

# CFGs and PCFGs

(Probabilistic)  
Context-Free  
Grammars



# A phrase structure grammar

Context Free Grammar (回想Compiler)

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$VP \rightarrow V NP PP$

$NP \rightarrow NP NP$

$NP \rightarrow NP PP$

$NP \rightarrow N$

$NP \rightarrow e$

$PP \rightarrow P NP$

$N \rightarrow \text{people}$

$N \rightarrow \text{fish}$

$N \rightarrow \text{tanks}$

$N \rightarrow \text{rods}$

$V \rightarrow \text{people}$

$V \rightarrow \text{fish}$

$V \rightarrow \text{tanks}$

$P \rightarrow \text{with}$

*people fish tanks*

*people fish with rods*



# Phrase structure grammars = context-free grammars (CFGs)

- $G = (T, N, S, R)$ 
  - $T$  is a set of terminal symbols
  - $N$  is a set of nonterminal symbols
  - $S$  is the start symbol ( $S \in N$ )
  - $R$  is a set of rules/productions of the form  $X \rightarrow \gamma$ 
    - $X \in N$  and  $\gamma \in (N \cup T)^*$
- A grammar  $G$  generates a language  $L$ .



# Phrase structure grammars in NLP

T: 無法再拆成更小單位的符號，只會出現在RHS

C:

N: 可以再拆成更小單位的符號，出現在LHS或RHS

S: Start Symbol，代表句子的開頭

L: Phrase改寫成特定的字，例如DT  $\rightarrow$  the, NN  $\rightarrow$  man

R: Phrase改寫成Phrase，例如NP  $\rightarrow$  DT NN

- $G = (T, C, N, S, L, R)$ 
  - T is a set of terminal symbols
  - C is a set of preterminal symbols
  - N is a set of nonterminal symbols
  - S is the start symbol ( $S \in N$ )
  - L is the lexicon, a set of items of the form  $X \rightarrow x$ 
    - $X \in P$  and  $x \in T$
  - R is the grammar, a set of items of the form  $X \rightarrow \gamma$ 
    - $X \in N$  and  $\gamma \in (N \cup C)^*$
- By usual convention, S is the start symbol, but in statistical NLP, we usually have an extra node at the top (ROOT, TOP)
- We usually write  $e$  for an empty sequence, rather than nothing
  - $e$ 代表Empty Symbol，代表句子的結束



# A phrase structure grammar

## Grammar

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$VP \rightarrow V NP PP$

$NP \rightarrow NP NP$

$NP \rightarrow NP PP$

$NP \rightarrow N$

$NP \rightarrow e$

$PP \rightarrow P NP$

## Lexicon

$N \rightarrow \textit{people}$

$N \rightarrow \textit{fish}$

$N \rightarrow \textit{tanks}$

$N \rightarrow \textit{rods}$

$V \rightarrow \textit{people}$

$V \rightarrow \textit{fish}$

$V \rightarrow \textit{tanks}$

$P \rightarrow \textit{with}$

*people fish tanks*

*people fish with rods*



# Probabilistic – or stochastic – context-free grammars (PCFGs)

對於每一組Grammar Rule

找出機率和最高的組合

來代表這一句話的詞性結構

- $G = (T, N, S, R, P)$ 
  - T is a set of terminal symbols
  - N is a set of nonterminal symbols
  - S is the start symbol ( $S \in N$ )
  - R is a set of rules/productions of the form  $X \rightarrow \gamma$
  - P is a probability function
    - $P: R \rightarrow [0,1]$
    - $\forall X \in N, \sum_{X \rightarrow \gamma \in R} P(X \rightarrow \gamma) = 1$
- A grammar G generates a language model L.

$$\sum_{\gamma \in T^*} P(\gamma) = 1$$



# A PCFG

$S \rightarrow NP VP$  1.0

$VP \rightarrow V NP$  0.6

$VP \rightarrow V NP PP$  0.4

$NP \rightarrow NP NP$  0.1

$NP \rightarrow NP PP$  0.2

$NP \rightarrow N$  0.7

$PP \rightarrow P NP$  1.0

$N \rightarrow \textit{people}$  0.5

$N \rightarrow \textit{fish}$  0.2

$N \rightarrow \textit{tanks}$  0.2

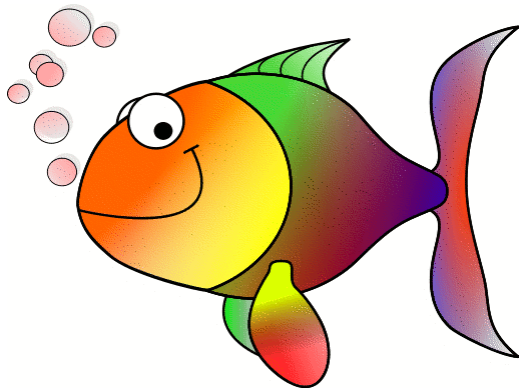
$N \rightarrow \textit{rods}$  0.1

$V \rightarrow \textit{people}$  0.1

$V \rightarrow \textit{fish}$  0.6

$V \rightarrow \textit{tanks}$  0.3

$P \rightarrow \textit{with}$  1.0



[With empty NP removed  
so less ambiguous]



# The probability of trees and strings

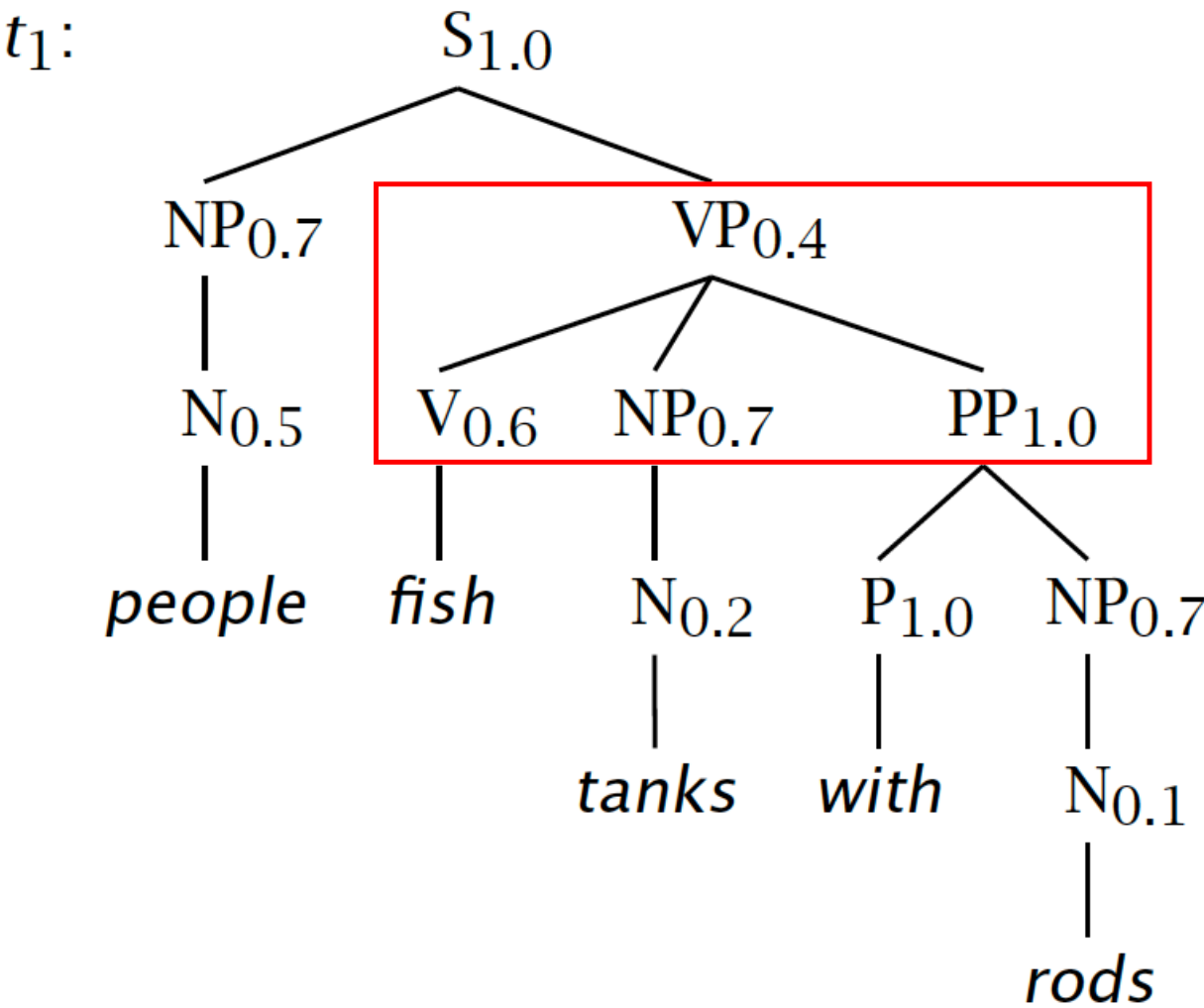
- $P(t)$  – The probability of a tree  $t$  is the product of the probabilities of the rules used to generate it.
- $P(s)$  – The probability of the string  $s$  is the sum of the probabilities of the trees which have that string as their yield

