

Theorem) $g_n = F_{n+2} - 1$

Base Case

$$g_0 = 0$$

$$F_2 - 1 = 1 - 1 = 0$$

Induction Case

$$\begin{aligned} g_{n+1} &= F_{(n+2)+1} - 1 = F_{n+3} - 1 = (F_{n+1}) + (F_{n+2} - 1) \\ &= F_{n+1} + g_n \quad (\text{by IH}) \end{aligned}$$

$$g_{n+1} = g_n + F_{n+1}$$

□