Discovering Data-Driven Nudges to Help Students Learn More Math

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Introduction

The decline in math performance among American students has been a critical issue exacerbated by the COVID-19 pandemic, with some reporting an alarming half-year lag in math achievement among U.S. public school students in grades 3-8 (Fahle et al. 2023). Learning losses and disparities, particularly among younger students, have been significant due to reduced instructional time and remote learning challenges (Zierer 2021; Di Pietro 2023). Moreover, Ewing and Green (2021) note that repeated school closures compounded these issues, emphasizing that the pandemic is not the only problem affecting math performance. Historical trends have also played a role, indicating a need for comprehensive solutions.

Emphasizing quality math instruction is paramount for improving math performance. Teacher proficiency in pedagogical strategies, guidance, and teacher-student relations is crucial, especially for students who have lost interest in mathematics or lack a sufficient foundation (Wang et al. 2023; Battey et al. 2016). The correlation between teachers' deep mathematical knowledge and student success highlights the importance of specialized training (Hill et al. 2008; Battey et al. 2016).

The pandemic also accelerated the adoption of digital platforms for math education. Indeed, adaptive practice software can mitigate the impact of school closures and possibly reverse their adverse effects (Meeter 2021; Alabdulaziz 2021). Recent meta-analyses demonstrate that integrating digital tools and blended learning approaches improves student outcomes significantly (Ran, Kim, and Secada 2021; Ran, Kasli, and Secada 2020; Sadaf, Wu, and Martin 2021). Leveraging these tools and insights gained during the pandemic should help address longstanding educational challenges (Ewing and Green 2021). In particular, integrating technology, pedagogy, and content knowledge is essential, and professional development programs are most effective when they focus on using technology to foster a more engaging and effective learning environment (Young 2016; Blanchard et al. 2016).

The rise of digital platforms has also underlined the importance of student engagement in online learning. Blending online and traditional teaching methods effectively enhances engagement and understanding (Chiang, Lin, and Tseng 2016; Sadaf, Wu, and Martin 2021). Teachers are essential in helping students develop meta-cognitive skills to enhance student engagement (Haleva, Hershkovitz, and Tabach 2020). Furthermore, teachers' beliefs and self-efficacy toward technology integration influence their willingness to adopt innovative teaching practices (Ertmer et al. 2012; Liljedahl and Oesterle 2020). This relationship between student engagement and teacher attitudes highlights the importance of providing cost-effective, high-quality solutions that can be implemented across diverse educational settings.

In this study, we partner with Zearn to address these topics. This math education platform reaches approximately 25% of United States elementary schools and over a million middle-school students. Zearn's approach integrates interactive digital lessons with hands-on teaching, aligning with the Common Core State Standards and providing a comprehensive educational experience ("Zearn Math: Top-Rated Math Learning Platform" 2023).

Our study leverages this rich resource to offer an innovative approach to educational interventions using behavioral science principles. Focusing on teacher quality, we align with Hanushek (2020), who maintains that the efficacy of resource utilization supersedes quantity. We also follow current trends in providing cost-effective, easy-to-implement interventions that align engagement incentives (Lavecchia, Liu, and Oreopoulos 2016; Koch, Nafziger, and Nielsen 2015).

Our two-step approach initially employs unsupervised learning techniques to analyze behavioral patterns in teacher activity on Zearn, aligning with the data mining value in educational research (Salazar, Serrano, and Vergara 2007; Hershcovits, Vilenchik, and Gal 2020; Qiu et al. 2022; Al-Shabandar et al. 2018; Shin and Shim 2020). Subsequently, we aim to establish the causality of our interventions through a large-scale field experiment guided by recommendations from Greene (2022) for holistic, transparent, and reproducible research. Our unique integration of behavioral science with digital education seeks to provide impactful insights into math education and offer a blueprint for similar studies in other fields.

Results

Study 1: Data-Driven Nudge Engineering

Our study used a comprehensive dataset from Zearn's educational platform, encompassing the 2019-2020 academic year and spanning multiple schools in Louisiana. Zearn's approach combines concrete, pictorial, and abstract methods for teaching mathematics, offering a personalized experience that allows teachers to track student progress effectively. Key features like the Badge system, which tracks student lesson completion, and Tower Alerts, which notifies teachers when students repeatedly struggle with a given concept, motivate students and provide valuable insights for educators.

Key variables from the dataset included teacher logins, file downloads, and specific interactions with educational content. Additionally, we accessed alongside student data at the classroom-week level, encompassing metrics such as lesson completion (i.e., Badges) and instances of learning difficulties (i.e., Tower Alerts). This granularity allowed for an in-depth analysis of both teacher behaviors and student performance.

The data revealed diverse engagement patterns. Teachers logged into Zearn multiple times weekly, with notable variations in interaction frequency and duration. Student data reflected a broad spectrum of performance levels across different classrooms and schools. The standard deviations of key variables underscored this diversity, as seen in Table 1, which displays comprehensive summary statistics. This rich combination of teacher behaviors and student performance metrics, carefully matched with a weekly frequency for each classroom, allowed for a thorough analysis while upholding privacy standards.

Table 1: Pre-experimental Intervention Data. This table summarizes key educational metrics for Zearn teachers. The data was collected from July 2019 to June 2020.