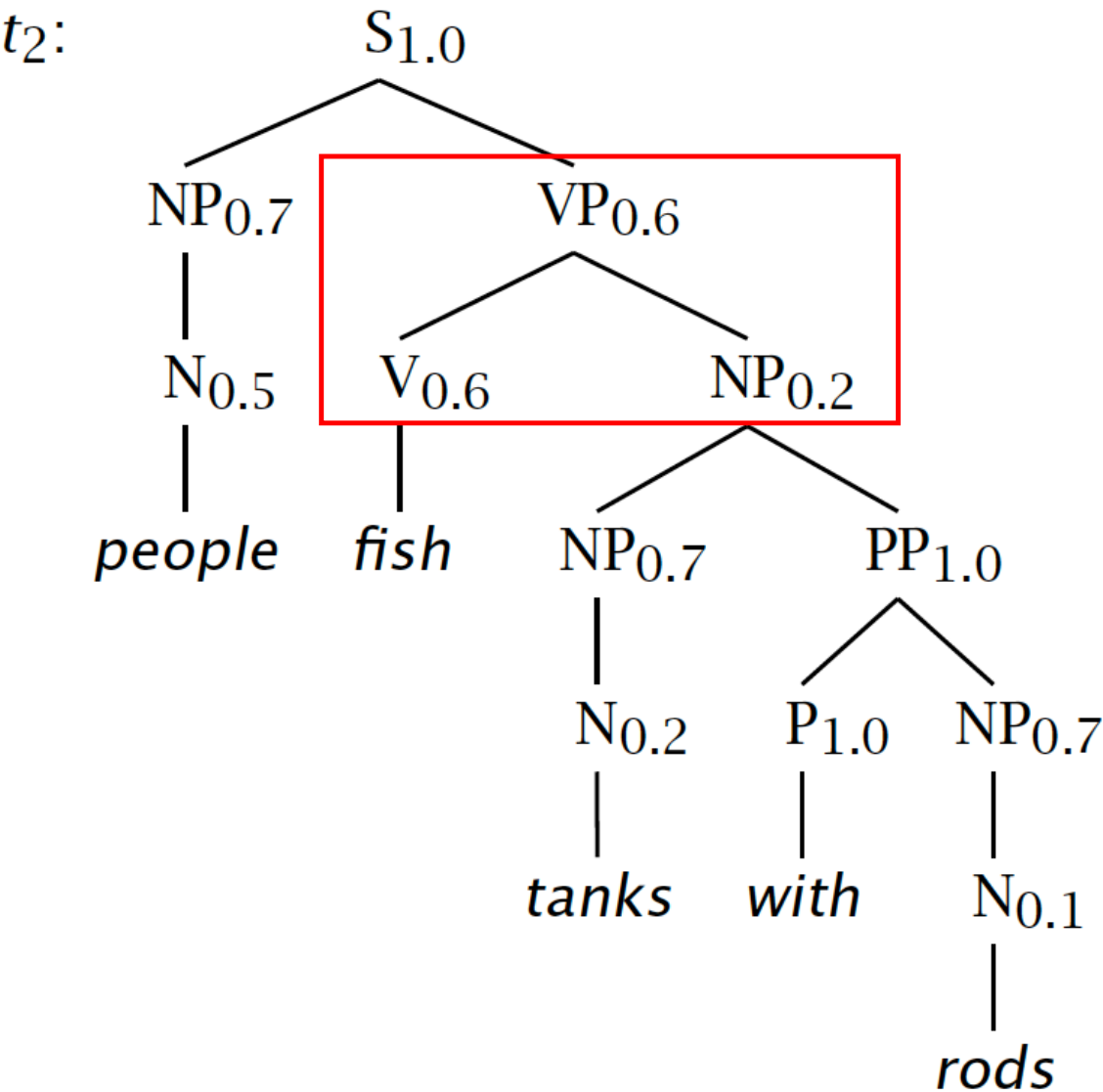
$$P(s) = \sum_j P(s, t) \text{ where } t \text{ is a parse of } s$$
$$= \sum_j P(t)$$

$P(t)$ 代表這一句話的詞性結構所畫出的樹為 $t$ 的機率，用前一頁的機率去算

$P(s)$ 代表這一句話具有正確文法結構的機率（？）









# Tree and String Probabilities

- $s = \textit{people fish tanks with rods}$

- $P(t_1) = 1.0 \times 0.7 \times 0.4 \times 0.5 \times 0.6 \times 0.7$   
 $\times 1.0 \times 0.2 \times 1.0 \times 0.7 \times 0.1$   
 $= 0.0008232$

Verb attach

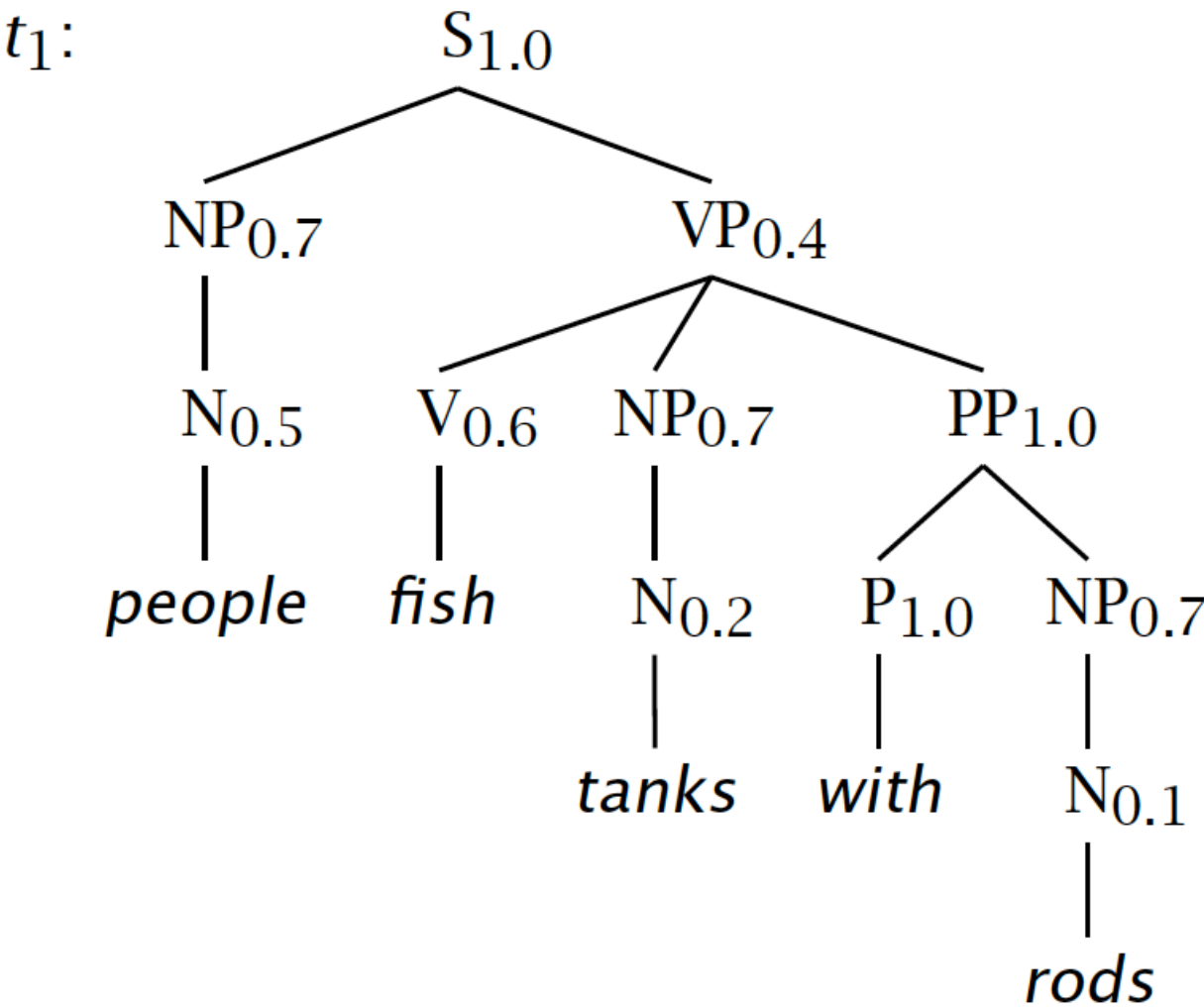
- $P(t_2) = 1.0 \times 0.7 \times 0.6 \times 0.5 \times 0.6 \times 0.2$   
 $\times 0.7 \times 1.0 \times 0.2 \times 1.0 \times 0.7 \times 0.1$   
 $= 0.00024696$

Noun attach

- $P(s) = P(t_1) + P(t_2)$   
 $= 0.0008232 + 0.00024696$   
 $= 0.00107016$

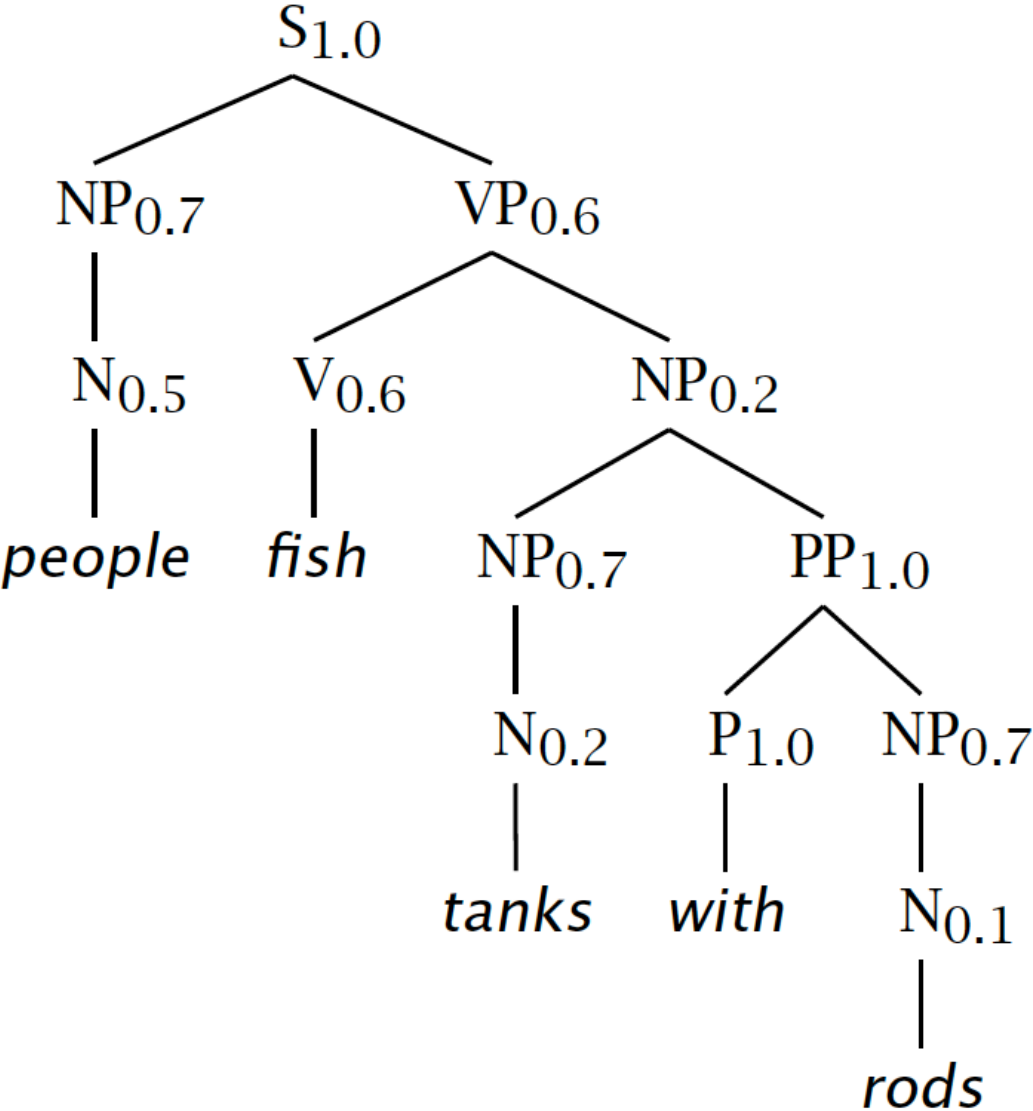
$$P(t_1) > P(t_2)$$

所以t1會是比較可能的詞性排列組合





$t_2$ :





# CFGs and PCFGs

# (Probabilistic) Context-Free Grammars

[illegible]

# Restricting the grammar form for efficient parsing



# Chomsky Normal Form

試著把大量的Rules做合併  
做出數量較少的Rules

- All rules are of the form  $X \rightarrow YZ$  or  $X \rightarrow w$ 
  - $X, Y, Z \in N$  and  $w \in T$
- A transformation to this form doesn't change the weak generative capacity of a CFG
  - That is, it recognizes the same language
    - But maybe with different trees
- Empties and unaries are removed recursively
- n-ary rules are divided by introducing new nonterminals ( $n > 2$ )

