * matrix([[0.0010230526, 0.000195000, 0.0001472105, -0.0002766842,
0.0003207895],
                    [0.0001950000, 0.018573621, 0.0189497579, 0.0195837053,
0.0052181263],
                    [0.0001472105, 0.018949758, 0.0201629974, 0.0225924711,
0.0045108789],
                    [-0.0002766842, 0.019583705, 0.0225924711, 0.0326845763,
0.0043572474],
                    [0.0003207895, 0.005218126, 0.0045108789, 0.0043572474,
0.0064332737]])
p= matrix([-1.07800, -1.15240, -1.15945, -1.16545, -1.09970])
G= matrix([[-1.0, 0.0, 0.0, 0.0, 0.0],
           [0.0, -1.0, 0.0, 0.0, 0.0],
           [0.0, 0.0, -1.0, 0.0, 0.0],
           [0.0, 0.0, 0.0, -1.0, 0.0],
           [0.0, 0.0, 0.0, 0.0, -1.0]
h=matrix([0.0, 0.0, 0.0, 0.0, 0.0])
A=matrix([1.0, 1.0, 1.0, 1.0, 1.0], (1, 5))
b=matrix(1.0)
sol = solvers.qp(Q, p, G, h, A, b)
print(sol["primal objective"])
print(sol["x"])
```

```
dcost
   dres
     pcost
                             gap
                                    pres
 0: -1.1244e+00 -2.1419e+00
                                    7e-16
                             1e+00
   1e+00
 1: -1.1253e+00 -1.1515e+00
                             3e-02
                                    1e-16
   3e-02
 2: -1.1355e+00 -1.1399e+00
                             4e-03
                                    1e-16 2e-03
 3: -1.1391e+00 -1.1398e+00 7e-04
                                    3e-16 2e-16
 4: -1.1393e+00 -1.1393e+00
                            2e-05
                                    2e-16 2e-16
 5: -1.1393e+00 -1.1393e+00
                             6e-07
                                    1e-16
   1e-16
Optimal solution found.
-1.1393294355920018
[ 5.02e-07]
[ 9.52e-07]
[ 9.25e-01]
[ 7.49e-02]
[ 6.95e-07]
```

For lambda=1 as the penalization/tuning parameter on the variance of return, the optimal allocation of the investment in the portfolio is to put 5.02e-07 of the investment in US 3-month T-Bills, 9.52e-07 of the investment in S&P 500, 9.25e-01 of the investment into Wilshire 5000, 7.49e-02 of the investment into NASDAQ Composite, and 6.95e-07 of the investment into EAFE. The optimal value is -1.1393 which means that the previous allocation along with the penalization of lambda=1 minimized the objective function to get to this value.