		Aggregated Statistics per Teacher				
	N	Mean	Standard Deviation	1st Quartile	Median	3rd Quartile
# of teachers	4,698	-	-	-	-	-
# of classes	7,788	1.66	1.05	1.0	1.0	2.0
# of badges	7,263,099	1,546.00	1,608.52	366.0	1,089.8	2,144.4
Total minutes on Zearn	2,944,282	133.52	669.98	3.1	14.3	64.1

Study 1a

Zearn offers a variety of student and teacher activities, which, in our data, can be simplified through dimensionality reduction techniques, enhancing interpretability. We employed Independent Component Analysis (ICA) on teacher behavioral variables due to their non-Gaussian nature. This statistical approach allowed us to uncover latent variables that may have been obscured with traditional methods. These components are weighted vectors of specific teacher activities and were estimated to maximize statistical independence in the data-generating process. Our findings revealed three pivotal independent components, as indicated by the 'elbow' of Figure 1, which represents the optimal number of components that best portray the underlying dimensions of teacher behavior. (For more details, please refer to the SI section.)

The significant finding from the ICA was the prominence of IC1, accounting for 14.1498735 14.12% of the variance in teacher Zearn activity, as indicated in Figure 2. This result is significant in nudge engineering, highlighting the need to focus interventions on elements encapsulated by IC1. Conversely, IC2 and IC3, with 11.5189174 7.27% and 6.0842 6.56% variance explained, play lesser but still noteworthy roles.

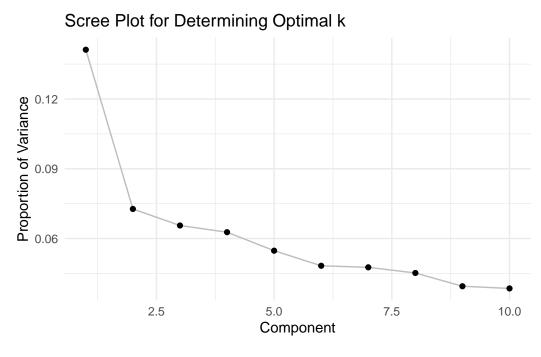
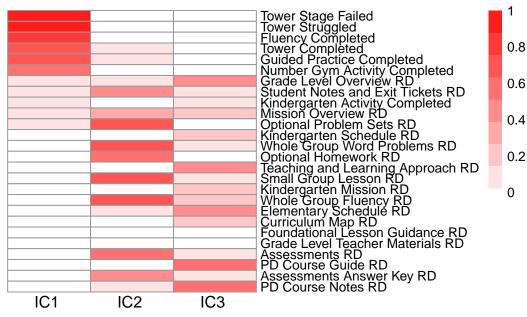


Figure 1: Scree Plot for Determining Optimal Number of Components. This plot displays the proportion of variance explained by each independent component (IC). The optimal number of components is indicated by the 'elbow' of the plot, where the variance explained by each additional component is minimal.



14.1% of vaf.1.5% of var6.1% of var.

Figure 2: Independent Component Analysis (ICA) Results. The heatmap displays teacher actions in each row, while the columns represent the three independent components (IC1, IC2, IC3) that explained the most variance in the teacher behavioral data. The color gradient on the heatmap indicates the relative importance of each activity within these components. Note that these metrics pertain to teacher activity on the platform, not student actions.

The activities with the highest weighting in IC1 were measures associated with Towers (Struggled, Failed, and Completed) in conjunction with other completed student activities such as Fluency, Guided Practice, and Number Gym Activity. Notably, these metrics pertain to teacher logins, not student actions, implying a direct link between teacher engagement and student performance, suggesting an educational "scaffolding" effect.

Further examination revealed an empathy-driven process behind this trend. In this context, "empathy" refers to teachers' ability to comprehend and engage with students' challenges, as demonstrated by their focused attention on areas where students struggled or succeeded. The weightings in IC1 for activities like Tower Struggles (0.8938868), Fluency Completed (0.7779199), Guided Practice Completed (0.7300542), and Number Gym Activity Completed (0.6083755) were significantly higher compared to other activities. This pattern underlines the empathy-driven process in teaching, where teachers' engagement with the platform is focused on understanding student challenges.

The second component, IC2, showed substantial weightings on variety of Resource Downloads (RD), particularly Small Group Lessons (0.7172712) and Whole Group (i.e., the entire class) Word Problems (0.6443804). We labeled this component as "Groups." IC3, with strong weights on Resource Download (RD) for the Professional Development (PD) course guide (0.6198419) and course notes (0.6127954), was labeled "Guidance." This component, accounting for a smaller variance (6.0842%), was harder to interpret due to its complexity and diverse activities.

Building upon these findings, we advanced to a censored (lower bound = 0) panel regression model that accounted for various temporal and subject-specific variables and the potential impact of teachers handling multiple classes. This model applied a first-difference approach with a lower bound of zero for the dependent variable. The regression formula incorporated changes in independent components (IC1, IC2, IC3) as predictors for the change in the logarithm of badges (+1), accounting for individual teachers, classes, and weeks. As presented in **?@tbl-pldv**, the regression highlighted a strong positive contemporaneous correlation between IC1 and badges, with a coefficient of 0.0125066 (p < 0), suggesting a significant impact on student achievement.

The correlation we discovered has significant implications. It indicates that an increase in activities associated with IC1, such as encountering one more 'Tower Struggle' than average, can lead to an approximate 1.5945587 percent increase in student badges. Although this increase may seem small, it may be substantial in the context of complex human behaviors, where even modest changes often lead to far-reaching effects. This observation highlights the potential of targeted interventions based on our findings to yield significant improvements in educational outcomes.