修改規則：

1. RHS只有e的要被拿掉
2. RHS只有一項，且可以被改寫為terminal的，要改寫
3. RHS有三項或以上的，要改成兩項

結果：所有Rules的RHS都應該只有兩項，或是只有一項terminal





# A phrase structure grammar

$S \rightarrow NP VP$	$S \rightarrow NP VP$	
$VP \rightarrow V NP$	$S \rightarrow VP$	$N \rightarrow people$
$VP \rightarrow V NP PP$		$N \rightarrow fish$
$NP \rightarrow NP NP$		$N \rightarrow tanks$
$NP \rightarrow NP PP$		$N \rightarrow rods$
$NP \rightarrow N$		$V \rightarrow people$
<del><math>NP \rightarrow e</math></del>		$V \rightarrow fish$
$PP \rightarrow P NP$		$V \rightarrow tanks$
		$P \rightarrow with$



# Chomsky Normal Form steps

S → NP VP

~~S → VP~~

VP → V NP

VP → V

VP → V NP PP

VP → V PP

NP → NP NP

NP → NP

NP → NP PP

NP → PP

NP → N

PP → P NP

PP → P

S -> V NP

S -> V

S -> V NP PP

S -> V PP

N → people

N → fish

N → tanks

N → rods

V → people

V → fish

V → tanks

P → with



# Chomsky Normal Form steps

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$S \rightarrow V NP$

$VP \rightarrow V$

~~$S \rightarrow V$~~

$VP \rightarrow V NP PP$

$S \rightarrow V NP PP$

$VP \rightarrow V PP$

$S \rightarrow V PP$

$NP \rightarrow NP NP$

$NP \rightarrow NP$

$NP \rightarrow NP PP$

$NP \rightarrow PP$

$NP \rightarrow N$

$PP \rightarrow P NP$

$PP \rightarrow P$

$N \rightarrow people$

$N \rightarrow fish$

$N \rightarrow tanks$

$N \rightarrow rods$

$V \rightarrow people$

$V \rightarrow fish$

$V \rightarrow tanks$

$P \rightarrow with$

$S \rightarrow people$   
 $S \rightarrow fish$   
 $S \rightarrow tanks$



# Chomsky Normal Form steps

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$S \rightarrow V NP$

~~$VP \rightarrow V$~~

$VP \rightarrow V NP PP$

$S \rightarrow V NP PP$

$VP \rightarrow V PP$

$S \rightarrow V PP$

$NP \rightarrow NP NP$

$NP \rightarrow NP$

$NP \rightarrow NP PP$

$NP \rightarrow PP$

$NP \rightarrow N$

$PP \rightarrow P NP$

$PP \rightarrow P$

$N \rightarrow \textit{people}$

$N \rightarrow \textit{fish}$

$N \rightarrow \textit{tanks}$

$N \rightarrow \textit{rods}$

$V \rightarrow \textit{people}$

$S \rightarrow \textit{people}$

$V \rightarrow \textit{fish}$

$S \rightarrow \textit{fish}$

$V \rightarrow \textit{tanks}$

$S \rightarrow \textit{tanks}$

$P \rightarrow \textit{with}$

$VP \rightarrow \textit{people}$   
 $VP \rightarrow \textit{fish}$   
 $VP \rightarrow \textit{tanks}$



# Chomsky Normal Form steps

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$S \rightarrow V NP$

$VP \rightarrow V NP PP$

$S \rightarrow V NP PP$

$VP \rightarrow V PP$

$S \rightarrow V PP$

$NP \rightarrow NP NP$

~~$NP \rightarrow NP$~~

$NP \rightarrow NP PP$

$NP \rightarrow PP$

~~$NP \rightarrow N$~~

$PP \rightarrow P NP$

$PP \rightarrow P$

$N \rightarrow people$

$N \rightarrow fish$

$N \rightarrow tanks$

$N \rightarrow rods$

$V \rightarrow people$

$S \rightarrow people$

$VP \rightarrow people$

$V \rightarrow fish$

$S \rightarrow fish$

$VP \rightarrow fish$

$V \rightarrow tanks$

$S \rightarrow tanks$

$VP \rightarrow tanks$

$P \rightarrow with$

$NP \rightarrow people$

$NP \rightarrow fish$

$NP \rightarrow tanks$

$NP \rightarrow rods$



# Chomsky Normal Form steps

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$S \rightarrow V NP$

~~$VP \rightarrow V NP PP$~~

$S \rightarrow V NP PP$

$VP \rightarrow V PP$

$S \rightarrow V PP$

$NP \rightarrow NP NP$

$NP \rightarrow NP PP$

$NP \rightarrow P NP$

$PP \rightarrow P NP$

把RHS有三項的改成兩項

$VP \rightarrow V @VP\_V$

$@VP\_V \rightarrow NP PP$

$NP \rightarrow people$

$NP \rightarrow fish$

$NP \rightarrow tanks$

$NP \rightarrow rods$

$V \rightarrow people$

$S \rightarrow people$

$VP \rightarrow people$

$V \rightarrow fish$

$S \rightarrow fish$

$VP \rightarrow fish$

$V \rightarrow tanks$

$S \rightarrow tanks$

$VP \rightarrow tanks$

$P \rightarrow with$

$PP \rightarrow with$



# Chomsky Normal Form steps

$S \rightarrow NP VP$   
 $VP \rightarrow V NP$   
 $S \rightarrow V NP$   
 $VP \rightarrow V @VP\_V$   
 $@VP\_V \rightarrow NP PP$   
 $S \rightarrow V @S\_V$   
 $@S\_V \rightarrow NP PP$   
 $VP \rightarrow V PP$   
 $S \rightarrow V PP$   
 $NP \rightarrow NP NP$   
 $NP \rightarrow NP PP$   
 $NP \rightarrow P NP$   
 $PP \rightarrow P NP$

$NP \rightarrow people$   
 $NP \rightarrow fish$   
 $NP \rightarrow tanks$   
 $NP \rightarrow rods$   
 $V \rightarrow people$   
 $S \rightarrow people$   
 $VP \rightarrow people$   
 $V \rightarrow fish$   
 $S \rightarrow fish$   
 $VP \rightarrow fish$   
 $V \rightarrow tanks$   
 $S \rightarrow tanks$   
 $VP \rightarrow tanks$   
 $P \rightarrow with$   
 $PP \rightarrow with$



# A phrase structure grammar

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$VP \rightarrow V NP PP$

$NP \rightarrow NP NP$

$NP \rightarrow NP PP$

$NP \rightarrow N$

$NP \rightarrow e$

$PP \rightarrow P NP$

$N \rightarrow \textit{people}$

$N \rightarrow \textit{fish}$

$N \rightarrow \textit{tanks}$

$N \rightarrow \textit{rods}$

$V \rightarrow \textit{people}$

$V \rightarrow \textit{fish}$

$V \rightarrow \textit{tanks}$

$P \rightarrow \textit{with}$





# Chomsky Normal Form steps

$S \rightarrow NP VP$

$VP \rightarrow V NP$

$S \rightarrow V NP$

$VP \rightarrow V @VP\_V$

$@VP\_V \rightarrow NP PP$

$S \rightarrow V @S\_V$

$@S\_V \rightarrow NP PP$

$VP \rightarrow V PP$

$S \rightarrow V PP$

$NP \rightarrow NP NP$

$NP \rightarrow NP PP$

$NP \rightarrow P NP$

$PP \rightarrow P NP$

$NP \rightarrow \textit{people}$

$NP \rightarrow \textit{fish}$

$NP \rightarrow \textit{tanks}$

$NP \rightarrow \textit{rods}$

$V \rightarrow \textit{people}$

$S \rightarrow \textit{people}$

$VP \rightarrow \textit{people}$

$V \rightarrow \textit{fish}$

$S \rightarrow \textit{fish}$

$VP \rightarrow \textit{fish}$

$V \rightarrow \textit{tanks}$

$S \rightarrow \textit{tanks}$

$VP \rightarrow \textit{tanks}$

$P \rightarrow \textit{with}$

$PP \rightarrow \textit{with}$

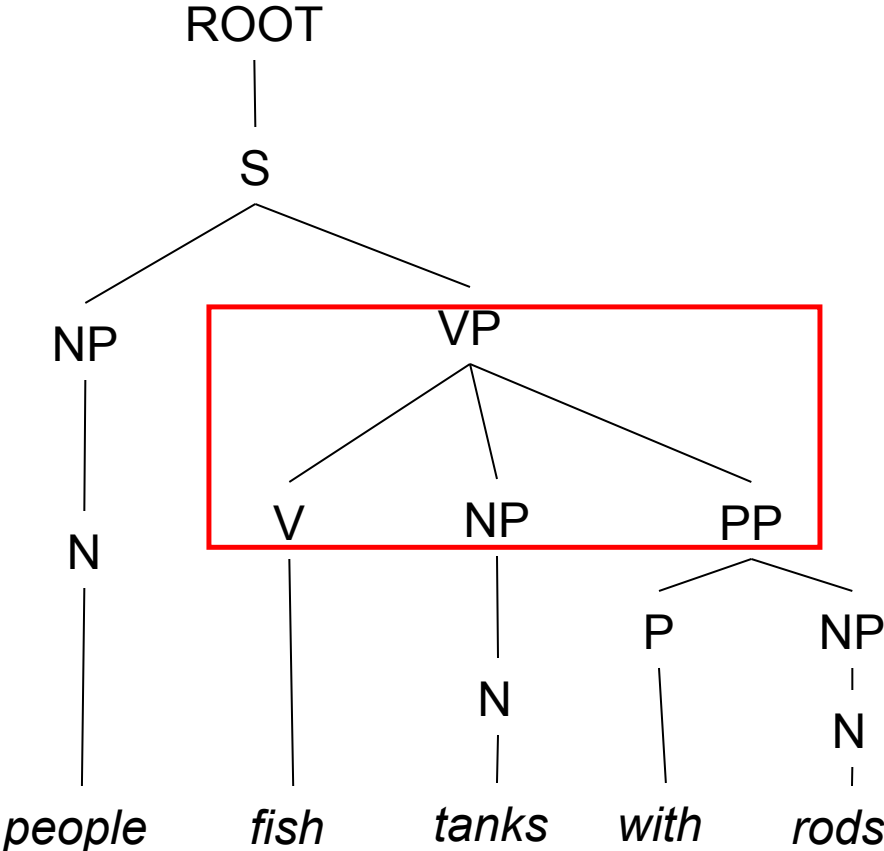


# Chomsky Normal Form

- You should think of this as a transformation for efficient parsing
- With some extra book-keeping in symbol names, you can even reconstruct the same trees with a detransform
- In practice full Chomsky Normal Form is a pain
  - Reconstructing n-aries is easy
  - Reconstructing unaries/empties is trickier
- **Binarization is crucial for cubic time CFG parsing**
- The rest isn't necessary; it just makes the algorithms cleaner and a bit quicker



