

Example 1: Estimating the Probability of Success

Scenario: You are a basketball coach interested in estimating the probability that your star player will make a free throw.

- **Prior Model:** Beta distribution, $\text{Beta}(\alpha = 2, \beta = 2)$, representing an initial belief that the player is equally likely to succeed or fail.
- **Likelihood Model:** Binomial distribution, where the number of successful free throws in a series of attempts follows $\text{Binomial}(n, p)$.
- **Data:** The player makes 8 successful free throws out of 10 attempts.
- **Question:** How do you update your belief about the player's free throw success probability after observing these 10 attempts?

Example 2: Predicting Arrival Times

Scenario: You are a transportation analyst predicting bus arrival times, and you want to update your belief about the mean arrival time of buses.

- **Prior Model:** Normal distribution, $\mathcal{N}(\mu = 15, \sigma^2 = 4)$, representing your initial belief that the average arrival time is 15 minutes with some uncertainty.
- **Likelihood Model:** Normal distribution, where observed arrival times follow $\mathcal{N}(\mu, \sigma^2 = 4)$.
- **Data:** Observed arrival times of 13, 14, 15, 16, and 14 minutes.
- **Question:** How do you update your belief about the mean arrival time after these observations?

Example 3: Estimating Event Rates

Scenario: You are a quality control manager in a factory interested in estimating the rate of defective items produced per hour.

- **Prior Model:** Gamma distribution, $\text{Gamma}(\alpha = 2, \beta = 1)$, representing your initial belief about the defect rate.
- **Likelihood Model:** Poisson distribution, where the number of defective items follows $\text{Poisson}(\lambda)$.
- **Data:** Observations show 3, 4, 2, and 5 defective items in four consecutive hours.
- **Question:** How do you update your belief about the defect rate per hour after these observations?