Table 2: ?(caption)

```
Oneway (individual) effect First-Difference Model
Note: Coefficient variance-covariance matrix supplied: vcovHC(ica_fd_model, type = "HC3", cl
Call:
pldv(formula = fmla, data = pUsage, model = "fd", lower = 0,
    objfun = "lsq", sample = "cens")
Unbalanced Panel: n = 7788, T = 12-53, N = 270532
Observations used in estimation: 262744
Residuals:
  Min. 1st Qu. Median Mean 3rd Qu.
                                          {\tt Max.}
 -1.621 0.401 0.911 0.883 1.381
                                          4.185
Coefficients:
              Estimate Std. Error t-value Pr(>|t|)
(Intercept) 0.00490541 0.01127130 0.4352
            0.01250661 0.00304065 4.1131 3.904e-05 ***
ic1
ic2
            0.03591921 0.00203289 17.6691 < 2.2e-16 ***
ic3
            0.00084636 0.00065654 1.2891
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
Total Sum of Squares:
Residual Sum of Squares: 303930
R-Squared:
                0.0027047
Adj. R-Squared: 0.0026933
F-statistic: 116.604 on 3 and 7787 DF, p-value: < 2.22e-16
<div id="kwmpaeobvq" style="padding-left:0px;padding-right:0px;padding-top:10px;padding-bott</pre>
  <style>#kwmpaeobvq table {
  font-family: system-ui, 'Segoe UI', Roboto, Helvetica, Arial, sans-serif, 'Apple Color Emo
  -webkit-font-smoothing: antialiased;
  -moz-osx-font-smoothing: grayscale;
}
#kwmpaeobvq thead, #kwmpaeobvq tbody, #kwmpaeobvq tfoot, #kwmpaeobvq tr, #kwmpaeobvq td, #kw
  border-style: none;
}
#kwmpaeobvq p {
 margin: 0;
 padding: 0;
}
                                      7
#kwmpaeobvq .gt_table {
  display: table;
  border-collapse: collapse;
  line-height: normal;
  margin-left: auto;
```

margin-right: auto;
color: #333333;

Study 1b

Prompted by Buyalskaya et al. (2023), we sought to uncover the subtleties of habitual behaviors within an educational setting. Our primary goal was to understand how regular teacher interactions with the Zearn platform impacted student learning outcomes. To measure this, we used the log-transformed average weekly badges per student over the entire school year as our dependent variable. We constructed our explanatory variables with careful consideration of the patterns that could identify habitual engagement and their relationship to student performance:

- 1. Login Percentages across months and days of the week: The frequency of logins across different time periods. For example, among all the logins for a teacher, we assess how many are from January or Tuesdays, compared to all other months and days of the week, respectively. Note that our regression omitted July and Sunday, periods with low login incidence.
- 2. Time Spent on Zearn: Teachers' weekly average time spent on the platform, measured in minutes.
- 3. Average Streak: Average number of consecutive weekdays in which the teacher logs in.
- 4. Average Days Between Logins: Average number of days between two login instances.

To account for any school-specific factors that may have influenced the relationship between teacher behavior and student achievement, we estimated a school-fixed-effects regression model. Unlike study 1, this regression did not follow each teacher or class across weeks, as our unit of analysis was a teacher summed across classrooms and averaged across all weeks.

The regression results, as detailed in **?@tbl-habit-regression** (see Table S2 for full results), revealed that all weekday login percentages positively affected student badges. However, Friday logins stood out significantly, supporting our hypothesis that specific days of the week have more influence on habitual engagement.

In particular, our analysis indicates that shifting 10% of logins from other weekdays to Fridays, without increasing overall platform usage, could boost student lesson completion by around 2.9737312%. For the typical teacher, this means switching from one Friday login per month to two while reducing one login from another weekday during that month, resulting in an increase of 4.5498088% in average weekly lesson completion. This outcome underscores the importance of strategic engagement rather than just login frequency.

We attribute this pronounced effect to two key factors. First, Friday logins facilitate "Reflective Catch-Up," enabling teachers to review and analyze the previous week's activities and make necessary adjustments. Second, "Foresighted Planning" occurs on Fridays as teachers proactively plan for the upcoming week, a practice less common on weekends.

Table 3: ?(caption)

```
Oneway (individual) effect Within Model
Note: Coefficient variance-covariance matrix supplied: vcovHC(habits_fe_model, type = "HC3",
Call:
plm(formula = logBadges ~ Mon + Tue + Wed + Thu + Fri + Sat +
   month_1 + month_2 + month_3 + month_4 + month_5 + month_6 +
   month_8 + month_9 + month_10 + month_11 + month_12 + Total_Minutes_on_Zearn +
   mean_streak + mean_time_lag, data = feUsage, model = "within")
Unbalanced Panel: n = 775, T = 1-32, N = 4698
Residuals:
      Min.
              1st Qu.
                         Median
                                    3rd Qu.
                                                 Max.
-1.42318280 -0.20293285 -0.00035107 0.19518856 1.66840513
Coefficients:
                       Estimate Std. Error t-value Pr(>|t|)
Mon
                     0.00291674 0.00148990 1.9577 0.0503397 .
Tue
                     0.00174982 0.00153990 1.1363 0.2558910
Wed
                     0.00193440 0.00145613 1.3284 0.1841075
                     0.00211710 0.00138744 1.5259 0.1271145
Thu
Fri
                     Sat
month_1
                     0.00365410 0.00302303 1.2088 0.2268315
month_2
                     0.00338642 0.00311206 1.0882 0.2765917
month_3
                     0.00439358 0.00303125 1.4494 0.1472979
month_4
                     0.00497739 0.00299245 1.6633 0.0963291 .
                     0.00493384 0.00330591 1.4924 0.1356671
month_5
                     0.00351744 0.00330185 1.0653 0.2868087
month_6
month_8
                     0.00269614 0.00317083 0.8503 0.3952137
month_9
                     month_10
                     0.00361004 0.00317933 1.1355 0.2562471
```