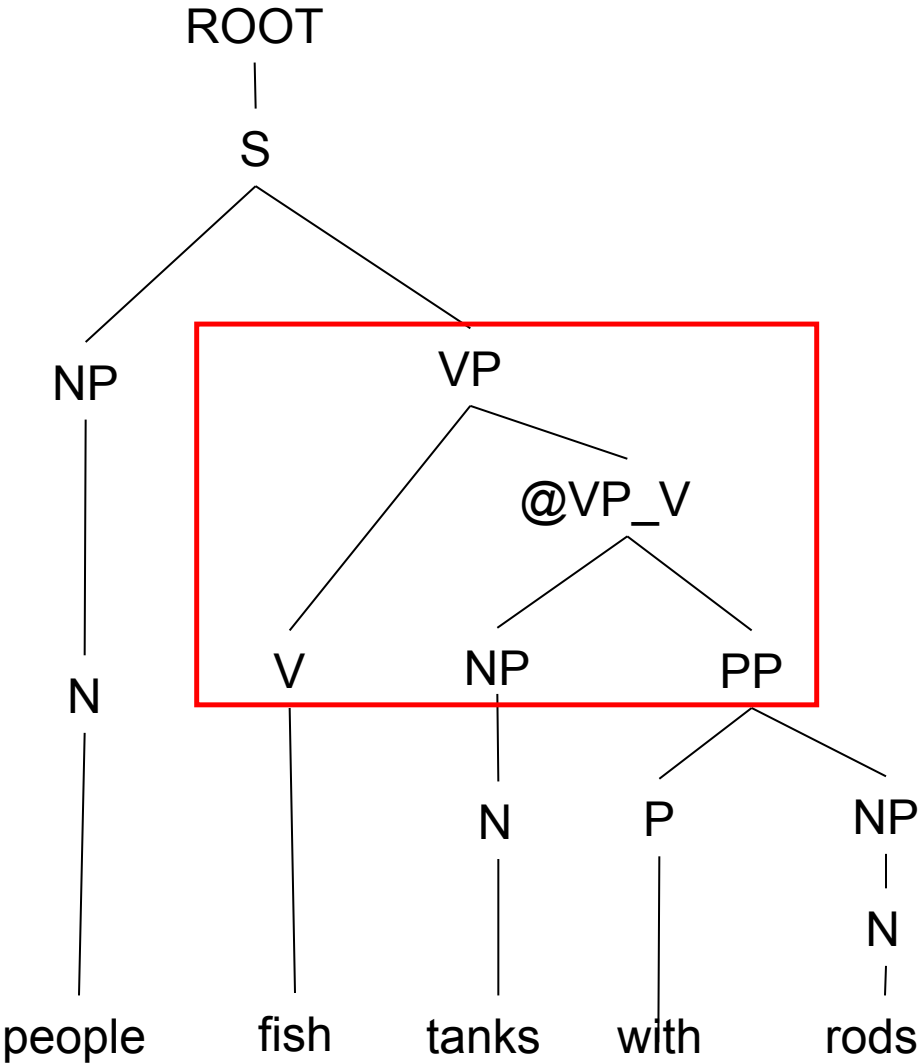
# An example: before binarization...



Subtree with 3 or more children  
這種文法就需要被修改

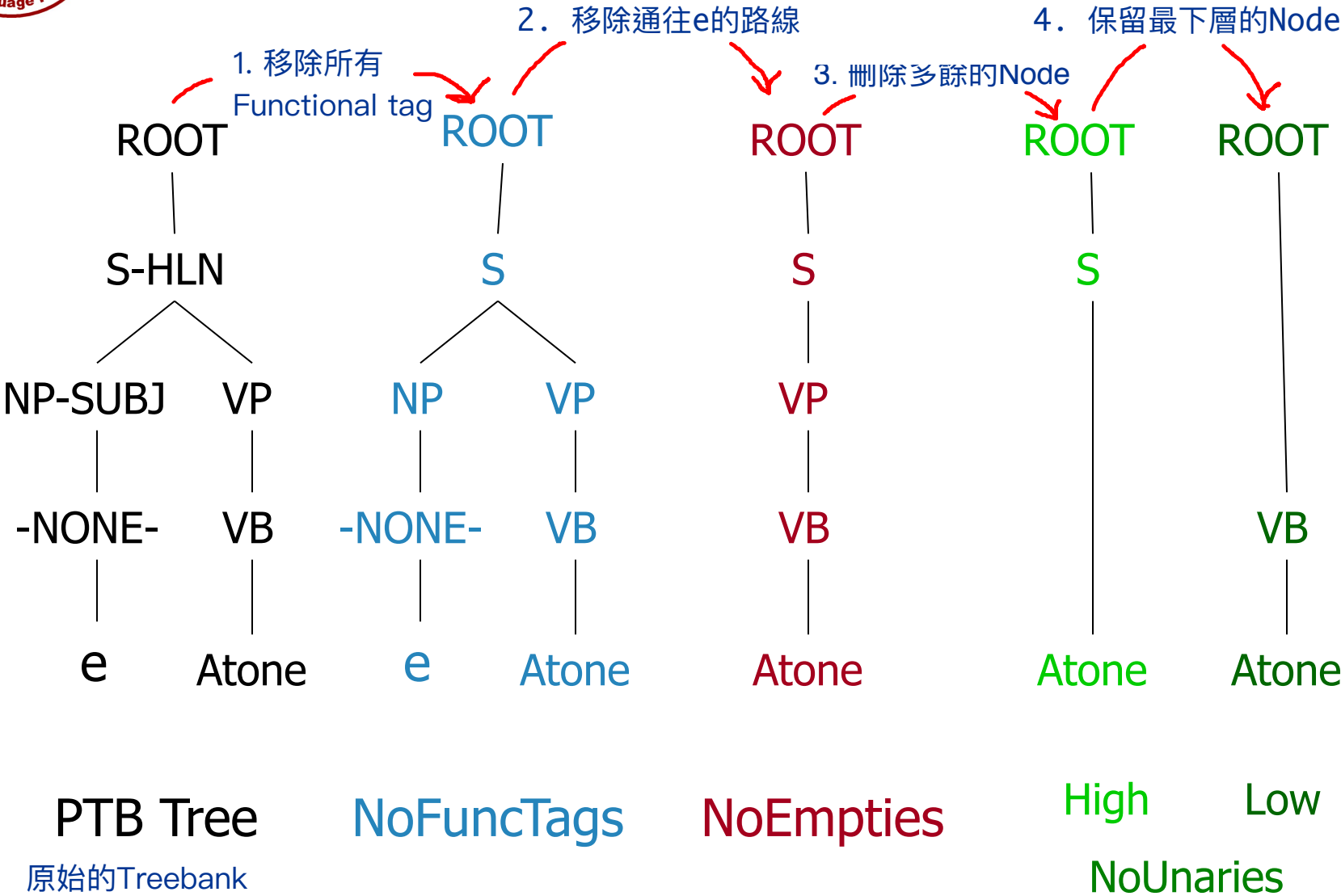


# After binarization...

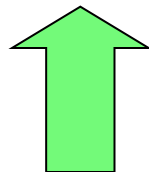




# Treebank: empties and unaries

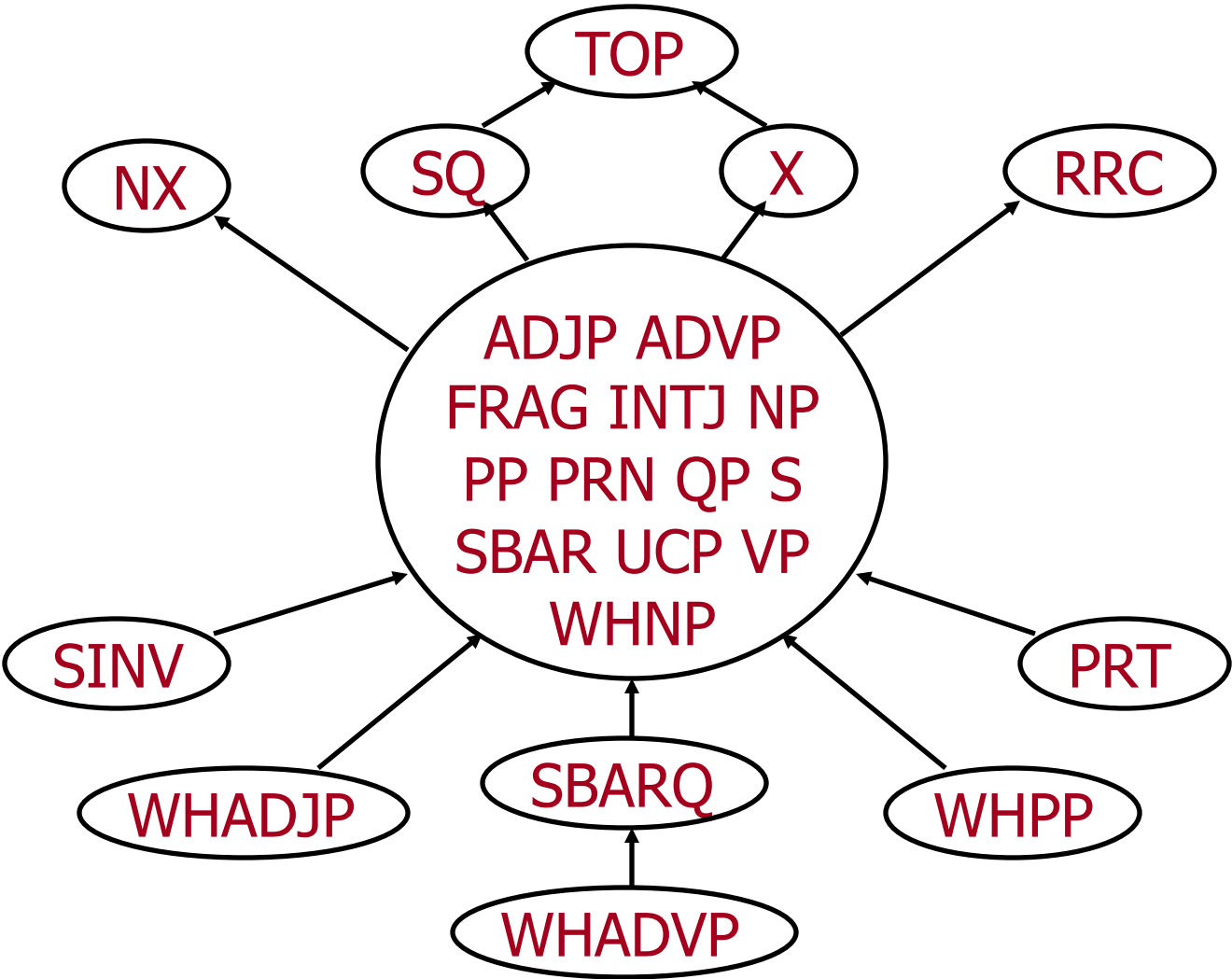


# Unary rules: alchemy in the land of treebanks





# Same-Span Reachability



NoEmpties

- LST
- CONJP
- NAC

[illegible]

# Restricting the grammar form for efficient parsing



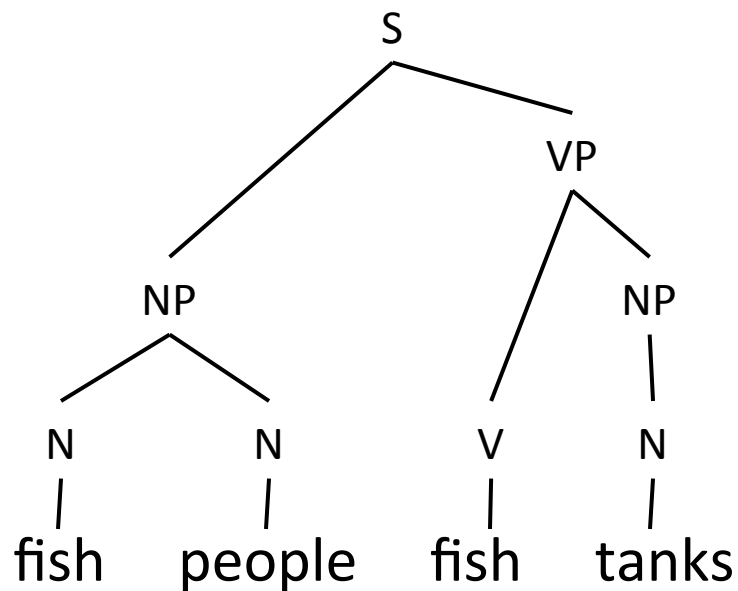


# CKY Parsing

# Exact polynomial time parsing of (P)CFGs



# Constituency Parsing



## PCFG

### Rule Prob $\theta_i$

$S \rightarrow NP VP$   $\theta_0$

$NP \rightarrow NP NP$   $\theta_1$

...

$N \rightarrow \text{fish}$   $\theta_{42}$

$N \rightarrow \text{people}$   $\theta_{43}$

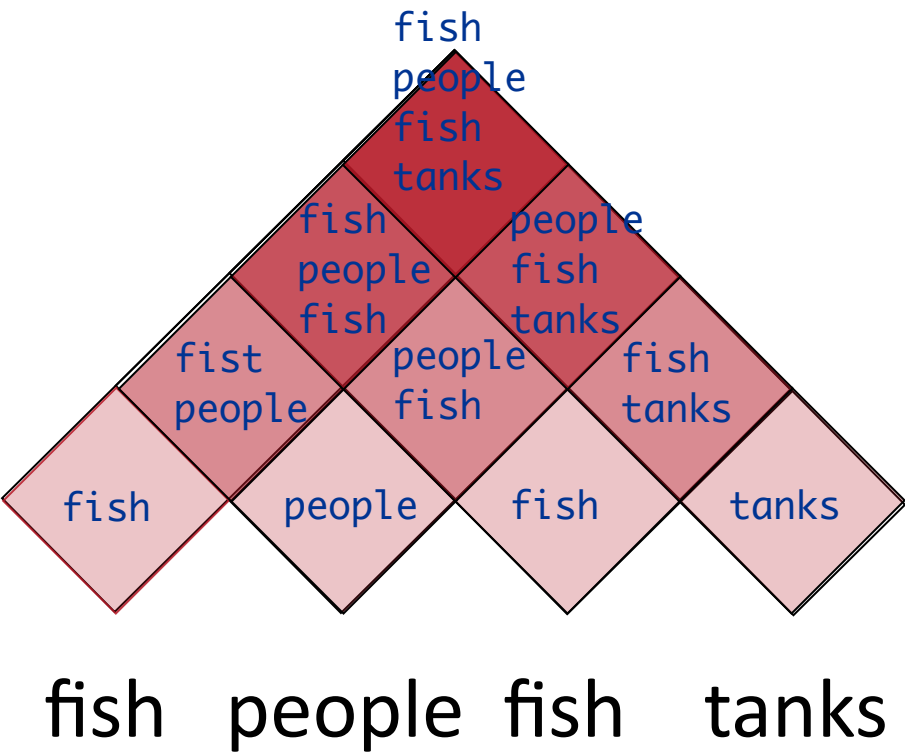
$V \rightarrow \text{fish}$   $\theta_{44}$

...



# Cocke-Kasami-Younger (CKY) Constituency Parsing

Parse Triangle / Chart





# Viterbi (Max) Scores

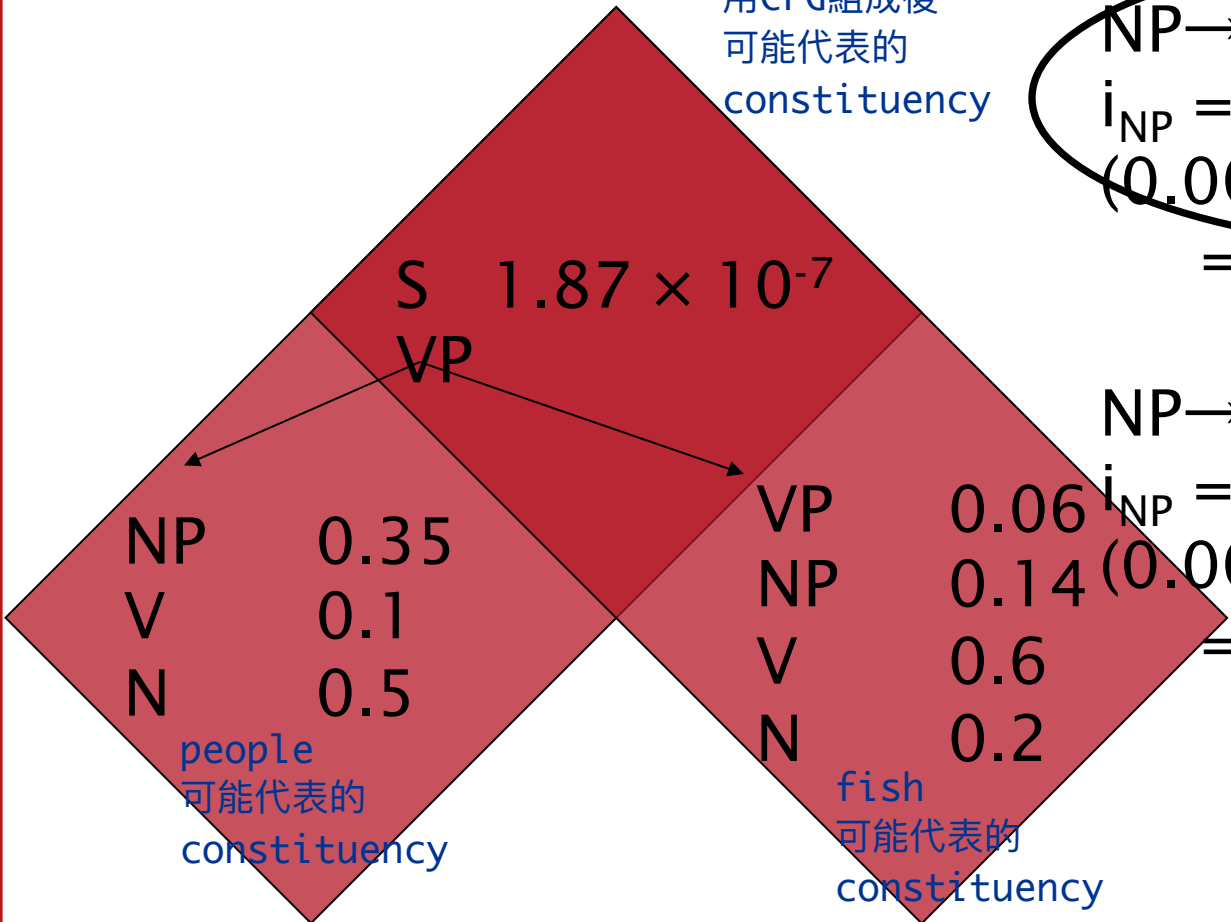
people fish  
用CFG組成後  
可能代表的  
constituency

NP→NN NNS0.13

$$i_{NP} = (0.13)(0.0023)$$
$$(0.0014)$$
$$= 1.87 \times 10^{-7}$$

NP→NNP NNS0.056

$$i_{NP} = (0.056)(0.001)$$
$$(0.0014)$$
$$= 7.84 \times 10^{-8}$$



people  
可能代表的  
constituency

fish  
可能代表的  
constituency

people

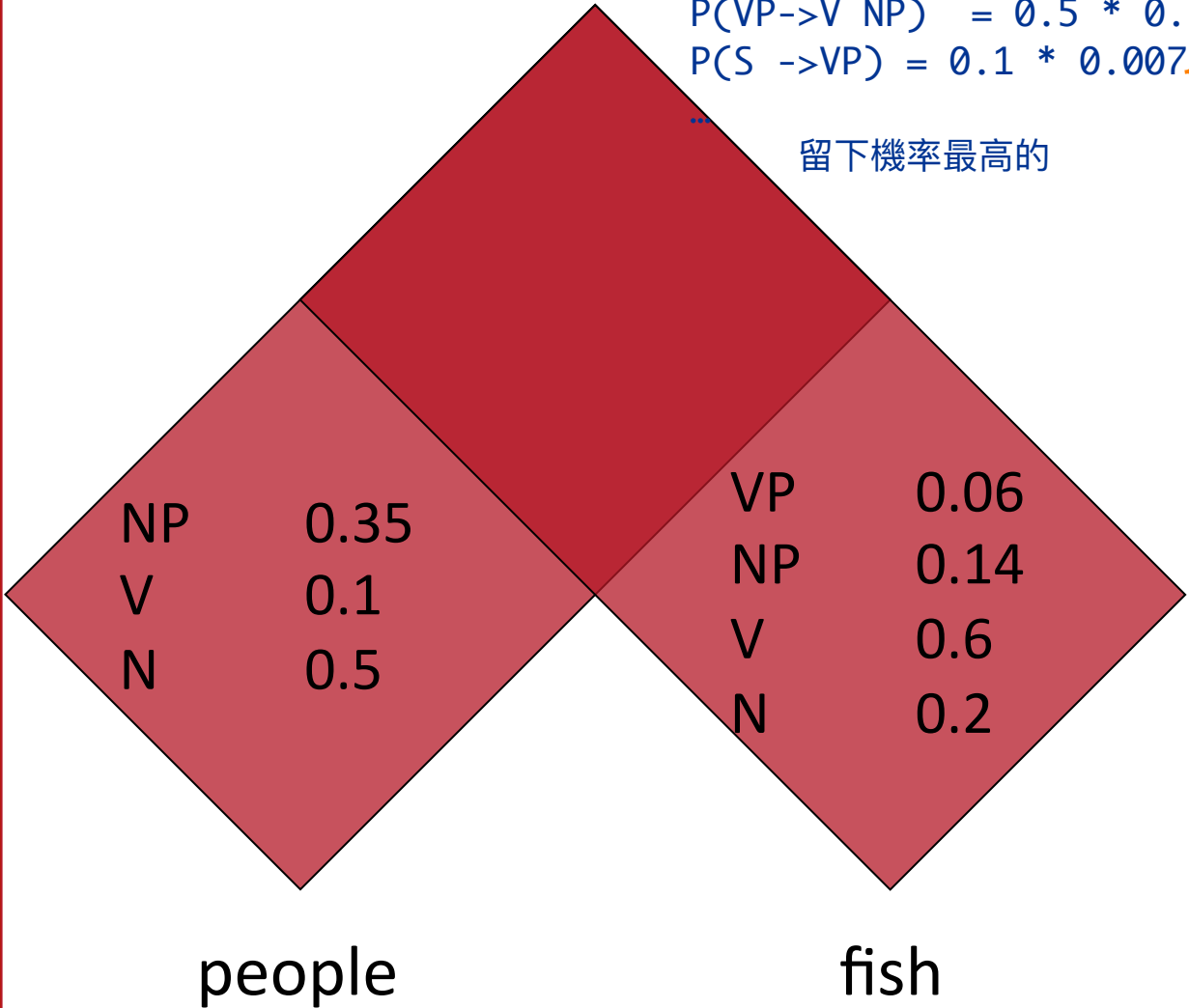
fish



# Viterbi (Max) Scores

$P(NP \rightarrow NP \ NP) = 0.1 * 0.35 * 0.14$   
 $P(VP \rightarrow V \ NP) = 0.5 * 0.1 * 0.14 = 0.007$   
 $P(S \rightarrow VP) = 0.1 * 0.007$