0.00332315 0.00323016 1.0288 0.3036431

 $0.00392722 \quad 0.00323669 \quad 1.2133 \quad 0.2250722$

Total_Minutes_on_Zearn 0.00228729 0.00041558 5.5039 3.954e-08 ***
mean_streak 0.05957129 0.02393196 2.4892 0.0128447 *

mean_time_lag -0.00342982 0.00079935 -4.2908 1.824e-05 ***

month_11

month_12

Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Total Sum of Squares: 488.87 Residual Sum of Squares: 453.27

R-Squared: 0.072807 Adj. R-Squared: -0.11581 9

F-statistic: 9.35671 on 20 and 774 DF, p-value: < 2.22e-16

Study 2: Nudge Engineering - From Data to Intervention Design

In Study 1, we discovered that specific psychopedagogical strategies, habits, and timing of teacher interactions were crucial in promoting student success on digital platforms. With this foundation, we could craft effective educational interventions and strategies. By leveraging the insights from the first study, we aimed to develop targeted interventions that maximize student learning outcomes.

Study 2a

We used the first component from the ICA in Study 1a to design an "empathy" nudge. This intervention involved sending emails to teachers, encouraging them to adopt a more student-centered teaching approach by viewing math problems from their students' perspective. Our analysis in Study 1a indicated that this process was linked to the highest weighted behaviors in IC1. The emails contained key messages emphasizing the importance of empathy in teaching math, advice from other teachers, and helpful tips for assisting students who struggle with a lesson. The emails also suggested specific actions, including using Zearn's features to view lessons from a student's perspective and checking the Tower Alerts Report (see SI for the complete emails). We hypothesized that this empathy approach would significantly enhance student performance.

Study 2b

In Study 2b, we aimed to test whether nudging teachers to log in on Fridays, as opposed to an unspecified day, would improve student performance. Our approach involved sending emails to teachers, encouraging them to log in on Fridays, highlighting effective teaching habits, and emphasizing the benefits of regular check-ins on student progress. These emails included testimonials from other teachers, research insights, and motivational messages to encourage habit formation (see SI for the complete emails). Our rationale was that Friday logins would aid in reflecting on the week's activities and proactive planning for the following week.

In addition, we designed a control email tailored to this treatment. These emails were sent every Wednesday to remind Zearn teachers to review their Zearn Pace Report without the personalized behavioral prompts or motivational materials in the Friday treatment group emails. Although the control emails prompted participation with Zearn, they did not provide information on Friday logins, introspection, or strategic preparation.

We hypothesized that the Friday approach to teacher engagement with the platform would yield a greater impact on student achievement than the Wednesday control or our study-wide control.

Impact Evaluation

In collaboration with Zearn, Study 2 became part of a large multi-arm "megastudy," set in motion during a critical four-week period in 2021, involving over 140,000 teachers and nearly 3 million students. Teachers in our study taught a median of 20 students (mean = 21.30, SD = 15.31) in a median of 1 classroom (mean = 1.15, SD = 0.59). Before the intervention, teachers in our study had, on average, logged into Zearn a total of 3.62 times between July 1, 2021, and September 14, 2021 (SD = 8.49) (see SI for a complete description of the sample).

file study2regress.xlsx saved

Table 4: Efficacy of Different Teacher Engagement Interventions on Student Learning Outcomes. The table showcases the effects of the 'Empathy' and 'Friday Login' interventions compared to the study-wide control, along with the results from the Friday-specific control group. We measure student achievement by the number of badges students earned.

Treatment	Coefficient	Std. Error	t value	P Value
Empathy	0.0721	0.0304	2.3748	0.0176
Friday	0.0550	0.0301	1.8250	0.0680
Friday-control	0.0898	0.0299	2.9988	0.0027
Intercept	2.0056	0.0343	58.5501	0.0000

As stated in our preregistration, we evaluated the impact of intervention messages on the number of math lessons completed by students during the four-week intervention period. Students in the megastudy control condition completed a regression-estimated 1.761 lessons during the 4-week intervention period. Table 4 shows that our interventions increased the number of math lessons completed by students during the intervention period by a regression-estimated average of 0.0487 lessons, which is a 2.77% increase over the megastudy control condition. Specifically, the empathy treatment increased the number of lessons completed by students by 0.0721 lessons, or a 4.09% increase over the megastudy control condition (d = , p = .018). The Friday treatment increased the number of lessons completed by students by 0.0550 lessons, or a 1.81% increase over the megastudy control condition (d = , p = .068). The Friday treatment was not significantly different from the Friday-specific control (F(1,118137) = 0.85, p = .357).

Table S3 presents unstandardized coefficients from our primary regression analysis and unadjusted, robust SEs and CIs. Additionally, we utilized the Benjamini-Hochberg (BH) procedure to compute adjusted p-values, which help to control for false discovery rates when conducting multiple comparisons (Benjamini & Hochberg, 1995). Before adjusting for multiple hypothesis testing, all treatments exhibited significant benefits. However, only our treatment-specific

control had a BH-adjusted p-value of less than 0.05. This intervention involved encouraging teachers to log in to Zearn weekly to receive updated student performance reports. Although reliable, the effect of this intervention was small, resulting in an estimated 0.0898 extra lessons completed in four weeks, or a 5.10% increase over the megastudy control (d = , p = .003). Even after applying the James-Stein shrinkage procedure to adjust for the winner's curse (i.e., the maximum of 15 estimated effects is upward biased), we estimated that this intervention still produced 0.061 extra lessons completed, or a 3.46% increase over the control condition (James & Stein, 1992).

Discussion

Materials and Methods

Study 1

Data Collection

The Zearn math educational learning platform provided us with administrative data spanning the 2019-2020 academic year (September 2019 to May 2020). This dataset included detailed teacher actions and classroom-week-level student achievement metrics. Teacher actions on the platform were timestamped to the second. For privacy considerations, Zearn aggregated student data at the classroom-week level, including student achievement measures and indications of student struggles. The dataset covered various schools across Louisiana (?@fig-teachers-map).

To promote transparency and replicability of our study, we have deposited the data and code used in our analyses in a publicly accessible database. Researchers and interested parties can access the complete dataset and all related processing scripts at the GitHub repository: $\frac{1}{1000} \frac{1}{1000} \frac{1}{100$

Inclusion Criteria

We aggregated teacher behavior data to the weekly level and merged it with the student data at the classroom-week level. We also excluded inactive teachers (those with no recorded activity for over two months) from the dataset.

We defined the inclusion criteria for classrooms strictly as such:

- 1. Classrooms linked to a single teacher
- 2. Classrooms with no more than seven months of inactivity during the academic year

3. Classrooms with an average of no less than five actively engaged students

Study 1a

Independent Component Analysis (ICA)

We extracted all teacher behavioral variables from the dataset that displayed non-zero variance and standardized them to have a zero mean and unit variance. To determine the ideal number of independent components, we performed ICA using a range of components from 1 to 10. Our decision on the optimal number was informed by recognizing the 'elbow' on the scree plot generated from the results, yielding three independent components. We used the icafast function from the R ica package for all ICAs conducted (Helwig 2022).

Censored Panel Regression

We estimated a censored panel regression model using a first difference approach, with the pldv function from the plm package (Croissant and Millo 2008) in R and a lower bound of 0 for the dependent variable:

$$\Delta \ln(\mathrm{Badges}_{itc} + 1) = \beta_0 + \beta_1 \Delta (\mathrm{IC1})_{it} + \beta_2 \Delta (\mathrm{IC2})_{it} + \beta_3 \Delta (\mathrm{IC3})_{it} + \Delta \epsilon_{itc}$$

where i, c, t index the teacher, class, and week, respectively. Standard errors were clustered at the teacher level with the vcovHC function (Millo 2017) in R.

Study 1b

Fixed-Effects Regression

We estimated a fixed effects model using the plm function (Croissant and Millo 2008) in R:

$$\begin{split} \ln\left(\sum_{c=1}^{C_i}\left(\text{Avg. Badges}\right)_c + 1\right) &= \beta_0 + \sum_{m=1, m \neq 7}^{12} \beta_m \text{Login}\%_{m,i} + \sum_{d=1, d \neq 7}^{7} \beta_{12+d} \text{Login}\%_{d,i} \\ &+ \beta_{19} \big(\text{Avg. Minutes}\big)_i + \beta_{20} \big(\text{Avg. Streak}\big)_i \\ &+ \beta_{21} \big(\text{Avg. Days Between Logins}\big)_i + \text{School FEs} + \epsilon_i \end{split}$$

for teacher i with C_i classes. Login% represents the percentage of logins by teacher i during each month m and on each day of the week d relative to other months and days, respectively. We exclude July and Sunday to prevent multicollinearity. Standard errors are clustered by school with the vcovHC function (Millo 2017) in R.

Study 2

We used the findings from Study 1 to inform the creation of two interventions as part of a larger multi-arm "mega study" that involved 15 sets of nudges.

Implementation

Study 2 was conducted in collaboration with Zearn ("Zearn Math: Top-Rated Math Learning Platform," n.d.) and was preregistered for the fall of 2021. To incentivize teacher participation during our "intervention period" from September 15, 2021, to October 12, 2021, all teachers on the platform received two messages on September 1 and 8, 2021. These messages informed them they had been enrolled in the "Zearn Math Giveaway" and that every email opened until October 12, 2021, would earn them tickets. These tickets were used to enter drawings for various prizes, such as autographed children's books, stickers, and gift cards.

Data

We excluded Zearn elementary school teachers from the study if they (a) taught grades other than first through eighth (n = X), (b) lacked a valid email address in the Zearn system as of September 8, 2021 (n = X), (c) had less than one or more than 150 students associated with their Zearn account as of October 18, 2021 (n = X), (d) had fewer than one or more than six classrooms associated with their Zearn account as of DATE (n = X), (e) had other teachers associated with their Zearn classroom(s) as of DATE (n = X), or (f) had not logged onto the Zearn platform (or had no associated student who logged onto the platform) between March 1, 2021 and September 14, 2021 (n = X).

After exclusions, we randomized N=140,461 teachers across 22,281 schools who served 2,992,077 students in 161,722 classrooms into one of the intervention conditions or the control condition ($N_{\rm control}=29,513$). The control condition was larger than the interventions to account for multiple comparisons. Among n=16,372, or 11.66%, of teachers, at least one of two problems occurred in the emails sent by Zearn Math during the intervention period: an email message that was intended but not sent (n=13,568, or 9.66% of teachers), or an email message that was sent but not intended (i.e., from a different treatment condition; n=2,804, or 2.00% of teachers). As these email problems were systematically related to treatment assignment ($\chi^2=33.01,\ df=15,\ p=.005$), we did not exclude these participants and conducted intent-to-treat analyses. Refer to the SI for email problem prevalence by condition and study analyses that exclude or adjust for email problems, respectively.

Impact Assessment

We followed our preregistered analysis plan to assess the effect of each treatment on the primary outcome of interest: math lessons completed by students during our four-week intervention period (Gallo et al. 2022b, 2022a). We estimated a weighted ordinary least squares (OLS) regression with the areg command in Stata (StataCorp 2023). Each teacher's observations were weighted proportionally to the total number of students in their Zearn classroom(s).