留下機率最高的



S → NP VP	0.9
S → VP	0.1
VP → V NP	0.5
VP → V	0.1
VP → V @VP_V	0.3
VP → V PP	0.1
@VP_V → NP PP	1.0
NP → NP NP	0.1
NP → NP PP	0.2
NP → N	0.7
PP → P NP	1.0



# Extended CKY parsing

- Unaries can be incorporated into the algorithm
  - Messy, but doesn't increase algorithmic complexity
- Empties can be incorporated
  - Use fenceposts 把s加到句子最前面，所以那張Chart的邊長就會是n+1
  - Doesn't increase complexity; essentially like unaries
- Binarization is *vital*
  - Without binarization, you don't get parsing cubic in the length of the sentence and in the number of nonterminals in the grammar
    - Binarization may be an explicit transformation or implicit in how the parser works (Early-style dotted rules), but it's always there.



# The CKY algorithm (1960/1965) ... extended to unaries

```
function CKY(words, grammar) returns [most_probable_parse, prob]
  score = new double[#(words)+1][#(words)+1][#(nonterms)]
  back = new Pair[#(words)+1][#(words)+1][#(nonterms)]
  for i=0; i<#(words); i++
    for A in nonterms
      if A -> words[i] in grammar
        score[i][i+1][A] = P(A -> words[i])
      //handle unaries
      boolean added = true
      while added
        added = false
        for A, B in nonterms
          if score[i][i+1][B] > 0 && A->B in grammar
            prob = P(A->B)*score[i][i+1][B]
            if prob > score[i][i+1][A]
              score[i][i+1][A] = prob
              back[i][i+1][A] = B
              added = true
```

*A -> terminal 0.3*

*unary rules*



# The CKY algorithm (1960/1965)

## ... extended to unaries

```

for span = 2 to #(words) start from length >= 2
  for begin = 0 to #(words)- span 從左到右
    end = begin + span
    for split = begin+1 to end-1
      for A,B,C in nonterms
        prob=score[begin][split][B]*score[split][end][C]*P(A->BC)
        if prob > score[begin][end][A]
          score[begin][end][A] = prob
          back[begin][end][A] = new Triple(split,B,C)
        //handle unaries
        boolean added = true
        while added
          added = false
          for A, B in nonterms
            prob = P(A->B)*score[begin][end][B];
            if prob > score[begin][end][A]
              score[begin][end][A] = prob
              back[begin][end][A] = B
              added = true
        return buildTree(score, back)

```

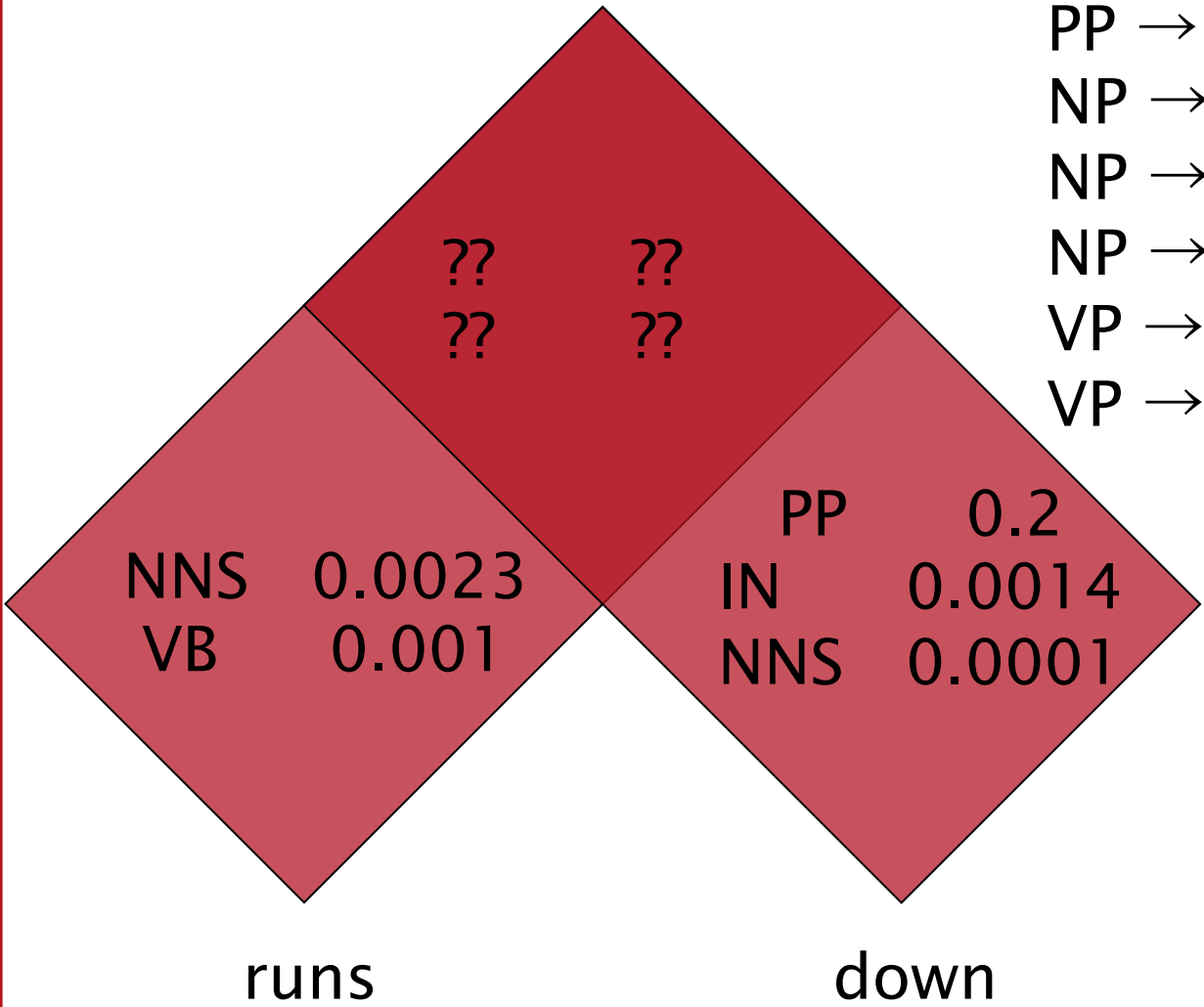
*binary rules*

*unaries rules*





# Quiz Question!



PP → IN	0.002
NP → NNS NNS	0.01
NP → NNS NP	0.005
NP → NNS PP	0.01
VP → VB PP	0.045
VP → VB NP	0.015

What constituents (with what probability can you make?



# CKY Parsing

# Exact polynomial time parsing of (P)CFGs



# CKY Parsing

## A worked example



# The grammar: Binary, no epsilons,

$S \rightarrow NP VP$	0.9
$S \rightarrow VP$	0.1
$VP \rightarrow V NP$	0.5
$VP \rightarrow V$	0.1
$VP \rightarrow V @VP\_V$	0.3
$VP \rightarrow V PP$	0.1
$@VP\_V \rightarrow NP PP$	1.0
$NP \rightarrow NP NP$	0.1
$NP \rightarrow NP PP$	0.2
$NP \rightarrow N$	0.7
$PP \rightarrow P NP$	1.0

$N \rightarrow \textit{people}$	0.5
$N \rightarrow \textit{fish}$	0.2
$N \rightarrow \textit{tanks}$	0.2
$N \rightarrow \textit{rods}$	0.1
$V \rightarrow \textit{people}$	0.1
$V \rightarrow \textit{fish}$	0.6
$V \rightarrow \textit{tanks}$	0.3
$P \rightarrow \textit{with}$	1.0

	fish	1	people	2	fish	3	tanks	4
0	score[0][1]	score[0][2]	score[0][3]	score[0][4]				
1		score[1][2]	score[1][3]	score[1][4]				
2			score[2][3]	score[2][4]				
3				score[3][4]				
4								

$S \rightarrow NP VP$	0.9
$S \rightarrow VP$	0.1
$VP \rightarrow V NP$	0.5
$VP \rightarrow V$	0.1
$VP \rightarrow V @VP\_V$	0.3
$VP \rightarrow V PP$	0.1
$@VP\_V \rightarrow NP PP$	1.0
$NP \rightarrow NP NP$	0.1
$NP \rightarrow NP PP$	0.2
$NP \rightarrow N$	0.7
$PP \rightarrow P NP$	1.0
$N \rightarrow people$	0.5
$N \rightarrow fish$	0.2
$N \rightarrow tanks$	0.2
$N \rightarrow rods$	0.1
$V \rightarrow people$	0.1
$V \rightarrow fish$	0.6
$V \rightarrow tanks$	0.3
$P \rightarrow with$	1.0

	fish	1	people	2	fish	3	tanks	4
0	fish可能是N或V  N->fish 0.2 V->fish 0.6							
1								
2								
3								
4								

總之長度為一的  
直接查表填入就可以了

```

for i=0; i<#(words); i++
  for A in nonterms
    if A -> words[i] in grammar
      score[i][i+1][A] = P(A -> words[i]);
  