The primary predictors were indicators for each intervention, omitting the control condition. The regression also included the following control variables: (1) school fixed effects, (2) an indicator for the teacher's account type (free or paid), (3) the number of times the teacher logged into Zearn prior to the study, from August 1 to September 14, 2021, (4) the total number of students in the teacher's classroom(s) as of October 18, 2021, (5) the number of classrooms associated with the teacher as of October 18, 2021, (6) the number of days since the teacher obtained a Zearn account prior to the study's launch, (7) the number of days separating the study's launch and the start of the teacher's school year, (8) the average number of lessons completed by a teacher's students from the start of their school year to the start of the intervention (or from July 14, 2021, if the school year start was not known), (9) whether the teacher opened our September 1, 2021 email announcing the upcoming Zearn Math Giveaway, (10) a similar indicator for our September 8, 2021, email reminding them of the giveaway, and (11) the percentage of a teacher's students in each grade except for third grade to avoid multicollinearity, since for most teachers, students were in a single grade.

Study 2a: Empathy-Based Intervention Design and Implementation

We designed four emails highlighting empathy and encouraging teacher engagement, specifically regarding pedagogical content knowledge. A group of 7,443 teachers were chosen at random to receive these emails.

Study 2b: Habitization-Based Intervention Design and Implementation

We designed four emails to be sent on Fridays, emphasizing the importance of regular Friday logins and student progress tracking. A total of 7,476 teachers were randomly selected to receive these emails. Additionally, we created a condition-specific control group that received messages on Wednesdays without Friday-specific content but with links to specific actions on Zearn. A group of 7,577 teachers were randomly chosen to participate in this control group.

Ethical Considerations and IRB Approval

This study was conducted in accordance with ethical standards and received exempt status from the Institutional Review Board (IRB) at the University of Pennsylvania. The study's

methodologies were designed to ensure the confidentiality and anonymity of all participants involved, adhering strictly to ethical guidelines for educational research. Data received from Zearn was aggregated at the classroom-level, with no identifying information about teachers or students.

Supplementary Information

Supplementary Methods

Independent Component Analysis (ICA)

We implement the FastICA algorithm (Hyvärinen and Oja 2000) to estimate independent components from our dataset. In this model, matrix $X = \{x_{ij}\}_{I \times J}$, consisting of I samples across J random variables, is expressed as a linear mixture of independent components C, represented by:

$$X = C'M + E.$$

Here, C holds the independent components, M is a mixing matrix, and E denotes the noise. The aim is to minimize mutual information between components in C, which is achieved by maximizing their marginal negentropy, thereby rendering the columns of C statistically independent.

The FastICA process begins by transforming X into a whitened matrix Y, ensuring uncorrelated variables with unit variance. This transformation is achieved through eigenvalue decomposition:

$$Y = X \times \begin{bmatrix} \frac{\mathbf{v_1}}{\sqrt{\lambda_1}} & \frac{\mathbf{v_2}}{\sqrt{\lambda_2}} & \frac{\mathbf{v_3}}{\sqrt{\lambda_3}} \end{bmatrix},$$

where $(\lambda_1, \lambda_2, \lambda_3)$ and $(\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3})$ are, respectively, the eigenvalues and eigenvectors of $\frac{X'X}{I}$.

Afterward, the algorithm approximates the negentropy with

$$\hat{\theta}(c_n) = (\mathbb{E}[\ln(\cosh(c_n))] - \mathbb{E}[\ln(\cosh(z))])^2$$

where $c_n, n \in {1,2,3}$, is one of the components, and z is a Gaussian variable with zero mean and unit variance. FastICA iteratively maximizes this value across all components, producing an orthogonal rotation matrix $R_{3\times3}$ such that C=YR.

For more details on ICA and the FastICA algorithm, see Hyvärinen and Oja (2000) and Helwig and Hong (2013).

Table 5: ?(caption)

	[,1]	[,2]	[,3]
RD.elementary_schedule	0.0012815550	0.024040580	0.4442745132
RD.whole_group_fluency	0.0037699842	0.636178448	0.1546448043
RD.mission_overview	0.0281164599	0.295381216	0.2043542544
Guided.Practice.Completed	0.7300541628	0.043633836	0.0004274325
RD.small_group_lessons	0.0088585241	0.717271198	0.0905434930
Tower.Completed	0.7312323777	0.077117696	0.0091871723
Fluency.Completed	0.7779199059	0.008809797	0.0118221307
Number.Gym.Activity.Completed	0.6083754834	0.010052721	0.0158928978
RD.grade_level_overview	0.1232709831	0.130587368	0.4127310926
Tower.Stage.Failed	0.8983722277	-0.013551158	0.0118462789
RD.optional_problem_sets	0.0231728708	0.671794921	-0.0137884304
${\tt RD.student_notes_and_exit_tickets}$	0.0962966593	0.459484346	0.0374053612
Kindergarten.Activity.Completed	0.0487249545	-0.015386722	0.0374616168
Tower.Struggled	0.8938867758	-0.023919326	0.0076762215
RD.k_mission	0.0045031259	-0.004215659	0.2083578081
RD.whole_group_word_problems	0.0134765775	0.644380435	0.1212289942
RD.assessments	-0.0075946115	0.534308157	0.0903171307
${\tt RD.teaching_and_learning_approach}$	0.0117546131	-0.018464740	0.3984898923
RD.optional_homework	0.0132968499	0.624794755	-0.0197029670
RD.k_schedule	0.0221229961	-0.037948643	0.2647603896
RD.curriculum_map	0.0002141569	0.012212114	0.2553372458
RD.assessments_answer_key	-0.0126052545	0.450399168	0.1261182866
RD.pd_course_notes	-0.0157565564	0.044441361	0.6127954259
RD.pd_course_guide	-0.0121737892	-0.034075702	0.6198419192
RD.grade_level_teacher_materials	-0.0014068009	0.017765216	-0.0019681835
RD.foundational_lesson_guidance	-0.0010920758	0.005465543	0.0155101461

Censored Panel Regression

We can't really use fixed effects because there does not exist a sufficient statistic allowing the fixed effects to be conditioned out of the likelihood (Honore 1992).