```

$S \rightarrow NP VP$	0.9
$S \rightarrow VP$	0.1
$VP \rightarrow V NP$	0.5
$VP \rightarrow V$	0.1
$VP \rightarrow V @VP\_V$	0.3
$VP \rightarrow V PP$	0.1
$@VP\_V \rightarrow NP PP$	1.0
$NP \rightarrow NP NP$	0.1
$NP \rightarrow NP PP$	0.2
$NP \rightarrow N$	0.7
$PP \rightarrow P NP$	1.0
$N \rightarrow people$	0.5
$N \rightarrow fish$	0.2
$N \rightarrow tanks$	0.2
$N \rightarrow rods$	0.1
$V \rightarrow people$	0.1
$V \rightarrow fish$	0.6
$V \rightarrow tanks$	0.3
$P \rightarrow with$	1.0

	fish	1	people	2	fish	3	tanks	4
0	$N \rightarrow fish\ 0.2$ $V \rightarrow fish\ 0.6$ $S \rightarrow VP = 0.006$ $VP \rightarrow V = 0.06$ $NP \rightarrow N = 0.14$							
1		$N \rightarrow people\ 0.5$ $V \rightarrow people\ 0.1$						
2				$N \rightarrow fish\ 0.2$ $V \rightarrow fish\ 0.6$				
						$N \rightarrow tanks\ 0.2$ $V \rightarrow tanks\ 0.1$		

對於長度為1的  
遞迴向前改寫所有grammar  
rules的可能

```
// handle unaries
boolean added = true
while added
    added = false
    for A, B in nonterms
        if score[i][i+1][B] > 0 && A->B in grammar
            prob = P(A->B)*score[i][i+1][B]
            if(prob > score[i][i+1][A])
                score[i][i+1][A] = prob
                back[i][i+1][A] = B
            added = true
```

**S → NP VP**

S → VP

**VP → V NP**

VP → V

VP → V @VP\_V

VP → V PP

@VP\_V → NP PP

**NP → NP NP**

NP → NP PP

NP → N

PP → P NP

N → *people*

N → *fish*

N → *tanks*

N → *rods*

V → *people*

V → *fish*

V → *tanks*

P → *with*

**0.9**

0.1

**0.5**

0.1

0.3

0.1

1.0

**0.1**

0.2

0.7

1.0

0.5

0.2

0.2

0.1

0.1

0.6

0.3

1.0

	fish	1	people	2	fish	3	tanks	4
0								
1	N → fish 0.2 V → fish 0.6 <b>NP → N 0.14</b> VP → V 0.06 S → VP 0.006		S → NP VP = 0.9*0.14*0.01 VP → V NP = 0.5*0.6*0.35 NP → NP NP = 0.1*0.14*0.35					
2			N → people 0.5 V → people 0.1 NP → N 0.35 <b>VP → V 0.01</b> S → VP 0.001					
3				N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006				
4							N → tanks 0.2 V → tanks 0.1 NP → N 0.14 VP → V 0.03 S → VP 0.003	

fish和people

都有可能是

N, V, NP, VP, S找出所有可能  
向前改寫的排列組合，並算機率

prob=score[begin][split][B]\*score[split][end][C]\*P(A->BC)  
if (prob > score[begin][end][A])  
score[begin][end][A] = prob  
back[begin][end][A] = new Triple(split,B,C)



$S \rightarrow NP VP$	0.9
$S \rightarrow VP$	0.1
$VP \rightarrow V NP$	0.5
$VP \rightarrow V$	0.1
$VP \rightarrow V @VP\_V$	0.3
$VP \rightarrow V PP$	0.1
$@VP\_V \rightarrow NP PP$	1.0
$NP \rightarrow NP NP$	0.1
$NP \rightarrow NP PP$	0.2
$NP \rightarrow N$	0.7
$PP \rightarrow P NP$	1.0
$N \rightarrow people$	0.5
$N \rightarrow fish$	0.2
$N \rightarrow tanks$	0.2
$N \rightarrow rods$	0.1
$V \rightarrow people$	0.1
$V \rightarrow fish$	0.6
$V \rightarrow tanks$	0.3
$P \rightarrow with$	1.0

	fish	1	people	2	fish	3	tanks	4
0	<div>N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006</div>	<div>NP → NP NP 0.0049 VP → V NP 0.105 S → NP VP 0.00126</div>						
1			<div>N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 S → VP 0.001</div>	<div>NP → NP NP 0.0049 VP → V NP 0.007 S → NP VP 0.0189</div>				
2				<div>N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006</div>	<div>NP → NP NP 0.00196 VP → V NP 0.042 S → NP VP 0.00378</div>			
3						<div>N → tanks 0.2 V → tanks 0.1 NP → N 0.14 VP → V 0.03 S → VP 0.003</div>		
4								

//handle unaries  
boolean added = true  
while added  
  added = false  
  for A, B in nonterms  
    prob = P(A->B)\*score[begin][end][B];  
    if prob > score[begin][end][A]  
      score[begin][end][A] = prob  
      back[begin][end][A] = B  
      added = true

S->VP  
=0.105\*0.1>0.00126  
所以把原本的S->NP VP换掉

Diagram illustrating the dynamic programming algorithm for parsing the sentence "fish people fish tanks" using a grammar with rules like  $S \rightarrow NP VP$ ,  $NP \rightarrow N NP$ ,  $V \rightarrow fish$ , etc.

The diagram shows a 4x4 grid of subproblems. The top-left cell (0,0) contains the full sentence. The top-right cell (0,3) contains the subproblem "fish people fish". The bottom-left cell (3,0) contains the subproblem "fish people fish". The bottom-right cell (3,3) contains the subproblem "fish people fish".

The diagram highlights two possible parse trees for the sentence, one in red and one in green, and shows how the algorithm computes the probability of each parse tree by combining the probabilities of its subproblems.

for split = begin+1 to end-1  
 for A,B,C in nonterms  
 prob=score[begin][split][B]\*score[split][end][C]\*P(A->BC)  
 if prob > score[begin][end][A]  
 score[begin][end][A] = prob  
 back[begin][end][A] = new Triple(split,B,C)

$S \rightarrow NP VP$	0.9
$S \rightarrow VP$	0.1
$VP \rightarrow V NP$	0.5
$VP \rightarrow V$	0.1
$VP \rightarrow V @VP\_V$	0.3
$VP \rightarrow V PP$	0.1
$@VP\_V \rightarrow NP PP$	1.0
$NP \rightarrow NP NP$	0.1
$NP \rightarrow NP PP$	0.2
$NP \rightarrow N$	0.7
$PP \rightarrow P NP$	1.0
$N \rightarrow people$	0.5
$N \rightarrow fish$	0.2
$N \rightarrow tanks$	0.2
$N \rightarrow rods$	0.1
$V \rightarrow people$	0.1
$V \rightarrow fish$	0.6
$V \rightarrow tanks$	0.3
$P \rightarrow with$	1.0

	fish	1	people	2	fish	3	tanks	4
0	$N \rightarrow fish$ 0.2 $V \rightarrow fish$ 0.6 $NP \rightarrow N$ 0.14 $VP \rightarrow V$ 0.06 $S \rightarrow VP$ 0.006	$NP \rightarrow NP NP$ 0.0049 $VP \rightarrow V NP$ 0.105 $S \rightarrow VP$ 0.0105	$NP \rightarrow NP NP$ 0.0000686 $VP \rightarrow V NP$ 0.00147 $S \rightarrow NP VP$ 0.000882					
1		$N \rightarrow people$ 0.5 $V \rightarrow people$ 0.1 $NP \rightarrow N$ 0.35 $VP \rightarrow V$ 0.01 $S \rightarrow VP$ 0.001	$NP \rightarrow NP NP$ 0.0049 $VP \rightarrow V NP$ 0.007 $S \rightarrow NP VP$ 0.0189					
2				$N \rightarrow fish$ 0.2 $V \rightarrow fish$ 0.6 $NP \rightarrow N$ 0.14 $VP \rightarrow V$ 0.06 $S \rightarrow VP$ 0.006	$NP \rightarrow NP NP$ 0.00196 $VP \rightarrow V NP$ 0.042 $S \rightarrow VP$ 0.0042			
3						$N \rightarrow tanks$ 0.2 $V \rightarrow tanks$ 0.1 $NP \rightarrow N$ 0.14 $VP \rightarrow V$ 0.03 $S \rightarrow VP$ 0.003		
4								

這邊要像49頁一樣  
 找出unaries  
 然後看看能不能取代原本的  
 例如 $S \rightarrow VP =$   
 $0.1 * 0.00147 < 0.000882$   
 無法取代

```

for split = begin+1 to end-1
  for A,B,C in nonterms
    prob=score[begin][split][B]*score[split][end][C]*P(A->BC)
    if prob > score[begin][end][A]
      score[begin][end][A] = prob
      back[begin][end][A] = new Triple(split,B,C)
  
```

**S → VP** **0.1**

0	N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006	NP → NP NP 0.0049 VP → V NP 0.105 S → VP 0.0105	NP → NP NP 0.0000686 VP → V NP 0.00147 S → NP VP 0.000882	
1		N → people 0.5 V → people 0.1 NP → N 0.35 VP → V 0.01 S → VP 0.001	NP → NP NP 0.0049 VP → V NP 0.007 S → NP VP 0.0189	NP → NP NP 0.0000686 VP → V NP 0.000098 S → NP VP 0.01323
2	這邊要像49頁一樣 找出unaries 然後看看能不能取代原本的 例如S→VP = 0.1*0.000098<0.000882 無法取代			
3		N → fish 0.2 V → fish 0.6 NP → N 0.14 VP → V 0.06 S → VP 0.006	NP → NP NP 0.00196 VP → V NP 0.042 S → VP 0.0042	
4	for split = begin+1 to end-1 for A,B,C in nonterms prob=score[begin][split][B]*score[split][end][C]*P(A->BC) if prob > score[begin][end][A] score[begin][end][A] = prob back[begin][end][A] = new Triple(split,B,C)			
			N → tanks 0.2 V → tanks 0.1 NP → N 0.14 VP → V 0.03 S → VP 0.003	

$S \rightarrow NP VP$	0.9
$S \rightarrow VP$	0.1
$VP \rightarrow V NP$	0.5
$VP \rightarrow V$	0.1
$VP \rightarrow V @VP\_V$	0.3
$VP \rightarrow V PP$	0.1
$@VP\_V \rightarrow NP PP$	1.0
$NP \rightarrow NP NP$	0.1
$NP \rightarrow NP PP$	0.2
$NP \rightarrow N$	0.7
$PP \rightarrow P NP$	1.0
$N \rightarrow people$	0.5
$N \rightarrow fish$	0.2
$N \rightarrow tanks$	0.2
$N \rightarrow rods$	0.1
$V \rightarrow people$	0.1
$V \rightarrow fish$	0.6
$V \rightarrow tanks$	0.3
$P \rightarrow with$	1.0

	fish	1	people	2	fish	3	tanks	4
0	$N \rightarrow fish$ 0.2 $V \rightarrow fish$ 0.6 $NP \rightarrow N$ 0.14 $VP \rightarrow V$ 0.06 $S \rightarrow VP$ 0.006		$NP \rightarrow NP NP$ 0.0049 $VP \rightarrow V NP$ 0.105 $S \rightarrow VP$ 0.0105		$NP \rightarrow NP NP$ 0.0000686 $VP \rightarrow V NP$ 0.00147 $S \rightarrow NP VP$ 0.000882		$NP \rightarrow NP NP$ 0.0000009604 $VP \rightarrow V NP$ 0.00002058 $S \rightarrow NP VP$ 0.00018522	
1			$N \rightarrow people$ 0.5 $V \rightarrow people$ 0.1 $NP \rightarrow N$ 0.35 $VP \rightarrow V$ 0.01 $S \rightarrow VP$ 0.001		$NP \rightarrow NP NP$ 0.0049 $VP \rightarrow V NP$ 0.007 $S \rightarrow NP VP$ 0.0189		$NP \rightarrow NP NP$ 0.0000686 $VP \rightarrow V NP$ 0.000098 $S \rightarrow NP VP$ 0.01323	
2					$N \rightarrow fish$ 0.2 $V \rightarrow fish$ 0.6 $NP \rightarrow N$ 0.14 $VP \rightarrow V$ 0.06 $S \rightarrow VP$ 0.006		$NP \rightarrow NP NP$ 0.00196 $VP \rightarrow V NP$ 0.042 $S \rightarrow VP$ 0.0042	
3							$N \rightarrow tanks$ 0.2 $V \rightarrow tanks$ 0.1 $NP \rightarrow N$ 0.14 $VP \rightarrow V$ 0.03 $S \rightarrow VP$ 0.003	