Fixed-Effects Regression

Hausman Test

Table 6: ?(caption)

	Marginal	Effect of 1 SD
RD.small_group_lessons	Ü	2.595123570
RD.optional_problem_sets		2.440848518
RD.whole_group_word_problems		2.341678459
RD.whole_group_fluency		2.302906102
RD.optional_homework		2.259175505
RD.assessments		1.917338372
RD.student_notes_and_exit_tickets		1.774031716
RD.assessments_answer_key		1.612707395
Tower.Completed		1.192302333
RD.mission_overview		1.113445881
Tower.Stage.Failed		1.075887414
Guided.Practice.Completed		1.069816022
Tower.Struggled		1.032683056
Fluency.Completed		1.005559077
Number.Gym.Activity.Completed		0.798325432
RD.grade_level_overview		0.658161788
RD.pd_course_notes		0.191788497
RD.elementary_schedule		0.125556423
RD.curriculum_map		0.065743595
RD.grade_level_teacher_materials		0.061885238
RD.foundational_lesson_guidance		0.019578702
Kindergarten.Activity.Completed		0.008841148
RD.k_mission		0.008124217
RD.teaching_and_learning_approach		-0.017896113
115. Todomino_ana_roarmino_approacm		
RD nd course guide		-0.085161352
RD.pd_course_guide RD.k schedule		-0.085161352 -0.086231789
RD.pd_course_guide RD.k_schedule	Marginal	-0.086231789
RD.k_schedule	Marginal	-0.086231789 Effect of 1 Unit
RD.k_schedule RD.small_group_lessons	Marginal	-0.086231789 Effect of 1 Unit 3.63007427
RD.k_schedule RD.small_group_lessons RD.optional_problem_sets	Marginal	-0.086231789 Effect of 1 Unit 3.63007427 3.40997992
RD.k_schedule RD.small_group_lessons RD.optional_problem_sets RD.whole_group_word_problems	Marginal	-0.086231789 Effect of 1 Unit 3.63007427 3.40997992 5.06075475
RD.k_schedule RD.small_group_lessons RD.optional_problem_sets RD.whole_group_word_problems RD.whole_group_fluency	Marginal	-0.086231789 Effect of 1 Unit 3.63007427 3.40997992 5.06075475 5.81199097
RD.k_schedule RD.small_group_lessons RD.optional_problem_sets RD.whole_group_word_problems RD.whole_group_fluency RD.optional_homework	Marginal	-0.086231789 Effect of 1 Unit 3.63007427 3.40997992 5.06075475 5.81199097 4.27637694
RD.k_schedule RD.small_group_lessons RD.optional_problem_sets RD.whole_group_word_problems RD.whole_group_fluency RD.optional_homework RD.assessments	Marginal	-0.086231789 Effect of 1 Unit 3.63007427 3.40997992 5.06075475 5.81199097 4.27637694 4.95885656
RD.k_schedule RD.small_group_lessons RD.optional_problem_sets RD.whole_group_word_problems RD.whole_group_fluency RD.optional_homework RD.assessments RD.student_notes_and_exit_tickets	Marginal	-0.086231789 Effect of 1 Unit 3.63007427 3.40997992 5.06075475 5.81199097 4.27637694 4.95885656 3.02595854
RD.k_schedule RD.small_group_lessons RD.optional_problem_sets RD.whole_group_word_problems RD.whole_group_fluency RD.optional_homework RD.assessments RD.student_notes_and_exit_tickets RD.assessments_answer_key	Marginal	-0.086231789 Effect of 1 Unit 3.63007427 3.40997992 5.06075475 5.81199097 4.27637694 4.95885656 3.02595854 6.43624937
RD.k_schedule RD.small_group_lessons RD.optional_problem_sets RD.whole_group_word_problems RD.whole_group_fluency RD.optional_homework RD.assessments RD.student_notes_and_exit_tickets RD.assessments_answer_key Tower.Completed	Marginal	-0.086231789 Effect of 1 Unit 3.63007427 3.40997992 5.06075475 5.81199097 4.27637694 4.95885656 3.02595854 6.43624937 1.01429677
RD.k_schedule RD.small_group_lessons RD.optional_problem_sets RD.whole_group_word_problems RD.whole_group_fluency RD.optional_homework RD.assessments RD.student_notes_and_exit_tickets RD.assessments_answer_key Tower.Completed RD.mission_overview	Marginal	-0.086231789 Effect of 1 Unit 3.63007427 3.40997992 5.06075475 5.81199097 4.27637694 4.95885656 3.02595854 6.43624937 1.01429677 2.08870535
RD.k_schedule RD.small_group_lessons RD.optional_problem_sets RD.whole_group_word_problems RD.whole_group_fluency RD.optional_homework RD.assessments RD.student_notes_and_exit_tickets RD.assessments_answer_key Tower.Completed RD.mission_overview Tower.Stage.Failed	Marginal	-0.086231789 Effect of 1 Unit 3.63007427 3.40997992 5.06075475 5.81199097 4.27637694 4.95885656 3.02595854 6.43624937 1.01429677 2.08870535 0.46889535
RD.k_schedule RD.small_group_lessons RD.optional_problem_sets RD.whole_group_word_problems RD.whole_group_fluency RD.optional_homework RD.assessments RD.student_notes_and_exit_tickets RD.assessments_answer_key Tower.Completed RD.mission_overview Tower.Stage.Failed Guided.Practice.Completed	Marginal	-0.086231789 Effect of 1 Unit 3.63007427 3.40997992 5.06075475 5.81199097 4.27637694 4.95885656 3.02595854 6.43624937 1.01429677 2.08870535 0.46889535 0.74828333