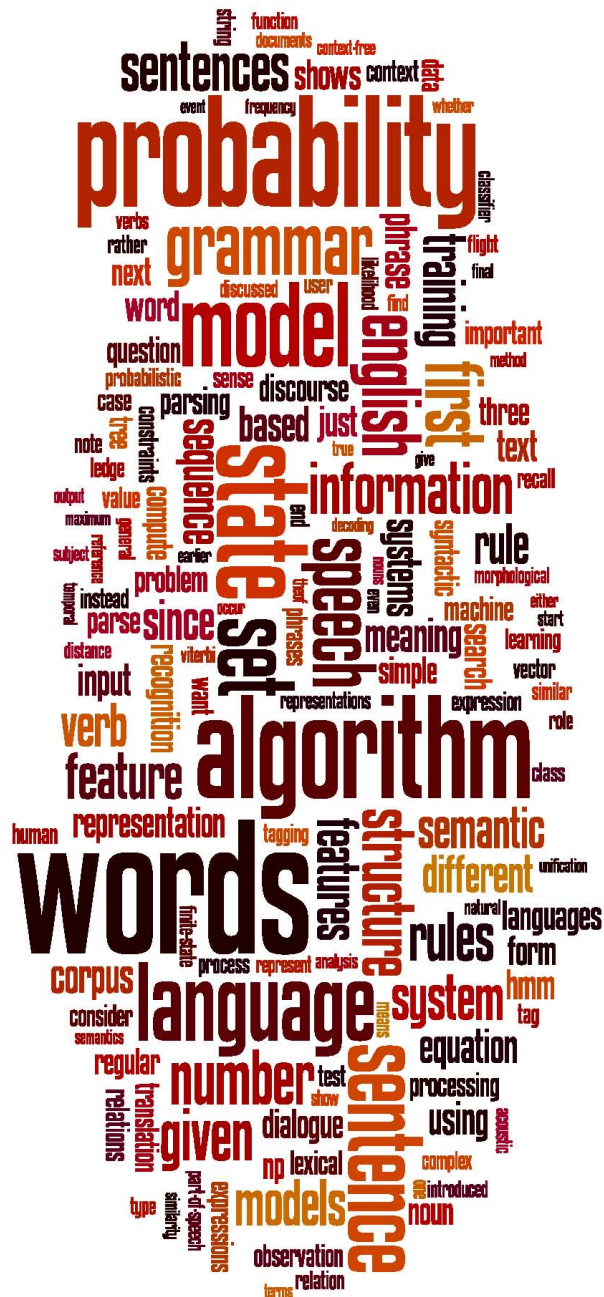
- 2 這邊就像50頁那樣  
fish people fish tanks  
有以下三種解讀方法  
[fish people fish] [tanks] (紅框)  
[fish people] [fish tanks] (綠框)  
[fish] [people fish tanks] (紫框)
- 3 一樣列出所有排列組合後，把機率最高的填入  
最後得到這句話的最佳排列为S->NP VP  
問題來了，TREE呢？？？  
哪一塊代表NP，哪一塊代表VP？  
所以要呼叫最後一行的buildTree，進行backtracking
- 4 就能找到路徑，畫出最終的樹

Call buildTree(score, back) to get the best parse



# CKY Parsing

## A worked example



# Constituency Parser Evaluation

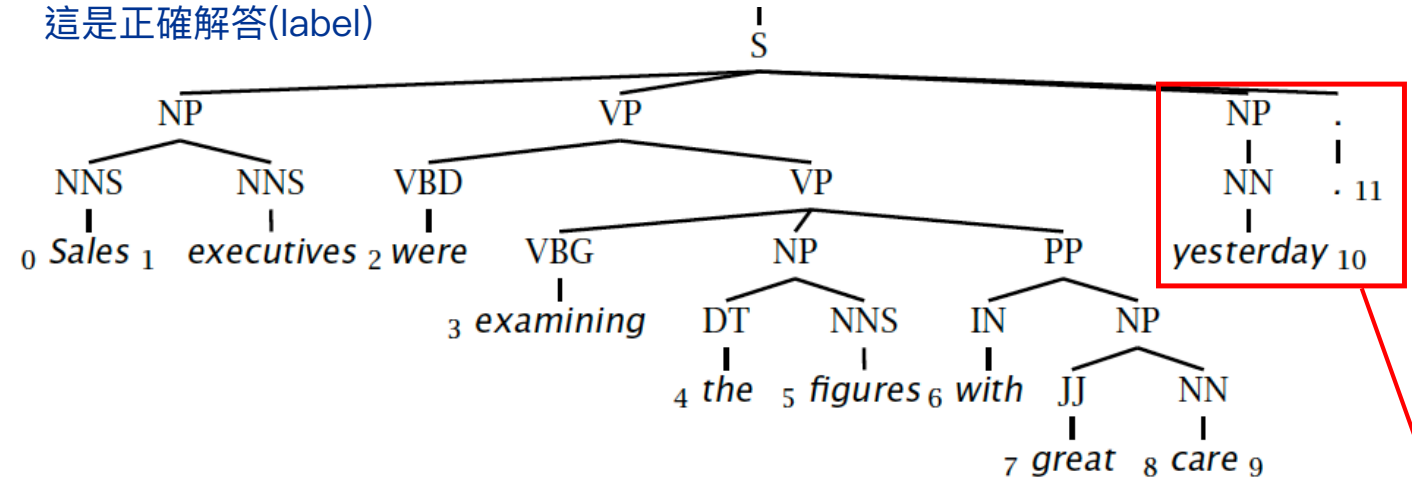




# Evaluating constituency parsing

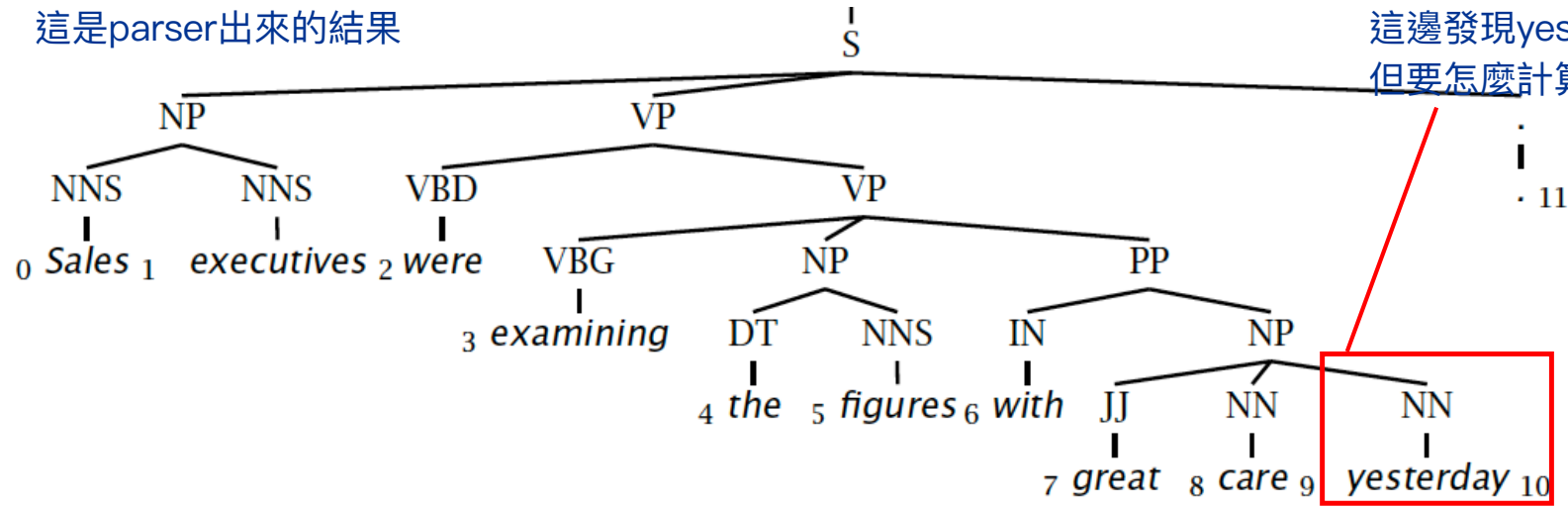
Gold standard brackets: S-(0:11), NP-(0:2), VP-(2:9), VP-(3:9), NP-(4:6), PP-(6:9), NP-(7,9), NP-(9:10)

這是正確解答(label)



Candidate brackets: S-(0:11), NP-(0:2), VP-(2:10), VP-(3:10), NP-(4:6), PP-(6:10), NP-(7,10)

這是parser出來的結果



這邊發現yesterday錯了  
但要怎麼計算他的error





# Evaluating constituency parsing

## Gold standard brackets:

S-(0:11), NP-(0:2), VP-(2:9), VP-(3:9), **NP-(4:6)**, PP-(6-9), NP-(7,9), NP-(9:10)

## Candidate brackets:

S-(0:11), NP-(0:2), VP-(2:10), VP-(3:10), **NP-(4:6)**, PP-(6-10), NP-(7,10)

Labeled Precision	$3/7 = 42.9\%$	找到的7個有3個是全對的
Labeled Recall	$3/8 = 37.5\%$	正解的8個有3個被找到了
LP/LR F1	40.0%	
Tagging Accuracy	$11/11 = 100.0\%$	總共11個字各自都標對了

缺點：像這個例子只錯了一個yesterday  
但因為會導致其他tag的位置錯很多  
所以效能會比預期低很多



# How good are PCFGs?

- Penn WSJ parsing accuracy: about 73% LP/LR F1
- Robust
  - Usually admit everything, but with low probability
- Partial solution for grammar ambiguity
  - A PCFG gives some idea of the plausibility of a parse
  - But not so good because the independence assumptions are too strong
- Give a probabilistic language model
  - But in the simple case it performs worse than a trigram model
- The problem seems to be that PCFGs lack the lexicalization of a trigram model



# Constituency Parser Evaluation