RD.k_schedule RD.small_group_lessons RD.optional_problem_sets RD.whole_group_word_problems RD.whole_group_fluency RD.optional_homework RD.assessments RD.student_notes_and_exit_tickets RD.assessments_answer_key Tower.Completed RD.mission_overview Tower.Stage.Failed Guided.Practice.Completed Tower.Struggled	Marginal	-0.086231789 Effect of 1 Unit 3.63007427 3.40997992 5.06075475 5.81199097 4.27637694 4.95885656 3.02595854 6.43624937 1.01429677 2.08870535 0.46889535 0.74828333 1.59455873
RD.k_schedule RD.small_group_lessons RD.optional_problem_sets RD.whole_group_word_problems RD.whole_group_fluency RD.optional_homework RD.assessments RD.student_notes_and_exit_tickets RD.assessments_answer_key Tower.Completed RD.mission_overview Tower.Stage.Failed Guided.Practice.Completed Tower.Struggled Fluency.Completed	Marginal	-0.086231789 Effect of 1 Unit 3.63007427 3.40997992 5.06075475 5.81199097 4.27637694 4.95885656 3.02595854 6.43624937 1.01429677 2.08870535 0.46889535 0.74828333 1.59455873 0.75066416
RD.k_schedule RD.small_group_lessons RD.optional_problem_sets RD.whole_group_word_problems RD.whole_group_fluency RD.optional_homework RD.assessments RD.student_notes_and_exit_tickets RD.assessments_answer_key Tower.Completed RD.mission_overview Tower.Stage.Failed Guided.Practice.Completed Tower.Struggled Fluency.Completed Number.Gym.Activity.Completed	Marginal	-0.086231789 Effect of 1 Unit 3.63007427 3.40997992 5.06075475 5.81199097 4.27637694 4.95885656 3.02595854 6.43624937 1.01429677 2.08870535 0.46889535 0.74828333 1.59455873 0.75066416 1.31408880
RD.k_schedule RD.small_group_lessons RD.optional_problem_sets RD.whole_group_word_problems RD.whole_group_fluency RD.optional_homework RD.assessments RD.student_notes_and_exit_tickets RD.assessments_answer_key Tower.Completed RD.mission_overview Tower.Stage.Failed Guided.Practice.Completed Tower.Struggled Fluency.Completed Number.Gym.Activity.Completed RD.grade_level_overview		-0.086231789 Effect of 1 Unit 3.63007427 3.40997992 5.06075475 5.81199097 4.27637694 4.95885656 3.02595854 6.43624937 1.01429677 2.08870535 0.46889535 0.74828333 1.59455873 0.75066416 1.31408880 5.49711821
RD.k_schedule RD.small_group_lessons RD.optional_problem_sets RD.whole_group_word_problems RD.whole_group_fluency RD.optional_homework RD.assessments RD.student_notes_and_exit_tickets RD.assessments_answer_key Tower.Completed RD.mission_overview Tower.Stage.Failed Guided.Practice.Completed Tower.Struggled Fluency.Completed Number.Gym.Activity.Completed RD.grade_level_overview RD.pd_course_notes	Marginal	-0.086231789 Effect of 1 Unit 3.63007427 3.40997992 5.06075475 5.81199097 4.27637694 4.95885656 3.02595854 6.43624937 1.01429677 2.08870535 0.46889535 0.74828333 1.59455873 0.75066416 1.31408880 5.49711821 1.86623832
RD.k_schedule RD.small_group_lessons RD.optional_problem_sets RD.whole_group_word_problems RD.whole_group_fluency RD.optional_homework RD.assessments RD.student_notes_and_exit_tickets RD.assessments_answer_key Tower.Completed RD.mission_overview Tower.Stage.Failed Guided.Practice.Completed Tower.Struggled Fluency.Completed Number.Gym.Activity.Completed RD.grade_level_overview RD.pd_course_notes RD.elementary_schedule		-0.086231789 Effect of 1 Unit 3.63007427 3.40997992 5.06075475 5.81199097 4.27637694 4.95885656 3.02595854 6.43624937 1.01429677 2.08870535 0.46889535 0.74828333 1.59455873 0.75066416 1.31408880 5.49711821 1.86623832 1.78431556
RD.k_schedule RD.small_group_lessons RD.optional_problem_sets RD.whole_group_word_problems RD.whole_group_fluency RD.optional_homework RD.assessments RD.student_notes_and_exit_tickets RD.assessments_answer_key Tower.Completed RD.mission_overview Tower.Stage.Failed Guided.Practice.Completed Tower.Struggled Fluency.Completed Number.Gym.Activity.Completed RD.grade_level_overview RD.pd_course_notes RD.elementary_schedule RD.curriculum_map		-0.086231789 Effect of 1 Unit 3.63007427 3.40997992 5.06075475 5.81199097 4.27637694 4.95885656 3.02595854 6.43624937 1.01429677 2.08870535 0.46889535 0.74828333 1.59455873 0.75066416 1.31408880 5.49711821 1.86623832 1.78431556 1.82063347
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RD teaching and learning approach

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Supplementary Tables

Marginal Effects

Supplementary Discussion

Robustnes Checks

Additional Analyses: Detailed list of supplementary analyses to be conducted.

Supplementary Equations

Supplementary Notes

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